Dynamic Tailings Basin Study: Final Report

by

Kelin X. Whipple, Gary Parker, Chris Paola, and David Mohrig

Prepared for

HIBBING TACONITE COMPANY
Hibbing, Minnesota

January 1996
Minneapolis, Minnesota
The University of Minnesota is committed to the policy that all persons shall have equal access to its programs, facilities, and employment without regard to race, religion, color, sex, national origin, handicap, age or veteran status.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXECUTIVE SUMMARY</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>5</td>
</tr>
<tr>
<td>Problem Definition</td>
<td>5</td>
</tr>
<tr>
<td>Overall Objectives of Research Project</td>
<td>5</td>
</tr>
<tr>
<td>Previous Work and Reports</td>
<td>6</td>
</tr>
<tr>
<td>Preliminary Report, January 1994 - Summary</td>
<td>6</td>
</tr>
<tr>
<td>Objectives of the 1994-1995 Campaign</td>
<td>7</td>
</tr>
<tr>
<td>METHODOLOGY DEVELOPED IN STAGE 1 INVESTIGATION</td>
<td>10</td>
</tr>
<tr>
<td>Framework for the Study</td>
<td>10</td>
</tr>
<tr>
<td>Relation for Fan Elevation below the Launders</td>
<td>13</td>
</tr>
<tr>
<td>STAGE 1: EXPERIMENTARY PROGRAM</td>
<td>19</td>
</tr>
<tr>
<td>Motivation and Experimental design</td>
<td>19</td>
</tr>
<tr>
<td>Scaling Relationships</td>
<td>19</td>
</tr>
<tr>
<td>Experimental Procedure</td>
<td>20</td>
</tr>
<tr>
<td>Summary of Runs Completed</td>
<td>20</td>
</tr>
<tr>
<td>Results</td>
<td>21</td>
</tr>
<tr>
<td>STAGE 1: FIELD PROGRAM</td>
<td>24</td>
</tr>
<tr>
<td>Objectives</td>
<td>24</td>
</tr>
<tr>
<td>Data Collected</td>
<td>24</td>
</tr>
<tr>
<td>Results</td>
<td>25</td>
</tr>
<tr>
<td>STAGE 1: THEORETICAL PROGRAM</td>
<td>28</td>
</tr>
<tr>
<td>Objective: Relation for Upstream Fan Slope</td>
<td>28</td>
</tr>
<tr>
<td>Formulation</td>
<td>28</td>
</tr>
<tr>
<td>Reduction of the Relations</td>
<td>32</td>
</tr>
<tr>
<td>Test of Relation for Upstream Slope against Field Data and Adjustment</td>
<td>32</td>
</tr>
<tr>
<td>Modification of the Formulation Using an Existing Sediment Transport Relation</td>
<td>33</td>
</tr>
</tbody>
</table>
STAGE 1: APPLICATION TO WEST AREA #1 ...................... 34
  Scenario Analysis .................................. 34
  Discharge Oscillation .................................. 36
  Uncertainty in the Predictions ......................... 36

STAGE 2: GENERALIZED METHOD: ENTIRE TAILINGS POND ........ 38
  The Proposed Scheme .................................. 38
  Analysis ............................................ 38
  Calibration .......................................... 40
  Implementation: Stacking of Hydroseparated Tailings ....... 40
  Spreadsheet Program: Application to Design ............... 41

REFERENCES .............................................. 43
  Tables 1 through 4
  Figures 1 through 15
List of Figures

Figure 1a. Schematic of a hypothetical pond with emplacement of hydrosedimented material in a pie-shaped region near the launders. A channel with length $L_c$ and width $B_c$ is maintained within the region to allow for delivery of non-separated material to the distal part of the pond. The vertical cut A - A' is shown in Figure 1b.

Figure 1b. Vertical cut A - A' through the pond showing the present gradeline of the fan, the ultimate gradeline, and the pie-shaped region in which the hydrosedimented material is to be placed. The material must be built at least up to the ultimate gradeline of the channel; in addition it may be stacked some height $z$ above the gradeline.

Figure 2. The Hibbing Taconite Company's tailings basin at Hibbing, Minnesota.

Figure 3. The nature of the problem at the HTC basin. Continued aggradation at the present rate will lead to burial of the emergency launders in sediment before the design lifetime of the basin is reached. The problem could be ameliorated by reducing either the overall slope or the concavity of the topographic profile in the basin.

Figure 4. Schematic of the pie-shaped basin used for theoretical analysis.

Figure 5. Sequential topographic profiles taken along the length of West Area #1 showing that aggradation has occurred at a constant rate with respect to basin length, maintaining a self-similar topographic profile.

Figure 6. Schematic illustration of the small experimental basin ($L=6.9$ ft $2.1$ m). CHT indicates constant head tank. SF indicates auger-type sediment feeder. The large experimental basin was of similar design, but was fitted with a single, continuous weir gate at the downstream end of the fan.

Figure 7a. Fan slope as a function of the volumetric ratio of sediment feed rate ($Q_{so}$) to water discharge ($Q_w$) for the Sand I experimental series. $Q_w$ is the independent variable in all of these runs. Median size ($D_{50}$) of sediments used is indicated in $\mu$m. Data plotted for the 70 and 160 $\mu$m experiments represent averages of two sets of experiments (see Fig. K1B).
Replicate sets of experiments with the 70 and 160 μm sediments. Temporal and spatial fluctuations in fan slope occur as channels shift across the sediment surface, introducing some error into our estimates of “average” fan slopes. However, the results are generally reproducible.

Fan slope as a function of the ratio $Q_{so}/Q_w$ in runs in which $Q_{so}$ was the independent variable.

Fan slope as a function of the ratio $Q_{so}/Q_w$ for the coarse sand mixtures of the Sand IV experimental series ($D_{50}$ greater than or equal to 160 μm). Unlike experiments with either finer-grained sand or crushed coal, fan slope is found to be independent of grain size.

Fan slope as a function of the volumetric ratio of sediment feed rate ($Q_{so}$) to water discharge ($Q_w$) for the crushed coal experimental series. Median size ($D_{50}$) of sediments used is indicated in microns. As in the finer sand experiments, fan slope is sensitive to grain size. The independent variable in each set of experiments is indicated in parentheses in the legend.

Photographs of bed configuration taken during one of the oscillating $Q_w$ experiments (fluorescent green dye is used to highlight wetted areas against the black (coal) sediment background). Green dye also colors the standing water in the pond at the downstream end. A. Channels during high-flow stage. Note major channel running down fan center. B. Channels during low-flow stage. The effective confinement of flow during low stage periods leads to the favorable biasing of fan slope towards equilibrium conditions associated with the high-stage flows. C. Example of flow configuration if low-flow stage is maintained too long; the channels backfill and the upper fan aggrades rapidly.

Response of fan slope to oscillating water discharges as a function of the frequency of oscillation. The solid line indicates the slope associated with constant flow at the mean discharge ($S/Sm=1.0$). The dashed line indicates the slope associated with constant flow at the peak discharge. Arrows indicate the range of fan slope fluctuations observed for long oscillation periods.

Transect lines along which field measurements were made on the HTC fan.

A. Total load (bedload plus suspended load) field data plotted in non-dimensional form against excess Shields stress. Note that material less than 40 μm is designated wash load and is excluded from the reported total load. Fitted regression line is shown. B. Data from A plotted on a logarithmic scale. The Meyer-Peter/Müller bedload equation is shown for comparison. C. Data from A differentiated by sample location.
Figure 13. Friction factors (Cf) determined from field depth, slope, and velocity data plotted against the ratio of flow depth to median grain size (H/D₅₀). Regression line and equation are shown. The familiar Manning-Strickler resistance equation is shown for comparison, plotting well below the field data as explained in the text.

Figure 14. Field data on characteristic channel form. The ratio of flow depth to median grain size (H/D₅₀) is plotted against water surface slope. A linear relationship such as that defined by the data indicates a channel closure similar to that developed by Parker (1978) for gravel-bedded rivers.

Figure 15. Actual (1995) versus calibrated bed profile, West Area #1.
EXECUTIVE SUMMARY

The Hibbing Taconite Company operates an iron mine and mill in northern Minnesota, USA. The tailings produced by the mine are disposed in a tailings basin. The point of feed is the upstream end of one of the sub-basins within the tailings pond, the West Area #1. The tailings deposit forms an aggrading alluvial fan within each sub-basin.

Projections of tailings deposition rates within the West Area #1 indicate an imminent over-filling at the upper end of the basin. This premature filling, which threatens to reduce the lifetime of the tailings pond below design expectations, is the result of slopes in the upper part of the deposit which are steeper than anticipated at the time of basin design. St. Anthony Falls Laboratory, University of Minnesota, under contract from Barr Engineering, has undertaken a combined field, experimental, and theoretical study of the dynamics of tailings-basin sedimentation, with a particular focus on the physical controls (i.e., water discharge, sediment load, and sediment size distribution) on upper fan slope. This study has provided the tools necessary to evaluate the relative effectiveness of several tailings management schemes intended to alleviate the problem.

The tools were developed in two stages. In Stage 1, the analysis was restricted to West Area #1 due to the availability of specific data pertaining to that area. The results for Stage 1 were based on the assumption that all future tailings would be stored in West Area #1. The assumption was made for illustrative purposes only. The results of the calculation allow, however, for inferences concerning the performance of the entire pond. In Stage 2, a numerical model was developed to describe filling of the entire basin. The model allows for the testing of a variety of schemes for tailings management. The results of each of the two stages are summarized below.

STAGE 1. A number of tailings management schemes were identified. Each was evaluated using at least one, and in most cases all of the following three models developed during the study: a) a regression model derived from experimental data; b) a numerical model based on field relationships for sediment transport, channelization, and flow resistance; and c) a numerical model based on field data combined with a theoretical
suspension-dominated sediment transport model. While the results differ according to the
model used to evaluate an option, the results of the three models do not differ greatly, thus
lending support to each other.

The effectiveness of each scheme was evaluated in the following way. The
expected increase in bed elevation at the upstream end of the West Area #1 was computed
30 years into the future under the assumption of no tailings management scheme; i.e.
continued operation in the present mode. This is referred to as the null scheme below.
The corresponding elevation 30 years hence was then predicted for each scheme. The
benefit was quantified in terms of the amount of bed lowering the scheme would achieve
in 30 years compared to the null scheme. A positive benefit implies bed lowering; a
negative benefit implies that the bed elevation with the scheme would be higher than for
the null scheme. The results are presented below and summarized in Table 1.

1. **Null scheme.** In the case of the null scheme the bed elevation at the upstream
end of the West Area #1 is computed to be 1744 ft 30 years hence. This number is well in
excess of the prevailing upstream bed elevation of 1625 ft at the time of writing of this
report, and is also well in excess of the maximum permissible upstream bed elevation of
1635 ft. It must be remembered, however, that the result is overly pessimistic by a
considerable margin in that it is based on the assumption that all tailings would be placed
only in West Basin #1. Because of this it is of more value to consider the benefit of each
scheme rather than the absolute upstream bed elevation predicted by it in 30 years. This
elevation can be obtained, however, by subtracting the benefit in ft from the null upstream
elevation of 1744 ft.

2. **25% reduction in water discharge.** It has been suggested that a 25%
decrease in water discharge to the tailings basin would have the beneficial effect of
allowing Hibbing Taconite Company to save on electrical costs. The benefit to the tailings
basin is negative, with the bed elevation at the upstream building 11 to 18 ft higher than
the null scheme in 30 years.

3. **50% increase in water discharge.** An increase in water discharge would
lower the upstream slope of the fan even with no hydroseparator. The benefit is positive,
with the bed elevation near the launders building up 13 to 18 ft less than the null scheme in
20 years.

4. **100% increase in water discharge.** The benefit is increased relative to
Scheme 3, with a buildup of bed elevation near the launders that is 21 to 28 ft less than the
null scheme in 20 years.

5. **Install hydroseparator.** According to this option, all the material coarser than
150 μm in the tailings would be removed. It is assumed here that the material so removed
would not be delivered to the downstream end of the West Area #1, but would rather be
stored at a site where it would not interfere with the functioning of the basin itself. This
scheme has the largest benefit, with a bed elevation near the launders that is 46 to 61 ft
less than the null scheme. The benefit of the hydroseparator accrues in three ways. The
reduction in sediment feed to the basin reduces the overall aggradation rate within it. The combination of reduced feed and unchanged water discharge reduces the upstream slope. Finally, the finer grain size of the tailings reduces the upstream slope. The last effect is the weakest.

6. Install hydroseparator but reduce water discharge by 25%. The benefits associated with this scheme are only slightly less than Scheme 5, with a reduction in bed elevation near the launders of 39 to 58 ft compared to the null scheme.

7. Install hydroseparator but reduce water discharge by 50%. The benefits are somewhat less than Scheme 6, with a reduction in bed elevation near the launders of 28 to 51 ft compared to the null scheme.

8. Surging the water discharge. According to this scenario, which is not included in Table 1, the water discharge would be cyclically surged. This scheme would yield a benefit only if the high discharge is the effective one in setting upstream fan slope. While both numerical models suggest that the higher discharge should be the more effective one, neither is capable of addressing the problem quantitatively. The problem could only be addressed experimentally. Experiments suggested that surging the flow would be an effective, cost-efficient method for reducing the deposit slope. In experiments where water discharge was surged to 150% of the current operational level for 50% of the time, the fan regressed to a slope profile that approached that associated with the higher discharge (i.e., the fan slope obtained by running at 150% discharge continuously). The implication is that nearly all the benefit of Option 2 could be obtained with a considerably smaller increase in electrical cost associated with pumping. Due to modeling limitations, it is not possible to specify optimum surge discharges and durations. This option, however, can be field-tested as described in the full report.

A tentative implication of the result for Scheme 8 is that the null state could likely be maintained by cycling the water discharge such that the present water discharge would be released for 50% of the time, and two thirds of this discharge would be released for the remaining 50% of the time.

STAGE 2. The above results for Schemes 1 to 7 of Stage 1 cannot be used for design purposes for the entire basin, as they apply only to West Area #1. They nevertheless are encouraging, indicating that either increased discharge or hydroseparation can have a strongly beneficial effect on upstream fan elevation at the launders. With this in mind a numerical model was developed for the entire basin. The model has been cast in a form that can be used directly by the personnel of Hibbing Taconite Company. It is described in more detail below. Of relevance here are the results obtained from the model for the entire basin for Schemes 1 to 7. The results are cast in terms of the number of years that would elapse until the basin is “filled,” i.e. fan elevation at the launders reaches 1635 ft. An acceptable scheme fills the basin in 30 years or more starting from 1995. The results are summarized in Table 2.
1. **Null scheme.** The basin is found to fill in 6.4 years beyond 1995.

2. **25% reduction in water discharge.** The basin fills in 1.2 years.

3. **50% increase in water discharge.** The basin fills in 16.9 years.

4. **100% increase in water discharge.** The basin fills in 23.8 years.

5. **Install hydroseparator.** As in Stage 1, the calculation is based on the hydroseparation of all tailings coarser than 150 \(\mu\)m. Based on information from Hibbing Taconite Company, it is assumed that the hydroseparated material would not be removed from the basin, but would be used to fill a pie-shaped region close to the launders with radius \(L_c\). At any given time a channel with length \(L_c\) and width \(B_c\) would be maintained through the hydroseparated material to deliver the non-separated material to the distal parts of the basin. The geometry is shown in Figure 1. The hydroseparated material must fill the basin at least to the grade line of the channel; it could also be stacked some height above the channel. Two cases were examined. A value of \(L_c\) of 6600 ft yields a filling time of 32.6 years, with stacking height of hydroseparated material of 40 ft above channel grade. A value of \(L_c\) of 5750 ft yields a filling time of 34.5 years, with a stacking height of 30.0 ft.

6. **Install hydroseparator but reduce water discharge by 25%.** The general scheme is the same as that illustrated in Figure 1. For a channel length \(L_c\) of 6500 ft the basin fills in 28.4 years with a stacking height of 6.0 ft; a value of \(L_c\) of 5500 ft yields a filling time of 30.2 years and a stacking height of 34.0 ft.

7. **Install hydroseparator but reduce water discharge by 50%.** Again the general scheme is the same as that illustrated in Figure 1. A channel length of 6500 ft yields a filling time of 26.1 years and a stacking height of 5.0 ft; a channel length of 5500 ft gives a filling time of 27.8 years and a stacking height of 30.0 ft.

The above analysis indicates that either hydroseparation or water discharge increase is an effective mechanism for extending pond lifetime to near or beyond 30 years beyond 1995. Some combination of hydroseparation and water discharge decrease can also allow a lifetime near 30 years beyond 1995. Mean water discharge may be further decreased by a surging scheme as described in Scheme 8 of Stage 1.
INTRODUCTION

Problem Definition

Hibbing Taconite Company presently disposes of its entire production of tailings by means of deposition in a tailings basin. Current disposal rates are near 21 million long tons per annum. The size of the tailings ranges from in excess of 1000 μm to finer than 10 μm. The material is delivered to the basin as a slurry discharge from two launders. The mass concentration of tailings in the slurry is near 8.6 percent by weight. A diagram of the tailings basin is shown as Figure 2. The basin contains a number of sub-basins. At present, West Area #1 is the prime site for disposal; the East Area is used for three months during the winter. The water used to slurry the tailings down the launders is collected at the reclaim intake at the downstream end of the basin and recirculated. The basin is designed so that no sediment escapes. The water is also recirculated completely. Water may be added to account for evaporation losses, or drained off to account for direct precipitation on the surface of the basin. The elevation of the basin is constantly increasing due to sediment deposition. The dams defining the perimeter of the basin are periodically raised in order to prevent overflow. The profile of the sediment surface is markedly concave down the basin, with bed slopes above 1% at the upstream end and below 0.1% near the dams. This concavity is associated with sediment sorting as well as downstream decrease in sediment flux. Typical grain size is as coarse as 500 μm near the launders, but in clay, size ranges (slimes) in ponded water near the dams.

The main problem with tailings basin performance at present is illustrated in Figure 3. If the present profile of bed elevation of the basin is extrapolated upward based on predicted disposal rates of tailings, the emergency launders will become unusable before the basin has reached its design lifetime. It is thus desirable to find a means for limiting the buildup of sediment at the upstream end of the basin. This could be accomplished in several possible ways. The first of these is the reduction in the concavity of the profile. This would imply somewhat smaller bed slope near the launders. The resulting profile is illustrated in Figure 3. Another possibility is the reduction of average bed slope over most of the length of the basin.

Overall Objectives of the Research Project

Present mine plans involve consideration of a lowering of water discharge in order to save pumping costs, and the use of a hydroseparator in order to remove the coarsest sediment. It can be predicted that the former would raise basin slope and the latter would lower it. Existing understanding of fan dynamics at the start of this study did not allow evaluation of the net effect of both schemes in a quantitative sense, although our own
preliminary research suggested the form of the relation required to make the prediction—it would give the overall slope of the fan in terms of the basic governing variables water discharge, sediment discharge, and grain size. It is the necessity of evaluating this relation that formed the core of the research reported here. The basic theoretical background as well as the form of the design relation that we were aiming for are developed in the "Framework" section below. The goal of the project is to develop a design relation tailored for the HTC fan, based on field observations on the fan, experimental modeling, and existing engineering and geomorphic literature.

It is useful to describe how knowledge of such relations could be used to design measures to improve basin performance. Suppose that water discharge is to be reduced to save money on pumping. This would increase basin slope. If at the same time a hydroseparator is used to remove the coarsest 20–40% of the tailings, both the total rate of supply of sediment to the basin and the characteristic grain size of the coarser material would be reduced. This would tend to reduce basin slope. The net effect of both discharge change and the use of a hydroseparator could be evaluated with an equation that expresses the dependence of fan slope on all these factors. Various options could be tried until an optimal design is attained.

**Previous Work and Reports**

St. Anthony Falls Laboratory has performed a series of investigations for Barr Engineering Company starting from the late summer of 1993. The investigations include field work, study of field data, the performance of a series of experiments in the laboratory on basin sedimentation, and the development of several computer models describing basin filling. The results of this work are summarized in this report. Preliminary results were reported in the preliminary report, "Modeling of Tailings Basin Performance, Hibbing Taconite Company" dated January 1994, and an addendum to that report issued in February 1994.

**Preliminary Report, January 1994—Summary**

In this section we present a brief synopsis of the preliminary report dated January, 1994. Our purpose in doing so is to provide a context and motivation for the more extensive study undertaken during the period of March 1994 to July 1995.

The January 1994 report, together with the February 1994 addendum, presented findings of (1) a preliminary experimental study, and (2) initial versions of analytical and theoretical models of tailings basin deposition. The work in question was performed in 1993. The preliminary experimental study included experiments performed in a pie-shaped model basin with a radius \(L\) of 2.1 meters with well sorted quartz (silica) sands and silts with mean grain sizes in the range of 70-550 \(\mu\text{m}\) (micron). The main dependent variable was mean fan slope \(S\). Preliminary analysis of the experimental data on uniform-size sediment mixtures yielded a strong dependence of \(S\) on water discharge \(Q_w\) and a weak dependence on median grain size \(D_{50}\) reported as:

\[ S = f(Q_w, D_{50}) \]
Subsequent runs with poorly sorted mixtures of sediment sizes, reported in the February 1994 addendum, indicated a stronger dependence on grain size:

\[
\overline{S} \propto Q_w^{-0.51} D_{50}^{0.15},
\]

(1)

where the dependence on water discharge has been recast in terms of mass concentration \(X\) given by

\[
X = \frac{\rho_s Q_{so}}{(\rho Q_w + \rho_s Q_{so})}.
\]

(3)

In the above relation \(\rho\) denotes the density of water, \(\rho_s\) denotes the density of the sediment (rock material density rather than bulk deposited density) and \(Q_{so}\) denotes the volume feed rate of sediment at the upstream end (mass feed rate/material density).

Theoretical analysis indicated an even stronger dependence of \(S\) on water discharge (or mass concentration) and a weak dependence on grain size, relative to the experimental findings:

\[
\overline{S} \propto Q_{so}^{0.757} Q_w^{-0.857} D_{50}^{0.15} L^{0.11}.
\]

(4)

Thus, the strongest conclusions of the preliminary study were that: (1) reducing water discharge would result in a steepening of the fan, (2) fan slope is strongly controlled by mass concentration, (3) fan slope is influenced to a lesser extent by grain size. Considerable uncertainty, however, remained as to the magnitude of the grain size dependence.

**Objectives of the 1994-1995 Campaign**

The main objective of the 1994-1995 campaign was to obtain the data needed to refine and improve on the models we had developed based on the 1993 data. Specific needs included: larger-scale experiments that would be less influenced by basin size limitations; experiments that were better scaled to conditions at HTC; and field data on flow resistance, sediment flux and channel distribution. These target areas are discussed in more detail below. Another objective was to evaluate the efficacy of surging water discharge as a means of reducing fan slope.

**Experimental program.** The more extensive experimental program of the 1994-1995 campaign was designed with a number of objectives: (1) to replicate selected previous runs to examine reproducibility of the main observations; (2) to examine fan dynamics in a much larger basin that would minimize wall interference and allow better
reproduction of length-sensitive effects such as downstream sediment sorting, profile concavity, and formation of incisions and depositional lobes; (3) to gather more and better data on the relation between overall fan slope and the key independent variables: water and sediment discharge, and grain size; (4) to measure the degree and distribution of channelization as an aid to developing the theoretical model; and (5) to evaluate the utility of surging water discharge as a means of reducing slope.

In addition to the use of a much larger facility, a major distinguishing feature of the 1995 experiments was the use of crushed coal sediment in some of the experiments to enable us to achieve a more accurate scale model of the HTC fan than was possible using silica-based model sediment. The rationale and methods for this scaling technique are discussed in more detail below (Experimental Program).

**Field program.** Field data are needed in a study such as this both to constrain essential constituent relations of the theoretical model and to provide a basis for evaluation of the accuracy of the experimental simulations. The complexity of the HTC system, with its generally high transport rates, distributed, weakly channelized flows, and abundance of very fine sediment, means that use of semi-empirical relations in the model is unavoidable. The objective of the field program for 1994-1995 was to provide an adequate data base for development of semi-empirical relations for (1) flow resistance and sediment transport and (2) flow depth as a function of local grain size and slope.

**Theoretical program.** The theoretical models provide a means of evaluating the influence of effects that, because of scale, cannot be adequately represented in a scale model, and also of extrapolating model results to conditions not explicitly examined in the model study. Hence the theoretical model is the tool that will be used ultimately to evaluate different strategies for reducing the slope of the HTC fan. In the present case, some of the key questions that must be addressed in developing and calibrating the theoretical model are: (1) what are the best relations to use to model sediment transport and flow resistance on the fan? (2) what is the appropriate channel closure to apply to the system (i.e. what sets the overall flow width and depth at any point on the fan)? (3) how does fan slope change during incision events? This last question is of particular importance to determining the effects of discharge oscillation on the overall slope of the system.

**Summary of work done, 1994-1995 campaign.** The objectives of the 1994-1995 campaign as outlined above were generally met. Overall, the findings of the preliminary report are generally borne out in the more extensive 1994 - 1995 study, though in detail predictions differ quantitatively, and are supported by a much more complete understanding of the problem. The principal conclusion that fan slope is most responsive to changes in mass concentration, and less so to grain size, is firmly upheld. The experimental and field results have been incorporated into a generalized analytical/numerical model. The model has a modular structure that can accept specified forms of resistance equation, channel width closure, and transport law. Both field-based laws from regression of field data (guided by theory) and existing relationships from the literature have been explored. This flexible structure allows interpretation of both field and experimental data and understanding of linkages among relations. Three versions of
the model bracket the range of uncertainty in field and theoretical relationships, and therefore in basin response.
METHODOLOGY DEVELOPED IN STAGE 1 INVESTIGATION

Framework for the Study

We discuss the theoretical model we have developed for this project in detail in a later section. It is helpful, however, to have an outline of the analysis in mind in order to provide a context for discussion of the field and laboratory programs, and to indicate how the results can be applied to the problem at hand.

Simplifications concerning fan configuration and operation. As noted previously, the engineering problem of interest at Hibbing Taconite Company concerns the alluvial fan in West Area #1 of the tailings pond. The geometry of this basin is shown in Figure 1. The configuration is sufficiently close to a pie-shaped segment of a circle to allow for approximation as such. The result is the simplified basin shape shown in Figure 4; it has radius $L$ and angle $\theta$ (radians).

The tailings pond fills with sediment by means of actively shifting braided channels. At any given point, then, bed elevation is likely to vary in an erratic way. When averaged over time, however, the fluctuations give rise to a consistent fan morphology. Here it is assumed that the braid channels can work their way across the fan with no lateral restrictions other than the dams themselves, so that average conditions obey axial symmetry.

In the last several years of mine operation the water discharge $Q_w$ and sediment disposal rate $Q_{so}$ into the tailings basin have been held relatively constant near 265 ft$^3$/s (7.50 m$^3$/s) and 8.1 ft$^3$/s (0.23 m$^3$/s), respectively. The sediment disposal rate, and indeed all sediment transport rates reported here are given in volumes of pure sediment without pores, so that the sediment mass can be obtained by multiplying by sediment density.

Another simplification involves the inner and outer dams, or dikes. These have been raised in time so as to a) prevent water and sediment from escaping over the outer dam, and to b) either prevent or allow the delivery of water and sediment into West Area #2, as desired, by means of the adjustment of the height of a weir along an interior dam. The result has been an approximately constant rate of vertical aggradation, or rate of increase in bed elevation $v_b$ in the West Area #1 in recent years of 4.0 ft/year (1.2 m/year), as shown in Figure 5. A reasonable simplification of this mode of operation in the context of Figure 4 is the assumption that the margins are raised just fast enough to prevent sediment from escaping the basin. This corresponds to the most cost-effective scheme of dike raising. (While this is true for the entire tailings pond, it is not necessarily true for

Hibbing Taconite Co. Final Report 10
West Area #1, in that tailings are often allowed to pass to West Areas #2 and #3 by means of a breach in the dike.)

From the point of view of design, the question of interest is as follows. For a given basin of the type shown in Figure 4 subjected to constant rates of water and sediment discharge $Q_w$ and $Q_{so}$ from its launders, and further subjected to a rate of dike-raising $v_b$ that just prevents sediment overflow from the basin, what is the resulting profile of bed slope and elevation of the fan within?

**Relation for fan slope under conditions of equilibrium aggradation.** Let $r$ denote a radial coordinate from the apex of the fan, as shown in Figure 4, $t$ denote time and $\eta(r,t)$ denote mean bed elevation. Mean bed slope $S$ is given by the relation

$$S = -\frac{\partial \eta}{\partial r}$$

at distance $r$ down the fan. Under conditions of constant $Q_w$ and $Q_{so}$, the slope profile should approach an asymptotic profile $S = S(r)$ independent of time. Here the goal is to predict this slope.

The technique of dimensional analysis allows for a determination of the general form of the relation for fan slope. Let $D$ denote the geometric mean grain size and $\sigma_g$ denote the geometric standard deviation of the feed sediment. ($D$ is replaced with median size $D_{50}$ in some of the discussion below.) In addition, let $g$ denote the acceleration of gravity, $\nu$ denote the kinematic viscosity of water, $\rho$ denote the density of water, and $\rho_s$ denote the density of the rock of which the disposed tailings is composed. In the case of the Hibbing Taconite Company, $\rho_s$ takes a value of 3.0 grams/cm$^3$. Under the imposed constraints, the most general possible functional relation for fan slope takes the form

$$S = f_{n1}(r, L, v_b, Q_w, Q_{so}, \theta, D, \sigma_g, g, \nu, \rho_s, \rho)$$

where $f_{n1}$ denotes a functional relationship.

The parameter $v_b$ corresponding to the rate of dike raising cannot be chosen arbitrarily. Here it is chosen so that it is no larger than the value sufficient for all the sediment delivered to the basin to be just deposited within it. Based on a simple mass balance, then, it is found that

$$v_b = \frac{Q_{so}}{(1 - \lambda_p) \frac{L^2}{2} \theta}$$

where $\lambda_p$ denotes bed porosity, here taken to be constant. After $v_b$ is eliminated from (6) with the use of (7), $S$ is found to be a function of 11 other parameters. The standard techniques of dimensional analysis can be used to reduce the relation to the following simpler form:
\[ S = f_{n_4}(\bar{\rho}; \bar{Q}_w, \bar{Q}_\infty, \theta, R, Re_p, \sigma_g, \frac{D}{L}) \]  
\( \tag{8} \)

where \( R = (\rho_s - \rho) / \rho \) denotes the submerged specific gravity of the sediment, and

\[ \bar{r} = \frac{r}{L}; \quad \bar{Q}_w = \frac{Q_w}{\sqrt{gD D^2}}; \quad \bar{Q}_\infty = \frac{Q_\infty}{\sqrt{RgD D^2}}; \quad Re_p = \sqrt{RgD D} \quad V \]  
\( \tag{9a,b,c,d} \)

denote the following dimensionless parameters; distance down the fan, water discharge, sediment feed and particle Reynolds number, respectively.

If the slope does not vary too greatly in the downstream direction (a condition that may not hold for the entire basin but may hold for parts of it), (8) can be integrated to determine an average fan slope \( \bar{S} \) given by

\[ \bar{S} = \int_{\bar{r}_1}^{\bar{r}_2} S(\bar{r}) d\bar{r} = f_{n_4}(\bar{Q}_w, \bar{Q}_\infty, \theta, R, Re_p, \sigma_g, \frac{D}{L}). \]  
\( \tag{10} \)

Some of the independent variables in the functional relationship are not likely to be important, at least for the problem of interest. The parameter \( D/L \) is extremely small both for the experimental basins and for the basins of Hibbing Taconite Company. The value of the angle \( \theta \) is \((1/2) \pi \) in one of the experimental basins and \((3/8) \pi \) in the other; a value appropriate to West Area #1 would be close to either of these. In addition, a theoretical development presented later assigns no role to either \( D/L \) or \( \theta \). With this in mind, these parameters are dropped from the list. The parameter \( R \) takes the value 1.65 for the sediment used in most (but not all) of the experiments and 2.0 for the tailings at Hibbing. This variation may be small enough to allow for it to be dropped from the list. Note, however, that dropping it from the list in (10) is not equivalent to ignoring it completely, as it occurs in the definition of \( \bar{Q}_\infty \) given in (9c).

In most of the experiments the sediment was sufficiently uniform for \( \sigma_g \) to be approximated as unity, and thus dropped from the list. With this in mind, the goal of both the theoretical and experimental work was the determination of laws of the form

\[ \bar{S} = f_{n_4}(\bar{Q}_w, \bar{Q}_\infty, Re_p). \]  
\( \tag{11} \)

In point of fact the bed slope of the experimental fans shows little variation in the downstream direction, whereas the bed slope in the West Area #1 declines from over 0.010 to below 0.001 in the downstream direction, as shown in Figure 5. Of most relevance to the problem at Hibbing Taconite Company is the slope \( S_u \) at the upstream end of the fan. Here this region is taken to be the first 1640 ft (500 m) of the fan downstream of the launders, over which the average slope \( S_u \) is 0.0131. One engineering solution to the problem of overfilling of the basin near the launders would be the lowering of this slope.
The strong decline in bed slope in the downstream direction in the West Area #1 indicates that relations of the form of (11) are not directly applicable to the problem. In addition, the large geometric standard deviation $\sigma_g$ associated with the feed sediment ensures a strong change in the characteristic size of the bed material in the downstream direction. At the upstream end of the fan, however, most of the finer grain sizes are acting as wash load; that is, they are simply being transported downstream with little or no deposition on the bed. The material that constitutes the bed material itself at the upstream end is much better sorted than the feed material. With this in mind, it is reasonable to apply (11) directly to the determination of the upstream fan slope, with the proviso that the grain size $D$ used to determine $Q_w$, $Q_{so}$, and $Re_p$ be identified with the geometric mean size of the bed material at the upstream end of the fan rather than that of the feed material. In the case of the West Area #1, the appropriate value of $D$ for the present condition is 400 $\mu$m.

Relation for Fan Elevation below the Launders

The actual design parameter of interest for Hibbing Taconite Company is the fan elevation below the launders. A successful design strategy for tailings management would result in a bed elevation 30 years hence, and would not exceed 1635 ft, the maximum elevation allowing for use of the emergency launders. The elevation in 30 years would likely be significantly greater if no management scheme were undertaken. For the sake of illustration, the method described below is developed only for West Area #1 based on a continuation of the feed rate and depositional volume of tailings for 1992. The values obtained from this method are highly conservative in that the rest of the basin is not used to fill sediment. It serves, however, to set the stage for a more complete model for the entire tailings pond.

The most direct way to achieve the goal of predicting bed elevation below the launders would be a prediction for the complete slope and elevation profile of the fan. Such a prediction, while possible in principle, has not been achieved in the course of this project due to the complexity of predicting the differential transport of material ranging from the clay sizes to in excess of 1000 $\mu$m down the length of the fan. Instead, a somewhat simpler approach was devised, and the basin has been divided accordingly into two regions with differing modes of sediment deposition.

Coarse silt, sand, and fine gravel are readily eroded and deposited by fluvial action. Fine silt and clay sizes behave somewhat differently; once mobilized, they deposit only under conditions of still or very slowly moving water. This is the reason for the maintenance of ponded water at the downstream end of a tailings basin. The cutoff between the two ranges is somewhat ambiguous. Here, however, a cutoff of 40 $\mu$m is adopted. A length $L_c < L$ is assumed such that all material coarser than this cutoff size deposits upstream of the point $r = L_c$ in a "fluvial zone" and all material finer deposits downstream in a "ponded zone".
Sample application. The parameters in (20) that might be changed by any tailings management scheme are the upstream slope $S_u$, (so that $\lambda$ differs from zero), vertical aggradation rate $v_b$ and length of the fluvial zone $L_s$. These changes might be driven by, among other factors, changes in the total feed rate of tailings $Q_{so}$, as well as the feed rate of tailings $Q_{soc}$ that are in excess of 40 $\mu$m and thus expected to deposit in the fluvial zone. The subscript "i" is here used to denote the existing (or "initial") values for these parameters in the absence of modification due to a management scheme:

\[
\begin{align*}
Q_{so} & = 5.54 \text{ ft}^3/\text{s} (0.157 \text{ m}^3/\text{s}) \\
Q_{soc} & = 2.90 \text{ ft}^3/\text{s} (0.082 \text{ m}^3/\text{s}) \\
S_i & = 0.0131; \\
\lambda_{psa} & = 0.423; \quad \lambda_{psl} = 0.555.
\end{align*}
\]

The area of West Area #1 $A_{wbl}$ is near $8.86 \times 10^6 \text{ yd}^2 \left(8.10 \times 10^6 \text{ m}^2\right)$. Using the above-quoted value of $L$, the relation $A_{wbl} = (1/2) \theta L^2$ and (7) as applied to both the zones of "coarse" sand and silt and "slimes," it is found that

\[
\begin{align*}
\theta & = 0.942 \text{ radians}; \\
v_b & = 3.97 \text{ ft/year} \left(1.21 \text{ m/year}\right); \\
L & = 9209 \text{ ft} \left(2807 \text{ m}\right).
\end{align*}
\]

52.3% of the tailings are in excess of 40 $\mu$m and 23% are in excess of 150 $\mu$m. The appropriate form for (20) for the case in which nothing is done (null case) is, then,

\[
\eta_{il} = \eta_{Eo} + v_b t + S_i (L - L_s) + S_i I L_s
\]

(21)

where $\eta_{il}$ denotes the elevation at the launders at time $t$ in the case that no management scheme is implemented.

The null case (no scheme for tailings management) is considered first here. For simplicity the elevation at the downstream end of the pond at time $t = 0$, i.e. $\eta_{Eo}$ is set equal to 1555 ft (474 m), or 70 ft (21.3 m) below the current (1995) bed elevation at the launders of 1625 ft (495.3 m), in accordance with (21) and the above-quoted value of $I$ of 0.50. The computed value is very close to the current measured value of 1556 ft (474.3 m) near the Western Dam of West Area #1. According to (21) this elevation difference would be maintained in time. In 30 years the elevation of the bed adjacent to the launders would be 1744 ft (531.6 m), or 119 ft (36.3 m) above their present elevation.

A scheme for tailings management might involve the use of a hydroseparator to extract tailings coarser than a given size, a change in water discharge or both. Here it is assumed that any hydroseparated material is removed from West Area #1 and deposited elsewhere. An appropriate form of (11) is used to compute the resulting upstream bed slope $S_u$, and thus the value of $\lambda$. In computing $v_b$ from (12a) and $L_s$ from (12b), $Q_{so}$ and $Q_{soc}$ now take values of the tailings feed rate and the feed rate of material in excess of 40
μm, respectively corresponding to the exclusion of any hydroseparated portion. The parameter \( \lambda_{psa} \) takes its previous value of 0.555. The value of \( \lambda_{psa} \) must, however, be prorated according to the following formula based on field information:

\[
\lambda_{psa} = \frac{f_1 \lambda_1 + f_2 \lambda_2}{f_1 + f_2}
\]  

(22)

where \( f_1 \) denotes the fraction of the material composing \( Q_{soc} \) in excess of 150 μm, \( f_2 \) denotes the corresponding fraction between 40 and 150 μm, \( \lambda_1 = 0.385 \) is a bulk porosity for deposits of material with a characteristic size in excess of 150 μm and \( \lambda_2 = 0.452 \) is the corresponding material for deposits with characteristic sizes between 40 and 150 μm. The fractions \( f_1 = 0.23 \) and \( f_2 = 0.293 \) yield the previously-quoted value of \( \lambda_{psa} \) of 0.423 for “coarse” feed material that has not been hydroseparated. The calculation of \( \lambda_{psa} \) proceeds easily from (22) and the grain size distribution of the tailings once the “cut” size above which all material is to be hydroseparated is specified.

As a sample case here, it is assumed that all of the tailings in excess of 150 μm are removed by a hydroseparator and delivered to a location other than West Area #1. Using the fractions 0.23, 0.293 and 0.477 for the portions of the tailings before hydroseparation in excess of 150 μm, between 40 and 150 μm and less than 40 μm, respectively, the new values of \( Q_{soc} \) and \( Q_{soc} \) are now 0.77 x 0.157 = 0.121 m³/s (4.27 ft³/s) and 0.293 x 0.157 = 0.046 m³/s (1.62 ft³/s). It is further assumed that this, perhaps in combination with an increase in water discharge, gives rise to a 50% decrease in upstream slope, so that \( \lambda = -0.50 \). Using the numbers above, (12a,b) and (22) (with the value \( f_1 = 0 \) for feed sediment after hydroseparation) it is quickly found that

\[
\lambda_{psa} = 0.452 \\
v_b = 3.22 \text{ ft/year (0.98 m/year)}; \\
L_s = 7850 \text{ ft (2393 m)}.
\]

Applying these numbers to (20), it is found that in 30 years from 1995 the bed elevation at the upstream end would be 1690 ft (515 m) or 54 ft (16.4 m) below the elevation that would be achieved if no scheme were implemented. The elevation difference between the upstream and downstream end of the fan would be 39.0 ft (11.9 m), or 31.4 ft (9.6 m) less than the values that would prevail if no scheme were implemented.

It can be seen from these calculations that a lowering of the bed elevation of the upper fan can be achieved in two ways. If the total delivery of sediment to the basin is reduced by e.g. using a hydroseparator, the aggradation rate of the fan as a whole is reduced. By reducing the upstream slope, on the other hand, the entire fan is forced to regrade, resulting in a one-time lowering of bed elevation at the upstream end. This “one time” benefit is not realized instantaneously. Rather, the fan must regrade itself to the changed conditions over a period of months, or perhaps years. The response time of the
fan to imposed changes is not well known at this time, but it is likely to be on the order of several years.

The above method was implemented in spreadsheet form in the subsequent section *Scenario Analysis* to yield the results for Schemes 2 - 7 of Table 1. The values of the fraction change in upstream fan slope \( \lambda \) used in that analysis were obtained from the results of the experimental, field and theoretical research of Stage 1 described below. It is again emphasized that the results in Table 1 apply only to West Area #1 under the assumption that the sediment delivery to the basin continues to be equal to the values recorded in 1992. All of Schemes 2 to 7 yield bed elevations below the launders in 30 years well in excess of 1635 ft. This is because the storage space in the rest of the basin is not used in the calculation, rendering the calculation highly conservative. It can be seen, however, that Scheme 5 (hydroseparation of all material in excess of 150 \( \mu \)m with no change in water discharge) is particularly effective in lowering bed elevation relative to the null case.

A more detailed methodology using the entire basin is presented below in the section describing the Stage 2 analysis.
STAGE 1: EXPERIMENTAL PROGRAM

Motivation and Experimental Design

Strategy for 1994-1995 Campaign. As mentioned earlier, the objectives of the experimental program were: (1) to replicate selected previous runs to examine reproducibility of the main observations; (2) to examine fan dynamics in a much larger basin that would minimize wall interference and allow better reproduction of lengthsensitive effects such as downstream sediment sorting, profile concavity, and flow channelization; (3) to gather more and better data on the relation between overall fan slope and the key independent variables: water discharge, sediment discharge, and grain size; (4) to measure the degree and distribution of channelization as an aid to developing our theoretical model; and (5) to evaluate the utility of oscillating water discharge as a means of reducing slope.

Experimental Facility. Two sedimentation basins were used for experimental study of tailings delta dynamics. Preliminary experiments were conducted in a quarter-circle basin with radius \( L \) of 6.9 feet (2.1 meters). The outer perimeter of this basin was fitted with three adjustable weir gates (Fig. 6). Additional experiments were conducted in a larger basin \( (L = 17.1 \text{ feet} (5.2 \text{ meters})) \) fitted with a single weir gate. The larger basin was somewhat narrower, with an expansion angle of 3/16 of a circle. This afforded considerable savings in sediment volumes and run times without altering experimental results. In both cases a constant head tank was used to supply steady water discharges \( Q_w \). Steady sediment supply rates \( Q_{so} \) were delivered by an auger-type dry sediment feeder. Water and sediment inputs were allowed to mix before being delivered to the apex of the basin through a flow splitter (Fig. 6). Three lines of ruled stakes spaced 38.1 inches (15 cm) apart allowed rapid measurement of radial profiles without interruption of the experiments (Fig. 6). The quarter-circle basin geometry approximates the geometry of the Hibbing Taconite tailings basin (West Area #1).

Scaling Relationships

Prediction of field responses on the basis of experimental results must consider scaling relationships between the model and the prototype. The relatively fine sediment sizes and large basin scale at Hibbing Taconite limit our ability to produce accurate dynamically-scaled laboratory models of the tailings basin. The fundamental problem in physical scale modeling of relatively fine-grained systems such as the HTC tailings basin is that satisfaction of Froude scaling relations generally require the use in the scale model of sediment that is so fine that it would behave cohesively. Accordingly, preliminary experiments conducted with silica sand as a model sediment can be thought of as
approximate scale models of much coarser gravel fan systems. To some extent this
problem can be avoided by using sediment with a smaller specific gravity \( \rho_s/\rho \) than the
prototype, which allows a closer approach to Froude similarity while maintaining grain
size above the cohesive limit. With this in mind, we performed the main body of large-
scale experiments using crushed coal with a specific gravity of 1.35 - 1.50. This allowed
us to construct approximate Froude-scale models of the uppermost part of the HTC
tailings basin (strictly only the upper 1640 feet (500 m) are accurately modeled).

Fortunately, although predicting different fan slopes, regression equations derived
from the sand experiments predict similar system responses to imposed changes
(increasing water discharge or decreasing sediment discharge, for example) as those
derived from the coal experiments. The entire body of experimental data is therefore used
in this report to gauge the magnitude of expected field responses, and also in the
development of the theoretical model presented herein. Thus, despite certain unavoidable
scale effects related to reduced Reynolds numbers and sediment size, the experimental
findings effectively bracket the range of plausible system responses.

**Experimental Procedure**

Experiments generally started with an initial bed slope of near zero, although a
gently-sloping sediment ramp was used as an initial condition in some runs. After an initial
progradational stage in which the fan advances across the basin under conditions of
constant base level, the weir gates were raised incrementally to impose a rate of base level
rise chosen to just barely keep all sediment confined in the basin, and thus to
approximately equal the mean surface aggradation rate. This technique results in a steady-
state overall aggradation profile that is a precise analogy to conditions observed at the
Hibbing Taconite facility.

After the initial progradational stage, each stage of fan building was continued for
long enough for quasi-steady state aggradation to be at least approximately achieved.
During each run, surface elevation profiles were recorded every 15 to 30 minutes and
overhead photographs were taken every 5 to 15 minutes to document changes in bed
configuration. Occasional spot measurements of channel widths and depths were also
recorded.

**Summary of Runs Completed**

In total 63 experiments were completed (Table 3). These can be divided into 6
different run series, each with a different set of objectives. Run series include 5 series
employing silica sand (Sand I, II, III, IV, V) and one series employing crushed coal (Coal
I).

The Sand I series constitutes the experiments reported in the preliminary report of
January 1994 and addendum of February, 1994 and the subsequent replicate experiments
conducted in fall of 1994 (22 total experiments). The Sand I series included experiments
with four unimodal grades of sediment (70, 110, 160, and 550 μm), and two poorly sorted mixtures. These experiments addressed the dependence of fan slope on water discharge $Q_w$ and grain size $D_{50}$ and tested experimental reproducibility. The Sand II series used the 160 μm sand in the small basin in a test of the dependence of fan slope on sediment discharge $Q_s$ (3 experiments). The Sand III series used 120 μm sand in the large basin with the objective of testing for any basin-scale effects and was designed to address the dependence of fan slope on both $Q_s$ and $Q_w$ (7 experiments). The Sand IV series used 270 and 460 μm sediment sizes in the small basin, and was designed to augment the analysis of the dependence of fan slope on $Q_w$ and $D_{50}$ completed in the Sand I series (6 experiments). The Sand V series used the 160 μm sand and was designed to test the efficacy of a 2% (by weight) clay concentration in augmenting the efficiency of sediment transport by suspension. Finally the Coal I series was designed to test the dependence of fan slope on $Q_s$, $Q_w$, $D_{50}$ and basin size ($L = 6.9$ and 17.1 feet) in a more accurate scale model of HTC conditions. In addition, the Coal I series included a set of experiments to evaluate the efficacy of surging water discharge periodically as a mechanism for reducing fan slope (27 experiments total).

Results

Constant Discharge Experiments. In all cases radial profiles of the experimental fans built with uniform sediment sizes were approximately linear (constant slope). Therefore all analyses are reported in terms of the average fan slope $S$, determined by linear regression through radial profile elevation data recorded in the experiments and averaged in space and time.

Experimental data indicate a strong dependence of fan slope on sediment discharge $Q_{so}$, water discharge $Q_w$, submerged specific gravity $R$, and a weaker dependence on grain size $D_{50}$ (Figs. 7 and 8; Table 3). The experimental data are usefully cast in terms of the general “design equation” derived earlier (Framework), in which fan slope is shown to be a function of water discharge, sediment discharge, submerged specific gravity, and grain size,

$$ S \propto Q_w^{\delta W} Q_{so}^{\delta s} R^{\delta p} D_{50}^{\delta D}, \quad (23) $$

where $\delta W$, $\delta s$, $\delta p$, and $\delta D$ are constants. It was not feasible to conduct experiments with more than two model sediments, and it is not possible to evaluate the exponent $\delta p$ on the basis of the experimental results, although a strong dependence on $R$ is indicated (Table 3). However, as shown in Table 4, analysis of the crushed coal and sand experiments yielded similar values of the exponents $\delta W$, $\delta s$, and $\delta D$, indicating (1) that the power-law form of equation (23) is generally valid (the exponents are constants), and (2) that both sets of experiments are potentially useful for interpretation of field conditions at the HTC facility.

In the Sand V experiments we compared fan slopes for silica sand input with and without the addition of 2% clay. We found, contrary to expectations, that the fan slope...
actually decreased slightly when we removed the clay from the feed. We believe this small effect to have been an artifact of the extremely low Reynolds numbers of the experiments. Our interpretation of all the data, including the observation that the transport rates measured in the field are well fit by the Meyer-Peter/Müller (1948) equation (Field Results section), is that the effect of the clay on overall fan slopes is negligible.

Together the silica sand and coal experiments form a basis for predicting system response at the HTC facility. Analyses of data from the various experimental series yield similar, but quantitatively different, results, as summarized in Table 4. Given the range of predicted exponents, expected field response to proposed tailings basin operations can be bracketed. Example calculations are presented below (Discussion section) and are summarized in the Executive Summary. However, as the experiments were conducted almost exclusively with uniform sediment mixtures, and due to scaling limitations, these calculations only apply to the upper fan section, as discussed in more detail below.

**Oscillating water discharge.** On the basis of preliminary work, oscillating the water discharge was identified as a potentially cost-effective strategy for reducing fan slope at the HTC facility. Consider a simple step function oscillation between a peak discharge \( Q_{wh} \) and a minimum discharge \( Q_{wl} \), such that mean discharge \( Q_{wm} \) is the average of \( Q_{wh} \) and \( Q_{wl} \). Let \( S_h, S_i, S_m \) denote the fan slope under conditions of steady discharge at \( Q_{wh}, Q_{wl}, Q_{wm} \), respectively, and \( S_f \) denote the resultant fan slope under the fluctuating discharge. The essential hypothesis to be tested, and quantified, is that \( S_f \) is less than \( S_m \). Such a condition is likely to hold true under conditions in which channel dimensions are set primarily by the peak flows. A series of experiments using the crushed coal model sediment was devoted to testing this hypothesis. It is important to note that effects associated with non-steady water discharges are beyond the scope of theoretical analyses presented in this report. Thus, experimental findings summarized here constitute the only quantitative basis for interpreting the effects of oscillating the water discharge.

Observation of the formation and avulsion of individual channels in the constant-discharge experiments indicated that there should be an optimal oscillation frequency, \( F_o \), for which a maximal influence on fan slope is obtained. For instance, for a very rapid oscillation, no change in channelization or surface morphology occurs and \( S_f \) equals \( S_m \). At the other extreme, where oscillations are long enough for the fan to reach quasi-equilibrium, fan slope itself fluctuates between \( S_h \) and \( S_i \), potentially aggravating sedimentation problems at the upper end of the tailings basin in the case of the HTC facility.

In the oscillating discharge experiments, \( Q_{wh} \) was set to 150% of \( Q_m \) and \( Q_{wl} \) set to 50% of \( Q_m \). This choice was made in order to model the effect of fluctuating between running 2 pumps and 6 pumps (compared to the current constant operation of 4 pumps) at the HTC facility to evaluate the potential reduction in upper fan slope at no increase in average energy consumption. Oscillation periods \((1/F_o) \) of 5, 10, and 20 minutes were used. Experimental results indicate that under the right conditions fan slope can be significantly reduced below \( S_m \) with little temporal fluctuation in fan slope. The optimal effect is obtained for an intermediate oscillation frequency, \( F_o \), which is long enough to
allow the incision of deep channels during the high flow stage, but short enough to prevent
the infilling and over-topping of these deep channels during the low flow stage (Figs. 9 and 10).
STAGE 1: FIELD PROGRAM

Objectives

As discussed above, the objective of the field program for 1994-1995 was to provide an adequate data base for development of semi-empirical relations for (1) flow resistance and sediment transport and (2) flow depth as a function of local grain size and slope.

Data Collected

Data were collected at the HTC fan over the course of three field campaigns. The measurements were collected with the aid and consultation of Mr. Marty Halverson of HTC and consisted of:

velocity: 64 flow velocities measured with an electromagnetic current meter.

depth: 69 flow depths, measured either directly with a staff or indirectly as part of the total-station surveys of the site

bedload: 71 samples collected with a standard Helley-Smith sampler and analyzed for mass transport and size distribution

suspended load: 67 measurements collected with a USGS type DH-48 bottle-type sampler and analyzed for mass transport and size distribution

bed material size distribution: 73 measurements

topography: maps including elevations of bed and water surface and channel locations produced by Marty Halverson of HTC using a total station

channel form and distribution: measurements of channel dimensions were obtained both from the site maps and by direct high-resolution aerial photography using a small balloon suspended over the channel network

The data were obtained along three transect lines (Fig. 11): one near the apex of the fan where flow was highly concentrated and velocities and transport rates were at a maximum; another further downstream where the flow had distributed itself into channels but the bed material was still coarse; and a third further downstream where grain size was reduced to a silt-sand mixture. Monitoring of topography along survey lines for two
months showed clearly that most transport and deposition occurs along the main flow lanes, which are in the range of 50-200 m wide at HTC.

**Results**

**Total Load.** Total-load (bedload plus suspended load) data from the HTC fan are shown in Figure 12 in nondimensional form, as a function of excess Shields stress. The Shields stress was calculated from the shear stress, estimated from measured water-surface slope and flow depth, and local bed-material median size. Critical Shields stress was estimated from the relationship given by Brownlie (1981). The fraction finer than 40 μm, as determined from the grain-size analyses, is considered to move as wash load and is therefore removed from the calculated total load. As is typically the case for transport rates measured in the field, there is considerable scatter in the data. Nonetheless, there is a clear increase in total load with excess stress. The best-fit regression line for the data is given by

$$q^* = 7.5(\tau^* - \tau_c^*)^{1.1}$$

with an \(r^2\) value of 0.2067. This is statistically indistinguishable from the well-known Meyer-Peter/Müller (MPM) bedload relation:

$$q^* = 8.0(\tau^* - \tau_c^*)^{1.5}$$

Both equations are shown in Figure 12b. In the above relations, \(q^*\) denotes the dimensionless Einstein sediment transport rate and \(\tau^*\) denotes the dimensionless Shields stress, defined respectively in (38a,b) below, and \(\tau_c^*\) denotes the critical Shields stress at the threshold of sediment motion. The observation that the data are fairly well predicted by the MPM relation suggests that the fine-grained washload component of the mixture does not strongly influence the transport of the coarser material.

Figure 12c shows the same data but sorted by measurement site. There are no obvious trends except that two outlier data points (high transport rate, low stress) are seen to come from one sample location near the apex of the fan. This was a site where rapid spatial deceleration of the flow is likely to bias the shear-stress estimates, making them too small.

Accurate measurement of sediment transport in the field is especially difficult, in shallow flows where large quantities of sediment are transported in suspension. The expected error is to underestimate the transport rate. In this sense the agreement of the observations with the MPM relation, which has repeatedly been shown to work well in predicting bedload rates, makes sense. Nonetheless, in view of possible undersampling of the suspended load, we have viewed estimates obtained this way as lower limits.
**Resistance.** Data on flow resistance are shown in terms of drag coefficient $C_f$ as a function of relative roughness $h/D_{50}$ in Figure 13. This drag coefficient is defined in (26) below. Data points for which field observation suggested that shear stresses were influenced by backwater effects have been excluded. The best-fit regression line for these data is

\[ C_f^{1/2} = 4.8 \left( \frac{H}{D_{50}} \right)^{0.11} \]  

(26a)

with an $r^2$ value of 0.0259. A re-analysis of the data with the exclusion of a somewhat suspect subset yielded the similar relation

\[ C_f^{1/2} = 4.5 \left( \frac{H}{D_{50}} \right)^{0.12} \]  

(26b)

The dependence on relative roughness is sufficiently weak that it is not significant statistically. The best-fit constant value for $C_f^{1/2}$ is 7.7.

Also shown in Figure 13 is a standard resistance relation (the Manning-Strickler formula), which tends to underpredict the observed values of the drag coefficient. This is probably due to at least two effects not included in the Manning-Strickler relation: increased effective viscosity of the fluid due to high suspended load, and effective flow resistance induced by trains of antidunes and associated breaking standing waves. We believe that these effects satisfactorily account for the observed friction factors and that they do not represent a significant source of error in predicting the dynamics of the fan.

**Channel Form.** These measurements are shown in Figure 14 as depth relative to median grain size versus bed slope. They reflect self-adjustment of channel flow depth (and hence shear stress) via bank erosion and widening in a system in which the channels are free to find an equilibrium width unconstrained by lateral banks or walls. The best-fit regression line in Figure 14 is

\[ \frac{H}{D_{50}} = 1.75^{1.05} \]  

(27)

with an $r^2$ value of 0.5354. The near-exact dependence on the inverse of the slope would be expected for channels near bedload equilibrium in the sense of Parker (1978), but the coefficient in the relation is much larger than would be predicted from Parker’s analysis, which fixes the Shields stress at 1.2 times the critical value, or about 0.06. The observed average Shields stress on the HTC fan is approximately 1.2, so it is about 20 times higher than in the Parker theory. This is not surprising, given that theory is for bedload-dominated transport of a single grain size in equilibrium channels, whereas at HTC the transport is dominantly in suspension, a wide range of grain sizes is present, and the
channels probably do not reach equilibrium in many cases. In modeling the fan, we used the observed average Shields stress of 1.2. We also note that, although we did not observe any downstream change in the nondimensional stress, for safety reasons we could not sample the channels in the lower part of the fan. We believe that the approximation of constant Shields stress is reasonable for the upper part of the fan, however.
STAGE 1: THEORETICAL PROGRAM

Objective: Relation for Upstream Fan Slope

As explained above, theoretical formulations provide a means of evaluating the influence of effects that, because of scale, cannot be adequately represented in a scale model, and also of extrapolating model results to conditions not explicitly examined in the model study. Some of the key questions that might be addressed by a physically based theoretical model include the following: (1) how do the large quantities of suspended fines affect sediment transport and flow resistance on the fan? (2) what is the appropriate channel closure to apply to the system (i.e. what sets the distribution of flow widths among the braided channels at any point on the fan)? (3) how can downstream sediment sorting be included, and how strong an effect does it have on profile concavity? and (4) how do channel width and depth vary dynamically during incision events? This last question is of particular importance in determining the effects of discharge oscillation on the overall slope of the system.

A complete answer to the above questions is beyond the scope of the present analysis. This notwithstanding, an attempt will be made here to sketch out a first theoretical model of fan evolution under the condition of a constant rate of bed aggradation that captures much of the essence of the phenomenon. This model should provide a basis for an extension that does indeed address the above questions. The single greatest drawback of the model presented here is the inability to treat downstream fining associated with sediment mixtures. The reason for this is the inability to specify a model of suspension-dominated sediment transport that could be expected to be accurate on a grain size-specific basis for heterogeneous material. With this in mind, the model assumes a uniform sediment size $D$, but otherwise assumes conditions that are sufficiently general to encompass field conditions at Hibbing Taconite Company. The analysis is directed toward the determination of a relation for upstream fan slope $S_u$ near the launders.

Formulation

The configuration of Figure 4 is considered. The vertex of the basin is supplied with sediment of size $D$ at the feed rate $Q_{so}$, and water at the rate $Q_w$ at the upstream end, causing a fan to develop within the basin. The elevation of the downstream margin is raised at precisely the rate $v_b$ necessary to keep sediment from exiting the system; water is allowed to overflow freely. The fan develops as a series of braided channels. At any given time only a portion of the fan is inundated, but the entire fan is constantly reworked due to channel migration and avulsion. Here only quantities that are time-averaged over
many fan reworkings are considered, allowing the assumption of axial symmetry. The mean volume sediment transport rate per unit width in the channels is denoted as $q$. Fan width at radial distance $r$ from the vertex $B_f$ is given by the relation

$$B_f = r \theta$$  \hspace{1cm} (28)

The total sediment transport across the fan width is given by $Q_s$, where

$$Q_s = q B_{ac}$$  \hspace{1cm} (29)

and $B_{ac}$ denotes the total width of the active channels along an arc perpendicular to the radial coordinate. The Exner equation of sediment conservation takes the form

$$(1 - \lambda_p)B_f \frac{\partial \eta}{\partial t} = - \frac{\partial q B_{ac}}{\partial r}$$  \hspace{1cm} (30)

where $t$ denotes time, $\eta$ denotes bed elevation and $\lambda_p$ denotes bed porosity, here assumed to be constant. The condition of constant vertical aggradation at rate $v_b$ can be expressed as

$$\frac{\partial \eta}{\partial t} = v_b.$$  \hspace{1cm} (31)

The two boundary conditions on the Exner equation are as follows;

$$Q_s|_{r=0} = Q_{so}, \quad Q_s|_{r=L} = 0.$$  \hspace{1cm} (32)

That is, the upstream sediment transport rate should be equal to the feed rate and the downstream sediment transport rate should be zero. The latter boundary condition insures that the solution domain has the character of a tailings basin.

The Exner relation (30) can be reduced with the aid of (28), (29), (31) and (32) to yield the following forms;

$$\frac{Q_s}{Q_{so}} = 1 - \hat{r}^2, \quad \hat{r} = \frac{r}{L}$$  \hspace{1cm} (33a,b)

and

$$v_b = \frac{Q_{so}}{(1 - \lambda_p) \theta L^2}. \hspace{1cm} (34)$$

The first two of the above relations indicate that the sediment transport should decline parabolically down the fan to zero at the downstream end as sediment deposits.
The third was introduced as (7) earlier; it is essentially a geometric relation specifying the aggradation rate that must result if all the sediment is to deposit within the basin.

In the present simple theory quasi-normal flow is assumed in the braid branches, so that the relation between boundary shear stress $\tau_b$, channel depth $h$ and channel bed slope $S$, where

$$
S = -\frac{\partial \eta}{\partial r}
$$

is given by

$$
\tau_b = \rho gh S
$$

Further progress requires the introduction of internal relations for a) sediment transport, b) resistance and c) composite self-formed channel width. Here the sediment transport relation is assumed to have the general form

$$
q^* = \alpha_c \alpha_s (\tau^* - \tau_c^*)^n
$$

where $\tau^*$ denotes the dimensionless Shields stress and $q^*$ denotes the dimensionless Einstein sediment transport, defined respectively as

$$
q^* = \frac{q}{\sqrt{RgD}} \quad \tau^* = \frac{\tau_b}{\rho RgD}
$$

and $\tau_c^*$ denotes the critical value of the Shields stress at the threshold of motion. Here $\alpha_s$ and $n$ are dimensionless parameters; $\alpha_c$ is a calibration coefficient, as explained below. In the case of the Meyer-Peter/Muller sediment transport relation, for example, $\alpha_s = 8$, $\alpha_c = 1$ and $n = 1.5$.

The relation for resistance is assumed to be of the form

$$
\tau_b = \rho C_f u^2, \quad C_f^{1/2} = \alpha_c (\frac{h}{D})^p
$$

Here $C_f$ is a dimensionless resistance coefficient; the form of (39b) is that of a generalized Manning-Strickler relation with dimensionless coefficients $\alpha_c$ and $p$. The form of the Manning-Strickler relation itself is recovered for the choice $p = 1/6$; a Chezy-type relation is obtained for the choice $p = 0$.

The final relation is that for the cumulative width of the active channels. It would seem most obvious to introduce an assumption that directly involves $B_{ac}$. This does not, however, turn out to be the most effective way to specify width. There are both theoretical and empirical grounds for seeking instead a relation between the depth, slope and grain size of the anabranches themselves, i.e. a relation for channel form. In particular, a relation of the following type is sought;

Hibbing Taconite Co. Final Report 30
\[
\left(\frac{h}{D}\right)^m = \alpha_b
\]

(40)

where \(m\) and \(\alpha_b\) are again dimensionless coefficients.

**Evaluation of the Internal Relations using Field Data**

The field data necessary to evaluate the internal relations of the theory were collected during the course of the field monitoring program, and were reported earlier in the report. It was found from Figure 14 that \(m\) could be set equal to 1.05 and \(\alpha_b\) equal to 1.7 in (40). The relation can alternatively be cast in terms of the Shields stress; a similar regression then yields \(m = 1\) and \(\alpha_b = 2.4\). The form so obtained was the strongest of all the field relations, so the latter coefficients are not modified hence. As noted above, the resistance relation shows considerable scatter, but can be fitted almost equally well to a Chezy-type relation with \(\alpha_r = 7.7\) and \(p = 0\) or a generalized Manning-Strickler relation with \(\alpha_r = 4.5\) and \(p = 0.12\). Here the Manning-Strickler relation is chosen.

The most problematic relation is the one for sediment transport. Using the evaluation \(m = 1\) in (40), the expression for the Shields stress within a channel can be rewritten in the following simple form using (36) and (38b);

\[
\tau^* = \frac{hS}{RD} = \frac{\alpha_b}{R}
\]

(41)

Recalling that the submerged specific gravity of the sediment \(R\) takes the constant value of 2.0 for tailings at Hibbing Taconite, (41) indicates that within a range of scatter all the braid channels have a constant Shields stress of 1.2. If this is true, then it should be difficult to extract the range of values of \(\tau^*\) necessary to determine (21) from field data.

The relevant diagram is shown as Figure 12b. While, as expected, the data cloud does not exhibit a strong trend, (37) defines a relation passing through the middle of the cloud if the choices \(n = 1.5\), \(\alpha_s = 8\) and \(\alpha_c = 1\) are made. These are in fact the original values from the Meyer-Peter/Muller equation itself. It is not meant to imply that the Meyer-Peter equation is being used here, as (37) represents an empirical equation for total (bedload plus suspended load) rather than bedload alone.

As discussed in the section on field data, there is reason to believe that the measurements underestimate sediment transport, so that \(\alpha_c\) may have to be adjusted upward in order to obtain accurate results. (It is also possible that the actual sediment feed to the basin was low at the time of measurement due to a partial mill shutdown.) In addition, arguments will be presented later for values of \(n\) larger than 1.5. The values for \(\alpha_s\), \(\alpha_c\) and \(n\) quoted above should be considered tentative at best.
Reduction of the Relations

In this section the exponent \(m\) is set equal to 1, but the other coefficients \(\alpha_b, \alpha_s\) and \(\alpha_r\) and exponents \(p\) and \(n\) are taken to be arbitrary. After some work the above relations can be reduced to the following forms for the slope \(S\) and the active width \(B_{ac}\) of the fan:

\[
S = \{(\alpha_c \alpha_r)^{-1} \alpha_b^{(3/2+p)} \alpha_r (\frac{\alpha_b}{R} - \tau_c^*)^{-n} j^{1/(1+p)} (\frac{Q_{so}}{Q_w}) (1 - j^2)^{1/(1+p)} \}
\]

\[
\frac{B_{ac}}{D} = \alpha_b^{-(3/2+p)} \alpha_r^{-1} \bar{Q}_w S^{1+p}
\]

where the parameter \(\bar{Q}_w\) is defined in (5b). The relation for upstream slope in particular takes the form

\[
S_u = \{(\alpha_c \alpha_r)^{-1} \alpha_b^{(3/2+p)} \alpha_r (\frac{\alpha_b}{R} - \tau_c^*)^{-n} j^{1/(1+p)} (\frac{Q_{so}}{Q_w})^{1/(1+p)} \}
\]

Test of Relation for Upstream Slope against Field Data and Adjustment

It will be recalled that (42) was derived under the condition of uniform sediment size \(D\), whereas characteristic sediment size in the West Basin \#1 is continuously changing in the downstream direction. With this in mind, (42) cannot be expected to be generally valid. On the other hand, the bed sediment in the vicinity of the upstream end of the fan may be sufficiently uniform to allow the application of (44) to the determination of the upstream slope \(S_u\).

To this end \(\{\alpha_s, \alpha_r, \alpha_b, n, p\}\) are first set equal to \(\{8.0, 4.5, 2.4, 1.5, 0.12\}\); in addition the critical Shields stress \(\tau_c^*\) is set equal to 0.035. The correct value for \(R\) for the Hibbing fan is 2.0, and \(Q_w\) takes the value 265 ft\(^3\)/s (7.5 m\(^3\)/s). Data for 1992 indicate a total tailings feed rate of 8.13 ft\(^3\)/s (0.230 m\(^3\)/s); 93\% of the tailings were introduced into West Area \#1, and 68\% of the tailings were deposited there. It can be reasonably assumed that all of the material in excess of 150 \(\mu\)m introduced into West Area \#1 deposited within it. This size range, which constitutes about 23\% of the feed material, might also be thought to be the range of sizes actively exchanging with the bed. The effective value of \(Q_{so}\) for use in (42) is then 0.93 x 0.23 x 8.13 = 1.74 ft\(^3\)/s (0.049 m\(^3\)/s). The value of \(S_u\) so obtained with a calibration coefficient \(\alpha_c = 1\) is equal to 0.0194, which compares with a measured value of 0.0131.

Perfect agreement with the measured value (to the extent that the input parameters are accepted as valid) can be obtained by increasing the calibration coefficient \(\alpha_c\) to 1.55. There is some justification for such an increase, due to the suspected under-measurement
of sediment load in the field alluded to earlier. Equally good agreement, however, could be obtained by setting the exponent $n$ in the transport relation equal to the higher value of 2. A value of $\alpha_c$ of 1 yields a value of $S_u$ of 0.0181; perfect agreement with the measured value is obtained for $\alpha_c = 1.44$. The relevance of this is explained below.

In summary, it appears that the theory can provide a reasonable relation for the upper fan slope $S_u$, although some calibration may be required at this point. The general form of the equation so obtained for upper fan slope can be written as

$$S_u = \beta \left( \frac{Q_{soc}}{Q_m} \right)^{1/(1+p)} \quad (45)$$

where $\beta$ is a coefficient and $Q_{soc}$ is an appropriately chosen "coarse" sediment feed rate that reflects the sediment sizes likely to deposit on the bed at the upstream end of the fan.

The form of (45) is used later in the analysis to evaluate various options for tailings management; for simplicity $p$ is set equal to 0. It is of value to note that according to (45), upper fan slope is completely independent of grain size $D$. This result is probably not entirely correct, but may be valid as a first approximation.

**Modification of the Formulation Using an Existing Sediment Transport Relation**

An attempt was made to improve upon the empirical sediment transport relation based on field data. In order to do this, the Van Rijn (1984a; 1984b) formulation was chosen, as it is one of the better relations in the literature for the case of dominant suspension. In order to implement the theory, the Van Rijn relation, which is rather complicated, was fitted to laws of the form of (37). The precise fit obtained varied with the grain size distribution chosen, but in general $n$ was found to be close to 2.0 and $\alpha_c$ was found to vary between 42 to 45. These choices underestimate upstream fan slope, but the underestimate is not grossly out of order. A variation on (45) was thus developed as a theoretical means of evaluating options for tailings management. The advantage of this scheme is that it brings in at least some dependence of fan slope on grain size.
STAGE 1: APPLICATION TO WEST AREA #1

The main goal of this project has been to obtain a design equation that could be used to evaluate various scenarios for changing the slope of the HTC fan. The results of the Stage 1 analysis are given in this section. In particular, we give some examples of the results of slope changes due to a variety of factors and evaluate their effects on the future evolution of the fan. It must be emphasized that all the results quoted here are predicated on the use only of West Area #1 to store tailings. It is assumed that the conditions prevailing in 1992, when 68% of the tailings deposited in West Area #1, are continued indefinitely. The results are thus illustrative but highly conservative. An analysis of the complete basin follows.

Scenario Analysis

At this point, it is not possible to provide a single precise value for the change in slope resulting from a given imposed change on the fan system. Hence, our strategy has been to develop a set of models that we believe bracket the range of likely slope responses. In the scenarios examined here, we have evaluated the changes in slope for all the models, and then report the maximum and minimum changes. We also report maximum and minimum estimates of the difference in elevation of the apex of the fan after 30 years beyond 1995, relative to continuing operation under present conditions, resulting from the imposed changes. The basis for these estimates is explained in the “Framework” section; note that the geometry used in the calculations is that of West Area #1. The results of the scenario analysis are given in Table 1.

The details of the models we have developed are presented in the Theoretical Program section. Recapitulating briefly, the models are: a set of relations derived directly from the experimental data; a numerical model based on the empirical transport and resistance relations derived from the field data; and a numerical model based on the transport relation of Van Rijn (1984a; 1984b), which includes sediment suspension and thus is more sensitive to grain-size changes than the field-based numerical model.

Changes in water discharge. Slope tends to change inversely to water discharge. In general both numerical models predict larger effects than the experimentally based slope relations. For a discharge reduction of 25%, the experimental results predict an increase in slope of about 20%, while the numerical models give about 30%. A 50% increase reduces the slope by about 22% based on the experimental results and about 31% based on the numerical models. A doubling of discharge (100% increase) likewise yields predicted slope decreases of about 35% and 46% from the experimental and numerical models, respectively.
Since the rate of deposition in the basin is unaffected by changes in water discharge, all of the change in estimated elevation of the fan apex 30 years into the future results from changes in the slope of the upper part of the fan. We estimate that the 25% discharge reduction would raise the apex elevation between 11 and 18 ft higher than if the basin were operated with no changes. The 50% increase would reduce the apex elevation between 13 and 18 ft, while the 100% increase would reduce the elevation between 21 and 28 ft.

**Installation of hydroseparator.** This scenario is based on the assumption that the hydroseparator would remove all material coarser than 150 μm (#100 mesh). This would reduce the range of sizes fed into the basin, the mean size of the feed, and the overall rate of sediment supply to the basin. These effects work to reduce the slope of the fan. The range of slope decreases predicted from the experimental data (34%-64%) is much wider than for the changes in discharge, and the maximum decrease (64%) is significantly greater than predicted from the field-based numerical model (36%) or the Van Rijn numerical model (53%). Both of these differences arise because the numerical model has no grain-size dependence (and hence responds only to the decrease in sediment supply rate) while some of the experimental data sets did show significant grain-size dependence. The Van Rijn numerical model includes a modest grain-size dependence and is thus closer to the maximum result predicted from the experimental data.

Installation of the hydroseparator affects the final predicted fan elevation both through the slope changes discussed above and through reduction in total rate of sediment supply to the basin, which reduces the aggradation rate. Considering both effects, we estimate a reduction in fan apex elevation of between 46 and 61 ft over 30 years. In our scenario analysis, we have assumed that the sediment taken out by the hydroseparator is removed permanently from the basin. This is important because most of the reduction in estimated apex elevation is caused by the reduction in aggradation rate rather than the reduction in slope.

**Combinations of effects.** Because the models are nonlinear, predictions for combinations of more than one type of change cannot be derived simply by adding the individual effects. All of our analyses suggest that addition of the hydroseparator would still cause a reduction in slope even if combined with a 25% reduction in water discharge; the minimum predicted slope decrease in this scenario is 22%, and the maximum is 57%. Again both numerical models give results between the extremes predicted from the experimental data. However, for a 50% discharge reduction, both the minimum experimentally based prediction and the field-based numerical model agree in giving no net change in slope (i.e. the slope reduction due to the hydroseparator is cancelled out exactly by the increase due to the discharge reduction). The result for the suspension-based numerical model for this case is also quite low (6.6%). The maximum predicted decrease from the experimental results is 45%.

In all of these scenarios, the predicted elevation of the fan apex after 30 years is substantially lower than it would be if the basin were operated in its present mode; as shown in Table 1, the minimum reduction in elevation is 28 ft, even for the case with no
slope change. This is because the hydroseparator always reduces the overall sediment supply rate to the fan and thus lowers the aggradation rate. This has a larger effect than any predicted slope changes. Again we stress that this is based on the assumption that whatever the hydroseparator removes is taken permanently out of West Area #1. If instead the separated sediment were simply trucked to another part of West Area #1 then the benefit would be reduced, in that space otherwise available for the storage of non-hydroseparated material would be occupied by hydroseparated material.

Discharge oscillation

One of the goals of the experimental program was to evaluate the efficacy of surging the water discharge as a low-cost means of reducing slope. The hope was that there would be a range of oscillation frequencies for which the higher discharge might predominate and reduce the slope, even though the discharge averaged over a complete oscillation period was unchanged. As discussed in the section on experimental results, the experiments suggest that this is indeed the case. Unfortunately, due to limitations in our ability to produce accurate Froude-scale models of the HTC tailings basin, it is not possible to determine an optimal oscillation frequency $F_o$ for the field case from the model results. Field experimentation with oscillation frequencies will be necessary to determine the most favorable operational procedure. Observations of morphological changes on the fan slope and in the channelization of the flows in particular can be used in a simple way to gauge an effective oscillation frequency. As in the experiments, the high stage discharge must be maintained long enough for deep channels to form over a significant proportion of the fan surface, say 30% of basin length $L$, and the low stage must not be maintained so long that significant infilling and overtopping of the high-flow channels occurs. In effect this is like using the high-stage discharges to flush out and maintain the "summer channel" tested with limited success in 1993.

The scenario analyses suggest that discharge increases can significantly reduce the overall fan slope and the expected elevation of the fan apex in the future. For instance, a 50% increase in discharge produces a gain of between 13 and 18 ft of vertical distance over 30 years (Table 1). The experiments indicate that with the optimal oscillation frequency the overall fan slope approaches that associated with the peak discharge. However, average pumping costs are not increased because the average discharge remains the same. Hence, discharge oscillation appears to be a cost-effective and relatively simple means of extending the operating lifetime of the basin, with or without the additional benefits of a hydroseparator.

Uncertainty in the predictions

The most important source of uncertainty in the predictions given above is lack of understanding of how grain-size distribution influences fan slope in systems such as the HTC fan that have a wide range of sizes in transport. It is still not possible to account completely for all effects of a wide size spectrum in either theoretical or experimental models. Based on quantitative exploration of the effects of the fine tail of the size
distribution on the transport dynamics, we believe that, if anything, we have probably underestimated the magnitudes of slope reduction that can be expected in the scenarios discussed above, especially those involving the use of a hydroseparato and consequent reduction in the coarse fraction of the sediment feed. Hence, the projections given above for reductions in slope and apex elevation should be interpreted as conservative in that they probably err on the side of being too small.
STAGE 2: GENERALIZED METHOD: ENTIRE TAILINGS POND

The Proposed Scheme

After completion of the above work, further consultation with the personnel of Hibbing Taconite Company and Barr Engineering allowed for the development of a predictive model for the entire tailings pond. In accordance with instructions from Hibbing Taconite Company, the schematization of Figure 1 was considered. Hydroseparated material is deposited in a pie-shaped region at the upstream end of the tailings pond. A channel with width $B_e$ and length $L_e$ is maintained within this zone to allow non-hydroseparated material to reach the distal part of the fan, where it may deposit.

Since the bed of the channel is to be covered with non-hydroseparated material, the hydroseparated material must fill the fan at least up to the level of the channel if the latter is not to overspill and avulse. This consideration places a maximum bound on channel length $L_e$. If desired, some of the hydroseparated material can be stacked to an elevation $z$ above the channel, as shown in Figure 1. This would allow for a shorter channel. In computing stacking height the side slopes are set equal to 1:3 here.

In implementing the scheme, there would be only one channel at a given time through the zone being filled with hydroseparated material. In order to promote uniform filling of the tailings pond, however, it would be desirable to switch the location of the channel between the West Area #1 and the East Area from time to time.

Analysis

The same techniques as those introduced in the section Theoretical Program above are used to solve for the following parameters under the condition of constant bed aggradation speed $\nu_b$; constant channel slope $S_c$ in the region $r < L_e$ of Figure 1 and varying fan slope $S_f(r)$ in the region $L_e < r < L_s$ of Figure 1. The analysis strictly applies to uniform sediment with characteristic size $D$. The results are found to be

$$S_c = \left\{ R \alpha_r^{1/(1.5+p)} \left( \frac{Q_w}{\sqrt{gDDB_e}} \right)^{-1/(1.5+p)} \left[ \tau_e + (\alpha_c \alpha_s)^{-1/n} \left( \frac{Q_w}{\sqrt{RgDDB_e}} \right)^{1/n} \right] \right\}^{(2+2p)/(2+2p)}$$

(46)
The slope $S_s$ of the slimes region $L_s < r < L$ is taken to be 0.0023 here, a value slightly different from the value of 0.00225 assumed previously.

The above relations for $S_o$, $S_f(r)$ and $S_s$ can be integrated to yield the elevation profile $\eta(r)$. In determining a filling time for the entire pond, the constant of integration (in $r$) is evaluated by imposing the following relation at the launders:

$$\eta(r = 0) = 1635 \text{ ft}$$

The profile resulting from this boundary condition will be the one when the tailings pond has attained its maximum filling. The parameters $v_b$ and $L_s$ are computed based on modified versions of (12a) and (12b);

$$v_b = \frac{Q_{soc}}{\frac{1}{2}(1 - \lambda_{pct}) \theta (L_e^2 - L_s^2)} = \frac{Q_{soc} - Q_{soc}}{\frac{1}{2}(1 - \lambda_{pct}) \theta (L_e^2 - L_s^2)}$$

$$L_s = \left(\frac{L_e^2 + \varphi L_e^2}{1 + \varphi}\right)^{1/2}, \quad \varphi = \frac{(Q_{soc} - Q_{soc})(1 - \lambda_{pct})}{Q_{soc}(1 - \lambda_{pct})}$$

In the above relations $\theta$ is chosen such that $A_p = \frac{1}{2} \theta L_e^2$, where $A_p$ is total tailings pond depositional area and $L$ is average radius from launders to outer dike.

In implementing the above scheme the following parameters were chosen.

$$L = 12772 \text{ ft (3893 m)}$$
$$A_p = 2.14 \times 10^8 \text{ ft}^2 (1.99 \times 10^7 \text{ m}^2)$$
$$\eta_{Lo} = 1625 \text{ ft (495.3 m)}$$

These values are appropriate for the tailings basin as a whole: different values introduced earlier must be used to describe only West Area #1. In addition, the following parameters were selected in light of previous work reported here.

$$Q_w = 265 \text{ ft}^3/\text{s (7.5 m}^3/\text{s)}$$
$$\lambda_{pct} = 0.555$$
$$R = 2.0$$
$$\tau_e^* = 0.035$$
\[ \alpha_a = 8 \]
\[ \alpha_r = 4.5 \]
\[ \alpha_b = 2.4 \]
\[ p = 0.12 \]

The parameters \( Q_{so} \) and \( Q_{soc} \) must be selected according to the design feed rate of 8.1 ft\(^3\)/s (0.230 m\(^3\)/s) of tailings in the absence of hydroseparation and the fraction of material hydroseparated. The truncation of the grain size distribution due to hydroseparation is determined by a “cut” grain size \( D_c \) such that all material coarser is removed from the grain size distribution of the tailings. The grain size \( D \) in (46) is taken to be the geometric mean size of the material greater than 40 \( \mu \)m but finer than \( D_c \). The porosity of deposits formed from the material in this size range \( \lambda_{psa} \) is calculated as a weighted average of the value 0.385 for material in excess of 150 \( \mu \)m and the value 0.452 for material between 150 \( \mu \)m and 40 \( \mu \)m, in the manner described by (22). The parameters \( B_e \) and \( L_e \) can be chosen at the discretion of the user in order to achieve an appropriate tailings pond design.

**Calibration**

The above model was calibrated using appropriate values for West Area #1 for 1992 as described above. Calibration was performed using the values, \( n = 1.5 \) and \( 2.0 \), yielding the values \( \alpha_e = 2.32 \) and 2.16, respectively. These values differ from the one quoted in *Testing Relation for Upstream Slope Against Field Data and Adjustment* because they were calibrated against the entire profile of West Area #1, whereas the latter one was calibrated only against the upstream slope near the launders. In light of the fact that it was found to provide an accurate approximation of the van Rijn (1984a, b) relation for sand-bed streams, the exponent 2.0 was selected for the model. The product \( \alpha_e \alpha_e \) thus takes the value 17.28.

The calibrated bed profile is compared with a recent measured one in Figure 15. The agreement is generally quite good. The calibrated profile is not as strongly upward concave as the measured one. This is likely due to the inability of the model described above to describe grain sorting on the fan.

**Implementation: Stacking of Hydroseparated Tailings**

When applied to the entire tailings pond, the ultimate elevation profile is computed from the slope profile by integrating and applying (48). As shown in Figure 1, the difference between the predicted ultimate bed and the existing bed defines the volume available for filling with sediment. The time for filling is then computed by dividing the available volume for storing non-hydroseparated material by its feed rate corrected for porosity. This time is measured in years from 1995.

The method fails if the available volume of hydroseparated material is insufficient to fill the volume below the ultimate grade line of the channel, as shown in Figure 1.
this case the channel must avulse and fill with non-hydroseparated material the zone reserved for hydroseparated material. The problem can be rectified by choosing a shorter channel length $L_e$. In the event that the hydroseparated tailings more than fill the allotted space below channel grade, they may be stacked above the channel up to height $z$, as illustrated in Figure 1. Let $V_s$ denote the volume of material (tailings + pores) available for stacking above the grade line of the channel and $m$ denote the side slopes of the stacked pile, here assumed to be equal to 1:3. The relation between $V_s$ and $z$ is found to be

$$V_s(z) = \frac{1}{2} \theta L_e^2 z - \frac{1}{2} (2 + \theta) L_e mz^2 + \frac{1}{3} (2 + \frac{1}{2} \theta) m^2 z^3$$  \hspace{1cm} (51)

The maximum possible height of stacking is given by $z_{\text{max}}$, where

$$z_{\text{max}} = \frac{\theta L_e}{4 + \theta m}$$  \hspace{1cm} (52)

and the maximum stacking volume $V_{s\text{max}}$ is given by

$$V_{s\text{max}} = \frac{\theta^2 (6 + \theta)}{6 (4 + \theta)^2 m} L_e^3$$  \hspace{1cm} (53)

**Spreadsheet Program: Application to Design**

The above scheme was implemented as a Microsoft Excel 5.0 spreadsheet workbook. Both a copy of the spreadsheet on disc and a user’s manual have been supplied to Hibbing Taconite Company with this report. The program was implemented for Schemes 2 - 7 described earlier, but this time for the entire tailings pond. The results are shown in Table 2. All options were run assuming a cutoff size $D_c$ for hydroseparation of 150 µm (#100 mesh). In addition, the width of the channel through the hydroseparated material placed in the tailings pond $B_e$ was assumed to be 15 ft. Both these parameters, however, and many more in the spreadsheet itself, can be freely changed by the user. Schemes 5, 6, and 7 show two suboptions, one with less stacking and one with more. The height of stacking, rather than being a parameter set by the user, is a computed parameter determined by the user’s choice for such input parameters as channel length and width, hydroseparation cutoff size, water discharge, etc. The user may iterate, however, to find the input parameters appropriate for a desired stacking height.

As can be seen by comparing Tables 1 and 2, the Stage 2 results exhibit the same trends as the Stage 1 analysis. The null state (continuation with no change) is found to have a predicted filling time of 6.3 years from 1995. Higher water discharges lower the bed elevation near the launders in 30 years (Table 1), and thus increase the expected filling time (Table 2); hydroseparation has the same effect. The Stage 2 analysis, however, offers specific indications as to whether a scheme is feasible or not. It is seen that a doubling of
discharge leads to a filling time of 24 years, a value not too far from the desired minimum value of 30 years. Hydroseparation at 150 μm with no change in water discharge increases the filing time to over 30 years; almost as good results are achieved with hydroseparation and a 25% decrease in water discharge. As noted earlier, surging may be a way to decrease the mean discharge (and thus mean energy consumption for pumping) further with little adverse effect on tailings pond filling time. A field test is recommended before implementing a surging scheme, however.

The results of the Stage 2 analysis are thus encouraging in regard to the extension of the lifetime of the tailings pond to at least 30 years beyond 1995. Other options such as different cut-sizes of hydroseparation, channel lengths, water discharges etc. can be explored with the software supplied with this report.

It should be pointed out that the prediction methods presented in this report are the first of their kind, and are thus subject to some degree of error. It is hoped, however, that the degree of error has been reduced by a) the use of a physically-based theory to explain fan evolution and b) the determination of the internal relations of the theory by means of direct field measurements at the tailings pond itself and c) a calibration process that yielded a coefficient of calibration in a range expected from more general considerations.
REFERENCES CITED


Table 1
Results from Stage 1 Analysis (West Basin #1 Only)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Action Taken</th>
<th>Experimental Findings</th>
<th>Numerical Model (Field Transport)</th>
<th>Numerical Model (van Rijn Transport)</th>
<th>Benefit, ft (30 yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>% Change Predicted for Upper Fan Slope</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Maximum**</td>
<td>Minimum**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>No change****</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Reduce Qw 25%</td>
<td>19.8</td>
<td>18.8</td>
<td>29.6</td>
<td>-18</td>
</tr>
<tr>
<td>3</td>
<td>Increase Qw 50%</td>
<td>-22.4</td>
<td>-21.6</td>
<td>-30.6</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>Increase Qw 100%</td>
<td>-35.2</td>
<td>-34.0</td>
<td>-46.4</td>
<td>28</td>
</tr>
<tr>
<td>5</td>
<td>Install Hydroseparator*</td>
<td>-64.4</td>
<td>-34.0</td>
<td>-36.4</td>
<td>61</td>
</tr>
<tr>
<td>6</td>
<td>Hydroseparator; Qw-25%</td>
<td>-57.4</td>
<td>-21.6</td>
<td>-30.6</td>
<td>58</td>
</tr>
<tr>
<td>7</td>
<td>Hydroseparator; Qw-50%</td>
<td>-45.0</td>
<td>0.0</td>
<td>0.0</td>
<td>51</td>
</tr>
</tbody>
</table>

Benefit given in terms of fan head elevation (ft) after 30 years time beyond 1995, measured relative to the null case
Benefit is defined as positive for a reduction in the predicted fan head elevation

* Hydroseparation is assumed to effectively remove all material coarser than 150 microns (#100 mesh).
** Maximum prediction is based on experiments with sand of 160 microns and finer and all coal experiments and therefore includes both grain-size and sediment load effects.
*** Minimum prediction is based on experiments with sand coarser than 160 microns.
**** The predicted fan elevation at the launders (fan head) is 1744 ft for the null case.

Note: Slopes of experimental fans with sand coarser than 160 microns were independent of grain-size.
Model grain sizes in excess of 160 microns scale as gravels in natural systems.
Table 2
Results from Stage 2 Analysis (Entire Tailings Pond)

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Action Taken</th>
<th>Channel Length** ( L_c ) (ft)</th>
<th>Filling Time (yrs beyond 1995)</th>
<th>Stacking Height (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No Change</td>
<td>0</td>
<td>6.3</td>
<td>N/A</td>
</tr>
<tr>
<td>2</td>
<td>Reduce Qw 25%</td>
<td>0</td>
<td>1.2</td>
<td>N/A</td>
</tr>
<tr>
<td>3</td>
<td>Increase Qw 50%</td>
<td>0</td>
<td>16.9</td>
<td>N/A</td>
</tr>
<tr>
<td>4</td>
<td>Increase Qw 100%</td>
<td>0</td>
<td>23.8</td>
<td>N/A</td>
</tr>
<tr>
<td>5a</td>
<td>Install Hydroseparator*</td>
<td>6600</td>
<td>32.6</td>
<td>4</td>
</tr>
<tr>
<td>5b</td>
<td>Install Hydroseparator*</td>
<td>5750</td>
<td>34.5</td>
<td>30</td>
</tr>
<tr>
<td>6a</td>
<td>Hydroseparator*; Qw-25%</td>
<td>6500</td>
<td>28.4</td>
<td>6</td>
</tr>
<tr>
<td>6b</td>
<td>Hydroseparator*; Qw-25%</td>
<td>5500</td>
<td>30.2</td>
<td>34</td>
</tr>
<tr>
<td>7a</td>
<td>Hydroseparator*; Qw-50%</td>
<td>6500</td>
<td>26.1</td>
<td>5</td>
</tr>
<tr>
<td>7b</td>
<td>Hydroseparator*; Qw-50%</td>
<td>5500</td>
<td>27.8</td>
<td>30</td>
</tr>
</tbody>
</table>

*Hydroseparation is assumed to effectively remove all material coarser than 150 microns (#100 mesh).

**Channel width \( B_c = 15 \) ft in all cases where \( L_c > 0 \).
<table>
<thead>
<tr>
<th>Series</th>
<th>Run</th>
<th>D50 microns</th>
<th>Qw cm³/s</th>
<th>Qso cm³/s</th>
<th>Qso/Qw</th>
<th>Ave Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand I</td>
<td>1</td>
<td>550</td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>160</td>
<td>475</td>
<td>7.55</td>
<td>0.016</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120</td>
<td>7.55</td>
<td>0.063</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>110</td>
<td>475</td>
<td>7.55</td>
<td>0.016</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.095</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120</td>
<td>7.55</td>
<td>0.063</td>
<td>0.089</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>70</td>
<td>475</td>
<td>7.55</td>
<td>0.016</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.091</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120</td>
<td>7.55</td>
<td>0.063</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>100</td>
<td>475</td>
<td>7.55</td>
<td>0.016</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.069</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120</td>
<td>7.55</td>
<td>0.063</td>
<td>0.099</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>60</td>
<td>475</td>
<td>5.28</td>
<td>0.011</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>5.28</td>
<td>0.023</td>
<td>0.073</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120</td>
<td>5.28</td>
<td>0.044</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>160</td>
<td>475</td>
<td>7.55</td>
<td>0.016</td>
<td>0.052</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120</td>
<td>7.55</td>
<td>0.063</td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>70</td>
<td>466</td>
<td>7.55</td>
<td>0.016</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>225</td>
<td>7.55</td>
<td>0.034</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>122</td>
<td>7.55</td>
<td>0.062</td>
<td>0.072</td>
</tr>
<tr>
<td>Sand II</td>
<td>9</td>
<td>160</td>
<td>225</td>
<td>3.77</td>
<td>0.017</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>225</td>
<td>7.55</td>
<td>0.034</td>
<td>0.084</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>225</td>
<td>15.09</td>
<td>0.067</td>
<td>0.107</td>
</tr>
<tr>
<td>Sand III</td>
<td>10</td>
<td>120</td>
<td>230</td>
<td>3.77</td>
<td>0.016</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120</td>
<td>3.77</td>
<td>0.031</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>465</td>
<td>7.55</td>
<td>0.016</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>465</td>
<td>15.09</td>
<td>0.032</td>
<td>0.065</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>15.09</td>
<td>0.066</td>
<td>0.093</td>
</tr>
<tr>
<td>Sand V</td>
<td>11</td>
<td>160</td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.083</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.072</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.068</td>
</tr>
<tr>
<td>Sand IV</td>
<td>12</td>
<td>270</td>
<td>465</td>
<td>7.55</td>
<td>0.016</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120</td>
<td>7.55</td>
<td>0.063</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>460</td>
<td>465</td>
<td>7.55</td>
<td>0.016</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>230</td>
<td>7.55</td>
<td>0.033</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>120</td>
<td>7.55</td>
<td>0.063</td>
<td>0.130</td>
</tr>
</tbody>
</table>
Table 3, cont.

<table>
<thead>
<tr>
<th>Series</th>
<th>Run</th>
<th>D50</th>
<th>R</th>
<th>Qw cm³/s</th>
<th>Qso cm³/s</th>
<th>Qso/Qw</th>
<th>Ave Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal I</td>
<td>1</td>
<td>125</td>
<td>0.50</td>
<td>321</td>
<td>2.77</td>
<td>0.009</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>267</td>
<td>2.77</td>
<td>0.010</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>214</td>
<td>2.77</td>
<td>0.013</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>160</td>
<td>2.77</td>
<td>0.017</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>550</td>
<td>0.50</td>
<td>321</td>
<td>2.77</td>
<td>0.009</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>267</td>
<td>2.77</td>
<td>0.010</td>
<td>0.018</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>214</td>
<td>2.77</td>
<td>0.013</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>160</td>
<td>2.77</td>
<td>0.017</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>550</td>
<td>0.50</td>
<td>419</td>
<td>2.77</td>
<td>0.007</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>214</td>
<td>2.77</td>
<td>0.013</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>125</td>
<td>0.35</td>
<td>419</td>
<td>2.77</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>321</td>
<td>2.77</td>
<td>0.009</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>214</td>
<td>2.77</td>
<td>0.013</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>108</td>
<td>2.77</td>
<td>0.026</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>130</td>
<td>0.38</td>
<td>419</td>
<td>3.45</td>
<td>0.008</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>321</td>
<td>3.45</td>
<td>0.011</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>214</td>
<td>3.45</td>
<td>0.016</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>108</td>
<td>3.45</td>
<td>0.032</td>
<td>0.023</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>130</td>
<td>0.38</td>
<td>214</td>
<td>3.45</td>
<td>0.016</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>214</td>
<td>3.45</td>
<td>0.016</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>214</td>
<td>6.9</td>
<td>0.032</td>
<td>0.017</td>
</tr>
</tbody>
</table>

F₀: 0.2

(1/min) 0.1 0.05

112 - 321  3.45 0.031 - 0.011 0.012
113 - 321  3.45 0.031 - 0.012 0.011 - 0.014
114 - 321  3.45 0.031 - 0.013 0.009 - 0.015
<table>
<thead>
<tr>
<th>Experimental Series</th>
<th>$\delta_w$</th>
<th>$\delta_s$</th>
<th>$\delta_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand I</td>
<td>-0.60</td>
<td>NA</td>
<td>0.50</td>
</tr>
<tr>
<td>Sand II</td>
<td>-0.63</td>
<td>0.45</td>
<td>NA</td>
</tr>
<tr>
<td>Sand III</td>
<td>-0.60</td>
<td>0.31</td>
<td>NA</td>
</tr>
<tr>
<td>Sand IV</td>
<td>-0.60</td>
<td>NA</td>
<td>0.00</td>
</tr>
<tr>
<td>Coal I</td>
<td>-0.68</td>
<td>0.48</td>
<td>0.41</td>
</tr>
<tr>
<td>Sand Composite (70 &lt; $D_{50}$ &lt; 160)</td>
<td>-0.60</td>
<td>0.45</td>
<td>0.50</td>
</tr>
<tr>
<td>Sand Composite (160 &lt; $D_{50}$ &lt; 460)</td>
<td>-0.60</td>
<td>0.60</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Figure 1a. Schematic of a hypothetical pond with emplacement of hydroseparated material in a pie-shaped region near the launders. A channel with length $L_c$ and width $B_c$ is maintained within the region to allow for delivery of non-separated material to the distal part of the pond. The vertical cut $A - A'$ is shown in Figure 1b.
Figure 1b. Vertical cut A - A' through the pond showing the present grade line of the fan, the ultimate grade line, and the pie-shaped region in which the hydroseparated material is to be placed. The material must be built at least up to the ultimate grade line of the channel; in addition it may be stacked some height $z$ above the grade line.
Figure 2. The Hibbing Taconite Company’s tailings basin at Hibbing, Minnesota.
Figure 3. The nature of the problem at the HTC basin. Continued aggradation at the present rate will lead to burial of the emergency launders in sediment before the design lifetime of the basin is reached. The problem could be ameliorated by reducing either the overall slope or the concavity of the topographic profile in the basin.
Figure 5. Sequential topographic profiles taken along the length of West Area #1 showing that aggradation has occurred at a constant rate with respect to basin length, maintaining a self-similar topographic profile.
Figure 6. Schematic illustration of the small experimental basin (L = 6.9 ft (2.1 m)).
CHT indicates constant head tank. SF indicates auger-type sediment feeder.
The large experimental basin was of similar design, but was fitted with a single, continuous weir gate at the downstream end of the fan.
Figure 7a. Fan slope as a function of the volumetric ratio of sediment feed rate (Qso) to water discharge (Qw) for the Sand I experimental series. Qw is the independent variable in all of these runs. A. Median size (D50) of sediments used is indicated in μm. Data plotted for the 70 and 160 μm experiments represent averages of two sets of experiments (see Fig. K1B).
Figure 7b. Replicate sets of experiments with the 70 and 160 μm sediments. Temporal and spatial fluctuations in fan slope occur as channels shift across the sediment surface, introducing some error into our estimates of “average” fan slopes. However, the results are generally reproducible.
Figure 7c. Fan slope as a function of the ratio $Q_{so}/Q_{w}$ in runs in which $Q_{so}$ was the independent variable.
Slope Dependence on $Q_w$ for Coarse Sediments

Figure 7d. Fan slope as a function of the ratio $Q_{so}/Q_w$ for the coarse sand mixtures of the Sand IV experimental series ($D_{50}$ greater than or equal to 160 μm). Unlike experiments with either finer-grained sand or crushed coal, fan slope is found to be independent of grain size.
Figure 8. Fan slope as a function of the volumetric ratio of sediment feed rate ($Q_{so}$) to water discharge ($Q_w$) for the crushed coal experimental series. Median size ($D_{50}$) of sediments used is indicated in microns. As in the finer sand experiments, fan slope is sensitive to grain size. The independent variable in each set of experiments is indicated in parentheses in the legend.
Figure 9. Photographs of bed configuration taken during one of the oscillating Qw experiments (fluorescent green dye is used to highlight wetted areas against the black (coal) sediment background). Green dye also colors the standing water in the pond at the downstream end. A. Channels during high-flow stage. Note major channel running down fan center. B. Channels during low-flow stage. The effective confinement of flow during low stage periods leads to the favorable biasing of fan slope towards equilibrium conditions associated with the high-stage flows. C. Example of flow configuration if low-flow stage is maintained too long; the channels backfill and the upper fan aggrades rapidly.
Figure 10. Response of fan slope to oscillating water discharges as a function of the frequency of oscillation. The solid line indicates the slope associated with constant flow at the mean discharge ($S/Sm = 1.0$). The dashed line indicates the slope associated with constant flow at the peak discharge. Arrows indicate the range of fan slope fluctuations observed for long oscillation periods.
Figure 11. Transect lines along which field measurements were made on the HTC fan.
Figure 12. A. Total load (bedload plus suspended load) field data plotted in non-dimensional form against excess Shields stress. Note that material less than 40 μm is designated wash load and is excluded from the reported total load. Fitted regression line is shown. B. Data from A plotted on a logarithmic scale. The Meyer-Peter/Müller bedload equation is shown for comparison. C. Data from A differentiated by sample location.
Figure 13. Friction factors (Cf) determined from field depth, slope, and velocity data plotted against the ratio of flow depth to median grain size (H/D50). Regression line and equation are shown. The familiar Manning-Strickler resistance equation is shown for comparison, plotting well below the field data as explained in the text.
Figure 14. Field data on characteristic channel form. The ratio of flow depth to median grain size (H/D50) is plotted against water surface slope. A linear relationship such as that defined by the data indicates a channel closure similar to that developed by Parker (1978) for gravel-bedded rivers.
Figure 15. Actual (1995) versus calibrated bed profile, West Area #1.