Stability Coefficients in Longitudinal Studies

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Longitudinal studies of personality traits and intelligence have used an exponential function to relate the magnitudes of the correlations between occasions to the time interval. This exponential function is shown to be equivalent to a quasi-simplex model of a stationary process with constant reliability. Two numerical illustrations are provided that compare the least-squares fit of the logarithm of the exponential function to an analysis of the covariance structure. *Index terms: change scores, covariance structure analysis, exponential function,* LISREL, *simplex, stability index.*

Change processes, such as decay, growth, and learning, may be expressed as exponential functions of time. Some longitudinal studies of personality traits and intelligence have employed an exponential function (suggested by Converse & Markus, 1979) that relates the correlation coefficients between measures to the time interval. These studies have been summarized by several authors (Conley, 1984; Smith, 1992). The expression that has been used is:

$$c_{ij} \approx rs^n \ , \tag{1}$$

where

c_{ij} is the correlation between the measures at times i and j,

r is the reliability of the measures (or the average reliability when it is not constant),

s is the stability coefficient (Conley, 1984), and

n is the time interval between the administration of tests i and j.

Because c and n are given, it is possible to solve for r and s. Conley (1984) and Smith (1992) investigated the stability coefficients of various personality traits.

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The Regression Solution and Covariance Structure Analysis

Assuming that the reliability of the measures does not change, it is possible to obtain approximate solutions for r and s using least-squares procedures. A direct application of Equation 1 is cumbersome, because it involves a polynomial of power $2n - 1$ in s. Taking logarithms transforms Equation 1 into a linear regression equation of $\log(c)$ on n, with $\log(r)$ as the intercept and $\log(s)$ as the slope. This regression does not satisfy the statistical assumptions for least squares because the correlation coefficients are not independent observations, and because the log transformation does not yield an additive error term. Nevertheless, the least-squares solution can be used as an approximation.

Equation 1 is equivalent to a stationary Markov simplex. Guttman (1954) introduced the simplex model, and Humphreys (1960, 1968) was among the first to apply it to follow-up studies of scholastic achievement. Two numerical illustrations are provided below in which the least-squares solution from Equation 1 is compared with the LISREL analysis of the simplex model. One example used Humphreys' (1968, Table 2, p. 377) data of a large-scale follow-up study of scholastic achievement; the other used a covariance matrix, which was reported by Bock & Bargmann (1966, Table 1, p. 524), of measures of proficiency in a two-hand coordination training program. The formulation of the covariance model is available in the LISREL user's guide; therefore, the relations needed for comparison with Equation 1 are outlined briefly (see also Jöreskog, 1970).

Let

$$y_i = \eta_i + e_i \ , \qquad (2)$$

where

y_i is the measurement of a variable at the ith occasion,

η_i is the corresponding variable of true scores, and

e_i is the error of measurement.

The simplex model relates the ith true score to the $(i-1)$th true score:

$$\eta_i = \beta_i \eta_{i-1} + \zeta_i \ , \qquad (3)$$

where β_i is the path coefficient, and ζ_i is the error term in the structural equation. When the process is stationary—all βs are equal for equal time intervals—the total effect of η_i on η_j is β^n, where n is the time interval between the ith and jth occasions.

To relate the simplex model to Equation 1, two further assumptions are necessary:

1. The variances of the y variables must be equal, and
2. The variances of the errors of measurement in all occasions must be equal—or, alternatively, the y variables must be given in standard scores and all the reliabilities must be equal.

It can be shown algebraically, or by reading the path diagram (Jöreskog & Sörbom, 1986, p. III.72), that the covariance between y_i and y_j is $\beta^n \sigma^2_\eta$, and the correlation is

$$r_{ij} = \beta^n \sigma^2_\eta / \sigma^2_y = k\beta^n \ , \qquad (4)$$

where

r_{ij} is the correlation,

k is the reliability,

β is the path coefficient, and

n is the time interval.

Obviously, Equation 4 is the same as Equation 1.

Numerical Illustrations

Humphreys Data

The correlation matrix of a follow-up study of college grades reported by Humphreys (1968, Table 2, p. 377), and also provided in the LISREL user's guide (Jöreskog & Sörbom, 1986, p. III.75), was reanalyzed. The correlation matrix was based on college grades, variables 7–14 (1–8 in LISREL), and the error terms and the path coefficients were constrained to be equal. The values obtained for k and β were .580 and .918, respectively, for the maximum likelihood solution. For the unweighted least-squares solution, k = .571 and β = .924. The goodness-of-fit indices were GFI = .995 and AGFI = .993 for the maximum likelihood solution, and GFI = .999 and AGFI = .999 for the unweighted least-squares solution. Using the logarithmic transformation to base 10 of Equation 1, the antilogs of the intercept and slope of the regression of log(c) on n were .567 and .924, respectively, and the correlation between the dependent and independent variables was .938.

Bock and Bargmann Data

This example used a correlation matrix computed from a covariance matrix reported by Bock & Bargmann (1966, Table 1, p. 524) of six measures of proficiency in a two-hand coordination training program (taken from a study by Mukherjee, 1963). The error terms and the path coefficients were constrained to be equal. The LISREL procedure yielded k and β values of .863 and .925, respectively, for the maximum likelihood solution, and .892 and .891, respectively, for the unweighted least-squares solution. The goodness-of-fit indices were GFI = .966 and AGFI = .936 for the maximum likelihood solution, and GFI = .999 and AGFI = .997 for the unweighted least-squares solution. Using the logarithmic transformation of Equation 1, the antilogs of the intercept and slope of the regression of log(c) on n were .875 and .900, respectively, and the correlation between the dependent and independent variables was .849.

In both illustrations, the solutions obtained using Equation 1 and LISREL yielded similar results, but the fit for the data of the regression solution was much lower in the second illustration.

Discussion

The results showed a high level of correspondence between the regression solutions and the LISREL least-squares solutions. This correspondence may be due to the good fit of the simplex model to the data in both illustrations. There are, however, several reasons to favor the covariance structure analysis. As indicated above, the application of the least-squares method to the logarithmic transformation of Equation 1 is theoretically inadmissible and may be used as an approximation only. The covariance structure analysis, in addition to its theoretical merits, also permits tests of fit (even when the assumptions of multivariate normality are not satisfied, generalized least squares may be used).

Another problem in using the regression solution is that the assumptions of Equation 1—the assumptions of a stationary process (equality of the path coefficients) and the equality of the reliabilities—are difficult to satisfy. Wiley & Wiley (1970) argued that the proper model for longitudinal data should assume that the error variance, which is a property of the measuring instrument, is stable and that the true variances change over time (which implies that the total variance and the reliability are not constant). Equation 1 can be modified to handle Wiley and Wiley's model. From Equation 3, the regression of η_j on η_i is β^n, which does not involve the true score variance. If the reliabilities are given, the regression of y_j on y_i can be corrected for attenuation and replaced for c_{ij} in Equation 1, which replaces *r* by unity. However, a correction for attenuation requires caution—because the choice of an unsuitable measure of reliability may be inconsistent with the correlations, and an improper correlation matrix may result (e.g., corrected matrices are sometimes not Gramian).

An advantage of covariance structure analysis is that it can be employed to analyze various simplex models, not only the stationary Markov simplex model (see Jöreskog, 1970); the regression model is inappropriate, by definition. In spite of its shortcomings, the regression equation obtained from Equation 1 can be useful when only the correlations of consecutive occasions are available or when the time intervals are not equal.

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