知床  Fall
Critical behavior in QED3 and the effects of Chern-Simon term

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1 Infrared dominance (dispersion like model)

Our main goal is to determine $S_F$ and $\rho$

$$S_F(p) = \int ds \frac{\rho_1(s) \gamma \cdot p + m \rho_2(s)}{p^2 - s + i\epsilon}.$$  \hspace{1cm} (1)

In QED$_3$ with 4-spinor, there are only infrared divergences. We may have the following type divergence

$$f = \frac{e^2}{p} \ln \left( \frac{m^2 - p^2}{4m^2} \right)$$ \hspace{1cm} (2)

and their summation

$$\frac{1}{m - \gamma \cdot p} \sum_{n=0}^{\infty} \frac{1}{n!} (\ln(f))^n \rightarrow \frac{1}{m - \gamma \cdot p} \exp(f) \rightarrow \frac{1 - \gamma \cdot p/m}{}.$$ \hspace{1cm} (3)

likely $\beta = -1$ may be realized in QED$_3$

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soft photon emission

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For two photon emission process, \( f \) is given by

\[
f = \frac{e^2}{(2\pi)^2} \int d^3k \delta(k^2) \theta(k_0) \sum_s T_1 \bar{T}_1 \tag{4}
\]

For evaluation of \( f \), there are some ways. Our method is based on dispersion-like model. The spectral function of 4-component fermion and photon propagator are defined as

\[
S_F(p) = -i \int d^3x e^{-ip \cdot x} \int \frac{d^2r}{(2\pi)^2} \frac{e^{ir \cdot x} e^F(x)}{\sqrt{r^2 + m^2}} \tag{5}
\]

\[
T_1 = -ie \frac{\epsilon_\mu(k, \lambda)}{\gamma \cdot (p + k) - m} \gamma^\mu \exp(i(p + k) \cdot x) U(p, s). \tag{6}
\]

We show \( O(e^2) \) perturbative results

\[
\frac{\gamma \cdot p A(p) + m B(p)}{2m} = -\sum_{\lambda, s} T_1 \bar{T}_1, \tag{7}
\]

\[
B(p) = \frac{(d + 1)m^2}{2(p \cdot k)^2} + \frac{d}{p \cdot k} + \frac{(d - 1)}{2k^2}, \tag{8}
\]

\[
A(p) = \frac{dm^2}{(p \cdot k)^2} + \frac{d}{p \cdot k} + \frac{(d - 1)}{k^2}. \tag{9}
\]

However after exponentiation it may change by the condition

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\[ p^2 = m^2 \]

\[ F = -e^2 \int \frac{d^3k}{(2\pi)^2} \frac{e^{ik \cdot x}}{\sqrt{k^2 + \theta^2}} \frac{(\gamma \cdot p + m)}{2m} \times \left\{ \frac{3d + 1}{4} \frac{m^2}{(p \cdot k)^2} + \frac{d}{p \cdot k} + \frac{3(d - 1)}{4k^2} \right\} \]  

(10)

Everything is simple in Feynman gauge \( d = 1 \).

1.1 explicit form of \( F \)

Here we use the retarded propagator to derive the function \( F \)

\[ D_+(x) = \int \frac{d^3k}{(2\pi)^2} \exp(ik \cdot x) \theta(k_0) \delta(k^2 - \theta^2) = \frac{\exp(-\theta \sqrt{-x^2})}{8\pi i \sqrt{-x^2}} \]

(11)

Two types of integral

\[ F_1 = -\int \frac{d^3k}{(2\pi)^2} \exp(ik \cdot x) \theta(k_0) \delta(k^2 - \theta^2) \frac{1}{p \cdot k}, \]

\[ F_2 = -\int \frac{d^3k}{(2\pi)^2} \exp(ik \cdot x) \theta(k_0) \delta(k^2 - \theta^2) \frac{1}{(p \cdot k)^2}, \]

can be expressed by parameter integral for the \( D_+(x) \) function by

\[ m \int_0^\infty D_+(x + \alpha p) d\alpha = -F_1, \]

\[ m^2 \int_0^\infty \alpha D_+(x + \alpha p) d\alpha = -F_2 \]
The function $F$ for pure QED$_3$ is
\[
F = \frac{e^2}{8\pi} \left[ \exp\left(-\theta|x|\right) - \frac{\theta|x|E_1(\theta|x|)}{\theta} - \frac{E_1(\theta|x|)}{m} \right].
\]

2 Phase structures

For short and long distance we have the approximate form of the function $F$
\[
F_S \sim A - \theta|x| + (D + C|x|) \ln(\theta|x|))
\]
\[
- \frac{(1 - 2\gamma)e^2|x|}{16\pi}, (\theta|x| \ll 1),
\]
\[
F_L \sim 0, (1 \ll \theta|x|).
\]

Most important
\[
\exp(F) = A(\theta|x|)^{D+C|x|}(\theta|x| \ll 1),
\]
\[
A = \exp\left(\frac{e^2}{16\pi\theta} + \frac{e^2\gamma}{8\pi m}\right),
\]
\[
C = \frac{e^2}{8\pi}, D = \frac{e^2}{8\pi m},
\]

where $m$ is the physical mass and $\gamma$ is an Euler’s constant. Here we see $D$ acts as anomalous dimension $\beta$. For $D = 1$, $S_F(0)$ is finite and we have $\langle \bar{\psi}\psi \rangle \neq 0$. Thus if we require $D = 1$, we obtain $m = e^2/8\pi$. Linear infrared divergence is avoided by vacuum polarization. $\langle \bar{\psi}\psi \rangle = -5 \times 10^{-3}e^4$ for weak coupling. There is
some infrared enhancement by \( \exp(\gamma) \approx 1.78 \) in comparison with S-D equation with vertex correction and vacuum polarization to satisfy Ward-Takahashi identity.

2.1 Critical behaviour of \( \langle \bar{\psi}\psi \rangle \)

In QED\(_3\) there is no infrared cut-off. We supply this by vacuum polarization of massive fermion loop.

\[
\rho_\gamma(s) = \frac{-1}{\pi} \text{Im} \left( \frac{1}{s - \Pi(s)} \right),
\]

\[
\Pi(k^2) = -\frac{e^2}{8\pi} \left( (\sqrt{-k^2} + \frac{4m^2}{\sqrt{-k^2}}) \ln \left( \frac{2m + \sqrt{-k^2}}{2m - \sqrt{-k^2}} \right) - 4m \right)
\]

Photon propagator and the function \( \exp(F(x, \mu)) \) are modified to

\[
D_F(p) = \int_{4m^2}^{\infty} \frac{ds \rho_\gamma(s)}{p^2 - s + i\epsilon} \exp(F(x)) = \int ds \rho_\gamma(s) \exp(F(x, \sqrt{s})).
\]

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3 Spectral function $\rho(s)$

We evaluate the spectral function $\rho(s)$ from function from $\exp(\tilde{F}(x))$

$$\rho(s) = \frac{1}{m^2} \int_{\infty}^{\infty} \frac{dx}{2\pi} \exp(i(s - 1) \cdot x) \exp(\tilde{F}(\frac{x}{m^2}))$$

(18)

$$\rho_{\lambda}(s), \ N = 1.$$  

In the above figure we see that the non-pole contribution which oscillates in the large $s$ region. $((x\theta)^{cf}$ part)
4 Effects of Chern-Simon term

In the same way we add the contribution of Chern-Simon term

$$\Delta L = \frac{\theta}{4} \epsilon^{\mu\nu\rho} A_\rho F_{\mu\nu}. \quad (24)$$

For simplicity we consider the 2-component spinors in (7). The term which depends on Chern-Simon has no infrared divergences. $O(e^2)$ contribution of Chern-Simon term to the fermion spectral function is

$$\sum_s \frac{T_s \overline{T}_s}{s} \frac{e^2}{8\pi 2m} \left( \gamma \cdot (p + k) + m \right) \gamma^\mu (\gamma \cdot p + m) \gamma^\nu$$

$$\times \left( \gamma \cdot (p + k) + m \right) \frac{-ie^{\mu\nu\rho} k_\rho}{4(p \cdot k)^2 \theta}$$

$$= \frac{e^2}{16\pi m \theta} (\gamma \cdot p + m). \quad (25)$$

This term has no one particle singularity. We see that this works for mass changing effects and mass generation. Solving Dyson-Schwinger equation for fermion self-energy, we obtained the solution which behaves as $1/p$ at high-energy.

4.1 S-D equation with Topological mass

The the vacuum expectation value of spin density $\langle \overline{\psi} \psi \rangle$ with finite $\theta$ has logarithmic divergence. This fact is numerically

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shown. Dyson-Schwinger equation is

\[ S^{-1}(p) = S^{0-1}(p) - ie^2 \int \frac{d^3 p'}{(2\pi)^3} \gamma^\mu S(p) \gamma^\nu D^0_{\mu\nu}(k). \]  

(21)

\[ B(p) = \frac{e^2}{8\pi^2} \int_0^\infty dp' \frac{p'^2}{B^2(p') + p'^2 A^2(p')} \times \{ B_\alpha(p, p', \mu) B(p') + B_\beta(p, p', \mu) A(p') \}, \]

(22)

\[ A(p) = 1 + \frac{e^2}{16\pi^2 p^2} \int_0^\infty dp' \frac{p'^2}{B^2(p') + p'^2 A^2(p')} \times \{ A_\alpha(p, p', \mu) B(p') + A_\beta(p, p', \mu) A(p') \}, \]

(23)

\[ B_\alpha(p, k, \mu) = \frac{2}{pk} [(a + 2)L_0(p, k) + L_1(p, k, \mu)], \]

(24)

\[ B_\beta(p, k, \mu) = \frac{\mu}{pk} [2L_0(p, k) + \frac{p^2 - k^2 + \mu^2}{\mu^2} L_1(p, k, \mu)], \]

(25)

\[ A_\alpha(p, k, \mu) = \frac{2\mu}{pk} [2L_0(p, k) - \frac{p^2 - k^2 - \mu^2}{\mu^2} L_1(p, k, \mu)], \]

(26)

\[ A_\beta(p, k, \mu) = 4(a - 1) + \frac{2}{pk} (\mu^2 - a(p^2 + k^2) L_0(p, k) + \frac{\mu^4 - (p^2 - k^2)^2}{\mu^2 pk} L_1(p, k; \mu), \]

(27)
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\[ L_0(p, k) = \ln \left| \frac{p + k}{p - k} \right|, \]

\[ L_1(p, k, \mu) = \ln \left| \frac{1 + \mu^2 / (p + k)^2}{1 + \mu^2 / (p - k)^2} \right|. \]

(28)

(29)

take \( A(p) \equiv 1, B(p) = m(p) \). The \( m \)-equation now is

\[ m(p) = 2e^2 \int \frac{d^3p'}{(2\pi)^3} \frac{m(p') - \mu (p' \cdot k) / k^2}{(k^2 + \mu^2)(p'^2 + m^2)}. \]

(30)

Second term in the RHS is order \( 1/p \) and perturbative. First term has iterated to \( 1/p^2 \). Thus we expect order parameter of the spin density

\[ \langle \bar{\psi} \psi \rangle \propto -\ln \Lambda_{UV}. \]

(31)

5 Summary

Summing infrared divergents term of the spectral function we obtain the full propagator. This has the similar properties of KT phase transition with anomalous dimension. Vacuum polarization has the role of effective infrared cut-off. If we introduce Chern-Simons term drastically modified.

6 References


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