QCD Effects in Exclusive Rare *B*-Meson Decays

Alexander Khodjamirian

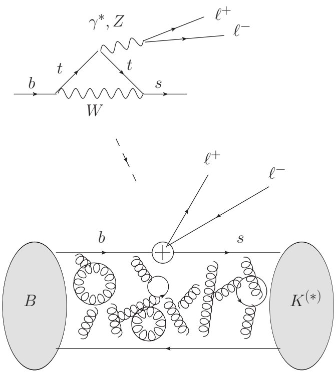
UNIVERSITÄT SIEGEN

Continuous Advances in QCD, Minneapolis 2011

[A.K., Th. Mannel, A. Pivovarov and Yu-M. Wang, JHEP09 (2010) 089]

Introduction

- $B \to K^{(*)}\ell^+\ell^-$ or $B \to K^*\gamma$,
  proceed via FCNC ,
  $BR \sim 10^{-7} - 10^{-6}$



- benchmark channels
  for flavour physics,
  $q^2 = (p_{\ell^+} + p_{\ell^-})^2$ or $q^2 = 0$

  first data: BABAR, Belle, CDF,
  precise LHCb data expected

- Standard Model: t, W, Z and high-virtuality gluons
  absorbed in effective operators $\Rightarrow H_{eff}$,
  the rest - hadronic matrix elements

$B \to K^{(*)}\ell^+\ell^-$ in the Standard Model

- Computation of the decay amplitude:

$$A(B \to K^{(*)}\ell^+\ell^-) = \langle K^{(*)}\ell^+\ell^- \mid H_{eff} \mid B\rangle \,,$$

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)O_i(\mu) \,,$$

- dominant contributions from the $b \to s\ell\ell$ or $b \to s\gamma$ operators:

$$O_{9(10)} = \frac{\alpha_{em}}{4\pi}[\bar{s}_L\gamma_\mu b_L]\ell\gamma^\mu(\gamma_5)\ell, \quad C_9(m_b) \simeq 4.4, \quad C_{10}(m_b) \simeq= -4.7,$$

$$O_7 = -\frac{em_b}{8\pi^2}[\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b]F^{\mu\nu} \,, \quad C_7(m_b) \simeq -0.3$$

- four-quark operators, combined with e.m. interaction:

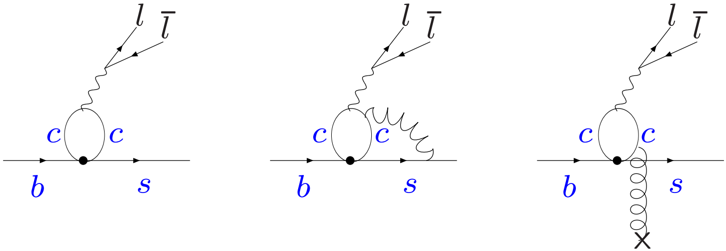
$$O_1^{(c)} = [\bar{s}_L\gamma_\rho c_L][\bar{c}_L\gamma^\rho b_L] \,, \qquad C_1(m_b) \simeq 1.1$$

$$O_2^{(c)} = [\bar{c}_L\gamma_\rho c_L][\bar{s}_L\gamma^\rho b_L] \,, \qquad C_2(m_b) \simeq -0.25$$

- $\oplus$ quark-penguin (O_{3-6}) and gluon-penguin (O_8) operators

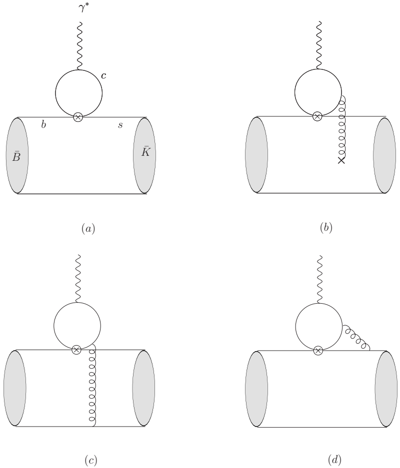
Charm-loops

- a combination of the $(\bar{s}c)(\bar{c}b)$ weak interaction ($O_{1,2}$) and e.m.interaction $(\bar{c}c)(\bar{\ell}\ell)$ "mimicking FCNC"
- Charm-loop effect:



- similar effects:
  u, d, s, c, b-quark loops (quark-penguin operators O_{3-6}),
  u-loops from $O^u_{1,2}$ (CKM suppressed in $b \to s$),
- $A(B \to K^{(*)}\ell^+\ell^-)$ include specific hadronic matrix elements, not just form factors

Charm-loop in $B \to K^{(*)}\ell^+\ell^-$



- factorizable c-quark loop
$C_9 \to C_9 + (C_1 + 3C_2)g(m_c^2, q^2)$

- perturbative gluon corrections being factorized in $O(\alpha_s)$ and added to C_9
[M. Beneke,T. Feldmann,D. Seidel (2001)]

- how important are the soft gluons (low-virtuality, nonvanishing momenta) emitted from the c-quark loop ?

Charm loop turns charmonium

- at $q^2 \to m^2_{J/\psi}, ...$ an on-shell hadronic state:
  $B \to J/\psi K \otimes J/\psi \to \ell^+\ell^-$
- $\oplus$ other ψ-resonances, open-charm states
  (e.g., $B \to \bar{D}DK \to K\ell^+\ell^-$), $M_{\bar{c}c} \leq m_B - m_K^{(*)}$
- $B \to \psi K$: the naive factorization fails, hinting at large nonfactorizable contributions
- to avoid huge backgr., J/ψ and $\psi(2S)$ -regions are subtracted from the q^2-distribution data in $B \to K^{(*)}\ell^+\ell^-$
- the effect of intermediate/virtual $\bar{c}c$ states remains at $q^2 \ll m^2_{J/\psi}$ (nonperturbative at $q^2 \sim 4m_c^2$)
- Can we use the { loop $\oplus$ corrections } ansatz?

Isolating the charm-loop in the decay amplitude

- the contribuiton of $O_{1,2}$ and e.m. interaction:

$$A(B \to K^{(*)}\ell^+\ell^-)^{(O_{1,2})} = -(4\pi\alpha_{em}Q_c)\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^*\frac{\bar{\ell}\gamma^\mu\ell}{q^2}\mathcal{H}_\mu^{(B\to K^{(*)})},$$

- the relevant hadronic matrix element:

$$\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q) = i\int d^4x e^{iq\cdot x}\langle K^{(*)}(p)|T\Big\{\bar{c}(x)\gamma_\mu c(x),$$
$$\Big[C_1O_1(0) + C_2O_2(0)\Big]\Big\}|B(p+q)\rangle,$$

- at small $q^2 \ll 4m_c^2$ use the operator-product expansion (OPE) for the T-product:

$$\mathcal{C}_\mu^a(q) = \int d^4x e^{iq\cdot x}T\Big\{\bar{c}(x)\gamma_\mu c(x), \bar{c}_L(0)\Gamma^a c_L(0)\Big\},$$

Expansion near the light-cone

- the dominant region in this T-product: $\langle x^2 \rangle \sim 1/(2m_c - \sqrt{q^2})^2$
- at $q^2 \ll 4m_c^2$: T- product of $\bar{c}c$-operators can be expanded near the light-cone $x^2 \sim 0$, schematically,

$$T\{\bar{c}(x)\gamma_\mu c(x), \bar{c}_L(0)\gamma_\rho c_L(0)\} = C_0^{\mu\rho}(x^2, m_c^2) + \text{two-gluon term} + ...$$

$$T\{\bar{c}(x)\gamma_\mu c(x), \bar{c}_L(0)\gamma_\rho \frac{\lambda^a}{2} c_L(0)\} = \int\limits_0^1 du C_1^{\mu\rho\alpha\beta}(x^2, m_c^2, u) G^a_{\alpha\beta}(ux) + ...$$

- after x-integration and taking hadronic matrix element: $O(C_1)/O(C_0) \sim O(C_{n+1})/O(C_n) \sim \Lambda^2_{QCD}/(4m_c^2 - q^2)$,
- but ! no local expansion possible in each term of LC OPE: $O(C_1) \sim \sum_{k=0}^{\infty}(q\Lambda_{QCD})^k/(4m_c^2 - q^2)^{k+1}$,

  $q \sim m_b/2$ and $m_b\Lambda_{QCD} \sim m_c^2$.

The resulting effective operators

- LO reduced to simple $\bar{c}c$ -loop,
  no difference between local and LC,

$$\mathcal{O}_\mu(q) = (q_\mu q_\rho - q^2 g_{\mu\rho})\frac{9}{32\pi^2}\, g(m_c^2, q^2)\bar{s}_L\gamma^\rho b_L \,.$$

- gluon emission: use c-quark propagator near the light-cone in the external gluon field [I. Balitsky, V. Braun (1999)]
- define LC kinematics ($n_\pm$) in the rest-frame of B,
  $q \simeq (m_b/2)n_+$
- one-gluon emission yields a new nonlocal operator:

$$\widetilde{\mathcal{O}}_\mu(q) = \int d\omega\, I_{\mu\rho\alpha\beta}(q, m_c, \omega)\bar{s}_L\gamma^\rho\delta[\omega - \frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L \;,$$

The local OPE limit

- $\omega \to 0$ in the nonlocal operator, no derivatives of $G_{\mu\nu}$

$$\widetilde{\mathcal{O}}_{\mu}^{(0)}(q) = I_{\mu\rho\alpha\beta}(q)\bar{s}_L\gamma^{\rho}\widetilde{G}_{\alpha\beta}b_L \;,$$

$$I_{\mu\rho\alpha\beta}(q,m_c) = (q_\mu q_\alpha g_{\rho\beta} + q_\rho q_\alpha g_{\mu\beta} - q^2 g_{\mu\alpha}g_{\rho\beta}) \times \frac{1}{16\pi^2}\int_0^1 dt\, \frac{t(1-t)}{m_c^2 - q^2 t(1-t)}$$

At $q^2 = 0$, the quark-gluon operator obtained
in $B \to X_s\gamma$ in *[M.Voloshin (1997)]*
in $B \to K^*\gamma$ *[A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]*

- the neccesity of resummation was discussed before
  [Z. Ligeti, L. Randall and M.B. Wise,(1997);
  A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei (1997);
  J. W. Chen, G. Rupak and M. J. Savage,(1997);
  G. Buchalla, G. Isidori and S.J. Rey (1997)]

Hadronic matrix elements for the charm-loop effect

- the LO: factorized $\bar{c}c$ loop

$$\left[\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q)\right]_{fact} = \left(\frac{C_1}{3}+C_2\right)\langle K^{(*)}(p)|\mathcal{O}_\mu(q)|B(p+q)\rangle\,,$$

- reduced to $B \to K^{(*)}$ form factors, nothing new
- The gluon emission yields:

$$\left[\mathcal{H}_\mu^{(B\to K^{(*)})}(p,q)\right]_{nonfact} = 2C_1\langle K^{(*)}(p)|\widetilde{\mathcal{O}}_\mu(q)|B(p+q)\rangle\,.$$

- new hadronic matrix element

$$\langle K^{(*)}(p)|\bar{s}_L\gamma^\rho\delta[\omega-\frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L|B(p+q)\rangle\,,$$

Charm-loop effect in $B \to K\ell^+\ell^-$

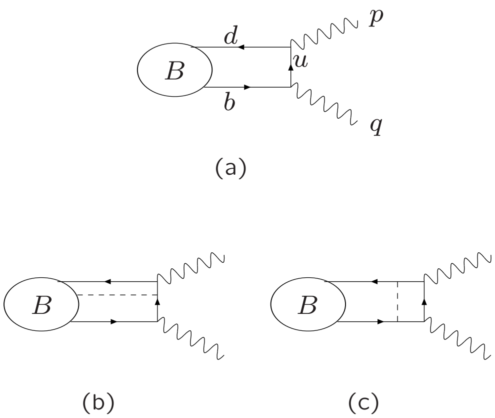
- The invariant amplitude:

$$\left[\mathcal{H}_\mu^{(B\to K)}(p,q)\right]_{fact.+nonfact.} = \left[(p\cdot q)q_\mu - q^2 p_\mu\right] \times \left[\left(\frac{C_1}{3}+C_2\right)A(q^2) + 2C_1\widetilde{A}(q^2)\right]$$

- the factorizable part $A(q^2) = \frac{9}{32\pi^2}\, g(m_c^2, q^2) f^+_{BK}(q^2)$
- Wilson coefficients enhance the nonfact. part $C_1/3 + C_2 \ll C_1$
- need nonperturbative QCD methods to calculate the form factor $f^+_{BK}(q^2)$ and the nonfactorizable amplitude $\widetilde{A}(q^2)$
- use one and the same LCSR approach

LCSR with B-meson distribution amplitudes

[A.K., T. Mannel, N.Offen,2005]



- a similar approach: LCSR for $B \to \pi$ in SCET

  [F.De Fazio, Th. Feldmann and T. Hurth, (2005)]

B-meson DA's

$$\langle 0|\bar{q}_{2\alpha}(x)[x,0]h_{v\beta}(0)|\bar{B}_v\rangle$$
$$= -\frac{if_B m_B}{4}\int\limits_0^\infty d\omega e^{-i\omega v\cdot x}\left[(1+\not v)\left\{\phi_+^B(\omega) - \frac{\phi_+^B(\omega)-\phi_-^B(\omega)}{2v\cdot x}\not x\right\}\gamma_5\right]_{\beta\alpha}$$

- defined in HQET; key input parameter: the inverse moment

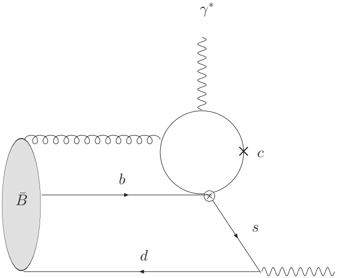
$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega,\mu)}{\omega}$$

- QCD sum rules in HQET: $\lambda_B(1\text{ GeV}) = 460 \pm 110$ MeV
  [V.Braun, D.Ivanov, G.Korchemsky,2004]
- all $B \to \pi, K^{(*)}, \rho$ form factors calculated
- so far only tree-level calculations, 2,3-particle DA's
- model for 3-particle DA's obtained from 2-point QCD sum rules in HQET *[A.K., T.Mannel, N.Offen (2007)]*

LCSR for the soft-gluon hadronic matrix element

- the correlation function:

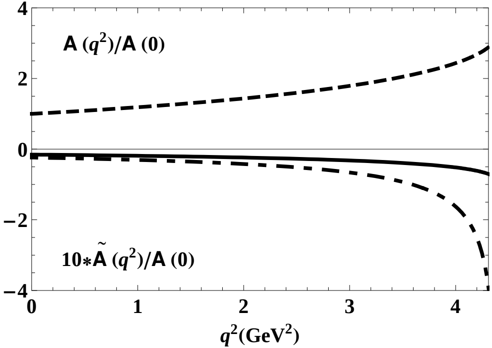
  $\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = i\int d^4y e^{ip\cdot y}$
  $\langle 0|T\{j^K_\nu(y)\widetilde{\mathcal{O}}_\mu(q)\}|B(p+q)\rangle\,,$



- hadronic dispersion relation in the kaon channel

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = \frac{if_K p_\nu}{m_K^2 - p^2}[(p\cdot q)q_\mu - q^2 p_\mu]\tilde{A}(q^2) + \int_{s_h}^{\infty} ds\,\frac{\tilde{\rho}_{\mu\nu}(s,q^2)}{s-p^2}$$

Light-cone vs local OPE



charm-loop hadronic matrix elements in $B \to K\ell^+\ell^-$:

simple loop: dashed;

soft-gluon emission from the loop:

light-cone (local) OPE- solid (dash-dotted)

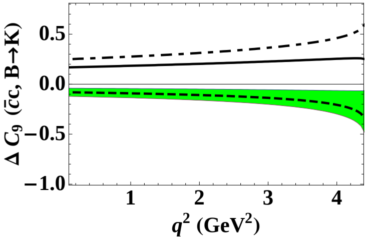
Charm-loop effect in $B \to K\ell^+\ell^-$ in terms of ΔC_9

- the effective coefficient $C_9(\mu = m_b) \simeq 4.4$
  a process-dependent correction to be added:

$$\Delta C_9^{(\bar{c}c, B\to K)}(q^2) = \frac{32\pi^2}{3} \frac{\mathcal{H}^{(B\to K)}(q^2)}{f_{BK}^+(q^2)}$$

$$= (C_1 + 3C_2)\, g(m_c^2, q^2) + 2C_1 \frac{32\pi^2}{3} \frac{\tilde{A}(q^2)}{f_{BK}^+(q^2)}$$

$\Delta C_9(0) = 0.17^{+0.09}_{-0.18}$,
$(\mu = m_b = 4.2\text{GeV})$



Charm-loop effect for $B \to K^* \ell^+ \ell^-$

- factorizable part determined by the three $B \to K^*$ form factors $V^{BK^*}(q^2)$, $A_1^{BK^*}(q^2)$, $A_2^{BK^*}(q^2)$,
- three kinematical structures for the nonfactorizable part:

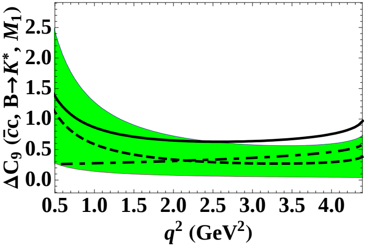
$$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(q^2) = (C_1 + 3C_2)\, g(m_c^2, q^2) - 2C_1 \frac{32\pi^2}{3} \frac{(m_B + m_{K^*})\widetilde{A}_V(q^2)}{q^2 V^{BK^*}(q^2)} ,$$

- nonfactorizable part enhances the effect, $1/q^2$ factor

$\Delta C_9^{(\bar{c}c, B\to K^*, V)}(1.0\mathrm{GeV}^2) = 0.7^{+0.6}_{-0.4}$

$\Delta C_9^{(\bar{c}c, B\to K^*, A_1)}(1.0\mathrm{GeV}^2) = 0.8^{+0.6}_{-0.4}$

$\Delta C_9^{(\bar{c}c, B\to K^*, A_2)}(1.0\mathrm{GeV}^2) = 1.1^{+1.1}_{-0.7}$



Charm-loop effect in $B \to K^*\gamma$

- By-product of our calculation for $B \to K^*\ell^+\ell^-$ at $q^2 = 0$
- factorizable part vanishes,
  nonfactorizable part yields a correction to $C_7^{eff}(m_b) \simeq -0.3$
  in the two inv. amplitudes:

$$C_7^{eff} \to C_7^{eff} + [\Delta C_7^{(\bar{c}c,B\to K^*\gamma)}]_{1,2}\,,$$

$$\left[\Delta C_7^{(\bar{c}c,B\to K^*\gamma)}\right]_1 \simeq \left[\Delta C_7^{(\bar{c}c,B\to K^*\gamma)}\right]_2 = (-1.2^{+0.9}_{-1.6}) \times 10^{-2}\,,$$

- the previous results in the local OPE limit , LCSR with K^* DA:

$$[\Delta C_7^{(\bar{c}c,B\to K^*\gamma)}]_1^{BZ} = (-0.39 \pm 0.3) \times 10^{-2}\,,$$
$$[\Delta C_7^{(\bar{c}c,B\to K^*\gamma)}]_2^{BZ} = (-0.65 \pm 0.57) \times 10^{-2}\,. \qquad (1)$$

[P.Ball, G. W. Jones and R. Zwicky (2007)]

- our result in the local limit is closer to 3-point sum rule estimate: [A.K.,G. Stoll,R. Rueckl,D. Wyler(1997)]

Accessing large q^2 with dispersion relation

- analyticity of the hadronic matrix element in q^2, dispersion relation:

$$\mathcal{H}^{(B\to K)}(q^2) = \mathcal{H}^{(B\to K)}(0) + q^2\Big[\sum_{\psi=J/\psi,\psi(2S)} \frac{f_\psi A_{B\psi K}}{m_\psi^2(m_\psi^2 - q^2 - im_\psi\Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s-q^2-i\epsilon)}\Big]$$

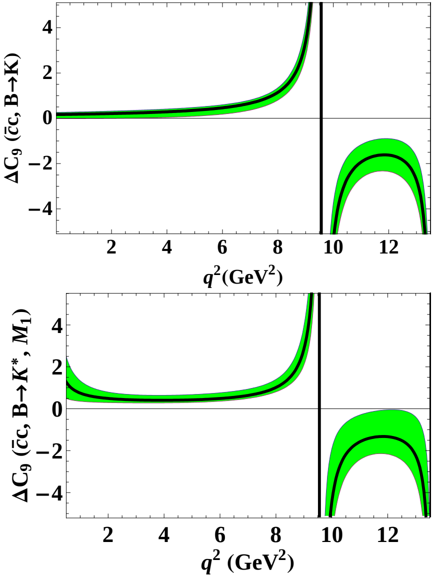
- absolute values of the residues $|f_\psi A_{B\psi K}|$ from exp. data
- the integral over $\rho(s)$ fitted as an effective pole
- previous uses of dispersion relation: only factorizable part, positive residues, k-factors to accomodate data

  [F. Krüger, L. Sehgal (1997); A. Ali, P. Ball, L. T. Handoko, G. Hiller (2000)]

Charm-loop effect at large q^2

solid- central input,
green-shaded - uncertainties

- ► the dispersion relation ansatz coinsides with OPE result at $q^2 < 4.0$ GeV^2 and is valid up to $s = 4m_D^2$ (at $q^2 < m^2_{J/\psi}$ largely independent of higher-states ansatz)



Summary

- soft-gluon emission from a c quark loop in $B \to K^{(*)}\ell^+\ell^-$ - a nonlocal operator,
  effective resummation of local operators,
  $\sim 1/(4m_c^2 - q^2)$-suppression
- LCSR with B meson DAs used to calculate the emerging hadronic matrix element
- accuracy can be improved by including $O(\alpha_s)$ effects
- analytical continuation using dispersion relation and data on $B \to \psi K$ allow to access $q^2 \leq 4m_D^2$
- $q^2 \sim (m_B - m_K)^2$ (low recoil) region not accessible
  $q^2 \to \infty$ a local expansion in $1/q^2$ possible ,
  [B.Grinstein, D. Pirjol (2004); Th. Feldmann, G. Buchalla (2011)]
  but the problem of hadronic matrix elements remains to be solved (no access with LCSR, lattice QCD ?)
- still a lot of work ahead to get a reliable estimate of QCD effects in $B \to K^{(*)}\ell^+\ell^-$, $B \to K^*\gamma$ amplitudes

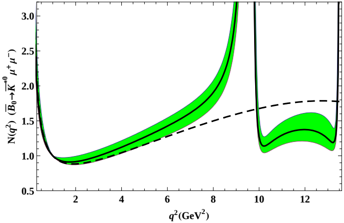
BACKUP

Form factors from LCSR with B-meson DA's

form factor	this work	LCSR with light-meson DA's
		[P.Ball and R.Zwicky] ([Duplancic et al])
$f^+_{B\pi}(0)$	0.25±0.05	0.258±0.031, $(0.26^{+0.04}_{-0.03})$
$f^+_{BK}(0)$	0.31±0.04	0.301±0.041±0.008
$f^T_{B\pi}(0)$	0.21±0.04	0.253±0.028
$f^T_{BK}(0)$	0.27±0.04	0.321±0.037±0.009
$V^{B\rho}(0)$	0.32±0.10	0.323±0.029
$V^{BK^*}(0)$	0.39±0.11	0.411±0.033±0.031
$A_1^{B\rho}(0)$	0.24±0.08	0.242±0.024
$A_1^{BK^*}(0)$	0.30±0.08	0.292±0.028±0.023
$A_2^{B\rho}(0)$	0.21±0.09	0.221±0.023
$A_2^{BK^*}(0)$	0.26±0.08	0.259±0.027±0.022
$T_1^{B\rho}(0)$	0.28±0.09	0.267±0.021
$T_1^{BK^*}(0)$	0.33±0.10	0.333±0.028±0.024

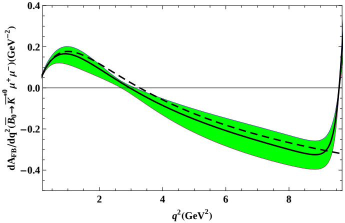
Observables for $B \to K^* \ell^+ \ell^-$

- differential distribution in q^2 with (solid) and without (dashed) charm-loop effect



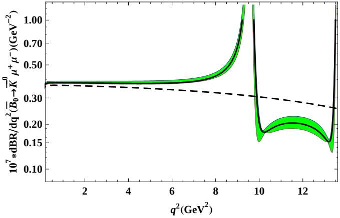
- forward-backward asymmetry : $q_0^2 = 2.9^{+0.2}_{-0.3}\mathrm{GeV}^2$

$\sim$ 10% larger without nonfactorizable correction



Influence on the observables for $B \to K\ell^+\ell^-$

- adding $\delta C_9(q^2)$ to the decay amplitude
- differential distribution in q^2 with (solid) and without (dashed) charn-loop effect



Summary

- other similar loop effects accessible with the same method
- the role of soft gluons in "weak annihilation" to be studied