Non-Abelian Duality and Monopole Confinement in supersymmetric QCD

from $\mathcal{N} = 2$ to $\mathcal{N} = 1$

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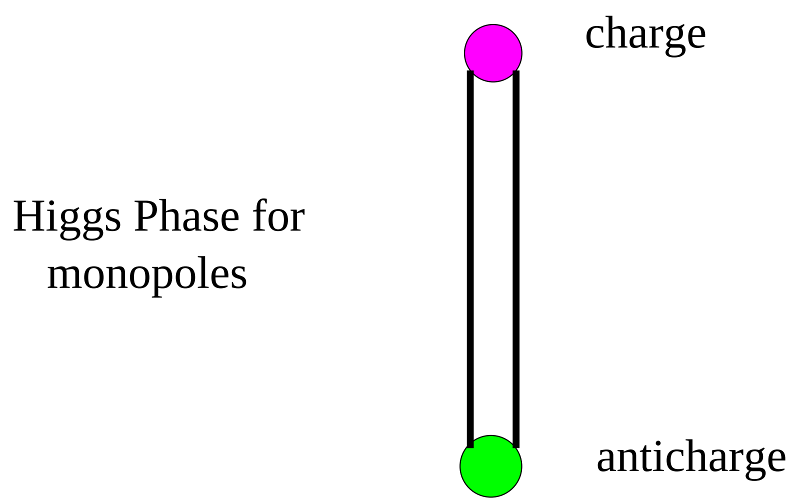
1 Introduction

Standard scenario of quark confinement:

Nambu, Mandelstam and 't Hooft 1970's:

Confinement is a dual Meissner effect upon condensation of monopoles.

Monopoles condense → electric Abrikosov-Nielsen-Olesen flux tubes are formed → electric charges are confined



Seiberg and Witten 1994 : Abelian confinement in $\mathcal{N}=2$ QCD

Cascade gauge symmetry breaking:

- SU(N)$\to$ U(1)$^{N-1}$ — VEV's of adjoint scalars
- U(1)$^{N-1}\to 0$ (or discrete subgroup) — VEV's of monopoles

At the last stage Abelian Abrikosov-Nielsen-Olesen flux tubes are formed.

$$\pi_1(U(1)^{N-1}) = \mathcal{Z}^{N-1}$$

In both QCD or $\mathcal{N}=1$ supersymmetric QCD there are no adjoint fields

Problem: Cannot decouple adjoint fields in monopole vacua

Masses in low energy U(1)$^{N-1}$ theory are $\sim \sqrt{\mu\Lambda}$

To have weak coupling we need $\mu \ll \Lambda$

Non-Abelian setup: *Shifman and Yung 2009*

$\mathcal{N} = 2$ QCD with $U(N)$ gauge group and $N_f > N$ fundamental flavors (quarks).

Fayet-Iliopoulos D-term ξ_3.

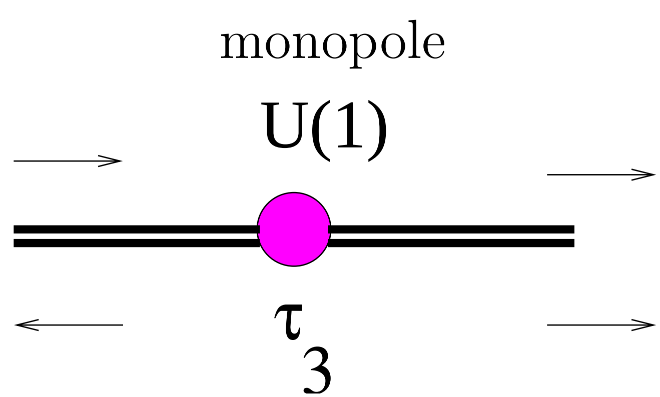
Quark vacuum

Different regimes separated by crossovers (= CMS)

Large ξ (weak coupling)

- N scalar quarks condense with VEV's $\sim \sqrt{\xi}$.
  $U(N)$ gauge theory with N_f quarks at weak coupling
- non-Abelian strings which confine monopoles

Example in $U(2)$



Small ξ (strong coupling)

- Gauge theory with dual gauge group

$$U(\tilde{N}) \times U(1)^{(N-\tilde{N})}$$

and N_f light dyons, $\tilde{N} = N_f - N$
(with *weight*-like electric charges)

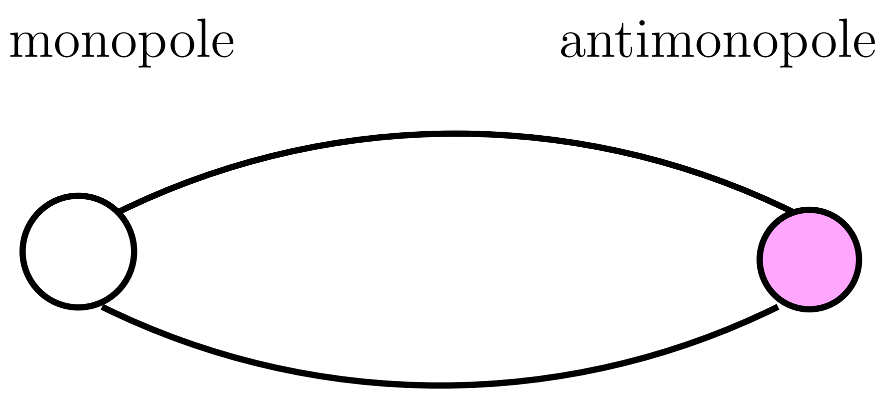
- non-Abelian strings which

still confine **monopoles**

(with *root*-like electric charges)

Instead of confinement of quarks at strong coupling (small ξ) we have :

- No confinement of color-electric charges. Quarks and gauge bosons are Higgs-screened
- Screened quarks and gauge bosons decay into monopole-antimonopole pairs at CMS. They form stringy mesons.



Now

$$N + 1 < N_f < \frac{3}{2}N$$

FI D-term $\xi_3 = 0$

$$\mathcal{W}_{[\mu]} = \sqrt{\frac{N}{2}}\,\frac{\mu_1}{2}\mathcal{A}^2 + \frac{\mu_2}{2}(\mathcal{A}^a)^2$$

At small μ this reduces to FI F-term $\xi = \xi_1 + i\xi_2 \sim \mu m$

All results above stay intact

Can we increase μ and decouple adjoint fields flowing to $\mathcal{N} = 1$ QCD?

2 Bulk theory with $N+1 < N_f < \frac{3}{2}N$

$\mathcal{N}=2$ QCD with gauge group $U(N) = SU(N)\times U(1)$ and N_f flavors of fundamental matter – quarks

+

Mass term for the adjoint chiral field $\mathcal{W}_{[\mu]}$

The bosonic part of the action

$$S = \int d^4x \left[\frac{1}{4g_2^2}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{4g_1^2}\left(F_{\mu\nu}\right)^2 + \frac{1}{g_2^2}\left|D_\mu a^a\right|^2 + \frac{1}{g_1^2}\left|\partial_\mu a\right|^2 \right.$$
$$\left. + \left|\nabla_\mu q^A\right|^2 + \left|\nabla_\mu \bar{\tilde{q}}^A\right|^2 + V(q^A, \tilde{q}_A, a^a, a)\right] .$$

Here

$$\nabla_\mu = \partial_\mu - \frac{i}{2}\,A_\mu - i A^a_\mu\,T^a\,.$$

The potential is

$$\begin{aligned}
V(q^A, \tilde{q}_A, a^a, a) &= \frac{g_2^2}{2}\left(\frac{i}{g_2^2}\, f^{abc}\bar{a}^b a^c + \bar{q}_A\, T^a q^A - \tilde{q}_A T^a\, \bar{\tilde{q}}^A\right)^2 \\
&+ \frac{g_1^2}{8}\left(\bar{q}_A q^A - \tilde{q}_A \bar{\tilde{q}}^A\right)^2 \\
&+ 2g_2^2 \left|\tilde{q}_A T^a q^A + \frac{1}{\sqrt{2}}\, \frac{\partial \mathcal{W}_\mu}{\partial a^a}\right|^2 + \frac{g_1^2}{2}\left|\tilde{q}_A q^A + \sqrt{2}\, \frac{\partial \mathcal{W}_\mu}{\partial a}\right|^2 \\
&+ \frac{1}{2}\sum_{A=1}^{N}\left\{\left|(a + \sqrt{2} m_A + 2T^a a^a) q^A\right|^2\right. \\
&+ \left.\left|(a + \sqrt{2} m_A + 2T^a a^a)\bar{\tilde{q}}^A\right|^2\right\} .
\end{aligned}$$

Large $\xi \sim \sqrt{\mu m}$

$r = N$ vacuum

Adjoint fields:

$$\langle \frac{1}{2}\, a + T^a\, a^a \rangle = -\frac{1}{\sqrt{2}} \left(\begin{array}{ccc} m_1 & \ldots & 0 \\ \ldots & \ldots & \ldots \\ 0 & \ldots & m_N \end{array} \right),$$

Quarks

$$\langle q^{kA} \rangle = \langle \bar{\tilde{q}}^{kA} \rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{cccccc} \sqrt{\xi_1} & \ldots & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & \sqrt{\xi_N} & 0 & \ldots & 0 \end{array} \right),$$

$$k = 1, ..., N\,, \qquad A = 1, ..., N_f\,,$$

$$\xi_P = 2 \left\{ \sqrt{\frac{2}{N}}\;\, \mu_1\, \hat{m} + \mu_2 (m_P - \hat{m}) \right\}, \qquad P = 1, ..., N$$

In the equal mass limit $\mathrm{U}(N)_{\text{gauge}} \times \mathrm{SU}(N_f)_{\text{flavor}}$ is broken down to

$$\mathrm{SU}(N)_{C+F} \times \mathrm{SU}(\tilde{N})_F \times \mathrm{U}(1)\,,$$

where $\tilde{N} = N_f - N$.

Instead of equal quark mass limit we consider "split limit"

$$m_P = m_{P'}, \qquad m_K = m_{K'}, \qquad m_P - m_K = \Delta m$$

$$P, P' = 1, ..., N \quad \text{and} \quad K, K' = N+1, ..., N_f$$

Then $r = N$ is isolated vacuum

Quarks and gauge fields fill following representations of the global group:

$$(1,1) \quad (N^2-1,1) \quad (\bar{N},\tilde{N}) \quad (N,\bar{\tilde{N}})$$

3 Non-Abelian duality at small μ

Small ξ

$$|\sqrt{\xi_P}| \ll \Lambda_{\mathcal{N}=2}\,, \qquad |m_A| \ll \Lambda_{\mathcal{N}=2}$$

Use Seiberg-Witten curve on the Coulomb branch at $\mu = 0$

We get theory of non-Abelian dyons and dual gauge fields with

$$U(\tilde{N}) \times U(1)^{(N-\tilde{N})}$$

gauge group and N_f non-Abelian dyons and $(N - \tilde{N})$ Abelian dyons.

The non-Abelian gauge factor $U(\tilde{N})$ is not broken by adjoint VEV's in the equal mass limit because this theory is not asymptotically free and stays at weak coupling

Argyres Plesser Seiberg:
$SU(\tilde{N}) \times U(1)^{(N-\tilde{N})}$ was identified at the root of the baryonic branch in SU(N) theory with massless quarks

Vacuum

$$\langle U(\tilde{N})\ adjoints\rangle = -\frac{1}{\sqrt{2}}\begin{pmatrix} m_{N+1} & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & m_{N_f} \end{pmatrix}$$

Dyons

$$\langle D^{lA}\rangle \; = \; \langle \bar{\tilde{D}}^{lA}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \dots & 0 & \sqrt{\xi_1} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & \dots & \sqrt{\xi_{\tilde{N}}} \end{pmatrix},$$

$$\langle D^{J}\rangle \; = \; \langle \bar{\tilde{D}}^{J}\rangle = \sqrt{\frac{\xi_J}{2}}, \qquad J = \tilde{N}+1, ..., N\,.$$

$$(1, ..., N)|_{I,II} \;\; \Rightarrow \;\; (N+1, ..., N_f, \tilde{N}+1, ..., N)|_{III}$$

$$\xi_P = 2\left\{\sqrt{\frac{2}{N}}\,\mu_1\,\hat{m} - \mu_2(\sqrt{2}e_P + \hat{m})\right\}$$

where e_P are the double roots of the Seiberg–Witten curve,

$$y^2 = \prod_{P=1}^{N}(x-\phi_P)^2 - 4\left(\frac{\Lambda}{\sqrt{2}}\right)^{N-\tilde{N}}\prod_{A=1}^{N_f}\left(x+\frac{m_A}{\sqrt{2}}\right)$$

At small masses the double roots of the Seiberg–Witten curve are

$$\sqrt{2}e_I = -m_{I+N}, \qquad \sqrt{2}e_J = \Lambda_{\mathcal{N}=2}\,\exp\left(\frac{2\pi i}{N-\tilde{N}}\,J\right)$$

for $\tilde{N} < N-1$, where

$$I = 1, ..., \tilde{N} \ \text{ and } \ J = \tilde{N}+1, ..., N\,.$$

The $\tilde{N}$ first roots are determined by the masses of the last $\tilde{N}$ quarks — a reflection of the fact that the non-Abelian sector of the dual theory is not asymptotically free and is at weak coupling in the domain.

In the "split mass" limit the global group is broken to

$$\mathrm{SU}(N)_F \times \mathrm{SU}(\tilde{N})_{C+F} \times \mathrm{U}(1)$$

Now dyons and dual gauge fields fill following representations of the global group:

$$\text{small } \xi : \qquad (1,1) \quad (1, \tilde{N}^2 - 1) \quad (\bar{N}, \tilde{N}) \quad (N, \bar{\tilde{N}})$$

Recall that quarks and gauge bosons of the original theory are in

$$\text{large } \xi : \qquad (1,1) \quad (N^2 - 1, 1) \quad (\bar{N}, \tilde{N}) \quad (N, \bar{\tilde{N}})$$

$(N^2 - 1)$ of $\mathrm{SU}(N)$ and $(\tilde{N}^2 - 1)$ of $\mathrm{SU}(\tilde{N})$

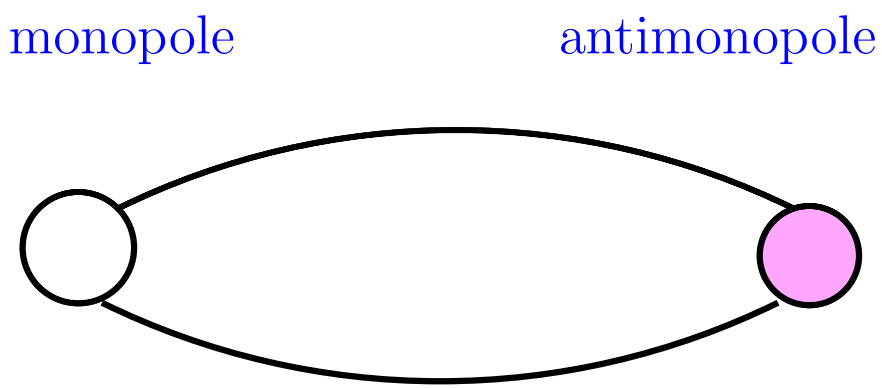
are different states

What is the physical nature of $(N^2 - 1)$ adjoints at small ξ?

Consider gauge boson or quark at large ξ and move to small ξ region

It decay to monopole and antimonopole (dyon with root-like electric charges) at CMS

At $\xi \neq 0$ monopoles are confined and cannot move apart



In the region of small ξ $(N^2 - 1)$ of SU(N) are stringy mesons formed by by pairs of monopoles and antimonopoles connected by two strings

They are Seiberg's neutral meson fields $M_P^{P'}$

4 Flowing to $\mathcal{N}=1$ QCD

Take

$$|\mu| \gg |m_A|, \qquad A = 1, ..., N_f$$

Step 1

$\tilde{N}$ non-Abelian dyons have VEV's $\sim \sqrt{\mu m}$

$(N - \tilde{N})$ Abelian dyons have VEV's $\sim \sqrt{\mu \Lambda_{\mathcal{N}=2}}$

Decouple Abelian dyons and $\mathrm{U}(1)^{N-\tilde{N}}$ gauge bosons at $m \ll \Lambda_{\mathcal{N}=2}$

Step 2

Decouple adjoint matter

Adjoint matter has mass $\sim \mu \gg \sqrt{\mu m}$

To stay at weak coupling we need

$$\sqrt{\mu m} \ll \tilde{\Lambda}\,, \qquad \tilde{\Lambda}^{N-2\tilde{N}} = \frac{\Lambda_{\mathcal{N}=2}^{N-\tilde{N}}}{\mu^{\tilde{N}}}$$

Effective superpotential of $\mathrm{U}(\tilde{N})$ gauge theory with N_f dyons

$$\mathcal{W} = -\frac{1}{2\mu_2}\left[(\tilde{D}_A D^B)(\tilde{D}_B D^A) - \frac{\alpha_D}{\tilde{N}}(\tilde{D}_A D^A)^2\right] + \left[m_A - \frac{\gamma\,(1+\frac{\tilde{N}}{N})}{1+\gamma\,\frac{\tilde{N}}{N}}\,m\right](\tilde{D}_A D^A),$$

where

$$\alpha_D = \frac{\gamma\,\frac{\tilde{N}}{N}}{1+\gamma\,\frac{\tilde{N}}{N}} \qquad \gamma = 1 - \sqrt{\frac{2}{N}}\,\frac{\mu_1}{\mu_2}$$

Vacuum

$$\langle D^{lA}\rangle = \langle \bar{\tilde{D}}^{lA}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \dots & 0 & \sqrt{\xi_1} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & \dots & \sqrt{\xi_{\tilde{N}}} \end{pmatrix},$$

where the first $\tilde{N}$ ξ's are of the order of μm

Spectrum in the "split mass" limit

- $(1, \tilde{N}^2 - 1)$ representation of the global group
  $\mathrm{SU}(N)_F \times \mathrm{SU}(\tilde{N})_{C+F} \times \mathrm{U}(1)$
  gauge fields: $\sim \tilde{g}\sqrt{\mu m_{\rm last}}$

  $2(\tilde{N}^2 - 1)$ dyons: $\sim m_{\rm last}$
- bifundamentals
  $4N\tilde{N}$ dyons: Δm
- $(N^2 - 1, 1)$ representation
  Stringy monopole-antimonopole mesons (= Seiberg's
  M-mesons): $\sim \sqrt{\mu m_{\rm last}}$

5 Relation to Seiberg's duality

Both dual theories have $U(\tilde{N})$ gauge group and N_f flavors of fundamental matter.

Identify

- Dyons D^{kA} $\equiv$ Seiberg's "dual quarks"
- Monopole-antimonopole stringy mesons $\equiv$ Seiberg's neutral M-meson

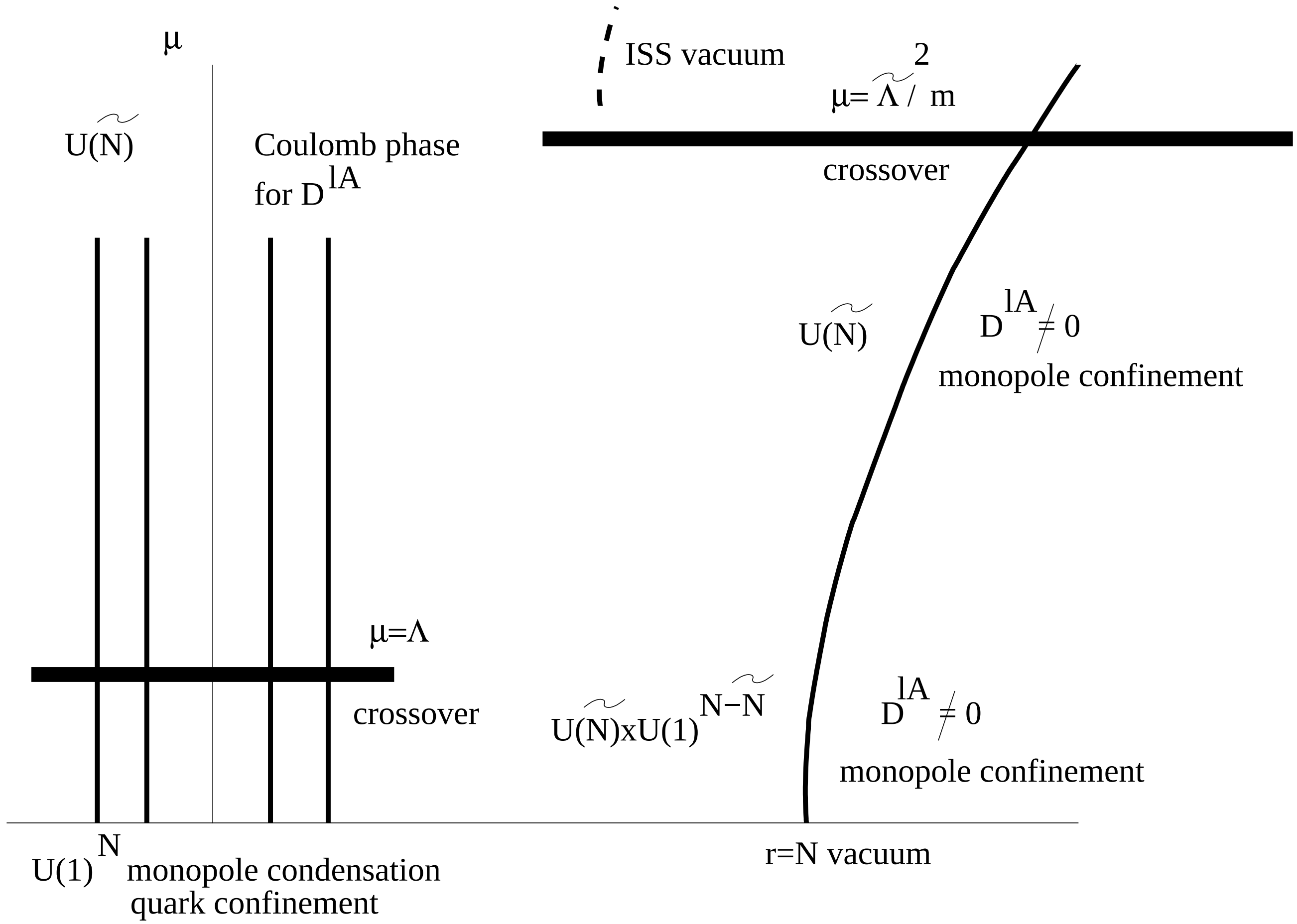
$$\mathcal{W}_{\text{Seiberg}} = \sqrt{2}\,(\tilde{D}_A D^B) M_B^A + \Lambda m_A\, M_A^A$$

The gluino condensation in the $U(\tilde{N})$ gauge theory with no matter induces the superpotential

$$\mathcal{W}^{\text{eff}}_{\text{Seiberg}} = \tilde{N}\,\Lambda^{\frac{2\tilde{N}-N}{\tilde{N}}}\,(\det M)^{\frac{1}{\tilde{N}}} + \Lambda m_A\, M_A^A\,.$$

Vacua:

$$\langle M \rangle \sim \Lambda^{\frac{N-\tilde{N}}{N}} m^{\frac{\tilde{N}}{N}} \qquad N \text{ "monopole" vacua}$$



μ
U(Ñ)
Coulomb phase
for D lA
μ=Λ
crossover
U(1) N
monopole condensation
quark confinement
ISS vacuum
μ= Λ̃ 2 / m
crossover
U(Ñ)
D lA ≠ 0
monopole confinement
U(Ñ)xU(1) N−Ñ
D lA ≠ 0
monopole confinement
r=N vacuum

6 Conclusions

- $\mathcal{N} = 2$ duality survives decoupling of the adjoint matter

  In $r = N$ quark vacuum of $\mathcal{N} = 1$ QCD

  - In both original and dual theories confined states are monopoles
  - Instead of confinement we have: Higgs screening + decay on CMS + magnetic string formation
  - Constituent quark = monopole

- Seiberg's duality describes N "monopole" vacua