An Economic Model for Vehicle Ownership Quota and Usage Restriction Policy Analysis

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Dedication

To my beloved grandparents.
ABSTRACT

Rationing policies, including vehicle ownership quota and vehicle usage restrictions, have been implemented in several megaregions to address congestion and other negative transportation externalities. However, no model is available in the literature that allows direct comparison of these rationing policies. To bridge this gap, this study develops an analytical framework for analyzing and comparing transportation rationing policies, which consists of a mathematical model of joint household vehicle ownership and usage decisions and welfare analysis methods based on compensating variation and consumer surplus. Under the assumptions of homogenous users and single time period, this study finds that vehicle usage rationing performs better when relatively small percentages of users (i.e. low rationing ratio) are rationed off the roads and when induced demand resulting from congestion mitigation is low. When the amount of induced demand exceeds a certain level, it is shown analytically that vehicle usage restrictions will always cause welfare losses. When the policy goal is to reduce vehicle travel by a large portion (i.e. high rationing ratio), the net social benefits of vehicle ownership quota rationing policy become more obvious. The optimal rationing ratios for both rationing policies can be determined by the model, and are influenced by network congestion and congestibility. A comparison with pricing policy is also provided to illustrate their difference under various conditions. Various policy implications, as well as future research directions, are also discussed.
Contents

Acknowledgements i
Dedication ii
Abstract iii
List of Figures vi

1 Introduction 1

2 Theoretical Framework 4
  2.1 Travel Demand ..................................................... 5
  2.2 Travel Supply ......................................................... 8
  2.3 Network Equilibrium .............................................. 9

3 Models for Vehicle Ownership and Usage Rationing Policy Analysis 11
  3.1 A Compensating Variation(CV) Approach for Welfare Analysis .... 12
  3.2 Vehicle Ownership Rationing ..................................... 13
    3.2.1 Welfare Impacts of Ownership Rationing ...................... 13
    3.2.2 Optimal Rationing Ratio ...................................... 15
  3.3 Vehicle Usage Rationing ........................................... 16
    3.3.1 Welfare Impacts of Vehicle Usage Rationing without Induced Demand ......................................................... 17
    3.3.2 Welfare Impacts of Vehicle Usage Rationing with Induced Demand ......................................................... 19

4 Policy Comparison: A Numerical Analysis 22
5 Road Pricing Policy

5.1 Formulation of Road Pricing ........................................ 27
5.2 Welfare Impacts of Road Pricing ................................... 29
5.3 Optimal Toll ......................................................... 29
5.4 Comparison between Road Pricing and Rationing Policy .... 30

6 Conclusion and Discussion ........................................... 32

Appendix A. Alternative Proof for Proposition 1 Based on Consumer Surplus ......................................................... 38

Appendix B. Summary of Notation ..................................... 41
List of Figures

2.1 Aggregate Demand Curve with Homogenous Travelers ............... 8
2.2 Network Equilibria with Different Levels of Supply ............... 9
3.1 New Equilibrium after Ownership Rationing with a Low Level of Congestion 14
3.2 New Equilibrium after Ownership Rationing with High Level of Congestion 15
3.3 Consumer Surplus Change after Usage Rationing without Induced Demand 18
4.1 Welfare Gain of the Two Rationing Policies using BPR Model .... 24
4.2 Welfare Gain of the Two Rationing Policies using Vickrey Model ... 25
4.3 Comparison of Welfare Gain under Ownership Rationing using Two Supply Models ................................. 26
5.1 New Equilibrium after Road Pricing ................................. 28
A.1 Consumer Surplus Change after Usage Rationing with Induced Demand 39
Chapter 1

Introduction

Various policy tools (e.g. priority transit, high occupancy vehicle lanes, congestion pricing) have been implemented to mitigate congestion by allocating scarce road space more efficiently. In theory, policy makers can reduce congestion and maximize social welfare by imposing marginal toll pricing on the entire network (Mohring and Harwitz, 1962, Arnott et al., 1994). However, this ideal pricing scheme is hard to implement because of 1) lack of information on demand structure; 2) hefty transaction cost; 3) and concerns on welfare distributional effects (Giuliano, 1994, Harrington et al., 2001, Yang et al., 2004, Zhang et al., 2008). Consequently, many second best pricing schemes have been proposed. Both theoretical and empirical studies on road pricing have been abundant in the literature (e.g. Verhoef, 2002, Zhang and Ge, 2004, Brownstone and Small, 2005). In contrast, few studies have been conducted to evaluate rationing as a policy tool to address congestion problem, although its role in resource allocation has been widely discussed in other industries (e.g. Evans, 1983). A pioneering study in transportation by Daganzo (1995) shows that a Pareto optimum congestion reduction scheme can be built by combining rationing and pricing. Nakamura and Kockelman (2002) tested this theoretical framework on the San Francisco Bay Bridge corridor. The authors pointed out several limitations in current studies and recommended further research in this field.

While the effects of rationing in transportation have not been well understood, it has been implemented in several metropolitan areas. For example, Singapore started the
Vehicle Quota System (VQS) in 1990 (Barter, 2005), which only releases a limited number of vehicle purchase permits each month through auctions. A similar quota system was adopted in Shanghai, China in 2001. A more drastic vehicle ownership rationing scheme, where residents can only acquire the vehicle purchase permits through monthly lottery, was recently implemented in Beijing, China (Lim, 2011). Rationing policies can be applied not only to vehicle ownership, but also to vehicle usage. For example, several Latin American countries introduced vehicle usage restriction measures due to emissions and air quality concerns in metropolitan area. Mexico City administration imposed a regulation banning each car from driving on a specific day of the week according to their license plate number in 1989 (dubbed as “Day without a Car”). Similarly regulation has been adopted by Sao Paulo, Brazil, Bogotá, Columbia, Quito, Ecuador, and Santiago, Chile (Davis, 2008). As congestion and air pollution problems deteriorate, vehicle usage restriction has been extended to some smaller cities, such as Medellín and Cali, with less than two million residents each. Outside Latin American, vehicle usage restriction is also seen in Beijing, China, Manila, Philippine, Lagos, Nigeria (Thomson, 1998), as well as Guangzhou, China during 2010 Asian Games (Hao et al., 2010).

The implementation of rationing policies in transportation has been controversial and inspired debates among policy makers, researchers and the general public. Findings about their merits and limitations from a few pilot programs are mixed. Smith and Chin (1997) concluded that the VQS in Singapore is effective in controlling traffic, but it has to be complemented by road pricing to yield higher social welfare. Eskeland and Feyzioglu (1997) find that the “Day without a Car” regulation in Mexico City has motivated some households to buy additional cars and consequently to increase their driving, both of which compromised the initial policy objective of reducing traffic. The authors also raise concerns about induced demand during weekdays, excessive weekend driving, and pollution brought by the second, and often older car. The vehicle usage decision is not explicitly modeled in this study (a uniform reduction in driving is assumed after usage rationing), which limits its applications in policy analysis. Davis (2008) also investigated the effectiveness of restriction policy in Mexico City using data from air quality, gasoline sale, and vehicle registration. They concluded some weekday travel demand has been moved to weekend and no significant reduction in total demand can be observed.

As road congestion and its adverse economic, social and environmental impact
worsen, vehicle ownership and/or usage rationing as a direct travel demand management tool could be adopted by more metropolitan areas, either as a permanent policy or as a temporary policy during special events. Theoretical models and applied research are both needed to evaluate various rationing schemes and support policy making. No previous study has explicitly compared the two different rationing schemes: the usage rationing that restricts the service flow on each vehicle and the ownership rationing that directly controls the total number of vehicles in the system. This study develops a theoretical model to evaluate the welfare effects of both rationing schemes. Following the indirect utility approach initiated by Dubin and McFadden (1984), this study extends the model previously proposed by De Jong (1990) to consider the joint decisions of vehicle ownership and usage under rationing policies. These demand-side models are then combined with supply-side models to capture the market equilibrium, for policy analysis and comparison. As far as the authors know, this is the first study of its kind which 1) develops a welfare analysis framework for rationing policies and 2) compares welfare effects of the two different rationing policies.

The rest of this thesis is organized as following:

- Chapter 2 presents theoretical framework for travel demand and network supply models, and the network equilibrium that can be achieved.

- Chapter 3 presents the two different rationing policies, Vehicle Ownership Quota and Vehicle Usage Restriction, and the modeling approach for these two policies in detail.

- In Chapter 4 numerical examples are used to compare these welfare impact of these two policies.

- Chapter 5 further compares the rationing policies with the more popular road pricing policy and illustrates their difference under various conditions.

- Chapter 6 concludes this thesis with a final discussion of the analysis presented in this thesis and the future research directions.
Chapter 2

Theoretical Framework

For a traveler who is facing various rationing policies, the decision of owning a vehicle and the decision of using a vehicle are interrelated. A vehicle usage rationing similar to the “Day without a Car” regulation will reduce the utility of owning a car by reducing its service flow (or usage). Consequently, some travelers may give up their personal vehicles and use alternative travel modes, while wealthier people could buy a second car to compensate the reduced utility from the first car. Decisions will be made based on individual values of time and willingness to pay. The total number of vehicles is not guaranteed to be reduced. In contrast, the vehicle ownership rationing will directly control the total number of vehicles. However, more driving could be expected on each vehicle due to reduced congestion and induced demand. Therefore, to correctly capture these behavioral dynamics in reaction to rationing policy, it is crucial to jointly model vehicle ownership and usage decisions.

The performance of the market dynamics should also depend on travel supply (the marginal cost of traveling). Rationing policies differ in cities with different levels of congestion. To correctly model induced demand and capture network equilibria under different policies, we must integrate travel demand and supply models in our analysis, which are discussed in the following subsections.
2.1 Travel Demand

This study follows the indirect utility approach initiated by Dubin and McFadden (1984) in their study about residential electric appliance holdings and consumption because of its solid foundation in consumer behavior theory. This approach was first introduced to the field of transportation by Mannering and Winston (1985) (an early version was presented at the 1982 Winter Meeting of the Econometric Society) in their seminal work on household vehicle ownership and utilization. They also extended this approach from static to dynamic models and addressed several econometric issues in model estimation. Following these early works, many researchers, including Winston and Mannering (1984), Hensher et al. (1992), De Jong (1990), Goldberg (1998), and West (2004) have further extended and applied this indirect utility approach in various context (De Jong et al. (2004) provided a review).

In this study, we consider a household who seeks to maximize its utility under budget constraints. We consider two goods: vehicle usage \((A)\) and all other goods \((X)\). The household faces a discrete choice of owning vehicles and a continuous choice of vehicle usage conditional on the ownership choice. This joint decision gives the consumption of vehicle usage and determines vehicle ownership. The consumption of vehicle usage and all other goods yield positive marginal utility.

To derive the model, we must first specify a functional form for conditional indirect utility, which has a one-to-one correspondence with the demand function. A demand function linear in income and price has been used by Mannering and Winston (1985), Winston and Mannering (1984), Hensher et al. (1992), Goldberg (1998), and West (2004). Most of these studies focus on the issue of household vehicle type choice, and thus households who choose not to own vehicles and to utilize other transportation modes are usually not considered in those models. To consider such an option, the utility level for the choice of surrendering driving and adopting other means of transportation has to be decided externally. With a demand function linear in income and price, the demand could become negative as price increases under this specification. De Jong (1990, 1996) proposed a double log specification, with which the utility level of choosing not to own a vehicle becomes endogenous. The double-log demand function is also bounded to be positive in utility and performs well in an early empirical study according
to the authors. Therefore, this paper follows the second approach, while maintaining consistency in connotation with other literature whenever possible.

From De Jong (1990), we assume the demand for driving $A_i$ units of distance (e.g. measured in annual Vehicle Mile Traveled (VMT)) by a household $i$ with annual income $Y_i$ is determined by:

$$\ln A_i = \alpha_i \ln(Y_i - C) - \beta_i p + \eta_i$$  \hspace{1cm} (2.1)

where: $p$ is the operating cost per mile for the vehicle;
$C$ represents the annualized capital cost of owning a car;
$\eta_i$ summarizes household socio-economic and demographic characteristics;
and $\alpha_i$ and $\beta_i$ are parameters.

Following Burtless and Hausman (1978), the corresponding indirect utility function is:

$$V(p, Y_i - C) = \frac{1}{\beta_i} \exp(\eta_i - \beta_i p) + \frac{1}{1 - \alpha_i} (Y_i - C)^{1-\alpha_i}$$  \hspace{1cm} (2.2)

We can easily verify this correspondence by applying Roy’s identity to 2.2, which gives the demand function 2.1. Equation 2.2 gives the maximum utility on the downward-sloping part of the budget line (where people choose to own a vehicle). For each point on that line, we have to compare its utility with the utility of owning no vehicle (which is always an option) to decide the optimal decision. To derive the utility of owning no vehicle, De Jong (1990) observed that the optimal decision would be to not drive at all when operating cost per mile $p$ goes to infinite. Therefore, we have:

$$\lim_{p \to \infty} V(p, Y_i) = U(0, Y_i)$$  \hspace{1cm} (2.3)

which yields:

$$U(0, Y_i) = \frac{1}{1 - \alpha_i} Y_i^{1-\alpha_i}$$  \hspace{1cm} (2.4)

In this paper, we will only focus on the homogeneous user group. Even with this assumption, the analysis is already complex and produces important insights and findings. Planned future research will relax this assumption to consider heterogeneous users. The sub-scripts for individual household $i$ will therefore be omitted for the rest of the paper.
However, we shall see that such theoretical framework can also be applied to the heterogeneous user group. Also, we will first only consider one traveler/vehicle per household and leave the complexity of intra-household travel behavior for future research. In the following sections, the word traveler or household will be used interchangeably.

Under the assumption of homogenous users, all people will choose to own a car if \( V(p, Y - C) > U(0, Y) \). The demand of driving is conditional on the decision of owning a car, following equation 2.1.

\[
V(p, Y - C) = U(0, Y) \tag{2.5}
\]

When equation 2.5 is satisfied, people are indifferent between owning a car and driving \( A_{min} \) miles with it, and owning no car. This minimal driving distance and the corresponding maximal operating cost \( p_0 \) can be derived by combining 2.2 and 2.4. Therefore, the curve of aggregate driving demand (see Figure 2.1) will comprise two parts: a downward sloping curve which represents the driving demand when everyone chooses to own a car and a horizontal line which summarizes the situation when the operating cost is so high that only part \( (P \in [0, 1]) \) of the entire population choose to own a vehicle. The latter situation represents a market equilibrium where everyone in the market enjoys the same level of utility \( (U(0, Y)) \) and nobody can benefit by unilaterally switching between the decision of owning a car or not owning a car, while the former situation represents a simpler equilibrium in a less congested condition where everyone chooses to own a vehicle and drives the same amount.

\( p_0 \) in this graph indicates a maximal operation cost beyond which nobody is willing to drive. The total driving demand at the kicking point \( (q_0, p_0) \) is \( q_0 = A_{min} \times H \), where \( H \) is the number of households. For each point on the plane left of this point, the total driving demand \( q \) can be calculated following \( q = A_{min} \times H \times P \), where \( P \in [0, 1] \) is the portion of travelers who choose to own a car. The curve on the right of \( (q_0, p_0) \) represents situation when operation cost is lower than \( p_0 \). In this case, all households will choose to own a car (and enjoy a utility higher than \( U(0, Y) \)). The amount of driving per household \( A(p) \) is given by 2.1. Total amount of driving \( q = A \times H \).
2.2 Travel Supply

To maintain the tractability of the analysis, we consider a stylized network with only one link and one origin-destination pair. The total demand is carried by this idealized network, with road capacity \( F \). The generalized travel cost \( p \) is then a function of travel demand \( q \).

\[
p = \phi t(q)
\]

where \( \phi \) is the value of time. To evaluate policies on the network with different levels of congestion, this study employs a generalized Bureau of Public Roads (BPR) function to describe the supply side. The total travel cost \( p \) is thus:

\[
p = \phi T_0 (1 + \xi (\frac{q}{F})^\varphi)
\]

where: \( T_0 \) captures the free flow time. \( \xi \) and \( \varphi \) are parameters whose values allow us to model networks with different congestability (travel time elasticities). In the most common BPR function, \( \xi = 0.15, \varphi = 4 \). As this paper focuses on rationing policies
and their impacts on market equilibrium, we assume other long term costs such as fuel price remain constant and become part of $\phi T_0$.

With both the demand and supply curves ready, we can solve for the equilibrium point $(p^*, q^*)$.

### 2.3 Network Equilibrium

Based on the demand and supply models introduced in the previous subsection, this subsection analyzes market equilibrium conditions. The location of an equilibrium point depends on both travel demand and network supply.

As illustrated in Figure 2.2, there are two distinct equilibrium situations corresponding to different level of network congestion:

- When the network is not very congestable (travel time increase slowly as demand increases), the slope of supply curve is relatively flat. Consequently, the supply curve will intersect the demand curve on the downward sloping part and the equilibrium point $(q_1^*, p_1^*)$ is on the right of $(q_0, p_0)$. This means that each household
will choose to own a vehicle and drive $A(p_1^*)$ . $(q_1^*, p_1^*)$ is given by solving the following equation set:

\begin{align*}
q &= HA(p) = H(Y - C)^α \exp(-βp) \\
p &= \phi T_0(1 + ξ(q_F))^{q^*}.
\end{align*}

(2.8)

- When the network is very congestable (travel time increases fast as demand increase), the supply curve becomes very steep. The equilibrium point $(q_2^*, p_2^*)$ is on the left of $(q_0, p_0)$. We can verify that $p_2^* = p_0$ from Figure 2.2. This indicates that only a portion of households will choose to own a vehicle and put $A_{min}(p_0)$ on it. The percentage is $P = \frac{q_2^*}{HA_{min}}$. $(q_2^*, p_2^*)$ is given by the following equation:

\begin{align*}
\phi T_0(1 + ξ(q_F))^{q^*} = p_0
\end{align*}

(2.9)

These two scenarios represent networks of different congestability. We will then analyze policies under both scenarios in the following sections.
Chapter 3

Models for Vehicle Ownership and Usage Rationing Policy Analysis

In this section, we will analyze the two distinct rationing policies: vehicle ownership rationing and vehicle usage rationing.

For vehicle ownership rationing, we consider a rationing policy that directly limits the total number of vehicles in the system (although most of such policies target the newly added vehicles through license quota, they could be interpreted as a control of total vehicles in the future). Under this regulation, only part of the population ($\theta$) who are willing to own a vehicle can actually own one, regardless of their willingness to pay, while the $1 - \theta$ portion of potential drivers will be rationed out from the market.

For vehicle usage rationing, we consider a “Day without a Car” type of rationing policy under which each vehicle can only be on road for $\lambda (0 < \lambda < 1)$ portion out of all days. This is also the current practice of some metropolitan areas, including Beijing, where each vehicle can only drive four out of five workdays.
3.1 A Compensating Variation (CV) Approach for Welfare Analysis

In previous studies, the majority of researchers have used consumer surplus (CS) for welfare analysis. Consumer surplus is an approximation for the willingness-to-pay (WTP) welfare measurement. Its popularity in transportation studies is largely due to the fact that it has a very straightforward graph implementation, which is the triangular area below demand curve and above the price line. However, in this study, consumer surplus is not applicable and might lead to a false result for several reasons.

When modeling ownership rationing, we should note that for the $1 - \theta$ portion of people, by not allowing them to buy a car and drive, their real income will be switched from $Y - C$ back to $Y$. This shift in income does not generate any increase in driving (since they are not allowed to own a vehicle at the beginning), thus can not be captured by any triangular area on the demand-supply graph. Therefore, consumer surplus cannot take into account such a shift, and the welfare loss for those people will be over-estimated by applying the CS measure.

The CS measure also encounters difficulty when applied for usage rationing analysis. Please note that limiting each vehicle to driving four out of five weekdays is different from simply reducing VMT by $1 - \lambda$ because in the former case people can’t decide which portion of travel they want to sacrifice. Applying consumer surplus measure will automatically assume that the $1 - \lambda$ least wanted amount will be abandoned, and thus under-calculate the welfare losses.

To overcome these difficulties, the measure of Compensating Variation (CV) from Hicks (1946) will be used in this study. It is defined as the money to be taken away from (or paid to when it’s negative) the individual after an economic change, that leaves the individual with the same utility value as before. Small and Rosen (1981) extended such welfare calculation method to discrete choice models and provided a thorough discussion about the condition under which individual CV can be aggregated. They pointed out that CV can be aggregated as long as individual income is the same for everyone and individual share of total income is fixed under policy changes. With homogenous users, both conditions hold. In the field of transportation, applications of CV in welfare changes can be found in Mannering and Winston (1987), Winston and
Mannering (1984). CS will be used as approximate welfare measure only if CV is not applicable in section 3.3.1.

3.2 Vehicle Ownership Rationing

Following the theoretical modeling framework established in previous sections, this subsection models and evaluates the ownership rationing policy under which $1 - \theta$ portion of the initial driving group (the proportion of the households that choose to drive before the implementation of rationing policy) will be rationed out from driving. As a result, roads in the network will be less congested and $\theta$ portion of $P \times H$ households who still own a vehicle will be better off and potentially take advantage of this by driving more. The total travel demand after implementing the rationing policy becomes:

$$q_o = \theta PHA(p)$$  \hspace{1cm} (3.1)

Subscript $o$ represents the case of ownership rationing. By substituting 3.1 into equation 2.2 and then combining with supply function 2.7, the new equilibrium point, $(q_{o*}, p_{o*})$, can be obtained by solving the following equation set:

$$\begin{cases} q_o = \theta PHA(p) = \theta PH(Y - C)^{\alpha} \exp(-\beta p), \\ p = \phi T_0(1 + \xi(q_o F)^{\gamma}). \end{cases}$$ \hspace{1cm} (3.2)

3.2.1 Welfare Impacts of Ownership Rationing

The welfare change for the $\theta$ portion of households who no longer own a vehicle after rationing policy implementation could be captured by calculating their compensating variation $CV_{o,1-\theta}$ through the following equation:

$$U(0, Y - CV_{o,1-\theta}) = V(p^*, Y - C)$$ \hspace{1cm} (3.3)

Equation 3.3 ensures that by compensating each household $CV_{o,1-\theta}$, their utility stays the same as the level before the rationing policy is implemented. Similarly, for
the $\theta$ portion of households who still keep their vehicles, compensating variation $CV_{o,\theta}$ is calculated by following the same logic:

\[
V(p_{o}^{**}, Y - CV_{o}^{o} - C) = \frac{1}{\beta} \exp(-\beta p_{o}^{**}) + \frac{1}{1 - \alpha} (Y - C - CV_{o,\theta})^{1-\alpha} = V(p^{*}, Y - C) \quad (3.4)
\]

By combining the welfare impacts for these two group of households, the overall welfare change is:

\[
CV_{o} = PH(\theta CV_{o,\theta} + (1 - \theta)CV_{o,1-\theta}) \quad (3.5)
\]

As previously discussed, rationing policies may perform differently with different levels of congestion. First consider the case when the congestion level is relatively low. Then, initially, every household owns a vehicle, thus $P = 1$, $CV_{o,1-\theta} < 0$, $CV_{o,\theta} > 0$. The new equilibrium point after ration is shown in figure 3.1

![Figure 3.1: New Equilibrium after Ownership Rationing with a Low Level of Congestion](image)

Then consider the case when the congestion level is high. In this situation, only
part of the population own a vehicle \((0 < P < 1)\). \(CV_{o,1-\theta} = 0\); In this case, the new equilibrium point after ration is given out by the the figure 3.2

\[
\begin{align*}
\frac{1}{\beta} \exp(-\beta p_o^{**}) + \frac{1}{1 - \alpha} (Y - C - CV_{o,\theta})^{1-\alpha} &= \frac{1}{1 - \alpha} Y^{1-\alpha} \\
\text{Overall CV is:} \quad CV_o &= PH\theta CV_{o,\theta}
\end{align*}
\]

3.2.2 Optimal Rationing Ratio

When considering vehicle ownership rationing, policy makers often want to know the optimal rationing rate that would generate the greatest welfare gain. Since when roads in the network are not congested, ownership rationing will most certainly result in a welfare loss, in this subsection we will only focus on the more congested conditions when only part of the population initially own a vehicle\((P < 1)\).

To approach this, we take first order derivatives with regard to \(\theta\) on both sides of 3.7:
\[
\frac{\partial CV_o}{\partial \theta} = PH [CV_{o,\theta} + \theta \frac{\partial CV_{o,\theta}}{\partial \theta}] \quad (3.8)
\]

\[
\frac{\partial CV_{o,\theta}}{\partial \theta} \text{ can be obtained by calculating first order derivatives for both sides of 3.6:}
\]

\[
\frac{\partial CV_{o,\theta}}{\partial \theta} = -(Y - C - CV_{o,\theta})^\alpha \exp(-\beta p_o^{**}) \frac{\partial p_o^{**}}{\partial \theta} \quad (3.9)
\]

where, \( \frac{\partial p_o^{**}}{\partial \theta} \) is derived from 3.2:

\[
\frac{\partial p_o^{**}}{\partial \theta} = \frac{1}{\theta [\beta + \frac{1}{\xi \varphi \phi_o (\frac{q^{**}_o}{\delta})^{-\varphi}]} \quad (3.10)
\]

Unfortunately, a closed form result can not be obtained by setting \( \frac{\partial CV_o}{\partial \theta} = 0 \). In other words, the optimal ownership rate ratio does not have a closed form solution.

We will introduce a numerical example later to gain better insight of its property in Chapter 4.

### 3.3 Vehicle Usage Rationing

In this subsection, we will analyze usage rationing policy when each vehicle can only be used on \( \lambda (0 < \lambda < 1) \) portion of the weekdays. Some households may forego their vehicles as the availability for utilization (thus overall utility) from each vehicle decrease due to the rationing policy, which others may purchase a second vehicle to circumvent the policy. However, as we only investigate homogenous households in this study, we assume that vehicle ownership remains unchanged and leave the long term ownership shifts for future study when heterogenous households are considered.

As noted in the beginning of section 3, the vehicle usage rationing is different from decreasing the least-wanted (i.e. with the least willingness-to-pay) \( 1 - \lambda \) of total travel, which is usually assumed in CS-based analysis.

To address this difference, we derive the indirect utility after usage rationing as follows:

\[
V_{usage}(p, Y - C) = \lambda V(p, Y - C) + \frac{1}{1 - \alpha} \left(1 - \lambda\right)U(0, Y - C)
\]

\[
= \frac{\lambda}{\beta} \exp(\eta - \beta p) + \frac{1}{1 - \alpha} (Y - C)^{1-\alpha} \quad (3.11)
\]
where, $\lambda V(p, Y - C)$ is the contribution of the $\lambda$ portion of the days when a vehicle is allowed to be driven, while $(1 - \lambda)U(0, Y - C)$ is the contribution of the $\lambda$ portion of the days when this vehicle usage is rationed.

Applying Roy’s Identity, we can verify that the demand function for individual household derived from this indirect utility function is consistent with our assumption of the usage rationing policy:

$$A_u(p) = \lambda A(p) = \lambda(Y - C)^\alpha \exp(-\beta p) \tag{3.12}$$

The new aggregated travel demand under rationing policy is then given by:

$$q_u(p) = PHA_u(p) = \lambda PH(Y - C)^\alpha \exp(-\beta p) \tag{3.13}$$

Travel is an induced demand which allows people to fulfill all other activities occurring at different time and locations. Under vehicle usage rationing, people can not drive during certain days. In the short term, it may be difficult to find replacement activities during the driving days and people would stay at the same level of travel demand in days when they are allowed to drive. One example is commute driving. People are unlikely to commute longer or more often during the days when they are allowed to drive. In other words, such travel demand is not substitutable, especially for the short term. However, in the long term, people may engage in more activities during the driving days to benefit from the traffic reduction due to usage rationing policy. These induced demand may compromise the benefit from initial traffic reduction. The following subsections will investigate scenarios with and without induced demand separately.

### 3.3.1 Welfare Impacts of Vehicle Usage Rationing without Induced Demand

First we consider a situation when no induced demand is introduced. In this case, consumer’s decision is not optimized under the given utility function (the optimal decision implies that users will benefit from the initial traffic reduction and drive more). Therefore, the individual utility level is unknown (the indirect utility function only provides individual household utility under optimized consumption), the compensation variation
measure can not be applied. Instead, we will use consumer surplus as an approximation for welfare calculation in this instance.

\[
q = \lambda PHA(p)
\]

\[
q^* = \frac{1}{\lambda} q^*
\]

\[
\text{Area } 1 = \lambda q^0 \left[ p_0 - \phi T_0 \left( 1 + \xi \left( \frac{q^0}{\phi} \right) \phi \right) \right]
\]

in which Area 1 indicates welfare gain for those still allowed to drive due to reduced level of congestion, while Area 2 indicates the loss for those rationed off the network due to the reduction of VMT.

\[
Area2 = \lambda q^0 \left( p_0 - \phi T_0 \right)
\]

\[
(3.15)
\]

Figure 3.3: Consumer Surplus Change after Usage Rationing without Induced Demand

In figure 3.3, we have a new demand curve from 3.13. Given the same operating cost, the aggregate demand corresponding to this new curve is only \( \lambda \) of the initial demand. The consumer surplus gain, from this graph, is given by:

\[
CS_u = \text{Area1} - \text{Area2} \tag{3.14}
\]
\[\text{Area} 2 = (1 - \lambda) \int_{p^*}^{+\infty} PH(Y - C)^{\alpha} \exp(-\beta p) dp\]

\[= \frac{1 - \lambda}{\beta} PH(Y - C)^{\alpha} \exp(-\beta p_0)\]  

(3.16)

The overall welfare changes depend on the rationing ratio (\(\lambda\)), the shape of the demand curve, and the level of network congestion. No closed form solution is available. We will further investigate the optimal usage rationing ratio and welfare consequences in a numerical example in section 4.

### 3.3.2 Welfare Impacts of Vehicle Usage Rationing with Induced Demand

We now consider the impact of induced demand to welfare changes. In this case, households will drive more every day when they are allowed to drive, due to the reduced level of network congestion. As each household would drive optimally, their utility can be obtained through the indirect utility function 3.11. The compensating variation corresponding to long-term welfare impacts can then be derived.

First, a new equilibrium point \((q_u^{**}, p_u^{**})\) will be obtained from the following equations:

\[
\begin{aligned}
q_u &= \lambda PHA(p) = \lambda PH(Y - C)^{\alpha} \exp(-\beta p), \\
p &= \phi T_0(1 + \xi \left(\frac{q_u}{F}\right)^{\varphi})
\end{aligned}
\]

(3.17)

For the \(P\) portion of population who own a vehicle before the usage rationing policy, their compensating variation is calculated using the following equation:

\[
\lambda V(p_u^{**}, Y - C - CV_{u,\lambda}) + (1 - \lambda)U(0, Y - C - CV_{u,\lambda}) = V(p^*, Y - C)
\]

(3.18)

which is equivalent to:

\[
\frac{\lambda}{\beta} \exp(-\beta p_u^{**}) + \frac{1}{1 - \alpha} (Y - C - CV_{u,\lambda})^{1-\alpha} = \frac{1}{\beta} \exp(-\beta p^*) + \frac{1}{1 - \alpha} (Y - C)^{1-\alpha}
\]

(3.19)
Compensation variation allows the utility of each individual household to stay at the same level. For the remaining population who do not own a vehicle before the rationing policy, their utility is unchanged since their income stays the same. Therefore, total user welfare gain is then $CV_u = PHCV_{u,\lambda}$.

Similar to the analysis of ownership rationing, we are more interested in situation when congestion is high. So we will focus on the case where $P < 1$. In this case, $V(p^*, Y - C) = U(0, Y)$. We shall show that with induced demand, the long term welfare changes will always be losses through the following proof.

**Proposition 1.** When induce demand is taken into account, which is the satisfaction of equation 3.17, vehicle usage rationing policy will always results in a user welfare loss

**Proof.** To prove this, we stake first-order derivatives on both sides of 3.19 with respect to $\lambda$.

$$\frac{\partial CV_u}{\partial \lambda} = (Y - C - CV_{u,\lambda})^\alpha \exp(-\beta p_u^{**})\left[\frac{1}{\beta} - \lambda \frac{\partial p_u^{**}}{\partial \lambda}\right]$$ \hspace{1cm} (3.20)

Also we can derive $\frac{\partial p_u^{**}}{\partial \lambda}$ from 3.17:

$$\frac{\partial p_u^{**}}{\partial \lambda} = \frac{1}{\lambda [\beta + \frac{1}{\epsilon_0 \lambda T_0 (\frac{2\pi}{F})^{-\epsilon}] < \frac{1}{\lambda \beta}$$ \hspace{1cm} (3.21)

Thus, substituting 3.21 into 3.20, we see $\forall \lambda \in [0, 1)$$\frac{\partial CV_u}{\partial \lambda} > 0$ \hspace{1cm} (3.22)

However, in the extreme case, when the policy rations 0 percentage of vehicle usage ($\lambda = 1$), the utility level should be unchanged and we have $CV_u = 0$. Therefore, we obtain:

$$CV_u(\lambda) < 0, \hspace{1cm} \forall \lambda \in [0, 1)$$ \hspace{1cm} (3.23)

The same conclusion can be reached if we conduct analysis based on the more popular consumer surplus measure. This proof will be provided in the appendix A.
Vehicle usage restriction may seem attractive because of its significant initial success in reducing congestion and pollution. For example, the success of vehicle usage restriction during Beijing Olympics encouraged policy makers to permanently adopt this policy as the major solution to fight increasing congestion in Beijing, China (Wang, 2010). However, people’s adaptive behavior and the induced travel demand associated with it could significantly compromise the initial gain. Theoretical reasoning in this section illustrates how such policy could lead to long term losses in social welfare.

Empirical evidence also raises concerns about its long-term effect. For example, the vehicle restriction policy based on license plate was only applied from 7:00am to 9:00am and from 5:30am and 7:30am when it was first implemented in Bogotó, 1998. As its effectiveness fades, the policy was then extended to 6:00am-9:00am and 4:00pm-7:30pm in 2003 and further to practically the entire day (6:00am-8:00pm). Although many factors, such as traffic due to population increase and economic development, may have contributed to the extension of the restriction measure, the impact of induced demand can not be under-estimated. Although the induced demand is not necessarily completely detrimental (the substitution of peak traffic with non-peak traffic and weekend traffic could imply more efficient usage of existing network capacity), it does deviate from the initial policy objective of reducing traffic. Therefore, a comprehensive welfare analysis is needed before any policy should be implemented. And the model proposed in this paper provides policy makers with a useful tool for such analysis.
Chapter 4

Policy Comparison: A Numerical Analysis

Closed form results are not available for several scenarios in previous analysis. In this section, numerical analysis will be conducted to provide additional insights on welfare implications of both rationing policies.

Ideally, model parameters should be calibrated with vehicle ownership and usage data, which is beyond the scope of this paper with a theoretical focus. In this study, we follow the parameters reported by De Jong (1990):

- Income elasticity of driving $\alpha = 0.49$
- Price elasticity of driving $\beta = 0.028$
- Average annual income $Y = 35000$
- Vehicle Price averaged in years $C = 2536$

We also set the parameters for supply-side function as: $\varphi = 4$ and $\xi = 0.15$ in equation 2.7, which is a typical BPR function. We assume the free flow operation cost $\gamma T_0$ to be $2 and capacity $F$ to be $14PH$ for convenience (as we are only considering an idealized network, the choice of these two parameters only reflects the network capacity relative to travel demand, but does not have a strict physical meaning).

With this set of parameters, we can get the critical operating price $p_0 = 30.2$. Since this study only considers the homogeneous user group, when operation cost per 100 kilometer is over $30.2$, no household will choose driving. When operation cost is below
$30.2, all households will choose to own a car. When cost is exactly at $30.2, people will be indifferent with either choices.

As discussed in previous sections, we focus on the more congested situation when only part of the population own a vehicle. Under the set of parameters we choose, we will solve the network equilibria numerically and compare the performance of both vehicle ownership and usage rationing policies. As revealed in section 3.3.2, when we assume that households will adjust their VMT to reach the maximum utility in usage rationing condition, welfare change under usage rationing will always be a loss. So in this case, ownership rationing outperforms usage rationing. The more interesting scenario will be to compare the welfare impacts under the two rationing policies without induced demand. In this scenario, households won’t adjust their VMT choice, at least in short term, after usage rationing is adopted. An example for this scenario could be temporary vehicle usage rationing during special events. It could also become more relevant when trips are not substitutable during different days (such as commute trips).

We first investigate how two policies perform as the rationing ratio, the most important policy parameter, varies. Figure 4.1 gives out the usage welfare gain with regard to the remaining portion after rationing ($\theta$ for the case of ownership rationing and $\lambda$ for the case of usage rationing). In other words, 0.8 on the horizontal axis means:

- For ownership rationing policy, only 80% of households who want to purchase vehicles are allowed to do so.

- For usage rationing, vehicle owners can only use their vehicles in 80% of the time (one in every five days).

For vehicle ownership rationing, when $\theta = 0.40$, which means to ration out 60% of the vehicles, consumer welfare gain reaches the maximum.

In contrast, with vehicle usage rationing, when $\lambda = 0.82$, which means to limit each vehicle roughly to driving 4 out of 5 weekdays, consumer welfare gain reaches the maximum.

With the same rationing ratio, vehicle usage rationing enjoys a greater social welfare gain when rationing ratio is small (less than 0.15 under this set of parameter, right side in Figure 4.1), while ownership rationing policy offers a greater social welfare gain as rationing ratio increases. This is intuitive because under vehicle ownership rationing,
potential drivers who are rationed out of the market are partially compensated by not
paying the capital cost of buying a vehicle. However, all drivers under vehicle usage
rationing have already paid the initial capital cost. As the rationing ratio increases,
the service flow, or utilization, from owning a vehicle keeps on dropping from vehicle
usage rationing. When the utility losses become larger than the gain from initial capital
cost savings, the vehicle usage rationing policy becomes less attractive than vehicle
ownership rationing. In contrast, when the rationing ratio remains small, vehicle usage
rationing policy is more attractive because it offers a deeper cut of travel demand (vehicle
ownership rationing is long-term, thus also inspires induced demand in our analysis).
Moreover, people could drive more during weekends, off-peak time periods, and under
emergency needs, once they own a vehicle. During these time periods, network could
exhibit different level of congestion and people could have different values of time.
Vehicle usage rationing could become more advantageous because it offers certain level
of flexibility, which can not be captured in the single period analysis in this paper.

Another interesting question is how policy performance varies as network congesta-
bility differs. We expect both policies to perform better in a network where marginal cost
of driving increases significantly as demand increases, compared to a single-bottleneck network where marginal cost remains constant. As a comparison to the BPR type of supply curve, this study compares the two policies with Vickrey’s bottle-neck model, which implies constant marginal cost (Vickrey, 1963, Zhang, 2008). Similar to those for BPR function, we assume $\varphi = 1$, free flow operation cost $\gamma T_0 = \$2$, and capacity $F = 14PH$ as in equation 2.7. However, $\varphi$ now equals to one, which differs its congestability from the BPR function. In order to produce the same amount of total driving $q^*$ in equilibrium for both supply curves, $\xi$ is set to be 9.06 for Vickrey’s model. Given these parameters, we can again solve the welfare gains numerically, as illustrated in figure 4.2:

![Figure 4.2: Welfare Gain of the Two Rationing Policies using Vickrey Model](image)

With a less congestable network, the vehicle ownership rationing policy always dominates the vehicle usage rationing. The vehicle usage rationing policy which always generates welfare losses under the selected parameters. In this case, vehicle ownership rationing reaches maximal welfare gain when $\theta = 0.38$.

Since vehicle ownership rationing can also generate welfare gain under Vickrey’s supply function, it would be useful to compare the magnitude of this welfare gain to that can be achieved with BPR type of supply function. Figure 4.3 compares the welfare
gain of ownership rationing under two supply models as rationing ratio varies from 0 to 1:

Figure 4.3: Comparison of welfare Gain under Ownership Rationing using Two Supply Models

From the figure 4.3, we see that under the same rationing ratio, the model using BPR function as a supply function always offers a bigger consumer welfare gain compared to that using Vickrey equation. Therefore, the rationing policy can perform better under network conditions associated with higher marginal cost. Consequently, rationing policies are more likely to succeed in highly congested mega-cities where demand exceeds capacity.
Chapter 5

Road Pricing Policy

5.1 Formulation of Road Pricing

Compared to vehicle ownership and/or usage rationing policies, road pricing as a way to allocate scarce infrastructure resources is much more popular and has been widely deployed. Therefore, it is interesting to compare these two policies, which may have important implications for future decision-making.

To consider road pricing under the framework set up in section 2, we add a new term \( \tau \) in the link performance function to represent toll charge. For simplicity, we will first consider a constant toll for all drivers and for all time periods. These constraints can be later relaxed. Thus, the new generalized travel cost \( p \) is:

\[
p = \tau + \phi T_0 (1 + \xi \left( \frac{q}{F} \right)^\phi)
\]  

(5.1)

We assume the consumer (traveler) behavior still follows the demand model proposed in this study. Therefore, the same aggregate demand curve can be derived and the market equilibrium can be solved by combining the supply curve and the demand curve. Similar to our analysis for rationing policies, we are more interested in cases with high congestion, which locate on the horizontal part to the right of the kicking point on demand curve. Before any toll charge is implemented, the market is in an equilibrium where only a part of the total population choose to own a car and drive \( A_{\text{min}} \) annually. The utility is the same for both drivers and people who choose not to drive. After a toll \( \tau \) is imposed, the higher driving cost forces some people to abandon driving. As a
result, the level of congestion decreases and people remaining in the driving market will drive more to benefit from lower congested travel time. In equilibrium, the generalized cost for driving must come back to $p_0$. Otherwise, if the generalized cost is higher than this value, people can increase their utility (thus be better off) by choosing not to own a car. The decreasing driving demand will push down the driving cost, which will reach $p_0$ eventually. Similarly, the generalized cost will not be lower than $p_0$ either because in that case, people will increase their driving consumption according to the indirect utility function (which defines the optimal consumption bundle for a given price $p$) by either driving more or switching to drive if not so previously.

Therefore, under these assumptions, we can solve a new equilibrium point $q^{**}_p, p^{**}_p$ by the following equation set:

\[
\begin{align*}
    p &= p_0 \\
    p &= \tau + \phi T_0 (1 + \xi (\frac{q}{E}))^{\phi}.
\end{align*}
\] (5.2)

The above reasoning can be illustrated by the following figure:

![Figure 5.1: New Equilibrium after Road Pricing](image)

So after the road pricing policy is implemented, the portion of drivers among the
entire population will drop to $q^*_{p,H_{min}}$, while all the driving households will still drive $A_{min}$ at an operating cost of $p_0$.

5.2 Welfare Impacts of Road Pricing

This section analyzes the welfare changes after a uniform toll $\tau$ is charged for the entire population and for all time periods.

For the households who will still drive, since their driving amount and operation cost will remain unchanged under the assumed demand pattern, their utility will be same as before.

For the households who give up owning vehicles, their utility will also remain unchanged because the market is in equilibrium (their utility becomes $U(0,Y)$ and as previously indicated, $U(0,Y) = V(p^*, Y - C)$).

Therefore, the only difference comes from the charged toll. If we assume the collected toll will be completely used for the benefit of society and no transaction cost is applied, the overall social welfare gain with road pricing will be:

$$W_p = \tau q^*_{p}$$

5.3 Optimal Toll

With the modeling framework set up in this study, we will derive the theoretical optimal toll fee. To achieve this, we take first order derivative with regard to $\tau$ in equation 5.3:

$$\frac{\partial W_p}{\partial \tau} = q^*_{p} + \tau \frac{\partial q^*_{p}}{\partial \tau}$$

where $\frac{\partial q^*_{p}}{\partial \tau}$ is calculated by taking first derivatives with regard to $\tau$ of equation set 5.2:

$$\frac{\partial q^*_{p}}{\partial \tau} = -\frac{F}{\phi T_0 \xi \varphi(q^*_{p})^\varphi - 1}$$

We will obtain the optimal toll fee by setting $\frac{\partial W_p}{\partial \tau} = 0$, which is:
\[ \tau = \frac{\varphi(p_0 - \phi T_0)}{\varphi + F^\varphi} \quad (5.6) \]

### 5.4 Comparison between Road Pricing and Rationing Policy

As discussed in our previous study, vehicle usage rationing policy will always cause a social welfare loss when induced demand is considered. Thus, in this section, we will only compare road pricing and vehicle ownership rationing (which is always superior than vehicle usage rationing policy when long-term induced demand is considered).

To give a fare comparison, we compare the welfare impacts of the two policies when the same amount of driving demand is trimmed from the original demand, which is \( q_o^{**} = q_p^{**} \). This is equivalent to setting the toll rate at \( \tau = p_0 - p_o^{**} \).

From our previous paper, we concluded that the welfare gain after vehicle ownership rationing to be:

\[ CV_o = PH\theta CV_{o,\theta} \quad (5.7) \]

where \( CV_{o,\theta} \) is solved from:

\[ \frac{1}{\beta} \exp(-\beta p_o^{**}) + \frac{1}{1-\alpha} (Y - C - CV_{o,\theta})^{1-\alpha} = \frac{1}{1-\alpha} Y^{1-\alpha} \quad (5.8) \]

Thus, with equation 5.3, we obtained the difference in social welfare gain for the two policies:

\[ \Delta W = (p_0 - p_o^{**}) q_o^{**} - PH\theta CV_{o,\theta} \quad (5.9) \]

\[ = \theta PH[(p_0 - p_o^{**})(Y - C)^\alpha \exp(-\beta p_o^{**}) - CV_{o,\theta}] \]

To evaluate the sign of \( \Delta \), we only need to consider the part within the parenthesis. We define \( h(p_o^{**}) \), such that:

\[ h(p_o^{**}) = (p_0 - p_o^{**})(Y - C)^\alpha \exp(-\beta p_o^{**}) - CV_{o,\theta} \quad (5.10) \]

We take first derivatives with regard to \( p_o^{**} \) in equation 5.10 (this is doable because \( CV_{o,\theta} \) is a function of \( p_o^{**} \)),
\[
\frac{dh(p_o^*)}{dp_o^*} = \theta PH\left[-(Y - C)^\alpha \exp(-\beta p_o^*)(1 + \beta(p_0 - p_o^*)) - \frac{\partial CV_{o,\theta}}{\partial p_o^*}\right] \tag{5.11}
\]

where \(\frac{\partial CV_{o,\theta}}{\partial p_o^*}\) is obtained by taking first derivatives in equation 3.6:

\[
\frac{\partial CV_{o,\theta}}{\partial p_o^*} = -(Y - C - CV_{o,\theta})^\alpha \exp(-\beta p_o^*) \tag{5.12}
\]

Thus,

\[
\frac{dh(p_o^*)}{dp_o^*} = \exp(-\beta p_o^*)[\alpha(Y - C - CV_{o,\theta}) - (Y - C)^\alpha(1 + \beta(p_0 - p_o^*))] < 0 \tag{5.13}
\]

Since \(h(p_0) = 0\),

\[
h(p) > 0, \forall p \in (0, p_0) \tag{5.14}
\]

Thus, following equality always holds:

\[
\Delta W > 0 \tag{5.15}
\]

Therefore, we can conclude that when road pricing and vehicle ownership rationing are set up in such a way that both policies reduce travel demand with the same amount (or have the same congestion mitigation effects), road pricing will always generate a bigger social welfare gain. This conclusion is not surprising since pricing policy does not distort the market and everyone can maximize their utility. In contrast, under rationing policy, some people who are willing to drive under economic consideration can not drive. However, this does not guarantee the superiority of pricing policy since, in reality, the transaction cost of road pricing is very significant (sometimes the collected toll can not even cover the operation cost of running such a program). Moreover, different conclusions can be drawn if heterogeneous drivers, more time periods, and multiple origin-destination pairs are considered.
As aversive environmental and economical impacts of excessive vehicle usage become more severe, some mega-cities adopt vehicle usage and/or vehicle ownership rationing policies to directly control overall travel demand. These policies have not been adequately studied in literature. To bridge this gap, this study proposes to analyze rationing policies by extending the joint model of vehicle ownership and mileage models. The numerical analysis in this study suggests that the vehicle usage rationing policies can yield a higher welfare gain when rationing ratio remains small, while vehicle ownership rationing becomes more advantageous when the rationing ratio becomes sufficiently large.

The numerical example also reveals that on a congested network where marginal cost of driving increases rapidly, both rationing policies perform much better compared to a less congested network. This helps to explain why such policies are currently only implemented in mega-cities such as Mexico City, Beijing, and San Polo. However, as urban population increase, more cities could face the same policy choices. Findings from this study could provide insights for future policy decisions.

Analysis in this paper shows that with homogeneous users and the assumed demand function, the vehicle usage could yield short term social welfare gains, but will unavoidably lead to long-term social welfare losses. The reason for this phenomenon is due to induced demand, which is the excessive driving people would take part in to benefit from the initial traffic reduction imposed by rationing policy. Under the assumption of homogeneous users, in the long term, any rationing policies simply replace the excessive
driving by one driver with the excessive driving by a different driver or the same driver during a different time. However, rationing policies do have their merits in controlling excessive demand, when other policy tools such as pricing are not feasible or effective due to various reasons. Actually, rationing is not that unusual in transportation. The queue, or the first come, first serve process, is a widely used demand rationing process. They are used more common for rail or bus tickets (whose price is rigid), but still visible for urban highways (such as ramp metering). The usage of direct vehicle usage or ownership rationing may complement those less visible rationing process to better mitigate congestion.

This study also compares rationing policies with the more popular road pricing policy. When both policies achieve the same congestion mitigation effects, road pricing will always generate a bigger social welfare gain if no transaction cost is assumed. In reality, transaction cost is a major obstacle for implementing a pricing scheme and is less a problem for rationing policies (which requires some enforcement expenses). A hybrid scheme that combines both rationing and pricing policies could be an attractive direction for future research.

It has to be pointed out that this paper only considers homogeneous users and one time period. Since vehicle usage rationing policy does offer travelers some flexibility and people could drive more during weekends, off-peak time periods, and under emergency needs. Therefore, it may exhibit more advantages when heterogeneous drivers and peak versus off-peak periods are considered. Analysis under these alternative assumptions may better help future policy design. However, framework set up in this paper will provide a good starting point for more comprehensive policy analysis.
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Appendix A

Alternative Proof for Proposition 1 Based on Consumer Surplus

Proof. Having obtained the demand function 3.13 after usage rationing, we can start calculating the consumer surplus (CS) change after rationing policy is adopted. In fact, a new equilibrium point will be obtained. Detailed implementation is shown in the graph A.1:

As a result, consumer surplus change ($CS_u$) will be given by:

$$CS_u(\lambda) = S_{\text{Area}_1} - S_{\text{Area}_2} \quad (A.1)$$

where,

$$S_{\text{Area}_1} = \int_{p_u^*}^{p_u} \lambda PHA(p)dp \quad (A.2)$$

$$S_{\text{Area}_2} = (1 - \lambda) \int_{p_u^{**}}^{+\infty} PHA(p)dp \quad (A.3)$$

Do a first derivative with regard to $\lambda$ in equation A.1, we will have:

$$\frac{\partial CS_u(\lambda)}{\partial \lambda} = \frac{\partial S_{\text{Area}_1}}{\partial \lambda} - \frac{\partial S_{\text{Area}_2}}{\partial \lambda} \quad (A.4)$$
where,

\[
\frac{\partial S_{\text{Area1}}}{\partial \lambda} = PH\left[ \int_{p_u^*}^{p_{u}^*} A(p) dp - \lambda \frac{\partial p_u^{**}}{\partial \lambda} A(p_u^{**}) \right] \quad (A.5)
\]

\[
\frac{\partial S_{\text{Area2}}}{\partial \lambda} = -PH \int_{p_u^{**}}^{+\infty} A(p) dp \quad (A.6)
\]

\(A(p)\) is obtained from equation 2.1,

Thus,

\[
\frac{\partial CS_u(\lambda)}{\partial \lambda} = PH(Y - C)^{\alpha} \exp(-\beta p_u^{**}) (\frac{1}{\beta} - \lambda \frac{\partial p_u^{**}}{\partial \lambda}) \quad (A.7)
\]

From 3.21, we get:

\[\forall 0 < \lambda < 1\]

\[
\frac{\partial CS_u(\lambda)}{\partial \lambda} > 0 \quad (A.8)
\]

Since when \(\lambda = 1\), \(CS_u = 0\),

\[\forall 0 < \lambda < 1\]

\[
CS_u(\lambda) < 0 \quad (A.9)
\]
This is the end of proof.
Appendix B

Summary of Notation

$\alpha$: Income elasticity of driving;

$\beta$: Price elasticity of driving;

$\eta$: household socio-economic and demographic characteristics;

$\phi$: Value of time;

$1 - \theta$: Vehicle ownership rationing ratio;

$1 - \lambda$: Vehicle usage rationing ratio;

$\xi, \varphi$: Parameters in BPR function;

$A(p)$: Annual VMT by a household;

$A_u(p)$: Annual household VMT after vehicle usage rationing;

$C$: Annualized capital cost of owning a car;

$CS_u$: Aggregated consumer surplus gain after vehicle usage rationing without induced demand;

$CV_{o,\theta}$: Compensating variation for individual household who keeps their vehicles after vehicle ownership rationing;

$CV_{o,1-\theta}$: Compensating variation for individual household who no longer owns vehicles after rationing;

$CV_o$: Aggregated compensating variation after vehicle ownership rationing;

$CV_{u,\lambda}$: Compensating variation for individual driving household after vehicle usage rationing with induced demand;

$CV_u$: Aggregated compensating variation after vehicle usage rationing with induced demand;
$F$: Road capacity;
$H$: Number of households;
$p$: operation cost per mile;
$p_0$: Maximal operation cost beyond which nobody is willing to drive;
$P$: Percentage of households who own vehicles;
$q$: Aggregated travel demand;
$q_0$: Total driving amount at the critical price $p_0$;
$q_u$: Aggregated travel demand under vehicle usage rationing;
$(q^*, p^*)$: Network equilibrium point;
$(q_0^{**}, p_0^{**})$: Network equilibrium point after vehicle ownership rationing;
$(q_u^{**}, p_u^{**})$: Equilibrium point under vehicle usage rationing with induced demand;
$t(q)$: Travel time;
$T_0$: Free-flow travel time;
$U(A, X)$: Direct utility function;
$V(p, Y − C)$: Indirect utility function;
$V_{usage}(p, Y − C)$: Indirect utility function after vehicle usage rationing;
$Y$: Annual household income;