Magnetoacoustic Tomography with Magnetic Induction for Electrical Conductivity

Imaging of Biological Tissue

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Dedication

This dissertation is dedicated to my parents, Guang Li and Fujin Liu, for their love and support throughout my life, and for encouraging me to do my best along my career path. I would also like to dedicate this dissertation to my wife, Shang Zhou, for her love, patience, support and encouragement during all these years.
Abstract

Electrical properties of biological tissue including conductivity and permittivity play important roles in many biomedical and clinical researches such as modeling neural or cardiac electrical activities and management of electromagnetic energy delivery to the body during clinical diagnosis and treatment. More importantly, these electrical properties may serve as an intrinsic contrast for anatomical or functional imaging. It is therefore of great value to noninvasively image the electrical properties of biological tissue with good accuracy and high spatial resolution. This dissertation research aims at developing and evaluating a new modality i.e. magnetoacoustic tomography with magnetic induction (MAT-MI), for imaging electrical conductivity distribution of biological tissue. In MAT-MI, a conductive object is placed in a static magnetic field and a time-varying magnetic stimulation is applied to induce eddy current inside the object volume. Within the static magnetic field, the Lorentz force acting on the induced eddy current causes mechanical movement of those charged particles in the object and leads to detectable ultrasound signals. These ultrasound signals can be acquired by ultrasound probes and used to reconstruct a high spatial resolution image that indicates the object’s electrical conductivity contrast. We have proposed and investigated two types of MAT-MI approaches i.e. single-excitation MAT-MI and multi-excitation MAT-MI. The corresponding image reconstruction algorithms, simulation protocols and experiment systems have been developed for feasibility testing and performance evaluation. It is shown in our computer simulation and experiment studies that using the single-excitation MAT-MI we are able to image the conductivity boundaries of the object with several
millimeter spatial resolution. In addition, we have also demonstrated that the multi-excitation MAT-MI approach allows us to further extract the internal information and reconstruct more completely the conductivity contrast of the object. For both approaches, two-dimensional (2D) and three-dimensional (3D) images of physical or tissue phantoms have been acquired and showed promising agreement with the target conductivity distribution. All the results we have collected so far from simulations and experiments suggest that the MAT-MI approach is potential to become an important noninvasive modality for electrical conductivity imaging of biological tissue.
# Table of Contents

<table>
<thead>
<tr>
<th>CHAPTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Tables</td>
</tr>
<tr>
<td>List of Figures</td>
</tr>
<tr>
<td>Introduction</td>
</tr>
<tr>
<td>1.1 Overview</td>
</tr>
<tr>
<td>1.2 Motivation and Significance</td>
</tr>
<tr>
<td>1.3 Scope of the Dissertation</td>
</tr>
<tr>
<td>Background</td>
</tr>
<tr>
<td>2.1 Bioimpedance and its Biomedical Relevance</td>
</tr>
<tr>
<td>2.2 Bioimpedance Measurements</td>
</tr>
<tr>
<td>2.3 Bioimpedance Imaging</td>
</tr>
<tr>
<td>2.3.1 Electromagnetic Methods</td>
</tr>
<tr>
<td>2.3.2 Hybrid Methods</td>
</tr>
<tr>
<td>Single-Excitation MAT-MI</td>
</tr>
<tr>
<td>3.1 Introduction</td>
</tr>
<tr>
<td>3.2 Imaging Problem Description</td>
</tr>
<tr>
<td>3.2.1 Forward Problem</td>
</tr>
<tr>
<td>3.2.2 Inverse Problem</td>
</tr>
<tr>
<td>3.3 Reconstruction Algorithms</td>
</tr>
<tr>
<td>3.3.1 Scalar Algorithm</td>
</tr>
<tr>
<td>Section</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>3.3.2</td>
</tr>
<tr>
<td>3.4</td>
</tr>
<tr>
<td>3.4.1</td>
</tr>
<tr>
<td>3.4.2</td>
</tr>
<tr>
<td>3.4.3</td>
</tr>
<tr>
<td>3.5</td>
</tr>
<tr>
<td>3.5.1</td>
</tr>
<tr>
<td>3.5.2</td>
</tr>
<tr>
<td>3.5.3</td>
</tr>
<tr>
<td>3.5.4</td>
</tr>
<tr>
<td>3.6</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>4.1</td>
</tr>
<tr>
<td>4.2</td>
</tr>
<tr>
<td>4.2.1</td>
</tr>
<tr>
<td>4.2.2</td>
</tr>
<tr>
<td>4.3</td>
</tr>
<tr>
<td>4.4</td>
</tr>
<tr>
<td>4.4.1</td>
</tr>
<tr>
<td>4.4.2</td>
</tr>
<tr>
<td>4.4.3</td>
</tr>
<tr>
<td>4.5</td>
</tr>
</tbody>
</table>
4.5.1 2D Multi-Excitation Experiment System Design .......................................... 122
4.5.2 2D Experiment Results ............................................................................. 124
4.5.3 3D Multi-Excitation Experiment System Design .......................................... 128
4.5.4 3D Experiment Results ............................................................................. 130
4.6 Discussion ...................................................................................................... 133

Conclusions and Future Work ............................................................................. 138

5.1 Conclusions .................................................................................................. 138
5.2 Future Work .................................................................................................. 141

Literature Cited .................................................................................................. 144

Appendix A - Copyright Permissions ................................................................. 157
Appendix B - VITA ............................................................................................. 161
List of Tables

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Simulation Results of Conductivity Reconstruction</td>
<td>67</td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Diagram of a four-electrode impedance measurement cell</td>
<td>14</td>
</tr>
<tr>
<td>2. Illustration of MAT-MI</td>
<td>30</td>
</tr>
<tr>
<td>3. Schematic diagram of acoustic tomography with pulsed driving force field ( \mathbf{F} ) as acoustic sources</td>
<td>49</td>
</tr>
<tr>
<td>4. Diagram of the two layer concentric spherical model</td>
<td>57</td>
</tr>
<tr>
<td>5. Example of single-excitation MAT-MI forward solution using concentric spherical model</td>
<td>61</td>
</tr>
<tr>
<td>6. Example of single-excitation MAT-MI inverse solution using concentric spherical model and the scalar reconstruction algorithm</td>
<td>62</td>
</tr>
<tr>
<td>7. Single-excitation MAT-MI simulation results using different transducer numbers and concentric spherical models with different geometries</td>
<td>64</td>
</tr>
<tr>
<td>8. Single-excitation MAT-MI simulation results with concentric spherical models with different conductivity contrast</td>
<td>66</td>
</tr>
<tr>
<td>9. Original and reconstructed force fields using the vector acoustic reconstruction algorithm</td>
<td>69</td>
</tr>
<tr>
<td>10. Image reconstruction performance of the vector algorithm as a function of the ratio between the detecting radius and the imaging area radius</td>
<td>71</td>
</tr>
<tr>
<td>11. MAT-MI Simulations of current density and conductivity reconstruction using the vector algorithm with concentric spherical model</td>
<td>73</td>
</tr>
<tr>
<td>12. Diagram of a 2D single-excitation MAT-MI system setup</td>
<td>75</td>
</tr>
</tbody>
</table>
13. MAT-MI 2D images of saline samples with different salinities…………………78
14. 2D MAT-MI image of a gel phantom………………………………………………79
15. 2D MAT-MI images of two tissue phantoms………………………………………80
16. Schematic diagram of the focused cylindrical scanning mode MAT-MI for 3D imaging……………………………………………………………………………..82
17. Setup of the 3D single-excitation MAT-MI experimental system…………………83
18. Experiment results of 3D single-excitation MAT-MI with a gel phantom………85
19. System diagram of the 2D multi-excitation MAT-MI system…………………..104
20. 2D finite element mesh used for the computer simulation study of multi-excitation MAT-MI………………………………………………………………………………106
21. Example of computer simulations of 2D multi-excitation MAT-MI with broadband acoustic measurement data and under noise free condition……………108
22. Computer simulation results of 2D multi-excitation MAT-MI with limited bandwidth data and various SNR…………………………………………………………….110
23. Computer simulation results of 2D multi-excitation MAT-MI with models of various object size for test of imaging resolution………………………………………111
24. Diagram of the 3D multi-excitation MAT-MI system…………………………….113
25. 3D finite element mesh used in the 3D multi-excitation MAT-MI computer simulation study……………………………………………………………………………..115
26. 3D conductivity volume model used in the 3D multi-excitation MAT-MI computer simulation………………………………………………………………………………116
27. Forward simulated eddy current and MAT-MI acoustic source distributions within the 3D multi-excitation MAT-MI simulation...........................................117
28. Reconstructed 3D conductivity distribution under noise free condition and with broadband acoustic measurements in the 3D multi-excitation MAT-MI........118
29. Computer simulation results of 3D multi-excitation MAT-MI with limited bandwidth data and various SNR.................................................................119
30. Reconstructed 3D conductivity distribution with limited bandwidth data and SNR to be 30.........................................................................................120
31. Experiment results of 2D multi-excitation MAT-MI.................................125
32. Target conductivity distribution and computer simulated acoustic source maps according to the 2D multi-excitation MAT-MI experiment setup.............127
33. Photos and diagram of a 3D gel phantom.............................................130
34. Reconstructed MAT-MI acoustic source images at slice 1 corresponding to different magnetic excitations............................................................131
35. Reconstructed MAT-MI acoustic source images from slice 1 to slice 5 obtained with excitation group C.................................................................131
36. Reconstructed relative conductivity images of the gel phantom from slice 1 to slice 5 using the multi-excitation MAT-MI algorithm.................................132
Chapter 1

Introduction

1.1 Overview

Electrical properties of biological tissue including conductivity and permittivity are important biophysical parameters in modeling living systems. For example, an accurate volume conductor model plays an important role in understanding the electrophysiological activities of excitable tissues such as the heart and the brain (Malmivuo and Plonsey 1995). This is especially the case in electroencephalography (EEG) and electrocardiogram (ECG) signal modeling and source localization using these non-invasive body surface potential measurements (He 2004). In addition, the exact information about the electrical properties of different tissues can help manage all kinds of electromagnetic energy delivered to the human body during research, clinical diagnosis and treatment such as in high field MRI and electrical neural stimulations. Furthermore, electrical properties of biological tissue may serve as an indicator of certain physiological parameters and an intrinsic imaging contrast with diagnostic value. Many previous studies have correlated the electrical properties of tissue to physiological or pathological events. For example, measurements of thorax impedance are often used for monitoring cardiac functions and estimating stroke volume in impedance cardiography (Kubicek et al 1970). On the other hand, a bioimpedance image with high spatial resolution could complement existing imaging modalities such as x-ray computer tomography (CT), ultrasound (US) and magnetic resonance imaging (MRI) and help manage different diseases. It may for instance be used for early breast cancer screening as
many researchers have show that cancerous breast tissue has significantly higher conductivity than its surrounding tissues (Surowiec et al 1988, Jossinet 1998).

Over decades, many researchers have investigated varieties of ways to measure or image electrical impedance of biological tissue. Direct bioimpedance measurements over in vitro tissue samples can generally be done using two or four electrode measurement cells (Baker 1989, Gabriel et al 1996b). Direct in vivo measurement however is complicated. Localized bioimpedance change is usually measured using small electrode probes over different locations on the exposed tissue surface (Fallert et al 1993, Cinca et al 1997). Global impedance change over the entire tissue volume such as in impedance plethysmography can be measured using non-invasive surface electrodes, but the reliability and accuracy of such methods are generally controversial and the result is sometimes hard to interpret (Malmivuo and Plonsey 1995). On the other hand, non-invasive bioimpedance imaging methods have gained a lot of research interest since early 1980’s. Different modalities have been developed since then such as electrical impedance tomography (EIT) (Barber and Brown 1984), magnetic induction tomography (MIT) (Griffiths et al 1999) and magnetic resonance electrical impedance tomography (MREIT) (Khang et al 2002). However, so far none of these methods have gained broad clinical applications due to their limitations either in spatial resolution or in the need of large current injection. Some alternative hybrid approaches have also been proposed to conduct biological current or bioimpedance imaging through coupling between electromagnetic field and acoustic field such as magnetoacoustic tomography (MAT) (Towe and Islam 1988) and Hall effect imaging (HEI) (Wen et al 1998). Based on the similar coupling
mechanism, magnetoacoustic tomography with magnetic induction (MAT-MI) was proposed (He 2005) for high spatial resolution bioimpedance imaging by combining magnetic stimulations and ultrasound measurements. The development and evaluation of this new imaging modality is the major research work of this dissertation research.

In MAT-MI, a conductive object is placed in a static magnetic field and a time-varying magnetic stimulation is applied to induce eddy current inside the object volume. Within the static magnetic field, the Lorentz force acting on the induced eddy current will mechanically move those charged particles in the object and leads to detectable ultrasound signals. These ultrasound signals can be acquired by ultrasound probes and used to reconstruct a high spatial resolution image that indicates the object’s electrical conductivity contrast. As investigations of other imaging modalities, we start with theoretical modeling and analysis of its physics, followed by imaging algorithm development. Computer simulation and experiment studies were then used for concept validation and performance evaluation. We have developed and investigated two types of MAT-MI approaches i.e. single-excitation MAT-MI and multi-excitation MAT-MI. For both approaches, two-dimensional (2D) and three-dimensional (3D) images of physical or tissue phantoms have been acquired and showed promising agreement with the target conductivity distribution. All the results that we have collected so far from simulations and experiments suggest that the MAT-MI approach is potential to become an important noninvasive modality for electrical conductivity imaging of biological tissue.
1.2 Motivation and Significance

As mentioned above, bioimpedance information is important for modeling living systems and bioimpedance images with high spatial resolution may provide valuable information for clinical diagnosis. Therefore, a cost-effective non-invasive imaging modality that can give accurate reconstruction of electrical properties of biological tissue with high spatial resolution is desired and is potential to make big impact on many biomedical research and clinical fields.

The MAT-MI imaging approach investigated in this dissertation research was proposed to fulfill this need. As compared to other existing non-invasive bioimpedance imaging modalities, the MAT-MI approach has the following advantages and benefits. First, the use of magnetic stimulation in MAT-MI makes it immune to the “shielding effect” (Wen 2000) associated with the use of contact surface electrodes for current injection/voltage measurements. Most other bioimpedance imaging modalities such as EIT, MREIT, MAT/HEI have this “shielding effect” problem which indicates that the imaging sensitivity will degrade in those areas surrounded by low conductivity layers. For example, these imaging modalities will have decreased imaging sensitivity in human brain and breast because these areas are surrounded by low conductive skull and fat layers, respectively. In addition, ultrasound measurements used in MAT-MI make it capable of generating a high spatial resolution image. Ultrasound measurements can effectively decouple the signal over space while the sound propagates in the sample volume. After a simple back projection, MAT-MI acoustic source distributions which contain information of the sample’s conductivity distribution can be easily reconstructed.
on each spatial location inside the region of interest. On the contrary, imaging modalities use boundary electrical or magnetic measurements such as EIT or MIT can only obtain signal measurements which are global integrations of their source field over the entire sample space. This in turn leads to an ill-posed inverse problem that these modalities have to solve and the imaging spatial resolution is therefore quite limited. Furthermore, as the MAT-MI approach uses relatively low cost ultrasound measurements and does not have a stringent requirement on the static magnetic field homogeneity, it is expected to have much lower cost than MREIT which needs to use high cost MRI machines to obtain a high spatial resolution bioimpedance image.

1.3 Scope of the Dissertation

In chapter 2, brief background knowledge of bioimpedance and its possible roles in biomedical and clinical researches are presented. The methods to make direct bioimpedance measurements are then briefly described. After that, a literature review is given on existing non-invasive bioimpedance imaging modalities.

In chapter 3, theoretical derivations and validation studies on the single-excitation MAT-MI are described. We start with the forward and inverse problem description of this imaging approach followed by the derivations of two reconstruction algorithms. One of them is a scalar algorithm in which we reconstruct the MAT-MI acoustic source first and then the electrical conductivity distribution. The other algorithm is a vector algorithm in which measured acoustic signals are vectorized and back projected to directly reconstruct the Lorentz force vector. In order to validate the algorithms in computer simulation, we
used a concentric spherical conductivity volume model. The MAT-MI forward solution using this model is derived and the numerical simulation results using the two reconstruction algorithms are presented. We have also conducted 2D and 3D single-excitation MAT-MI experiments using physical phantoms and biological tissue phantoms. Design of both the 2D and 3D systems and the corresponding experiment results we collected through these systems are presented in this chapter.

In chapter 4, we present the validation and evaluation study on the multi-excitation MAT-MI approach. Similarly, we describe the imaging problem and derive its reconstruction algorithm first. A finite element method (FEM) based forward solver is developed in order to conduct MAT-MI forward simulation with arbitrary geometry. With this forward solver, we validated the multi-excitation MAT-MI algorithm in both 2D and 3D computer simulations. The corresponding 2D and 3D multi-excitation MAT-MI experiment systems were also built for experimental evaluations. Experiment results obtained using these systems are shown in this chapter to demonstrate the merit of the multi-excitation MAT-MI approach.

In chapter 5, the major conclusions and contributions of the present dissertation research are summarized. Future work and investigations are also recommended.

The cited literatures, copyright transfer approval and author’s vita are listed at the end of this dissertation.
2.1 Bioimpedance and its Biomedical Relevance

Bioimpedance is defined as the electrical impedance of biological tissues. It is usually measured by introducing an electric current at certain frequency into the tissue volume and measuring the corresponding voltages. The voltage divided by the current then gives the impedance value. However, other than the bulk bioimpedance which depends on the size and volume of the biological tissue sample, it is of more interest to find out the tissue electrical properties including electrical conductivity $\sigma$ and permittivity $\varepsilon$ that can be used to characterize different tissues and their physiological and pathological statuses. Actually, finding out the conductivity or permittivity values from impedance measurements of tissues are often performed in bioimpedance researches (Surowiec et al 1988, Baker 1989). Other researches may also use resistivity, admittivity or complex relative permittivity, but all these physical quantities can be expressed as different combinations of conductivity and permittivity (Foster and Schwan 1989). Generally, electrical conductivity is a measure of a material’s ability to conduct electrical current; while permittivity is a measure of a material’s ability to polarize under external electrical field and thereby to reduce the total electric field inside the material. In SI unit system, conductivity is in the unit of Siemens per meter (S/m); while permittivity is in the unit of Farads per meter (F/m).

The electrical properties of biological tissues are closely related to the conductive ions in both intracellular and interstitial spaces, macromolecules such as proteins and cell
membranes (Foster and Schwan 1989). It is also well known that the electrical properties of biological tissues are temperature and frequency dependent. The temperature coefficients for both conductivity and permittivity however are also tissue type and frequency dependent and have not been generalized from the limited amount of data from literature. The highest temperature coefficients are around 1-2%°C⁻¹ (Gabriel et al 1996a). One possible generalization is that for most body fluid and tissues the conductivity increases with increasing temperature (Geddes and Baker 1967). For frequency dependence, generally as the frequency increases, the electrical conductivity of biological tissues increase and the permittivity values decrease. Parameter models such as the Debye model and the Cole-Cole model can be used to describe the frequency spectrum of the electrical properties of different types of tissues (Gabriel et al 1996c). The finite conductivities of biological tissues are related to the nature and extent of their ionic content and ionic mobility. As compared to materials classified as good conductors (e.g. copper) which have conductivity values above $10^6$ S/m, biological tissues are poor conductors which generally have conductivity values about or less then 1 S/m under 1 MHz and may exceed 10 S/m above GHz. Within low frequency range, i.e. from direct current (DC) to 1 MHz, most tissues except skin have a slow conductivity changing rate over frequency. This changing rate becomes significantly larger above GHz. The relative permittivity of a tissue may reach up to $10^6$ at frequency below 100 Hz and it decreases at high frequencies in three main steps known as the $\alpha$, $\beta$, and $\gamma$ dispersions. Among them, the low frequency $\alpha$ dispersion is associated with ionic diffusion at cellular membranes. The $\beta$ dispersion in the KHz to MHz region is mainly due to the polarization of cell
membranes and partly due to the polarization of macromolecules such as proteins. The $\gamma$ dispersion in the GHz region is due to the polarization of water molecules (Foster and Schwan 1989, Gabriel et al 1996a).

Besides the temperature and frequency dependence of the electrical properties of tissues, the dynamic range of these properties among different tissue types are pretty large. In general, those body fluid inside human body such as cerebrospinal fluid (CSF) and plasma have relatively high conductivity, while the bone and fat tissues have relatively low conductivity values. This conductivity difference may reach up to 100 folds in frequency region below 1 MHz. The relative permittivity values of different tissues may also vary more than 10 folds around 1 MHz with fat and bone tissue having the lowest relative permittivity values. In addition, it should be noted that for certain tissue types such as muscle and white matter, significant anisotropy can be observed. A comprehensive dataset about the electrical properties of different normal tissues can be found in some review articles (Gabriel et al 1996b, Foster and Schwan 1989, Gabriel et al 1996a, Geddes and Baker 1967, Gabriel et al 1996c).

As important biophysical parameters, the conductivity and permittivity of biological tissues play important roles in modeling living systems and understanding how they interact with all kinds of electromagnetic (EM) energy. Possible types of EM energy that are of biomedical interest include the internal electrophysiological activities generated by excitable tissues such as the heart and the brain, the external applied EM energy used in clinical diagnosis and treatment such as the RF field in high field MRI, electrical neural stimulation and RF ablation. While modeling the electrical behavior of
biological system and its response to different applied EM energy, we should note that unlike discrete models used in electrical circuits or networks most biological tissue models in bioelectromagnetism are distributed models. In another word, the biological tissue volume is modeled as a media that extends continuously throughout the 3D volume with continuously distributed electrical properties. This is the so called “volume conductor” (Malmivuo and Plonsey 1995). It should also be noted that within low frequency range, i.e. under MHz, the capacitive component of tissue impedance is negligible and conduction current is much larger than displacement current. The quasistatic condition is therefore satisfied in this frequency band and electromagnetic wave propagation can be neglected. In such low frequency band, only conductivity is required to specify different tissues in the volume conductor model. The volume conductor model of the human body is an important component in EEG/ECG signal modeling and source localization, i.e. the forward problem and inverse problem in bioelectromagnetism. Some researchers have shown that the accuracy of the volume conductor model in terms of geometry and tissue conductivity values may have nontrivial influences on the signal modeling of body surface potentials generated by neural or cardiac sources (Haueisen et al 1997, Rudy et al 1979, Klepfer et al 1997). In addition, the tissue conductivity uncertainties may also limit the accuracy of EEG source localization to certain degree (Awada et al 1998). Therefore researches trying to model and localize the brain electrical activities have tried different ways in order to get a more accurate estimation of the tissue conductivity properties especially the scalp to skull conductivity ratio through both in vitro and in vivo experiments (Rush and Driscoll 1968, }
Lai et al 2005, Zhang et al 2006). Other than the internal electrophysiological activities of excitable tissues, the human body may also be exposed to other externally applied EM energies in different clinical settings such as the RF field used in MRI study, electrical current injection used in cardiac shocks and neural stimulations and RF ablation used in surgery. Under all these clinical settings, accurate models of the electrical properties of the human body or the corresponding tissues of interest (Sadleir et al 2010) would definitely help manage the applied EM energy dose in a better controlled way and help generate more efficient system designs. This can be seen in the estimations of the specific absorption rate (SAR) and field inhomogeneity in high field MRI (Yang et al 2002, Katscher et al 2009b, Zhang et al 2010). An accurate and patient specific volume conductor model would also help better estimate current path in cardiac electroshocks therapy (Jorgenson et al 1995) and help design better electrodes and stimulation patterns in transcranial direct current stimulation (tDCS) therapy in order to produce more focal and efficient stimulations (Sadleir et al 2010, Bikson et al 2009, Datta et al 2009, Datta et al 2010). Tissue conductivity estimation may also play a role in planning the RF induced heating ablation used in treatment of focal tumors (Solazzo et al 2005).

As biological tissues exhibit a broad range of conductivity and permittivity and these properties are known to change according to different physiological and pathological conditions, theoretically the electrical properties can be a good imaging contrast for diagnosis and research. For example, many researches have shown that cancerous breast tumor tissue has significantly different electrical properties or impedance spectrum parameters than normal breast tissue or benign tumors (Surowiec et
al 1988, Jossinet 1998, Jossinet 1996, Jossinet and Schmitt 1999, Morimoto et al 1990, Zou and Guo 2003, Kerner et al 2002, Poplack et al 2007). Significant electrical conductivity difference has also been found between liver tumors and normal liver tissue (Haemmerich et al 2003, Haemmerich et al 2009). There are also evidence showing that the electrical properties of prostate cancer tissue and skin basal cell carcinoma are significantly different than the benign or normal prostate and skin tissue, respectively (Halter et al 2009, Beetner et al 2003). Generally such kind of differences between carcinoma and normal tissue are attributed to different cellular water content, amount of extracellular fluid, membrane permeability, packing density and orientation of the malignant cells (Zou and Guo 2003). Other than carcinomas, tissues under conditions of ischemia, hemorrhage or edema are expected to exhibit different electrical properties as blood and most body fluid have quite different conductivity and permittivity than most other soft tissues (Fallert et al 1993, Cinca et al 1997). There are also researchers (Brown et al 1985) trying to track or image the fast neural activities and evoked hemodynamic responses of the brain by recording the bioimpedance changes of the brain tissue or through certain noninvasive impedance imaging method (Klivington and Galambos 1967, Klivington and Galambos 1968, Galambos and Velluti 1968, Holder et al 1996, Tidswell et al 2001, Bagshaw et al 2003, Gilad et al 2009a, Gilad et al 2009b). Therefore, a noninvasive bioimpedance imaging modality with good accuracy and high spatial resolution could complement or even compete with existing imaging modalities such as x-ray CT, ultrasound and MRI. Possible clinical applications of imaging methods using electrical conductivity or permittivity as imaging contrast include pneumography as for
detection and monitoring of pulmonary edema, plethysmography as for monitoring cardiac function or peripheral blood flow, cerebrography as for detection of cerebral ventricular hemorrhage, abdominal measurements as for monitoring gastric emptying and tumor imaging as for breast cancer detection and for monitoring of tumor growth (Brown et al 1985, Dawids 1987, Holder 2002).

2.2 Bioimpedance Measurements

Techniques for direct measurements of bioimpedance generally involve injecting electrical current into the biological tissue volume through electrodes and measuring the corresponding electrical potentials. Depending on the number of electrodes used, they can be categorized as two-electrode or four-electrode measurements (Baker 1989). For the two-electrode technique, voltages are measured across two current carrying electrodes. This leads to frequency dependent polarization impedance on the electrode-tissue interface. This contact impedance is largely capacitive, and becomes apparent at the low end of the frequency range. This effect is more prominent in ionic solutions than in biological tissues because tissue cells will shield part of the ionic current which causes the polarization effect (Schwan 1992). The polarization impedance can be reduced by using carefully chosen electrode material (Gabriel et al 1996b) or through certain compensation methods (Schwan 1963). In comparison, the four-electrode technique uses separate sets of electrodes for current injection and voltage measurements. A diagram of a four-electrode impedance measurement cell is shown in Fig. 1. Electrical current is
injected into the sample volume through the two plate electrodes A and B, while two additional electrodes C and D are placed in the middle part of the measurement cell for voltage sensing. Electrodes C and D are generally very thin, embedded in the measurement cell wall and connected to ultrahigh input impedance of the voltmeter for the purpose that these electrodes will not affect the current density distribution in the measurement cell and will not draw significant current. Using the four-electrode technique, the impedance measurements will not be contaminated by the contact impedance. In practice, impedance measurements over wide frequency range are often implemented using commercialized impedance analyzer (Gabriel et al 1996b).

For *in vitro* measurements, if the tissue sample is homogeneous, using the four-electrode technique, it is easy to derive the conductivity or resistivity value of the sample.
as the current density inside the measurement cell is generally uniform (Baker 1989). For in vivo measurements, because most tissue samples are inhomogeneous, sometimes anisotropic and have complex geometry, the injected current flow pattern is generally not known and it is difficult to derive tissue conductivity directly. Such kind of in vivo measurement is generally implemented using four-electrode system consists of small probe or needle electrodes and only possible over multi-layer tissue with big extent such as the myocardial tissue (Fallert et al. 1993, Cinca et al. 1997, Tsai et al. 2000, Paulson et al. 2004). Theoretically, the interelectrode spacing needs to be smaller than one third of the thickness of the tissue layer in order to accurately derive tissue conductivity (Robillard and Poussart 1979). In practice, calibration of the electrode constant is required and is often done using standard solution with known conductivity. However, as the electrode constant is determined by the object geometry, electrode configurations and the relative position of the electrodes in the tissue, an ideal calibration which has the minimum measurement error may be hard to obtain (Tsai et al. 2000).

2.3 Bioimpedance Imaging

In order to image the interior distribution of conductivity or permittivity of the human body noninvasively, several different imaging modalities have been proposed and investigated since last century. Here we categorize them into two major types based on the stimulation and measurement techniques utilized in these modalities. The first type is the electromagnetic method, which includes electrical impedance tomography (EIT), magnetic induction tomography (MIT), magnetic resonance electrical impedance
tomography (MREIT) and magnetic resonance electrical property tomography (MREPT). All these modalities use electromagnetic stimulations such as current injection or magnetic induction and conduct the corresponding electromagnetic measurements such as electrical voltage sensing through electrodes or magnetic sensing through coils. The second type is the hybrid method, which employs different forms of energy other than conventional electromagnetic energy into the stimulation or measurement technique. All the hybrid methods we introduce here use the same Lorentz force based coupling mechanism between electromagnetic energy and acoustic/mechanical energy. These hybrid methods include magnetoacoustic tomography (MAT), Hall effect imaging (HEI) and magnetoacoustic tomography with magnetic induction (MAT-MI). In the following sections of this chapter, we will give a more detailed background review of all these impedance imaging modalities.

2.3.1 Electromagnetic Methods

2.3.1.1 EIT

Among all the electromagnetic methods for noninvasive bioimpedance imaging, EIT was first developed using current injection and noninvasive surface voltage measurement (Barber and Brown 1984, Brown et al 1985, Metherall et al 1996, Jossinet et al 2002, Brown 2003). In this modality, multiple electrodes are attached to the surface of a conductive object and current is injected into object volume denoted as $\Omega$ through these electrodes. During current injection, measurements of the electrical voltage via selected pairs of electrodes are collected on the volume boundary surface $\partial \Omega$. With a set
of collected voltage measurements due to different current injection patterns, an image is then reconstructed. In EIT, for a source free volume, Poisson’s equation holds as in Eq. (2.1):

$$\nabla \cdot (\gamma(\mathbf{r}) \nabla \phi(\mathbf{r})) = 0 \quad \text{in } \Omega$$

$$\int_{\Gamma_1} \gamma(\mathbf{r}) \nabla \phi(\mathbf{r}) \cdot \mathbf{n} = \pm I \quad \Gamma_1 \subset \partial \Omega$$

$$\int_{\Gamma_2} \gamma(\mathbf{r}) \nabla \phi(\mathbf{r}) \cdot \mathbf{n} = 0 \quad \Gamma_2 \subset \partial \Omega$$

where $\phi(\mathbf{r})$ is the electrical potential over the volume $\Omega$, $\gamma(\mathbf{r}) = \sigma(\mathbf{r}) + j\omega\varepsilon(\mathbf{r})$ is the complex admittivity which depends on both the conductivity $\sigma$ and the permittivity $\varepsilon$, $\omega$ is the angular frequency of the applied current and $\mathbf{r}$ is the position vector in the domain $\Omega$. $\Gamma_1$ includes the boundary areas in contact with current injection electrodes through which current is flowing into ($-I$) or out from ($+I$) the object volume. $\Gamma_2$ denotes the rest boundary of the object volume without current injection. $\mathbf{n}$ is the unit norm of the boundary surface pointing outward the volume. Forward solution of this imaging problem, i.e. solving the potential/voltage distribution $\phi$ with a prior knowledge about the electrical property distribution and current injection setup, is readily available through numerical method such as the finite element method. However the inverse problem of EIT is well known to be ill-posed because of the limited number of surface voltage measurements, each of which is a volume integration of the sensitivity to tissue impedance throughout the volume. In terms of image reconstruction, both linearized single pass algorithms based on the Jacobian of the forward solver (Metherall et al 1996) and iterative reconstruction algorithms (Woo et al 1992) exist. Because EIT image reconstruction is a nonlinear process in nature, iterative algorithms are more accurate but
on the other hand more time consuming and harder to apply to in vivo situations. The ill-posed inverse problem of EIT in turn leads to the low spatial resolution in all the EIT images. In principle, if the number of surface electrodes is $N$, there will be $N \times (N - 1)/2$ independent voltage measurements. This number sets a limit on the amount of information available for image reconstruction. Theoretically, large number of electrodes could help improve the spatial resolution of EIT, but the quality of the images may not improve significantly after certain number because of the increased correlation between individual data. In addition, in practice large number of electrodes will increase instrumental complexity and may introduce larger measurement errors including the errors in modeling the boundary and electrode positions. Spatial resolution around 5% to 10% of the object dimension is commonly achieved for EIT technique (Malmivuo and Plonsey 1995).

Ever since the first EIT system was developed (Barber and Brown 1984), several different EIT data acquisition hardware designs have been proposed and investigated including the neighboring method, cross method, opposite method and adaptive method (Malmivuo and Plonsey 1995). Basically, different designs use different patterns of current injection and voltage measurement and have different imaging sensitivity patterns. The neighboring method has better sensitivity at peripheral regions but degraded sensitivity in the object center. Adaptive method on the other hand has more uniform but lower sensitivity over the space if similar amount to current is injected into the body. In addition, besides single frequency EIT systems, many multi-frequency systems such as the electrical impedance spectroscopy (EIS) system have been developed in order to
better characterize tissues using their electrical impedance spectrum (Kerner et al 2002, Yerworth et al 2003, Romsauerova et al 2006, Boverman et al 2008). The frequency range is generally from 1 KHz to 2 MHz.

In spite of its low spatial resolution, EIT technique has its advantages of low cost, safety and high speed. Long term monitoring of physiological functions is also possible using EIT technique. Applications of EIT such as monitoring the pulmonary function of neonates (Brown 2003, Hampshire et al 1995) still hold their promises as no competing technique is available. In addition, researches on breast tumor imaging and head imaging using EIT/EIS technique are still ongoing (Kerner et al 2002, Poplack et al 2007, Romsauerova et al 2006, Boverman et al 2008, McEwan et al 2006).

2.3.1.2 MIT

Magnetic induction tomography (MIT) is another bioimpedance imaging modality developed in the 1990s (Griffiths et al 1999, Scharfetter et al 2001, Korjenevsky et al 2000). In MIT, the conductive object is exposed to an alternating magnetic field generated by an array of transmitter coils that induces eddy currents in the object. These eddy currents cause their own secondary magnetic fields, which theoretically contain information about the electrical properties of the conductive object. Measurements of the secondary magnetic fields due to different transmitting patterns through an array of receiver coils are then collected for image reconstruction. However, as the secondary magnetic fields are much weaker than the primary excitation fields, cancellation of the main excitations through back off coils (Griffiths et al 1999), special oriented coils
(Watson et al 2004, Scharfetter et al 2005, Scharfetter et al 2001, Karbeyaz and Gencer 2003) and gradiometers (Scharfetter et al 2001, Karbeyaz and Gencer 2003) have been used in order to control the dynamic range of the collected signal from the receiver coils. As all the developed MIT systems use under MHz excitations, the magnetic quasistatic condition holds for most biomedical application of MIT. This condition indicates that the contribution from the induced eddy current to the primary magnetic excitations can be ignored and the induced electric potentials in the conductive object volume $\Omega$ can be described as in Eq. (2.2):

\[
\begin{align*}
\nabla \cdot (\gamma(r)\nabla \phi(r)) &= -j\omega A(r) \cdot \nabla \gamma(r) \quad \text{in } \Omega \\
(\nabla \phi(r) + j\omega A(r)) \cdot n &= 0 \quad \text{at } \partial \Omega
\end{align*}
\]  

(2.2)

where $\phi(r)$ is the electrical potential over the conductive object volume $\Omega$, $\gamma(r) = \sigma(r) + j\omega \varepsilon(r)$ is the complex admittivity, $A(r)$ is the primary magnetic vector potential, $r$ is the position vector and $n$ is the unit norm of the boundary $\partial \Omega$. With a discretized volume using finite element the electrical potential can be solved and used to estimate the secondary magnetic field $B_2$ from Biot-Savart Law. Assuming a perfect cancellation of the primary excitation signals, the measured MIT signal through the receiver coils can be described using Lenz’s Law as in Eq. (2.3) (Gursoy and Scharfetter 2009a):

\[
v = -j\omega \oint_S B_2 \cdot dS
\]  

(2.3)

where $v$ is the voltage signal detected in the receiver coils induced by the secondary magnetic field. $S$ denotes the surface vector of the receiver coils. This part of the forward problem can also be described using the reciprocity theorem as in Ref. (Karbeyaz and
In most MIT systems only the real part of the voltage data is used which only contains the information of the conductivity $\sigma(\mathbf{r})$ distribution. For the inverse problem of MIT, i.e. reconstruction of the electrical property images, a linearized sensitivity matrix based solution is generally used (Karbeyaz and Gencer 2003, Gursoy and Scharfetter 2009a, Merwa et al 2005). The sensitivity matrix is generally derived by taking the Jacobian of the MIT forward solver at a given conductivity distribution. This matrix then represents the voltage changes due to any small local conductivity changes in the volume. The inverse problem of MIT is also well known to be ill-posed and regularization is often needed in solving it.

As compared to the EIT technique, the MIT method uses contactless coils for excitations and measurements. It avoids some disadvantages associated with the use of surface electrodes such as contact impedance and the “shielding effect” caused by the low conductive areas surrounding the object (Wen 2000). As mentioned above, the cancellation of the primary excitation signal is important for any MIT systems. The primary excitation signal is generally 100 times larger than the useful secondary signal. MIT instruments are generally more complex than EIT systems. In addition, as the transmitter and receiver coil arrays are generally arranged around the object, limited number of independent measurements are available for image reconstruction. This leads to the ill-posed inverse problem of MIT and limited spatial resolution that can be achieved in reconstructed MIT images. Though at its early stage of development and its inherent low spatial resolution, the MIT technique may have some potential biomedical applications such as monitoring of the pulmonary functions and detecting hemorrhagic
stroke in the brain (Gursoy and Scharfetter 2009b, Zolgharni et al 2009a, Zolgharni et al 2009b).

2.3.1.3 MREIT

In order to overcome the technical difficulties of EIT and to produce high spatial resolution conductivity images, magnetic resonance electrical impedance tomography (MREIT) was proposed in the early 1990s. The key idea is to use a MRI scanner to measure the internal magnetic flux density $\mathbf{B} = (B_x, B_y, B_z)$ due to current injection in a conductive object and use these measurements to reconstruct high spatial resolution conductivity images (Woo and Seo 2008). Using a MRI scanner with its main magnetic field sitting in the $z$ direction, we can obtain an image representing the $B_z$ field due to injected current flow inside an object. By rotating the imaging object twice in the MRI bore, we can obtain $B_x$ and $B_y$ in a similar way. This allows us to calculate the current density field $\mathbf{J} = (J_x, J_y, J_z)$ according the Ampere’s Law as $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$, where $\mu_0$ is the magnetic permeability of the free space. This technique, named magnetic resonance current density imaging (MRCDI) was originally proposed to noninvasively image current density distribution $\mathbf{J}$ (Joy et al 1989, Scott et al 1991, Scott et al 1992, Scott et al 1995, Scott et al 1995, Joy et al 1999, Patriciu et al 2005, Wang et al 2009). Early MREIT methods use the current density map $\mathbf{J}$ obtained from MRCDI technique to reconstruct the conductivity images (Khang et al 2002, Kwon et al 2002, Ider et al 2003, Oh et al 2003, Birgul et al 2003). However, these methods are difficult to apply in practice because of the need to rotate the imaging object in MRI bore. In order to solve
this problem several approaches have been proposed to reconstruct conductivity images from just the $B_z$ data without object rotation (Oh et al. 2003, Seo et al. 2003, Ider and Onart 2004, Muftuler et al. 2004, Birgul et al. 2006, Hamamura et al. 2006).

In MREIT, single or multiple cycles of DC current pulses are injected into a conductive volume through surface electrodes. The current injection process is similar as that in the EIT technique and can be described as in Eq. (2.1). Because in most MREIT systems, very low frequency current injections are utilized, the tissue capacitance effect can be ignored and the admittivity $\gamma(r)$ in Eq. (2.1) is generally replaced by conductivity $\sigma(r)$. As compared to EIT, the current injections in MREIT are synchronized with a MRI scanner and the corresponding MR pulse sequence. As the injected current in the imaging object will produce a magnetic flux density $\mathbf{B}$, it will then produce extra phase shifts in the measured MRI signals. With the main magnetic field of the MRI scanner sitting in the $z$ direction, these phase shifts are proportional to the $B_z$ component of current injection related $\mathbf{B}$. The raw MRI k-space data $S^\pm$ corresponding to positive or negative current injections $I^+$ or $I^-$ can be described as in Eq. (2.4) (Woo and Seo 2008):

$$S^\pm(m,n) = \iiint M(x,y)e^{i\delta(x,y)} e^{\pm j\pi B_z(x,y)T_c} e^{j(xm\Delta k + yn\Delta k)} dx dy$$

(2.4)

where $M(x,y)$ is a conventional MR magnitude image; $\delta(x,y)$ denotes any systematic phase artifact; $\gamma_h$ is the gyromagnetic ratio of proton and $T_c$ is the pulse width of the injected current. Dividing the complex image corresponding to $I^+$ by that corresponding
to $I^−$, one can then derive the $B_z$ image through the phases as in Eq. (2.5) (Woo and Seo 2008):

$$B_z(x, y) = \frac{1}{2\gamma_n T_c} \text{arg} \left( \frac{M^+(x, y)}{M^-(x, y)} \right) \tag{2.5}$$

where $M^±(x, y)$ are the complex images obtained by taking Fourier transform of $S_z^±(m,n)$. For the inverse problem of MREIT, because the measurements of magnetic flux density are obtained on each pixel/voxel in the volume, it is not ill-posed as compared to the EIT inverse problem. The spatial resolution of MREIT is mainly determined by the MRI scanner and several millimeter (mm) or below spatial resolution can be easily achieved. There are two categories of reconstruction algorithms that have been developed for MREIT, i.e. $J$-based and $B_z$-based algorithms. Most $J$-based algorithms assume that the current density distribution $J$ is completely obtained as in MRCDI (Khang et al 2002, Kwon et al 2002, Ider et al 2003, Birgul et al 2003, Hasanov et al 2008). However, as mentioned above, this would require rotating the imaging object in the MRI bore at least twice and it is hard to apply in medical field especially with human subject. The $B_z$-based algorithms in comparison use $B_z$ data corresponding to different current injections and do not need to rotate the imaging object. These algorithms include the harmonic $B_z$ algorithm (Oh et al 2003, Seo et al 2003), variational gradient $B_z$ algorithm (Park et al 2004), local harmonic $B_z$ algorithm (Seo et al 2008), projected current density algorithm (Nam et al 2008, Gao and He 2008, Nam and Kwon 2010) and sensitivity matrix based algorithms (Muftuler et al 2004, Hamamura et al 2006). Most
Based algorithms are iterative and require a MREIT forward solver, which is generally implemented using finite element method (Lee et al. 2003).

After the development of those $B_z$-based reconstruction algorithms together with developments in MREIT hardware, sequence designs and other experiment techniques (Kwon et al. 2007, Lee et al. 2007, Park et al. 2007, Hamamura and Muftuler 2008, Jeon et al. 2009, Muftuler et al. 2009), the MREIT experiment studies have gained rapid progress from tissue phantom imaging, to animal and human studies (Hamamura et al. 2006, Oh et al. 2004, Oh et al. 2005, Lee et al. 2006, Sadleir et al. 2006, Kim et al. 2007, Kim et al. 2009). In addition, MREIT studies with animal tumor models also showed promising results (Muftuler et al. 2006). However, one limitation of the MREIT technique is its need of relatively high amplitude of current injection (generally around 10mA or above) in order to obtain sufficient signal to noise ratio (SNR). This level of current injection may cause muscle or neural stimulations and should be decreased for future MREIT human studies.

2.3.1.4 MREPT

Magnetic resonance electrical property tomography (MREPT) is a recently proposed imaging modality that aims to noninvasively extracting the electrical properties of biological tissues through the MRI B1 mapping technique (Katscher et al. 2009a, Katscher et al. 2009b, Zhang et al. 2010). The MRI B1 mapping provides a measurement of the active magnetic components of the applied RF field in MRI, which is a function of the electrical properties of the imaging object loaded in the MRI bore. The knowledge
about the perturbed RF field can then be used to derive the conductivity and permittivity of biological tissue. The RF field distortion caused by the imaging object inside the scanner is more obvious as the major magnetic field intensity goes high (e.g. 3T) and the MR Larmor frequency increases. The signal detection of MREPT does not require extra instrument other than the standard MRI scanner. Its image reconstruction only involves differentiation and integration of the measured B1 map data and no ill-posed inverse problem needs to be solved. The spatial resolution of the reconstructed MREPT image is mainly determined by the resolution of the MRI image and the quality of the B1 mapping data. Therefore, MREPT is considered as a high resolution bioimpedance imaging modality.

As this technique does not need any surface electrode mounting and current injection, it has better flexibility and may have broader applications than the other bioimpedance imaging methods if successfully developed. However, as an emerging technique, more studies are still needed to fully demonstrate its feasibility and medical values.

2.3.2 Hybrid Methods

2.3.2.1 MAT

Magnetoacoustic tomography (MAT) is a technique originally developed for noninvasive measurements of bioelectric currents (Towe and Islam 1988, Islam and Towe 1988, Roth et al 1994). It is found in MAT that application of an oscillating magnetic field to current carrying media or an oscillating current flowing in a static
magnetic field can generate detectable acoustic vibrations through the Lorentz force based coupling mechanism. The collected MAT acoustic signals using microphone can then be used to determine the magnitude of the internal current flow as the acoustic response is linearly correlated to it (Towe and Islam 1988). A simulation study also showed that current dipole sources in a volume conductor could be reconstructed using the MAT approach (Islam and Towe 1988). Reversed MAT technique, using ultrasound excitation and electrical voltage measurements for imaging current density or conductivity were also investigated later on (Haider et al 2008). Although, the original MAT technique has not been implemented for practical bioelectrical current detection, it provides the basic idea of coupling the electromagnetic energy and acoustic energy through the Lorentz force. This coupling mechanism is the fundamental principle for all the hybrid methods introduced below.

2.3.2.2 HEI

Hall effect imaging (HEI) was developed in the late 1990s for imaging the electrical properties of biological tissues using the similar Lorentz force based coupling mechanism as MAT (Wen et al 1998, Wen 2000, Wen 1999). Hall effect describes the charge separation phenomenon in a conductive object moving in magnetic field. Accordingly, HEI utilizes ultrasound to induce localized mechanical vibrations in a conductive tissue sample sitting in a static magnetic field and detect the corresponding Hall voltages using surface electrodes. For biological tissue samples, the Hall voltages/currents are caused by the Lorentz force induced separation of conductive ions
in intra- and extra-cellular space (Wen et al 1998, Montalibet et al 2001b). As the ultrasound wave packet propagates through the sample, the conductivity distribution of the sample along the ultrasound beam is then encoded in the time course of the measured Hall voltages (Wen et al 1998, Montalibet et al 2001a). Assuming a ultrasound transducer generates a longitudinal wave packet along the $z$ direction and it is perpendicular to the magnetic field $B_0$, using the ultrasound momentum

$$ M(z,t) = \int_{-\infty}^{\infty} p(z,\tau) d\tau, $$

where $p(z,\tau)$ denotes the ultrasound pressure wave, the Hall voltage measurements can be described as in Eq. (2.6) (Wen et al 1998):

$$ V_h(t) = aWR_d B_0 \int_{\text{soundpath}} M(z,t) \frac{\partial}{\partial z} \left[ \frac{\sigma(z)}{\rho(z)} \right] dz $$

where $\alpha$ is a portion constant representing how much current is collected by the electrodes; $W$ is the ultrasound beam width and $R_d$ is the detection circuit impedance; $\sigma(z)$ and $\rho(z)$ are the conductivity and density distribution of the imaging object along the $z$ direction, respectively. As we can see from Eq. (2.6) that the Hall voltage measured in HEI is sensitive to the conductivity and density gradient. This is consistent with multiple experiment studies (Wen et al 1998, Wen 1999, Montalibet et al 2001a).

The collected Hall voltage time course with the ultrasound transducer placed at one location forms a line-scan that is similar to an A-line in ultrasound imaging. A 2D Hall effect image can be formed by scanning the transducer at raster grids and record the corresponding Hall voltages at each location. According to the reciprocity theorem, HEI can also operate in a reverse mode, in which a pulsed electrical field is introduced in the imaging object through surface electrodes and the Lorenz force induced acoustic signals
are picked up by an ultrasound probe. Theoretically, the forward and reverse mode produce the same HEI images. In practice, the reverse mode may have some advantages in its lower noise level and its ability to use phase-array transducers and those fast 2D or 3D ultrasound imaging techniques (Wen et al 1998).

The HEI technique can produce high spatial resolution images representing the conductivity boundaries inside biological tissue samples. The spatial resolution of HEI is close to ultrasound imaging and is mainly determined by the central frequency and band width of the ultrasound system. However, as HEI uses surface electrodes for applying/measuring electrical field, it is limited by the shielding effect (Wen 2000). In addition, it is believed that no quantitative measurements of the electrical properties can be obtained using HEI in real biomedical applications because of the limited acoustic window, its internal shielding effect and the dispersion of the tissue electrical properties over frequency (Wen 2000).

2.3.2.3 MAT-MI

In order to solve the problem of the shielding effect existed in other hybrid bioimpedance imaging methods as MAT and HEI, magnetoacoustic tomography with magnetic induction (MAT-MI) approach was proposed by Dr. Bin He and co-workers (He 2005, Xu and He 2005). The basic idea of MAT-MI is illustrated in Fig. 2. As shown in Fig. 2, a conductive tissue object is placed in a static magnetic field. The application of a pulsed magnetic field induces eddy current in this object. Similarly as in MAT and HEI, Lorentz force acting on the induced eddy current causes ultrasound vibrations that can be
detected using ultrasound transducers placed around the object. The collected ultrasound signals are then used to reconstruct the conductivity image of the object.

The forward problem of MAT-MI consists of two physical processes, i.e. magnetic induction and Lorentz force induced acoustic wave propagation. The magnetic inductions in a conductive tissue object volume $\Omega$ can be described as in Eq. (2.7):

$$
\begin{cases}
\nabla \cdot (\sigma(r) \nabla \phi(r)) = -\nabla \cdot (\sigma(r) \mathbf{A}(r)) & \text{in } \Omega \\
(\nabla \phi(r) + \mathbf{A}(r)) \cdot \mathbf{n} = 0 & \text{at } \partial \Omega
\end{cases}
$$

(2.7)

where $\mathbf{A}(r)$ is the spatial function of the magnetic vector potential as $\nabla \times \mathbf{A}(r) = \mathbf{B}_1(r)$. In current MAT-MI, we consider applying around MHz magnetic stimulations to biological tissue samples and the quasistatic condition holds. Therefore, the tissue capacitance effect is ignored and we only consider the conductivity property of the
sample. We also have $B_1(r,t) = B_1(r)f(t)$ and $\phi(r,t) = \phi(r)f'(t)$ according to the quasistatic condition, where $f(t)$ is the temporal function of the time varying magnetic field excitation $B_1$, and the prime denotes the first order time derivative. For simplicity, we often let $f'(t) = \delta(t)$ to represent the pulsed magnetic stimulation used in MAT-MI which is different from the time harmonic magnetic stimulations used in the MIT technique as indicated in Eq. (2.2). Within a static magnetic field $B_0$, the Lorentz force induced acoustic wave propagation can be described by the wave equation as in Eq. (2.8) (Roth et al. 1994):

$$\nabla^2 p - \frac{1}{c_s^2} \frac{\partial^2 p}{\partial t^2} = \nabla \cdot (J \times B_0)$$  \hspace{1cm} (2.8)

where $p$ is the acoustic pressure and $c_s$ denotes the acoustic speed. The right hand side of Eq. (2.8), i.e. $\nabla \cdot (J \times B_0)$, is a term representing the divergence of the Lorentz force over unit volume. This term is also considered as the acoustic source of the MAT-MI acoustic signal. The inverse problem of MAT-MI concerns how to reconstruct the conductivity distribution $\sigma(r)$ of a biological tissue sample from the collected MAT-MI acoustic signals $p$. As we will see in the later chapters, the inverse problem of MAT-MI is not ill-posed as in EIT or MIT techniques because the ultrasound measurements collected around the sample can be used to derive the Lorentz force or the Lorentz force induced acoustic sources, which contain the information about the conductivity distribution, in each pixel/voxel in the internal imaging area.

The basic imaging theory and reconstruction algorithm of the MAT-MI approach have been developed in previous pilot study (Xu and He 2005). Afterwards, computer
simulations using concentric spherical models (Li et al 2007) and multiple layer cylindrical models (Ma and He 2007) have been conducted to demonstrate the feasibility of the MAT-MI approach to perform high resolution conductivity imaging. Both 2D and 3D MAT-MI systems with single magnetic excitation have been developed and used for feasibility testing and performance evaluation in MAT-MI experiments with saline and tissue phantoms (Li et al 2006, Xia et al 2007). It is shown in these studies that using the single-excitation MAT-MI approach, we can noninvasively image the conductivity boundaries of tissue object with spatial resolution close to ultrasound imaging (Li and He 2009). In addition to the scalar acoustic source based algorithm (Xu and He 2005), reconstruction algorithms based on vector acoustic source and potential energy have also been developed and validated in computer simulations (Li et al 2008, Xia et al 2009, Xia et al 2010). In order to reconstruct the complete conductivity contrast, multi-excitation MAT-MI was recently proposed (Li and He 2010a, Li and He 2010b, Li et al 2010). In this method, multiple magnetic excitations using different coil configurations are employed and ultrasound measurements corresponding to each excitation are collected to derive the conductivity distribution inside the sample. Computer simulations and phantom experiment studies on both 2D and 3D multi-excitation MAT-MI approaches have been conducted and it is shown that with well designed magnetic excitations, more complete conductivity contrast information can be obtained in practical experiments. There are also studies shown that conductivity anisotropy that is often seen in muscle and neural tissues may introduce significant difference in generated MAT-MI signals as compared to those from isotropic media (Brinker and Roth 2008) which is assumed in all
the developed MAT-MI reconstruction algorithms and experiment systems so far. MAT-MI reconstruction algorithms that can deal with conductivity anisotropy are therefore needed for future MAT-MI studies on muscle or neural tissues.
Chapter 3
Single-Excitation MAT-MI

3.1 Introduction

Magnetoacoustic tomography with magnetic induction (MAT-MI) is a high resolution non-invasive conductivity imaging method that integrates magnetic stimulation and ultrasound measurements through the Lorentz force based coupling mechanism. Based on its forward modeling, two major reconstruction algorithms have been developed for MAT-MI with single magnetic excitation. One of them is a scalar algorithm in which we reconstruct the MAT-MI acoustic source first and then reconstruct the electrical conductivity distribution (Xu and He 2005). The other algorithm is a vector algorithm in which measured acoustic signals are vectorized and back projected to directly reconstruct the Lorentz force vector (Li et al 2008, Xia et al 2009). We have validated the two algorithms in computer simulations using a concentric spherical volume conductor model with which the MAT-MI forward problem has an analytical solution (Li et al 2007, Xia et al 2009). All the simulation results show that it is feasible to conduct non-invasive conductivity imaging of biological tissues with high spatial resolution using the single excitation MAT-MI method. We have also conducted imaging experiments with saline or biological tissue phantoms using the developed 2D or 3D single excitation MAT-MI systems (Li et al 2006, Xia et al 2007, Li and He 2009). These experiment studies further validate the feasibility of the MAT-MI approach and demonstrated that a conductivity boundary image with spatial resolution close to ultrasound imaging can be obtained using practical single-excitation MAT-MI systems. Most materials in this
chapter have been previously published (Li et al 2007, Li et al 2006, Xia et al 2007, Xia et al 2009).

3.2 Imaging Problem Description

In this section, we describe in detail the MAT-MI imaging problem with single magnetic excitation including the corresponding forward problem and inverse problem. The forward problem of MAT-MI is the problem in which the static field, the magnetic stimulation and the volume conductor are known but the generated MAT-MI acoustic signal is unknown. We will derive the two major governing equations as shown in Eq. (2.7) and Eq. (2.8) in this section. The forward solution with a concentric spherical volume conductor model will be discussed in section 3.4.1. The inverse problem of MAT-MI is the problem with known static magnetic field, dynamic magnetic stimulations and the corresponding acoustic signal measurements and unknown conductivity distribution of the volume conductor. We will introduce the general strategy and comment on the inverse problem of MAT-MI. Detailed reconstruction algorithm will be derived in section 3.3.

3.2.1 Forward Problem

The forward problem of the MAT-MI approach describes two major physical processes in its signal generation mechanism, i.e. magnetic induction in a conductive sample and acoustic wave propagation with the Lorentz force induced acoustic sources.

We consider a sample domain $\Omega$ with isotropic conductivity $\sigma(r)$, where $r$ is
the position vector. The sample is placed in a static magnetic field with flux density $\mathbf{B}_0(\mathbf{r})$. Denote the stimulating time-varying magnetic field as $\mathbf{B}_1(\mathbf{r},t)$ which can also be represented by the curl of its vector potential $\mathbf{A}(\mathbf{r},t)$ as $\mathbf{B}_1 = \nabla \times \mathbf{A}$. As in MAT-MI we are considering around $\mu s$ level current pulses for driving the stimulating coil, the corresponding MHz skin depth in general biological tissue (assuming conductivity of 0.2 S/m and relative permeability of 1) is at the level of meters, so the magnetic induction problem in MAT-MI can be considered quasi-static and magnetic diffusion can be ignored. This condition also indicates that the magnetic vector potential $\mathbf{A}$ in the tissue can be very well approximated by the vector potential produced by the coil in the absence of the tissue (Wang and Eisenberg 1994). The quasistatic condition allows us to separate the spatial and temporal functions of the time-varying magnetic field, i.e. $\mathbf{B}_1(\mathbf{r},t) = \mathbf{B}_1(\mathbf{r}) f(t)$ and $\mathbf{A}(\mathbf{r},t) = \mathbf{A}(\mathbf{r}) f(t)$. In addition, the displacement current can be ignored as it is much smaller than the conductive current in biological tissue at MHz frequency (Xu and He 2005). Therefore, the tissue capacitance effect is ignored here and only conductivity of the tissue is considered.

According to Faraday’s law, the curl of the electrical field intensity $\mathbf{E}(\mathbf{r},t)$ is equal to the negative time varying rate of the magnetic flux density $\mathbf{B}_1(\mathbf{r},t)$. Using the magnetic vector potential, we can have Eq. (3.1):

$$\nabla \times (\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}) = 0$$

(3.1)

Therefore the electrical field intensity $\mathbf{E}(\mathbf{r},t)$ can be written as in Eq. (3.2):
\[ E = -\nabla \phi - \frac{\partial A}{\partial t} \quad (3.2) \]

where \( \phi(r,t) \) is the electrical scalar potential. According to Ampere’s law and because we ignore the displacement current, the current density \( J(r,t) \) is solenoidal as in Eq. (3.3):

\[ \nabla \cdot J = 0 \quad (3.3) \]

According to Ohm’s law, the current density is related to the electrical field through conductivity as in Eq. (3.4):

\[ J = \sigma E \quad (3.4) \]

Combining Eqs. (3.2)-(3.4), we can derive Eq. (3.5):

\[ \nabla \cdot (\sigma \nabla \phi) = -\nabla \cdot (\sigma \frac{\partial A}{\partial t}) \quad (3.5) \]

For simplicity, we often let \( f'(t) = \delta(t) \) to represent the pulsed magnetic stimulation used in MAT-MI, where \( \delta(t) \) is an ideal delta function. According to the quasistatic condition and Faraday’s Law and Ohm’s Law, the similar spatial and temporal separation holds for the induced electrical field and eddy current density, i.e. \( \phi(r,t) = \phi(r)f'(t) \), \( E(r,t) = E(r)f'(t) \) and \( J(r,t) = J(r)f'(t) \) where the prime denotes the first order time derivative. After cancelling out the time function, Eq. (3.5) takes the form as in Eq. (2.7).

As we will see in section 3.4.1, with a two layer concentric spherical model and uniform magnetic stimulation Eq. (3.5) has an analytical solution. The natural boundary conditions on all the conductivity boundaries are used there. For arbitrary geometry, Eq. (3.5) must be solved in the whole conductive sample domain \( \Omega \) with a Neumann boundary condition on the current density at the outer boundary surface as in Eq. (3.6):
\[ \mathbf{J} \cdot \mathbf{n} = 0 \]  

(3.6)

where \( \mathbf{n} \) is the unit vector norm of the outer boundary \( \partial \Omega \). This boundary condition requires the current density component that is normal to the bounding surface to vanish. Combining with Eq. (3.6), Eq. (3.5) has a unique solution for the electrical potential \( \phi \) inside the conductive domain \( \Omega \) if a reference point is chosen. This solution can generally be obtained in arbitrary geometry by using numerical method such as finite element method (FEM) (Wang and Eisenberg 1994). In Eq. (3.6), substituting \( \mathbf{J} \) with Eqs. (3.2) and (3.4) and cancelling out the time function, Eq. (3.6) takes the form as in Eq. (2.7).

With the magnetically induced eddy current \( \mathbf{J} \) and the static magnetic field \( \mathbf{B}_0 \), the Lorentz force acting on the eddy current over unit volume can be written as \( \mathbf{J} \times \mathbf{B}_0 \). According to Newton’s second law of motion and assuming the particle velocity \( \mathbf{v} \) caused by the Lorentz force is small, we have the following Eq. (3.7) (Roth et al 1994, Xu and He 2005):

\[
\frac{\partial (\rho_0 \mathbf{v})}{\partial t} = -\nabla p + \mathbf{J} \times \mathbf{B}_0
\]

(3.7)

where \( \rho_0 \) is the density of the material at rest and \( p \) is acoustic pressure. Taking the divergence of both sides of Eq. (3.7), we have Eq. (3.8):

\[
\frac{\partial (\nabla \cdot (\rho_0 \mathbf{v}))}{\partial t} = -\nabla^2 p + \nabla \cdot (\mathbf{J} \times \mathbf{B}_0)
\]

(3.8)

In addition, we have the conservation of mass as in Eq. (3.9) and the definition of the adiabatic compressibility of the medium \( \beta \) as in Eq. (3.10):
\[ \nabla \cdot (\rho_0 \mathbf{v}) = -\frac{\partial p}{\partial t} \]  
(3.9)

\[ \beta, p = \frac{p}{\rho_0} \]  
(3.10)

where \( \rho \) is the density variation. Combining Eqs. (3.8)-(3.10) and using the relationship

\[ c_s = \frac{1}{\sqrt{\rho_0 \beta_s}} \]

where \( c_s \) is the acoustic speed in the medium, we can derive the wave equation with the Lorentz force induced acoustic source as in Eq. (3.11):

\[ \nabla^2 p - \frac{1}{c_s^2} \frac{\partial^2 p}{\partial t^2} = \nabla \cdot (\mathbf{J} \times \mathbf{B}_0) \]  
(3.11)

This is exactly the same equation as Eq. (2.8). We should also note that in MAT-MI the static magnetic field is generally generated from some external sources such as permanent magnets placed outside the imaging object volume, thus \( \nabla \times \mathbf{B}_0 = 0 \) inside the imaging object volume (Xu and He 2005) and Eq. (3.11) can be further simplified as Eq. (3.12):

\[ \nabla^2 p - \frac{1}{c_s^2} \frac{\partial^2 p}{\partial t^2} = (\nabla \times \mathbf{J}) \cdot \mathbf{B}_0 \]  
(3.12)

Eq. (3.11) or (3.12) can generally be solved using Green’s function method. Assuming the medium is acoustically homogeneous, using the 3D Green’s function, the solution to Eq. (3.11) can be written as in Eq. (3.14) (Xu and He 2005):

\[ p(r_0, t) = -\frac{1}{4\pi} \int \int d^3 \mathbf{r} \nabla_\mathbf{r} \cdot [\mathbf{J} \times \mathbf{B}_0] \frac{\delta(t - |\mathbf{r}_0 - \mathbf{r}|/c_s)}{|\mathbf{r}_0 - \mathbf{r}|} \]  
(3.14)
where \( \mathbf{r}_0 \) is a position located on certain ultrasound detection aperture. With known MAT-MI acoustic sources induced by Lorentz force we can then use Eq. (3.14) to calculate out the acoustic pressure.

### 3.2.2 Inverse Problem

In solving the inverse problem of MAT-MI, we need to reconstruct the conductivity distribution of the volume conductor \( \sigma(\mathbf{r}) \) with the knowledge of the static magnetic field \( \mathbf{B}_0(\mathbf{r}) \), the dynamic magnetic stimulation \( \mathbf{B}_1(\mathbf{r}, t) \) or its vector potential \( \mathbf{A}(\mathbf{r}, t) \) and the measured MAT-MI acoustic signals \( p(\mathbf{r}_0, t) \).

As in its forward modeling, the signal generation mechanism of MAT-MI includes both the processes of magnetic induction and acoustic wave propagations. Accordingly solving the inverse problem of MAT-MI often involves two steps. In the first step, we reconstruct the map of the Lorentz force induced acoustic sources (Xu and He 2005, Li et al. 2007), or the Lorentz force distribution (Li et al. 2008, Xia et al. 2009) or the associated potential energy (Xia et al. 2010) from the measured acoustic signals. In the second step, we then reconstruct the conductivity distribution from the result we get from the first step. In section 3.3, we will show the detailed derivation of two reconstruction algorithms based on the first two strategies.

Overall, the inverse problem of MAT-MI is not ill-posed as in EIT or MIT techniques because the ultrasound measurements collected around the sample can be used to derive the Lorentz force or the Lorentz force induced acoustic sources, which contain the information about the conductivity distribution, in each pixel/voxel in the internal
imaging area. This is because the acoustic sources over space are time resolved in the measured acoustic signal due to their acoustic time of flight difference. In comparison, the electromagnetic measurements used in EIT or MIT at each location are always a volume integration of the product between the current density and the lead field of the probes, e.g. surface electrode or coil.

3.3 Reconstruction Algorithms

In this section, we derive two reconstruction algorithms for single-excitation MAT-MI. The first algorithm is a scalar algorithm, in which we first reconstruct the Lorentz force induced MAT-MI acoustic sources \( \nabla \cdot (\mathbf{J} \times \mathbf{B}_0) \) and then derive the conductivity distribution \( \sigma(r) \). We will first derive the acoustic source reconstruction part of this algorithm using the Green’s function technique (Norton and Linzer 1981, Xu and Wang 2002) under a special measurement aperture i.e. a closed spherical surface. After that its general form with arbitrary measurement aperture will be given. The conductivity reconstruction part of this algorithm will be derived after that. The second algorithm is a vector algorithm which only applies to special measurement aperture e.g. close spherical or cylindrical detection surfaces. This algorithm takes the vectorized acoustic pressure measurements to reconstruct the Lorentz force vector \( \mathbf{J} \times \mathbf{B}_0 \) followed by the reconstruction of the current density \( \mathbf{J} \) and conductivity distribution \( \sigma(r) \).
3.3.1 Scalar Algorithm

In MAT-MI, with a pulsed magnetic stimulation and letting \( f'(t) = \delta(t) \) we can rewrite Eq. (3.11) as in Eq. (3.15):

\[
\nabla^2 p(\mathbf{r}, t) - \frac{1}{c_s^2} \frac{\partial^2}{\partial t^2} p(\mathbf{r}, t) = \nabla \cdot (\mathbf{J}(\mathbf{r}) \times \mathbf{B}_0) \cdot \delta(t) \quad (3.15)
\]

Here we assume the medium is acoustically homogeneous and \( c_s \) is a constant. Taking Fourier transform on both sides of Eq. (3.15) over variable \( \mathbf{r} = c_s t \), Given Fourier transform pair:

\[
\tilde{p}(\mathbf{r}, k) = \int_{-\infty}^{\infty} p(\mathbf{r}, \mathbf{\tau}) \exp(ik\mathbf{\tau}) d\mathbf{\tau} \quad (3.16)
\]

\[
p(\mathbf{r}, \mathbf{\tau}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{p}(\mathbf{r}, k) \exp(-ik\mathbf{\tau}) dk \quad (3.17)
\]

where \( k = \omega/c_s \) is the wave number. Letting \( AS(\mathbf{r}) = \nabla \cdot (\mathbf{J}(\mathbf{r}) \times \mathbf{B}_0) \), we have a non-homogeneous Helmholtz equation as in Eq. (3.18):

\[
(\nabla^2 + k^2) \tilde{p}(\mathbf{r}, k) = AS(\mathbf{r})c_s \quad (3.18)
\]

If the acoustic source is spatially bounded by a radius \( R \), which is generally the case in practice, we have \( AS(\mathbf{r}) = 0 \) for \( r > R \). With a bounding surface \( S_0 \) that encloses the source field volume \( V \), using Green’s function, the signal measured at position \( \mathbf{r}_0 \) on the surface \( S_0 \) can be expressed as in Eq. (3.19):

\[
\tilde{p}(\mathbf{r}_0, k) = -c_s \int_{V} \int_{S_0} \int_{V} AS(\mathbf{r}) \tilde{G}_k(\mathbf{r}_0, \mathbf{r}) d\mathbf{r} \quad (3.19)
\]
where in 3D, the Green’s function of Helmholtz equation is \( \tilde{G}_k (\mathbf{r}_0, \mathbf{r}) = \frac{\exp(ik |\mathbf{r}_0 - \mathbf{r}|)}{4\pi |\mathbf{r}_0 - \mathbf{r}|} \).

Equation (3.19) is the forward solution of the MAT-MI wave equation in frequency domain. If we take the inverse Fourier transform on Eq. (3.19) we can then get Eq. (3.14).

If the bounding surface \( S_0 \) on which the acoustic measurements are collected is a spherical surface with radius \( r_0 \), using the spherical coordinate \( \mathbf{r}_0 = (r_0, \theta_0, \varphi_0) \), we have \( AS(\mathbf{r}) = AS(r, \theta, \varphi) \) where \( r < r_0 \) and \( AS(\mathbf{r}) = 0 \) when \( r > r_0 \). Under spherical coordinate the Green’s function can be expanded using spherical Bessel function of the first kind \( j_l(\cdot) \), spherical Hankel function of the first kind \( h_l^{(1)}(\cdot) \) and the Legendre polynomial \( P_l(\cdot) \) as in Eq. (3.20) (Arfken and Weber 1995):

\[
\tilde{G}_k (\mathbf{r}_0, \mathbf{r}) = \frac{\exp(ik |\mathbf{r}_0 - \mathbf{r}|)}{4\pi |\mathbf{r}_0 - \mathbf{r}|} = \frac{ik}{4\pi} \sum_{l=0}^{\infty} (2l + 1) j_l(kr) h_l^{(1)}(kr_0) P_l(n \cdot n_0), \quad (k > 0) \quad (3.20)
\]

where \( n = \mathbf{r}/r \), and \( n_0 = \mathbf{r}_0/r_0 \). The Legendre polynomial can be further expanded as in Eq. (3.21):

\[
P_l(n \cdot n_0) = \frac{4\pi}{2l + 1} \sum_{m=-l}^{l} Y_l^m(\theta, \varphi) Y_l^{m*}(\theta_0, \varphi_0) \quad (3.21)
\]

Where \( Y_l^m(\theta, \varphi) \) is the spherical harmonics and the star denotes conjugate. Substitute Eqs. (3.20) and (3.21) into (3.19), we can get Eq. (3.22):

\[
\bar{p}(\mathbf{r}_0, k) = -ikc_s \int \int \int \ AS(\mathbf{r}) d\mathbf{r} \sum_{l=0}^{\infty} j_l(kr) h_l^{(1)}(kr_0) \sum_{m=-l}^{l} Y_l^m(\theta, \varphi) Y_l^{m*}(\theta_0, \varphi_0) \quad (3.22)
\]

Multiplying both sides of Eq. (3.22) with \( Y_l^m(\theta_0, \varphi_0) \) and integrating over the normalized spherical surface \( \Omega_0 \), noticing that \( d\Omega_0 = \sin \theta_0 d\theta_0 d\varphi_0 \), we can get:
\[ \int_{\Omega_0} \int \int d\Omega_0 \tilde{p}(r_0, k) Y_{lm}^m(\theta_0, \varphi_0) = -i c r \int \int \int \int \int d\Omega_0 \tilde{p}(r_0, k) Y_{lm}^m(\theta_0, \varphi_0) \] 

According to the orthogonal integral of spherical harmonics as in Eq. (3.24), Eq. (3.23) can be simplified as in Eq. (3.25):

\[ \int_{\Omega_0} \int d\Omega_0 \tilde{p}(r_0, k) Y_{lm}^m(\theta_0, \varphi_0) Y_{lm}^m(\theta_0, \varphi_0) = \delta(l, l') \delta(m, n) \] (3.24)

\[ \int_{\Omega_0} \int d\Omega_0 \tilde{p}(r_0, k) Y_{lm}^m(\theta_0, \varphi_0) \] 

\[ = -i c r \int \int \int \int \int d\Omega_0 \tilde{p}(r_0, k) Y_{lm}^m(\theta_0, \varphi_0) \] 

\[ = -i c r \int \int \int \int \int d\Omega_0 \tilde{p}(r_0, k) Y_{lm}^m(\theta_0, \varphi_0) \] (3.25)

Dropping the prime and rearrange Eq. (3.25) we can get Eq. (3.26):

\[ \int_{\Omega_0} \int d\Omega_0 \tilde{p}(r_0, k) Y_{lm}^m(\theta_0, \varphi_0) \frac{1}{h_{l}^{(1)}(k r_0)} = -i c r \int \int \int \int \int d\Omega_0 \tilde{p}(r_0, k) Y_{lm}^m(\theta_0, \varphi_0) \] (3.26)

Multiplying both sides of Eq. (3.26) with \( k \cdot j_i(k r') \) and integrating with respect to \( k \) from zero to \( +\infty \), and then multiplying both sides of it with \( Y_{lm}^m(\theta', \varphi') \) and summing \( n \) from \(-l\) to \( l \), \( l \) from zero to \( +\infty \), we can derive Eq. (3.27):

\[ \int_{\Omega_0} \int d\Omega_0 \int^{+\infty}_{r_0} dr \tilde{p}(r_0, k) \sum_{l=0}^{\infty} j_i(k r') \sum_{m=-l}^{+l} Y_{lm}^m(\theta', \varphi') Y_{lm}^m(\theta, \varphi) \] 

\[ = -i c r \int \int \int \int \int d\Omega_0 \tilde{p}(r_0, k) \sum_{l=0}^{\infty} j_i(k r') \sum_{m=-l}^{+l} Y_{lm}^m(\theta', \varphi') \frac{1}{r} \] 

\[ = -i c r \int \int \int \int \int d\Omega_0 \tilde{p}(r_0, k) \frac{\pi}{2 r^2} \delta(r-r') \frac{\delta(\theta-\theta') \delta(\varphi-\varphi')}{\sin \theta} \] 

\[ = -i c \frac{\pi}{2} A S(r', \theta', \varphi') \]
In the derivation of Eq. (3.27), the identity of the spherical Bessel function as in Eq. (3.28)
\[ \int_{0}^{\infty} dk (k^2) j_{l}(kr) j_{l}(kr') = \frac{\pi}{2r^2} \delta(r-r') \]  
(3.28)
and the identity of the spherical harmonics as in Eq. (3.29)
\[ \sum_{l=0}^{\infty} \sum_{n=-l}^{l} Y_{l}^{n*} (\theta, \varphi) Y_{l}^{n*} (\theta', \varphi') = \frac{\delta(\theta - \theta') \delta(\varphi - \varphi')}{\sin \theta} \]  
(3.29)
were used. Finally, dropping the prime in Eq. (3.27), we can get Eq. (3.30):
\[ AS(r) = -\frac{2}{i c \pi} \iiint_{\Omega_0} d\Omega_0 \int_{0}^{\infty} dk \vec{p}(\mathbf{r}_0, k) \cdot k \sum_{l=0}^{\infty} \sum_{n=-l}^{l} j_{l}(kr) h_{l}^{(1)}(kr_0) Y_{l}^{n*} (\theta, \varphi) Y_{l}^{n} (\theta_0, \varphi_0) \]  
(3.30)
This is the exact solution for reconstructing the acoustic source \( AS(r) = \nabla \cdot (\mathbf{J}(r) \times \mathbf{B}_0) \) from pressure measurements \( \vec{p}(\mathbf{r}_0, k) \). Assuming the detection radius \( r_0 \) is much larger than the wavelength of the MAT-MI acoustic wave signals for imaging i.e. \( k|\mathbf{r}_0| > 1 \), we can have the approximation as in Eq. (3.31) (Xu and Wang 2002):
\[ h_{l}^{(1)}(kr_0) \approx \frac{1}{h_{l}^{(2)}(kr_0)} \left( \frac{1}{(kr_0)^2} + O(\frac{1}{(kr_0)^2}) \right) \]  
(3.31)
Using this approximation Eq. (3.30) becomes Eq. (3.32):
\[ AS(r) = -\frac{2}{i c \pi} \iiint_{\Omega_0} d\Omega_0 \int_{0}^{\infty} dk \vec{p}(\mathbf{r}_0, k) \cdot k^3 r_0^2 \sum_{l=0}^{\infty} j_{l}(kr) h_{l}^{(2)}(kr_0) \sum_{n=-l}^{l} Y_{l}^{n*} (\theta, \varphi) Y_{l}^{n} (\theta_0, \varphi_0) \]  
(3.32)
Similar to Eq. (3.20) we have another expansion identity as in Eq. (3.33)
\[ \frac{\exp(-ik|\mathbf{r}_0 - \mathbf{r}|)}{4\pi|\mathbf{r}_0 - \mathbf{r}|} = -\frac{ik}{4\pi} \sum_{l=0}^{\infty} (2l+1) j_{l}(kr) h_{l}^{(2)}(kr_0) P_{l}(\mathbf{n} \cdot \mathbf{n}_0) \]  
(3.33)
\[ = -ik \sum_{l=0}^{\infty} j_{l}(kr) h_{l}^{(2)}(kr_0) \sum_{n=-l}^{l} Y_{l}^{n*} (\theta, \varphi) Y_{l}^{n} (\theta_0, \varphi_0) \]  
(3.33)
Substitute Eq. (3.33) into (3.32) we can get Eq. (3.34):

\[
AS(\mathbf{r}) = -\frac{1}{2\pi^2 c_s} \int_{\Omega_0} d\Omega_0 \int_{-\infty}^{+\infty} dk \bar{p}(\mathbf{r}_0, k) \cdot k^2 r_0^2 \exp(-ik|\mathbf{r}_0 - \mathbf{r}|) \frac{\exp(-ik|\mathbf{r}_0 - \mathbf{r}|)}{|\mathbf{r}_0 - \mathbf{r}|} \tag{3.34}
\]

Because \(p(\mathbf{r}_0, t)\) is a real function, \(\bar{p}^*(\mathbf{r}_0, k) = \bar{p}(\mathbf{r}_0, -k)\). Taking summation of Eq. (3.34) with its complex conjugate and divided by two, we can get Eq. (3.35):

\[
\begin{align*}
AS(\mathbf{r}) & = -\frac{r_0^2}{2\pi c_s} \int_{\Omega_0} d\Omega_0 \int_{-\infty}^{+\infty} dk \bar{p}(\mathbf{r}_0, k) \cdot k^2 \frac{\exp(-ik|\mathbf{r}_0 - \mathbf{r}|)}{|\mathbf{r}_0 - \mathbf{r}|} \\
& = \frac{r_0^2}{2\pi c_s} \int_{\Omega_0} d\Omega_0 \int_{-\infty}^{+\infty} dk \bar{p}(\mathbf{r}_0, k) \cdot (-ik)^2 \frac{\exp(-ik|\mathbf{r}_0 - \mathbf{r}|)}{|\mathbf{r}_0 - \mathbf{r}|} 
\end{align*}
\tag{3.35}
\]

Taking the inverse Fourier transform we then get the reconstruction formula for MAT-MI acoustic source in time domain as in Eq. (3.36):

\[
\begin{align*}
AS(\mathbf{r}) & = -\frac{r_0^2}{2\pi c_s} \int_{S_d} dS_d \int_{\Omega_0} d\Omega_0 \frac{1}{|\mathbf{r}_0 - \mathbf{r}|} \frac{\partial^2 p(\mathbf{r}_0, t)}{\partial t^2} \bigg|_{t=|\mathbf{r}_0 - \mathbf{r}|/c_s} \\
& = \frac{1}{2\pi c_s} \int_{S_d} dS_d \int_{\Omega_0} d\Omega_0 \frac{1}{|\mathbf{r}_0 - \mathbf{r}|} \frac{\partial^2 p(\mathbf{r}_0, t)}{\partial t^2} \bigg|_{t=|\mathbf{r}_0 - \mathbf{r}|/c_s} 
\end{align*}
\tag{3.36}
\]

With arbitrary measurement geometry \(S_d\), a derivation using the time reversal technique (Xu and Wang 2004) can be applied as in Ref. (Xu and He 2005). The resultant reconstruction algorithm is given in Eq. (3.37):

\[
\begin{align*}
AS(\mathbf{r}) & = -\frac{1}{2\pi c_s^3} \int_{S_d} dS_d \int_{S_d} dS_d \frac{\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_d)}{|\mathbf{r} - \mathbf{r}_d|^2} \frac{\partial^2 p(\mathbf{r}_d, t)}{\partial t^2} \\
& \bigg|_{t=|\mathbf{r}_d - \mathbf{r}|/c_s} 
\end{align*}
\tag{3.37}
\]

where \(\mathbf{r}_d\) is a point on the detection surface \(S_d\) and \(\mathbf{n}\) is a unit vector normal to the surface \(S_d\) at \(\mathbf{r}_d\). When the detection surface is spherical and taking the far field assumption i.e. \(\mathbf{r}_d >> \mathbf{r}\) Eq. (3.37) will then degrade to Eq. (3.36).
With the reconstructed acoustic source \( \nabla \cdot (\mathbf{J}(\mathbf{r}) \times \mathbf{B}_0) \), we can then further derive the conductivity distribution \( \sigma(\mathbf{r}) \). As mentioned in section 3.2.1, because \( \nabla \times \mathbf{B}_0 = 0 \), \( \nabla \cdot (\mathbf{J}(\mathbf{r}) \times \mathbf{B}_0) = (\nabla \times \mathbf{J}(\mathbf{r})) \cdot \mathbf{B}_0 \) and according to both Ohm’s law and Faraday’s law the acoustic source can be expanded as in Eq. (3.38):

\[
\begin{align*}
A S(\mathbf{r}) &= (\sigma(\nabla \times \mathbf{E}(\mathbf{r}))) + \nabla \sigma \times \mathbf{E}(\mathbf{r})) \cdot \mathbf{B}_0 \\
&= (-\sigma \mathbf{B}_1(\mathbf{r}) + \nabla \sigma \times \mathbf{E}(\mathbf{r})) \cdot \mathbf{B}_0
\end{align*}
\] (3.38)

Note here because of the quasistatic condition, \( \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) f'(t) \), \( \mathbf{B}_1(\mathbf{r}, t) = \mathbf{B}_1(\mathbf{r}) f(t) \) and \( \nabla \times \mathbf{E}(\mathbf{r}) = -\mathbf{B}_1(\mathbf{r}) \). Assuming the conductive sample is piecewise homogeneous, the second term on the right hand side of Eq. (3.38) can then be ignored except at conductivity boundaries. This assumption then allows us to get Eq. (3.39) for reconstructing the conductivity distribution inside any smooth piece (Xu and He 2005, Li et al 2007).

\[
\sigma = -\frac{A S(\mathbf{r})}{\mathbf{B}_1(\mathbf{r}) \mathbf{B}_0(\mathbf{r})}
\] (3.39)

Note here Eq. (3.39) does not hold on boundaries between regions of different conductivity. In theory, as will be shown in the computer simulation study in section 3.4.2, if the acoustic measurements are broadband, using this algorithm together with a median filter we can reduce the reconstruction errors at the conductivity boundaries and correctly reconstruct the conductivity maps. However, with real experiment data that is collected with narrowband ultrasound transducers, we can only reconstruct the conductivity boundaries as illustrated in section 3.5.
3.3.2 Vector Algorithm

In comparison to the MAT-MI scalar acoustic source \( AS(r) = \nabla \cdot (J(r) \times B_0) \), the Lorentz force \( F = J(r) \times B_0 \) acting on the induced electrical current can be considered as a vector driving force field for the MAT-MI acoustic signal. The divergence of this vector field serves as the conventional conceived acoustic source. The vector algorithm derived in this section aims at reconstructing this vector field of Lorentz force directly from scalar acoustic measurements before reconstructing the conductivity. Theoretically, the present method also expands the application domain of the existing acoustic reciprocity principle from a scalar field to a vector field. As stated in the traditional acoustic reciprocity principle, the acoustic source and the transmitted acoustic wave at the receiver are interchangeable (Morse and Ingard 1968). Similarly, we will show that the stimulating vector source and the transmitted acoustic pressure vector (acoustic pressure vectorized according to certain measurement geometry) are also interchangeable (Xia et al 2009).

As shown in Fig. 3, here we consider a tomographic problem in which the acoustic vibration or displacement is driven by a force field \( F(r,t) \) and at the same time the force source in time domain was designed as a short positive pulse which can be approximated as a delta function. Defining \( F(r) = J(r) \times B_0 \), Eq. (3.15) can be rewritten as in Eq. (3.40)

\[
\nabla^2 p(r,t) - \frac{1}{c_s^2} \frac{\partial^2}{\partial t^2} p(r,t) = (\nabla \cdot F(r))\delta(t)
\]  

(3.40)
In the following, we will show that under the far-field condition, a rigorous reconstruction of the driving force field \( F \) directly from the acoustic measurements \( p(r_0, t) \) is available by employing Green’s function in the time domain (Norton and Linzer 1981, Xu and Wang 2002).

Taking the similar derivation process from Eq. (3.15) to Eq. (3.19), we can get Eq. (3.41)

\[
\tilde{p}(r_0, k) = -c_s \iiint \nabla \cdot \mathbf{F} \tilde{G}_d (\mathbf{r}, \mathbf{r}_0) d\mathbf{r} \tag{3.41}
\]
where $\tilde{G}_k(r, r_0)$ is the same Green’s function as in Eq. (3.19). Because the divergence of curl of a vector constantly equals zero, the force field is not uniquely defined by its divergence, except with the condition that the curl component of the force vector equals zero. In general, the curl of $F$ does not go to zero. Only under certain circumstances will this curl-free condition be met. For example, in MAT-MI, if the object is homogeneous in $z$ direction, and the static magnetic field and dynamic magnetic field are also in this direction, i.e. the whole problem is $z$ independent, this curl-free condition can be met.

The curl of $F$, according to elasticity, may serve as a shear wave source. As most biological soft tissue has resistance to compression, but not to shear deformation, the shear wave in soft tissue is much smaller than the longitudinal wave. In addition, the attenuation of shear wave in tissue is generally two to three magnitudes larger than that of longitudinal wave which makes the shear wave hard to detect in practice.

On the right hand side of Eq. (3.41), because $\tilde{G}_k \nabla \cdot F = -F \cdot \nabla \tilde{G}_k + \nabla \cdot (F \tilde{G}_k)$, and the force is zero outside the source region, according to the divergence theorem the integral of $\tilde{G}_k \nabla \cdot F$ over the whole source region is equal to the integral of $-F \cdot \nabla \tilde{G}_k$ over the same region. Thus we can rewrite Eq. (3.41) as follows

$$\bar{p}(r_0, k) = c_s \iiint d\mathbf{r} (\mathbf{F} \cdot \nabla \tilde{G}_k (\mathbf{r}, \mathbf{r}_0))$$

(3.42)

where $\nabla \tilde{G}_k (\mathbf{r}, \mathbf{r}_0) = -(jk - \frac{1}{|\mathbf{r}_0 - \mathbf{r}|}) \tilde{G}_k \mathbf{n}_{(r, r_0)}$, with $\mathbf{n}_{(r, r_0)}$ denoting the unit vector in the direction of $\mathbf{r}_0 - \mathbf{r}$. Expanding $\nabla \tilde{G}_k$ into Eq. (3.42), we get
\[ \tilde{p}(\mathbf{r}_0, k) = -c_s \int \int \int d\mathbf{r} \left( jk - \frac{1}{|\mathbf{r}_0 - \mathbf{r}|} \right) (\mathbf{F} \cdot \mathbf{n}_{(r_0)}) G_k (\mathbf{r}, \mathbf{r}_0) \]  \hspace{1cm} (3.43)

For a relatively large measurement geometry with appropriate acoustic frequency, (e.g. if $|\mathbf{r} - \mathbf{r}_0| = 10\text{cm}$ and frequency $f = 500$ KHz, $|\mathbf{r} - \mathbf{r}_0| = 200 >> 1$) the $1/|\mathbf{r}_0 - \mathbf{r}|$ term in Eq. (3.43) can be ignored. Meanwhile, as shown in Fig. 3, if $r_0 >> r$, as stated in the far field condition, $\mathbf{F} \cdot \mathbf{n}_{(r_0)}$ can be approximated as $\mathbf{F} \cdot \mathbf{n}_0$, where $\mathbf{n}_0$ is the unit vector in the direction of $\mathbf{r}_0$ and can also be considered as a unit normal vector of a normalized surface $\Omega_0$ centered at the origin.

Taking a spherical coordinate $(\rho, \theta, \phi)$, multiplying $r_0$ and taking the gradient on both sides of Eq. (3.43) with respect to $\mathbf{r}_0$, the right hand side becomes

\[ -(jk)c_s \int \int \int d\mathbf{r} \nabla (\tilde{G}_k (\mathbf{F}(\rho, \theta, \phi) \cdot \mathbf{r}_0)) \]

\[ = jk\tilde{p}(\mathbf{r}_0, k)\mathbf{r}_0 - (jk)c_s \int \int \int d\mathbf{r} \mathbf{F}(\rho, \theta, \phi) \tilde{G}_k - (jk)c_s \int \int \int d\mathbf{r} \left( \frac{\partial \tilde{G}_k}{\partial \theta_0} \nabla \theta_0 + \frac{\partial \tilde{G}_k}{\partial \phi_0} \nabla \phi_0 \right) (\mathbf{F}(\rho, \theta, \phi) \cdot \mathbf{r}_0) \]  \hspace{1cm} (3.44)

And the left hand side becomes

\[ \nabla (\tilde{p}(\mathbf{r}_0, k) \cdot \mathbf{r}_0) = jk\tilde{p}(\mathbf{r}_0, k)\mathbf{r}_0 + \tilde{p}(\mathbf{r}_0, k)\mathbf{n}_0 - (jk)c_s \int \int \int d\mathbf{r} \left( \frac{\partial \tilde{G}_k}{\partial \theta_0} \nabla \theta_0 + \frac{\partial \tilde{G}_k}{\partial \phi_0} \nabla \phi_0 \right) (\mathbf{F}(\rho, \theta, \phi) \cdot \mathbf{r}_0) \]  \hspace{1cm} (3.45)

From Eqs. (3.44) and (3.45), we get a vector equation as in Eq. (3.46):

\[ \tilde{p}(\mathbf{r}_0, k)\mathbf{n}_0 = -jkc_s \int \int \int d\mathbf{r} \mathbf{F}(\rho, \theta, \phi) \tilde{G}_k \]  \hspace{1cm} (3.46)

Multiplying both hand sides of Eq. (3.46) by $Y^*_l (\theta_0, \phi_0)$ and integrating over the normalized spherical surface $\Omega_0$, we have
\[
\int_{\Omega^0} \int_{\Omega_0} \tilde{p}(r_0, k) n_0 Y^n_i(\theta_0, \varphi_0) d\Omega_0 = -(jk)c_i \int_{\Omega^0} \int_{\Omega_0} d\Omega_0 \tilde{G}_k F(\rho, \theta, \varphi) Y^n_i(\theta_0, \varphi_0) \quad (3.47)
\]

where \( Y^n_i(\theta_0, \varphi_0) \) is the spherical harmonics function, \( d\Omega_0 = \sin(\theta_0) d\theta_0 d\varphi_0 \), the source point and measurement point are represented in spherical coordinate at \((\rho, \theta, \varphi)\) and \((\rho_0, \theta_0, \varphi_0)\), respectively. Taking the Green’s function expansion identity as in Eqs. (3.20) and (3.21), Eq. (3.47) can be rewritten as in Eq. (3.48):

\[
\int_{\Omega^0} \int_{\Omega_0} \tilde{p}(r_0, k) n_0 Y^n_i(\theta_0, \varphi_0) d\Omega_0 = k^2 c_i \int_{V} d\rho \sum_{m=0}^{+\infty} h^{(1)}_m(k \rho_0) j_j(k \rho \rho' \varphi') \sum_{n=-l}^{l} Y^n_i(\theta, \varphi) \int_{\Omega^0} \int_{\Omega_0} d\Omega_0 Y^{m*}_v(\theta_0, \varphi_0) Y^n_i(\theta_0, \varphi_0) \quad (3.48)
\]

Employing the orthogonal relationship of spherical harmonics as in Eq. (3.24), we can then derive

\[
\int_{\Omega^0} \int_{\Omega_0} d\Omega_0 \tilde{p}(r_0, k) n_0 Y^n_i(\theta_0, \varphi_0) = k^2 c_i \int_{V} d\rho \sum_{m=0}^{+\infty} h^{(1)}_m(k \rho_0) j_j(k \rho \rho' \varphi') \sum_{n=-l}^{l} Y^n_i(\theta_0, \varphi_0) \quad (3.49)
\]

By dropping the primes, multiplying both hand sides of Eq. (3.49) by \( j_j(k \rho \rho' \varphi') / h^{(1)}_m(k \rho_0) \), integrating \( k \) from 0 to +\( \infty \) and then multiplying both sides by \( Y^n_i(\theta', \varphi') \) and summing \( n \) from \(-l\) to \(+l\), and \( l \) from 0 to +\( \infty \), one can get

\[
\int_{0}^{+\infty} dk \int_{\Omega^0} \int_{\Omega_0} d\Omega_0 \tilde{p}(r_0, k) n_0 \sum_{n=-l}^{+l} Y^n_i(\theta_0, \varphi_0) Y^{m*}_v(\theta_0, \varphi_0) j_j(k \rho \rho' \varphi') / h^{(1)}_m(k \rho_0) = c_i \int_{V} d\rho \int_{\rho_0}^{+\infty} dk (k^2) j_j(k \rho \rho' \varphi') \sum_{n=-l}^{+l} Y^n_i(\theta, \varphi) Y^{m*}_v(\theta', \varphi') \quad (3.50)
\]

\[
=c_i \int_{V} d\rho (F(\rho, \theta, \varphi)) \frac{\pi}{2 \rho^2} \delta(\rho - \rho') \delta(\theta - \theta') \delta(\varphi - \varphi') \quad (3.50)
\]

\[
= \frac{\pi}{2} c_i F(\rho', \theta', \varphi')
\]
In the derivation of Eq. (3.50), we utilized the identity of the spherical Bessel function and spherical harmonics as in Eq. (3.28) and Eq. (3.29), respectively.

Using the same approximation as in Eq. (3.31), considering the expansion of the complex conjugate of Green’s function as in Eq. (3.33) and dropping the primes in Eq. (3.50), we can get

\[
F(\rho, \theta, \phi) = \frac{2}{\mathcal{A}_s} \int_{\Omega_0} dk \int_{\Omega_0} d\Omega_0 \bar{p}(\mathbf{r}_0, k)n_n(k^2) \sum_{l=0}^{\infty} \sum_{m=-l}^{l} Y_l^m(\theta_0, \varphi_0)Y_l^m(\theta, \varphi) \rho_0(2)(k\rho_0) j_l(k\rho - k^2),
\]

where \( S_0 \) is a spherical surface with radius \( \rho_0 \). Because \( \bar{p}(\mathbf{r}_0, t) \) is a real function, the integral of \( k \) can be extended to \(-\infty\) together with dividing the whole integral by two to hold Eq. (3.51). We can then expand the vectors in Eq. (3.51) to the Cartesian coordinate system after applying the inverse Fourier transformation and the final reconstruction formula can be written as

\[
\left[ F(\rho, \theta, \phi) \right] = \frac{1}{2\mathcal{A}_s} \int_{S_0} \frac{1}{t} \frac{\partial p(\mathbf{r}_0, t)}{\partial t} \left( \sin(\theta_0) \cos(\varphi_0) \mathbf{i} + \sin(\theta_0) \sin(\varphi_0) \mathbf{j} + \cos(\theta_0) \mathbf{k} \right)
\]

where \( (\theta_F, \varphi_F) \) and \( (\theta_o, \varphi_o) \) are the zenith and azimuth of vector \( \mathbf{F} \) and \( \mathbf{n}_n \), respectively, and \([F]\) is the amplitude of the force vector \( \mathbf{F} \). Unit norms along the \( x, y, z \) axis are denoted as \( \mathbf{i}, \mathbf{j}, \mathbf{k} \). As indicated in Eq. (3.52), we can reconstruct the vectorial acoustic source field by time reversing the vectorized acoustic pressure measurements. Analysis and discussion about the reconstruction errors brought by the approximations used in this derivation can be found in Ref. (Xia et al 2009).
If we rewrite Eq. (3.52) in its vector form and replace the time item by Green’s function, we can get Eq. (3.53) as shown below.

\[
\mathbf{F}(\rho, \theta, \varphi) = -\frac{2}{c^2} \iint dS_0 \frac{\partial p(r_0, t)}{\partial t} \tilde{G}_k^*(r, r_0)
\]

Comparing Eqs. (3.41) and (3.53), they are quite similar in form. It can be interpreted in the similar way as the principle of acoustic reciprocity. The acoustic reciprocity principle states that an acoustic response remains the same when the source and receiver are interchanged. In the view of classical principle of point source, integrating all of the acoustic point sources and multiplying by their corresponding Green’s function can be used to describe the transmitted acoustic pressure at the receiver. Likewise, the acoustic pressure at each receiving point multiplied by the conjugate of the Green’s function can be used to reconstruct the acoustic pressure at each source point. Correspondingly, applying acoustic reciprocity to explain Eq. (3.53), we can consider the driving force field \(\mathbf{F}\) and the vectorized pressure item \(\partial p(r_0, t)/\partial t\) as two counterparts in the principle of acoustic reciprocity. Integrating the divergence of the force sources leads to the transmitted acoustic pressure at the receivers. Correspondingly integrating the time derivative of the vectorized acoustic pressure can be used to reconstruct the vector force field at each source point. In theory, the present reconstruction method expands the application domain of the existing principle of acoustic reciprocity.

In addition, in MAT-MI, the Lorentz force acting on the induced current is always perpendicular to the static magnetic field. Taking the direction of the static magnetic field as the \(z\) axis in a cylindrical coordinate system \((\rho, \varphi, z)\) and applying a similar
derivation procedure in cylindrical coordinates, the reconstruction formula of the Lorentz force field $\mathbf{F}$ can be derived as in Eq. (3.54) (Xia et al 2009, Norton and Linzer 1981, Xu et al 2003):

$$
\int \int_{s} \mathbf{F}(r, \varphi, z) \exp(i\varphi_r) = -\frac{1}{2\pi \varepsilon_0} \int_{s} dS_{\rho} \sqrt{1 - \left(\frac{z_0 - z}{r} \right)^2} \left( \frac{1}{t} \frac{\partial \rho(r_0, t)}{\partial t} \right) \exp(i\varphi_0) \quad (3.54)
$$

where $\varphi_0$ and $\varphi_r$ are the azimuth of the vectors $r_0$ and $\mathbf{F}$, respectively.

With the reconstructed Lorentz force field and the known static magnetic field, the induced eddy current in biological tissue can be estimated based on $\mathbf{F} = \mathbf{J}(\mathbf{r}) \times \mathbf{B}_0$.

However, as we often assume the static magnetic field is uniform and pointing in the $z$ direction, only the $x$ and $y$ components of the eddy current can be estimated because the generated Lorentz force is restrained in the XY plane. The conductivity distribution can be roughly estimated from Ohm’s Law with simple models (Xia et al 2009). However, in general this is not as straightforward as it looks because the electrical field depends on the conductivity distribution and is generally not known. Forward modeling and numerical iteration is often needed as used in the J-substitution algorithm in MREIT (Khang et al 2002).

### 3.4 Computer Simulation Study

In this section, we demonstrate the feasibility of the single-excitation MAT-MI using computer simulations. The MAT-MI forward solution with a concentric spherical conductive volume model is first derived. Numerical simulations using the scalar and vector reconstruction algorithms are both presented.
3.4.1 MAT-MI Forward Solution with Concentric Spherical Model

Here we derive an analytical solution for MAT-MI using a two-layer concentric spherical volume conductor model. The model geometry is illustrated in Fig. 4, where $r_1$ and $r_2$ are radii of the inner and outer layers of the spherical conductive object, respectively. $\sigma_1$ and $\sigma_2$ are the corresponding conductivity values of the two layers. $\sigma_3$ is the background conductivity. The coordinate origin is at the spherical center. We assume there is a homogeneous static magnetic field in the whole space domain oriented to the positive $z$ direction with magnetic flux density of $B_0$. An excitation coil is placed around the sample spheres with its axis pointing in the $z$ direction and going through the XY plane at point $(0, \alpha)$. For the stimulating magnetic flux density $B_t(r,t)$ produced by the coil, we assume that it is homogeneous, points in the $z$ direction, and covers a spatial domain containing the entire conductive object. In addition, we assume that the concentric spheres and the surrounding media are acoustically homogeneous, indicating that the acoustic speed $c_0$ is a constant value and there is no acoustic scattering and attenuation. Under these assumptions, according to the definition of magnetic vector potential and the Coulomb gauge condition, the magnetic vector potential $A$ of the excitation field can be expressed as in equation Eq. (3.55).

$$A = \frac{1}{2} B_t [xj - (y - \alpha)i]$$  \hspace{1cm} (3.55)

Combining Eqs. (3.5) and (3.55) we can derive the Laplace equation as in Eq. (3.56) for the electric potential in those areas with homogeneous conductivity.
At conductivity boundaries, we have both the Dirichlet and Neumann boundary conditions:

\[ \phi_1 = \phi_2 \]
\[ \mathbf{J}_1 \cdot \mathbf{n} = \mathbf{J}_2 \cdot \mathbf{n} \]  \hspace{1cm} (3.57)

where \( \mathbf{n} \) is the unit normal vector of the conductivity boundaries. In the concentric spherical model, the solution of the Laplace equation in a spherical coordinate system \((r, \theta, \phi)\) can be represented in Legendre series as in equation (3.58) where \( Y^m_l \) is the Legendre spherical harmonics, and \( A_{lm}, B_{lm}, C_{lm} \) and \( D_{lm} \) are coefficients.

\[
\phi(r) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} Y^m_l (\theta, \phi) \cdot \begin{cases} 
A_{lm} r^l & \text{if } r < r_1 \\
B_{lm} r^l + C_{lm} r^{-(l+1)} & \text{if } r_1 < r < r_2 \\
D_{lm} r^{-(l+1)} & \text{if } r > r_2 
\end{cases}
\]  \hspace{1cm} (3.58)
According Eqs. (3.2) and (3.4) and the boundary condition as in Eq. (3.57), we can see that the term $\nabla \phi \cdot \mathbf{n}$ and $\frac{\partial \mathbf{A}}{\partial t} \cdot \mathbf{n}$ should have the similar angular portion. Considering Eq. (3.55) and the unit norm $\mathbf{n}$ in the concentric spherical model, i.e. $\mathbf{n} = (\sin \theta \cos \phi) \mathbf{i} + (\sin \theta \sin \phi) \mathbf{j} + (\cos \theta) \mathbf{k}$, we can see that only those items with $l = 1$ and $m = \pm 1$ are nonzero in Eq. (3.58) which leads to an angular portion that is proportional to $\sin \theta \cos \phi$. The two corresponding spherical harmonics are $Y_{l}^{-1} = \frac{3}{\sqrt{8\pi}} \sin \theta \exp(-i \phi)$ and $Y_{l}^{1} = -\frac{3}{\sqrt{8\pi}} \sin \theta \exp(i \phi)$. In addition, the coefficients have the relationships as $A_{l,l} = -A_{l,-l}$, $B_{l,l} = -B_{l,-l}$, $C_{l,l} = -C_{l,-l}$, $D_{l,l} = -D_{l,-l}$.

Expanding both the Dirichlet and Neumann boundary conditions on the two conductivity boundaries in the two-layer model, we can then get four equations and solve for the four coefficients $A_{l,l}$, $B_{l,l}$, $C_{l,l}$ and $D_{l,l}$. Note that the Neumann boundary conditions in Eq. (3.57) contain conductivity terms and the coefficients $A_{l,l}$, $B_{l,l}$, $C_{l,l}$ and $D_{l,l}$ are determined by the conductivity values in each layer as well as by other geometric parameters. After solving these coefficients, we can calculate the electrical field $\mathbf{E}$ and the current density distribution $\mathbf{J}$ according to Eqs. (3.2) and (3.4).

With simulated current density, the MAT-MI forward solution of the pressure signal $p$ over all the transducer positions can be simulated using Eq. (3.14), which is a spherical integration of the MAT-MI acoustic source over the conductivity volume. This can be easily implemented using numerical method (Li et al 2007).
3.4.2 Scalar Algorithm Validation

In this validation study, we use the two-layer concentric spherical model to validate the scalar reconstruction algorithm derived in section 3.3.1. In the forward simulation, the acoustic pressure signal is calculated on a spherical detection surface with its center located at the origin. Detection sites where ultrasound transducers are placed are uniformly distributed on the detection surface. Broadband acoustic measurements are assumed. In the inverse simulation, MAT-MI acoustic sources \( AS(\mathbf{r}) = \nabla \cdot (\mathbf{J}(\mathbf{r}) \times \mathbf{B}_0) \) are first estimated using Eq. (3.36) or Eq. (3.37) with simulated pressure measurements. Following that, the electrical conductivity distribution is estimated using Eq. (3.39).

In the present simulation study, the amplitude of the static magnetic field flux density \( \mathbf{B}_0 \) and the pulsed magnetic field flux density \( \mathbf{B}_1 \) are both set to be 1 Tesla. This is an achievable field level by current commercial MRI system and magnetic stimulator. The acoustic speed is set to be 1.5 \( mm/\mu s \), which is around the sound speed in water and normal soft tissue. The displacement \( \alpha \) as in Eq. (3.55) is set to 1mm. Unless there is an explicit description, the radius of the outer sphere \( r_2 \) is set to 60mm and the radius of the detection surface is set to 140mm. The forward and inverse calculation was implemented on a 160x160x160 \( mm^3 \) cube, with a calculation grid of 1x1x1 \( mm^3 \). The temporal calculation grid was set to be 0.67 \( \mu s \), which corresponds to a transducer sampling frequency of 1.5MHz.

To evaluate the conductivity image reconstruction, Correlation Coefficient (CC), Relative Error (RE), and Average Conductivity Error (ACE) are used as quantitative performance evaluation. The CC is defined as follows.
where $\sigma_{r,n}$ are the target and reconstructed conductivity value at the $n$th pixel/voxel, and $\bar{\sigma}$, $\bar{\sigma}_r$ are the mean conductivity value for the target and reconstructed image, respectively. $N$ is the total number of pixels/voxels in the image. CC is used to assess the similarity in spatial distribution between the reconstructed and target conductivity images. RE is defined as

$$RE = \frac{\sum_{n=1}^{N} (\sigma_{n} - \sigma) \cdot (\sigma_{r,n} - \bar{\sigma}_r)}{\sqrt{\sum_{n=1}^{N} (\sigma_{n} - \bar{\sigma})^2 \cdot \sum_{n=1}^{N} (\sigma_{r,n} - \bar{\sigma}_r)^2}}$$

and is used to estimate the reconstruction error. ACE is defined as

$$ACE = \frac{\left| \sigma - \frac{1}{M} \sum_{n=1}^{M} \sigma_{r,n} \right|}{\sigma}$$

where $\sigma$ is the target conductivity value in the region of interest, $\sigma_{r,n}$ is the reconstructed conductivity value at the $n$th pixel/voxel and $M$ is the number of elements in that region of interest. ACE can be used to evaluate the reconstruction errors in different regions within a piecewise homogeneous conductor model.

A MAT-MI simulation example using the two layer spherical model is shown in Fig. 5 and Fig. 6. Figure 5 shows the real conductivity distribution and simulated MAT-MI acoustic source in the Z=0 plane. In this example, the radius of the inner sphere $r_i$
was set to 30mm and the conductivity values $\sigma_1$, $\sigma_2$, and $\sigma_3$ were set to 0.25, 0.04 and 0.4 $S/m$ respectively. This conductivity configuration is analogous to a piece of muscle embedded in a fat layer which has a lower conductivity value. From Fig. 5, we can see that the MAT-MI acoustic source $\nabla \cdot (J \times B_0)$ has large peaks at conductivity boundaries, with each peak extending approximately 3mm. This peak size is related to the numerical calculation grid, while in the continuous case, it would reduce to a pulsed function. In experiments, this is related to the pulse width of the magnetic excitation pulse sent
by the coil and the ultrasound transducer's central frequency and bandwidth. The estimated pressure in this simulation (not shown in figure) was on the order of 0.01 Pascal and is in the detectable range of current commercial transducers. Figure 6 shows the reconstructed image of the conductivity distribution with 4,902 sampling positions under noise free condition. Figures 6(a) and 6(c) are obtained from the direct reconstruction using Eqs. (3.37) and (3.39). The conductivity boundaries can be easily seen in the reconstructed image in Fig. 6(a) but the internal contrast is much weaker. In

Fig. 6, Reconstructed conductivity distribution with 4,902 transducers in (a) the Z=0 plane and at (c) Z=0, X=0. Reconstructed conductivity images after using a median filter to remove the boundary-peak-noise in (b) the Z=0 plane and at (d) Z=0, X=0. (From Ref. (Li et al 2007) with permission © 2007 IEEE)
addition, there is some projection noise in the background area, which is mainly introduced by the discrete surface integration of the backprojection algorithm given in Eq. (3.37). Figures 6(b) and 6(d) were obtained by applying a median filter with 17mm widow width to remove the boundary peaks. The CC and RE between the reconstructed image in Fig. 6(a) and the target image in Fig. 5(a) are 0.17 and 1.86, respectively. In comparison, the corresponding CC and RE between Fig. 6(b) and the target image in Fig. 5(a) are 0.81 and 0.31, respectively. The overall conductivity distribution is well reconstructed as shown in Figs. 6(b) and 5(a). This result indicates that the median filter can effectively suppress the boundary peaks and the conductivity contrast can be correctly reconstructed. However, it is also observed in Fig. 6(b) that some boundary shifts are introduced by the use of the median filter.

Similar to other back-projection algorithms used in CT or MRI, using more detection data or measurements for MAT-MI image reconstruction can enhance image quality. Here we use transducer number to represent the amount of sampling positions. The effect of using different numbers of transducers is shown in Fig. 7, where reconstructed images in the Z=0 plane using different numbers of transducers (182, 762, 1742, 4902) are compared to the target conductivity distribution in terms of CC and RE. In addition, conductive objects with different inner layer sizes are tested. In this simulation, the radius of the inner sphere $r_i$ was set to be 5, 10, 20 30 and 40mm and the conductivity values $\sigma_1, \sigma_2$ and $\sigma_3$ were set to 0.25, 0.04 and $0.4 \text{ S/m}$ respectively. No noise is added to the simulated acoustic measurements. As shown in Fig. 7, when the number of transducers is increased to 4902, the average CC value goes up to 0.83 and the
Fig. 7, (a) Reconstructed conductivity images in the Z=0 plane for the two layer spherical model with the inner layer radius set to 5, 10, 20, 30, 40 mm, respectively. Images reconstructed using different transducer number (182, 762, 1742, 4902) are compared with the target conductivity image. (b) Correlation coefficient and (c) relative error are shown to evaluate the reconstructed images. (From Ref. (Li et al 2007) with permission © 2007 IEEE)
RE value goes down to 0.31. The large RE (even when using a large number of transducers) is mainly caused by the back-projection artifact and the boundary shift introduced by the use of the median filter. In addition, it is observed that the median filter causes a further loss of detailed structure in the reconstructed images. As shown in Fig. 7, when the inner layer radius is 5mm or 10mm, the inner sphere is barely visible in the reconstructed images. In this simulation, as we assumed broadband acoustic measurements, the imaging resolution depends mainly on the size of the spatial and temporal calculation grids (Li et al 2007). In practice, however, the MAT-MI imaging resolution mainly depends on the central frequency and bandwidth of the system components including both the magnetic stimulation part and the ultrasound sensing part (Li et al 2006, Xia et al 2007).

One of the advantageous features of bioimpedance imaging is the unique contrast it can provide. In order to find out how accurately the scalar algorithm can reconstruct the conductivity values of the sample, spherical models with different conductivity contrast were tested in simulation. We set the conductivity value of the inner layer $\sigma_1$ to 0.21, 0.25, 0.3, 0.4 and 0.6 $S/m$, while the conductivities of the outer layer and surrounding media $\sigma_2$ and $\sigma_3$ were set to 0.2 and 0.4 $S/m$, respectively. The radius of the inner sphere $r_i$ was set to be 30mm. Image reconstructions were conducted under noise free condition. Figure 8(a) shows the reconstructed images in the Z=0 plane using 4,902 transducers and the corresponding absolute error images. The absolute error image was obtained by subtracting the reconstructed conductivity image from the target image and taking the absolute value. Figure 8(a) suggests that theoretically the conductivity contrast
can be correctly reconstructed using the scalar algorithm. In addition, it is shown that the reconstruction errors are mainly focused on the conductivity boundary areas using the scalar algorithm together with a median filter. This kind of error is mainly due to the boundary shift introduced by the use of the median filter. Figures 8(b) and 8(c) show the corresponding CC and RE curve under different conductivity contrast. The mean

Fig. 8, (a) Reconstructed conductivity images and Absolute Error Images in the Z=0 plane for concentric spherical models with various conductivity contrast (4,902 transducers were used). The target conductivity values of the inner layer are set to 0.21, 0.25, 0.3, 0.4, and 0.6 S/m, respectively. The conductivity value of the outer layer and the background medium were set to 0.2 and 0.4 S/m respectively. (b) and (c) show the corresponding correlation coefficients and relative errors for the reconstructed images. (From Ref. (Li et al 2007) with permission © 2007 IEEE)
reconstructed conductivity value in each layer and the related average conductivity error is presented in Table I. In the estimation of the mean reconstructed conductivity value and ACE, the large boundary errors shown in Fig. 8(a) are excluded. The inner layer area refers to the spherical region of $r < 26\text{mm}$, the outer layer refers to the region $30\text{mm} \leq r < 56\text{mm}$. From Table I, we can see that the average conductivity error is no more than 5% for each case, indicating the accuracy of conductivity value reconstruction using the scalar reconstruction algorithm. However, as we mentioned before, in this simulation we assumed broadband and noise free acoustic measurements, which is an ideal case. In practice, the MAT-MI contrast sensitivity and reconstruction accuracy would depend on the strength of the static magnetic field and dynamic magnetic stimulations, the receiving sensitivity of the ultrasound transducer and the instrument noise level.

### 3.4.3 Vector Algorithm Validation

In this section, we validate the vector reconstruction algorithm proposed in section 3.3.2 through 3D numerical simulations. As a general validation, we first set up

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**Table I: Simulation Results of Conductivity Reconstruction**

<table>
<thead>
<tr>
<th>Inner layer Area</th>
<th>Outer layer Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_t$</td>
<td>$\sigma_{ra}$</td>
</tr>
<tr>
<td>0.21</td>
<td>0.2059</td>
</tr>
<tr>
<td>0.25</td>
<td>0.2432</td>
</tr>
<tr>
<td>0.3</td>
<td>0.2905</td>
</tr>
<tr>
<td>0.4</td>
<td>0.3857</td>
</tr>
<tr>
<td>0.6</td>
<td>0.5768</td>
</tr>
</tbody>
</table>

$\sigma_t$: target conductivity value  
$\sigma_{ra}$: reconstructed average conductivity value  
ACE: Average Conductivity Error
an arbitrary curl-free force vector field as the target vector source. Its divergence was then used as the scalar acoustic source in the forward calculation as in Eq. (3.41). Finite difference method was used to calculate the divergence of the vector field. Inverse simulations were carried out according to Eq. (3.52) or (3.54). In this simulation we have assumed that the acoustic wave is omnidirectional (i.e. spherical wave) and the media is acoustically homogeneous. We have also conducted simulations with different detection radii in order to test out the condition when the far field assumption is valid and the reconstruction is reasonably good. After that, we applied the vector reconstruction algorithm to single excitation MAT-MI simulation using the two layer concentric spherical model. Simulation results with assumptions of broadband and noise-free acoustic measurements are promising. However, as this algorithm requires a rigid three dimensional measurement geometry which is complicated in experiment, it has only been validated in computer simulations.

As shown in Fig. 9 (a), the original force field has two spherical regions. One region is centered at (7, 0, 3) mm and has a radius of 10 mm while the other region is centered at (-10, -2, -2) mm and has a radius of 6 mm. Each region has a force distribution along the radial direction from the center to the periphery. The force amplitude at each spherical center is zero and increases linearly along the radius. The simulation was done using an imaging area of 38.4mm×38.4mm×38.4mm with calculation grids of 128×128×128. The acoustic signals driven by the force field were computed at 7080 detection positions that were evenly distributed on a spherical surface with a radius of 128 mm. Here we used idealized measurement surface or aperture to
Fig. 9, (a) The original force vector distribution, and (b) the corresponding reconstructed vector distribution of force; (c) and (d) are images of original and reconstructed amplitude of force distribution at Z=0 slice, respectively. (e) is the profile of force amplitude along the horizontal central line at (Z=0, Y=0) of (c) and (d). The red line is for reconstructed amplitude value and the blue one is for original value. (From Ref. (Xia et al 2009) with permission © 2009 IEEE)
verify the proposed reconstruction algorithm. Finite and limited aperture would, as expected, give imperfect reconstruction and the corresponding effects will be evaluated in future investigations. As shown in Fig. 9 (b), the reconstructed distribution of force vector is well-correlated with the actual distribution. In order to give a more clear comparison, Fig. 9(c) and Fig. 9(d) illustrate the original and reconstructed force amplitude in the Z=0 plane. Fig. 9(e) displays the profile of the force amplitude along the axis of Z=0 and Y=0. It can be seen from these figures that the reconstructed force field agrees well with the original field. Only near the edge of the spherical object there are some smooth effects, which are caused by numerical computation.

In the derivation procedure of Eq. (3.52), approximations were applied while evaluating the gradient of Green’s function, complex conjugate of Green’s function and unit vector direction of \( n_{(r_0)} \). All of them are introduced under the far-field condition where \( r_0 >> r \). In order to get a better understanding on how big the difference between \( r_0 \) and \( r \) is sufficient to make the reconstruction algorithm valid, we performed a simulation study with measurement radii varying from 1.1 to 10 times the radius of the imaging area. The correlation coefficient (CC), relative error (RE) and average angular error (AAE) were used to evaluate the reconstruction performance. Fig. 10(a) is the curve of CC between the reconstructed force field and the original field as a function of the ratio between detecting radius and the radius of the imaging area. Fig. 10(b) shows the curves of amplitude REs. Red, green, blue, and black color lines represent the corresponding amplitude REs of \( F_x, F_y, F_z \) and \( \|F\| \), respectively. The AAE, which is defined as the average of all the angular difference between the reconstructed vectors and
original vectors, was less than 0.5° and varied by only 0.03°, as shown in Fig. 10 (c). The CCs between the x, y, z components or the amplitude of the original and reconstructed force field were close to 98%. The corresponding REs were approximately 18%. The CCs and REs both varied less than 1% as the measuring radius became larger. This simulation study suggests that the reconstruction of the vectorial source field using the proposed method is not sensitive to changes in the measuring radius. It also suggests that
the proposed reconstruction method can provide a fairly good reconstruction of the force field if the observing radius is larger than twice the radius of the imaging area.

In order to demonstrate the merits of the present method as applied to magnetoacoustic tomography, a simulation study using the concentric spherical model was conducted. The conductivity values referring to different concentric spherical layers from inside to outside are 0.6, 0.2 and 0 S/m. Simulated pressure signals from 6600 transducer sites distributed on a cylindrical surface with radius of 38.4 mm were used to feed into the proposed reconstruction algorithm. It can be shown that in this model the induced eddy current are mainly constrained in the XY plane. If \( B_0 \) is uniform and in the \( z \) direction, then \( \nabla \times F = \nabla \times (J \times B_0) = (B_{0z} \frac{\partial}{\partial z})J \). If the electric current changes slowly in the \( z \) direction, the curl component of \( F \) can be ignored and the proposed vector algorithm can reconstruct the distribution of force uniquely. Actually, in the concentric spherical model, the \( F \times \nabla \) is non-zero only at those conductivity boundaries and that is where the main reconstruction error occurs. Using the proposed vector algorithm, a current density distribution can be reconstructed without employing complicated experiment setup as mentioned in Ref. (Xu and He 2005). Based on the reconstructed current density distribution and Ohm’s law \( J = \sigma E \), assuming the electric field \( E \) is similar to that induced in a uniform conductive media by the magnetic stimulation, a better conductivity reconstruction was shown as Fig. 11. Among them Fig. 11(a) is the modeled conductivity image slice at \( Z=0 \) plane. The corresponding induced current density distribution is shown in Fig. 11(c). Using Eq. (3.54), the current density can be reconstructed and it is quite similar to the original current density distribution as shown in
Fig. 11, (a) and (b) are the images of original conductivity and reconstructed conductivity distribution at Z=0 slice of a spherical model in the MAT-MI simulation; (c) and (d) are the corresponding original and reconstructed current density distributions. See text for details. (From Ref. (Xia et al 2009) with permission © 2009 IEEE)

Fig. 11(d). The CC and RE between the original and reconstructed current density amplitudes in the whole volume are 97.8% and 22.6%, respectively. Fig. 11(b) shows the reconstructed conductivity image slice at Z=0 plane. The CC and RE between the conductivity image slices shown in Figs. 11(a) and 11(b) are 99.23% and 16.46%, respectively. As compared to the imaging results using the scalar algorithm as in section
3.4.2, the vector algorithm enables reconstruction of the source current density using a closed-form solution, leading to much better performance in conductivity reconstruction.

3.5 Experiment Study

In order to demonstrate the feasibility to image electrical conductivity related contrast of biological tissues using the single excitation MAT-MI method, we have built both 2D and 3D MAT-MI experiment systems. Experiment studies using well controlled saline and gelatin phantoms were first conducted to test the system sensitivity and performance. Highly conductive salted biological tissue phantoms have also been tested using these systems. Experiment system design and setup are introduced in this section followed by the experiment results obtained from 2D and 3D single excitation MAT-MI systems. All the MAT-MI images presented in this section are calculated using the scalar algorithm as in Eq. (3.37) and Eq. (3.39) or their simplified version. In addition, in comparison to computer simulation, the acoustic measurement data obtained in experiments are generally narrow band and from only limited scanning locations. As shown in the experiment results, we are only able to reconstruct boundaries at conductivity heterogeneity and the conductivity images calculated using Eq. (3.39) does not differ much from the acoustic source maps.

3.5.1 2D Experiment System Design

The diagram of a 2D single excitation MAT-MI experiment system is shown in Fig. 12. A permanent magnet (50 mm × 50 mm × 25 mm) is used to generate the static
magnetic field, which is about 0.1 Tesla at 2 cm away from its surface. A home made magnetic stimulator is used to send pulsed stimulation with 0.5 $\mu$s pulse width through a four turn coil with 4 cm radius above the sample. The stimulator uses a high voltage and high current IGBT switch to control capacitor discharge through a coil load. Similar design is also used in the magnetic stimulator for transcranial magnetic stimulation (TMS) (Malmivuo and Plonsey 1995). A 500 KHz flat ultrasound sensor (TC3029, Reson Inc.) mounted to a scanning system can scan around the sample in a circular orbit. Both the sample and transducer are submerged in distilled water for acoustic coupling. Data acquisition with sampling frequency of 5MHz is synchronized with the magnetic stimulation through a multi-functional data acquisition card (National Instrument PCI 6111). The signal collected from the transducer is amplified by 80 dB. Because of the
limited SNR, 100,000 times of averaging at the repetition rate of 500 Hz were used. This
composes a 2D MAT-MI system.

We could increase the system sensitivity and improve the field homogeneity by
using two permanent magnets and paired Helmholtz coil for magnetic stimulation as in
the 3D single excitation MAT-MI system shown in Fig. 17. Detailed system parameters
of that setup will be described in section 3.5.3.

The reconstruction algorithm shown in Eq. (3.37) is intrinsic 3D and requires
acoustic measurements on a closed surface surrounding the sample in order to
quantitatively reconstruct the conductivity distribution. However, with a phantom that is
uniform in the \( z \) direction, the MAT-MI imaging problem can also be downgraded to a
2D problem. The corresponding 2D algorithm in time domain for reconstructing MAT-
MI acoustic source from measurements collected on a circular orbit \( l_0 \) surrounding the
imaging object can be written as in Eq. (3.62) (Xia et al 2007):

\[
\text{AS}(\mathbf{r}) = \frac{1}{4\pi c_s^3} \int_{ \partial \Omega_0 } d\mathbf{l} \frac{1}{|\mathbf{r} - \mathbf{r}_0|} \frac{\partial^2 p(\mathbf{r}_0, |\mathbf{r} - \mathbf{r}_0|/c_s)}{\partial r^2}
\]  

(3.62)

where \( \mathbf{r}_0 \) is the position vector denoting where the ultrasound transducer is located.

A simpler method may also be used to reconstruct a 2D image slice if the distance
between the acoustic source and the transducer is much larger than the dimension of the
object. In this case, the acoustic wave front in the object can be approximated as a
crossing line parallel to the transducer surface. Therefore, the acoustic signal received by
the transducer surface at a certain time point can be considered as the summation of
acoustic sources located on a crossing line of the object parallel to the transducer surface. In such case, we can have a simplified back projection algorithm as in Eq. (3.63)

$$A\hat{S}(r) = \int_{l_0} dl_0 p(r_0, t)$$

(3.63)

Here \(A\hat{S}(r)\) represents the distribution of the total acoustic sources including the direct MAT-MI acoustic source and the scattered acoustic sources caused by the inhomogeneous media. An image reconstructed using Eq. (3.63) reflects the combined features of both the MAT-MI acoustic sources and acoustic scattering, while an image reconstructed using Eq. (3.62) represents the distribution of the MAT-MI acoustic sources associated with the electrical conductivity of the object.

### 3.5.2 2D Experiment Results

Using the 2D single excitation MAT-MI system, images of saline samples with different salinities i.e. 10%, 8%, 5%, 3%, 1% and 0%, are shown in Fig. 13. The corresponding conductivities of these saline samples are estimated to be 13.1, 10.3, 7.9, 5.6, 1.8, 5e-3 in the unit of S/m. The imaging center is at the origin of the circular scanning orbit. The saline samples were put in a plastic cup and emerged in water. This creates a conductivity step which is analog to a homogeneous tissue with higher conductivity embedded in a low conductive one. The transducer scanned around the sample with a 2.5 degrees scanning step size. The right bottom image in Fig. 13 (water sample) indicates that the plastic cup had little influence on the reconstructed image. It is shown that this 2D MAT-MI system can distinguish the conductivity difference between water and saline sample with salinity of 1% (corresponding to about 2 S/m conductivity.
contrast). In addition, it is obvious that the image boundary intensity positively correlates with the conductivity contrast between the sample and the background water medium.

Figure 14 shows an example MAT-MI image of a gel phantom. Two columns of gels (a cylinder shape and a square prism shape) with 0% salinity are embedded in gel of 10% salinity. All the gel blocks were made from mixture of water and 5% animal skin gelatin powder. Plastic film was inserted between these embedded blocks with the background gel to prevent ion diffusion. The scanning step is 1.25 degree. As seen in Fig. 14, the 2D MAT-MI image is consistent with the cross section of the phantom in terms of shape and size.

It is shown in Fig. 13 and Fig. 14 that the width of all the conductivity boundaries in the reconstructed images extends to about 3mm, indicating an “effective” spatial…
resolution of 3mm. Here we define the spatial resolution as the diameter of the smallest structure that can be reconstructed using this 2D MAT-MI setup. This is partly demonstrated in Fig. 14 that the shortest distance between the inner square prism boundary and the outer layer boundary is 4mm, which is clearly seen in the reconstructed image. Higher spatial resolution may be obtained by increasing the central frequency and bandwidth of the MAT-MI system.

Using a 2D MAT-MI system with improved sensitivity as in Fig. 17, we have also conducted an imaging study on tissue phantoms. Fig. 15 shows the 2D MAT-MI imaging
Fig. 15, (a) Photo of a salt pork tissue sample. (b) and (c) are reconstructed MAT-MI images of the tissue sample in (a) using the reconstruction algorithms listed in Eq. (3.62) and Eq. (3.63) respectively. (d) Photo of a pork tissue sample composed of a block of pork muscle embedded in a cylindrical fat layer. The surrounding material is animal gelatin. (e) and (f) are reconstructed MAT-MI images of the pork tissue sample in (d) using Eq. (3.62) and Eq. (3.63) respectively. (From Ref. (Xia et al 2007) with permission © 2007, American Institute of Physics)
results of two tissue phantoms. Figs. 15(a)–(c) are the imaging results of a salt pork tissue phantom. The phantom was composed of a piece of salted pork (1.2g sodium per 56g salted pork, the salted muscle tissue has conductivity of 2-4 S/m, the salted fat tissue has conductivity of 0.2-0.4 S/m) placed in a plastic cup filled with gel. Figs. 15(d)–(f) are the results of a tissue phantom made from fresh pork muscle and fat. As shown in Fig. 15(d), the inner part of the phantom is a block of saline soaked pork muscle (13mm×10mm) and the outer part is cylindrically shaped pork fat with a diameter of 25mm. The surrounding portion is animal gelatin made from mixture of 10% saline and 5% animal skin gelatin powder. Figs. 15(b) and (e) are reconstructed images of the two phantoms using Eq. (3.62), which is more accurate in the reconstruction of the MAT-MI acoustic sources (the divergence of the Lorentz force) within the tissue samples. Figs. 15(c) and (f) are reconstructed images using Eq. (3.63) which represent the distribution of the total acoustic sources in the samples. As shown in Fig. 15, the reconstructed images using either method are consistent with the tissue phantom geometry. Because the conductivity of muscle tissue is quite different from that of fat tissue, the boundaries between these two types of tissue are clearly seen in the reconstructed images. For example, in Figs. 15(a)-(c), a thin muscle layer can be identified in the reconstructed images (marked by the arrows).

The experiment results obtained using the 2D single-excitation MAT-MI systems with saline, gel and biological tissue phantoms demonstrate the feasibility to image with high spatial resolution the electrical conductivity related contrast through the MAT-MI approach. However, it is also demonstrated that with the 2D single-excitation MAT-MI
system, we can only reconstruct the conductivity boundaries of the sample in real experiments.

### 3.5.3 3D Experiment System Design

The 3D single-excitation MAT-MI system was developed by incorporating ultrasound focusing and 3D scanning into the 2D system. Fig. 16 shows the schematic diagram illustrating the concept of the 3D MAT-MI using a focused cylindrical scanning mode. Through ultrasound focusing in the \( z \) direction, we can localize the acoustic sources in a specific XY plane. A 2-dimensional (2D) MAT-MI image can be obtained at each cross section of the 3D object on which the acoustic sensors are focused. Furthermore, vertical scans in the \( z \) direction can provide a stack of 2D images, thus forming a 3D volume image of the object.

The practical 3D single-excitation MAT-MI system setup is shown in Fig. 17. The ultrasound transducer was immersed in distilled water. The employed transducer was a 500 KHz flat single element transducer with 29mm diameter (Panametrics, 513080).
Ultrasound focusing was implemented by adding an acoustic lens before the transducer. This transducer was mounted to a frame and can scan around the sample in a horizontal plane (the XY plane) to collect ultrasound signals from different angles. The scanning radius was approximately 150mm and the scanning angular range was from $1^\circ$ to $330^\circ$ with $1.25^\circ$ scanning step. In addition, the transducer can move in the vertical direction (the $z$ direction) with a 5mm scanning step. A home-made magnetic stimulator, which has a nine-turn coil with a radius of 40mm, was used to deliver magnetic pulses. In the experiment study, the stimulating pulse width was 1-µs with a pulse repetition frequency of 140 Hz. In order to get a more uniform time-varying magnetic field, two coils were placed symmetrically above and below the sample. This creates a Helmholtz coil, which produces a region with a nearly uniform magnetic field. According to the measured electrical current in one of the stimulating coils, the maximum magnetic flux density at

![Fig. 17, Setup of the 3D single excitation MAT-MI experimental system (From Ref. (Xia et al 2007) with permission © 2007, American Institute of Physics)](image)
the center of the sample was estimated to be 0.015T. In order to get a stronger and more uniform static magnetic field, magnets were also placed above and below the sample. Each magnet (36.5mm in radius) was placed 2 cm away from the sample and created a 0.2 Tesla static magnetic field near the sample. A computer controlled pulse generator was used to send trigger signals to the magnetic stimulator. After each stimulating pulse, signals from the transducer were amplified 90dB and recorded for 400 microseconds. The amplified signals were sampled at 5MHz using a data acquisition system (NI PCI6111), and averaged 50,000 times to increase SNR.

3.5.4 3D Experiment Results

To demonstrate the feasibility of the proposed focused cylindrical scanning mode MAT-MI in 3D imaging, an experiment study was conducted on a 3D gel phantom and the results are shown in Fig. 18. In this experiment, the ultrasound signal we collected was centered at 500 KHz, making its wavelength in water to be around 3mm. This indicates that the effective resolution in each layer is 3 mm. At the same time, because the beam width (6dB) of the focused transducer is 7mm, it indicates that the vertical resolution of the system is 7 mm. As shown in Fig. 18(a), the cylindrical shaped gel phantom is composed of two parts. The upper part has a 1cm×1cm×2.5cm cubic shaped hole on the right side and the lower part has a 1cm in diameter by 2.5cm high cylindrical shaped hole on the left side. Fig. 18(b) is a photo of the phantom. This gel phantom is made of gel with 10% salinity. During the imaging process the two holes were filled with vegetable oil with much lower conductivity than the surrounding gel. Fig. 18(c) shows
five slices of MAT-MI images collected at five successive layers from top to bottom. All of the images are reconstructed using Eq. (3.62). The distance between each layer is 5mm. From Fig. 18 it can be seen that the reconstructed MAT-MI images are consistent with the 3D geometry of the phantom. In the first two slices, only a rectangular shape is
clearly seen, which corresponds to the top part of the phantom. In the third slice, which is the middle layer containing the cubic and cylindrical structures, both shapes can be seen. In the fourth slice, as the scanning position moves lower, a clear circle with an indistinct rectangle are reconstructed. Only the circular structure is visible in the fifth slice.

For the focused cylindrical scanning mode MAT-MI with single excitation, in order to increase the resolution in the elevation direction (the \( z \) direction), a focused transducer with a narrower beam width should be used. Focus transducers with narrower beam generally have higher central frequency and smaller f-number (proportional to the ratio of focal length over aperture size). However, for focus transducer with a shorter focal length, a better EM shielding may need to be used as the EM interference increases when the transducer gets closer to the excitation coil. In addition, in 3D MAT-MI, since the collected acoustic signals are restrained in one single slice, more gain in signal strength is needed to obtain high resolution 3D MAT-MI images in biological tissue.

3.6 Discussion

In this chapter we have derived two reconstruction algorithms for the single-excitation MAT-MI imaging approach. Computer simulations using a concentric spherical volume conductor model have been used to validate these two algorithms. It is shown that with ideal broadband acoustic measurements collected on closed 3D surfaces, we are able to correctly reconstruct the conductivity distributions of the sample. This has in principle demonstrated the feasibility to image electrical conductivity related contrast of biological tissue with high spatial resolution. However, for the scalar algorithm, a
smoothing median filter needs to be used to extract the internal conductivity contrast and remove the boundary peaks. This generally would degrade the imaging resolution as shown in the simulation results in Fig. 7. The vector algorithm derived under a curl-free conduction and far-field approximations can be used to solve the MAT-MI Lorentz force source field in a close-form, but it is sort of hard to apply in practice as its need of fixed measurement geometries. In addition, the calculation of the conductivity distribution using this algorithm needs further modeling and iteration in general.

In all the algorithm derivations, we assumed that the medium is acoustically homogenous, and the effect of acoustic heterogeneity in soft tissues is negligible. The acoustic heterogeneity of soft tissue is less than 10% (Duck 1990), and the acoustic pathway in MAT-MI is about half the length of conventional diagnostic ultrasound. Furthermore, the effect of acoustic heterogeneity in MAT-MI is quite similar as that in photoacoustic tomography (PAT), which has been demonstrated negligible both theoretically (Xu and Wang 2003) and experimentally (Wang et al. 2003). Normally the acoustic heterogeneity could be considered as a secondary scattering acoustic source. If the amplitude of these secondary acoustic signals is small, acoustic signals generated by them can be taken as background noise. Corresponding random noise model may be used to investigate how acoustic heterogeneity influences the proposed reconstruction algorithm. More theoretical discussion about the effect of acoustic heterogeneity on thermoacoustic tomography (TAT) can be found in Ref. (Anastasio et al. 2005).

Besides the computer simulation studies, we have also demonstrated the feasibility of conductivity related imaging using the single-excitation MAT-MI approach
through 2D and 3D experiments. Spatial resolution of several millimeters was achieved in the reconstructed MAT-MI images and it has been demonstrated that this type of MAT-MI images is associated with electrical conductivity contrast as shown in Figs. 13-15. However, as in real experiment systems the acoustic measurements are generally narrow band and collected from limited scanning locations, both the 2D and 3D single-excitation MAT-MI systems are only able to reconstruct boundaries at conductivity heterogeneity. This leads to the development of the multi-excitation MAT-MI method as introduced in chapter 4.

In addition, the imaging sensitivity of current single-excitation MAT-MI systems is still not high enough to image normal in vitro or in vivo biological tissues which have electrical conductivity in the range of 0.01 S/m to 1 S/m. As indicated from its signal generation mechanism, i.e. the forward modeling as in section 3.2.1, besides the sample’s conductivity contrast the MAT-MI signal intensity is proportional to the strength of the static magnetic field and the dynamic magnetic excitations. In the developed 2D and 3D single-excitation MAT-MI systems, the static magnetic field is around 0.1-0.2 Tesla, while the maximum dynamic magnetic excitation is around 1e3-1e4 T/s with microsecond level pulse width. While this levels of field intensity is easily achievable with permanent magnets and home made device, further improvement on the field intensity and thus MAT-MI imaging sensitivity is feasible by incorporating standard MRI machine or high power magnetic stimulator.
Chapter 4
Multi-Excitation MAT-MI

4.1 Introduction

In order to achieve a more complete and accurate reconstruction of the conductivity contrast using the MAT-MI method, based on the analysis of the relationship between the conductivity distribution and the generated MAT-MI acoustic source, we recently proposed the multi-excitation MAT-MI approach and the corresponding reconstruction algorithms (Li and He 2010a, Li and He 2010b, Li et al 2010). In this approach, multiple magnetic excitations using different coil configurations are employed and ultrasound measurements corresponding to each excitation are collected to derive the conductivity distribution inside the sample. A modified reconstruction algorithm is also proposed for the multi-excitation MAT-MI imaging approach when only limited bandwidth acoustic measurements are available. We have conducted both 2D and 3D computer simulation and phantom experiment studies to test the performance of the proposed method. It is shown that if unlimited bandwidth (broadband) acoustic measurement data is available, we can accurately reconstruct the internal conductivity contrast of an object without using any filtering. With practical limited bandwidth (narrowband) data we can reconstruct the relative conductivity contrast of an object which still provides more useful information than the conductivity boundary images obtained from single-excitation MAT-MI. Benefits that come with this new method include better differentiation of tissue types with conductivity contrast,
specifically for potential breast cancer screening application in the future. Most materials in this chapter are taken from (Li and He 2010b, Li et al 2010).

4.2 Imaging Problem Description

4.2.1 Forward Problem

Similar as the forward problem of single-excitation MAT-MI, the forward problem of multi-excitation MAT-MI approach also describes two major physical processes in its signal generation mechanism, i.e. magnetic induction in the conductive sample and acoustic wave propagation with the Lorentz force induced acoustic sources. The only difference here is that multiple dynamic magnetic excitations are applied and multiple acoustic measurement data sets are generated accordingly.

We consider a sample domain $\Omega$ with isotropic conductivity $\sigma(r)$. The sample is placed in a static magnetic field with flux density $B_0(r)$. In the multi-excitation MAT-MI approach, we have $N$ different excitation coil setups with $N \geq 2$. Denote the stimulating time-varying magnetic field generated from the $j$th coil setup as $B_j^j(r, t)$ for $j = 1, \ldots, N$. The $j$th stimulating magnetic field applied to the conductive sample induces the corresponding electrical field $E^j(r, t)$ and eddy current density distribution $J^j(r, t)$. Similar as in the forward problem of single-excitation MAT-MI introduced in section 3.2.1, because we are considering $\mu s$ long magnetic stimulations in biological tissues we take the quasi-static condition and ignore the displacement current. The quasi-static condition allows us to separate the spatial and temporal function of the time-
varying magnetic field, i.e. $B'_j(r,t) = B'_j(r) f_j(t)$. According to Faraday's Law and Ohm's Law, the similar spatial and temporal separation holds for the induced electrical field and eddy current density, i.e. $E'_j(r,t) = E'_j(r) f'_j(t)$ and $J'_j(r,t) = J'_j(r) f'_j(t)$ where the prime denotes first order time derivative. The quasi-static condition also indicates that the stimulating magnetic field in the sample can be well approximated by the field generated by the same coil configuration in free space. Using the notations of magnetic vector potential $A^j(r,t)$ where $B'_j = \nabla \times A^j$ and electrical scalar potential $\phi^j(r)$, we can derive the governing equations for magnetic induction in multi-excitation MAT-MI with similar derivation process as in section 3.2.1. The major governing equations can then be written as in Eq. (4.1):

$$\nabla \cdot (\sigma \nabla \phi^j) = -\nabla \cdot (\sigma \frac{\partial A^j}{\partial t}) \quad j = 1, \ldots, N$$

Because of the quasi-static condition the magnetic vector potential $A^j$ and the corresponding flux density $B^j$ depend only on the $j$th coil configurations and can be estimated with known coil geometry. The magnetic vector potential $A^j$ in Eq. (4.1) is then considered to be known. Equation (4.1) subject to a Neumann boundary condition at the outer boundary $\partial \Omega$ on the current density $J^j$ as in Eq. (4.2) has a unique solution for electrical potential $\phi^j(r)$ inside the conductive domain $\Omega$ when we choose a reference position with zero potential (Wang and Eisenberg 1994).

$$J^j \cdot n = 0 \quad j = 1, \ldots, N$$

Here $n$ is the unit vector norm of the outer boundary surface $\partial \Omega$ of the conductive
object. This boundary condition requires the current density component that is normal to the bounding surface to vanish. With known $\sigma$ and $A_j$, we can solve for $\phi_j$ throughout the whole 3D conductive volume using the finite element method (FEM). The corresponding electrical field and current density can then be computed, as

$$E_j = -\frac{\partial A_j}{\partial t} - \nabla \phi_j, \text{ and } J_j = \sigma E_j.$$

With the magnetically induced eddy current $J_j$ and the static magnetic field $B_0$, the Lorentz force acting on the eddy current can be described as $J_j \times B_0$. As derived in section 3.2.1, in MAT-MI the divergence of the Lorentz force acts as acoustic source of propagating ultrasound waves that can be sensed by ultrasonic transducers placed around the sample. The wave equations governing the pressure distribution can then be written as in Eq. (4.3):

$$\nabla^2 p_j - \frac{1}{c_s^2} \frac{\partial^2 p_j}{\partial t^2} = \nabla \cdot (J_j \times B_0) \quad j = 1, \ldots, N \quad (4.3)$$

where $p_j$ is the pressure corresponding to the $j$th magnetic stimulation and $c_s$ is the acoustic speed in the media. Here we also assume the sample is acoustically homogeneous. Using the 3D Green’s function, the solution to Eq. (4.3) can be written as in Eq. (4.4):

$$p_j(r_0, t) = -\frac{1}{4\pi} \int_V d\mathbf{r} \nabla_r \cdot [J_j \times B_0] \frac{\delta(t - |r_0 - \mathbf{r}|/c_s)}{|r_0 - \mathbf{r}|} \quad (4.4)$$

where $r_0$ is a position located on certain ultrasound detection aperture. Depending on the ultrasound measurement scheme, the pressure signal $p_j$ at certain detection location may
be considered as a 3D volume integration or 2D surface integration of the acoustic sources multiplied with appropriate 3D or 2D Green’s function.

### 4.2.2 Inverse Problem

The inverse problem of multi-excitation MAT-MI concerns how to reconstruct the conductivity distribution $\sigma(r)$ of the sample with the obtained acoustic measurements $p_j$ with $j = 1, \cdots, N$ due to different magnetic excitations. First, with the acoustic measurements $p_j$ obtained on certain acoustic aperture around the sample, we can reconstruct the acoustic source map, i.e. distributions of $A S^j = \nabla \cdot (J^j \times B_0)$, in the 3D conductive volume using the time reversal back projection method (Xu and He 2005, Xu and Wang 2004) as in Eq. (3.36) or Eq. (3.37). In addition, multiplying both sides of Eq. (4.4) with $-4\pi t$, Eq. (4.4) becomes

$$\int_{-t}^{t} d\tau A S^j \delta(\tilde{r} - r) = g_j(r, \tilde{r})$$  \hspace{1cm} (4.5)

where $\tilde{r} = c_0 t$. Equation (4.5) takes the form of a spherical Radon transform and the corresponding expectation maximization algorithm developed for reflective tomography (Pan et al 2003) can also be used to reconstruct the acoustic source map. The EM algorithm generates fewer artifacts than the time reversal method when dealing with limited view angle data (Pan et al 2003), but it is computationally more demanding.

After reconstructing the acoustic source map, we can then derive the conductivity distribution of the object, which is of more interest from the clinic application perspective. Taking the fact that the static magnetic field $B_0$ in MAT-MI is generated
from sources outside the conductive object, for example from some permanent magnets, we have $\nabla \times \mathbf{B}_0 = 0$ inside the object volume and the acoustic source term on the right hand side of Eq. (4.3) can be further simplified as $(\nabla \times \mathbf{J}^j) \cdot \mathbf{B}_0$. Expanding this term using Ohm’s Law $\mathbf{J}^j = \sigma \mathbf{E}^j$, we can obtain Eq. (4.6)

$$AS^j = \sigma (-\frac{\partial \mathbf{B}^j}{\partial t}) \cdot \mathbf{B}_0 + (\nabla \sigma \times \mathbf{E}^j) \cdot \mathbf{B}_0$$  \hspace{1cm} (4.6)

Let the static magnetic field sit in the $z$ direction i.e. $\mathbf{B}_0 = B_0 \hat{z}$ and note that $\mathbf{B}^j (\mathbf{r}, t) = \mathbf{B}^j (\mathbf{r}) f_j(t)$ and $\mathbf{E}^j (\mathbf{r}, t) = \mathbf{E}^j (\mathbf{r}) f_j(t)$, Eq. (4.6) can be rewritten as Eq. (4.7):

$$AS^j = (\sigma (-B_{0z}^j) + (\frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y}) \cdot (E_x^j, -E_y^j)) B_0 \cdot f_j(t) \hspace{1cm} j = 1, \ldots, N$$  \hspace{1cm} (4.7)

where $E_x^j$ and $E_y^j$ are the $x$ and $y$ components of the induced electrical field vector $\mathbf{E}^j$, respectively. $B_{0z}^j$ is the $z$ component of $\mathbf{B}^j (\mathbf{r})$. Note here that this equation holds for every position inside the 3D sample volume and all acoustic sources have similar function in time i.e. $AS^j (\mathbf{r}, t) = AS^j (\mathbf{r}) f_j(t)$. In addition, as shown in Eq. (4.7) the generated acoustic source in MAT-MI is related to both the conductivity distribution of the sample and its spatial gradient in XY planes as well. For numerical stability consideration, in the following we consider solving $\nabla \sigma = \left( \frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y} \right)$ first and then derive the conductivity $\sigma (\mathbf{r})$ itself.

Using matrix form Eq. (4.7) can be written as in Eq. (4.8)

$$\mathbf{Ux} = \mathbf{b}$$  \hspace{1cm} (4.8)

where
With $N$ appropriately chosen coil setups, we can get the determinant of matrix $U$ to be nonzero and obtain $x$ using the regularized least square method

$$x = (U^T U + \lambda I)^{-1} U^T b$$

(4.9)

where $U^T$ is the transpose of $U$, $\lambda$ is a regularization parameter and $I$ is a $2 \times 2$ identity matrix. Many methods can be used to determine the regularization parameter $\lambda$ such as the L-curve method (Hansen 1994). We can also set it to be proportional to the condition number of the matrix $U$. However, if the condition number of the matrix $U$ is not large in the whole region of interest, we can select $\lambda = 0$ and Eq. (4.9) becomes the normal least square solution. Additionally, note here that the entries of matrix $U$ are components of the induced electrical field, which depends on the unknown conductivity distribution $\sigma(r)$ and the vector $b$ contains a term related to the conductivity distribution too. Therefore in order to calculate $x$ and derive the conductivity $\sigma(r)$, an iterative algorithm is required, as we will discuss later.

In order to compute $\sigma$ from $\nabla \sigma = \left(\frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y}\right)$ in all the imaging slices, a 2D layer potential integration technique can be used as in Eq. (4.10) (Oh et al 2003)

$$\sigma(r) = -\int_S \nabla_r \Phi(r - r') \cdot \nabla \sigma(r') dr' + \int_{\partial S} \mathbf{n}_r \cdot \nabla_r \Phi(r - r') \sigma_{\partial S}(r') dl_r'$$

(4.10)
where $\Phi(r-r') = \frac{1}{2\pi \log|r-r'|}$ is the two dimensional Green’s function of the Laplacian operator and $\nabla_r \Phi(r-r') = -\frac{1}{2\pi \log|r-r'|}$. $S$ denotes the 2D imaging region of interest (ROI) in the imaging slice where $\nabla \sigma$ is obtained and $\partial S$ denotes its boundary. $\sigma_{\partial S}$ is the conductivity value restricted at the boundary $\partial S$. The 2D integration as in Eq. (4.10) can be applied in a whole 3D volume slice by slice.

### 4.3 Reconstruction Algorithm

In this section we describe the image reconstruction algorithm for the proposed multi-excitation MAT-MI approach. A modified algorithm for conductivity reconstruction from limited bandwidth (narrowband) acoustic measurement data is also presented.

For $j = 1, \cdots, N$, we apply different magnetic excitations on the sample through different coil setups and collect the corresponding pressure measurements on certain acoustic aperture. The conductivity value $\sigma_{\partial S}$ at the boundary of the imaging ROI is measured experimentally. In practice, this can be done by applying certain coupling material with known conductivity value on the sample surface, and letting $\partial S$ reside in the area filled with this coupling material. Then the multi-excitation algorithm is as follows:

Step 1: Calculate the acoustic source map $A_S^j$ in the whole object volume using the time reversal back projection algorithm or EM algorithm for $j = 1, \cdots, N$. 

96
Step 2: Let $i = 0$ and assume an initial conductivity distribution $\sigma_0$.

Step 3: Solve the differential equations as in Eq. (4.1) combined with the Neumann boundary conditions as in Eq. (4.2) in the whole conductive volume for $j = 1, \cdots, N$.

Step 4: Compute $E_j$ based on the solution from Step 3.

Step 5: Compute $\sigma_{i+1}$ using Eqs. (4.9) and (4.10) on every imaging slice.

Step 6: If the relative error between $\sigma_{i+1}$ and $\sigma_i$ is larger than the given tolerance $\varepsilon$, i.e. \[
\frac{\|\sigma_{i+1} - \sigma_i\|_2}{\|\sigma_{i+1}\|_2} > \varepsilon,
\]
replace $i$ by $i+1$ and go to Step 3. Otherwise finish the procedure and use $\sigma_{i+1}$ as the solution.

One of the major technical limitations of the single-excitation MAT-MI approach comes from its narrowband acoustic measurements. Generally, in order to achieve millimeter level spatial resolution, ultrasound transducers with around MHz central frequency are needed. Using these transducers, DC to very low frequency signal components are usually not available in the collected pressure signal. This effect can be considered as a band pass filtering procedure in the MAT-MI forward problem. With this measurement data, denoting it $\tilde{p}_j$, we can only reconstruct part of the acoustic source distribution, denoting it $\tilde{A}\tilde{S}^j$. As shown in Eq. (4.6), the generated MAT-MI acoustic source is related to both the conductivity distribution and its spatial gradient. However, if only limited bandwidth acoustic measurements are available, these two subtypes of sources can not be detected equally. Assuming the sample is piecewise homogenous, the
gradient source (i.e. the acoustic source related to the conductivity gradient term $\nabla \sigma$, shown in the second term on the right hand side of Eq. (4.6)) can be considered as a broadband source. On the contrary, the conductivity source (i.e. the acoustic source related to the conductivity $\sigma$ itself, shown in the first term on the right hand side of Eq. (4.6)) is a narrow-band source whose central frequency depends on the object geometry. In addition, the gradient source is generally much larger than the conductivity source as shown in our previous computer simulation study as in section 3.4 and experiment studies as in section 3.5. In consequence, with limited bandwidth measurements, the acoustic source we can reconstruct will mainly be determined by the conductivity gradient term as in Eq. (4.11):

$$A \tilde{S}^j = \left( \frac{\partial \sigma}{\partial x}, \frac{\partial \sigma}{\partial y} \right) \cdot (E_j^i, -E_j^i) B_{0z} f_j^i(t) \quad j = 1, \ldots, N \quad (4.11)$$

The corresponding matrix form can be written as:

$$\mathbf{U} \mathbf{x} \approx \tilde{\mathbf{b}} \quad (4.12)$$

where $\mathbf{U}$ and $\mathbf{x}$ take the same definitions as in Eq. (4.8) and

$$\tilde{\mathbf{b}} = \begin{bmatrix} A \tilde{S}_1^i(r) \\ B_{0z} \\ \vdots \\ A \tilde{S}_N^i(r) \\ B_{0z} \end{bmatrix}$$

Replacing vector $\mathbf{b}$ with vector $\tilde{\mathbf{b}}$, a similar reconstruction procedure can be applied to estimate the conductivity distribution of the object. However, as we will show in the simulation and experiment studies, with limited bandwidth acoustic measurements, we
are not able to quantitatively reconstruct the absolute conductivity values. What will be visible in the reconstructed MAT-MI image is the relative conductivity contrast.

4.4 Computer Simulation Study

In order to validate the proposed multi-excitation MAT-MI approach and test its performance, we have conducted 2D and 3D computer simulations. We first developed a finite element method based MAT-MI forward solver in order to deal with volume conductor models that have arbitrary geometry. Two dimensional simulations were then performed under the assumption that the volume conductor is uniform in the $z$ direction and the magnetic fields are also approximately uniform in the $z$ direction. Magnetic excitations generated from a Helmholtz coil pair and two double figure eight coil pairs taking different orientations are employed in the multi-excitation system. Three dimensional simulations were conducted by combining the multi-excitation MAT-MI with the focused cylindrical scanning scheme. All the simulation results suggest a superior performance of the multi-excitation method over the single-excitation method. With broadband acoustic measurements, the multi-excitation approach can accurately reconstruct the conductivity distribution of both 2D and 3D models. With the simulated narrowband data, the multi-excitation MAT-MI method can still give a good reconstruction of the relative conductivity contrast.
4.4.1 MAT-MI FEM Based Forward Solver

As described in the MAT-MI forward problem, the forward solution of the MAT-MI acoustic signals with arbitrary geometry and inhomogeneous conductivity distribution can only be obtained using numerical methods such as the finite element method. The key problem is to solve the magnetic induction problem in a bounded region described by the governing equations as in Eqs. (4.1) and (4.2). After solving this problem, the MAT-MI acoustic source and generated acoustic pressure signals can be easily calculated with just numerical differentiation and integration as in Eq. (4.4).

In order to validate the multi-excitation MAT-MI method in more complicated models, we have developed both 2D and 3D FEM based MAT-MI forward solver. Finite element method was chosen over other possible numerical methods because of its ability to easily represent complex geometries and conductivity inhomogeneity.

In the finite element method, the solution to the governing equation in a bounded region is generally determined by minimizing an energy function (functional) or weighted residuals (Jin 2002). For the magnetic induction problem in MAT-MI, using variational principles we can show that the appropriate functional is the dissipated power in the conducting media (Wang and Eisenberg 1994) as in Eq. (4.13):

\[
W(\phi) = \int_{V} (J \cdot E) d\mathbf{r} = \int_{V} \sigma(\nabla \phi + \frac{\partial \mathbf{A}}{\partial t})(\nabla \phi + \frac{\partial \mathbf{A}}{\partial t}) d\mathbf{r}
\]  

(4.13)

Here we assume the conductivity \( \sigma \) is isotropic. The derivation of the FEM formula considering conductivity anisotropy can be found in Ref. (Wang and Eisenberg 1994). FEM formula using Galerkin’s method is also discussed in Ref. (Li et al 2009). In the developed MAT-MI forward solvers, we use linear triangular elements for the 2D solver.
and linear tetrahedral elements for the 3D solver. In the following part of this section the basic FEM formula for the 3D MAT-MI forward solver is given. The corresponding 2D formula can be readily downgraded from its 3D counterpart.

To minimize the functional shown in Eq. (4.13), we divide the conductive volume into small sub-volumes called finite elements. These elements are interconnected at points called nodes. The elements are generally of the same shape but differ in size. Within every element, using its interpolation functions/shape functions $h_i$, the electrical potential $\phi(\mathbf{r})$ in that element can be approximated in Cartesian coordinate as in Eq. (4.14):

$$\phi(x, y, z) = \sum_{i=1}^{4} \phi_i h_i(x, y, z)$$

(4.14)

where $\phi_i$ is the nodal potential on the $i$th node of the element. For linear tetrahedral element, its linear interpolation functions have the form as in Eq. (4.15):

$$h_i = a_i x + b_i y + c_i z + d_i$$

(4.15)

where $a_i, b_i, c_i, d_i$ are constant coefficients determined by the coordinates of the four nodes of that tetrahedral element. The known magnetic vector potential $\mathbf{A}(\mathbf{r})$ can also be interpolated over every element in the same way. With the interpolated electrical potential and magnetic vector potential, the dissipated power/functional in an element $W^e(\phi)$ can be described as in Eq. (4.16) (Wang and Eisenberg 1994):

$$W^e(\phi) = (\Phi^e)^T K^e \Phi^e + (\Phi^e)^T g^e$$

(4.16)
where \( \Phi = [\phi_1, \phi_2, \phi_3, \phi_4]^T \) is a column vector of the four nodal potentials. Note here, as compared to the functional shown in Eq. (4.13), a constant term that is not a function of nodal potentials and has no effect on the forward solution is omitted in Eq. (4.16). In Eq. (4.16) the matrix \( K \) and the vector \( g \) can be derived as in Eqs. (4.17) and (4.18) as follows:

\[
K_{ij} = \int_{V} \sigma \nabla h_i \cdot \nabla h_j \, d\mathbf{r} = V_{net} \sigma (a_i a_j + b_i b_j + c_i c_j) \quad (4.17)
\]

\[
g_j = 2f \sum_{j=1}^{4} \int_{V} \sigma \nabla h_i \cdot A_j h_j \, d\mathbf{r} = 2f \sum_{j=1}^{4} \sigma (a_i (A_i)_j + b_i (A_y)_j + c_i (A_z)_j) \quad (4.18)
\]

The total dissipated power \( W(\phi) \) over the entire conductive volume is then summed up over all finite elements. Assembling all the matrix equations as in Eq. (4.16) for all the elements give the final stiffness matrix \( K \), the load matrix \( g \) and the nodal potentials on all the nodes \( \Phi \) (Jin 2002). Minimizing the total dissipated power (functional) with respect to every nodal potential yield the matrix equation for solving the potentials as in Eq. (4.19):

\[
2K\Phi + g = 0 \quad (4.19)
\]

Choosing one reference point with zero potential, for example, \( \Phi_N = 0 \), we can uniquely solve the unknown nodal potentials on all other nodes by taking the matrix inversion. After solving the nodal potential the electrical field and current density distribution can be easily calculated according to Eqs. (3.2) and (3.4).
The 2D MAT-MI forward solver was implemented using the Matlab PDE Toolbox. For the 3D forward solver, COMSOL software is used for 3D meshing and Matlab codes were developed for finite element calculation.

4.4.2 2D Multi-Excitation MAT-MI Simulation

The diagram of the 2D multi-excitation MAT-MI system setup used in our computer simulation study is shown in Fig. 19. The static magnetic field is assumed to be uniform in the imaging area and pointing in \( z \) direction. The flux density \( B_0 \) is set to 1 Tesla. In this simulation study we considered a conductive sample that is homogeneous in \( z \) direction, i.e. the conductivity \( \sigma(r) \) is independent of \( z \). The conductive sample is placed around the center of the coordinate system. Three groups of coils are selected to sequentially send three different magnetic excitations, i.e. \( N = 3 \). As shown in Fig. 19(a), coil group A contains two figure eight coils located in planes of \( Z = 5 \) cm and \( Z = -5 \) cm, respectively. Each figure eight coil pair is arranged along the X axis and every coil in the group has a radius of 10 cm. The distance between the two coils in the figure eight coil pair is 4 cm. In addition, coils A-1 and A-3 are placed in the manner of a Helmholtz coil pair and similar arrangement is applied to coils A-2 and A-4. Coil group B is similar to group A, but is arranged along the Y axis. Coil group C contains a pair of Helmholtz coils with 10 cm coil radius and its axis is the Z axis. Fig. 19(b) shows the top view of the system and the directions of the stimulating current flow in each coil. We assume each coil has one turn and has the same current flow amplitude. For simplicity we set \( f'(t) = \delta(t) \) and the maximum current changing rate in every coil is set to be 1e8 A/s.
With this current changing rate, the stimulating magnetic field $B_{1z}$ generated by excitation Group C has a changing rate of 900 T/s at the coordinate center. Ultrasound transducers are assumed to be located on a circular orbit with radius of 20 cm around the sample in the $Z = 0$ plane.

Fig. 19, System diagram of the 2D multi-excitation MAT-MI system used in the computer simulation study. Each coil is labeled with its group number (A, B or C) and coil number in its group. Coils belonging to the same group are used together to generate a specific excitation pattern. (a) is the 3D view of the system and (b) is the top view showing the directions of stimulating current flow in each coil. (c) shows different regions of the object model used in the forward solver and reconstruction algorithm.
With this setup, the MAT-MI system can then be approximated as a two-dimensional system and the imaging slice at $Z = 0$ plane were selected for us to do the forward and inverse calculation. As shown in Fig. 19(c), the 2D object model is divided into three regions. The insulating region models the de-ionized water area surrounding the object in experiment. The coupling region is a conductive region with known conductivity value. The imaging region contains the conductive object and is our imaging region of interest, where the domain $S$ is defined.

In order to obtain the forward solution, the whole object model was discretized into a finite element mesh using three node triangular elements as shown in Fig. 20. The magnetic vector potential produced by each current carrying coil in one excitation group was calculated in each element in terms of elliptic integrals (Wang and Eisenberg 1994, Stratton 1941) and was added together to obtain the total magnetic vector potential produced by this excitation group. The magnetic induction problem was then solved in the non-insulating regions. The solution of electrical potential $\phi^j$ was obtained on each element node and the corresponding electrical field $\mathbf{E}^j$ and current density $\mathbf{J}^j$ were calculated at the center of each element. To calculate the acoustic source $AS^j = (\nabla \times \mathbf{J}^j) \cdot \mathbf{B}_0$, the current density value was interpolated to each node and the acoustic source was then calculated at the center of each element. With the simulated acoustic source distribution, the acoustic pressure measurements $p_j$ can then be calculated using the 2D version of Eq. (4.4). In addition, in order to simulate the limited bandwidth measurements $\tilde{p}_j$, we used an impulse response function $IR(t)$ that has a
central frequency at 500KHz and around 100% bandwidth to convolve with the pressure signal \( p_j \), i.e. \( \tilde{p}_j = p_j \otimes IR(t) \). Here the percentage bandwidth is defined as the ratio of the half strength frequency bandwidth over the transducer’s central frequency. For example, the 500KHz, 100% bandwidth transducer has a half amplitude bandwidth of 500KHz, i.e. from 250KHz to 750KHz.

In applying the proposed iterative algorithm, the initial conductivity distribution \( \sigma_0 \) was set to be uniform with the conductivity value to be \( \sigma_{as} \), i.e. the conductivity value of the media in the coupling region. Eqs. (4.9) and (4.10) were calculated only in the imaging region. The tolerance value \( \varepsilon \) was set to be 0.001. In the process of choosing the regularization parameter as in Eq. (4.9), we set a condition number threshold for

\[ \text{Fig. 20, 2D finite element mesh used for the computer simulation study of multi-excitation MAT-MI.} \]
matrix \( \mathbf{U} \). Basically, if the condition numbers of \( \mathbf{U} \) at all the pixels in the imaging region are smaller than 50, we set \( \lambda = 0 \) for all the pixels, otherwise we use the L-curve method to determine \( \lambda \). Actually, it is observed that the induced electric fields under different magnetic excitations would become parallel to each other mainly near the boundary between the insulating region and non-insulating region. In our simulation study the condition numbers of \( \mathbf{U} \) at all the pixels in the imaging ROI were always under the threshold. In the simulation using limited bandwidth data, a least-square deconvolution filter (Hayes 1996) was applied before doing the image reconstruction. The performance of the proposed algorithm was evaluated using simulated pressure data under different noise level. The SNR used here was defined as the ratio of the maximum pressure signal amplitude over the standard deviation of the added Gaussian random noise. In addition, a numerical phantom with objects of different sizes was also used to test the spatial resolution that can be achieved using the proposed algorithm.

To validate the proposed multi-excitation MAT-MI method, we first did a computer simulation using unlimited bandwidth data and under noise free conditions. The result is shown in Fig. 21. Figure 21(a) shows the target conductivity distribution and (b) shows the reconstructed conductivity image. Fig. 21(c) shows the profile comparison at \( y = 0.02 \) m. In this simulation, the whole model area is a circular region with 0.1m radius. The non-insulating area is a circular region with 0.085 m radius containing a circular imaging region that has 0.075 m radius. The conductivity values of the object are set to be in the same range of biological tissue conductivity. The finite element mesh of this
model has 17,109 nodes and 33,856 elements. The iterative reconstruction algorithm took 8 steps to converge to its final solution.

As shown in Fig. 21, the conductivity distribution is accurately reconstructed. The correlation coefficient (CC) between the target image and the reconstructed image in the ROI is 99.5% and the relative error (RE) is 6.5%. This result shows that a much better performance can be obtained using the proposed multi-excitation algorithm as compared
to the results obtained using either the scalar algorithm or the vector algorithm in single-excitation MAT-MI.

We have also conducted a simulation study to test the performance of the modified reconstruction algorithm when only limited bandwidth measurement data is available. Figure 22(a) shows the simulated impulse response function $IR(t)$ that centers at 500KHz. Using the simulated limited bandwidth pressure data $\hat{p}$, and the modified multi-excitation reconstruction algorithm, we reconstructed the corresponding conductivity image under different noise levels as shown in Fig. 22(b), 22(c) and 22(d). The SNR of the simulated pressure data used to calculate these images are 1000, 100 and 10, respectively. The CCs between the target conductivity image and the reconstructed images in the imaging ROI under these noise conditions are 76.7%, 72.7% and 23.1%, respectively. The corresponding REs are 32.6%, 32.6% and 34.2%. Note the different color scales used in these images. As compared with the target conductivity distribution shown in Fig. 21(a), only the relative conductivity contrast can be seen in these images. Quantitative conductivity values of different regions are not accurately reconstructed. In addition, some artifacts are seen at those conductivity boundaries and at the centers of some conductive pieces. Furthermore, it is shown that the error that comes from the bandwidth limitation is much larger than that comes from the added random noise. Even when the measurement SNR is 1000 the RE between the reconstructed image and the target image is still 32.6% and lower SNR values do not increase the relative errors significantly. With these limitations, however, the relative conductivity contrast shown in
these images still has values in certain potential clinical applications such as cancer detection.

To test the resolution that can be obtained by using the proposed method, a computer simulation was conducted using a numerical phantom containing objects of different sizes as shown in Fig. 23(a). The conductivity value in the background non-insulating region is set to be 0.2 S/m. Circular objects with radii to be 1mm, 3mm, 5mm,
8mm, 10mm and 15mm are presented in the imaging region. For each object size, the conductivity values are either 0.6 S/m or 0.04 S/m, creating different conductivity contrast. The simulation was done using bandwidth limited data with SNR=30 and the conductivity image was reconstructed using the modified multi-excitation algorithm. As shown in Fig. 23(b), the overall relative conductivity contrast pattern is well
reconstructed, with some artifact presented at the conductivity boundaries and at the centers of some internal conductive objects with large sizes. The small objects with 1mm radius can be clearly seen in the reconstructed conductivity image.

4.4.3 3D Multi-Excitation MAT-MI Simulation

In this section, we present the 3D computer simulation study conducted to further validate the multi-excitation MAT-MI method. In comparison to the 2D simulation in section 4.4.2 in which the conductive sample is assumed to be uniform in the \( z \) direction, we here assume the conductive imaging sample has a 3D inhomogeneous conductivity distribution.

Figure 24 shows the diagram of the 3D multi-excitation MAT-MI system setup used in our computer simulation study. Basically we use the same magnetic excitation designs as in the 2D system, together with the focused cylindrical scanning detection scheme in order to provide 3D acoustic measurement data set. In the simulation, the static magnetic field is assumed to be uniform in the imaging area and pointing in \( z \) direction. The flux density \( B_0 \) is set to 1 Tesla. Three groups of coils are utilized to sequentially send three different magnetic excitations, i.e. \( N = 3 \). Each coil is labeled with its group number (A, B or C) and coil number (1, 2, 3 or 4) in its group. Coils belonging to the same group are synchronized to generate a specific excitation pattern. Coil group A contains two figure eight coil pairs, i.e. A-1 with A-2 and A-3 with A-4, located in planes of \( Z = 4 \) cm and \( Z = -4 \) cm, respectively. Each figure eight coil pair is arranged along the X axis and every coil in the group has a radius of 8 cm. The distance between
the two coil centers in the figure eight coil pair is 19 cm. In addition, coils A-1 and A-3 are placed in the manner of a Helmholtz coil pair and coils A-2 and A-4 have similar arrangement. Coil group B is similar to group A, but is arranged along the Y axis. Coil group C contains a pair of Helmholtz coils with 8 cm coil radius and its axis is the Z axis. The current flow direction in each coil is marked by a red arrow. We assume each coil has one turn and has the same current flow amplitude. For simplicity we set \( f'(t) = \delta(t) \) and the maximum current changing rate in every coil is set to be 1e8 A/s. This current changing rate leads to a changing rate of stimulating magnetic field \( B_{i_0} \) generated by

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**Fig. 24.** Diagram of the 3D multi-excitation MAT-MI system used in the computer simulation study. Each coil is labeled with its group number (A, B or C) and coil number (1, 2, 3 or 4) in its group. Coils belonging to the same group are synchronized to generate a specific excitation pattern. Red arrow marked on each coil indicates the excitation current flowing direction. A focused transducer is used to scan around the sample to collect ultrasound signals. Each MAT-MI image slice is obtained by doing a horizontal scan. Horizontal scans at different vertical locations produce the 3D multi-slice volume data.
excitation Group C of 1124 T/s at the coordinate center. A focused ultrasound transducer scans around the sample in a cylindrical scheme, with the radius of the cylinder to be 20 cm. Each horizontal scan gives the data for reconstructing the MAT-MI acoustic source in the corresponding slice, while the vertical scan gives the multiple slice volume data for reconstructing the 3D conductivity distribution.

In order to obtain the forward solution for this 3D MAT-MI problem, a 12cm×12cm×6cm volume was first meshed to regular hexahedral elements with size of 2mm×2mm×6mm. Each hexahedral element was then divided into five linear tetrahedral elements. The 3D finite element mesh is shown in Fig. 25(a), while Fig. 25(b) shows how the hexahedral element is divided into five tetrahedral elements. The final FEM mesh has 180000 tetrahedral elements with 40931 nodes. All nodes with the same \( z \) coordinate are considered to be located in the same slice with slice thickness to be 6mm. A similar two layer setup including an imaging ROI and a layer of coupling region as shown in Fig. 19(c) is also utilized in this 3D multi-excitation MAT-MI simulation. A cylindrical non-insulating volume with radius of 0.055m is defined in this simulation study, while the imaging ROI is defined as a cylindrical volume with radius of 0.045m. Magnetic vector potential produced by each current carrying coil in certain excitation group was calculated in each element in terms of elliptic integrals and was added up according the principle of superposition. The solution to the magnetic induction problem in the form of electrical potential \( \phi_j \) was then obtained on each node and the corresponding electrical field \( E_j \) and current density \( J_j \) were calculated at the center of each element. The current density value was then interpolated to each node for
calculation of MAT-MI acoustic source in each slice. In the computer simulation, we assume the focused transducer has a sharp focusing gain profile along \( z \) direction that the acoustic signal it can detect at certain location only arises from sources in a single slice in its focal zone. Therefore, a 2D circular integration was used to simulate the complete acoustic measurements \( p_j \). In addition, in order to simulate the limited bandwidth measurements \( \tilde{p}_j \), we used the same impulse response function \( IR(t) \) as in the 2D simulation that has a central frequency at 500 KHz and around 100% bandwidth to convolve with the pressure signal \( p_j \), i.e. \( \tilde{p}_j = p_j \otimes IR(t) \).

The inverse reconstruction of MAT-MI acoustic sources in the simulation study was first conducted on each imaging slice with each magnetic excitation pattern. After
that, the multi-excitation MAT-MI iterative algorithm was applied. The tolerance value $\varepsilon$ was set to be 0.01. In the simulation using limited bandwidth data, a least-square deconvolution filter was applied before doing the image reconstruction. We have also conducted a performance testing under different SNR levels. In this simulation, the SNR was defined as the ratio of the maximum pressure signal amplitude over the standard deviation of the added Gaussian random noise.

In order to validate the 3D multi-excitation MAT-MI approach, we first conducted well controlled computer simulation studies. Figure 26 shows the 3D conductivity model used in our computer simulation study. As shown in Fig. 26(a) this 3D model has an overall cylindrical structure with four internal small structures located at different places in the 3D volume. Figure 26(b) shows a multiple axial slice representation of this model. Using the developed FEM forward solver, we can simulate the induced eddy current and MAT-MI acoustic sources in the 3D conductivity model under the three magnetic excitations described above. The result is shown in Fig. 27.
Figures 27(a), 27(b) and 27(c) illustrate the induced eddy current distributions in the Z=0 m slice corresponding to the magnetic excitations A, B and C, respectively. We can clearly see in this forward simulation the different current flow patterns and MAT-MI acoustic source patterns induced by different magnetic excitations.

After simulating the MAT-MI acoustic signals, we applied the proposed 3D multi-excitation MAT-MI reconstruction algorithm. An ideal case under the noise free condition and assuming unlimited bandwidth acoustic measurements is shown in Fig. 28. In this simulation, the algorithm converged after five times of iteration. Figures 28(a) and 28(b) show the 3D structure and multiple axial slice representation of the reconstructed conductivity distribution. The correlation coefficient (CC) and relative error (RE)
between the reconstructed 3D conductivity multiple slice data and the target multiple slice data are 97% and 6%, respectively. Figure 28(c) shows a profile comparison between the target and reconstructed conductivity distribution at Z=0 m, Y=0.01 m. As compared to the target conductivity distribution shown in Fig. 26, it is shown here that under these ideal conditions, we can accurately reconstruct the 3D conductivity distribution using the proposed multi-excitation MAT-MI approach. In addition, we can also see in Fig. 28(a) the impact of the limited slice thickness on the 3D reconstruction.
We have also conducted simulation to test the performance of the 3D multi-excitation MAT-MI approach with limited bandwidth acoustic measurements and under different SNR levels. Figure 29(a) shows the simulated transducer’s impulse response. Figures 29(b), 29(c) and 29(d) show the reconstructed conductivity image slices at Z=0 m under SNR level of 100, 30 and 10, respectively. The CCs between the reconstructed 3D conductivity multiple slice data and the target conductivity multiple slice data under these

![Fig. 29](image_url)

(a) Simulated impulse response of the ultrasound transducer that centers at 500KHz. (b), (c) and (d) are the reconstructed conductivity image slices at Z=0m with limited bandwidth data and SNR to be 100, 30 and 10, respectively.

Fig. 29, (a) Simulated impulse response of the ultrasound transducer that centers at 500KHz. (b), (c) and (d) are the reconstructed conductivity image slices at Z=0m with limited bandwidth data and SNR to be 100, 30 and 10, respectively.
SNR levels are 86%, 85% and 82%, respectively. The corresponding REs are 18.18%, 18.19% and 18.26%. It is observed that the major reconstruction error comes from the bandwidth limitation in the acoustic measurements instead of the added white noise. Note here we used different color scales as compared to target image slices shown in Fig. 26. The fact is that with the limited bandwidth acoustic measurements, we can only reconstruct the relative conductivity contrast instead of the absolute conductivity values. In addition, as shown in Fig. 29, some ringing artifacts around those conductivity

Fig. 30, (a) Reconstructed 3D conductivity distribution with limited bandwidth data and SNR to be 30. (b) Corresponding conductivity image slices at different locations along the Z direction. (c) Conductivity profile comparison between the target and the reconstructed conductivity distribution at Z=0 m, Y=0.01m.
boundaries can be observed in the reconstructed image slices. Figure 30 shows more comprehensive simulation results with limited bandwidth acoustic measurements and SNR of 30. Figures 30(a) and 30(b) show the 3D structure and multiple axial slice representation of the reconstructed conductivity distribution and Fig. 30(c) shows a profile comparison between the target and reconstructed conductivity distribution at Z=0 m, Y=0.01 m. As shown in this figure, the conductivity distribution reconstructed using limited bandwidth data has a scaling factor as compared to the target distribution. This scaling factor contributes a large amount of conductivity reconstruction errors calculated above. In addition, this effect is believed to be caused by the multi-excitation iterative algorithm with the presence of the reconstruction errors in the reconstructed MAT-MI acoustic source $A_{\tilde{\Sigma}}$ which is derived from limited bandwidth pressure measurement data. However, in spite of some error and artifact, the 3D relative conductivity distribution is well reconstructed.

### 4.5 Experiment Study

Taking the same magnetic stimulation and system designs shown in the computer simulation study, we have built 2D and 3D multi-excitation MAT-MI experiment systems. Experiment studies with controlled 2D and 3D gel phantoms were conducted and promising experiment results are obtained that clearly illustrates the merits of the multi-excitation MAT-MI approach. It is shown in the experiment results that the multi-excitation MAT-MI is superior to the single-excitation method in providing a more complete reconstruction of the electrical conductivity contrast. This capability gives the
multi-excitation method many advantages in differentiating tissue types especially for the future clinical applications such as breast tumor imaging.

4.5.1 2D Multi-Excitation Experiment System Design

The 2D experiment system setup is similar to the setup used in the 2D simulation study only with different parameters. The static magnetic field is generated from two permanent magnets and the field strength was measured to be 0.26 Tesla (Gaussmeter, Alpha Lab) at the coordinate center where the object is located. All the coils have radius of 4.45 cm. The Helmholtz coil pair of excitation group C has 3 turns in each coil and the figure eight coil pair of excitation group A and B has 2 turns in each coil. The distance between the upper coils and lower coils in each group is around 5 cm. The coils were driven by a home made stimulator, with 1 $\mu$s pulse width. The dynamic magnetic excitation was measured by a sensing coil with radius of 1.5cm connected to an oscilloscope. The estimated maximum current changing rate in the Helmholtz coil pair of excitation group C is 1.4e8 A/s, which corresponds to a magnetic field changing rate of 7e3 T/s at the coordinate center. Considering the 1 $\mu$s pulse width, the maximum dynamic magnetic field strength $B_t$ is around 0.007 T at the coordinate center. The maximum current changing rates of the other two excitation groups are at similar levels. A 500 KHz flat ultrasound transducer (Panametrics V301) with around 60% bandwidth was used in our MAT-MI experiment system. The transducer was mounted to a scanning frame and can scan around the sample with 330 degree view angle. The scanning step used in our experiment study was 2.5 degree. The scanning radius, i.e. the distance
between the transducer and the scanning center is 22.8 cm. A 3 cm thick sample which is uniform in z direction is submerged in 3 cm thick deionized water media for acoustic coupling. During the ultrasound scanning the magnetic excitation coil and the sample are both fixed in their positions to ensure reliable acoustic measurements. Acoustic data collections are synchronized with the magnetic excitation. This setup makes the corresponding MAT-MI problem valid to be simplified to a 2D problem both electrically and acoustically. The acoustic signal collected using the transducer was fed into preamplifiers with 90dB gain and digitized by a 5MHz data acquisition card. Signal averaging was used to increase SNR.

With the collected ultrasound data under each of the three excitation conditions, the corresponding acoustic source maps were firstly reconstructed using the time reversal back projection algorithm or the expectation maximization algorithm as mentioned in section 4.2.2. In order to build the FEM mesh for the proposed multi-excitation reconstruction algorithm, one of the reconstructed acoustic source images was post-processed and conductive regions including the coupling region and the imaging region were extracted and discretized into finite element meshes.

For comparison, we also performed ultrasound pulse echo imaging using the same transducer and a pulser-receiver (Panametrics 5077PR). The RF pulse echo data was collected at the same respective scanning positions MAT-MI data was collected. A simple back projection algorithm was used to form the pulse echo image.
4.5.2 2D Experiment Results

Using the developed multi-excitation MAT-MI experiment system, we have conducted phantom experiments to demonstrate the benefits of the proposed multi-excitation method. Results from an example gel phantom experiment are shown in Fig. 31. A photo of the gel phantom is shown in Fig. 31(a). The phantom contains a background region made from 5% salinity gel. Two cylindrical columns with diameter of 12 mm are embedded in the gel. Marked by the red and blue circles in the photo are two high conductive regions filled with 20% and 10% salinity gels, respectively. These two regions have diameter of 8 mm. The two annular areas sitting between the two high conductive regions and the background are made from beef suet, which has low conductivity value as fat tissue.

Figure 31(b) shows the ultrasound pulse-echo image we obtained from the gel phantom. This image indicates the acoustic impedance contrast of the phantom and boundaries between structures with different acoustic impedances can be seen in the image. As shown in this image, the echoes at the boundary between the 20% salinity gel and the fat layer are much stronger than echoes at other boundaries, indicating a larger acoustic impedance change here. This is consistent with the fact that the 20% salinity gel is much softer than the 10% and 5% salinity gels. However, the overall contrast of the pulse echo image is not strong and it would be challenging to differentiate tissue types from this type of image.

Using the multi-excitation MAT-MI system, we applied three different groups of magnetic excitations on the gel phantom and the corresponding acoustic source images
**Fig. 31,** (a) Photo of a gel phantom used in the 2D experiment study. The red circle marks a region containing 20% salinity gel and the blue circle marks a region containing 10% salinity gel. (b) An ultrasound pulse-echo image of the phantom showing the acoustic impedance contrast. (c), (d) and (e) are reconstructed MAT-MI acoustic source images under different magnetic excitations. (f) Reconstructed conductivity image showing the relative conductivity contrast. (g) Conductivity profile along $y = 0.01$ m showing the comparison between the target and reconstructed conductivity values.
were reconstructed as shown in Fig. 31(c), 31(d) and 31(e). Spatial resolution of 3 mm was achieved in these images. As expected, with limited bandwidth acoustic measurements the reconstructed acoustic sources are mainly distributed around conductivity boundaries i.e. where $\|\nabla \sigma\|$ is large. In addition, higher contrast can be seen in these images as compared to the pulse-echo image. This is mainly because of the stronger conductivity contrast existed in the gel phantom. However, as the reconstructed acoustic source maps emphasize conductivity boundaries, it is hard to tell which part of the object has high conductivity values. Using the modified multi-excitation algorithm, the conductivity image of the gel phantom was reconstructed as shown in Fig. 31(f). From this image, we can clearly see the relative conductivity contrast, while the fat layer shows lower conductivity than the surrounding background, the 10% salinity gel shows higher conductivity value and the 20% salinity gel shows the highest conductivity. A conductivity profile at $y = 0.01$ m is given in Fig. 31(g) showing the comparison between the target and reconstructed conductivity values. The geometry of the target distribution was estimated from the pulse-echo image and the conductivity values of each piece were estimated from corresponding conductivity measurements. As compared with the pulse echo image and the MAT-MI acoustic source images, the reconstructed conductivity image using the modified multi-excitation algorithm gives a more informative conductivity contrast map and enables us to better differentiate the material types in the phantom. In addition, we can also see similar artifacts at the conductivity boundaries and object centers in consistent with our computer simulation results.
For better comparison, we have also conducted a computer simulation using a conductivity distribution similar to that of the experiment phantom. The same excitation parameters as those used in the experiment were also employed. The simulated acoustic sources corresponding to the three magnetic excitations, i.e. $A S_j' = (\nabla \times J_j') \cdot B_0$ for $j = 1, 2, 3$, are shown in Fig. 32(a), 32(b) and 32(c), respectively. The target conductivity distribution is shown in (d).

**Fig. 32.** (a), (b) and (c) are computer simulated acoustic source distributions corresponding to three different magnetic excitations. These excitations have similar setups and parameters to those used in the phantom experiment. The target conductivity distribution is shown in (d).

For better comparison, we have also conducted a computer simulation using a conductivity distribution similar to that of the experiment phantom. The same excitation parameters as those used in the experiment were also employed. The simulated acoustic sources corresponding to the three magnetic excitations, i.e. $A S_j' = (\nabla \times J_j') \cdot B_0$ for $j = 1, 2, 3$, are shown in Fig. 32(a), 32(b) and 32(c), respectively. The target conductivity distribution is shown in (d).
conductivity distribution is shown in Fig. 32(d). It is shown that the reconstructed acoustic source patterns in Fig. 31(c), 31(d) and 31(e) are in general similar to those simulated source patterns shown in Fig. 32(a), 32(b) and 32(c), respectively. However, the reconstructed acoustic sources are lack of those low frequency components observed in the simulated acoustic sources. As we mentioned in the theory, this difference mainly comes from the bandwidth limitation in acoustic measurements. In spite of this, the reconstructed conductivity image shown in Fig. 31(f) represents well the relative conductivity contrast of the target distribution presented in Fig. 32(d).

**4.5.3 3D Multi-Excitation Experiment System Design**

The 3D multi-excitation MAT-MI experiment system setup has similar design as shown in Fig. 24. The static magnetic field is generated from two permanent magnets and the field strength was measured to be 0.2 Tesla (Gaussmeter, Alpha Lab) at the coordinate center where the object is located. All the coils have radius of 4.45 cm. All the coils in excitation group C have 3 turns, while all the coils in excitation group A and B have 2 turns. The distance between the upper coils and lower coils in each group is around 6 cm. A home made stimulator was developed to drive these coils. The current flowing in the coils has a waveform approximating a bipolar single cycle sinusoid which lasts 2 $\mu$s. The dynamic magnetic excitation generated by the stimulating coils was measured by a sensing coil with radius of 1.5 cm connected to an oscilloscope. The estimated maximum current changing rate in excitation group C is 1.5e8 A/s, which corresponds to a magnetic field changing rate of 6e3 T/s at the coordinate center. The
maximum dynamic magnetic field strength $B_{1z}$ is estimated to be less than 0.006 T at the coordinate center. Excitation group A and B have similar levels of stimulating current flowing in their coils.

The MAT-MI acoustic signal measurement was conducted using a cylindrical scanning scheme. During experiments, both the sample and the transducer were submerged in distilled water media for acoustic coupling. A 500 KHz flat transducer (Panametrics V301) with 29 mm diameter and around 60% bandwidth was employed in this study. An acoustic lens was placed in front of the transducer to implement ultrasound focusing. Both the lens and the transducer were mounted to a scanning frame that can scan around the sample. Each horizontal scan has 320 degrees view angle and 2.5 degrees scanning step. The scanning radius was about 16 cm. The vertical scan was done with a 5 mm scanning step. The focusing lens gives about 7 mm half strength beam width at the imaging center. Signals collected using this transducer was fed into preamplifiers with 90dB gain and digitized by a 5 MHz data acquisition card. Signal averaging was used to increase SNR.

With the ultrasound signal data acquired under each of the three magnetic excitations, we can first reconstruct the slice images of the MAT-MI acoustic source using the 2D time reversal back projection algorithm. In order to build the 3D FEM mesh for applying the proposed multi-excitation reconstruction algorithm, one of the MAT-MI acoustic source slice images was segmented to pick out the outmost boundary of the imaging sample and used to define the conductive object domain.
4.5.4 3D Experiment Results

Using the developed 3D multi-excitation MAT-MI system, we have conducted a phantom experiment study. Figures 33(a) and 33(b) show the top and side view photos of the gel phantom we used in this study. Figure 33(c) shows a diagram of the phantom geometry and the locations of the transducer’s central focus along $z$ direction where different image slices are obtained. In order to keep most part of the transducer and the acoustic lens in water, we only collected acoustic signals from the bottom part of the phantom. For the top part of the phantom, where no image slice is collected, we assume it has the same conductivity distribution as that in slice 5. The background material of the phantom is made from 15% salinity gel. All the white columns are made from tissue fat. The embedded gel column marked by the red circle in Fig. 33(a) is made by 20% salinity gel.

Figure 34 shows the reconstructed MAT-MI acoustic source images at the bottom slice i.e. slice 1, under three different magnetic excitations. It is shown that most

![Figure 33](image)

**Fig. 33,** (a) Top view photo of the 3D gel phantom. (b) Side view photo of the 3D gel phantom. (c) Diagram of the 3D structure of the gel phantom. Arrows on the right indicate the central positions where image slices from slice 1 to slice 5 are obtained.
significant MAT-MI acoustic sources are located at conductivity boundaries. Three different acoustic source patterns can be clearly seen that corresponds to the three different magnetic excitations. Fig. 35 shows the five image slices from bottom to top under the magnetic stimulation C. Those two small fat columns are reconstructed in image slice 1, 2 and 3, but not shown in the top two image slices. The big column on the contrary is consistently reconstructed in all the five image slices. The conductivity

Fig. 34, (a), (b) and (c) are reconstructed MAT-MI acoustic source images at slice 1 corresponding to excitation group A, B and C, respectively.

Fig. 35, Reconstructed MAT-MI acoustic source images from slice 1 to slice 5 obtained with excitation group C.
images obtained using the previous single excitation MAT-MI method would look exactly the same as images shown in Fig. 34 and Fig. 35 except with reversed color scale. With all the data collected under three different magnetic excitations, we applied the 3D multi-excitation MAT-MI algorithm and the reconstructed relative conductivity contrast image slices are shown in Fig. 36. Despite some imaging noise and artifact, the 3D relative conductivity contrast can be clearly identified and agrees well with the phantom geometry. Two small low conductive regions are reconstructed in slice 1, 2 and 3, but not visible in slice 4 and slice 5. On the contrary, the big column made from fat tissue with an embedded high conductive gel column is consistently reconstructed in all the image slices. As compared to the MAT-MI acoustic source images (similar to the conductivity images obtained using single excitation method), which emphasize the conductivity boundaries, the reconstructed conductivity image slices using the 3D multi-excitation
algorithm give a more informative conductivity contrast map and enables us to better differentiate the material types in the phantom based on their electrical conductivity properties.

4.6 Discussion

The MAT-MI imaging approach was previously proposed to do non-invasive conductivity imaging with high spatial resolution. However, as shown in chapter 3, using the single-excitation MAT-MI approach, we are not able to reconstruct the conductivity contrast accurately and only conductivity boundaries at the conductivity heterogeneity can be reconstructed in experiment. This is because the collected MAT-MI pressure signal is mainly determined by the conductivity gradient especially when using limited bandwidth ultrasound probes. Consequently, the reconstructed conductivity image using the single excitation MAT-MI method would mainly reflect the projection component of the conductivity gradient in the direction perpendicular to the induced electric field. In this study, we demonstrate in 2D and 3D computer simulations that using the proposed multi-excitation MAT-MI method, we can reconstruct the internal conductivity contrast of the object with good accuracy if unlimited bandwidth acoustic measurement data is available. In addition, in MAT-MI experiments, one of the major technical limitations comes from limited bandwidth acoustic measurements. As shown in our computer simulation and experiment studies in both 2D and 3D, using the modified multi-excitation MAT-MI algorithm, we can reconstruct the relative conductivity contrast fairly well. In consequence, this would bring us the potential ability to better identify different
tissue types based on their conductivity contrast and would significantly benefit potential MAT-MI applications such as cancer detection. For the fundamental principle, as the multiple magnetic excitations in the proposed 3D multi-excitation MAT-MI method utilizes different magnetic excitation patterns through different coil configurations, in comparison to the single excitation method, this method allows us to obtain more information about the conductivity gradient, i.e. its cross projection in different directions corresponding to the different induced electric fields. Therefore, using the multi-excitation method we can get a more complete reconstruction of the conductivity gradient and in turn improve conductivity contrast reconstruction in MAT-MI images.

In MAT-MI, reliable acoustic measurements are essential for reliable conductivity reconstruction. Generally it requires a stable excitation and scanning frame and good synchronization between magnetic excitation and acoustic data collection. Ultrasound transducers with different sensitivity and focusing patterns may need different calibrations in practice, but in general transducers with better receiving sensitivity and broader frequency bandwidth are expected to give a better MAT-MI image quality. In addition, a good EM shielding of the transducer is also needed to reduce signal contamination that comes from the excitation coil. This can also be achieved by holding the transducer at certain distance from the imaging ROI and let the EM interference fade out before the real MAT-MI signal arrives at the transducer. Furthermore, as in the present 3D experiment study, we used a 500 KHz MAT-MI system with the central frequencies of both magnetic stimulations and ultrasonic detections to be 500 KHz. With such a system, about 3 mm spatial resolution is achieved in each image slice because of
the tomographic reconstruction we used in each horizontal scan. However, the effective slice thickness, in another word, the spatial resolution along the $z$ direction, is determined by the focusing beam width and is estimated to be 7 to 8 mm. MAT-MI systems with higher frequencies and wider bandwidth would in turn give better in-slice spatial resolution and better ultrasound focusing would then give better spatial resolution in $z$ direction.

For the excitation number and coil configurations of the proposed multi-excitation MAT-MI method, we chose $N = 3$ in this study with one Helmholtz coil setup and two figure eight coil setups arranged at different directions. We have tested in simulation that using more figure eight coil setups arranged at different directions does not significantly decrease the condition number of the system inverse matrix and does not speed up the convergence in the iteration. Of course, using more excitation setups would add an average effect to help handle the measurement noise. Different coil configurations that generate different excitation patterns may need to be further explored and optimized in the future research.

The acoustic homogeneous assumption used in the theoretical derivation would still limit the application of the proposed multi-excitation MAT-MI approach to soft tissue imaging. As the acoustic heterogeneity in soft tissue is less than 10%, its effect can be considered negligible in MAT-MI (Xu and He 2005, Xu and Wang 2003).

From the bioimpedance imaging perspective, as the electrical properties of biological tissue are frequency dependent, the conductivity property obtained from the MAT-MI imaging approach only indicates the tissue conductivity value at a certain
frequency range determined by the central frequency of the magnetic excitation and acoustic measurements. Thus, the reconstructed MAT-MI image obtained in the present experiment study indicates conductivity properties of the phantom at around 500 KHz. Higher system frequencies thus would provide not only better spatial resolution, but also different spectrum information about the sample’s electrical conductivity contrast.

About the imaging sensitivity of the multi-excitation MAT-MI systems, because the figure eight coil configurations have lower EM energy delivery efficiency than the Helmholtz coil pair, the overall imaging sensitivity of the multi-excitation MAT-MI system is generally lower than the corresponding single-excitation system. This is also consistent with the collected MAT-MI acoustic signals and reconstructed acoustic source maps in experiments as shown in Figs. 31 and 34, while larger signal strength and better SNR are obtained when the Helmholtz coil excitation group is employed. With the imaging sensitivity of current multi-excitation system, we are not able to get reliable conductivity imaging on normal biological tissue samples. However, the imaging sensitivity of the MAT-MI technique is proportional to the field strength of both the static magnetic field and the dynamic magnetic excitation and still has big space of improvement. More than 20 times stronger static magnetic field is readily available in commercial MRI machines and the magnetic stimulation strength used in current MAT-MI systems is still more than 10 times smaller than the EM safety threshold that can be delivered to the human body.

In summary, we have developed the multi-excitation MAT-MI imaging approach and the corresponding reconstruction algorithms. Computer simulation and phantom
experiment studies have been conducted to demonstrate the promise of the proposed method in reconstructing the conductivity contrast using ultrasound measurements.
Chapter 5
Conclusions and Future Work

5.1 Conclusions

Non-invasive bioimpedance imaging has been actively investigated for decades because of the unique contrast information about electrical properties of biological tissues it can provide as physiological or pathological indicators. In this dissertation research we have studied the magnetoacoustic tomography with magnetic induction as a new hybrid modality for high resolution electrical conductivity imaging of biological tissue. The hybrid MAT-MI modality combines magnetic excitations and ultrasound measurements through the Lorentz force based coupling mechanism, therefore is able to perform conductivity imaging with spatial resolution close to ultrasound imaging.

After theoretical studies of the basic physical principles of MAT-MI imaging approach, its fundamental signal generation mechanism, i.e. its forward modeling is built up. Besides the conductivity distribution of the imaging sample, the MAT-MI signal is believed to be proportional to the field strength of the static magnetic field and the dynamic magnetic stimulations. In real MAT-MI imaging system, the receiving sensitivity and frequency bandwidth of the ultrasound transducer are also important factors that would influence the quality of the received MAT-MI signal and reconstructed MAT-MI images. In addition, it is suggested from our computer simulation and experiment studies that conductivity gradient of the conductive imaging object plays more important role in the MAT-MI acoustic signal generation. Therefore conductivity boundaries are the major informative contrast that can be produced by MAT-MI systems.
with single magnetic excitation. On the other hand, by employing multiple magnetic excitations with different spatial patterns, we have shown in our computer simulation and experiments that we are able to reconstruct a more complete conductivity contrast for better tissue type differentiation and thus better diagnosis in its imaging applications in the future.

We first investigated the single-excitation MAT-MI method. Based on its forward modeling, two imaging reconstruction algorithms were developed. By computer simulations with concentric spherical models, we have validated the feasibility to extract conductivity distributions using the MAT-MI acoustic measurements. We have demonstrated that the scalar reconstruction algorithm developed for single-excitation MAT-MI together with a median filter can correctly reconstruct the conductivity distributions with broadband acoustic measurement data. In addition, the vector algorithm has been demonstrated to be able to correctly reconstruct the Lorentz force and current density vector distributions from the vectorized broadband acoustic measurements in our computer simulation. Along with these promising computer simulation results, we have also evaluated the single-excitation MAT-MI method in phantom experiments with the developed experiment systems. With both the 2D and 3D single-excitation MAT-MI experiment systems, we have obtained high resolution MAT-MI images with saline, gel or salted biological tissue phantoms. These imaging results provide solid proof of the feasibility to extract electrical conductivity related maps from the magnetic induced ultrasound measurements. In addition, it was also observed that with the single-excitation MAT-MI system, as the acoustic measurements obtained from ultrasound probes are
generally narrowband which makes the dominant conductivity gradient source even more
dominant, we are only able to give an image showing boundaries at conductivity
heterogeneity.

According to a further theoretical analysis of the MAT-MI forward modeling, we
have developed a multi-excitation MAT-MI approach for the purpose of generating more
complete conductivity contrast reconstruction. Imaging algorithms for multi-excitation
MAT-MI with either broadband or narrowband acoustic measurements are developed and
validated using both computer simulations and phantom experiments. A finite element
method based MAT-MI forward solver has also been developed in order to conduct
simulations with arbitrary geometry. It is observed in our 2D and 3D computer
simulations that if broadband acoustic measurements are available, the multi-excitation
MAT-MI approach can accurately reconstruct the conductivity distribution of the
imaging samples. If on the contrary, only narrowband acoustic measurements are
available, the multi-excitation MAT-MI method still allows us to reconstruct the relative
conductivity contrast throughout the object volume. This finding is further verified in our
2D and 3D phantom experiment studies. Better imaging performance in terms of the
completeness of the conductivity contrast is always observed in multi-excitation MAT-
MI images as compared to those obtained from single-excitation MAT-MI method.

In summary, the present dissertation study developed and evaluated the MAT-MI
method as a high resolution conductivity imaging modality. Computer simulation and
phantom experiment studies have been conducted to demonstrate its feasibility and
performance. All the promising results we obtained suggest the MAT-MI method may
become an important method for noninvasive bioimpedance imaging with high spatial resolution.

5.2 Future Work

In spite of those promising results obtained from this dissertation study, limitations of the current MAT-MI method in the aspects of imaging sensitivity, speed and image qualities still exist and further improvement needs to be made in the future studies to make it possible to apply the MAT-MI method to in vivo imaging and possible clinic studies.

In the present dissertation study, most MAT-MI experiment systems have about 0.1-0.2 T static magnetic field around the imaging sample. The dynamic magnetic field around the imaging sample has about 5000T/s changing rate and 1-2 \( \mu \)s pulse width. With commercially available ultrasound transducers, we are able to detect MAT-MI acoustic signals generated from conductivity boundaries between different pieces with several millimeters in diameter and larger than 2 S/m conductivity contrast. This kind of imaging sensitivity, however, does not allow us to get reliable MAT-MI signal and image from normal soft biological tissues which generally have conductivity contrast below 1 S/m. However, the strength of the static field and dynamic excitations can still be largely enhanced under safety regulations. Static magnetic field of 3-4 T, which is about 20 times larger than the current MAT-MI field strength, is readily available in commercial or research MRI machines. For the dynamic magnetic field, we should use stimulation strength under safety limit which concerns about possible nerve stimulation. Considering
the imaging object used in our 3D multi-excitation MAT-MI simulation study, for example, assuming that the Helmholtz coil generates a 5000T/s dynamic field around the object and 1 µs long pulse is delivered, the maximum electrical field is estimated to be around 150V/m. This leads to a strength-duration constant of 1.5e-4 Vs/m, which is more than 10 time smaller than the nerve stimulation threshold of 2e-3 Vs/m (Wen et al 1998). Therefore, more than 10 times stronger dynamic magnetic stimulation could be used in future MAT-MI systems. With improved field strength and instrumentation, future MAT-MI experiment systems are expected to have the capability to image conductivity contrast of normal biological tissues.

Besides the relatively low imaging sensitivity, current MAT-MI systems have relatively low imaging speed because single element ultrasound transducer and mechanical scanning are used for current MAT-MI imaging. In addition, as the signal to noise ratio in raw MAT-MI data is low, large amount of signal averaging is generally needed. In the future work, electrically controlled phased array and other advanced signal processing technologies in ultrasound imaging may be incorporated in the MAT-MI method and significantly increase the imaging speed and signal quality. In addition, following the idea of using multiple magnetic excitations to get more complete reconstruction of the conductivity contrast, better designed coil systems may further improve the MAT-MI image quality and stability. Furthermore, better ways to avoid the EM contamination to the ultrasound transducer, such as better EM shielding or the use of optical fiber based transducers may be further explored to improve the MAT-MI signal quality.
Theoretically, MAT-MI image reconstruction algorithms that can take into account the acoustic heterogeneity and conductivity anisotropy need to be developed. Practical parameters such as the transducer’s finite aperture size and beam patterns need to be considered in the reconstruction algorithms in the future too.

Finally, after the technical improvement, \textit{in vitro} or \textit{in vivo} animal experiments may be conducted in the future to estimate the MAT-MI method for high resolution conductivity imaging. Different types of soft tissues, organs or the whole body may be explored in the future imaging studies. The most possible clinical application of MAT-MI in the future is believed to be tumor detection especially in breast cancer screening.
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156
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Best regards,

Xu

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Xu Li

FROM: jharsson@ieee.org
DATE: Friday, June 11, 2010 4:26 PM
TO: Xu Li
SUBJECT: Re: Ask for permission to reuse copyrighted material

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If there is any problem please let me know, thanks.

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5.

6. Xu
Appendix B - VITA

Xu Li

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EDUCATION
Ph.D. Candidate, Biomedical Engineering
Department of Biomedical Engineering, University of Minnesota       2004-present
Thesis: Magnetoacoustic Tomography with Magnetic Induction for Electrical Conductivity Imaging of Biological Tissue

M.S., Biomedical Engineering
Department of Biomedical Engineering, Zhejiang University, China       2004
Thesis: Network Modeling of Olfactory Neural System and its Application

B.S., Biomedical Engineering
Department of Biomedical Engineering, Zhejiang University, China       2001
Senior design project: Display module design of a portable EKG monitor

RESEARCH EXPERIENCE
Graduate Research Assistant              2004 – Present
Biomedical Functional Imaging & Computation Laboratory           University of Minnesota
• Developed a finite element method (FEM) based multi-excitation magnetoacoustic tomography with magnetic induction (MAT-MI) algorithm, implemented the algorithm and built the corresponding hardware systems. Significantly improved the MAT-MI imaging performance in both computer simulations and experiments.
• Conducted theoretical derivation of vector acoustic source reconstruction algorithm and verified it with numerical simulations.
• Developed the 2D and 3D MAT-MI experiment systems, major components of which includes ultrasonic scanning stage, magnetic stimulator, data acquisition system and data processing software. Summarized and published those pilot MAT-MI experiment studies using these systems.
• Simulated MAT-MI signal generation and validated the reconstruction algorithm using spherical models. Investigated the impact of a variety of parameters on the imaging performances.

Graduate Research Assistant         2001 – 2004
Neuroinformatics Group in the Interdisciplinary Lab in Zhejiang University
Department of Biomedical Engineering           Zhejiang University
• Simulated and analyzed K series model of olfactory neural system
• Extended the applications of KIII network model to pattern recognition
• Simulated a symbol space dynamic model of single neuron

TEACHING EXPERIENCE
Teaching Assistant
Responsibilities include leading and supervising experiment sessions, developing

161
experiment protocols, generating guide documents, purchasing and maintaining devices, office hours and grading.

- BMEn 3201, Bioelectricity and Bioinstrumentation, Fall 2007
- BMEn 4001/4002, Senior Design, Fall 2009, Spring 2010

HONORS AND AWARDS

- Best Poster Award in Design of Medical Devices Conference (DMD), 2007.
- NIH Neuro-Physical-Computational Sciences (NPCS) Graduate Training Fellowship, University of Minnesota, 2005-2006.
- Excellent Student Scholarship, Zhejiang University, China, 1997-2000.

JOURNAL PUBLICATIONS

2. X. Li and B. He, “Multi-Excitation Magnetoacoustic Tomography with Magnetic Induction for Bioimpedance Imaging,” *IEEE Transactions on Medical Imaging*, in press.

CONFERENCE PROCEEDINGS
7. X. Li and B. He, “Magnetoacoustic Tomography with Magnetic Induction (MAT-MI) for Impedance Imaging: Simulation Study,” *Proceedings of World Congress on Medical Physics and Biomedical Engineering*, (Abstract), 2006.