Local and quasi-local non-Gaussianity from inflation

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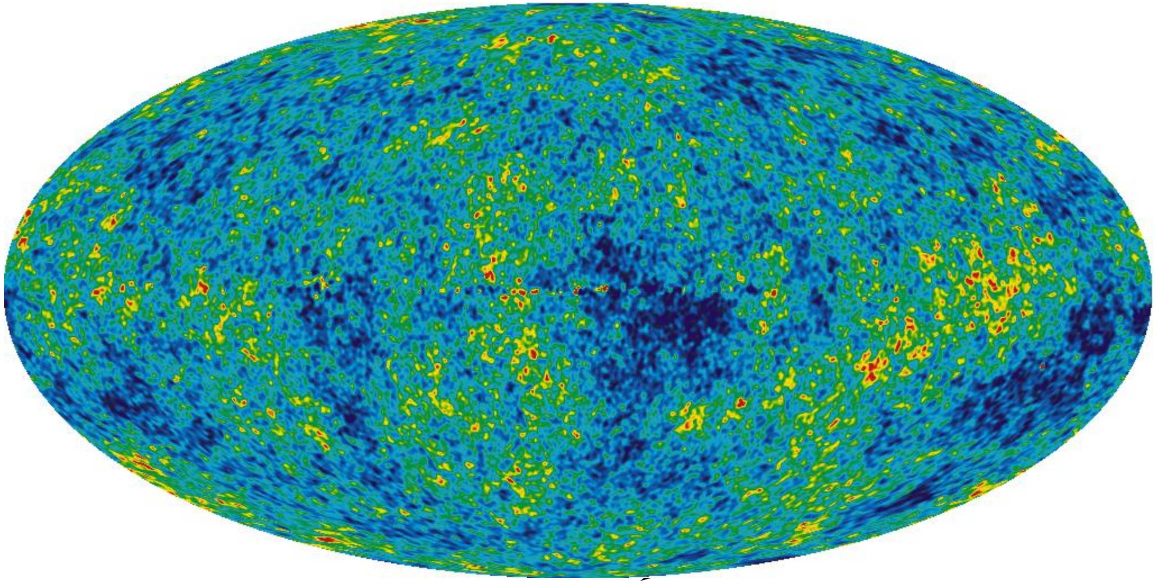
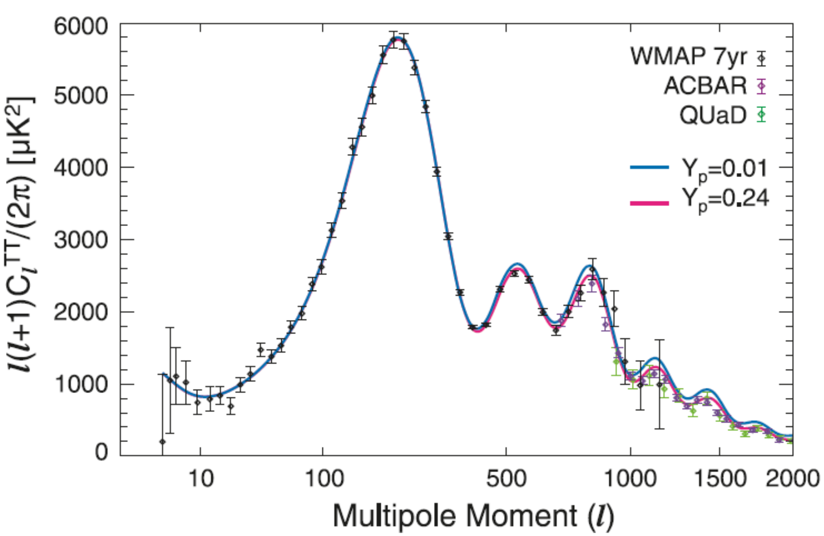
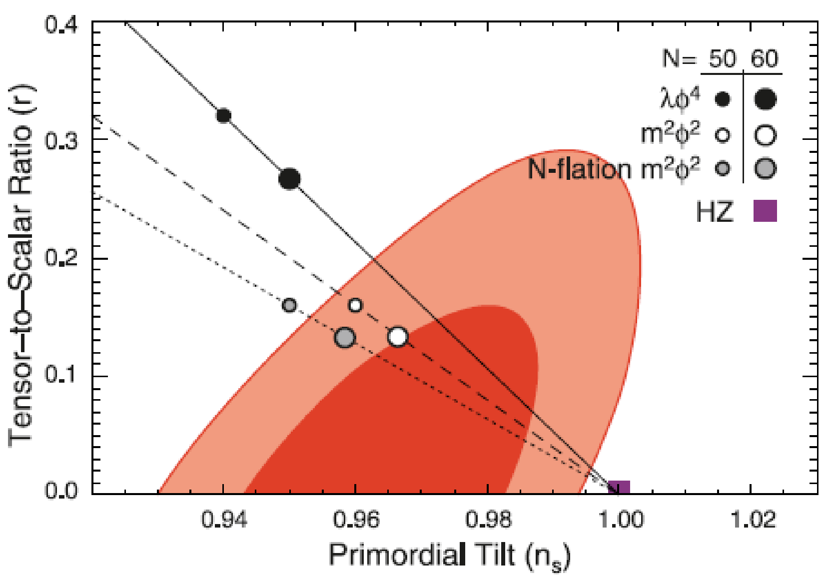
work with Chris Byrnes, Jose Fonseca, Kazuya Koyama, David Langlois, David Lyth, Shuntaro Mizuno, Misao Sasaki, Gianmassimo Tasinato, Jussi Valiviita, Filippo Vernizzi…

review: Classical & Quantum Gravity 27, 124002 (2010)
arXiv:1004.0818

pre-Planckian standard model of primordial cosmology

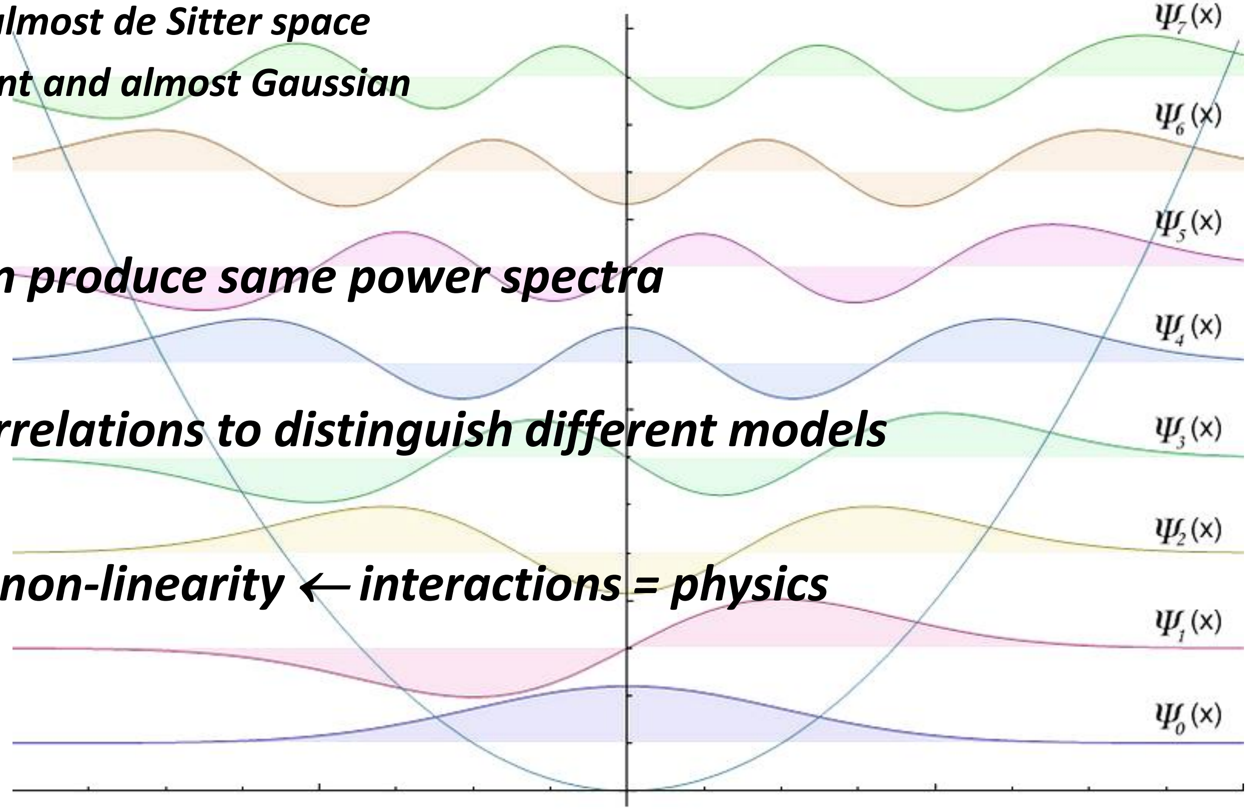






Primordial Gaussianity from inflation

- **Quantum fluctuations from inflation**
  - ***ground state of simple harmonic oscillator***
  - ***almost free field in almost de Sitter space***
  - ***almost scale-invariant and almost Gaussian***



- ***Different models can produce same power spectra***

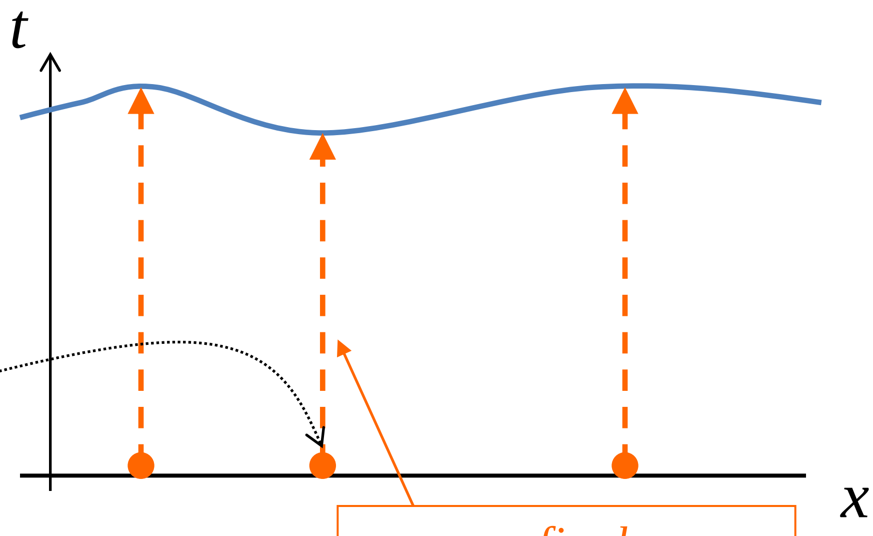
- ***Use higher-order correlations to distinguish different models***

- ***Non-Gaussianity $\leftarrow$ non-linearity $\leftarrow$ interactions = physics***

Wikipedia: AllenMcC

Primordial density perturbations quantum fluctuations

ζ = curvature perturbation on uniform-density hypersurface in radiation-dominated era



$$N = \int_{initial}^{final} H\, dt$$

during inflation
field perturbations $\phi(x,t_i)$ on initial spatially-flat hypersurface

on large scales, neglect spatial gradients, solve as "separate universes"

$$\zeta = N(\phi_{initial}) - \bar{N} \approx \sum_I \frac{\partial N}{\partial \phi_I} \delta\phi_I$$

Starobinsky \`85; Sasaki & Stewart \`96

Lyth & Rodriguez '05 – works to any order

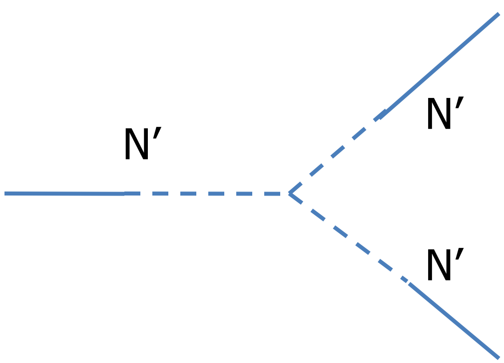
the δN formalism order by order at Hubble exit

$$\delta\phi_I = \delta_1\phi_I + \frac{1}{2}\delta_2\phi_I + \ldots$$

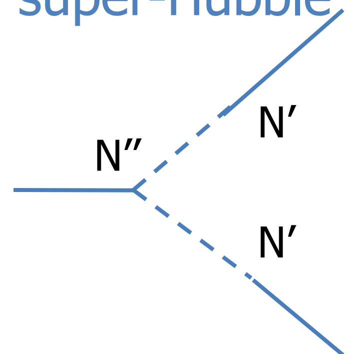
$$\zeta = \zeta_1 + \frac{1}{2}\zeta_2 + \ldots$$

$$= \left[\sum_I \frac{\partial N}{\partial \phi_I}\delta_1\phi_I\right] + \frac{1}{2}\left[\sum_I \frac{\partial N}{\partial \phi_I}\delta_2\phi_I + \sum_{I,J}\frac{\partial^2 N}{\partial\phi_I\partial\phi_I}\delta_1\phi_I\delta_1\phi_J\right] + \ldots$$

sub-Hubble quantum interactions



super-Hubble classical evolution



Byrnes, Koyama, Sasaki & DW (arXiv:0705.4096)

simplest local form of non-Gaussianity

applies to many models inflation including curvaton, modulated reheating, etc

$\zeta = \zeta(\delta\phi)$ is local function of ***single Gaussian random field***, $\delta\phi(x)$

$$\zeta(x) = N'\delta\phi(x) + \frac{1}{2}N''\left(\delta\phi(x)\right)^2 + \ldots$$

$$\Rightarrow \quad \langle\zeta(x_1)\zeta(x_2)\rangle = N'^2\langle\delta\phi(x_1)\delta\phi(x_2)\rangle + \ldots$$

$$\langle\zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle = \frac{1}{2}N'^2N''\langle\delta\phi(x_1)\delta\phi(x_2)\delta\phi^2(x_3)\rangle + \ldots$$

$$= \frac{3}{5}f_{NL}\langle\zeta(x_2)\zeta(x_3)\rangle\langle\zeta(x_1)\zeta(x_3)\rangle + \ldots$$

where $\boxed{f_{NL} = \frac{5}{6}\frac{N''}{(N')^2}}$

- odd factors of 3/5 because (Komatsu & Spergel, 2001, used) $\Phi_1 = (3/5)\zeta_1$

Simplest local form of non-Gaussianity to third order

$$\zeta = \zeta_1 + \frac{3}{5} f_{NL} \zeta_1^2 + \frac{9}{25} g_{NL} \zeta_1^3 + \cdots$$

$\Longrightarrow$ bispectrum

$$B_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{6}{5} f_{NL} \left[P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right]$$

trispectrum

$$\begin{aligned} T_\zeta(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) &= \tau_{NL} \left[P_\zeta(k_{13}) P_\zeta(k_3) P_\zeta(k_4) + (11 \text{ perms}) \right] \\ &\quad + \frac{54}{25} g_{NL} \left[P_\zeta(k_2) P_\zeta(k_3) P_\zeta(k_4) + (3 \text{ perms}) \right] \end{aligned}$$

where we have two independent parameters from δN calculation

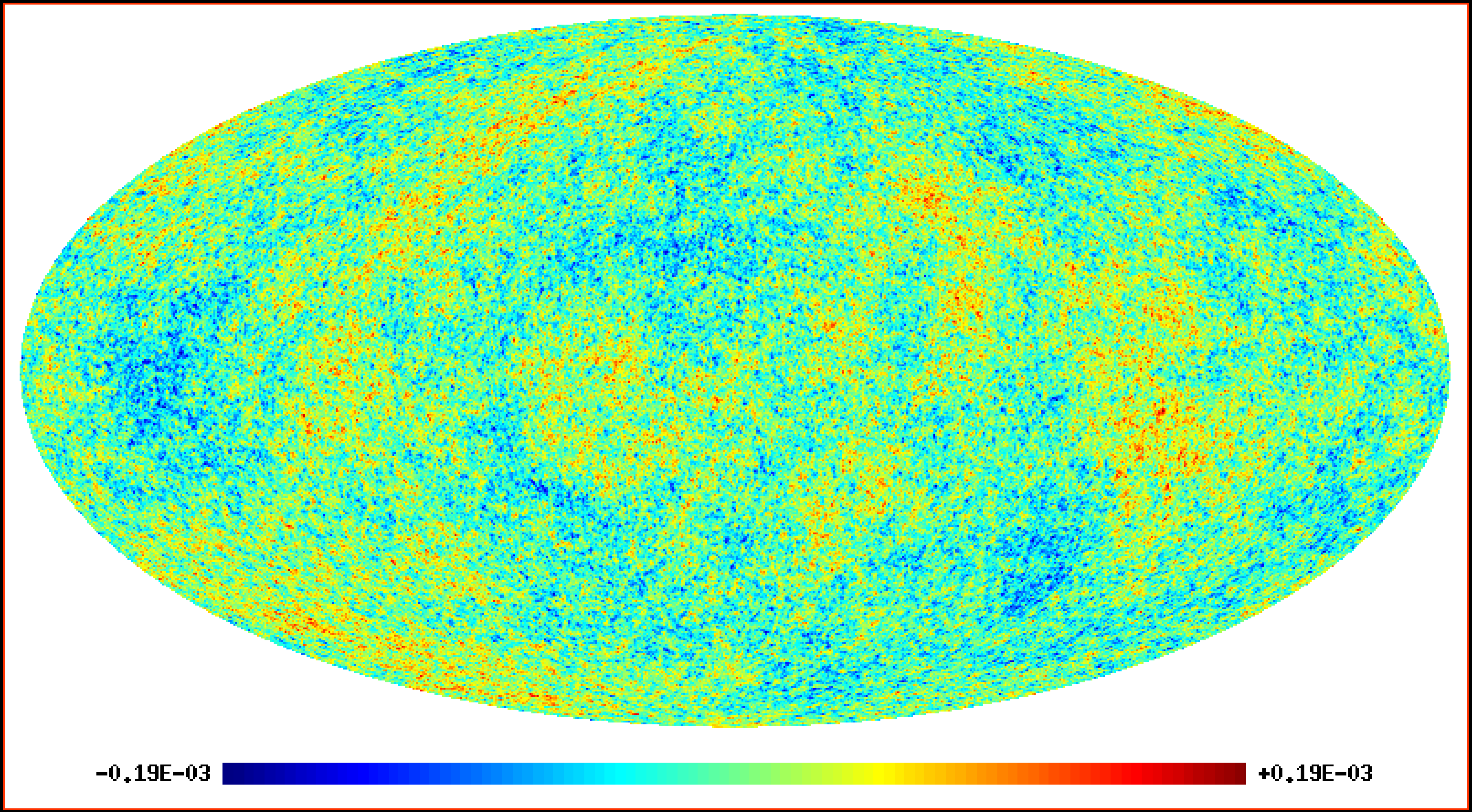
$\boxed{f_{NL} = \frac{5}{6} \frac{N''}{(N')^2}}$ $\boxed{g_{NL} = \frac{25}{54} \frac{N'''}{(N')^3}}$ and $\boxed{\tau_{NL} = \frac{(N'')^2}{(N')^4} = \frac{36}{25} f_{NL}^2}$

note: - both f_{NL} and g_{NL} appear at leading order in trispectrum

- have different k-dependence $P(k) \propto 1/k^3$

Newtonian potential a ***Gaussian random field***

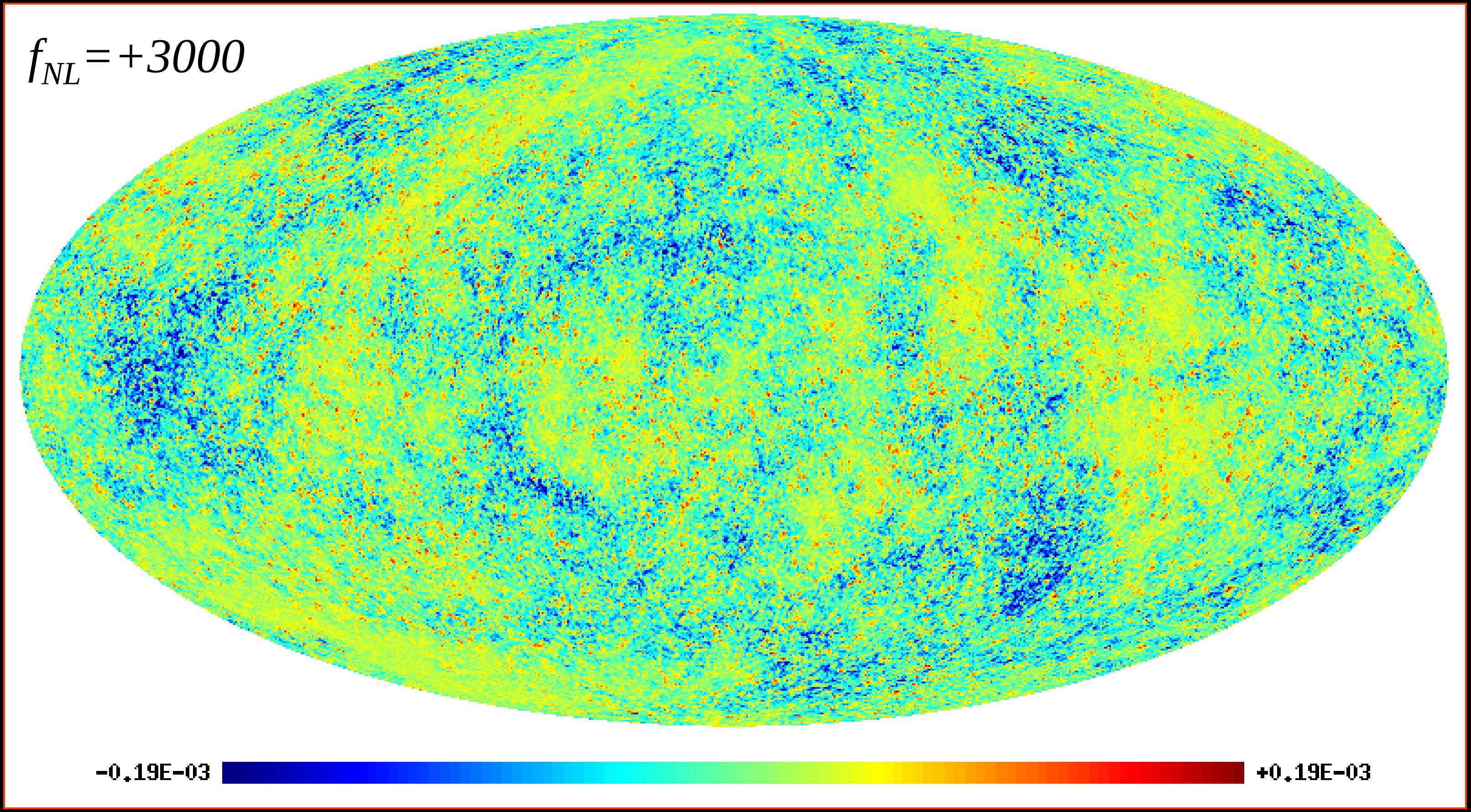
$$\Phi(x) = \phi_G(x)$$



Liguori, Matarrese and Moscardini (2003)

Newtonian potential ***a local function of Gaussian random field***

$$\Phi(x) = \phi_G(x) + f_{NL} \, (\phi_G^2(x) - <\phi_G^2>)$$

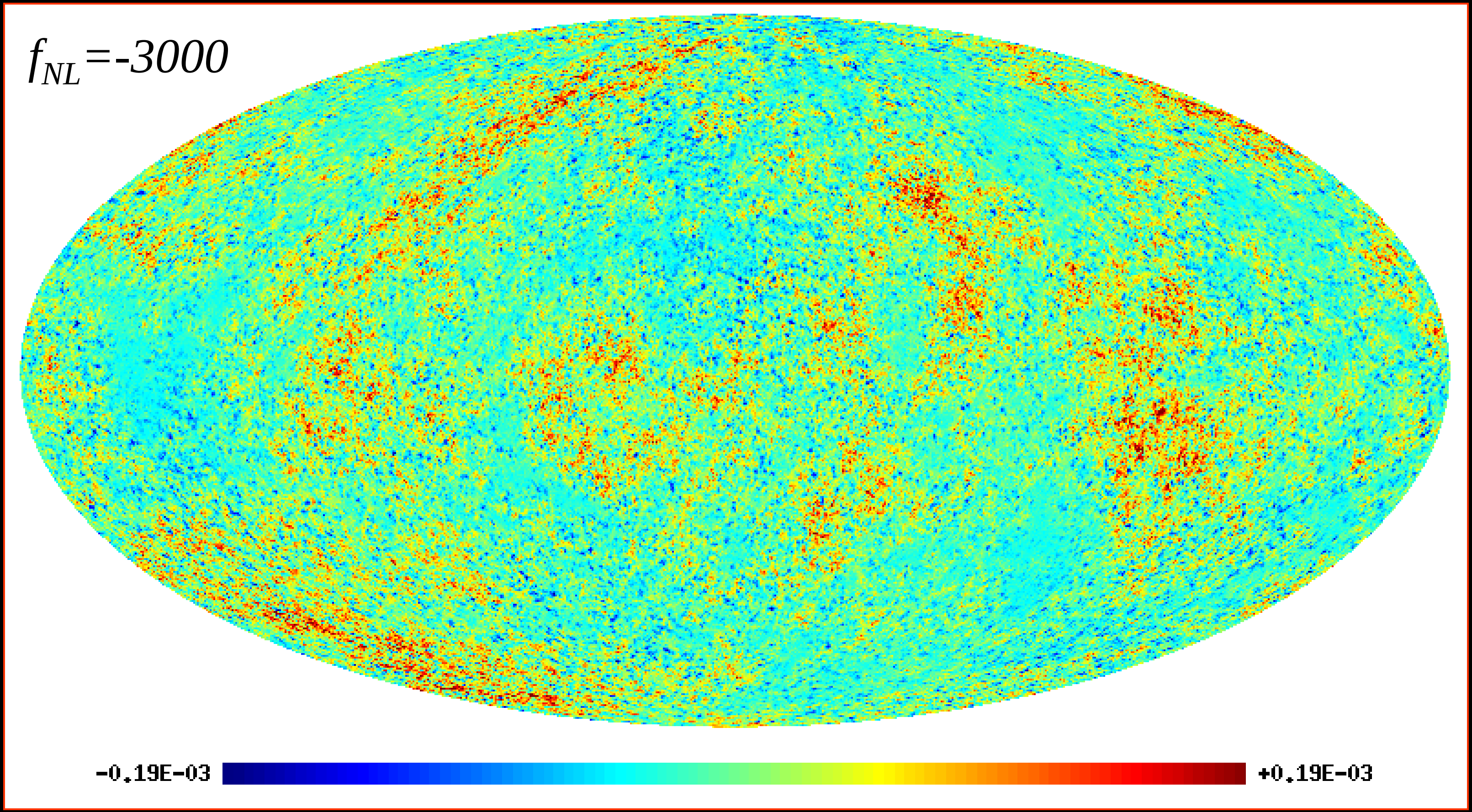


$\Delta T/T \approx -\Phi/3$, so positive $f_{NL} \Rightarrow$ more cold spots in CMB

Liguori, Matarrese and Moscardini (2003)

Newtonian potential ***a local function of Gaussian random field***

$$\Phi(x) = \phi_G(x) + f_{NL}\left(\phi_G^2(x) - \langle \phi_G^2 \rangle \right)$$



$\Delta T/T \approx -\Phi/3$, so negative $f_{NL} \Rightarrow$ more hot spots in CMB

Liguori, Matarrese and Moscardini (2003)

Newtonian potential a local function of ***Gaussian random field***

$$\Phi(x) = \phi_G(x) + f_{NL} \, (\, \phi_G^{\,2}(x) - < \phi_G^{\,2} > \,)$$

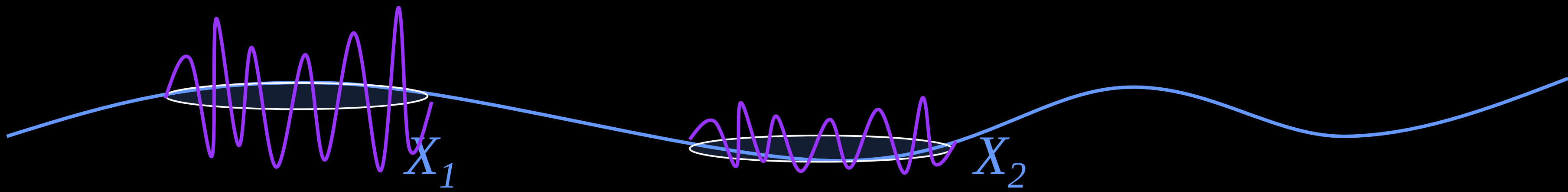
$\Rightarrow$ ***Large-scale modulation of small-scale power***

split Gaussian field into long (L) and short (s) wavelengths

$$\phi_G \, (X+x) = \phi_L(X) + \phi_s(x)$$

two-point function on small scales for given ϕ_L

$$< \Phi(x_1) \, \Phi(x_2) >_L = (1+4 \, f_{NL} \, \phi_L \,) < \phi_s \, (x_1) \, \phi_s \, (x_2) > + \ldots$$

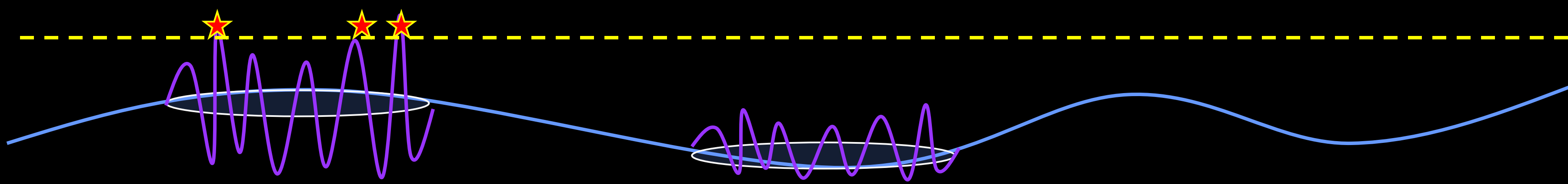


i.e., ***inhomogeneous modulation of small-scale power***

$$\mathbf{P \, (\, k \, , X \,) \, -> [\, 1 + 4 \, f_{NL} \, \phi_L(X) \,] \, P_s(k)}$$

peak – background split for galaxy bias *BBKS'87*

Local density of galaxies determined by number of peaks in density field above threshold => leads to galaxy bias



Poisson equation relates primordial density to Newtonian potential

$$\nabla^2 \Phi = 4\pi G \delta\rho \quad => \quad \phi_L = (3/2)\,(\,aH / k_L\,)^2\, \delta_L$$

so **local** $\Phi(x) \Rightarrow$ ***non-local form*** for primordial density field $\delta(x)$ from

+ inhomogeneous modulation of small-scale power

$$P_\delta(\,k\,, X\,) = [\,1 + 4\, f_{NL}\, \phi_L(X)\,]\, P_{\delta s}(k)$$

$\Rightarrow$ ***strongly scale-dependent bias on large scales***

Dalal et al, arXiv:0710.4560

Constraints on local non-Gaussianity

- WMAP CMB constraints using estimators based on optimal templates:
  - $-10 < f_{NL} < 74$ (95% CL) Komatsu et al WMAP7
- LSS constraints from galaxy power spectrum on large scales:
  - $-29 < f_{NL} < 70$ (95% CL) Slosar et al 2008
  - $27 < f_{NL} < 117$ (95% CL) Xia et al 2010 [NVSS survey of AGNs]

non-Gaussianity from inflation?

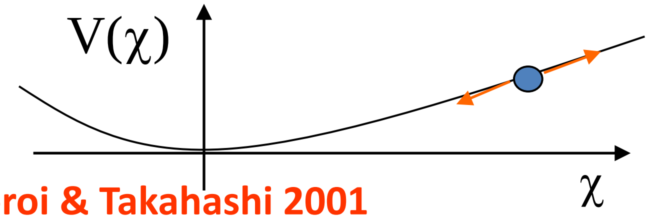
- **single slow-roll inflaton field**
  - adiabatic perturbations => ζ constant on large scales
  - during conventional slow-roll inflation $f_{NL}{}^{local} \approx N'' / N'^2 = O(\varepsilon) << 1$

- **sub-Hubble interactions**
  - e.g. DBI inflation, Galileon fields... $f_{NL}{}^{equil} \approx 1 / c_s{}^2$

- **super-Hubble evolution**
  - non-adiabatic perturbations during inflation => $\zeta \neq$ constant
  - many different models (during inflation, modulated reheating...)
  - e.g., **curvaton**

curvaton scenario:

Linde & Mukhanov 1997; Enqvist & Sloth, Lyth & Wands, Moroi & Takahashi 2001



curvaton χ = a weakly-coupled, late-decaying scalar field

- light during inflation (m<<H) hence acquires an almost scale-invariant, *Gaussian distribution of field fluctuations* on large scales
- energy density for massive field, $\rho_\chi = m^2\chi^2/2$
- spectrum of initially isocurvature density perturbations

$$\zeta_\chi \approx \frac{1}{3}\frac{\delta\rho_\chi}{\rho_\chi} \approx \frac{1}{3}\left(\frac{2\chi\delta\chi + \delta\chi^2}{\chi^2}\right)$$

- transferred to radiation when curvaton decays with some efficiency, $0<r<1$, where $r \approx \Omega_{\chi,decay}$

$$\zeta = r\zeta_\chi \approx \frac{r}{3}\left(2\frac{\delta\chi}{\chi} + \frac{\delta\chi^2}{\chi^2}\right)$$

$$= \zeta_G + \frac{3}{4r}\zeta_G{}^2 \quad \Rightarrow \quad f_{NL} = 5/4r$$

scale-dependence of f_{NL}?

Byrnes, Choi & Hall 2009
Khoury & Piazza 2009
Sefusatti, Liguori, Yadav, Jackson & Pajer 2009

Byrnes, Nurmi, Tasinato & Wands (2009); Byrnes, Gerstenlauer, Nurmi, Tasinato & Wands (2010)

- power spectrum $$P_\zeta(k) = \left[N'^2 P_{\delta\varphi}\right]_{k=aH}$$

  $\Rightarrow$ scale-dependence $$n_\zeta - 1 \equiv \frac{d \ln P_\zeta}{d \ln k} = H^{-1}\frac{d \ln N'^2}{dt} - 2\varepsilon$$

- bispectrum $$f_{NL}(k) = \frac{5}{6}\left[\frac{N''}{N'^2}\right]_{k=aH}$$

  $\Rightarrow$ scale-dependence $$n_{fNL} \equiv \frac{d \ln |f_{NL}|}{d \ln k} = \frac{N'}{N''}\left(\sqrt{2\varepsilon}(4\varepsilon - 3\eta) + \frac{V'''}{3H^2}\right)$$

- e.g., curvaton

$$\boxed{n_{fNL} = \frac{N'}{N''}\left(\frac{V'''}{3H^2}\right)}$$

scale-dependence probes self-interaction, not probed by power spectrum

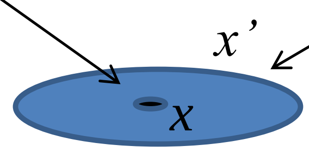
could be observable for curvaton models where $g_{NL} \sim \tau_{NL}$ (Byrnes et al 2011)

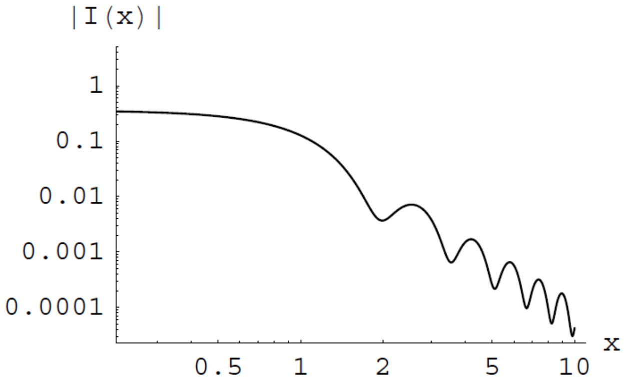
quasi-local model for scale-dependent f_{NL}

Byrnes, Gerstenlauer, Nurmi, Tasinato & Wands (2010)

- Fourier space: $$f_{NL}(k) = f_{NL}(k_p)\left(1 + n_{fNL} \ln\left(\frac{k}{k_p}\right)\right)$$
- quasi-local non-Gaussianity in real space:

$$\zeta(x) = \zeta_1(x) + \frac{3}{5} f_{NL} \zeta_1^{\,2}(x) + \frac{3}{5} n_{fNL} f_{NL} \int d^3x' \, \zeta_1^{\,2}(x') \, I(x - x')$$





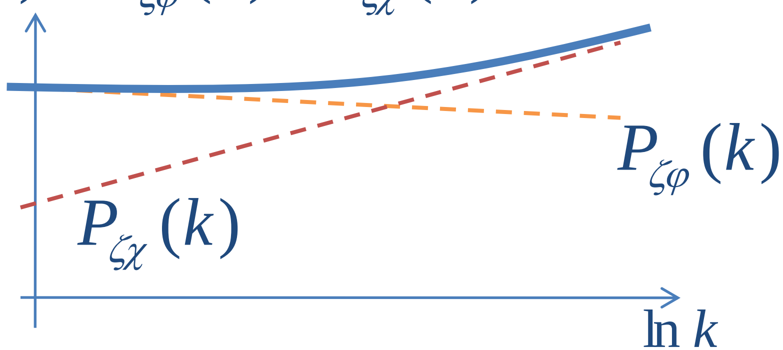
scale-dependent f_{NL} from a local two-field

Byrnes, Nurmi, Tasinato & Wands (2009)

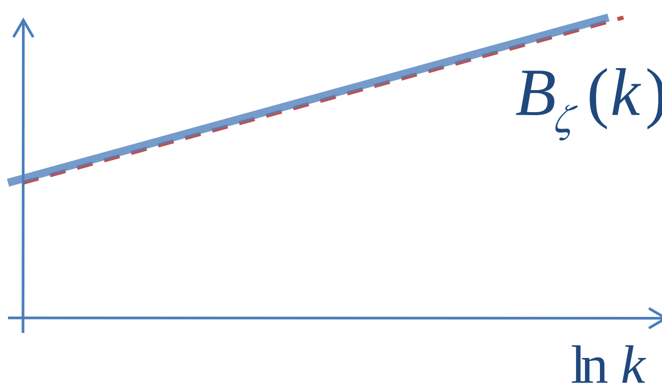
$$\boxed{\zeta(x) = \zeta_\varphi(x) + \zeta_\chi(x) + \frac{3}{5} f_{\chi\chi} \zeta_\chi{}^2(x)}$$

- power spectrum

$$P_\zeta(k) = P_{\zeta\varphi}(k) + P_{\zeta\chi}(k)$$



- bispectrum



$$B_\zeta(k) = B_{\zeta\chi}(k)$$

$$\rightarrow f_{NL}(k) \approx B_\zeta(k) \Big/ \left(P_\zeta(k)\right)^2$$

local two-field scale-dependent f_{NL}

Byrnes, Nurmi, Tasinato & Wands (2009)

$$\boxed{\zeta(x) = \zeta_\varphi(x) + \zeta_\chi(x) + \frac{3}{5} f_{\chi\chi} \zeta_\chi{}^2(x)}$$

- power spectrum $P_\zeta(k) = P_{\zeta\varphi}(k) + P_{\zeta\chi}(k)$

- bispectrum $f_{NL} = w_\chi{}^2(k) f_{\chi\chi}$ where $w_\chi(k) = P_{\zeta\chi}(k) \Big/ P_\zeta(k)$

scale-dependence $n_{fNL} = 2\left(n_{\zeta\chi} - n_\zeta\right)$

- e.g., inflaton + non-interacting curvaton

$$\boxed{n_{fNL} \equiv \frac{d \ln |f_{NL}|}{d \ln k} = 4(1 - w_\chi)\left(2\varepsilon + \eta_\chi - \eta_\varphi\right)}$$

for CMB+LSS constraints on this model see Tseliakhovich, Hirata & Slosar (2010)

outlook:

- **Great potential for discovery**
  - any nG close to current bounds would kill 95% of all known inflation models
  - requires multiple fields and/or unconventional physics
- **Scope for more theoretical ideas**
  - infinite variety of non-Gaussianity
  - new theoretical models require new optimal (and sub-optimal) estimators
- **More data coming**
  - final WMAP, Planck (end of 2012) + large-scale structure surveys
- **Non-Gaussianity will be detected**
  - non-linear physics inevitably generates non-Gaussianity
  - need to disentangle primordial and generated non-Gaussianity