Regge Trajectories in String and Large N Gauge Theories

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Outline

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This work done in collaboration with Francisco Rojas

Introduction

Infrared behaviors of planar open string theory and planar quantum field theory are identical. (Joel Scherk, 1971)

All physical phenomena in large N QCD should also appear in planar open string theory

There is a chance that in 't Hooft limit, confinement may be more tractable in string theory than in field theory:

e.g. for summing planar diagrams numerically, open string diagrams are better defined.

This talk: perturbative open string physics **or**
A closer look at results from 40 years ago

Focus: Open string Regge trajectory
$\alpha(t) = \alpha' t + 1 + \Sigma(t)$ with $\Sigma(t) = O(g^2)$

Projections: Even G-parity and S Massless Scalars

$$Z = \mathrm{Tr} w^{L_0 - 1/2} = \frac{P_+ - P_-}{2} \left(-\frac{\ln w}{2\pi} \right)^{4-D/2}$$

I. Nonabelian D-branes

$$P_\pm \equiv \frac{16\pi^4 w^{-1/2}}{\ln^4 w} (1 \mp w^{1/2})^{10-D-S} \frac{\prod_r (1 \pm w^r)^8}{\prod_n (1 - w^n)^8}$$

$n = 1, 2, \cdots$, $r = 1/2, 3/2, \cdots$

II. States even under $b_r^A, a_n^A \to -b_r^A, -a_n^A$ for $A > D + S$.

$$P'_\pm = \frac{16\pi^4 w^{-1/2}}{2 \ln^4 w} \left[\frac{\prod (1 \pm w^r)^8}{\prod (1 - w^n)^8} + \frac{\prod (1 \pm w^r)^{D-2+S} \prod (1 \mp w^r)^{10-D-S}}{\prod (1 - w^n)^{D-2+S} \prod (1 + w^n)^{10-D-S}} \right]$$

One Loop Correction

Open String Coupling:

$$\frac{g^2}{2\pi} = \alpha_s N = \frac{g_s^2 N}{4\pi}$$

One Loop M Gluon Neveu-Schwarz Even G-Parity Amplitude

$$\mathcal{M}_M = \frac{(g\sqrt{2\alpha'})^M}{2}(\mathcal{M}_M^+ - \mathcal{M}_M^-)$$

$$\mathcal{M}_M^\pm \quad = \quad 2^M \left(\frac{1}{8\pi^2\alpha'}\right)^{D/2} \int \prod_{k=2}^{M} d\theta_k \int_0^1 \frac{dq}{q} \left(\frac{-\pi}{\ln q}\right)^{(10-D)/2} P_\pm$$
$$\prod_{l<m} \left[\psi(\theta_m - \theta_l, q)\right]^{2\alpha' k_l \cdot k_m} \langle \hat{\mathcal{P}}_1 \hat{\mathcal{P}}_2 \cdots \hat{\mathcal{P}}_M \rangle^\pm$$

$+$: Even and odd G enter loop with same sign

$-$: Even and odd G enter loop with opposite sign

$$
\begin{aligned}
\psi(\theta, q) &= \sin\theta \prod_n \frac{(1-q^{2n}e^{2i\theta})(1-q^{2n}e^{-2i\theta})}{(1-q^{2n})^2} \\
\hat{\mathcal{P}} &= \epsilon\cdot\mathcal{P} + \sqrt{2\alpha'}k\cdot H\epsilon\cdot H,
\end{aligned}
$$

Jacobi Transform: $\ln q = 2\pi^2/\ln w$ under which

$$
\begin{aligned}
P_+ &\rightarrow q^{-1}(1-w^{1/2})^{10-D-S}\frac{\prod_r(1+q^{2r})^8}{\prod_n(1-q^{2n})^8} \\
P_- &\rightarrow 2^4(1+w^{1/2})^{10-D-S}\frac{\prod_n(1+q^{2n})^8}{\prod_n(1-q^{2n})^8}
\end{aligned}
$$

$$
0=\theta_1<\theta_2<\cdots<\theta_N<\pi, \qquad \theta_{ji}=\theta_j-\theta_i
$$

where the average $\langle \cdots \rangle$ is evaluated with contractions:

$$\begin{aligned}
\langle \mathcal{P}_l \rangle &= \sqrt{2\alpha'} \sum_i k_i \left[\frac{1}{2} \cot \theta_{il} + \sum_{n=1}^{\infty} \frac{2q^{2n}}{1-q^{2n}} \sin 2n\theta_{il} \right] \\
\langle \mathcal{P}_i \mathcal{P}_l \rangle - \langle \mathcal{P}_i \rangle \langle \mathcal{P}_l \rangle &= \frac{1}{4} \csc^2 \theta_{il} - \sum_{n=1}^{\infty} n \frac{2q^{2n}}{1-q^{2n}} \cos 2n\theta_{il} \\
\langle H_i H_j \rangle^+ &\equiv \chi_+(\theta_{ji}) = \frac{1}{2} \theta_2(0)\theta_4(0) \frac{\theta_3(\theta_{ji})}{\theta_1(\theta_{ji})} \\
&= \frac{1}{2 \sin \theta_{ji}} - 2 \sum_r \frac{q^{2r} \sin 2r\theta_{ji}}{1+q^{2r}} \\
\langle H_i H_j \rangle^- &\equiv \chi_-(\theta_{ji}) = \frac{1}{2} \theta_3(0)\theta_4(0) \frac{\theta_2(\theta_{ji})}{\theta_1(\theta_{ji})} \\
&= \frac{\cos \theta_{ji}}{2 \sin \theta_{ji}} - 2 \sum_n \frac{q^{2n} \sin 2n\theta_{ji}}{1+q^{2n}} \ .
\end{aligned}$$

Some comments:

- $\mathcal{M}^+ - \mathcal{M}^-$ difference projects out odd G-parity states.
- Open strings end on N Dp-branes for $p = D{-}1$. $\left(\frac{-\pi}{\ln q}\right)^{(10-D)/2}$
- S massless scalars: $(1 \mp w^{1/2})^{10-D-S}$
- q^{-2} UV divergence absorbable in g (Neveu-Scherk)
- For $4 < D < 8$ no other divergences!

Coupling Renomalization

GNS (Goddard,Neveu-Scherk): temporarily suspend momentum conservation $\sum_i k_i = p$, continue to $p = 0$ at end.

Neveu-Scherk counterterm $\rightarrow$ tree

If $D < 8$, no subleading ultraviolet divergences

$D > 4$: no infrared divergences:

θ integrals at $p = 0$: with $p \neq 0$ subtract and add Neveu-Scherk counterterm: $I(p) = (I(p) - C(p))_{p=0} + C(p)$.

Open String Regge Trajectory

$$(\beta(t)+\delta\beta)s^{\alpha(t)+\delta\alpha} \approx \beta s^{\alpha} + \delta\alpha\beta s^{\alpha}\ln s + \delta\beta s^{\alpha}$$

Compare loop at $s \to -\infty$ to Tree

$$\mathcal{M}^{\text{Tree}} \quad \sim \quad -2g^2\epsilon_2\cdot\epsilon_3\epsilon_1\cdot\epsilon_4\Gamma(-\alpha' t)(-\alpha' s)^{1+\alpha' t}$$

$$\begin{aligned}
\Sigma^{\pm} \quad &= \quad -\frac{8g^2\alpha'^{2-D/2}}{(8\pi^2)^{D/2}}\int_0^1 \frac{dq}{q}\left(\frac{-\pi}{\ln q}\right)^{(10-D)/2} P_{\pm} \\
&\quad \int_0^{\pi} d\theta\Bigg((-\psi^2(\theta)[\ln\psi]'')^{\alpha' t}\frac{\alpha' t}{[\ln\psi]''}X^{\pm} \\
&\quad\quad -\frac{1}{4}(-\psi^2(\theta)[\ln\psi]'')^{\alpha' t}[-\ln\psi]'' + \frac{1}{4\sin^2\theta}\Bigg) \\
X^{\pm} \quad &= \quad \frac{1}{4}[-\ln\psi]''^2 + \chi_{\pm}(\theta)\chi''_{\pm}(\theta) - \chi'^2_{\pm}(\theta) - 2\chi^2_{\pm}(\theta)[-\ln\psi]''
\end{aligned}$$

Next use the identity

$$\int_0^\pi d\theta \left([\ln \psi]'' + \frac{1}{\sin^2 \theta} \right) = \left(\sum_{n=1}^{\infty} \frac{4q^{2n} \sin 2\theta}{1 - 2q^{2n} \cos 2\theta + q^{4n}} \right) \Bigg|_0^\pi = 0$$

to replace each $\sin^{-2}\theta$ term by $[-\ln \psi]''$:

$$\Sigma^\pm = -\frac{8g^2 \alpha'^{2-D/2}}{(8\pi^2)^{D/2}} \int_0^1 \frac{dq}{q} \left(\frac{-\pi}{\ln q} \right)^{(10-D)/2} P_\pm$$

$$\int_0^\pi d\theta \left((-\psi^2(\theta)[\ln \psi]'')^{\alpha' t} \frac{\alpha' t}{[\ln \psi]''} X^\pm - \frac{1}{4} [(-\psi^2(\theta)[\ln \psi]'')^{\alpha' t} - 1][-\ln \psi]'' \right)$$

$\Sigma^\pm$ formally vanish as $t \to 0$: the gluon remains massless!

$$\alpha(t) = 1 + \alpha' t + \frac{1}{2}(\Sigma^+ - \Sigma^-) \equiv 1 + \alpha' t + \Sigma(t)$$

Contrary to appearances $X^{\pm}$ are θ independent

$$
\begin{aligned}
X^{+} &= \frac{1}{4}\theta_4(0)^4\theta_3(0)^4 - \frac{\boldsymbol{E}}{\pi}\theta_4(0)^4\theta_3(0)^2 + \frac{\boldsymbol{E}^2}{\pi^2}\theta_3(0)^4 = 4q + O(q^2) \\
X^{-} &= -\frac{1}{4}\theta_4(0)^4\theta_3(0)^4 + \frac{\boldsymbol{E}^2}{\pi^2}\theta_3(0)^4 = O(q^2) \\
\boldsymbol{E} &= \frac{\pi}{6\theta_3(0)^2}\left(\theta_3(0)^4 + \theta_4(0)^4 - \frac{\theta_1'''(0)}{\theta_1'(0)}\right) = \frac{\pi}{2} - 2\pi q + O(q^2)
\end{aligned}
$$

small q behavior shown on extreme right

$$
\frac{\theta_1'''(0)}{\theta_1'(0)} = -1 + 24\sum_{n=1}^{\infty}\frac{q^{2n}}{(1-q^{2n})^2}
$$

Small t and the Field Theory Limit

Small t probes the IR (hence field theory) regime.

Integer powers t^n, $n > 0$ in $\Sigma(t)$ depend on whole range of q.

Nonanalytic behavior at $t = 0$ comes from $q \sim 1$ or $w \sim 0$.

Change $q^{-1}dq \to 2\pi^2(w \ln^2 w)^{-1}dw$ to get $w \sim 0$ behavior:

$$\left(\frac{-\pi}{\ln q}\right)^{5-D/2} \frac{dq}{q}(P_+X^+ - P_-X^-) \quad \sim$$

$$\frac{dw}{w}\left(\frac{-2\pi}{\ln w}\right)^{-3+D/2}\left[4 + \frac{D+S-2}{\pi^2}\left(\frac{2\pi}{\ln w}\right)^2\right]$$

Small w contribution

$$\Sigma \approx -\frac{8g^2\alpha'^{2-D/2}}{(8\pi^2)^{D/2}}\int_0^\delta \frac{dw}{w}\left(\frac{-2\pi}{\ln w}\right)^{-3+D/2}\int_0^\pi d\theta$$
$$\left((-\psi^2(\theta)[\ln\psi]'')^{\alpha' t}\frac{\alpha' t}{[\ln\psi]''}\left[2+\frac{D+S-2}{2\pi^2}\left(\frac{2\pi}{\ln w}\right)^2\right]\right.$$
$$\left.-\frac{D+S-2}{8}\left(\frac{2\pi}{\ln w}\right)^4[(-\psi^2(\theta)[\ln\psi]'')^{\alpha' t}-1][-\ln\psi]''\right).$$

New variables: $w = e^{-T}$ and $\theta = \pi x$. As $T > e^{1/\delta}$

$$-\psi^2(\theta)[\ln\psi]'' \sim e^{x(1-x)T}(1-e^{-xT})^2(1-e^{-(1-x)T})^2$$
$$\left[\frac{1}{T}+\frac{e^{-xT}}{(1-e^{-xT})^2}+\frac{e^{-(1-x)T}}{(1-e^{-(1-x)T})^2}\right]$$

Nonanalyticity in t as $t \to 0$ from region $1 \ll \Lambda < x(1-x)T < \infty$:

$$\Sigma \approx -\frac{8g^2\alpha'^{2-D/2}}{(4\pi)^{D/2}}\left(\frac{-\alpha' t}{4}\frac{\Gamma(-2+\alpha' t+D/2)^2}{\Gamma(D-4+2\alpha' t)}\int_\Lambda^\infty du u^{2-\alpha' t-D/2}e^{-u\alpha'|t|}\right.$$
$$\left.-\frac{D+S-2}{4}\int_\Lambda^\infty du u^{-D/2}\left[\frac{\Gamma(\alpha' t+D/2)^2}{\Gamma(D+2\alpha' t)}u^{-\alpha' t}e^{-u\alpha'|t|}-\frac{\Gamma(D/2)^2}{\Gamma(D)}\right]\right)$$

If $p+1 \leq 0$, a few integration by parts shows that for small ξ

$$\int_\Lambda^\infty du u^p e^{-u\xi} \sim \xi^{-p-1}\Gamma(p+1)+\mathcal{P}(\xi,\Lambda)e^{-\xi\Lambda}$$

where $\mathcal{P}$ is a polynomial in ξ. In Σ this term is analytic at $t=0$ as is the contribution for w away from 0. The first term dominates only if $p+1>0$ ($D<6$ for 1st integral). 2nd integral is down by a factor of t.

$$\Sigma \sim -\frac{2g^2}{(4\pi)^{D/2}}\frac{\Gamma(3-D/2)\Gamma(-2+D/2)^2}{\Gamma(D-4)}(-t)^{-2+D/2}+O(\alpha' t\ln(-\alpha' t))$$

The $O(\alpha' t \ln(-\alpha' t))$ term is to remind that the displayed first term is dominant only for $D < 6$. IR divergences are absent for $D > 4$.

Reggeized gluon in D-dimensional gauge theory has a trajectory $\alpha(t) = 1 + C(-t/\mu^2)^{(D-4)/2}$.

As $D \to 4$ from above, a $\ln(-\alpha' t)$ dependence remains:

$$\Sigma_{D\to 4} \sim -\frac{g^2}{4\pi^2}\left[\frac{2}{D-4} + \ln(-\alpha' t)\right]$$

agreeing exactly with one loop gauge theory calculations.

Large t Behavior

Return to q variables since large t controlled by small q

Small q part of Σ:

$$\Sigma(t) \simeq -\frac{4g^2\alpha'^{2-D/2}}{(8\pi^2)^{D/2}}\pi^{25-5D/2-2S}\alpha' t\int_0^\delta \frac{dq}{q}\left(\frac{-1}{\ln q}\right)^{25-5D/2-2S}$$
$$\int_0^\pi d\theta\, e^{-\alpha'|t|16q^2\sin^4\theta}\sin^2\theta$$
$$+\frac{g^2\alpha'^{2-D/2}}{(8\pi^2)^{D/2}}\pi^{25-5D/2-2S}\int_0^\delta \frac{dq}{q}\left(\frac{-1}{\ln q}\right)^{25-5D/2-2S}$$
$$\int_0^\pi d\theta\left[e^{-\alpha'|t|16q^2\sin^4\theta}-1\right]\csc^2\theta$$

First integral larger than the second by a factor of $\sqrt{t}$

Consider the integral

$$I \equiv \int_0^{\delta} \frac{dq}{q} \left(\frac{-1}{\ln q} \right)^p \int_0^{\pi} d\theta \, \sin^2 \theta e^{-16q^2|\alpha' t| \sin^4 \theta}$$

Large $|\alpha' t|$ limits integation range to $-\ln q^2 > \ln(16|\alpha' t| \sin^4 \theta)$:

$$I \quad \sim \quad \frac{\pi}{2} \left(\frac{1}{\ln |\alpha' t|} \right)^{p-1} \frac{2^{p-1}}{p-1}$$

Applying this to Σ with $p = 25 - 5D/2 - 2S$ gives

$$\Sigma(t) \; \simeq \; \frac{2\alpha'^2 g^2}{(2\alpha')^{D/2}} \frac{(2\pi)^{2-D} \alpha' t}{24 - 5D/2 - 2S} \left(\frac{2\pi}{\ln |\alpha' t|} \right)^{24-5D/2-2S}$$

as $|\alpha' t| \to \infty$.

Summary of Results

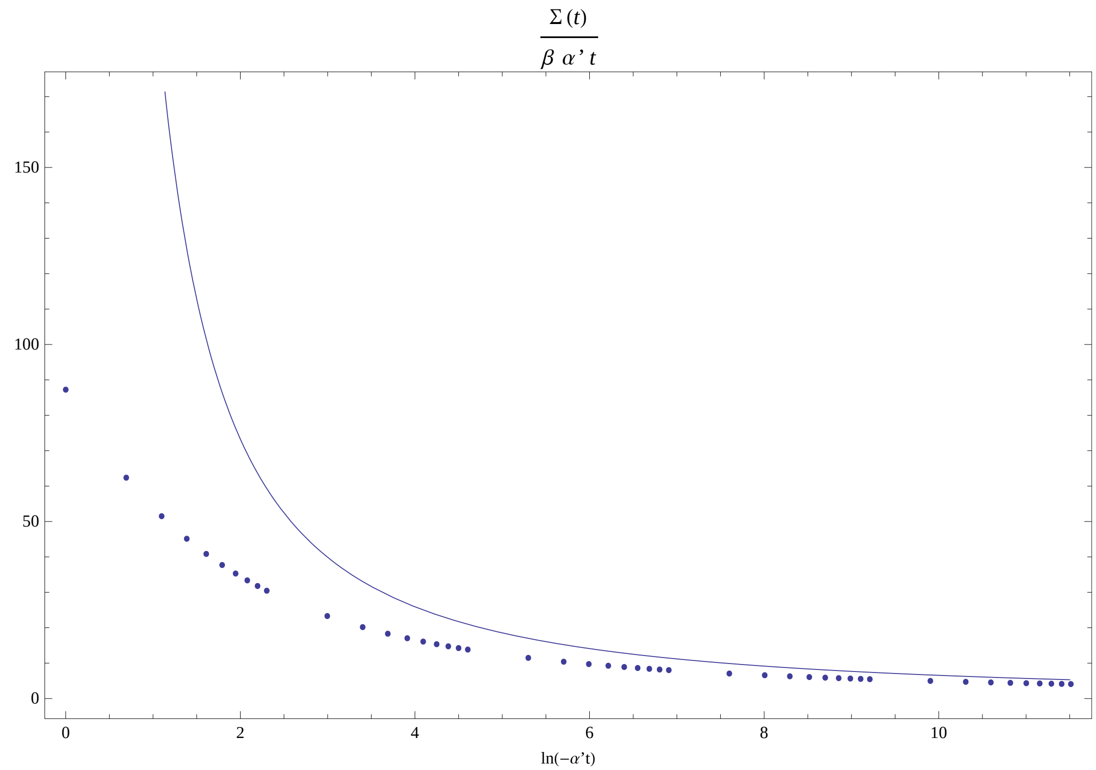
$$\alpha(t) = 1 + \alpha' t + \Sigma(t)$$

$$\Sigma(t) \sim -2g^2 \frac{\Gamma(3 - D/2)\Gamma(-2 + D/2)^2}{\Gamma(D - 4)(4\pi)^{D/2}} (-t)^{-2+D/2}, \qquad t \to 0$$

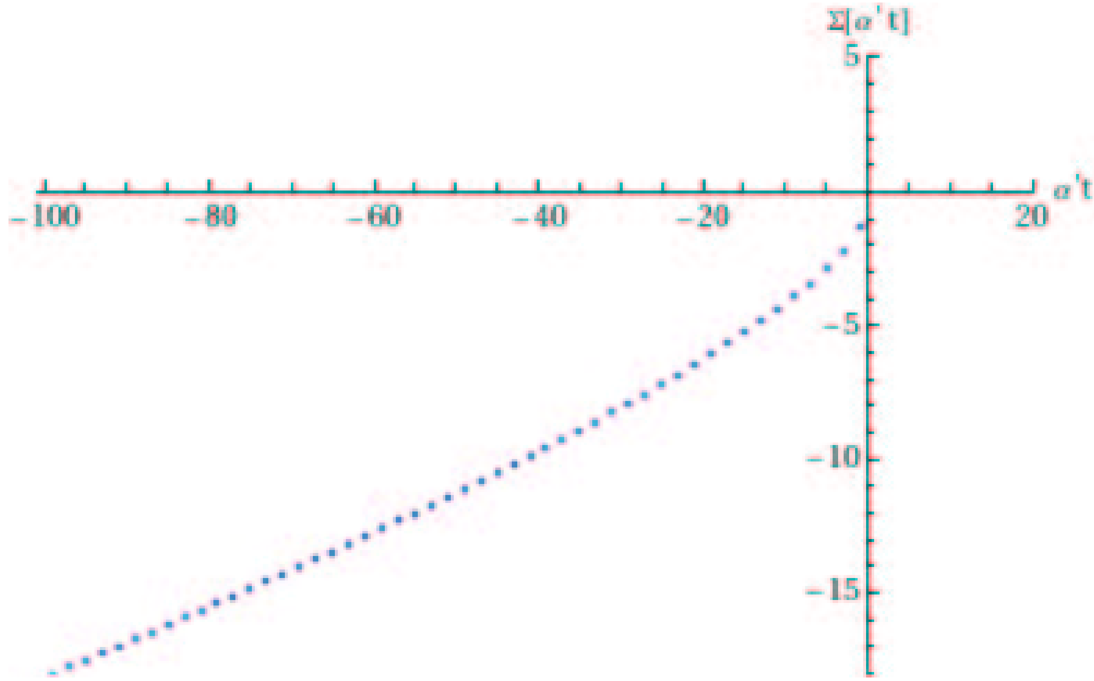
$$\Sigma(t) \sim \frac{2g^2 \alpha'^2}{(2\alpha')^{D/2}} \frac{(2\pi)^{2-D} \alpha' t}{24 - 5D/2 - 2S} \left(\frac{2\pi}{\ln|\alpha' t|} \right)^{24-5D/2-2S}, \quad t \to -\infty$$

Conclusion: Σ v.s. t Graphs $(D = 5)$

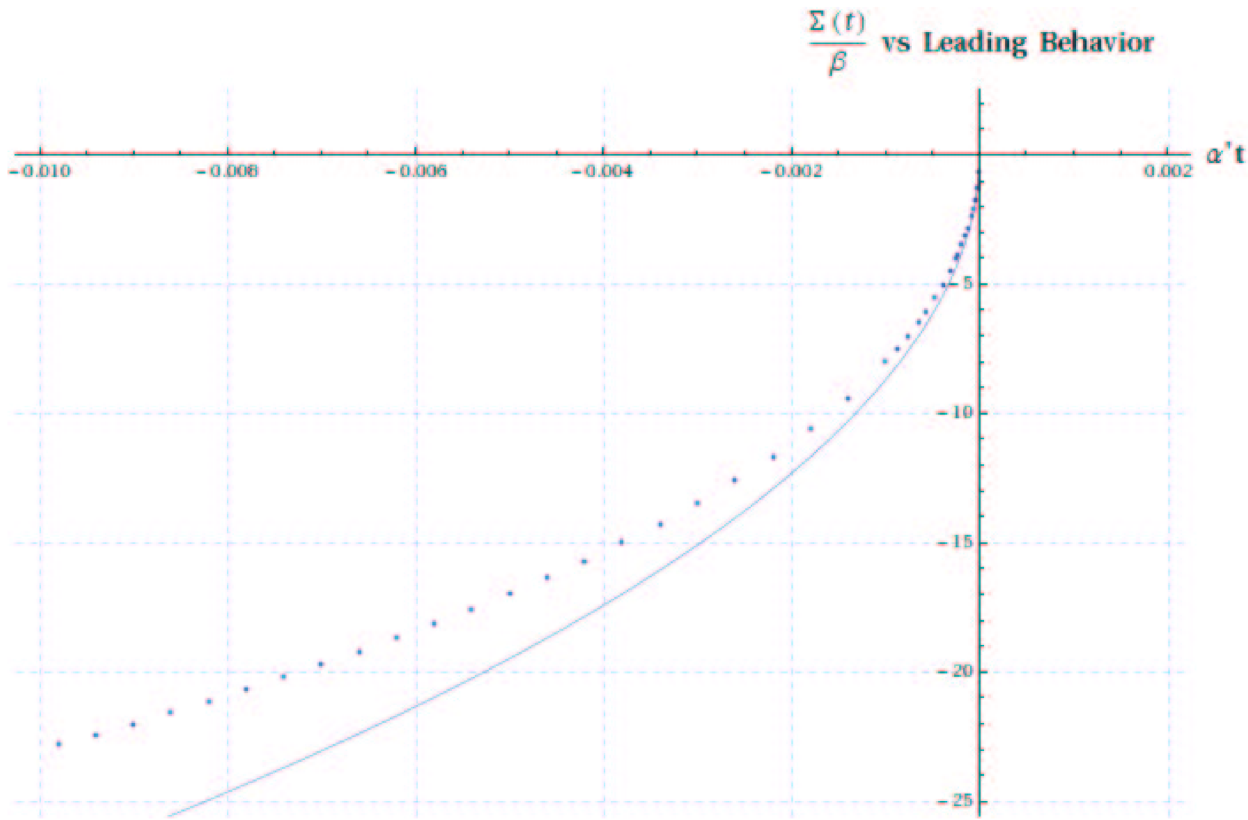
Large $\ln(-\alpha' t)$:



$$\beta = 4g^2 \alpha'^{2-D/2}/(8\pi)^{D/2}$$


Σ[α't]
5
α't
-100
-80
-60
-40
-20
20
-5
-10
-15

Blow up of small t region ($D = 5$ gauge theory)



References

[1] J. Scherk, Nucl. Phys. **B31** (1971) 222-234.

[2] A. Neveu and J. Scherk, Nucl. Phys. B **36** (1972) 155.

[3] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998), hep-th/9711200.

[4] G. 't Hooft, *Nucl. Phys.* **B72** (1974) 461.

[5] K. Bardakci and C. B. Thorn, Nucl. Phys. **B626** (2002) 287, hep-th/0110301.

[6] C. B. Thorn, Nucl. Phys. B **637** (2002) 272 [arXiv:hep-th/0203167].

[7] R. Giles and C. B. Thorn, *Phys. Rev.* **D16** (1977) 366.

[8] D. Chakrabarti, J. Qiu and C. B. Thorn, Phys. Rev. D **72** (2005) 065022, arXiv:hep-th/0507280.

[9] D. Chakrabarti, J. Qiu and C. B. Thorn, Phys. Rev. D **74** (2006) 045018 [Erratum-ibid. D **76** (2007) 089901] [arXiv:hep-th/0602026].

[10] S. G. Naculich and H. J. Schnitzer, Nucl. Phys. B **794** (2008) 189 [arXiv:0708.3069 [hep-th]]; R. C. Brower, H. Nastase, H. J. Schnitzer and C. I. Tan, Nucl. Phys. B **814** (2009) 293 [arXiv:0801.3891 [hep-th]].

[11] A. Neveu and J. H. Schwarz, Nucl. Phys. B **31** (1971) 86.

[12] A. Neveu, J. H. Schwarz and C. B. Thorn, Phys. Lett. B **35** (1971) 529.

[13] P. Goddard and R. E. Waltz, Nucl. Phys. B **34** (1971) 99.

[14] R. C. Brower and C. B. Thorn, Nucl. Phys. B **31** (1971) 163.

[15] P. Goddard and C. B. Thorn, Phys. Lett. B **40** (1972) 235.

[16] S. Mandelstam, private communication, April 1971.

[17] K. Kikkawa, B. Sakita, M. A. Virasoro, Phys. Rev. **184** (1969) 1701-1713.

[18] H. J. Otto, V. N. Pervushin, D. Ebert, Theor. Math. Phys. **35** (1978) 308.

[19] C. B. Thorn, Phys. Rev. **D78** (2008) 106008. [arXiv:0809.1085 [hep-th]].

[20] R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. B **298** (1988) 109.

[21] C. B. Thorn, Phys. Rev. **D78** (2008) 085022. [arXiv:0808.0458 [hep-th]].

[22] A. Neveu and J. Scherk, Nucl. Phys. B **36** (1972) 317.

[23] P. Goddard, Nuovo Cim. A **4** (1971) 349.

[24] A. Erdélyi, W. Magnus, F. Oberhettunger, F. G. Tricomi, *Higher Transcendental Functions*, Bateman Manuscript Project, Volume 2, Chapter XIII, McGraw-Hill (1953).

[25] Z. Kunszt, A. Signer and Z. Trocsanyi, Nucl. Phys. B **411** (1994) 397 [arXiv:hep-ph/9305239]; For earlier calculations, see R. K. Ellis and J. C. Sexton, Nucl. Phys. B **269**, 445 (1986); Z. Bern and D. A. Kosower, Nucl. Phys. B **379**, 451 (1992).

[26] C. B. Thorn, Phys. Rev. **D82** (2010) 125021. [arXiv:1010.5998 [hep-th]].