Magnetic Monopoles and Spin Ice

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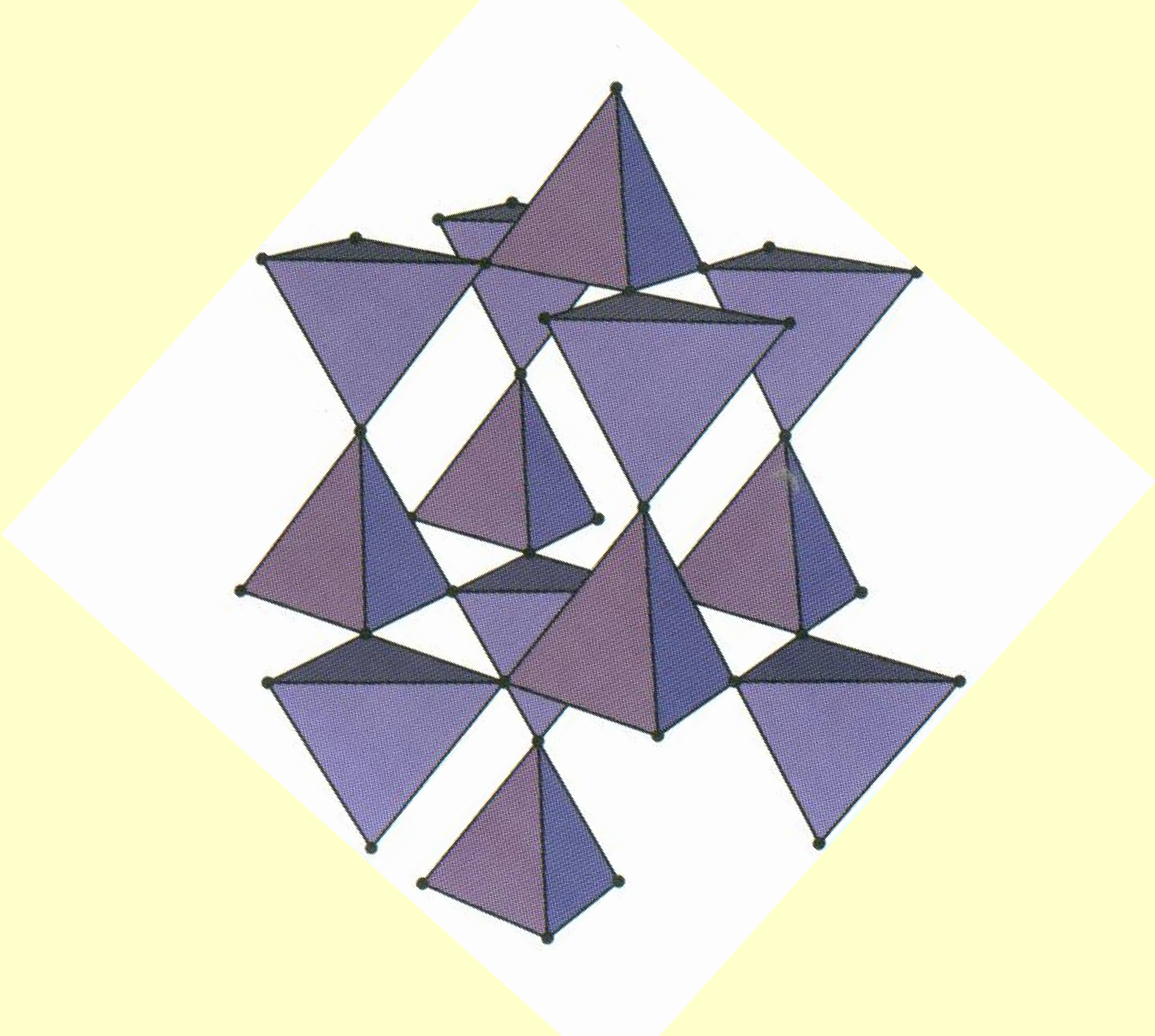
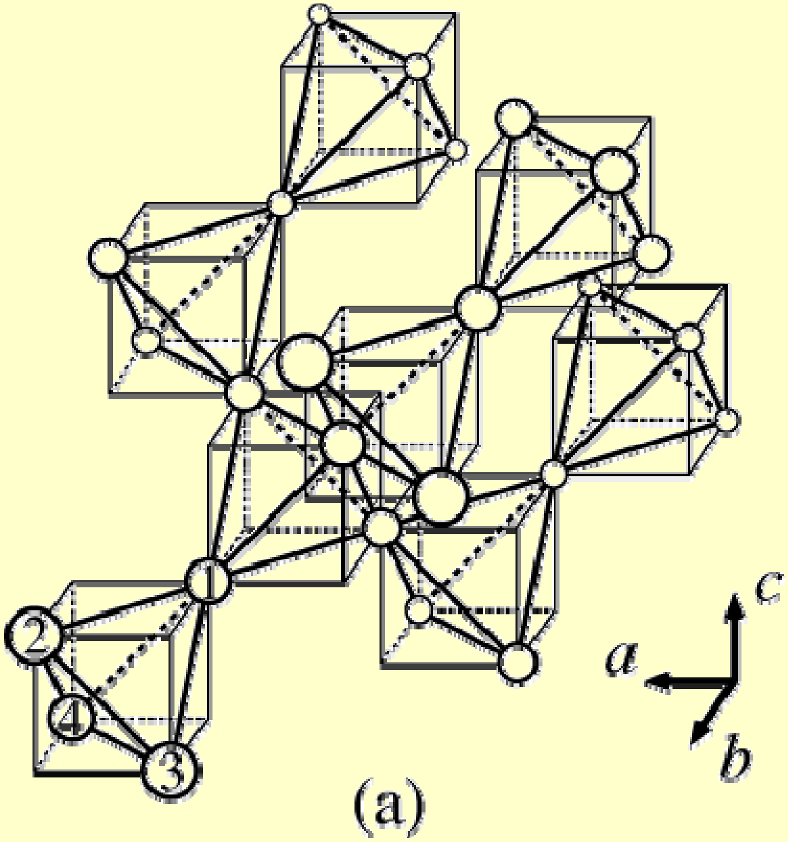
Outline

- Spin Ice
- The dipolar puzzle
- Dumbbells
- Monopoles
- Experiments
- A second gauge field

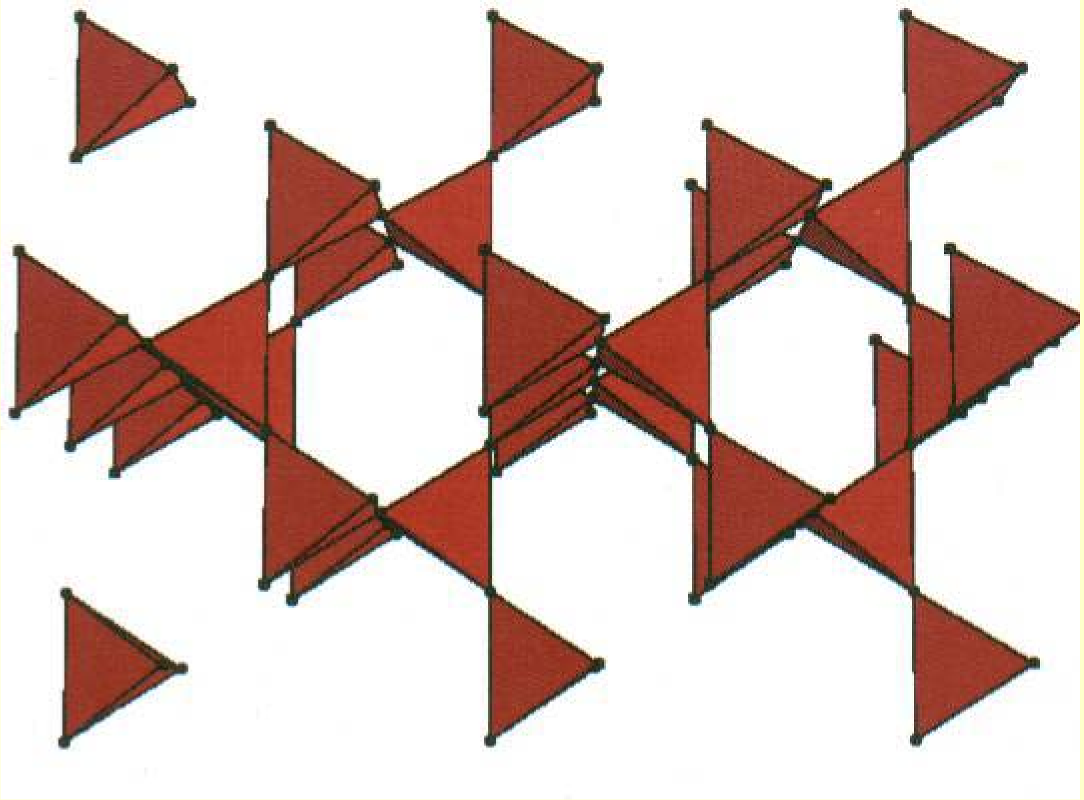


The pyrochlore lattice



- FCC lattice with a four site basis: “up” tetrahedra sit on the Bravais lattice
- If we look along the [111] direction,

http://ej.iop.org/images/0953-8984/13/50/102/Full/3059601.gif

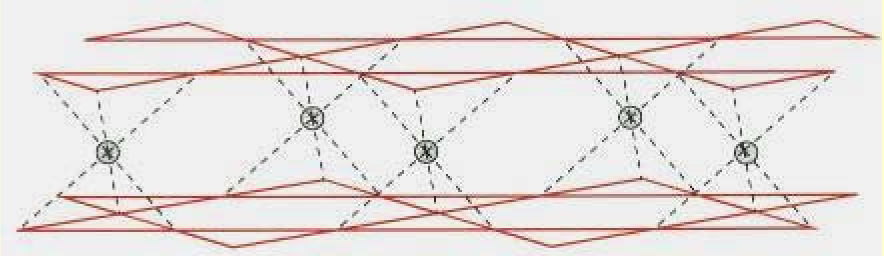


we get a set of interleaved Kagome and triangular planes

These are found in two large families of materials that take their names from two minerals:

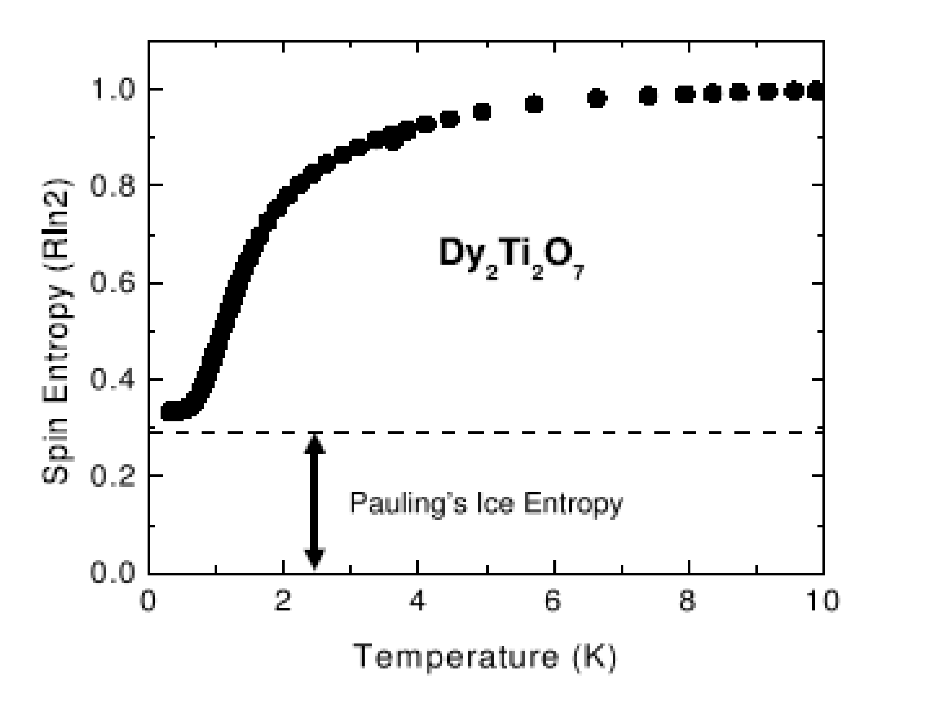
1) the pyrochlores $A_2B_2O_7$

2) the spinels AB_2O_4



Spin Ice

- The pyrochlores $Ho_2Ti_2O_7$ and $Dy_2Ti_2O_7$ do not order to the lowest temperatures. (*Bramwell + Harris, 1997)*
- Instead they exhibit a macroscopic low temperature entropy (*Ramirez et al, 1999*)



- Simplest explanation of this behavior is that we have an Ising antiferromagnet. However, the local easy axes are the [111] axes so we postulate

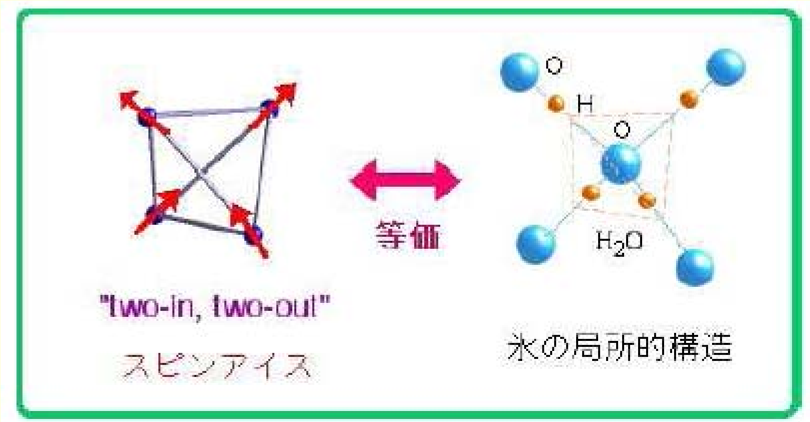
$$\mathcal{H} = -E\sum_i \left(\hat{\mathrm{d}}_{\kappa(i)} \cdot \mathbf{S}_i\right)^2 + J\sum_{\langle ij\rangle} \mathbf{S}_i \cdot \mathbf{S}_j = -(J/3)\sum_{\langle ij\rangle} \sigma_i\sigma_j$$

Anisotropy

FM exchange

Pseudospins: in and out

The two in and two out rule is precisely one of the Bernal-Fowler rules for ice and thus we can take over Pauling’s celebrated estimate for the entropy of ice S=(1/2) Log(3/2)



Why does spin ice obey the ice rules?

$$\mathcal{H} = \mathcal{H}_{nn} + \frac{\mu_0}{4\pi} \sum_{ij} \frac{\vec{\mu}_i \cdot \vec{\mu}_j - 3(\vec{\mu}_i \cdot \hat{r}_{ij})(\vec{\mu}_i \cdot \hat{r}_{ij})}{r_{ij}^3}$$

- The ferromagnetic exchange actually arises from the dipolar interaction – ferromagnetic when restricted to easy axes and nearest neighbor sites.
- But then why is the rest of the dipolar interaction innocuous? (*Siddarthan & Shastry, 1999; Gingras and co-workers, 2000-)*
- ANSWER The ice rules generate dipolar correlations among the spins – these are entirely consistent with the long ranged piece of the dipolar coupling!

Isakov, Moessner and Sondhi, 2005

Dipolar spin ice (cont'd)

- More specifically could establish a

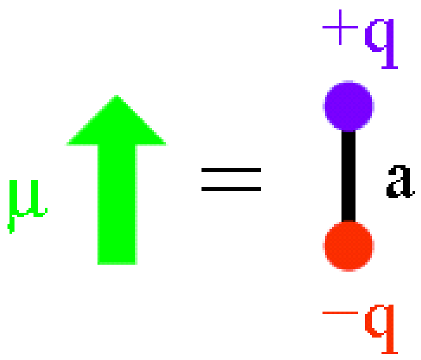
THEOREM There exists a model dipole interaction with the same long distance asymptotics which has exactly and only the ice rule ground states.

Now we can show this much more simply by a related but different route.

The 'dumbell' model

Dipole ≈ pair of opposite charges ($\mu = qa$):

- Sum over dipoles ≈ sum over charges:



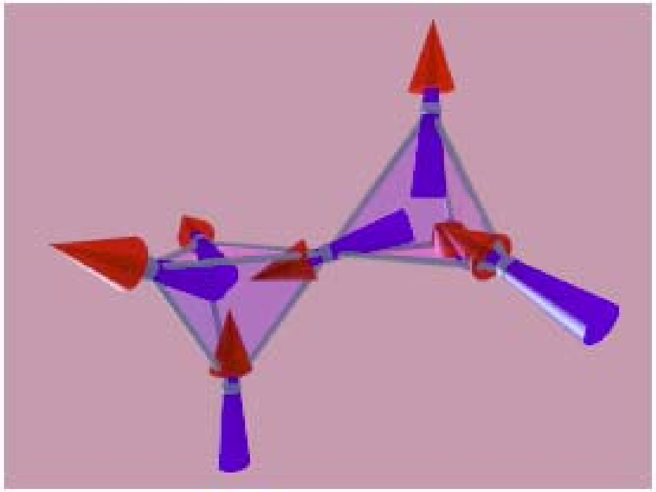
$$\mathcal{H}_{ij} = \sum_{m,n=1}^{2} v(r_{ij}^{mn})$$

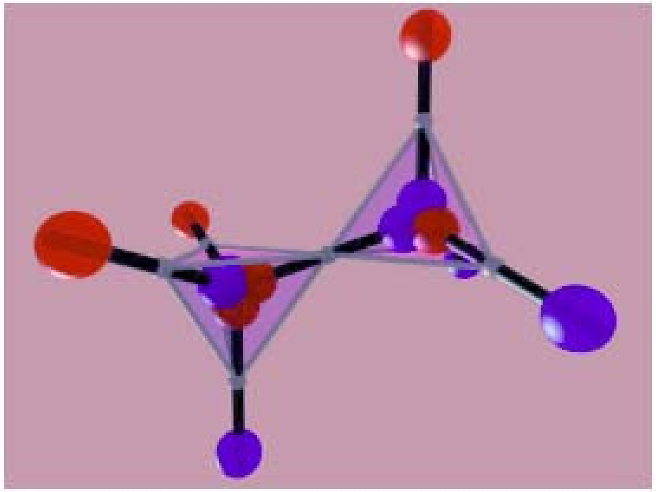
- $v \propto q^2/r$ is the usual Coulomb interaction (regularised):

$$v(r_{ij}^{mn}) = \begin{cases} \mu_0\, q_i^m q_j^n / (4\pi r_{ij}^{mn}) & i \neq j \\ v_o(\frac{\mu}{a})^2 = \frac{J}{3} + 4\frac{D}{3}(1+\sqrt{\frac{2}{3}}) & i = j, \end{cases}$$

Origin of the ice rules

Choose $a = a_d$, separation between centres of tetrahedra





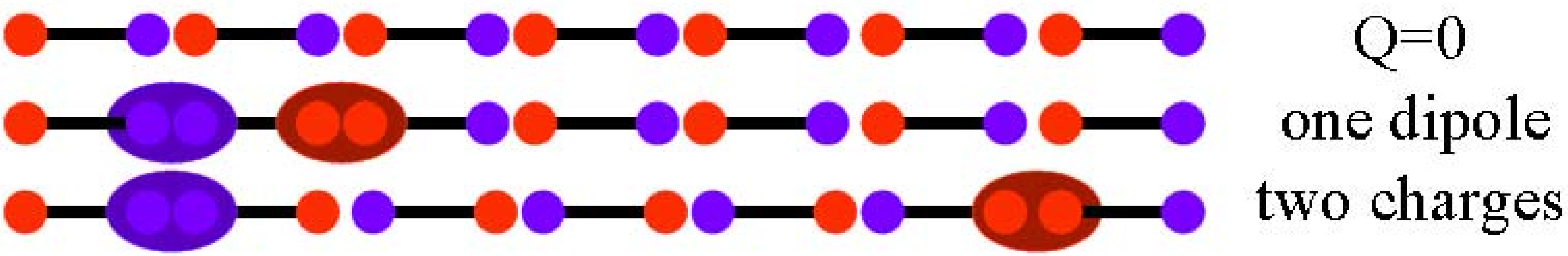
Resum tetrahedral charges $Q_\alpha = \sum_{r_i^m \in \alpha} q_i^m$:

$$\mathcal{H} \approx \sum_{ij}^{mn} v(r_{ij,mn}) \longrightarrow \sum_{\alpha\beta} V(r_{\alpha\beta}) = \begin{cases} \frac{\mu_0}{4\pi} \frac{Q_\alpha Q_\beta}{r_{\alpha\beta}} & \alpha \neq \beta \\ \frac{1}{2} v_o Q_\alpha^2 & \alpha = \beta \end{cases}$$

- Ice configurations ($Q_\alpha \equiv 0$) degenerate$\Rightarrow$ Pauling entropy!

Excitations: dipoles or charges?

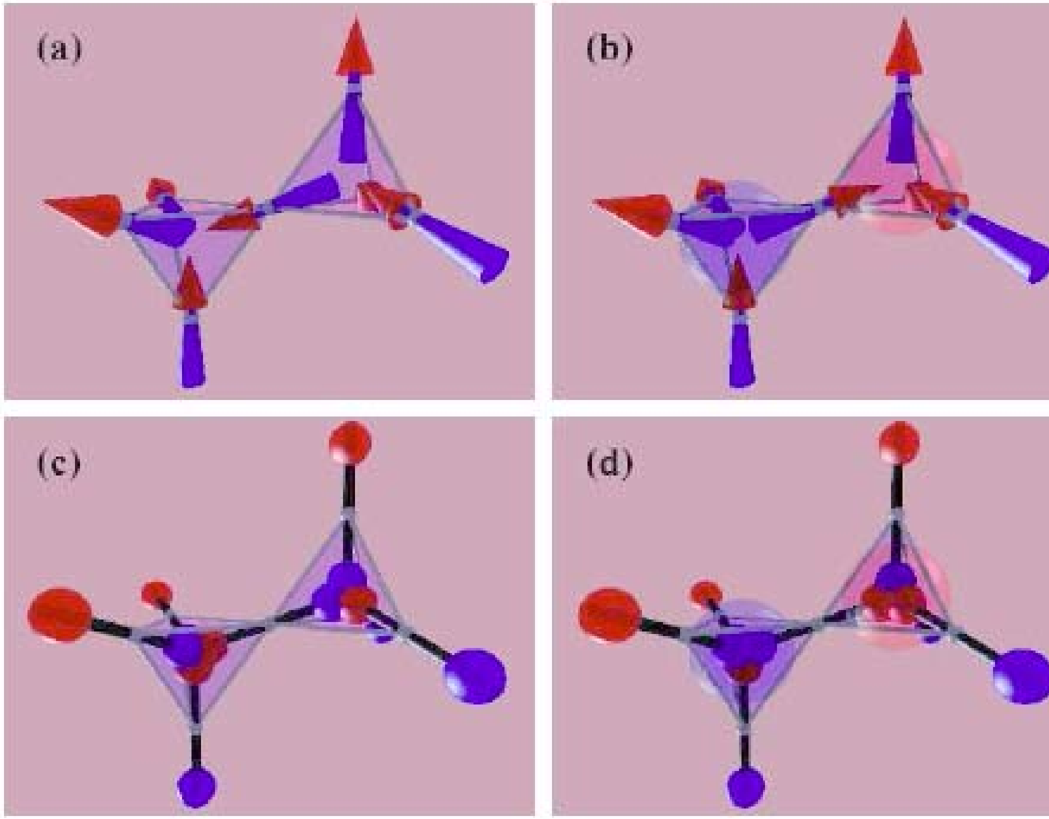
- Ground-state
  - no net charge
- Excited states:
  - flipped spin $\leftrightarrow$ dipole excitation
  - same as two charges?



Fractionalisation in $d = 1$

Excitations in spin ice: dipolar or charged?

Single spin-flip (dipole μ)
$\equiv$
two charged tetrahedra
(charges $q_m = 2\mu/a_d$)

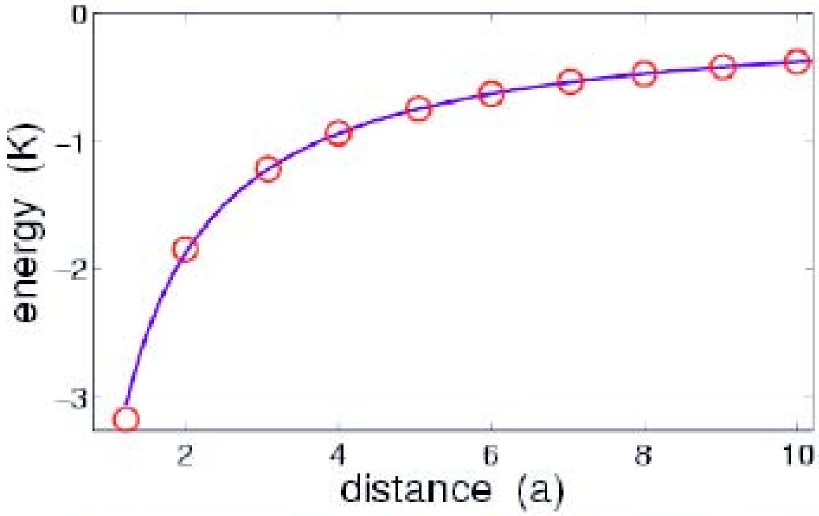


Are charges independent?
$\Rightarrow$ Fractionalisation in $d = 3$?

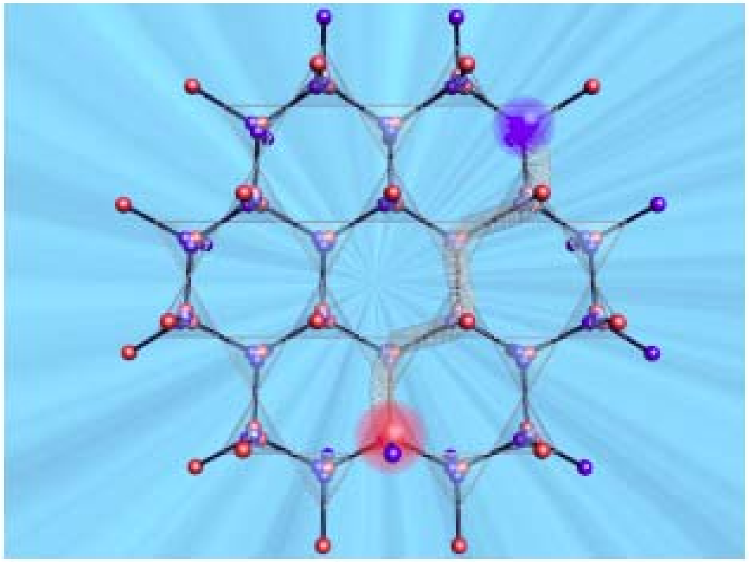
Deconfined magnetic monopoles

Dumbell Hamiltonian gives

$$E(r) = -\frac{\mu_0}{4\pi}\frac{q_m^2}{r}$$



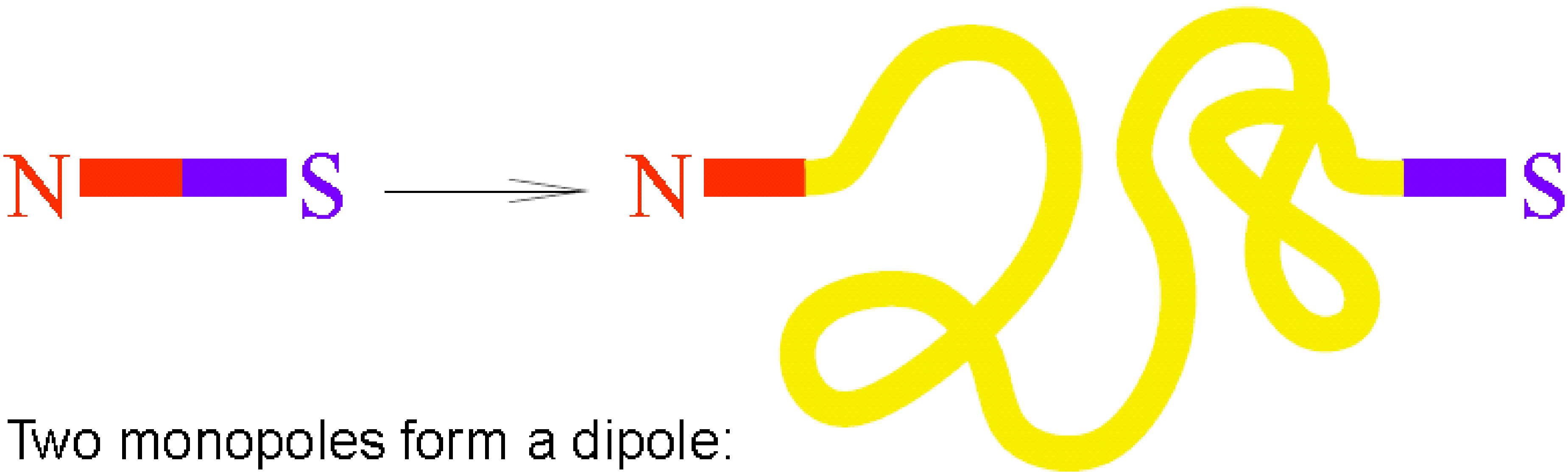
- magnetic Coulomb interaction
- deconfined monopoles
  - charge $q_m = 2\mu/a = (2\mu/\mu_b)(\alpha\lambda_C/2\pi a_d)q_D \approx q_D/8000$
  - monopoles in H, not B



Intuitive picture for monopoles

Simplest picture does not work: disconnect monopoles

Next best thing: no string tension between monopoles:



Two monopoles form a dipole:

- connected by tensionless 'Dirac string'
- Dirac string <u>is</u> observable

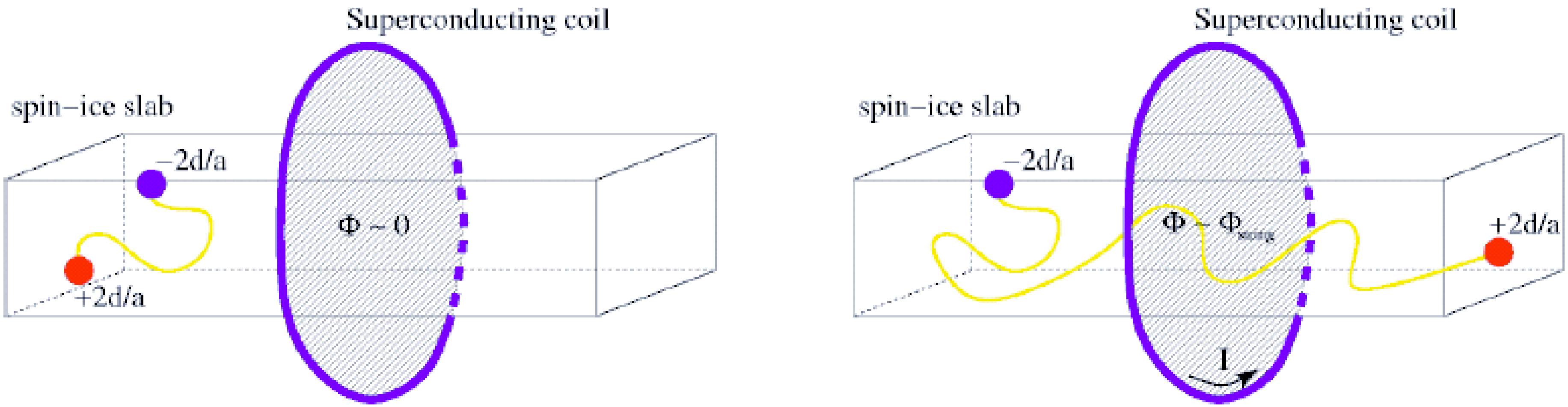
$\Rightarrow$ $q_m \approx q_D/8000$ not in conflict with quantisation of e

Experiment I: Stanford monopole search

Monopole passes through superconducting ring

$\Rightarrow$ magnetic flux through ring changes

$\Rightarrow$ e.m.f. induced in the ring $\Rightarrow$ countercurrent $\propto q_m$ is set up



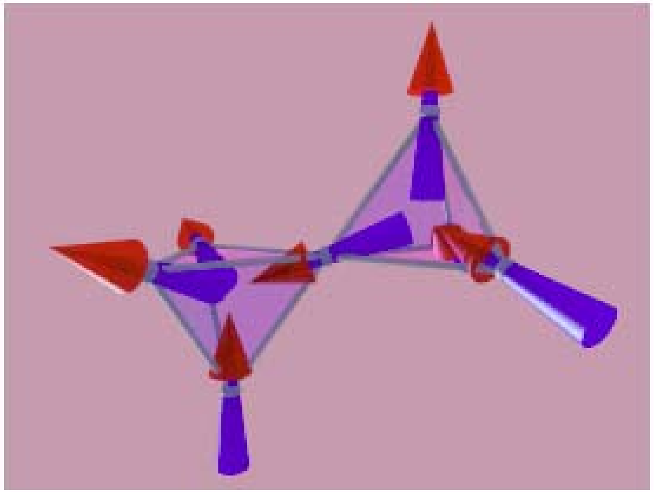
- 'Works' for both fundamental cosmic and spin ice monopoles
- signal-noise ratio a problem

Experiment II: interacting Coulomb liquid

Monopoles form a two-component liquid

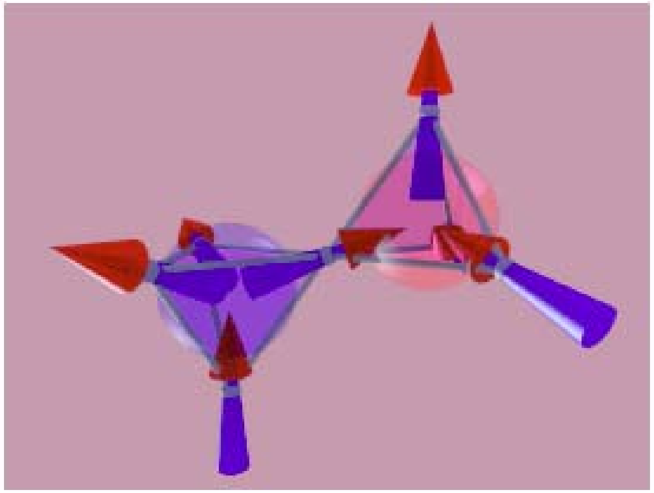
- any characteristic collective behaviour?
- interaction strength $\Gamma \propto (q_m^2/\langle r \rangle)/T \sim \exp[-c v_0/T]/T$ vanishes at both high and low T
- solution: [111] magnetic field acts as chemical potential

$\Rightarrow$ can tune $\langle r \rangle$ and T separately

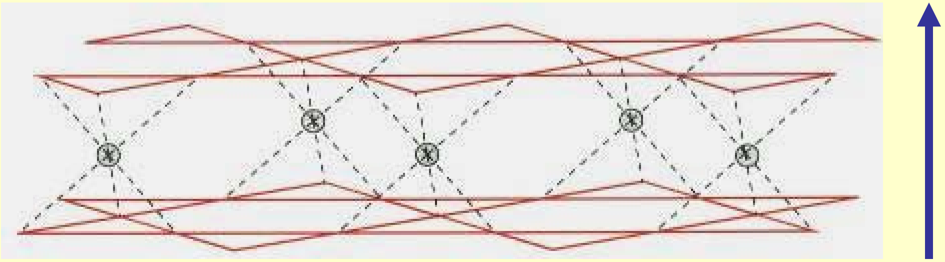


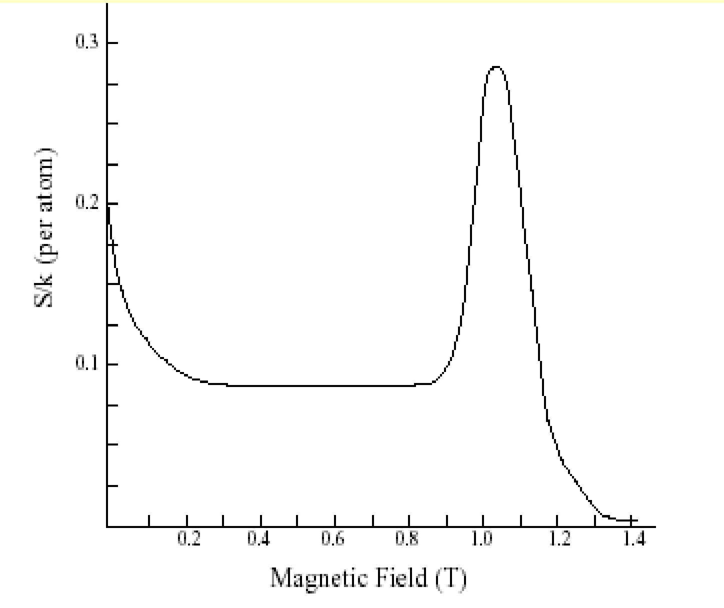
$\vec{B}$

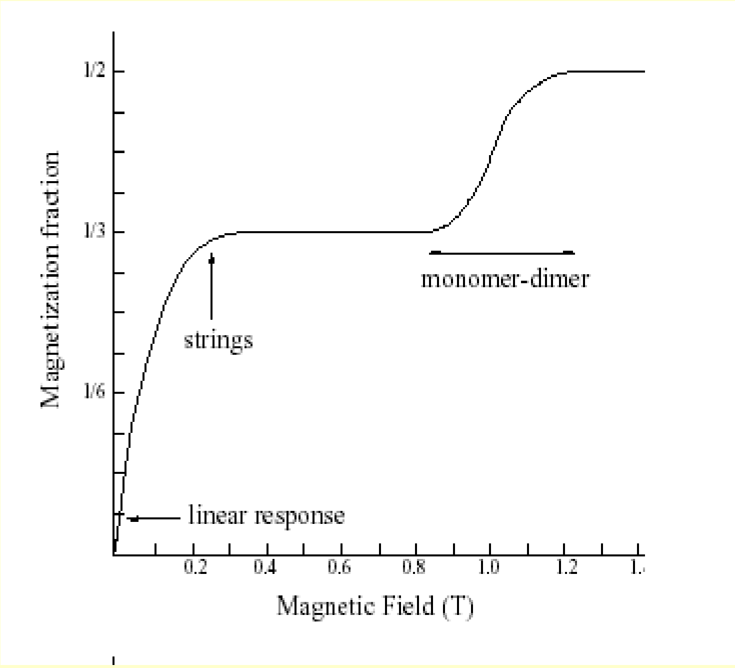
$\Uparrow$



A step back: the magnetization process





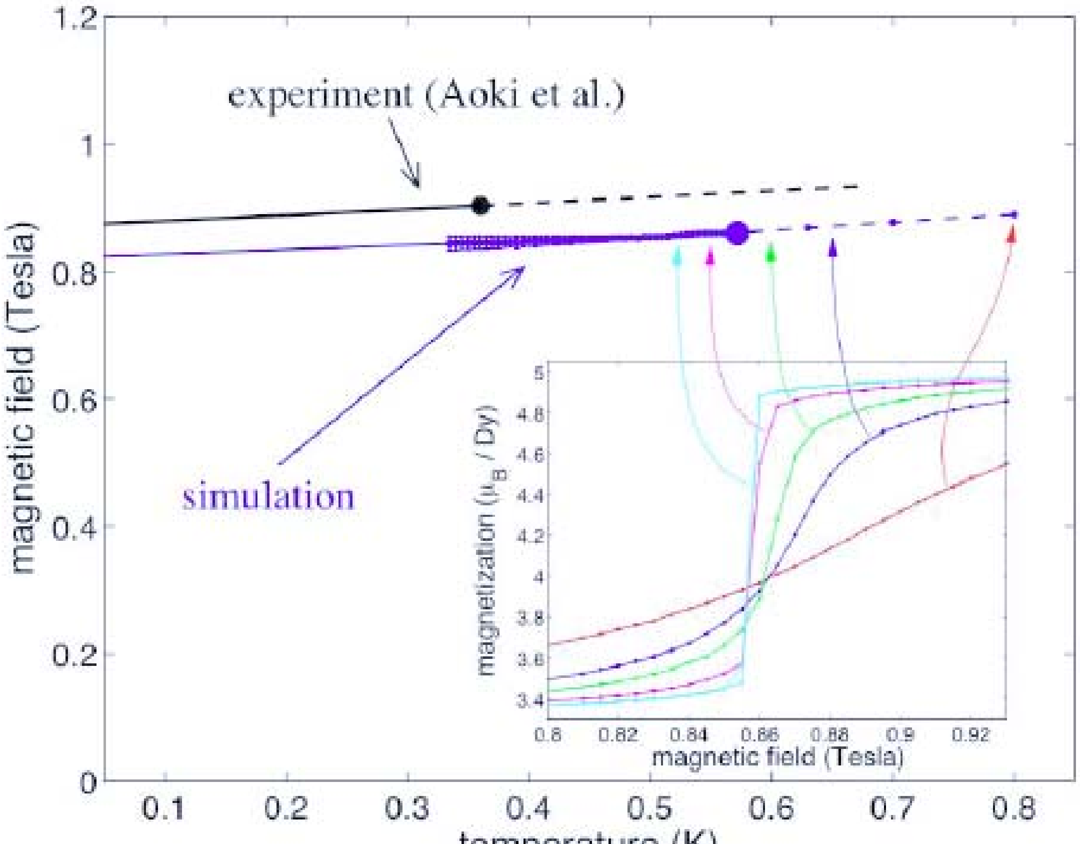


Moessner and Sondhi, 2003; Isakov, Raman, Moessner and Sondhi, 2004

Liquid-gas transition in spin ice in a [111] field

- $\mathcal{H}_{nn}$ predicts crossover to maximally polarised state
- dipolar $\mathcal{H}$: first-order transition with critical endpoint Fisher *et al.*

- observed experimentally
  Hiroi+Maeno groups
- confirmed numerically



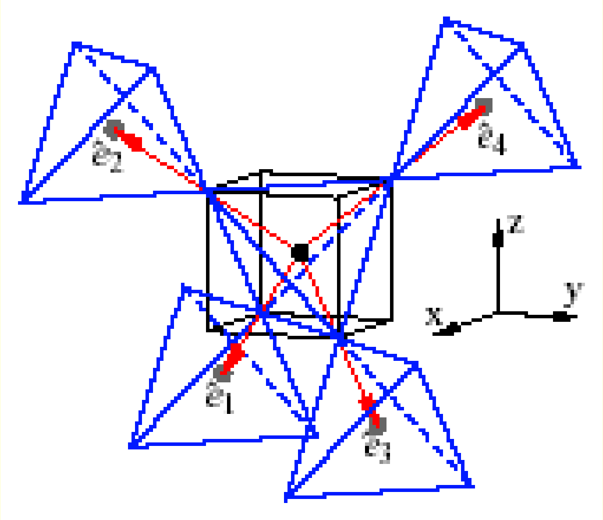
A Second Gauge Field

- Despite the lack of a phase transition there is a correlation length (thermal monopole separation) that diverges as T->0. This is associated with the emergence of a second gauge field that characterizes the statistical properties of the ground state manifold.
- To understand this think about the ground state constraint, which can be turned into a “conservation law”. This can be done for the O(N) case with vectors spins replacing Ising pseudospins:

(S. Isakov, K. Gregor, R. Moessner and S. L. Sondhi, 2004)
Also Henley, Hermele et al; Youngblood and Axe

Conservation law

Orient bonds on the dual diamond lattice from one sublattice to the other



Define N vector fields on each bond

$$\vec{B}^a(\mathbf{x}) = S^a(\mathbf{x})\widehat{e}(\mathbf{x})$$

$\sum S^a(\mathbf{x}) = 0$ on each tetrahedron in grounds states, implies

$\nabla \cdot \vec{B}^a = 0$ at each dual site

Second ingredient: rotation of closed loops of **B** connects ground states
Which implies large density of states near $\mathbf{B}_{av} = 0$

Using these “magnetic” fields we can construct a coarse grained partition function

$$\sum_{\text{spin configs}} \text{“1”} \rightarrow \sum_{\vec{B}^a(\mathbf{x})} \delta(\nabla \cdot \vec{B}^a)\; e^{-\frac{K}{2}\int d^3x \sum_a (\vec{\mathbf{B}}^a)^2}$$

Solve constraint $\vec{B} = \nabla \times \vec{A}$ to get Maxwell theory for N gauge fields

$$\sum_{\vec{A}^a(\mathbf{x})} e^{-\frac{K}{2}\int d^3x \sum_a (\nabla \times \vec{\mathbf{A}}^a)^2}$$

Leads to dipolar spin correlations and entropic Coulomb interactions between monopoles – now proportional to T.

Nature **451**, 42 (2008)

Alessandro Canossa

PWA

1956

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Information

KAGOME
トマトキッチンスタジオ

History of Kagome Co., Ltd.

Kagome Inc. was founded in 1989 as a subsidiary of Kagome Co., Ltd. This year Kagome Co., Ltd. celebrated it's 100th anniversary since its founding. Below is a brief history of the founding company.

1899 • Ichitaro Kanie begins cultivating Western vegetables in Japan and succeeds in growing tomatoes

1906 • Kanie establishes a factory in Tokai City, Aichi Prefecture, and begins full-scale production of tomato sauce

1933 • Tomato Juice is marketed

1959 • Tomato Paste is marketed

1964 • Chili Sauce and Tomato Sauce are introduced

1973 • Apple Juice and Mandarin Orange Juice are marketed

1978 • The Company?s shares are listed on the First Sections of the Nagoya and Tokyo Stock Exchanges

1988 • Kagome U.S.A., Inc. is established as a subsidiary

1989 • Kagome celebrates it?s 90th anniversary

1990 • Kagome U.S.A.?s plant in Los Banos, California, is completed

1998 • Kagome Inc. is established

1999 • All plants receive ISO9001 certification
• A new environmental policy is formulated