

# ***Does Nature Have a Preferred 1/Nc Expansion?***

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*(Of course, some limits  
do go both ways)*

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work with  
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and with  
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**Continuous Advances in QCD**

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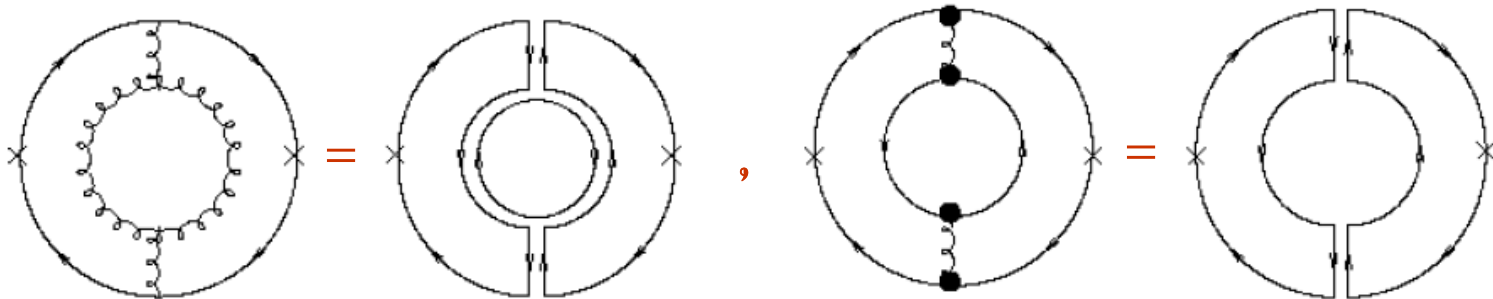
# Outline

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- 1) 't Hooft large  $N_C$  vs. orientifold large  $N_C$   
(fundamental vs. antisymmetric quarks)
- 2) How to build a baryon
- 3) Operator analysis and effective Hamiltonian
- 4) Probes: Masses vs. magnetic moments
- 5) Results: Which limit works better?

# In Standard 't Hooft Large $N_c$ , ...

- Quarks transform under  $N_c$ -dimensional fundamental representation  $\square$  of  $SU(N_c)$  Yang-Mills gauge group; each one carries a single color fundamental charge  $r, b, g, \dots$
- 't Hooft double-line notation: Each quark carries single directed line indicating color charge flow; gluons (in the adjoint) carry two oppositely-oriented lines  
→ Suppression of internal quark loops



## But when $N_C = 3, \dots$

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- The fundamental (F) and two-index antisymmetric (AS) representations are equivalent:

$$q_{ij} \equiv \varepsilon_{ijk} q^k \leftrightarrow (\text{anti-red})(\text{anti-green}) \equiv \text{blue} \leftrightarrow \begin{array}{|c|} \hline \bar{\square} \\ \hline \end{array} \equiv \square$$

- Equivalent AS representation for arbitrary  $N_C$  has  $N_C - 1$  indices
- But how do we know that quarks at arbitrary  $N_C$  live in the F and not the two-index AS representation?
- *Sociology*: Because Witten says so
- *Philosophy*: If quarks are not F, then what use is the fundamental representation?
- *Irritability*: Give me just one good reason to even consider it!

# Orientifold Large $N_c$

Armoni, Shifman, Veneziano (ASV):

Nucl. Phys. **B667**, 170 (2003); Phys. Rev. Lett. **91**, 191601 (2003)

- Pure gauge  $\mathcal{N} = 1$  SUSY for  $U(N_c)$ , in which a large number of nontrivial symmetry relations can be obtained, contains gluinos, which transform according to the *adjoint* (color-anticolor) representation
- As  $N_c \rightarrow \infty$ , one obtains a theory exactly equivalent in many sectors by replacing adjoint fermions with AS fermions:
- This “daughter” theory can be found to live on a brane configuration with an orientifold plane (a place where string lacks orientation)

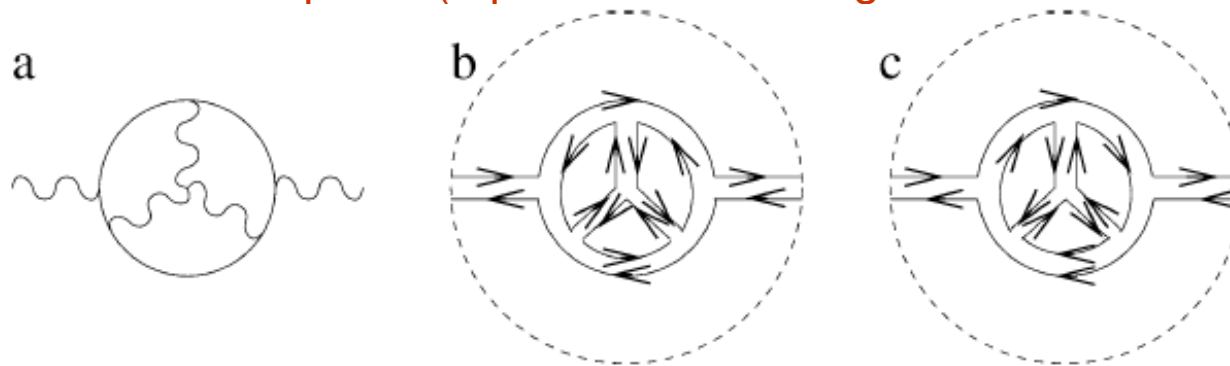
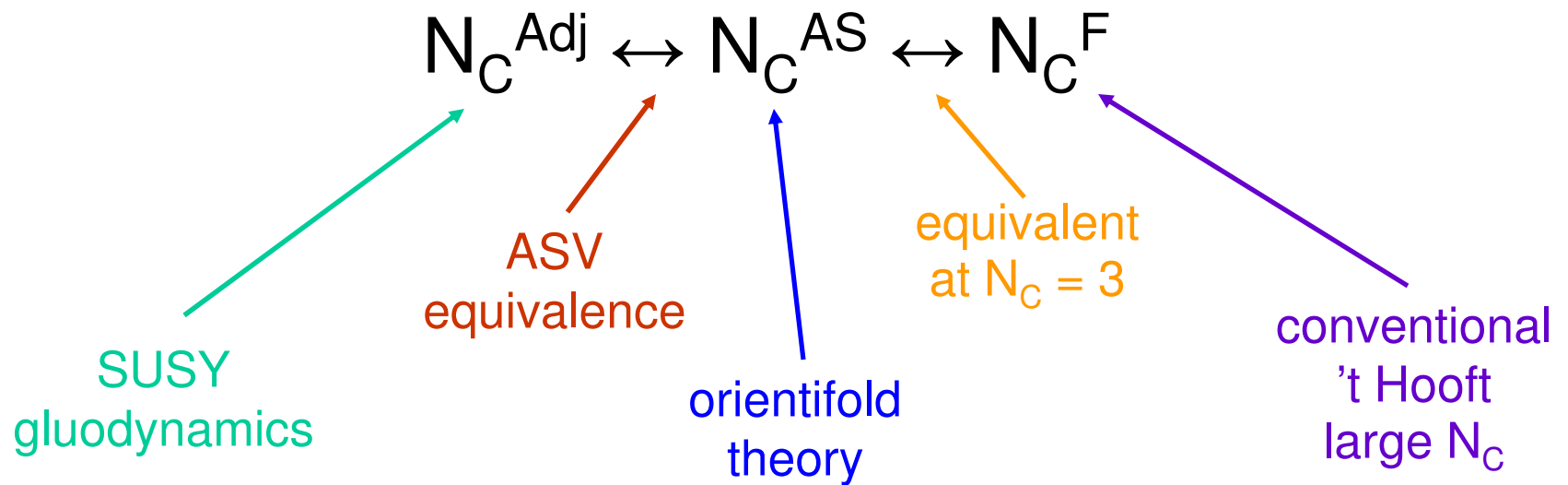


Fig. 2. (a) A typical planar contribution to the vacuum polarization. (b) For  $\mathcal{N} = 1$  SYM. (c) For the non-SUSY theory.

# The Three Large $N_C$ Limits

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# Baryons in the Two Limits

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- Baryon wave functions comprised of  $N_C^F$  quarks have been studied since Witten (Nucl. Phys. **B160**, 57 [1979]):

$$B_F \sim \epsilon^{i_1, i_2, \dots, i_{N_C}} q_{i_1} q_{i_2} \dots q_{i_{N_C}}$$

- With AS quarks, several constructions are possible (Bolognesi, Phys. Rev. D **75**, 065030 [2007]). For example,
  - Using the same  $\epsilon$  invariant with  $\frac{1}{2}N_C$  of the AS quarks:

$$B_\phi \sim \epsilon^{j_1, j_2, \dots, j_{N_C}} q_{j_1, j_2} q_{j_3, j_4} \dots q_{j_{N_C-1}, j_{N_C}}$$

but  $B_\phi$  baryons exist only for even  $N_C$ , and (as pointed out by Bolognesi) have other physical problems

# The $N_C^{AS}$ Baryon

- Bolognesi instead proposed a construction for baryon wave functions in which all AS quarks [ $\frac{1}{2}N_C(N_C - 1)$  in total] are completely antisymmetrized; for  $N_C = 3$  it reads:

$$B_\psi \sim (\epsilon_{i_2, j_2, i_1} \epsilon_{i_3, j_3, j_1} - \epsilon_{i_3, j_3, i_1} \epsilon_{i_2, j_2, j_1}) q^{i_1, j_1} q^{i_2, j_2} q^{i_3, j_3}$$

where, again,  $q_{ij} \equiv \epsilon_{ijk} q^k$

- This  $B_\psi$  construction reduces to  $B_F$  when  $N_C = 3$
- One can build a  $B_\psi$  baryon for every integer  $N_C \geq 2$
- The general  $N_C$  wave function can be expressed in closed form (RFL, unpublished)
- So let us compute observables for  $B_F$  (large  $N_C^F$  expansion) and  $B_\psi$  (large  $N_C^{AS}$  expansion) baryons and compare them



# Parametrizing Static Baryon Properties

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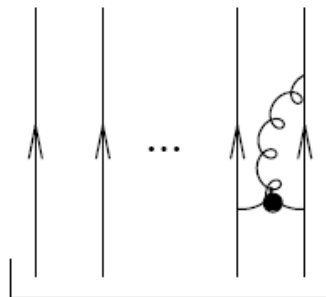
- The lightest ( $N$ ,  $\Delta$ ,  $\Sigma$ , etc.) baryons are degenerate as  $N_C \rightarrow \infty$  (for either the  $N_C^F$  or  $N_C^{AS}$  limit), and fill a multiplet that reduces for  $N_C = 3$  to the old SU(6) **56-plet**
- They differ only in quark flavor content or relative quark spin orientation, whose effects can be parametrized by operators using the basis (again, with either  $N_C^F$  or  $N_C^{AS}$  quarks):

$$J^i = \sum_{\alpha} q_{\alpha}^{\dagger} \left( \frac{\sigma^i}{2} \otimes \mathbb{1} \right) q_{\alpha},$$
$$T^a = \sum_{\alpha} q_{\alpha}^{\dagger} \left( \mathbb{1} \otimes \frac{\lambda^a}{2} \right) q_{\alpha},$$
$$G^{ia} = \sum_{\alpha} q_{\alpha}^{\dagger} \left( \frac{\sigma^i}{2} \otimes \frac{\lambda^a}{2} \right) q_{\alpha},$$

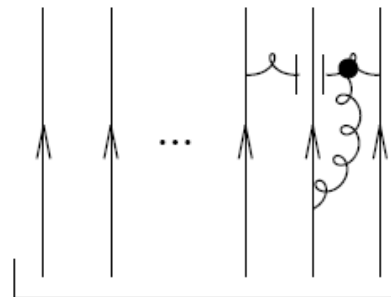
# The Effective Hamiltonian

Dashen, Jenkins & Manohar; Carone, Georgi & Osofsky; Luty & March-Russell  
(1994)

- Processes involving the (entangled) interaction of  $n$  quarks are represented by  $n$ -body operators; in  $N_C^F$ , typical diagrams are:



2-body



3-body

- Generic  $n$ -body operators are suppressed by  $1/N_C^n [N_C^F]$  or  $1/[\frac{1}{2}N_C(N_C-1)]^n \sim 1/N_C^{2n} [N_C^{AS}]$ , one factor for each  $J, T, G$
- From these operators construct a baryon Hamiltonian that is perturbative in powers of  $1/N_C$  [**Effective theory**]

# Calculating with the Hamiltonian

- For  $N_C^F$ ,

$$H = c_0 N_C \mathbf{1} + c_1^{(8)} N_C^0 T^8 + c_J J^2 / N_C + \dots$$

where  $T^8 = \sum_{\text{quarks } \alpha} q_\alpha^+ \frac{\lambda^8}{2} q_\alpha$  ,  $J^2 = \sum_\alpha \sum_\beta \left( q_\alpha^+ \frac{\sigma^i}{2} q_\alpha \right) \left( q_\beta^+ \frac{\sigma^i}{2} q_\beta \right)$

- For  $N_C^{AS}$ , just replace each  $N_C \rightarrow N_C^2$
- $c_k$ : dimensionless coefficients ( $\times \Lambda_{\text{QCD}}$ ), should be of order unity
- Easy to include SU(3) breaking: e.g.,  $c_1^{(8)} \rightarrow \epsilon c_1$ ,  $\epsilon \approx 0.25$
- Since the operators form a complete set, to each one corresponds a unique combination of baryon masses
- Compare to the average multiplet mass ( $N_C [N_C^F]$ ,  $N_C^2 [N_C^{AS}]$ )  
( $N_C^F$  Calculation performed by Jenkins & RFL [1995])

## $I = 0$ Baryon Mass Operators

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$$M = M_0^1 + M_0^8 + M_0^{27} + M_0^{64}$$

$$M_0^1 = c_{(0)}^{1,0} N_c \mathbb{1} + c_{(2)}^{1,0} \frac{1}{N_c} J^2,$$

$$M_0^8 = c_{(1)}^{8,0} T^8 + c_{(2)}^{8,0} \frac{1}{N_c} \{J^i, G^{i8}\} + c_{(3)}^{8,0} \frac{1}{N_c^2} \{J^2, T^8\},$$

$$M_0^{27} = c_{(2)}^{27,0} \frac{1}{N_c} \{T^8, T^8\} + c_{(3)}^{27,0} \frac{1}{N_c^2} \{T^8, \{J^i, G^{i8}\}\},$$

$$M_0^{64} = c_{(3)}^{64,0} \frac{1}{N_c^2} \{T^8, \{T^8, T^8\}\}, \quad (3.4)$$

# Isosinglet Mass Combinations

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$$N_0 = \frac{1}{2} (p + n),$$

$$\Sigma_0 = \frac{1}{3} (\Sigma^+ + \Sigma^0 + \Sigma^-), \text{ and } \Lambda$$

$$\Xi_0 = \frac{1}{2} (\Xi^0 + \Xi^-),$$

$$\Delta_0 = \frac{1}{4} (\Delta^{++} + \Delta^+ + \Delta^0 + \Delta^-),$$

$$\Sigma_0^* = \frac{1}{3} (\Sigma^{*+} + \Sigma^{*0} + \Sigma^{*-}),$$

$$\Xi_0^* = \frac{1}{2} (\Xi^{*0} + \Xi^{*-}), \text{ and } \Omega$$

# Scale of SU(3) flavor breaking

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- One of many possible measures:

$$\epsilon \equiv \frac{1}{3} \sum_{i=1}^3 \frac{B_i - N_0}{(B_i + N_0)/2} \approx 0.25$$

with  $B_i = \Sigma_0, \Lambda, \Xi_0$

- Any other reasonable definition should give  $\epsilon \approx 0.25\text{--}0.30$

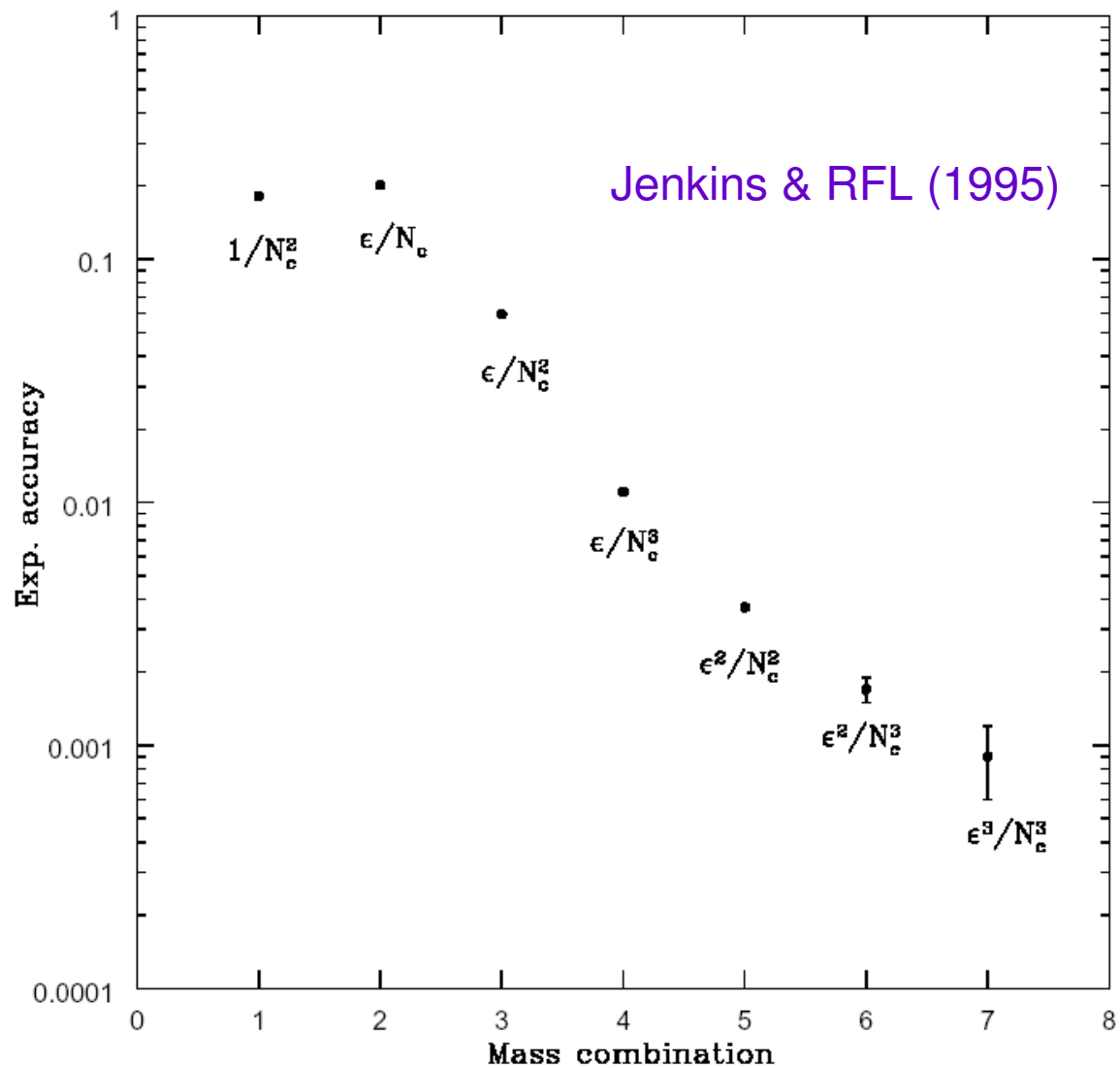
# The $I = 0$ Mass Combinations Special to $1/N_c$

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	Mass Combination	Large $N_c^F$ suppression	Large $N_c^{AS}$ suppression
$M_1$	$5(2N_0 + 3\Sigma_0 + \Lambda + 2\Xi_0) - 4(4\Delta_0 + 3\Sigma_0^* + 2\Xi_0^* + \Omega)$	$1/N_c$	$1/N_c^2$
$M_2$	$5(6N_0 - 3\Sigma_0 + \Lambda - 4\Xi_0) - 2(2\Delta_0 - \Xi_0^* - \Omega)$	$\epsilon$	$\epsilon$
$M_3$	$N_0 - 3\Sigma_0 + \Lambda + \Xi_0$	$\epsilon/N_c$	$\epsilon/N_c^2$
$M_4$	$(-2N_0 - 9\Sigma_0 + 3\Lambda + 8\Xi_0) + 2(2\Delta_0 - \Xi_0^* - \Omega)$	$\epsilon/N_c^2$	$\epsilon/N_c^4$
$M_5$	$35(2N_0 - \Sigma_0 - 3\Lambda + 2\Xi_0) - 4(4\Delta_0 - 5\Sigma_0^* - 2\Xi_0^* + 3\Omega)$	$\epsilon^2/N_c$	$\epsilon^2/N_c^2$
$M_6$	$7(2N_0 - \Sigma_0 - 3\Lambda + 2\Xi_0) - 2(4\Delta_0 - 5\Sigma_0^* - 2\Xi_0^* + 3\Omega)$	$\epsilon^2/N_c^2$	$\epsilon^2/N_c^4$
$M_7$	$\Delta_0 - 3\Sigma_0^* + 3\Xi_0^* - \Omega$	$\epsilon^3/N_c^2$	$\epsilon^3/N_c^4$

Cherman, Cohen & RFL, Phys. Rev. D **80**, 036002 [2009]:  
Compare these results for  $N_c^F$  and  $N_c^{AS}$

Mass  
difference  
quotient





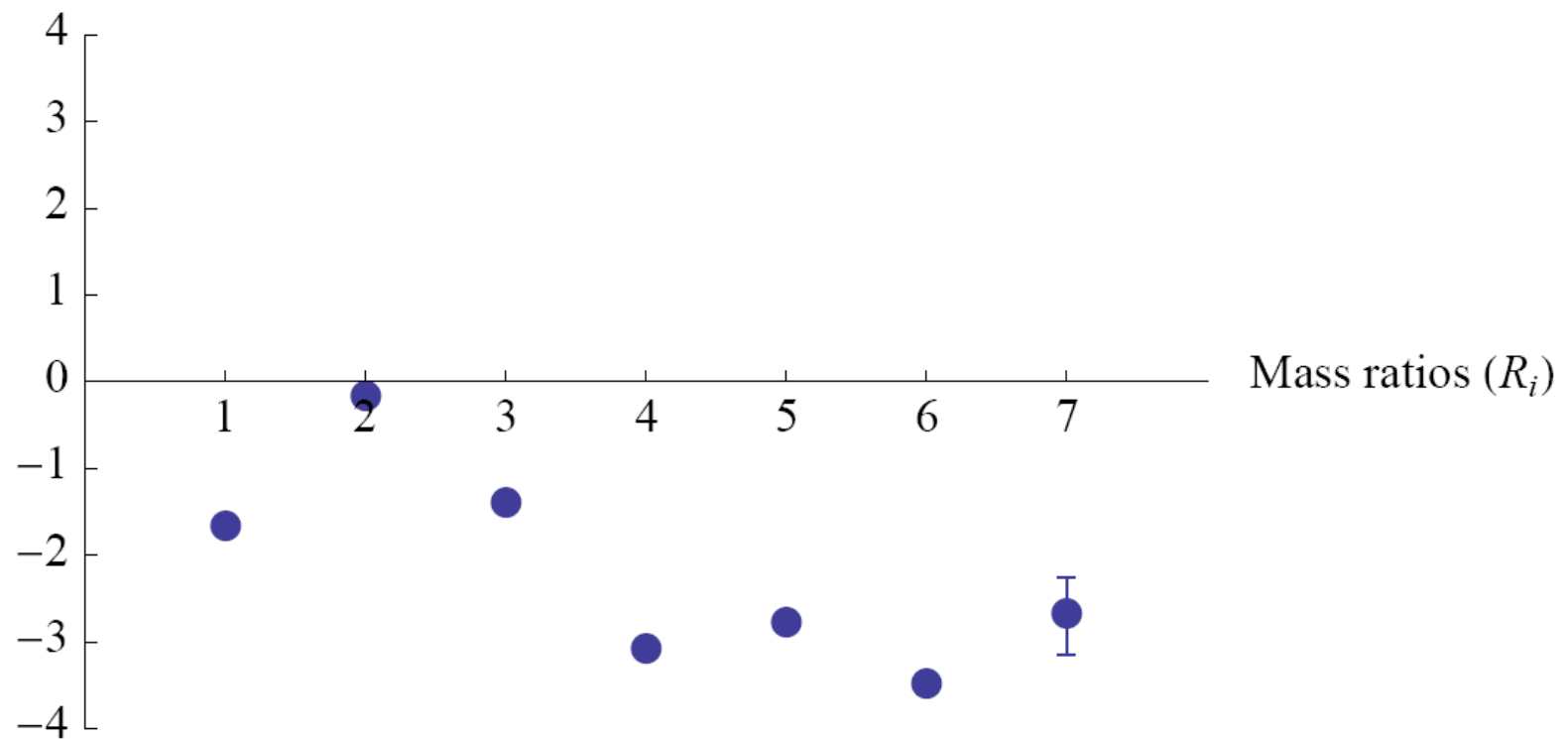
# There's *no way* $N_C^{AS}$ can give results that good. And yet, ...

- Take each  $M_i$  and form  $M_i'$ , the same combination with all “-” signs turned to “+” (Note that  $M_i'$  is  $O(N_C)$  [ $N_C^F$ ],  $O(N_C^2)$  [ $N_C^{AS}$ ])
- Define the scale-independent ratios  $R_i \equiv M_i / (1/2 M_i')$   
*e.g.*,  $M_3 = N_0 - 3\Sigma_0 + \Lambda + \Xi_0$   
 $\rightarrow R_3 = (N_0 - 3\Sigma_0 + \Lambda + \Xi_0) / [1/2 (N_0 + 3\Sigma_0 + \Lambda + \Xi_0)]$
- Compute the corresponding suppression factors  $S_i$  by replacing the masses in  $M_i$  and  $M_i'$ , with their  $N_C$  and  $\varepsilon$  scalings  
*e.g.*, in  $N_C^F$ ,  $M_3 \sim \varepsilon N_C^0$ ,  $M_3' \sim N_C \rightarrow S_3 = \varepsilon / N_C$
- How good is the expansion? Define *accuracy*  $A_i \equiv \ln(|R_i|/S_i)$   
A perfect prediction has  $|R_i| = S_i \rightarrow A_i = 0$   
A poor prediction has  $|R_i|/S_i > N_C$  or  $< 1/N_C$   
Since  $\ln(3) \approx 1$ , the figure of merit is whether all  $A_i$  turn out to lie in a band of  $< 2$  units wide around zero

# SU(3) Breaking Only, $\varepsilon = 0.25$

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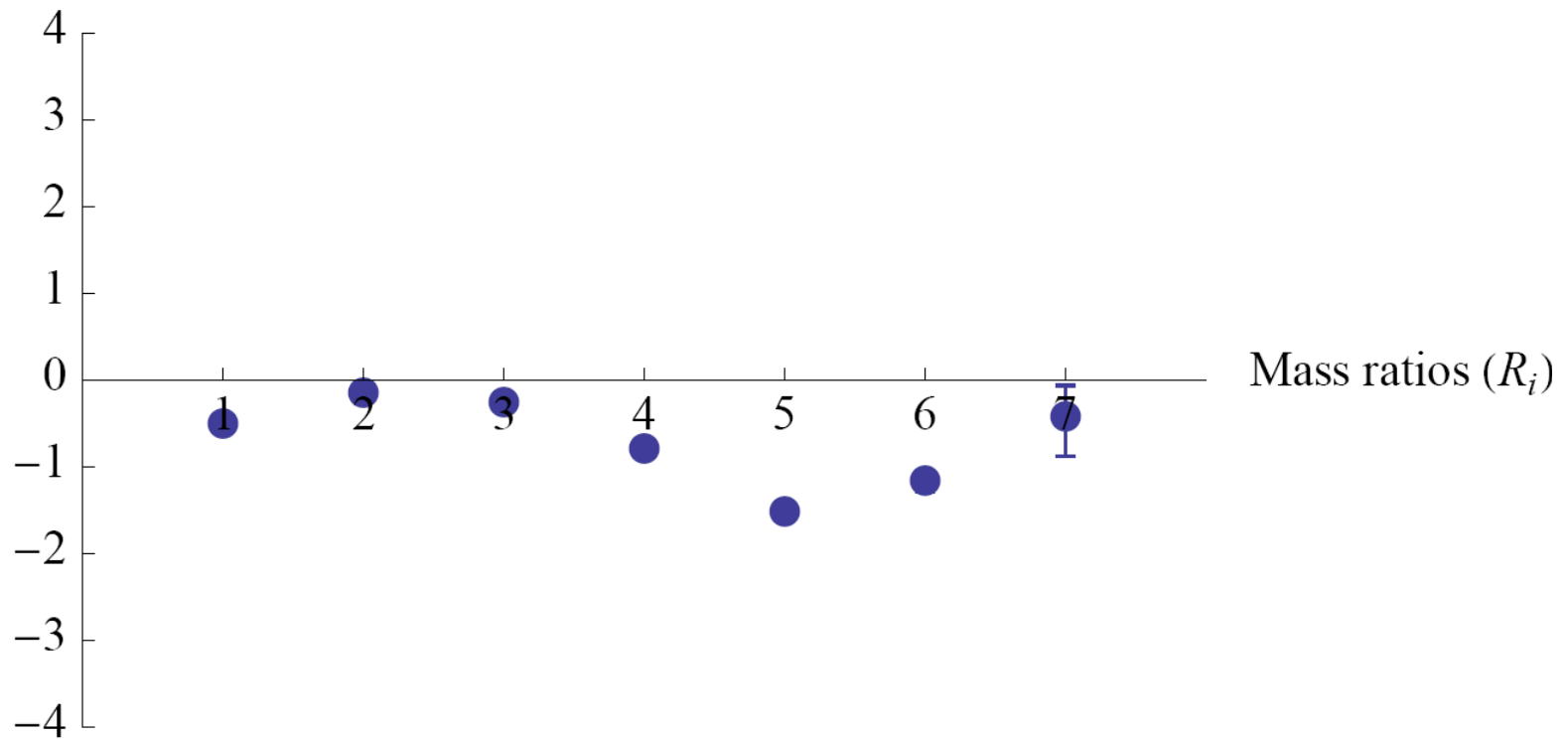
Accuracy ( $A_i$ )



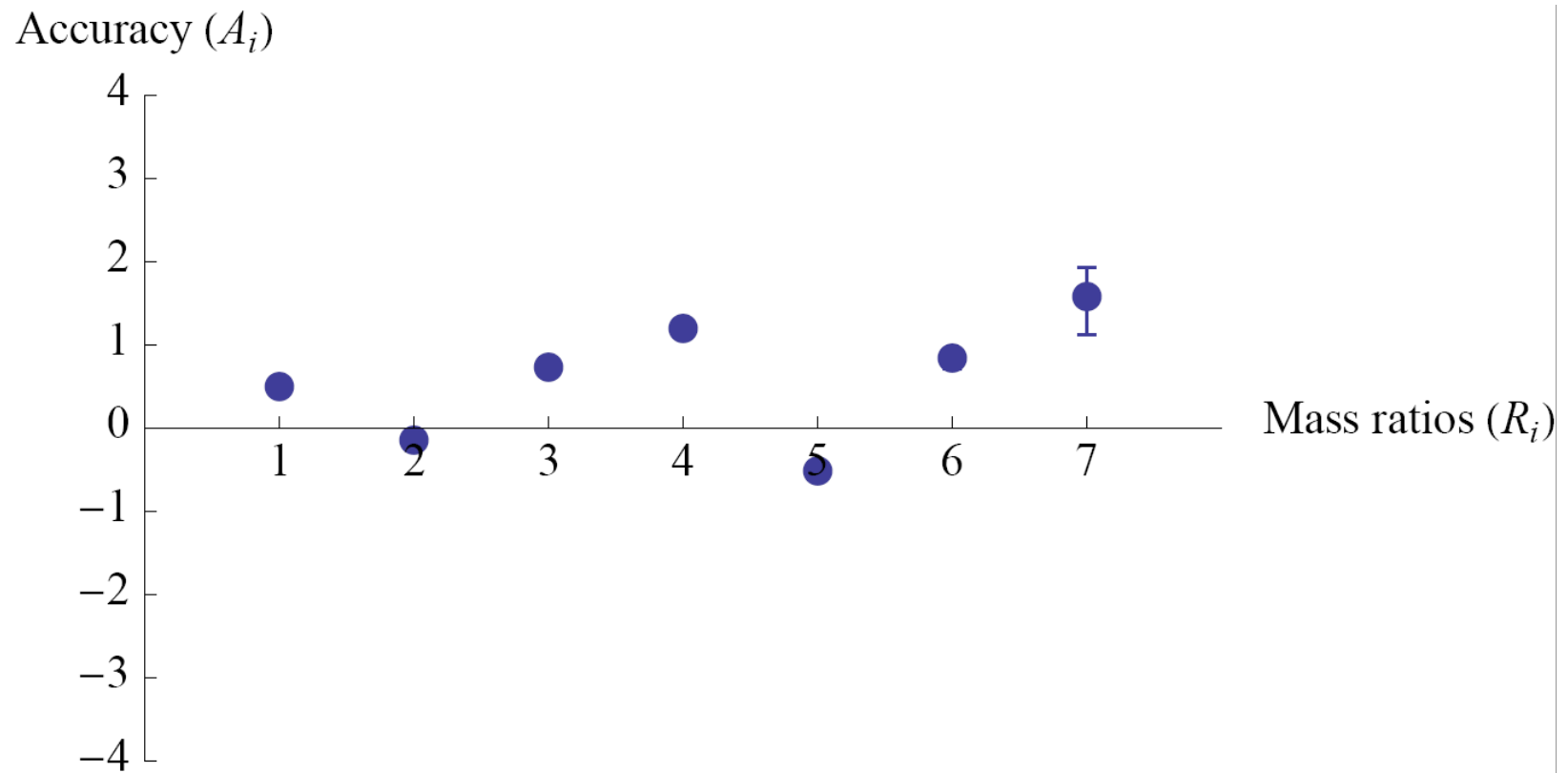
# Large $N_c^F$ Limit, $\varepsilon = 0.25$

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Accuracy ( $A_i$ )



# Large $N_c^{AS}$ Limit, $\varepsilon = 0.25$



## ***News Flash:***

The baryon mass spectrum demands a  $1/N_c$  expansion, but does not strongly prefer  $1/N_c^F$  to  $1/N_c^{AS}$

# What about the $I \neq 0$ splittings?

$$N_1 = (p - n),$$

$$\Sigma_1 = (\Sigma^+ - \Sigma^-),$$

$$\Xi_1 = (\Xi^0 - \Xi^-),$$

$$\Delta_1 = (3\Delta^{++} + \Delta^+ - \Delta^0 - 3\Delta^-),$$

$$\Sigma_1^* = (\Sigma^{*+} - \Sigma^{*-}),$$

$$\Xi_1^* = (\Xi^{*0} - \Xi^{*-}),$$

$$\Lambda\Sigma^0$$

$$\Sigma_2 = (\Sigma^+ - 2\Sigma^0 + \Sigma^-),$$

$$\Delta_2 = (\Delta^{++} - \Delta^+ - \Delta^0 + \Delta^-),$$

$$\Sigma_2^* = (\Sigma^{*+} - 2\Sigma^{*0} + \Sigma^{*-}).$$

$$\Delta_3 = (\Delta^{++} - 3\Delta^+ + 3\Delta^0 - \Delta^-)$$

**BUT:**

- $\Delta$  and  $\Sigma^*$  isospin splittings are poorly known
- $\Lambda\Sigma^0$  not directly measured

Eliminating them leaves just two  $I = 1$  combinations at  $O(1/N_C^F)$  and none at higher order  $\rightarrow$  Can just choose isospin violation parameter  $\varepsilon'$  to soak up extra  $N_C$  in  $N_C^{AS}$

Only one  $I = 2$  and no  $I = 3$  combinations remain

$\rightarrow$  *No decisive prediction*

# Magnetic moments: How many?

RFL & R. TerBeek, PRD 83, 016009 (2011)

- **Observables: 27**  
**9** (octet, incl.  $\Sigma^0\Lambda$ ), + **10** (decuplet), + **8** (octet-decuplet transitions)
- **Measured: 11**  
**8** (octet minus  $\Sigma^0$ ), + **2** (decuplet:  $\Delta^{++}$ ,  $\Omega^-$ ) + **1** transition ( $\Delta^+p$ )
- **Independent operators: 27**  
[RFL & Martin, PRD 70, 016008 (2004)]

$O(N_c^1)$	$G^{33}$
$O(N_c^0)$	$J^3, G^{38}, \frac{1}{N_c}T^3G^{33}, \frac{1}{N_c}N_sG^{33}, \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i3}, G^{33}\}$
$O(N_c^{-1})$	$\frac{1}{N_c}T^3J^3, \frac{1}{N_c}N_sJ^3, \frac{1}{N_c}T^3G^{38}, \frac{1}{N_c}N_sG^{38}, \frac{1}{N_c^2}\frac{1}{2}\{J^2, G^{33}\}, \frac{1}{N_c^2}(T^3)^2G^{33}, \frac{1}{N_c^2}N_s^2G^{33},$ $\frac{1}{N_c^2}T^3N_sG^{33}, \frac{1}{N_c^2}J^iG^{i3}J^3, \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i8}, G^{33}\}, \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i3}, G^{38}\}$
$O(N_c^{-2})$	$\frac{1}{N_c^2}J^2J^3, \frac{1}{N_c^2}N_s^2J^3, \frac{1}{N_c^2}(T^3)^2J^3, \frac{1}{N_c^2}T^3N_sJ^3, \frac{1}{N_c^2}\frac{1}{2}\{J^2, G^{38}\}, \frac{1}{N_c^2}(T^3)^2G^{38}, \frac{1}{N_c^2}N_s^2G^{38},$ $\frac{1}{N_c^2}T^3N_sG^{38}, \frac{1}{N_c^2}J^iG^{i8}J^3, \frac{1}{N_c^2}\frac{1}{2}\{J^iG^{i8}, G^{38}\}$

# The Single-Photon Ansatz

- Each quark in any magnetic moment operator couples proportionally to its electric charge:

$$Q = T^Q \equiv T^3 + \frac{1}{\sqrt{3}}T^8 \quad G^{iQ} \equiv G^{i3} + \frac{1}{\sqrt{3}}G^{i8}$$

- Only 4 indpt. operators otherwise conserving SU(3) flavor exist:

$$\mathcal{O}_1 \equiv G^{3Q}, \quad \mathcal{O}_2 \equiv \frac{1}{N_c}QJ^3, \quad \tilde{\mathcal{O}}_3 \equiv \frac{1}{N_c^2}\frac{1}{2}\{J^2, G^{3Q}\}, \quad \mathcal{O}_4 \equiv \frac{1}{N_c^2}J^iG^{iQ}J^3$$

- SU(3) flavor breaking enters as s quark number  $N_s$  or spin  $J_s$ :

$$\varepsilon\mathcal{O}_5 \equiv \varepsilon q_s J_s^3, \quad \varepsilon\mathcal{O}_6 \equiv \frac{\varepsilon}{N_c}N_s G^{3Q}, \quad \varepsilon\mathcal{O}_7 \equiv \frac{\varepsilon}{N_c}QJ_s^3 \quad \mathcal{O}(\varepsilon N_c^0)$$

$$\varepsilon\mathcal{O}_8 \equiv \varepsilon q_s \frac{N_s}{N_c}J^3, \quad \varepsilon\mathcal{O}_9 \equiv \varepsilon \frac{N_s}{N_c^2}QJ^3, \quad \varepsilon\mathcal{O}_{10} \equiv \frac{\varepsilon}{N_c^2}\frac{1}{2}\{\mathbf{J} \cdot \mathbf{J}_s, G^{3Q}\},$$

$$\varepsilon\mathcal{O}_{11} \equiv \frac{\varepsilon}{N_c^2}J_s^j G^{jQ}J^3, \quad \varepsilon\mathcal{O}_{12} \equiv \frac{\varepsilon}{N_c^2}\frac{1}{2}\{J^j G^{jQ}, J_s^3\}. \quad \mathcal{O}(\varepsilon N_c^{-1})$$



# Operator Demotion

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- If two operators  $X_1, X_2$  give the same  $O(1/N_c)$  matrix elements for each observable but give different ones at  $O(1/N_c^2)$ ,  $X_1 - X_2$  is called a *demoted* operator of  $O(1/N_c^2)$   
(More accurate and incisive accounting of  $1/N_c$  corrections)
- For magnetic moments:  $\frac{1}{2}\varepsilon\mathcal{O}_8 + \varepsilon\mathcal{O}_9$ ,  $-\frac{1}{3}\mathcal{O}_{11} + \mathcal{O}_{12}$ ,  $\mathcal{O}_{10} - \mathcal{O}_{12}$   
demoted to  $O(\varepsilon N_c^{-2})$ , hence neglected;  
 $\varepsilon\mathcal{O}_{13} \equiv \frac{1}{2}\varepsilon\mathcal{O}_5 + \varepsilon\mathcal{O}_7$  demoted to  $O(\varepsilon N_c^{-1})$
- Left with **9** operators: **1** at  $O(N_c)$  ( $G^{3Q}$ ),  
**1** at  $O(N_c^0)$ , **2** at  $O(\varepsilon N_c^0)$ , **2** at  $O(N_c^{-1})$ , **3** at  $O(\varepsilon N_c^{-1})$
- Since **11** observables, can perform least-squares fit to the **9** operator coefficients

# The Goldilocks fits

No  $1/N_c$  factors: This fit's too soft!

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$$\begin{array}{lll} d_1 = +0.995 \pm 0.116 & d_2 = -0.029 \pm 0.138 & d_3 = +0.150 \pm 0.075 \\ d_4 = +0.051 \pm 0.121 & d_5 = -1.708 \pm 1.593 & d_6 = -0.085 \pm 0.420 \\ d_8 = +0.535 \pm 0.829 & d_{10} = -0.420 \pm 0.845 & d_{13} = +0.178 \pm 0.420 \end{array}$$

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$1/N_c^{\text{AS}}$ : This fit's too hard!

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$$\begin{array}{lll} d_1 = +0.976 \pm 0.023 & d_2 = -0.188 \pm 0.176 & d_3 = +12.846 \pm 1.553 \\ d_4 = +5.289 \pm 2.743 & d_5 = -1.474 \pm 0.223 & d_6 = -1.147 \pm 0.491 \\ d_8 = +4.841 \pm 1.046 & d_{10} = -36.332 \pm 12.322 & d_{13} = +1.218 \pm 0.490 \end{array}$$

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$1/N_c^{\text{F}}$ : This fit's just right

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$$\begin{array}{lll} d_1 = +0.992 \pm 0.044 & d_2 = -0.078 \pm 0.148 & d_3 = +1.363 \pm 0.272 \\ d_4 = +0.461 \pm 0.489 & d_5 = -1.652 \pm 0.566 & d_6 = -0.288 \pm 0.438 \\ d_8 = +1.588 \pm 0.865 & d_{10} = -3.727 \pm 2.852 & d_{13} = +0.499 \pm 0.438 \end{array}$$

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## ***News Flash:***

Baryon magnetic moments, despite being a smaller data set than masses, strongly prefer  $1/N_c^F$  to  $1/N_c^{AS}$  or to no  $1/N_c$  expansion

# Taking stock

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- How can less data tell us more?
  - Sole leading mass operator,  $\mathbb{1}$ , gives same mass to all baryons
  - Sole leading magnetic moment operator,  $G^{3Q}$ , gives different values even for isospin multiplets (e.g.,  $\mu_n = -\frac{2}{3} \mu_p$ )
- What would it take to do better?
  - In the masses: Better decuplet ( $\Delta$ ,  $\Sigma^*$ ,  $\Xi^*$ ) isospin splittings
  - In the magnetic moments: Better values for  $\mu_{\Sigma^0 \Lambda}$ , measurements of a few octet-decuplet transitions (e.g.,  $\Sigma^* \Sigma$ )
- What if both results persist?
  - Resolved for philosophical discussion: Could different observables obey different  $1/N_c$  expansions, or is there a unique choice obeyed by all?

# Conclusions

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- The baryon mass spectrum demands a  $1/N_c$  expansion (as has been known for 16 years), but does not strongly prefer one based on fundamental representation quarks  $1/N_c^F$  to two-index antisymmetric representation quarks,  $1/N_c^{AS}$
- Baryon magnetic moments, despite being a smaller data set than masses, strongly prefer the  $1/N_c^F$  expansion to  $1/N_c^{AS}$  or to no  $1/N_c$  expansion
- Just a few additional data points in either set would greatly sharpen these conclusions
- Then we can argue about what the  $1/N_c$  expansion really *means*

# How to handle the $N_c$ 's

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- Denominator  $N_c$ 's come from 't Hooft scaling →
  - In going from  $1/N_c^F$  to  $1/N_c^{AS}$ , just replace  $1/N_c^1 \rightarrow 1/N_c^2$
- Numerator  $N_c$ 's come from combinatorics →
  - In going from  $1/N_c^F$  to  $1/N_c^{AS}$ , leave  $N_c(N_c-1)/2$  as is

# Using the magnetic moment fit, one can predict all the rest...

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TABLE VIII: Best fit values for the 16 unknown magnetic moments in units of  $\mu_N$  using the  $1/N_c^F$  expansion.

$\mu_{\Delta^+} = +3.09 \pm 0.16$	$\mu_{\Delta^0} = +0.00 \pm 0.10$	$\mu_{\Delta^-} = -3.09 \pm 0.16$	$\mu_{\Sigma^{*+}} = +2.62 \pm 0.35$
$\mu_{\Sigma^{*0}} = -0.06 \pm 0.32$	$\mu_{\Sigma^{*-}} = -2.73 \pm 0.35$	$\mu_{\Xi^{*0}} = -0.12 \pm 0.33$	$\mu_{\Xi^{*-}} = -2.37 \pm 0.39$
$\mu_{\Sigma^0} = +0.65 \pm 0.11$	$\mu_{\Delta^0 n} = +3.51 \pm 0.11$	$\mu_{\Sigma^{*0\Lambda}} = +2.65 \pm 0.32$	$\mu_{\Sigma^{*0\Sigma^0}} = +1.21 \pm 0.31$
$\mu_{\Sigma^{*+}\Sigma^+} = +2.69 \pm 0.32$	$\mu_{\Sigma^{*-}\Sigma^-} = -0.26 \pm 0.31$	$\mu_{\Xi^{*0}\Xi^0} = +2.30 \pm 0.33$	$\mu_{\Xi^{*-}\Xi^-} = -0.26 \pm 0.31$