Economic Modelling of Type 1 and Type 2 Error

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Introduction

Producers are constantly making decisions in the face of uncertainty in regards to the selection of feed products. Products purported to improve profit may be effective on some farms and yet fail on others due to differences in many factors. Differences in response and relative changes in prices and cost will greatly affect the profitability a product may have on a given farm. Still the producer and consultant must decide to use or not use these products. By using information about the historical success and failure of these products experienced by producers and researchers a quantitative approach to decision making can be undertaken. The purpose of this manuscript is to demonstrate this approach using simple statistical and economic principles.

Types of Errors in Decision Making

Producers seek products that increase their profits. They first seek products that offer the highest return per dollar of input cost. However, for a product to receive initial consideration, it must at least promote a production response (breakeven level) above the cost of the product. Hence producers are not only interested in whether a production response exists (ie >0) but if it's likely to be above the necessary level for making a profit. This level is called the breakeven production level. It is the production needed to be realized so that the producer can recover all costs associated with implementation of the product/service and can be described by:

\[
\text{Break-even Level} = \frac{\text{Product Cost}}{\text{Value of Production}} \quad [1]
\]

If a product costs $.05/d and milk value is $.10/lb then .5 lbs of production response is required to recover the cost associated with the product. Profit will only be realized when the response is above this level. At other levels of response, the profit realized would be dependent on the magnitude of the response.

\[
\text{Profit Change} = \text{production response value} - \text{product cost}[2]
\]
The frequency at which each response level is expected to occur can be determined by the normal density equation:

\[ Y = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2}(X-u)^2} \]

- \( Y \) = frequency of production level \( x \)
- \( x \) = production response (kg/d)
- \( u \) = mean response (kg/d)
- \( \sigma \) = SD of response (kg/d)

Products which have more variable responses will have larger standard deviations and thus have broader distribution curves. Hence products should be evaluated or ranked not only on their mean response but also on how variable the responses. In general risky (variable) products are less valuable because of the uncertainty of return that they produce. A producer uses a product or input in the hopes of realizing a response above the level needed to break-even. A product with a more certain outcome is more valuable than one with an uncertain outcome even if they both have the same mean response.

For each level of response a partial budget (changes in revenues and change in cost, equation 2) can be calculated (Figure 3). Note it is considered a cost in that the decision maker is committing the error-making choice. Also note that the cost equals zero at the break-even point. An economic expected value (probability times the cost) can be calculated for each response level (Figure 4). When the response is below break-even, the expected value will be negative and the producer would be losing profits. Summation of expected values below break-even level estimate the cost of a type 1 error: using a product whose response level is not sufficient to recover cost. Responses above breakeven ensure the producer of recovering more than the cost associated with the product or input. However if he/she fails to use the product or input, he/she will loose returns (type 2 error). By summing the response above breakeven, an estimate of the cost of a type 2 error can be made.

Once the two errors are calculated, the decision made is the one which has the minimum error cost. If type 1 error cost is lower then type 2 error cost the decision would be to use the product since the cost of product failure is less then the cost of foregone returns. When type 2 error is greater then 1, avoiding the product will potentially cost less.

In summary, it is important to assess both types of error. In general researchers and producers tend to concentrate on type 1 error (ignoring the cost of type 2). Furthermore it is important
Table 1. Data summary for bovine somatotropin and sodium bicarbonate.
1) Response to supplementation with .75% sodium bicarbonate in DM (9).

<table>
<thead>
<tr>
<th></th>
<th>BICARB (^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean difference 3.5% FCM (kg/d)</td>
<td>1.43</td>
</tr>
<tr>
<td>(weighted means)</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>1.13</td>
</tr>
<tr>
<td>Number of studies</td>
<td>12</td>
</tr>
<tr>
<td>Value of milk ($/kg)</td>
<td>.287</td>
</tr>
<tr>
<td>Feed intake increase (kg/d)</td>
<td>.36</td>
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<tr>
<td>Feed ($/kg)</td>
<td>.154</td>
</tr>
<tr>
<td>Product cost ($/cow/d)</td>
<td>.05</td>
</tr>
<tr>
<td>Feed cost ($/kg milk)</td>
<td>.07</td>
</tr>
<tr>
<td>$ of DM increase/kg milk</td>
<td>.039</td>
</tr>
<tr>
<td>Break-even value for milk (kg)</td>
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</tr>
<tr>
<td>Frequency below break-even</td>
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</tr>
<tr>
<td>Frequency above break-even</td>
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<tr>
<td>Expected value, $/cow/d</td>
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</tr>
<tr>
<td>Mean response level</td>
<td>.30</td>
</tr>
<tr>
<td>Type I Error (_{\text{Discrete}})</td>
<td>.02</td>
</tr>
<tr>
<td>Type II Error (_{\text{Discrete}})</td>
<td>.32</td>
</tr>
</tbody>
</table>

Table 1. Data summary for bovine somatotropin and sodium bicarbonate.
1) Response to supplementation with .75% sodium bicarbonate in DM (9).
**Figure 1**

**Type I and II Errors**

<table>
<thead>
<tr>
<th>Decision Results</th>
<th>Actual Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>USE PRODUCT</td>
<td>CORRECT/INCORRECT</td>
</tr>
<tr>
<td>NOT USE PRODUCT</td>
<td>INCORRECT/CORRECT</td>
</tr>
</tbody>
</table>

**Figure 2**

**Frequency Distribution**

**Sodium Bicarbonate**

**Figure 3**

**Cost**

**Sodium Bicarbonate**

**Figure 4**

**Expected Value**

**Sodium Bicarbonate**


32. Hansen, W. P. Unpublished data.


somatotropin. J. Dairy Sci. 74:945.


