Galilean symmetry in the EFT of inflation: new shapes of NG

Guido D'Amico

CCPP NYU

based on:
P. Creminelli, G. D'A., J. Noreña, M. Musso, E. Trincherini, ***JCAP*** **1102:006** [arXiv:**1011.3004**]

Outline

- Effective theory of inflation
- Galilean symmetry and action for perturbations
- Cubic lagrangian: new shapes of non-Gaussianities
- Observational limits
- Four-point function
- A better template for data analysis

Standard approach

Usual approach to inflation:

1) Build a Lagrangian for a scalar field: $\mathcal{L}(\phi, \partial_\mu\phi, \Box\phi, \ldots)$

2) Solve EOM of scalar + FRW to find an inflating solution $\ddot{a} > 0$

$$\phi = \phi_0(t) \qquad\qquad \mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2$$

3) Expand in perturbations around this solution

$$\phi(t, \vec{x}) = \phi_0(t) + \delta\phi(t, \vec{x}) \qquad\qquad g_{\mu\nu} = g_{\mu\nu}^{\mathrm{FRW}} + \delta g_{\mu\nu}$$

4) Solve equations, work out predictions

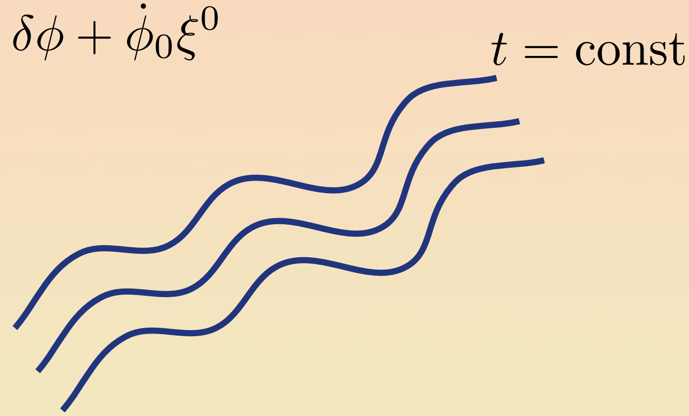
The EFT for inflation

Cheung et al. 2007

We can focus directly on the theory of perturbations around quasi de Sitter background, no need to solve for it!

- Bkg solution spontaneously breaks time diffs

$$t \to t + \xi^0(t, \vec{x}) \quad \Rightarrow \quad \delta\phi \to \delta\phi + \dot{\phi}_0 \xi^0$$



- Can choose unitary gauge $\delta\phi = 0$

The graviton describes 2+1 dofs,
like in a broken gauge theory.

- Action in unitary gauge is a sum of invariants of the 3D time slices:

$$S = \int \mathrm{d}t\mathrm{d}^3x\sqrt{-g}\Big[\boxed{\frac{M_{\rm Pl}^2}{2}R + M_{\rm Pl}^2\dot{H}g^{00} - M_{\rm Pl}^2(3H^2+\dot{H})}$$
$$\boxed{+\frac{M_2^4(t)}{2}(g^{00}+1)^2 + \frac{M_3^4(t)}{2}(g^{00}+1)^3} + \cdots \boxed{-\frac{\bar{M}_2^2(t)}{2}\delta K^2} + \ldots\Big]$$

Slow-roll

DBI, P(X)

Ghost infl.

Reintroducing the Goldstone

Stueckelberg trick: do a broken time diff and promote the parameter to a field

$$t \to \tilde{t} = t + \xi^0(x) \qquad \xi^0(x) \to -\pi(x)$$

Simple example (slow-roll inflation):

$$\int d^4x \sqrt{-g}\left[A(t) + B(t) g^{00}\right] \;\to\; \int d^4x \sqrt{-g}\left[A(t+\pi(x)) + B(t+\pi(x)) \frac{\partial(t+\pi(x))}{\partial x^\mu}\frac{\partial(t+\pi(x))}{\partial x^\nu} g^{\mu\nu}(x)\right]$$

Diff-invariant if π transforms non-linearly:

$$\pi(x) \to \pi(x) - \xi^0(x)$$

Decoupling limit: at high energy, no mixing with gravity!

$$S_\pi = \int d^4x \left[\frac{1}{2}M_{\rm Pl}^2 R - M_{\rm Pl}^2 \dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2}\right) + \boxed{2M_2^4\left(\dot{\pi}^2\right.} + \dot{\pi}^3 \boxed{\left. - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right)} - \frac{4}{3}M_3^4 \dot{\pi}^3 + \ldots\right]$$

$$c_s^{-2} = 1 - \frac{2M_2^4}{M_{\rm Pl}^2 \dot{H}}$$

Large NG from small c_s

Validity of the EFT

Cosmological perturbations probe the theory at E ~ H

Effective theory is valid for H << Λ

We probe small fluctuations $\phi_0(t+\pi(t,\vec{x}))$ $\quad H\pi = -\zeta \simeq 10^{-5}$

We are interested in theories of the form

$$M_{\rm Pl}^2 \dot{H} (\partial\pi)^2 + M(\partial^2\pi)^3 + \ldots$$

We want cubic term to be of order ~ 10^{-3} the kinetic one, from NG constraints.
In the H π >> 1 regime, we would have $10^5/\varepsilon$ boost -> outside the EFT!

Higher derivative terms *must* be small and *must* be evaluated on the lowest order e.o.m.
We cannot change the number of degrees of freedom.

Galilean symmetry

Nicolis, Rattazzi, Trincherini 2008

Shift symmetry on the gradient of a scalar

$$\phi \to \phi + b_\mu x^\mu + c$$

Lowest derivative galileons give 2nd order e.o.m!

$$\mathcal{L} \sim (\partial\phi)^2(\partial^2\phi)^n\ , \quad n \leq 3$$

Use these operators for an inflationary lagrangian (Burrage et al. 2010).
Bkg quite different from slow-roll, large non-Gaussianities given by cubic operators with 4 derivatives...

$$\ddot{\pi}\dot{\pi}^2\ , \quad \dot{\pi}^2\nabla^2\pi\ , \quad \dot{\pi}\nabla\dot{\pi}\nabla\pi\ ,\ \ddot{\pi}(\nabla\pi)^2\ ,\ \nabla^2\pi(\nabla\pi)^2$$

... but all these operators are equivalent to the ones with 3 derivatives arising in one-derivative EFT

Non-minimal galileons, at least 2 derivatives per field.
Is the effective theory consistent? YES!
Do we have interesting predictions? YES!

Building up the action

Perturbations endowed with a Galilean symmetry, which non-linearly realize Lorentz symmetry

Difficult to use the geometrical language.

Useful to introduce a “fake” scalar which linearly realizes Lorentz symmetry

$$\psi(t, \vec{x}) \equiv t + \pi(t, \vec{x})$$

Starting from 2 derivatives per field, do we generate the minimal galileons in curved spacetime?

$R(\partial\psi)^2\partial^2\psi$ is not generated, we will have at least the suppressed $R^2(\partial\psi)^2\partial^2\psi$

Building up the action (2)

Lorentz invariant operators for ψ are products of traces of the matrix $\nabla_\mu \nabla_\nu \psi$

We need to subtract from each trace its bkg value.
So we need to worry about single traces, which can change the tadpole terms: $[\Psi^n]$

Consider the sum, which contains the single trace operators

$$\sum_p (-1)^p g^{\mu_1 p(\nu_1)} \ldots g^{\mu_n p(\nu_n)} \nabla_{\mu_1} \nabla_{\nu_1} \psi \ldots \nabla_{\mu_n} \nabla_{\nu_n} \psi$$

For n>3, we have too many indices and single traces are just products of shorter ones.
Otherwise, we have a total derivative in Minkowski, which in de Sitter gives the minimal galileons:

$$H^2 \sum_p (-1)^p g^{\mu_1 p(\nu_1)} \ldots g^{\mu_n p(\nu_n)} \nabla_{\mu_1} \psi \nabla_{\nu_1} \psi \nabla_{\mu_2} \nabla_{\nu_2} \psi \ldots \nabla_{\mu_n} \nabla_{\nu_n} \psi$$

It is consistent to study the theory with all traces of Ψ, except the single traces, plus the minimal cubic and quartic Galileons, suppressed by H^2

New operators

$$\left(\left[\Psi\ldots\Psi\right]-c_1\right)\left(\left[\Psi\ldots\Psi\right]-c_2\right) \longrightarrow (\delta^{ij}\nabla_i\nabla_j\pi)^3$$

$$\left(\left[\Psi\ldots\Psi\right]-c_3\right)\left(\left[\Psi\ldots\Psi\right]-c_4\right)\left(\left[\Psi\ldots\Psi\right]-c_5\right) \longrightarrow \begin{array}{l}\nabla^2\pi(\nabla_i\nabla_j\pi)^2\\ \nabla^2\pi(\nabla_i\nabla_\mu\pi)^2\\ \nabla^2\pi(\nabla_\mu\nabla_\nu\pi)^2\end{array}$$

There is enough freedom to make these independent from quadratic operators.

We can check the mixing with gravity is subleading in slow-roll.

Final action has only 3 independent cubic operators:

$$\boxed{\begin{aligned} S = \int \mathrm{d}^4x a^3\Big[&-M_{\mathrm{Pl}}^2\dot{H}\left(\dot{\pi}^2-\frac{(\partial_i\pi)^2}{a^2}\right)+M_1\ddot{\pi}^3+M_2\,\ddot{\pi}\,\frac{(\partial_i\dot{\pi}-H\partial_i\pi)^2}{a^2}\\ &+M_3\left(\ddot{\pi}\,\frac{(\partial_i\partial_j\pi)^2}{a^4}-2H\dot{\pi}\ddot{\pi}^2+3H^3\dot{\pi}^3\right)\Big]\end{aligned}}$$

Non-Gaussianities

Almost free field in Bunch-Davies vacuum → almost Gaussian perturbations

Non-Gaussianities of paramount importance to discriminate different models

With EFT, approach very similar to particle physics (EWPT):
measure observables, constrain operators

What is the best observable? Bispectrum in Fourier space of a conserved quantity

$$\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \right\rangle = (2\pi)^3 \delta_D \left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 \right) B(k_1, k_2, k_3)$$

The function B is approximately homogeneous of degree -6.
In this scale-invariant limit, it depends just on two ratios of lengths of 3-momenta:

$$B(k_1, k_2, k_3) = k_1^6 B(1, r_2, r_3)$$

The shape of non Gaussianities

Babich, Creminelli, Zaldarriaga 2004

In the scale-invariant limit, we need just 1 number to specify the PS.

Instead, the bispectrum is a 2-d function. Different operators → different shapes!

How do we measure the non Gaussianity?

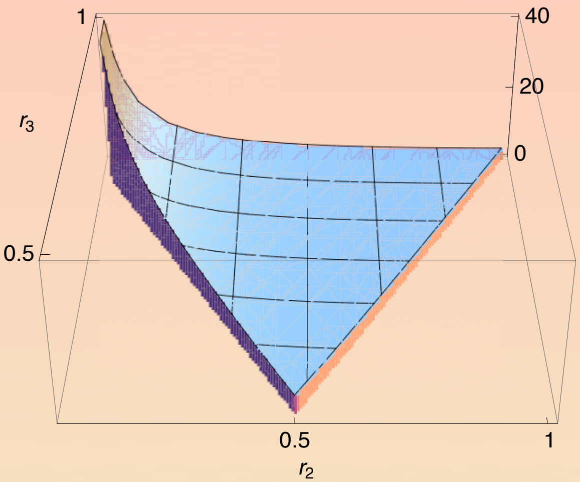
$$\hat{f}_{NL} = \frac{\sum_{\vec{k}_i} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} B(\vec{k}_1, \vec{k}_2, \vec{k}_3)/(\sigma_{k_1}^2 \sigma_{k_2}^2 \sigma_{k_3}^2)}{\sum_{\vec{k}_i} B(\vec{k}_1, \vec{k}_2, \vec{k}_3)^2/(\sigma_{k_1}^2 \sigma_{k_2}^2 \sigma_{k_3}^2)}$$

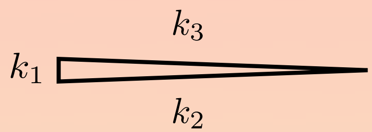
This suggests to quantify how similar are 2 shapes. Scalar product of bispectra:

$$B_1 \cdot B_2 = \int \mathrm{d}r_2 \mathrm{d}r_3 \, r_2^4 r_3^4 \, B_1(1, r_2, r_3) B_2(1, r_2, r_3)$$

Cosine of bispectra: $\cos(B_1, B_2) = \dfrac{B_1 \cdot B_2}{(B_1 \cdot B_1)^{1/2} (B_2 \cdot B_2)^{1/2}}$

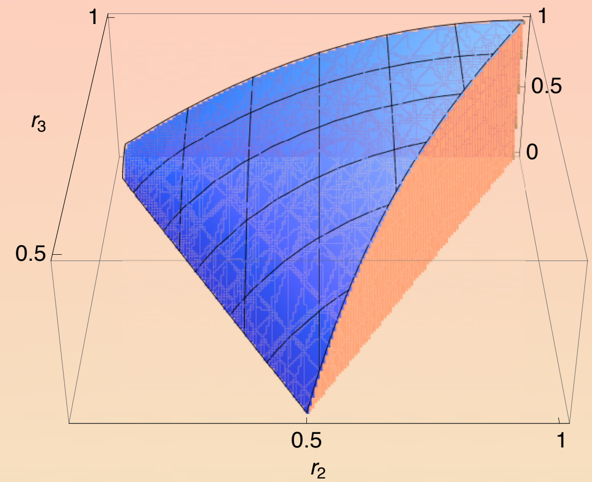
Shapes of non Gaussianities

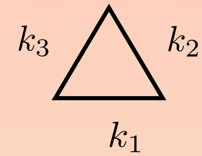




Local

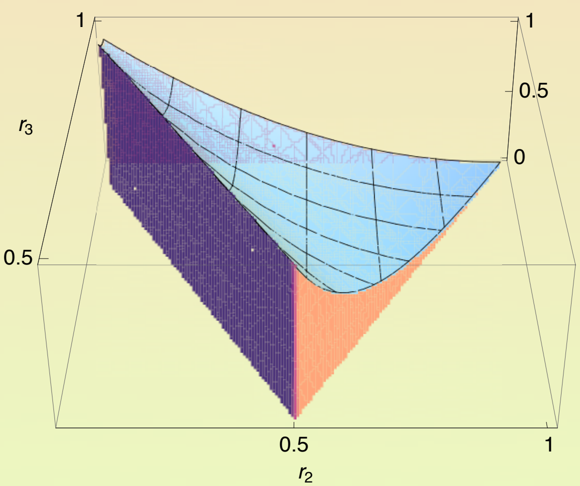
π^3

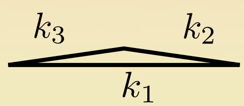




Equilateral

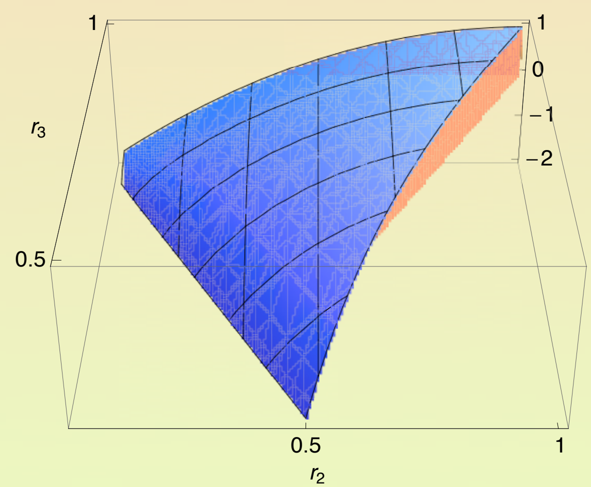
$(\partial\pi)^3$

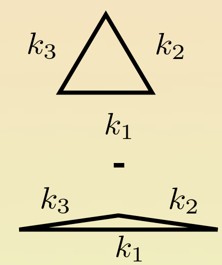




Enfolded

Modified vacuum





Orthogonal

$$\dot\pi\frac{(\partial_i\pi)^2}{a^2}+5.4\frac{2}{3}\dot\pi^3$$

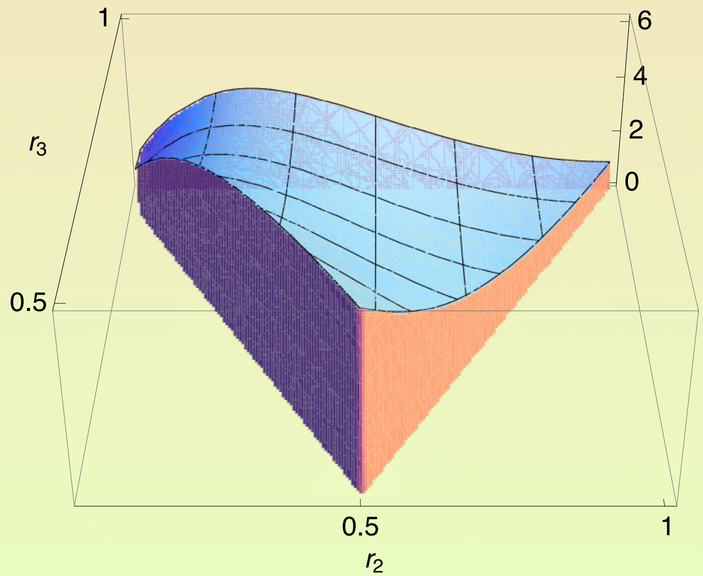
New shapes: M_3 operator

1-derivative EFT operators give 2 shapes, equilateral and orthogonal

$$S = \int \mathrm{d}^4 x a^3 \Big[- M_{\rm Pl}^2 \dot H \left(\dot\pi^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + M_1 \dot\pi^3 + M_2 \, \dot\pi \frac{(\partial_i \dot\pi - H \partial_i \pi)^2}{a^2} + M_3 \left(\ddot\pi \frac{(\partial_i \partial_j \pi)^2}{a^4} - 2 H \dot\pi \ddot\pi^2 + 3 H^3 \dot\pi^3 \right) \Big]$$

M_1 and M_2 operators give equilateral shape

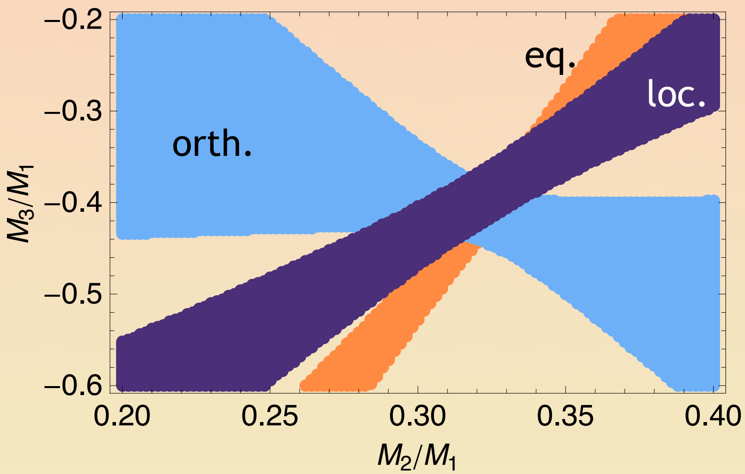
However, M_3 gives a "surfing" non Gaussianity



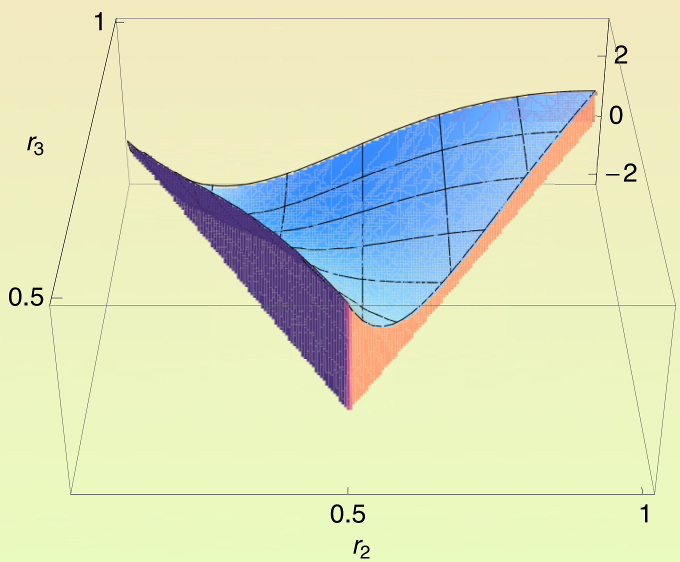
New shapes: orthogonal to standard ones

Orthogonal shape is found by tuning coefficients requiring small cosines with local and equilateral

Can we extend the space of shapes with our new operators? YES



Template	Cosine
Local	-0.15
Equilateral	0.03
Orthogonal	0.06
Enfolded	-0.03



Look where |cos| < 0.2

Intersection point at

$M_2 = 0.32\, M_1$, $M_3 = -0.42\, M_1$

This would require a dedicated template...

Constraints on parameters

Using the analysis of Smith et al. (2010) and WMAP7, we can put constraints on M_i

Choose equilateral template for M_1 and M_2: $f_{NL}^{\rm eq} \equiv \dfrac{k^6}{6\Delta_\Phi^2} B(k,k,k)$

$$f_{\rm NL}^{\rm eq} = 26 \pm 140\,(68\%\,{\rm CL}) \quad \longrightarrow \quad \frac{M_1 H}{\varepsilon M_{\rm Pl}^2} = 240 \pm 1280 \quad \frac{M_2 H}{\varepsilon M_{\rm Pl}^2} = -80 \pm 470$$

For M_3 we can use enfolded template (cos = 0.94): $f_{NL}^{\rm enf} \equiv \dfrac{k^6}{96\Delta_\Phi^2} B(k,k/2,k/2)$

$$f_{\rm NL}^{\rm enf} = 114 \pm 72\,(68\%\,{\rm CL}) \quad \longrightarrow \quad \frac{M_3 H}{\varepsilon M_{\rm Pl}^2} = 830 \pm 530$$

Four point function

Standard EFT: $\mathcal{L}_{1-\partial} = (\partial\pi_c)^2 + \frac{1}{\Lambda^2}(\partial\pi_c)^3 + \frac{1}{\Lambda^4}(\partial\pi_c)^4 + \ldots$

$$\mathrm{NG}_3 \equiv \frac{\langle\zeta^3\rangle}{\langle\zeta^2\rangle^{3/2}} \simeq \left.\frac{\mathcal{L}_3}{\mathcal{L}_2}\right|_{E\sim H} \simeq \left(\frac{H}{\Lambda}\right)^2 \qquad \mathrm{NG}_4 \equiv \frac{\langle\zeta^4\rangle}{\langle\zeta^2\rangle^2} \simeq \left.\frac{\mathcal{L}_4}{\mathcal{L}_2}\right|_{E\sim H} \simeq \left(\frac{H}{\Lambda}\right)^4$$

$$\Longrightarrow \quad \mathrm{NG}_4 \sim \mathrm{NG}_3^2$$

Non-minimal galilean action: $\mathcal{L} = (\partial\pi_c)^2 + \frac{1}{\Lambda^2}(\partial^2\pi_c)^2 + \frac{1}{\Lambda^5}(\partial^2\pi_c)^3 + \frac{1}{\Lambda^8}(\partial^2\pi_c)^4 + \ldots$

$$\mathrm{NG}_3 \simeq \left(\frac{H}{\Lambda}\right)^5 \qquad \mathrm{NG}_4 \simeq \left(\frac{H}{\Lambda}\right)^8$$

$$\Longrightarrow \quad \mathrm{NG}_4 \sim \mathrm{NG}_3^{8/5}$$

For a given cubic NG our model predicts a larger 4 pt function

Usual parametrization: $\mathrm{NG}_3 \simeq f_{\mathrm{NL}}\Delta_\zeta^{1/2} \quad \mathrm{NG}_4 \simeq \tau_{\mathrm{NL}}\Delta_\zeta$

$f_{\mathrm{NL}} = 100$ implies $\tau_{\mathrm{NL}} \sim 10^4$ vs. $\tau_{\mathrm{NL}} \sim 10^5$

A new template

Analysis of CMB is performed by using a sum of factorizable monomials in k's.

We choose the ones with a cosine close to unity w.r.t. the physical shape.

However, orthogonal and enfolded templates go to a constant in the squeezed limit, which is unphysical (Creminelli, G.D'A. Musso, Noreña, *1106.*).

For LSS observations, this gives wrong results! (e.g. bias at large scales)

Solution: we can introduce k^{-4} monomials and cancel divergences in the squeezed limit!

$$F_1(k_1,k_2,k_3) = \frac{16}{9\,k_1 k_2 k_3^4} + \frac{k_1^2}{9\,k_2^4 k_3^4} - \frac{1}{k_1^2 k_3^4} - \frac{1}{k_2^2 k_3^4} + \text{cycl.}$$

$$F_2(k_1,k_2,k_3) = \frac{1}{k_1^3 k_2^3} - \frac{1}{k_1 k_2^2 k_3^3} - \frac{1}{k_2 k_1^2 k_3^3} + \text{cycl.} \qquad F_3(k_1,k_2,k_3) = \frac{1}{k_1^2 k_2^2 k_3^2}$$

$$\boxed{F_\alpha(k_1,k_2,k_3) = N f_{\rm NL} \Delta_\Phi^2 \left[\alpha F_1 + F_2 + 2(1+\alpha) F_3\right]}$$

Model	α	|cos|
M_3	0.71	0.95
orth.	0.55	0.98
enf.	0.60	0.98
eq.	0	1

Conclusions and future work

- Additional operators in the EFT Lagrangian
- New shapes for the 3-point function
- Potentially large 4-point function
- New (1-parameter) template which goes to 0 in the squeezed limit
- Shape orthogonal to everything: put constraints on this?
- Initial conditions for LSS simulations using the new template (in preparation)

Thank you!