

Magnetic bions, multiple adjoints, and Seiberg-Witten theory

Erich Poppitz  Toronto

work with Mithat Ünsal SLAC/Stanford (in progress)

(will mention also some recent work with Mohamed Anber, Toronto, 1105.0940)

ABSTRACT:

(as submitted to CAQCD organizers)

We first study properties of magnetic bions in multi-adjoint-fermion theories on $R^3 \times S^1$, notably the flavor and radius dependence of the mass gap.

[Anber, EP, 2011]

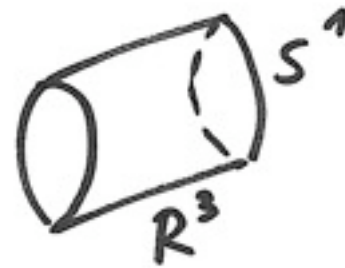
We will also argue that "Polyakov-like" magnetic bion confinement can be continuously related to the four-dimensional confinement mechanism in Seiberg-Witten theory by appropriately resumming the contributions of the 3d monopole-instanton tower.

[Unsal, EP, 2011]

But, first I really need to tell (remind) you what it's all about.

The theme of my talk is about inferring properties of infinite-volume theory by studying **(arbitrarily)** small-volume dynamics.

The small volume may be



← most of this talk

or



of characteristic size “L”

“Eguchi-Kawai” ... “large-N volume independence” ...

long history of stumbling (1980-2008) that I won't review

some recent (2008+) excitement:

remedy by Unsal, Yaffe 2008

EK reduction valid to arbitrarily small L (single-site) if either:

periodic adjoint fermions

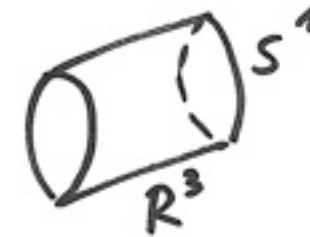
(more than one Weyl) - no center breaking, so EK reduction holds at all L



THIS TALK:

double-trace deformations
deform measure to prevent center breaking
at infinite- N , deformation does not affect
(connected correlators of “untwisted”) **observables**

theoretical studies



Unsal;
Unsal-Yaffe;
Shifman-Unsal;
Unsal-EP 2007-

used for current **lattice studies** of
“minimal walking technicolor”

is 4 ...3,5... Weyl adjoint theory
conformal or not?

small- $L(=1)$ large- N (~ 20 or more...) simulations (2009-)
Hietanen-Narayanan; Bringoltz-Sharpe; Catterall et al

small- N large- L simulations (2007-)
Catterall et al; del Debbio et al; Hietanen et al...

fix- N , take L -small: semiclassical studies of
confinement due to novel strange “oddball”
(nonselfdual) topological excitations, whose
nature depends on fermion content

- for vectorlike or chiral theories,
with or without supersymmetry
- a complementary regime to that
of volume independence, which
requires infinite N - a (calculable!)
shadow of the 4d “real thing”.

For this talk only consider 4d SU(2) theories
with N_W = multiple adjoints Weyl fermions

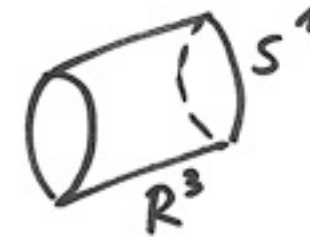
“applications”:

$N_W=1$ is $N=1$ SUSY YM \sim **Seiberg-Witten theory**
with soft-breaking mass

$N_W=4$
- “minimal walking technicolor”
- happens to be $N=4$ SYM
without the scalars

$N_W=5.5$ asymptotic freedom lost

theoretical studies



Unsal;
Unsal-Yaffe;
Shifman-Unsal;
Unsal-EP 2007-

fix- N , take L -small: semiclassical studies of confinement due to novel strange “oddball” (nonselfdual) topological excitations, whose nature depends on fermion content

In 4d theories with periodic adjoint fermions, for small- L , dynamics is semiclassically calculable (including confinement).

Polyakov’s 3d mechanism of confinement by “Debye screening” in the monopole-anti-monopole plasma extends to (locally) 4d theories.

However, the “Debye screening” is now due to composite objects, the “magnetic bions” of the title.

“Magnetic bions, multiple-adjoints, and Seiberg-Witten theory”

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with $N_W =$ multiple adjoints Weyl fermions

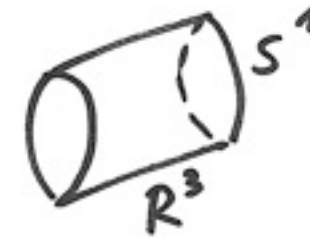
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In 4d theories with periodic adjoint fermions, for small-L, confining dynamics is semiclassically calculable.

$$S^1 : X^4 \sim X^4 + L$$

A_4 is now an adjoint 3d scalar Higgs field $\partial_4 + A_4 \longrightarrow \frac{2\pi n}{L} + A_4$

but it is a bit unusual - a compact Higgs field:

$$\langle A_4 \rangle \sim \langle A_4 \rangle + \frac{2\pi}{L}$$

such shifts of A_4 vev absorbed into shift of KK number "n" $A_4 \rightarrow A_4 + \partial_4 \left(\frac{2\pi x_4}{L} \right)$

thus, natural scale of "Higgs vev" is

$$\langle A_4 \rangle \sim \frac{\pi}{L} \text{ leading to}$$

"large" gauge transform

$$SU(2) \xrightarrow{\frac{1}{L}} U(1)$$

hence, semiclassical if $L \ll$ inverse strong scale

exactly this happens in theories with more than one periodic Weyl adjoints

follows from two things, without calculation:

1.) existence of deconfinement transition in pure YM and 2.) supersymmetry

in pure YM, at small L (high-T), V_{eff} min at $A_4=0$ & max at π/L (Gross, Pisarsky, Yaffe 1980s)

in SUSY $V_{\text{eff}}=0$, so one Weyl fermion contributes the negative of gauge boson V_{eff}

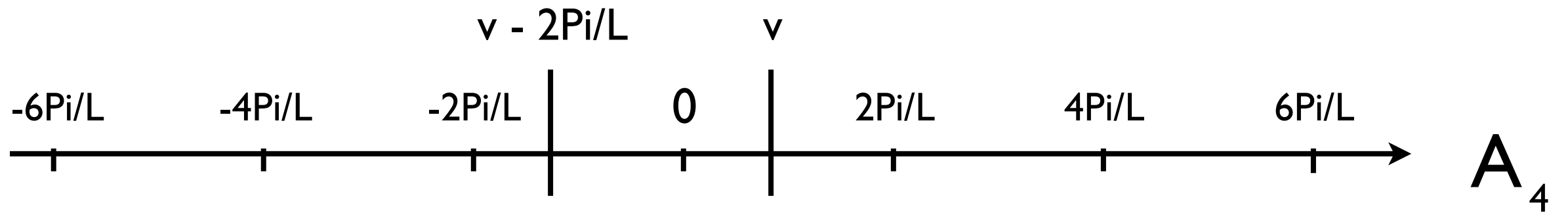
Q.E.D.

Polyakov's 3d mechanism of confinement by "Debye screening" in the monopole-anti-monopole plasma extends to (locally) 4d theories. However, the "Debye screening" is now due to composite objects, the "magnetic bions" of the title.

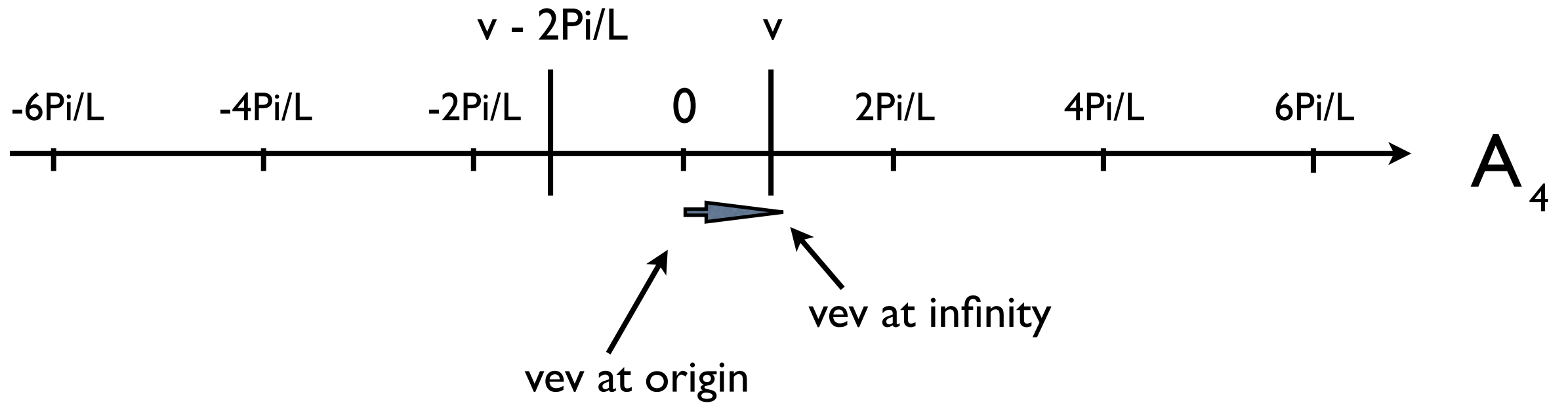
since $SU(2)$ broken to $U(1)$ at scale $1/L$

there are monopole-instanton solutions of finite Euclidean action, constructed as follows:

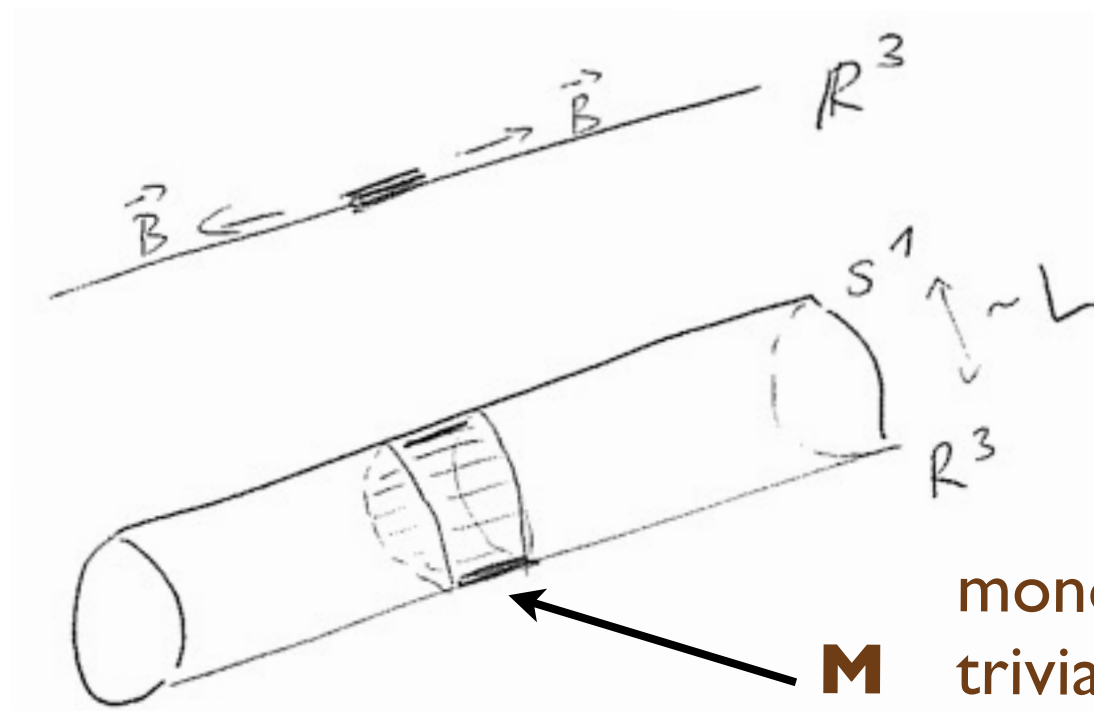
gauge equivalent vevs



gauge equivalent vevs

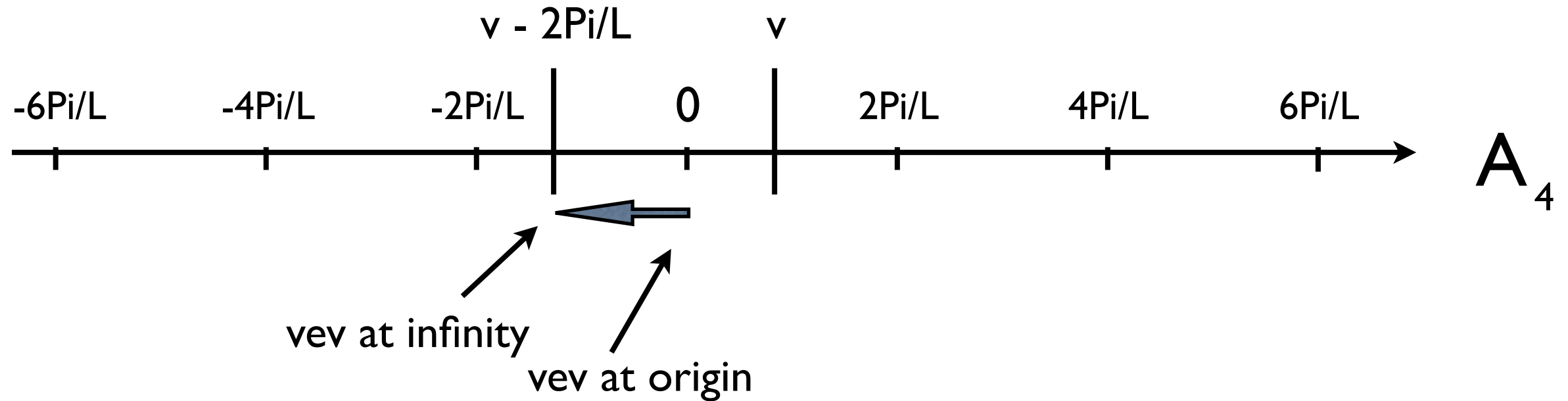


monopole-instanton of action $\sim v/g_3^2$



monopole trivially embedded in 4d

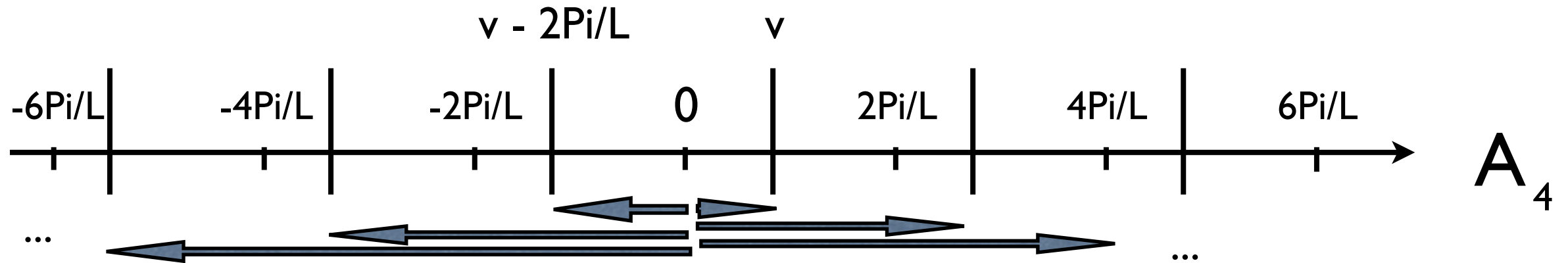
gauge equivalent vevs



monopole-instanton of action $\sim |2\pi/L - v|/g^2_3$

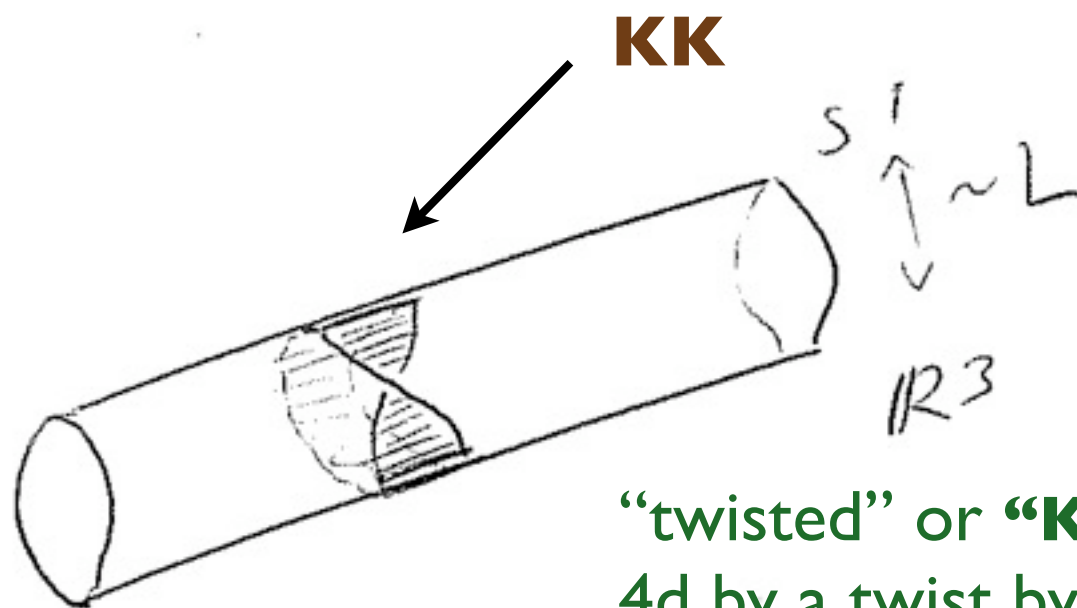
- use a large gauge transformation to make vev at infinity = v
- action does not change
- x_4 -dependence is induced, hence called "twisted"

gauge equivalent vevs

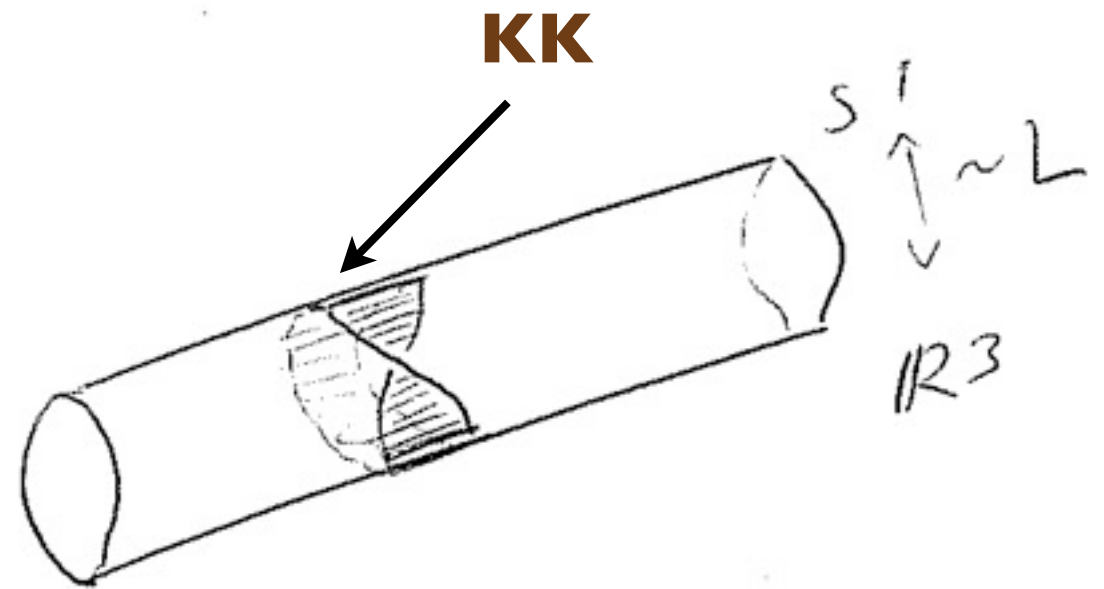
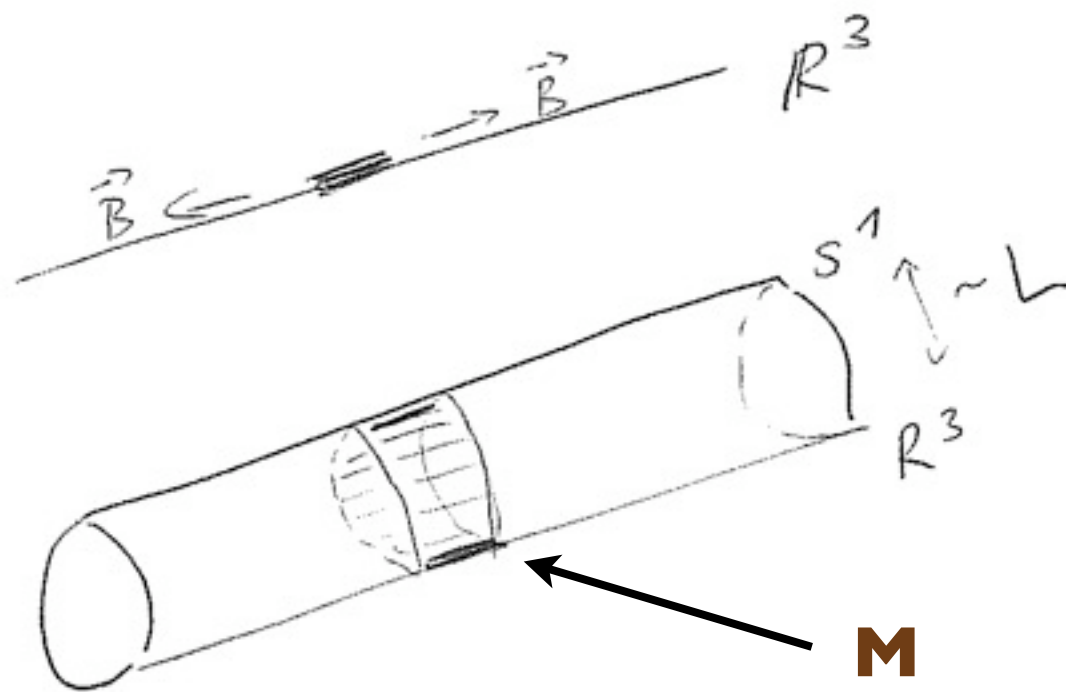


monopole-instanton tower; action $\sim |2k\pi/L - v|/g^2$

the lowest action member of the tower can be pictured like this (as opposed to the no-twist):

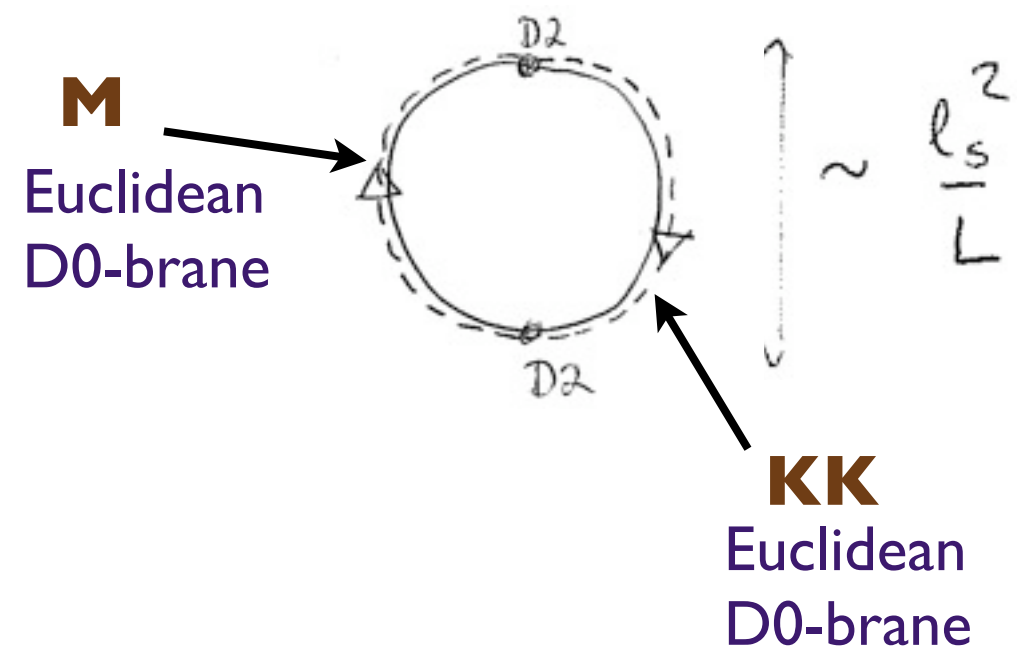


“twisted” or “**Kaluza-Klein**”: monopole embedded in 4d by a twist by a “gauge transformation” periodic up to center - in 3d limit not there! (infinite action)



K. Lee, P. Yi, 1997

	magnetic	topological	suppression
M	+1	1/2	e^{-S_0}
KK	-1	1/2	e^{-S_0}
B PST	0	1	e^{-2S_0}



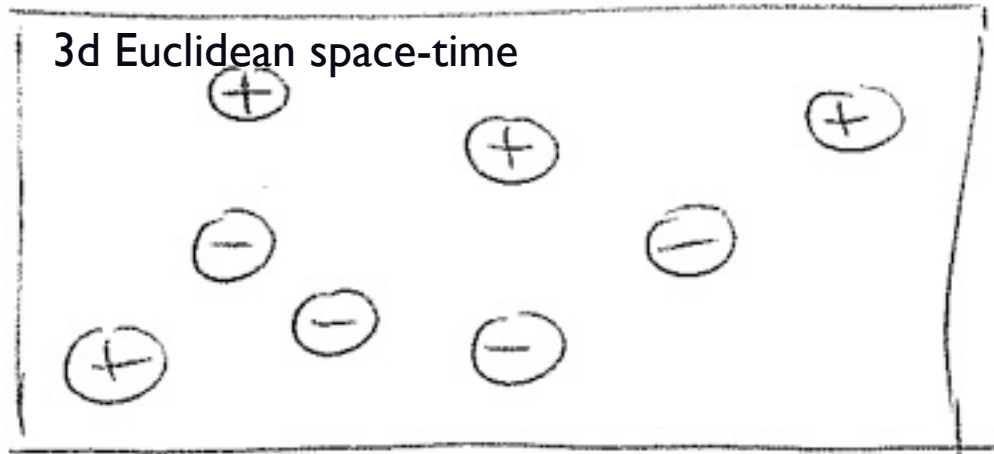
M & KK have 't Hooft suppression given by:

$$e^{-S_0} = e^{-\frac{4\pi v}{g_3^2}} = e^{-\frac{4\pi^2}{L g_3^2}} = e^{-\frac{4\pi^2}{g_4^2(L)}}$$

center-symmetric vev coupling matching

in SU(N), 1/N-th of the 't Hooft suppression factor

in a purely bosonic theory, vacuum would be a dilute M-M* plasma - but interacting, unlike instanton gas in 4d (in say, electroweak theory)



physics is that of Debye screening

analogy:

electric fields are screened in a charged plasma ("Debye mass for photon")
 in the monopole-antimonopole plasma, the dual photon (3d photon ~ scalar) obtains mass from screening of magnetic field:

$$\mathcal{L}_{\text{eff}} = g_3^2 (\partial\sigma)^2 + (\#) v^3 e^{-S_0} (e^{i\sigma} + e^{-i\sigma}) + \dots$$

also by analogy with Debye mass:

dual photon mass² ~ M-M* plasma density

$$m_\sigma \sim v e^{-S_0/2} = v e^{-\frac{4\pi v}{2g_3^2}} \quad (\text{for us, } v = \pi/L)$$

"(anti-)monopole operators"

aka **"disorder operators"** - not locally expressed in terms of original gauge fields (Kadanoff-Ceva; 't Hooft - 1970s)

Polyakov, 1977: **dual photon mass ~ confining string tension**

"Polyakov model" = 3d Georgi-Glashow model or compact U(1) (lattice)

but our theory has fermions and M and KK have zero modes

each have $2N_w$ zero modes

index theorem
Nye-Singer 2000,

disorder operators:

M:

$$e^{-S_0} e^{i\sigma} (\lambda\lambda)^{N_w}$$

KK:

$$e^{-S_0} e^{-i\sigma} (\lambda\lambda)^{N_w}$$

M*:

$$e^{-S_0} e^{-i\sigma} (\bar{\lambda}\bar{\lambda})^{N_w}$$

KK*:

$$e^{-S_0} e^{i\sigma} (\bar{\lambda}\bar{\lambda})^{N_w}$$

for physicists:
Unsal, EP 0812.2085

chiral symmetry $SU(N_w) \times U(1)$

U(1) anomalous, but $\mathbb{Z}_{4N_w}: \lambda \rightarrow e^{i\frac{2\pi}{4N_w}} \lambda \quad \sigma \rightarrow \sigma + \pi$ is not

topological shift symmetry is intertwined with exact chiral symmetry

~~$\cos\sigma$~~

$\cos(2\sigma)$ ✓

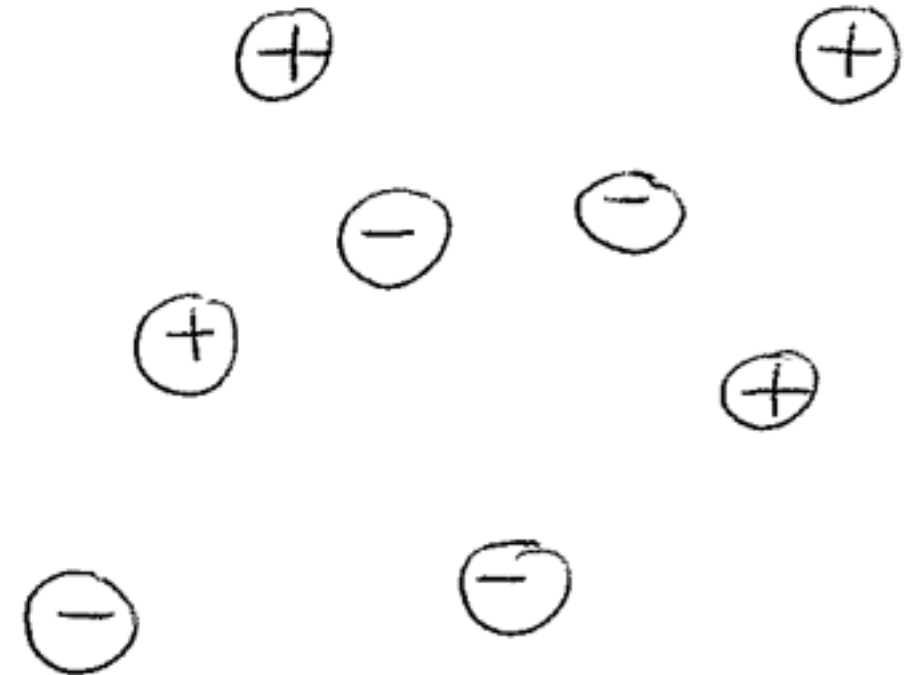
...

potential (and dual photon mass) allowed, but what is it due to?

Unsal 2007: **dual photon mass is induced by magnetic “bions” - the leading cause of confinement in SU(N) with adjoints at small L** (including SYM)

3d pure gauge theory vacuum monopole plasma

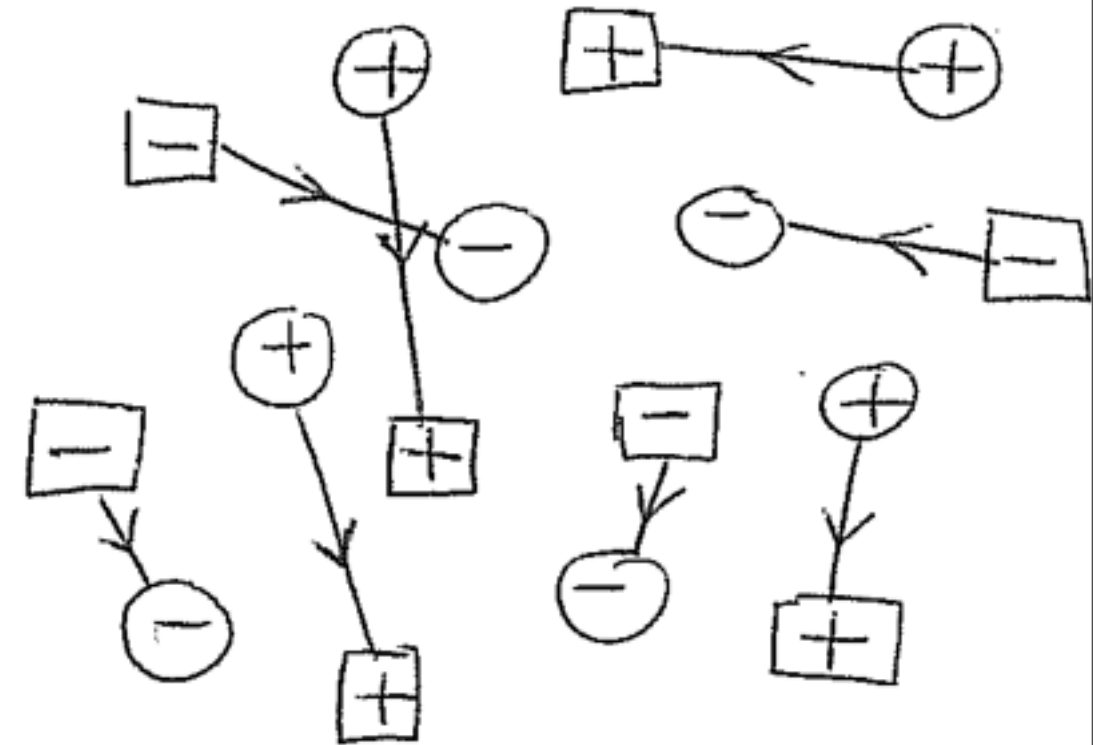
Polyakov 1977



circles = $M(+)/M*(-)$

4d QCD(adj) fermion attraction M - KK^* at small- L

Unsal 2007,

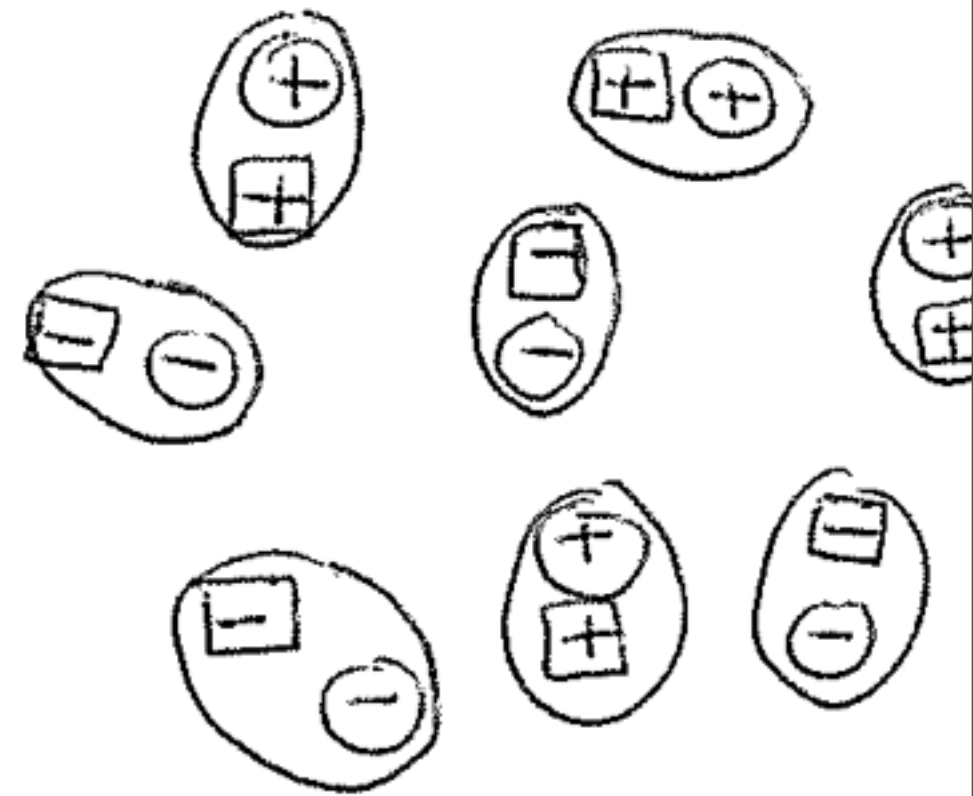


circles = $M(+)/M^*(-)$

squares = $KK(-)/KK^*(+)$

4d QCD(adj) bion plasma at small-L

Unsal 2007,



circles = $M(+)/M*(-)$

squares = $KK(-)/KK*(+)$

blobs = $Bions(++)/Bions*(-)$

4d QCD(adj) bion plasma at small-L

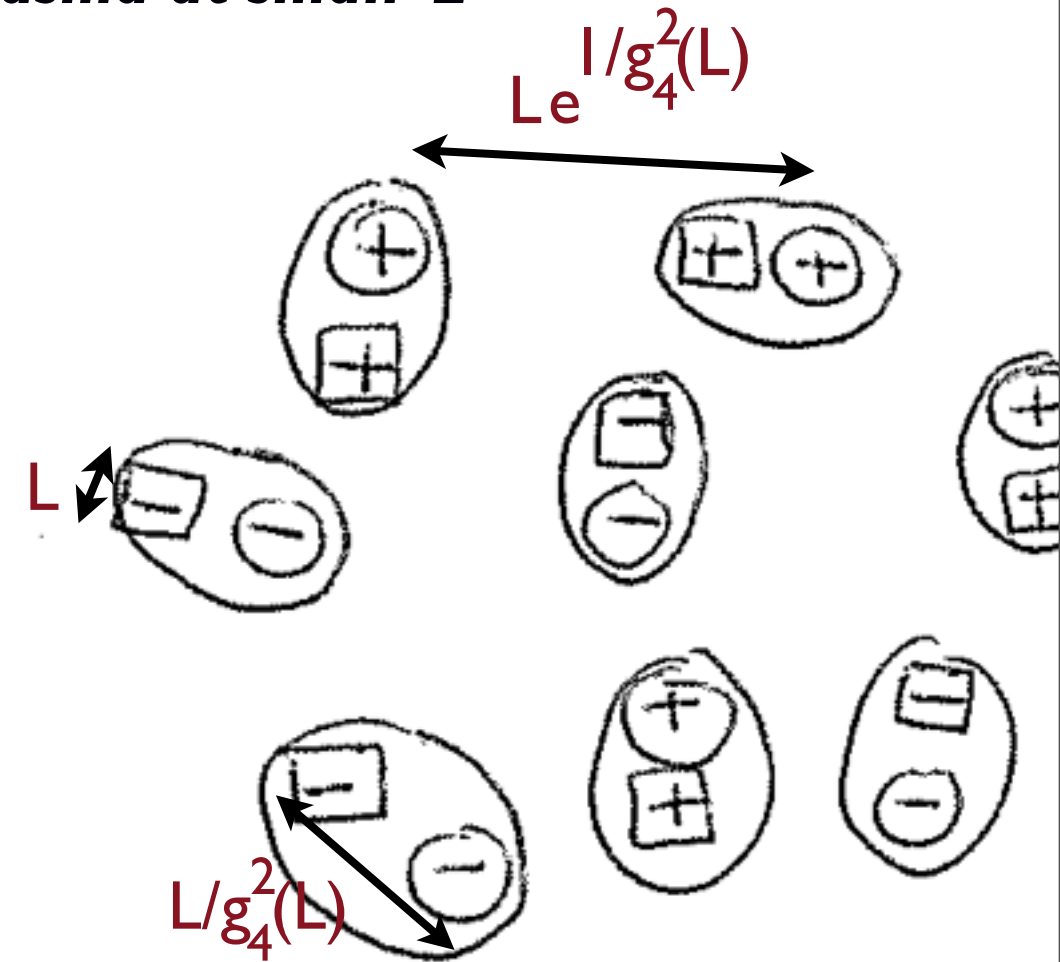
Unsal 2007, ...

$M + KK^* = B$ - magnetic “bions” -

- carry 2 units of magnetic charge
- no topological charge (non self-dual)
(locally 4d nature crucial: no KK in 4d)

bion stability is due to fermion attraction balancing Coulomb repulsion - results in scales as indicated

- bion/antibion plasma screening generates mass for dual photon



“magnetic bion confinement” operates at small-L in any theory with massless Weyl adjoints, including N=1 SYM (& N=1 from Seiberg-Witten theory)

it is “automatic”: no need to “deform” theory other than small-L

first time confinement analytically shown in a non-SUSY, continuum, locally 4d theory

can calculate mass gap, string tension...

Unsal, EP 2009, Anber, EP 2011

$$\frac{\mathcal{M}}{\Lambda} \sim (\Lambda L)^{\frac{8-2n_w}{3}} e^{-2\pi\tilde{c}(\log \frac{1}{\Lambda L})^{1/2}} \times (\text{less relevant contributions})$$

strong scale \rightarrow Λ

\tilde{c} \rightarrow $O(1)$, positive



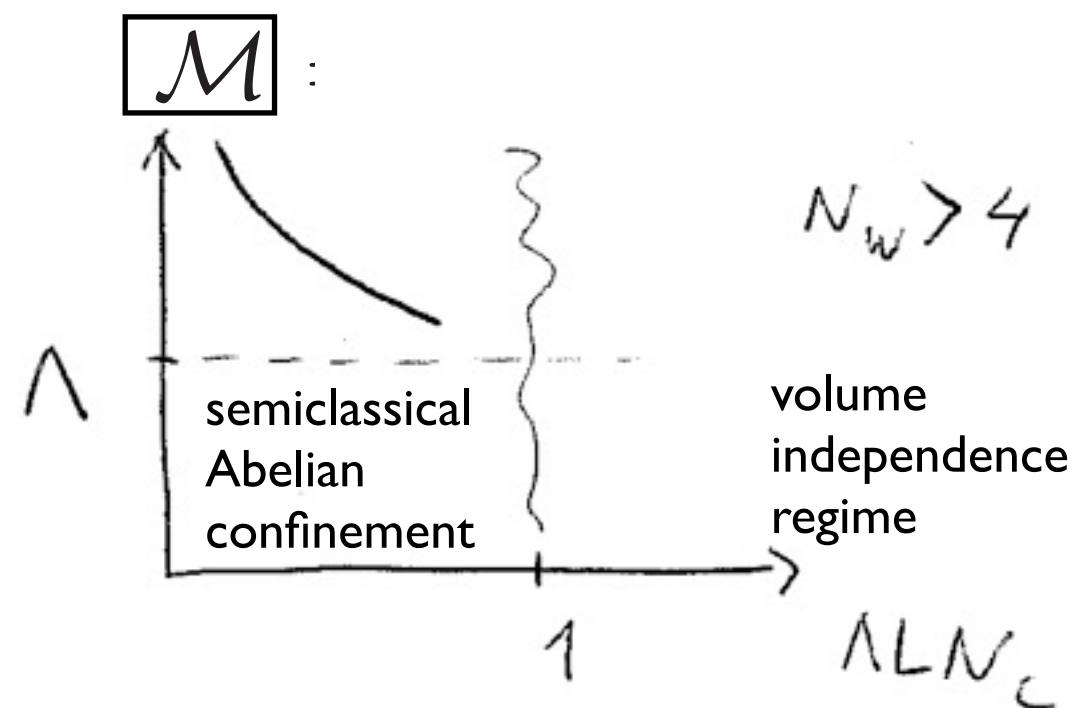
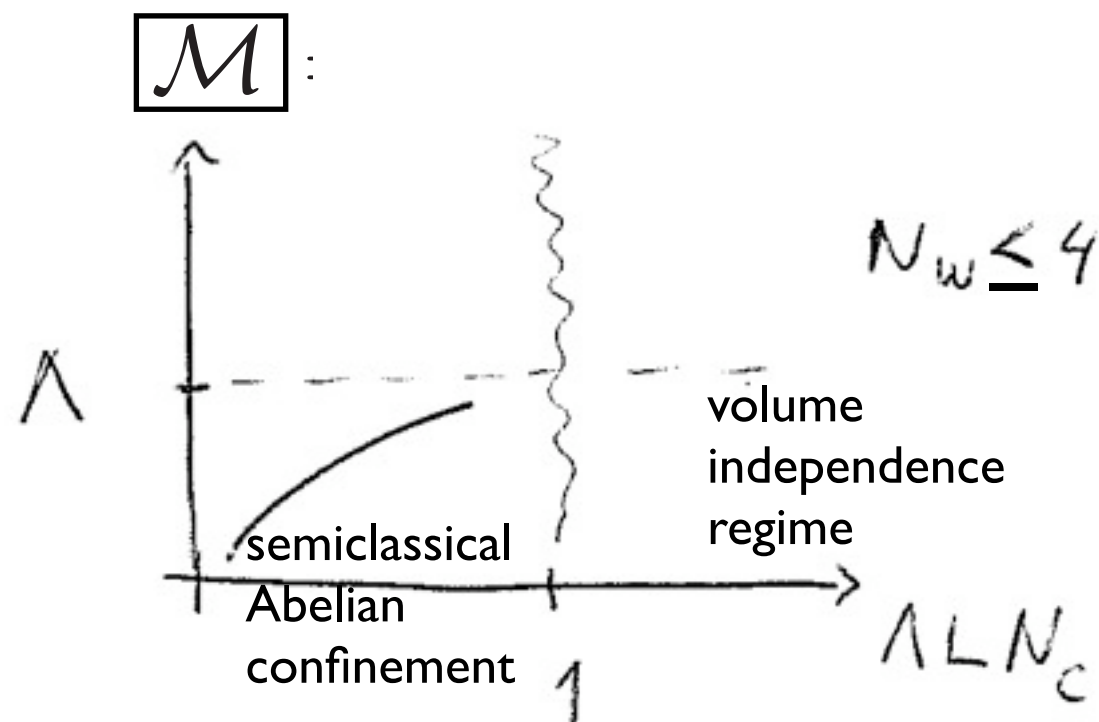
... how **dare** you study non-protected quantities?

can calculate mass gap, string tension...

Unsal, EP 2009, Anber, EP 2011

$$\frac{\mathcal{M}}{\Lambda} \sim (\Lambda L)^{\frac{8-2n_w}{3}} e^{-2\pi\tilde{c}(\log \frac{1}{\Lambda L})^{1/2}} \times (\text{less relevant contributions})$$

← strong scale
← $O(1)$, positive



Discussion on approach to R^4 in refs. - here only note for 4 and 5 massless Weyl adjoints appears that weak coupling IR fixed point at any L , hence Abelian confinement with exponentially small mass gap and string tension

$$\sim \frac{1}{L} e^{-\frac{\mathcal{O}(1)}{g_*^2}}$$

The question about the approach to infinite 4d in the non-SUSY case is very interesting...

... but let's turn to SUSY first:

We argued that “magnetic bions” are responsible for confinement in $N=1$ SYM at small L - a particular case of our Weyl adjoint theory - a “Polyakov like” confinement.

This remains true if $N=1$ obtained from $N=2$ by soft breaking.

On the other hand, we know monopole and dyon condensation is responsible for confinement in $N=2$ softly broken to $N=1$ at large L (Seiberg, Witten '94)

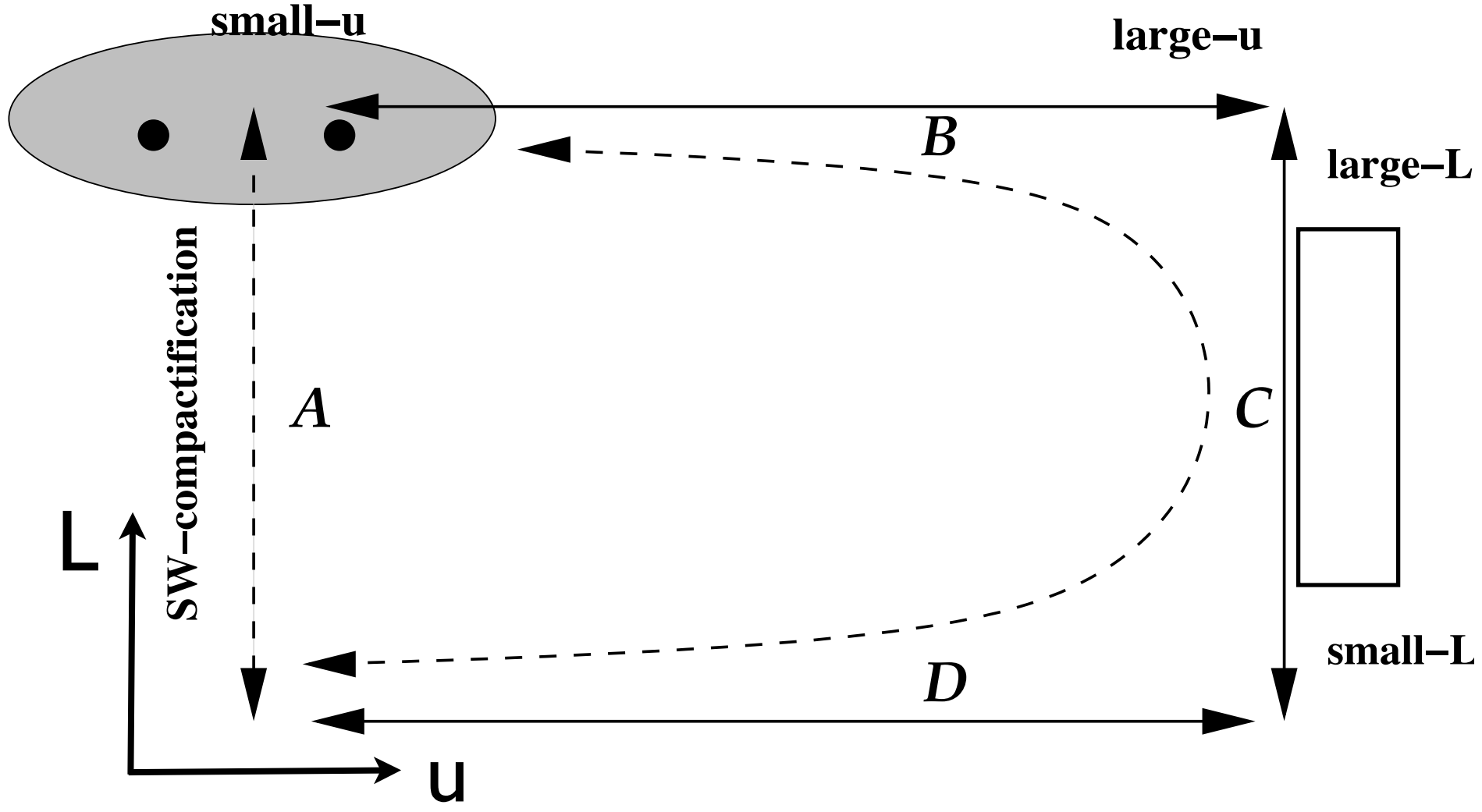
So, in different regimes we have different pictures of confinement in softly broken $N=2$ SYM.

(Both regimes are Abelian and quantitatively understood.)

Do they connect in an interesting way?

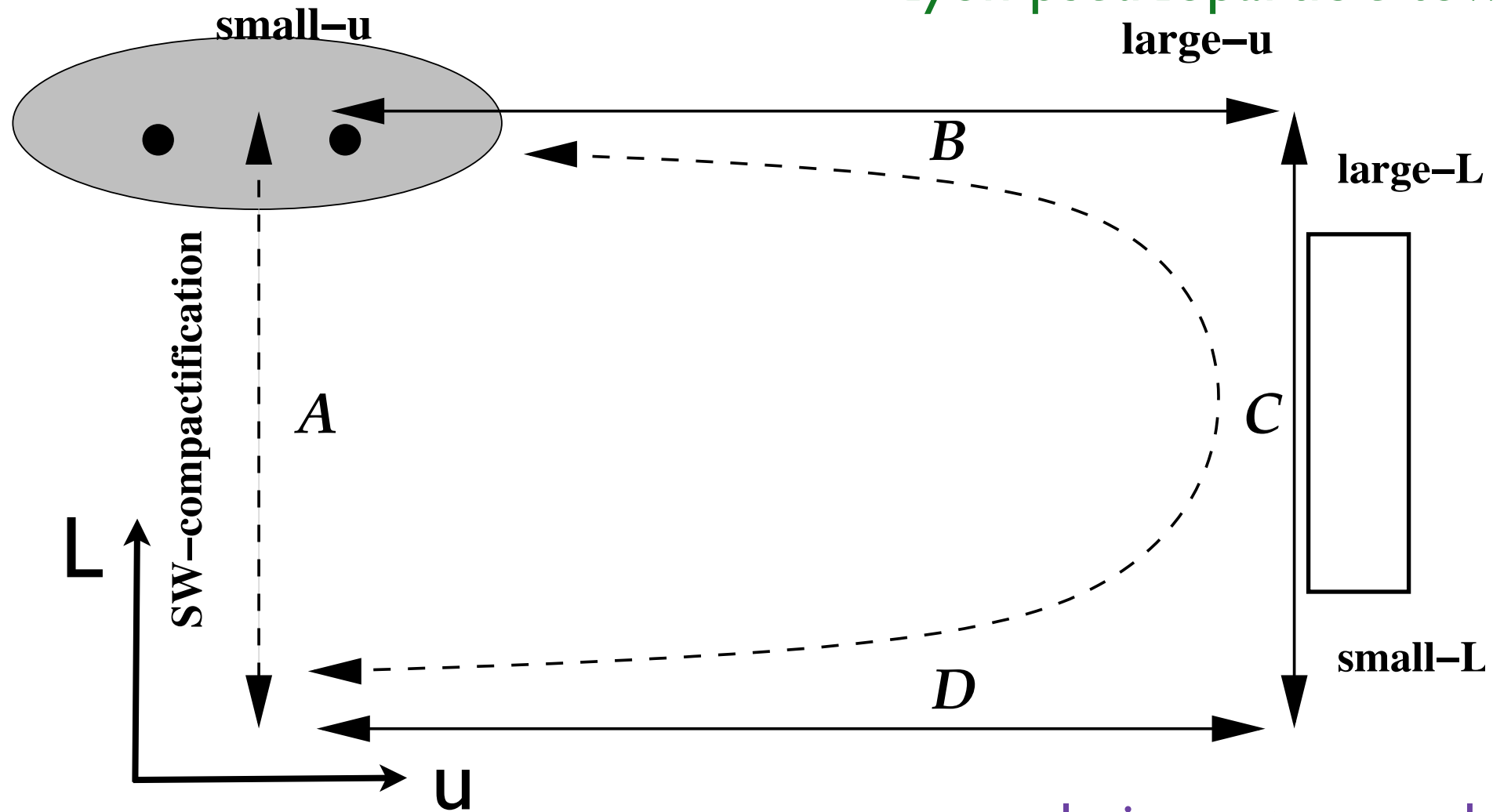
Unsal, EP in progress

path A - difficult: two mutually nonlocal descriptions at large L to merge into one at small L



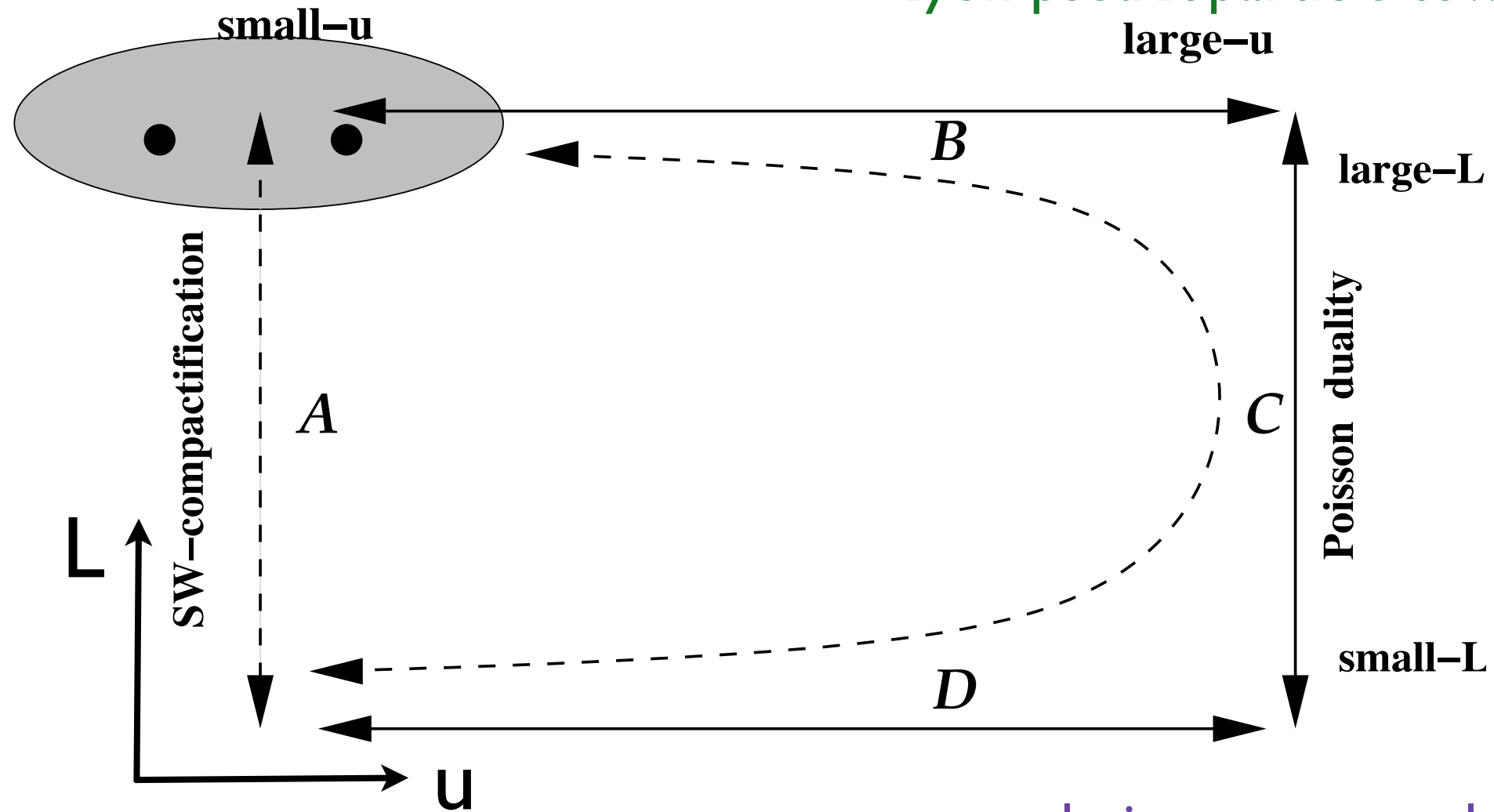
path BCD - easier: C, D can be arranged always semiclassical

dyon tower (sum over electric charges)
of particles with Euclidean worldlines
around S^1
= dyon pseudoparticle tower



monopole-instantons and twisted
monopole-instantons
=“KK tower” described earlier

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monopole-instantons and twisted
monopole-instantons
=“KK tower” described earlier

It turns out the sums over the instanton contributions of the two towers are identical and are related by Poisson duality:

Instanton corrections to K , in complex structure:

$$\sigma - i \frac{4\pi\omega}{g_4^2} \leftarrow LA_4 \quad v = \text{Sqrt}[u]$$

→ dual photon

can be inferred from solution by Chen, Dorey, Petunin (2010) of “wall-crossing” equations of Gaiotto, Moore, Neitzke (2008): an iterative solution, obtained at weak-coupling $v \gg \Lambda$, but **arbitrary** vL :

$$K_{dyon} = \frac{1}{\sqrt{2}\pi^{\frac{3}{2}} L^{\frac{3}{2}} |v|^{\frac{1}{2}}} \sum_{n_m = \pm 1} \sum_{n_e \in \mathbb{Z}} \frac{e^{-L|v| \sqrt{\left(\frac{4\pi}{g_4^2}\right)^2 + n_e^2 + i\omega n_e + i\sigma n_m}}}{\left[\left(\frac{4\pi}{g_4^2}\right)^2 + n_e^2\right]^{\frac{1}{4}}};$$

large- L sum over electric charges of dyon pseudoparticles

small- L sum over winding numbers of twisted monopole-instantons

$$K_{winding} = K_{dyon} = \frac{1}{\pi L^2 |v|} \sum_{n_m = \pm 1} \sum_{n_w \in \mathbb{Z}} e^{-\frac{4\pi L}{g_4^2} \sqrt{|v|^2 + \left(\frac{\omega + 2\pi n_w}{L}\right)^2} + i\sigma n_m}$$

Nontrivial to check their equivalence by a semiclassical calculation

(for general value of the moduli as fermion zero modes are different and only sums are equivalent)...

But, in an appropriate regime, same 4-fermi terms appear, and the “wall-crossing” consequence $K(\text{dyon})=K(\text{winding})$ can be semiclassically tested [Chen et al 2010]

- in this limit, Poisson duality can also be more simply understood, sans wallXing, but no time...

$$e^{-\frac{4\pi v L}{g_4^2} + i\sigma} \sum_{n_w \in \mathbb{Z}} e^{-\frac{1}{2} \frac{4\pi}{L v g_4^2} (\omega + 2\pi n_w)^2} \times (\text{four - fermion operator})$$

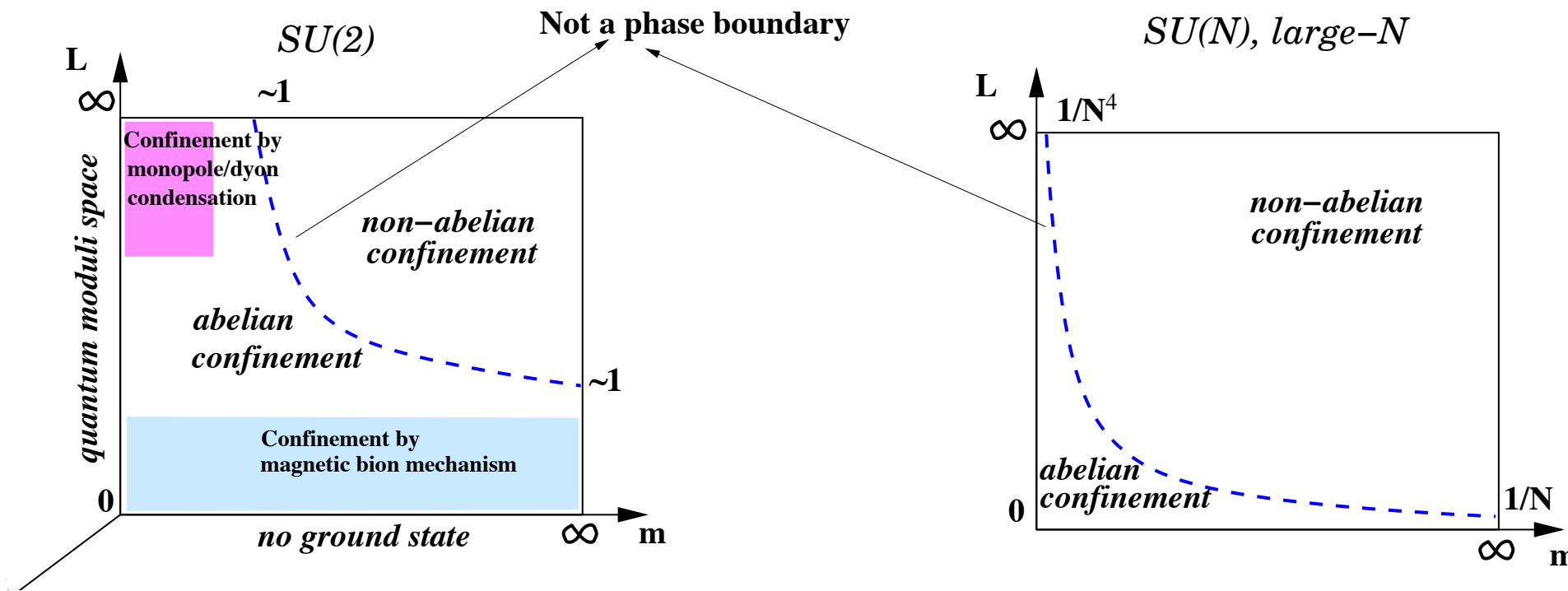
$L v g_4^2 \ll 1$ small-L physics well described by a few twisted monopole-instantons (as we'd already done)
- or an infinite sum over charged dyons

$L v g_4^2 \gg 1$ large-L physics well described by a few dyons
- or an infinite sum over twisted monopole instantons

$$: e^{-\frac{4\pi v L}{g_4^2} + i\sigma} \sum_{n_e \in \mathbb{Z}} \sqrt{\frac{L v g_4^2}{8\pi^2}} e^{-\frac{1}{2} \frac{v L g_4^2}{4\pi} n_e^2 + i n_e \omega} \times (\text{four - fermion operator})$$

The moral is that the dyons, whose condensation at large-L causes confinement, are related by Poisson resummation to the twisted monopole-instantons that form the small-L magnetic bions - which are responsible for confinement at small L, as was already described.

To conclude, we have found an - albeit indirect - relation between the 4d monopole/dyon condensation confinement of Seiberg and Witten and the small-L magnetic bion-induced “Polyakov-like” confinement.



The magnetic bion mechanism also applies to large classes of non-supersymmetric theories and can be used to study the approach to R^4 .

Does the relation between small and large L topological excitations in SUSY have anything to teach us about non-SUSY dynamics?

... it is perhaps early to tell, but the Poisson resummation of nonperturbative effects has interesting implications in finite-T YM

Unsal, EP, 11xx.yyyy - or next CAQCD...