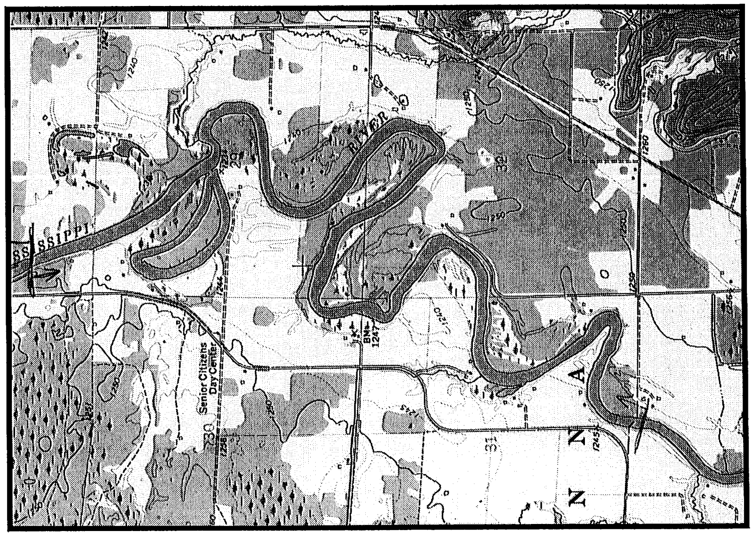
P.R. 321

INVENTORY AND ANALYSIS OF STREAM MEANDER PROBLEMS IN MINNESOTA

by Thomas E. MacDonald, Gary Parker,
Dave P. Leuthe



Abstract

Sixteen stream reaches in the state of Minnesota are analyzed for stream shift and meander properties. Aerial photographs and topographic maps are used with a digitizer to produce x-y coordinate data files for valley centerlines and stream centerlines. Photographs are chosen for a minimum of two different times, approximately twenty years apart. Data preparation is discussed in Chapter 4. After producing the data files, each stream reach is analyzed for channel shift and meander characteristics using a computer program called MEANDER. The program measures various components of stream shift, the most important of which is the average normal shift. It also measures sinuosity, time rate of change of sinuosity, average curvature, and rate of flood plain area reworking by the stream for the reach in question. A probability distribution curve for normal shift is produced for each stream analyzed. The shift measurement process is described in Chapter 5. Stream reaches are presented beginning on page 2. A ground photo, aerial photos, base map, and digitized map are included for each reach. Hydrologic and geomorphic data are also presented for each reach. The results of the computer analysis are presented as well. The results of the computer analysis for the stream reaches are used in regression analyses to try to determine relations between stream shift properties and stream parameters. Linear and log-linear regression was used in order to develop these relations. The most promising relations produced include average normal shift versus depth, average normal shift versus discharge, rate of area reworking versus depth, and rate of area reworking versus discharge. The discharges and depths in question refer to the two-year flood. The equations should be usable for other streams in the general region. The regressions are presented in Chapter 6.

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Acknowledgements

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The research was funded by the Legislative Commission on Minnesota Resources (Legal Citation: M.L. 89, Chapter 335, Sec. 29, Subd. lII), State of Minnesota.

Karl Wikstrom completed the extensive photographic work required for this study. Cliff Bentley computed all the stream discharges, as well as helped with the sieve analyses, stream digitizing, and site visits.

Patricia Swanson served as editor and handled, typesetting, keylining, and print production for this version of the report.

1 Introduction

Purpose

The purpose of this study is to identify meandering streams and rivers in the state of Minnesota which are experiencing bank erosion problems, and to analyze the erosion both quantitatively and qualitatively. This analysis should be a useful tool for those who work with actively or potentially eroding streams or rivers. This audience may include, but is certainly not limited to, transportation planners, bridge designers, erosion control designers, land planners, and conservation workers.

Focus

This report will focus on the analysis of 16 stream reaches which are experiencing problem erosion. The streams range from very small (Kanaranzi Creek), to very large (Minnesota and Mississippi Rivers). The streams were analyzed primarily for their shifting and meandering characteristics, and the results are presented in a manner that should be useful to the intended audience.

Review of Analysis

This analysis was conducted using aerial photographs and topographic maps of the selected sites. The photographs were enlarged to a common scale by a professional photographer and then digitized. Stream shift, and various components thereof, were then measured computationally for each reach, using data sets consisting of valley center line, and stream center lines at not less than two different times. The computer program MEANDER was used for this purpose. An attempt was then made to find a relation between stream shift and various stream parameters, and several useful relations were obtained. Results for each stream analysis are presented.

To conclude the analysis, the results are discussed and possible applications of this analysis are presented.

Booklet Format

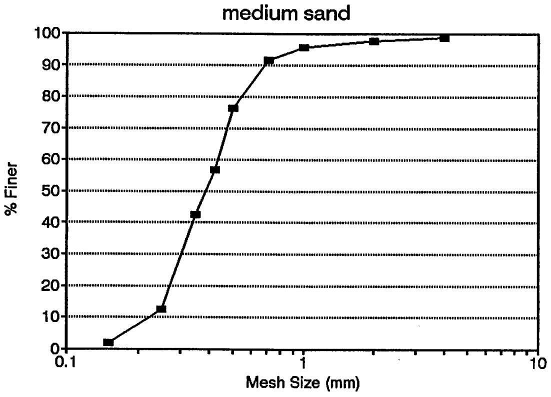
The left-hand side of the pages of this booklet contain information on each stream or river reach analyzed (see Contents, Chapt 2). The right-hand shaded areas of this booklet contain the sections of text which explain how the data and measurements were gathered, and how this information was used to arrive at the meander results for each of the reaches presented herein.

BUFFALO CREEK
at
McLeod County
T116N-R27W, secs. 26,35
T115N-R27W, sec. 2

HYDROLOGIC INFORMATION

2-Year Storm Discharge:	1739 cfs
Bankfull Stream Width:	57 ft
Stream Depth (Synthesized):	7.5 ft
Stream Slope:	0.00033

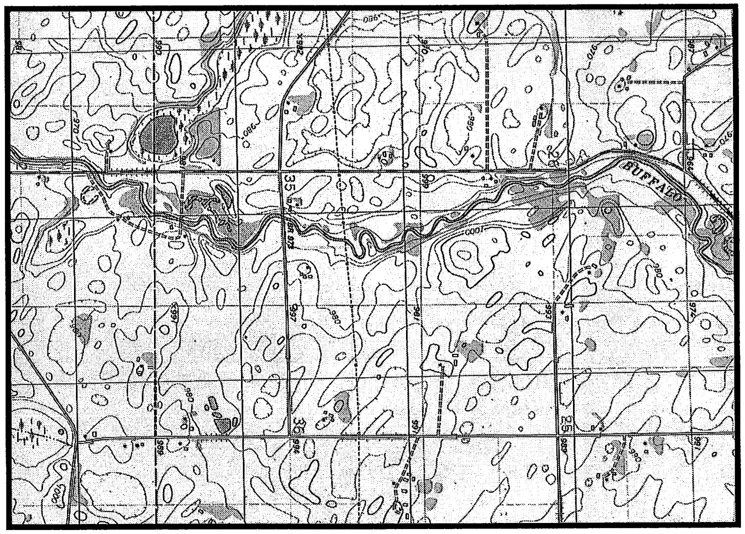
GEOMORPHIC INFORMATION



Grain size distribution, Buffalo Creek

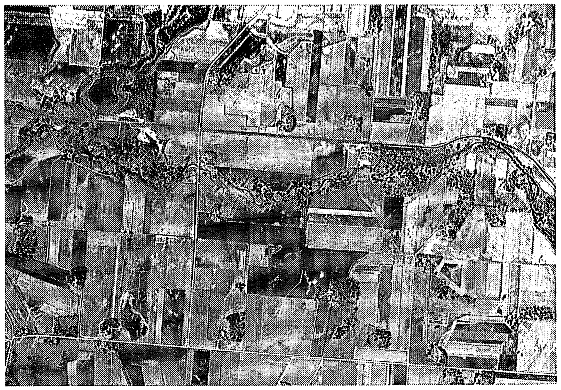
Ground photo of Buffalo Creek, 7/27/90

DESCRIPTION: Buffalo Creek is a small, sand-bed stream in south-central Minnesota, flowing northward into the South Fork Crow River, which flows into the Mississippi River.

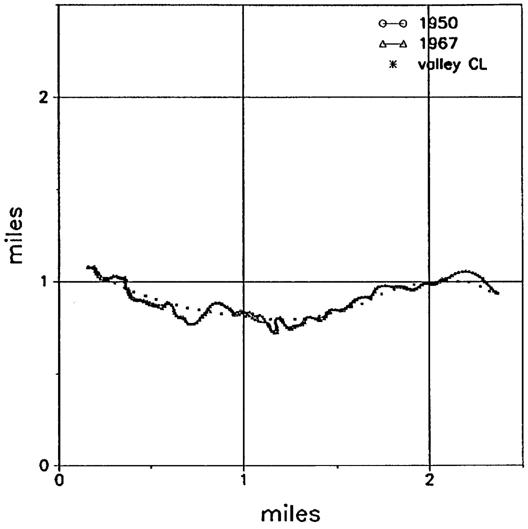


Base map, Buffalo Creek, 1982

Aerial photograph, Buffalo Creek, 1950



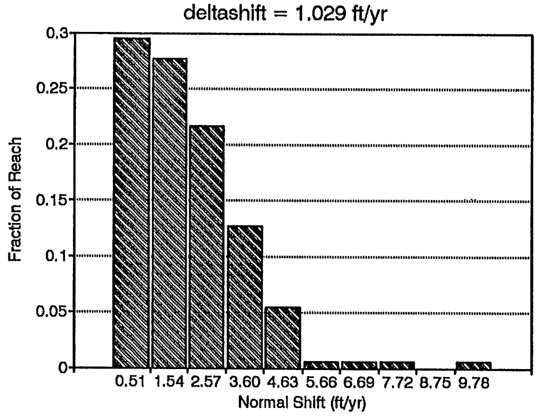
Aerial photograph, Buffalo Creek, 1967



Digitized centerlines, Buffalo Creek

PROGRAM RESULTS

SHIFT	
Average Normal Shift:	2.07 ft/yr
Standard Deviation of Normal Shift:	0.05
Average Absolute Transverse Shift:	1.66 ft/yr
Average Absolute Longitudinal Shift:	1.03 ft/yr
Average Transverse Shift:	0.01 ft/yr
Average Longitudinal Shift:	0.35 ft/yr
Shift Ratio:	0.88
SINUOSITY	
Average Stream:	1.21
Average Change:	0.00171
AVERAGE CURVATURE (ft^{-1}):	0.00419
AREA WORKED ($ft^2/ft/yr$):	2.51



Shift distribution, Buffalo Creek

3 Literature Review

During the past twenty years, much research has been conducted on the theory of stream meandering and migration. As a result, various techniques for classifying meanders and models for the prediction of stream channel migration rates have been developed. The emphasis has been placed on prediction, with actual measurements made to verify the accuracy of the models.

Brice (3)[1] classified meanders into four general groups: simple symmetrical, simple asymmetrical, compound symmetrical, and compound asymmetrical. He also proposes 16 form types from a study of the meandering patterns of 125 alluvial streams.

Brice traced the evolution of stream meander formation. According to Brice, an arc becomes a simple symmetrical meander loop when its length exceeds its radius. The simple loop becomes asymmetrical with the growth of an arc on its perimeter. When the arc becomes a secondary loop, the simple meander becomes compound, either symmetrical or asymmetrical. Brice theorized that although the radius, length, and height of compound meander loops are indefinite, they can be broken into simple meander loops, and analyzed for relations between meander size and river properties.

At a later time, Brice (4) classified meandering streams into three types: sinuous canaliform, sinuous point bar, and sinuous braided. Sinuous canaliform rivers tend to have narrow channels, cohesive banks, and strong meandering. Sinuous point bar streams are wider, with less cohesive banks, so that more bed material is transported. They are characterized by the presence of prominent point bars on the inside of bends. Sinuous braided streams have less cohesive banks, and more material transported, and thus contain braided forms within the channel. Point bars are still present, however.

Brice analyzed 174 rivers for stream shift, by comparing aerial photographs of the same reach at different times. Brice confirmed Hooke's (9) hypothesis that stream shift varies with the square root of the drainage area.

Hickin and Nanson (8) predicted stream migration rates for the Beatton River, in Northeast British Columbia, based on the ratio of stream radius of curvature to stream width. Lateral migration rates and incision were measured by performing dendrochronological surveys on ten point-bar complexes on the river. It was found that stream migration rates approach a maximum as the ratio approaches 3.0, then rapidly decreases.

Hickin and Nanson's model, however, did not predict downstream migration, merely transverse, or cross-valley migration. Ikeda, Parker, and Sawai (15) improved upon Hickin and Nanson's model by using non-linear techniques to relate stream migration to near-bank excess velocity. Parker (15) showed that a convolutional rather than algebraic relation exists between channel migration and curvature. An interesting finding of the study is that tight bends subside, while bends of longer wavelength grow in amplitude, at some point cutting themselves off.

Beck (1) used Ikeda, Parker, and Sawai's model to confirm the relation of meander growth and wavelength. In addition, he found that the speed of down-valley migration increases with sinuosity, that minimum radius of curvature increases with sinuosity, and that cut-off occurs at a sinuosity of 6.

Odgaard (13), in an attempt to improve upon Parker's (15) model, also developed a predictive stream shift model. Odgaard's model relates stream shift to various channel properties, including width, depth, curvature, arc angle of channel centerline, channel slope, friction factor, and amount of bank vegetation. Odgaard, like

(Cont'd on page 19)

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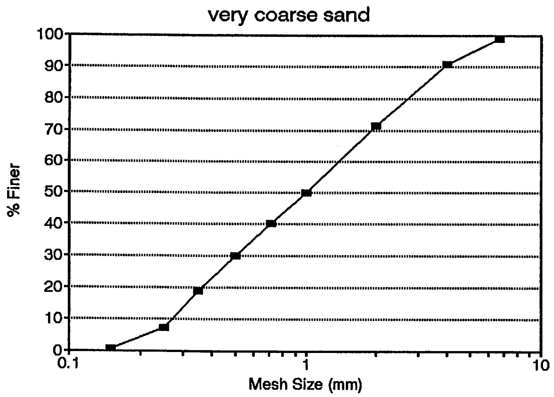
[1] Numbers refer to references.

COTTONWOOD RIVER
at
Brown County
T109N-R33W, secs. 21,22,27,28

HYDROLOGIC INFORMATION

2-Year Storm Discharge:	2296 cfs
Bankfull Stream Width:	80 ft.
Stream Depth (Synthesized):	6.2 ft.
Stream Slope:	0.00060

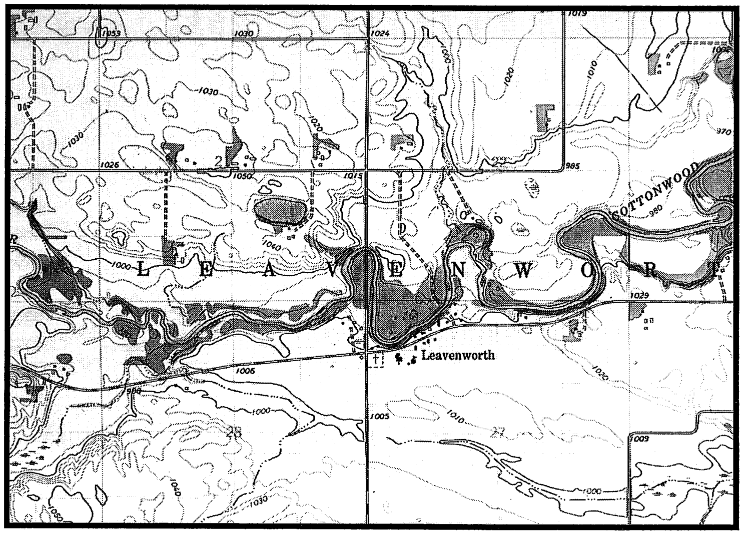
GEOMORPHIC INFORMATION



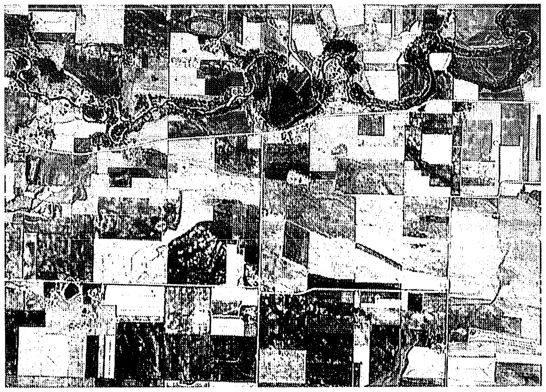
Grain size distribution, Cottonwood River

Ground photo of Cottonwood River, 10/21/89

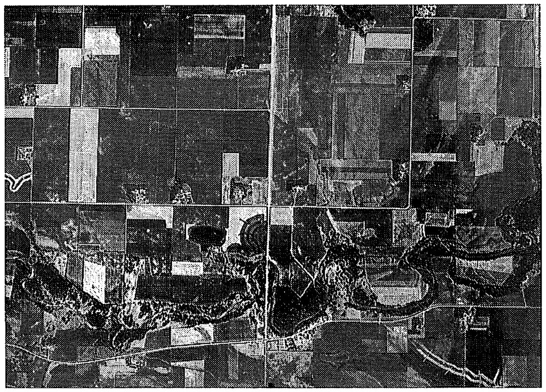
DESCRIPTION: The Cottonwood River is a sand-bed stream in south-central Minnesota, flowing eastward into the Minnesota River.



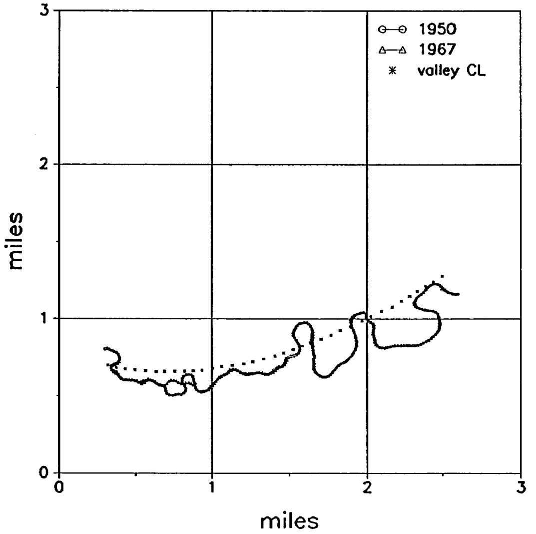
Base map, Cottonwood River, 1967



Aerial photograph, Cottonwood River, 1950



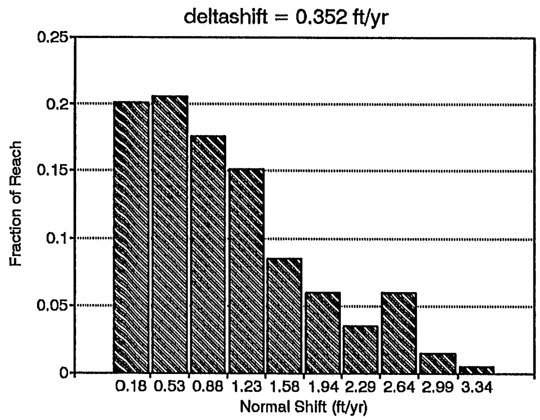
Aerial photograph, Cottonwood River, 1967



Digitized centerlines, Cottonwood River

PROGRAM RESULTS

SHIFT	
Average Normal Shift:	1.05 ft/yr
Standard Deviation of Normal Shift:	0.02
Average Absolute Transverse Shift:	0.65 ft/yr
Average Absolute Longitudinal Shift:	0.66 ft/yr
Average Transverse Shift:	0.16 ft/yr
Average Longitudinal Shift:	-0.11 ft/yr
Shift Ratio:	-1.79
SINUOSITY	
Average Stream:	1.86
Average Change:	0.00066
AVERAGE CURVATURE (ft^{-1}):	0.00179
AREA WORKED ($ft^2/ft/yr$):	1.94



Shift distribution, Cottonwood River

4 Data Collection

Stream Choice

There is no shortage of streams with erosion problems in the state of Minnesota. For this study, requests were made to DNR officers throughout the state for suggested river reaches with erosion problems. The response was very good, although some areas of the state were better represented than others. No response was received from certain areas of the state, which implies that there are few erosion problems, or simply a lack of interest. In addition, many of the suggested reaches were too small to work with from aerial photographs. In addition, there are differences of opinion as to what constitutes a "problem" stream. Indeed, some of the suggested reaches had no discernible problem.

The time period over which the channel shift was measured was chosen such that sufficient shift had occurred for accurate measurement. For most reaches, a time period of approximately 20 years was deemed satisfactory. Some streams (e.g. Root River), however, shift so quickly that a smaller time step had to be chosen, and, likewise, some shifted so slowly that a larger time step was acceptable. In any case, the chosen time phase can play a large role in the accuracy of the shift measurement.

Constructing Usable Data Sets

To construct usable data sets for analysis, aerial photographs were chosen for each reach of interest, over an appropriate time period. The photograph scales were typically 1:20,000. After the photographs were obtained, a base map of each reach was created by photographically enlarging a portion of a 7.5' USGS topographic map.

The aerial photographs were then rephotographed and enlarged to the same scale as the base map. 20 x 24 inch negatives were produced for each reach at each time. By overlaying the negatives, the stream and valley centerlines could be traced, and then digitized into coordinate data sets.

The creation of usable data sets required a significant amount of time, largely because the services of a professional photographer were required. It should be possible to eliminate the photography work, however, with the use of appropriate computer software, combined with a digitizer. With such software, the photographs could be digitized to a common scale directly, eliminating the photographic work. This may not be possible, however, if there is a significant amount of distortion in the aerial photographs.

In order for the program MEANDER to accurately measure shift from the stream and valley coordinates, the data first has to be smoothed to remove very small random errors from the digitizing process. The small random errors have to be removed because the initially small errors of digitization are amplified into large errors as the angular alignment θ of the channel centerline, and then curvature C, are computed. These are defined as follows (see also Fig. 11):

$$tan\ \theta = \frac{dy}{dx} \tag{1}$$

$$C = \frac{d\theta}{ds} \tag{2}$$

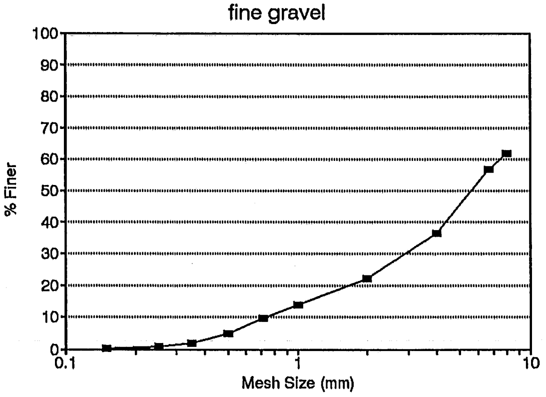
The smoothing procedure is implemented within the program, after the coordinates are read from data files. The procedure consists of converting the x-y coordinates to curvilinear s coordinates, equally spacing the coordinates using the Newton-Gregory method, and using a numerical smoothing filter to suppress the random errors. The filtering technique used is that described by Beck and Parker in their as yet unpublished paper, *Meandering River Patterns in Intrinsic Coordinates.*

HAWK CREEK
at
Renville County
T116N-R38W, sec. 21,28

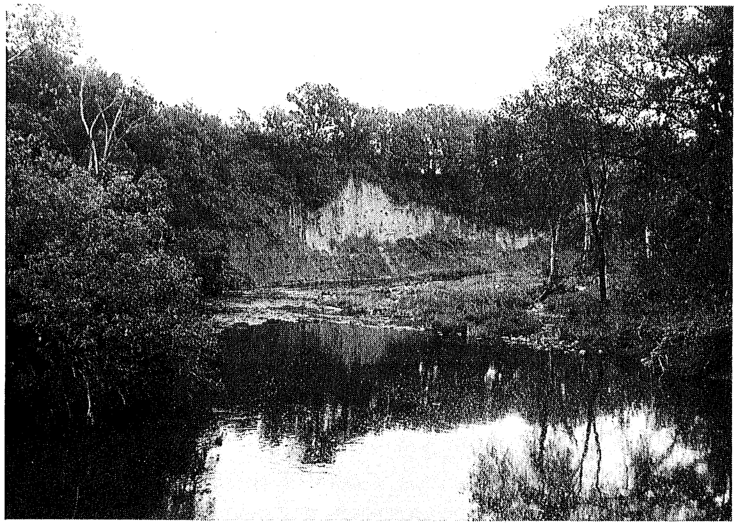
HYDROLOGIC INFORMATION

2-Year Storm Discharge:	1437 cfs
Bankfull Stream Width:	75 ft.
Stream Depth (Synthesized):	4.3 ft.
Stream Slope:	0.00063

GEOMORPHIC INFORMATION

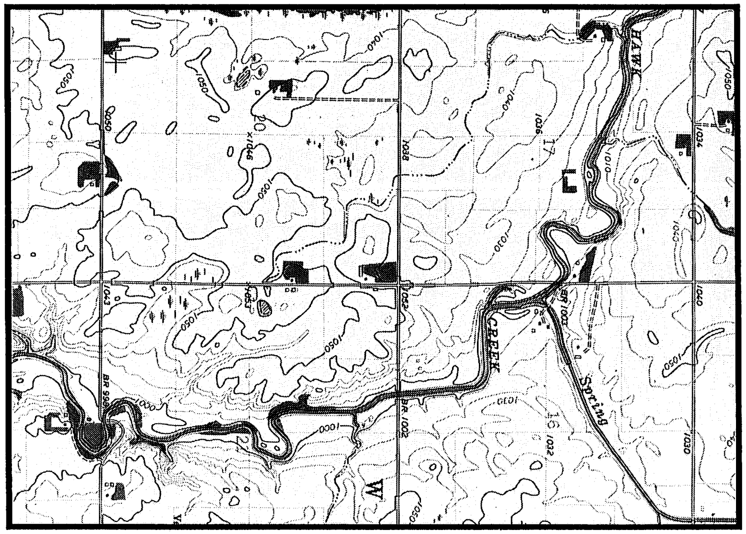


Grain size distribution, Hawk Creek



Ground photo of Hawk Creek, 7/27/90

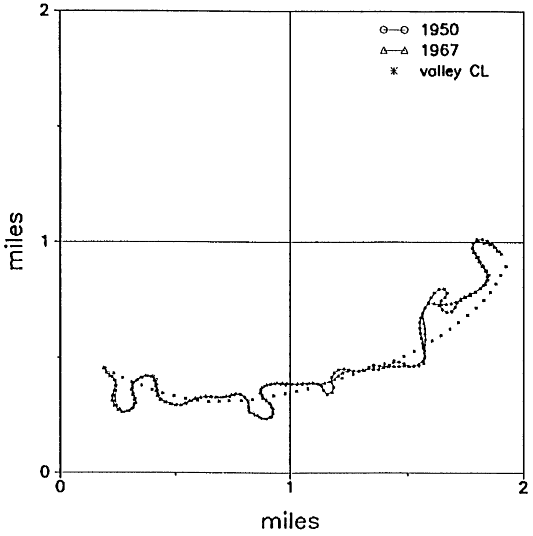
DESCRIPTION: Crooked Creek is a small, sand-bed stream in eastern Minnesota, flowing south-east into the St. Croix River.



Base map, Hawk Creek, 1965

Aerial photograph, Hawk Creek, 1950

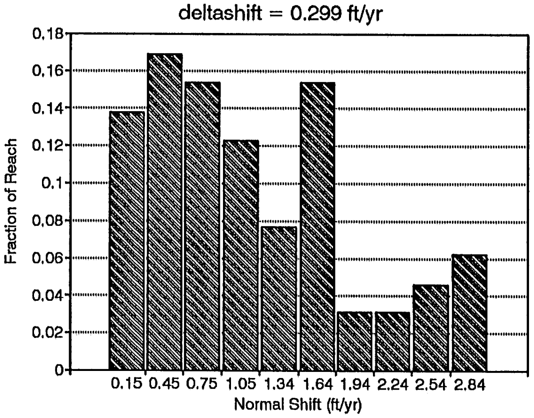
Aerial photograph, Hawk Creek, 1967



Digitized centerlines, Hawk Creek

PROGRAM RESULTS

SHIFT	
Average Normal Shift:	1.16 ft/yr
Standard Deviation of Normal Shift:	0.03
Average Absolute Transverse Shift:	0.69 ft/yr
Average Absolute Longitudinal Shift:	0.75 ft/yr
Average Transverse Shift:	0.07 ft/yr
Average Longitudinal Shift:	0.67 ft/yr
Shift Ratio:	0.33
SINUOSITY	
Average Stream:	1.70
Average Change:	-0.00011
AVERAGE CURVATURE (ft^{-1}):	0.00295
AREA WORKED ($ft^2/ft/yr$):	1.98



Shift distribution, Hawk Creek

The filter uses an exponential weighting function f_w, given by

$$f_w(s') = exp\left(\frac{-\xi\pi \mid s' \mid}{S\,L}\right) \qquad (3)$$

Here, s' is the streamwise coordinate, S is sinuosity, L is the cartesian meander wavelength, and ξ is a dimensionless weighting factor. The filter is used to smooth the x-y coordinates, angular alignment θ of channel centerline θ, and curvature C. Curvature, for example, is smoothed using the following filter:

$$C(s) = \frac{\int_{-\Delta s}^{\Delta s} f_w(s')C(s-s')ds'}{\int_{-\Delta s}^{\Delta s} f_w(s')ds'} \qquad (4)$$

where Δ s is the half the width of the filter "window," and is not to be confused with the distance between individual coordinates. Here, Δ s varies with the position of the coordinate being smoothed.

The removal of these errors does not produce an obvious visual change in the coordinates. Rather, the change is noticed to some extent in the angular alignment of the channel centerline, and to a larger extent in the curvature, where large errors can be suppressed. Plots of the x-y coordinates, angle θ, and curvature C are shown in figures 2, 3, and 4. Note that care must be used in choosing the factor ξ, as it controls the strength of the filter. Too strong of a filter will remove not only the error in the data, but also the substance. A higher value of ξ produces a weaker filter ($\xi = \infty$ corresponds to no filter). For this analysis, $\xi = 10$ was used, based on trial and error.

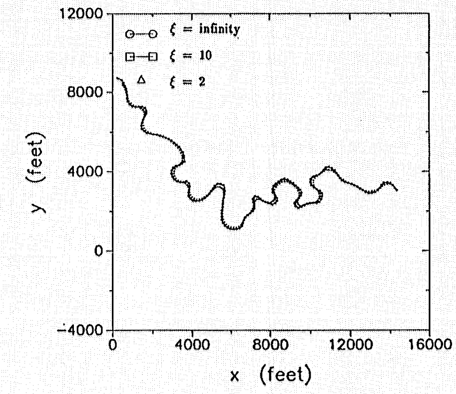


Figure 2: Comparison of smoothed versus unsmoothed x-y coordinate

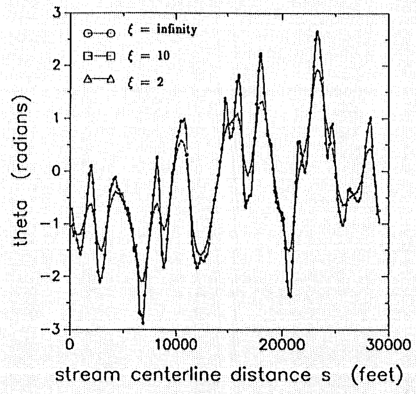


Figure 3: Comparison of smoothed versus unsmoothed theta

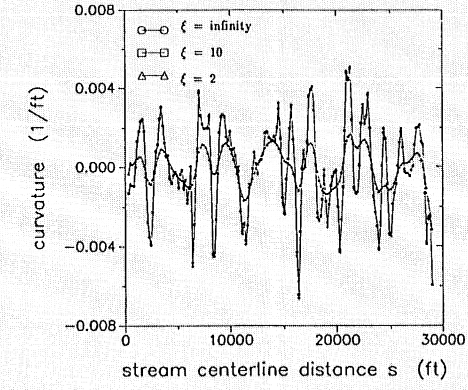


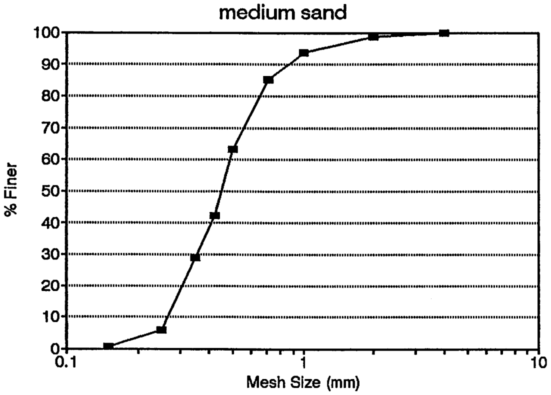
Figure 4: Comparison of smoothed versus unsmoothed curvature

Kanaranzi Creek
at
Rock County
T101N-R44W, secs. 25,26,35,36

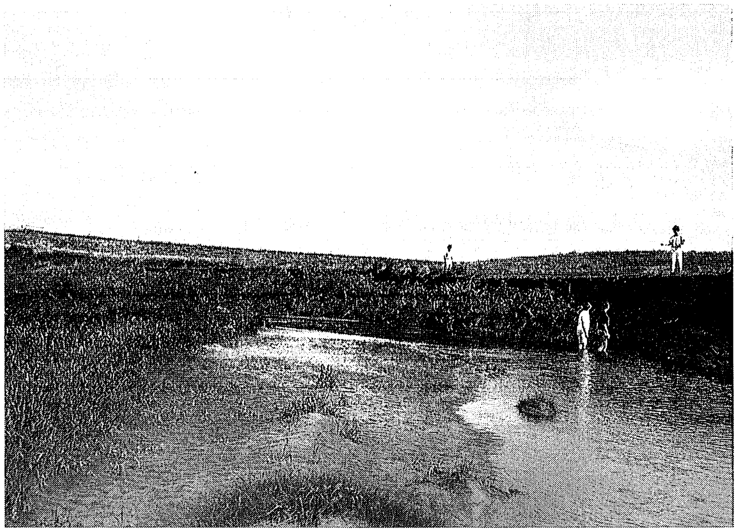
HYDROLOGIC INFORMATION

2-Year Storm Discharge:	826 cfs
Bankfull Stream Width:	25 ft.
Stream Depth (Synthesized):	4.6 ft.
Stream Slope:	0.00071

GEOMORPHIC INFORMATION

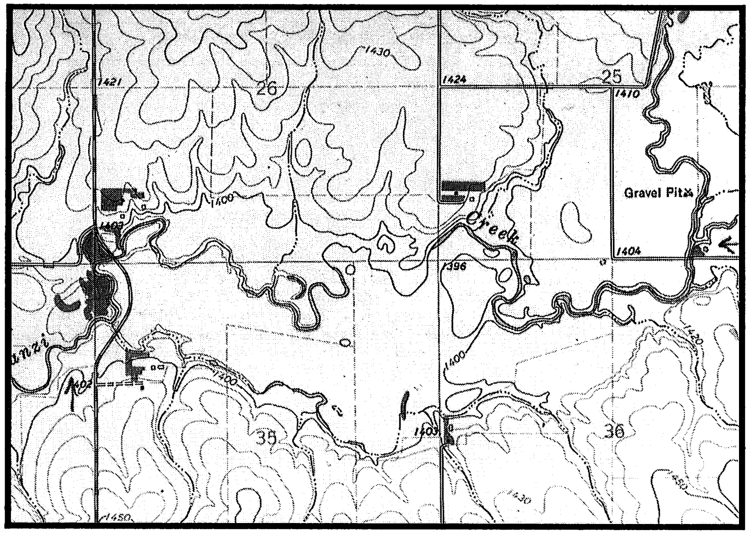


Grain size distribution, Kanaranzi Creek

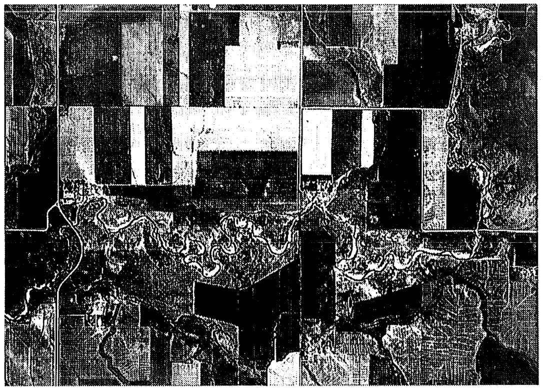


Ground photo of Kanaranzi Creek, 7/26/90

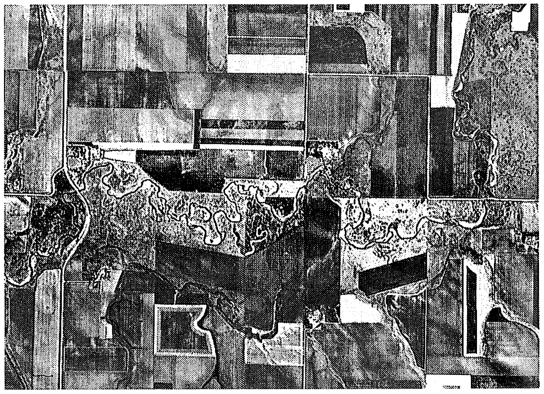
DESCRIPTION: Kanaranzi Creek is a very small, strongly meandering sand-bed creek in southwestern Minnesota. It lies in a poorly defined, wide and shallow valley.



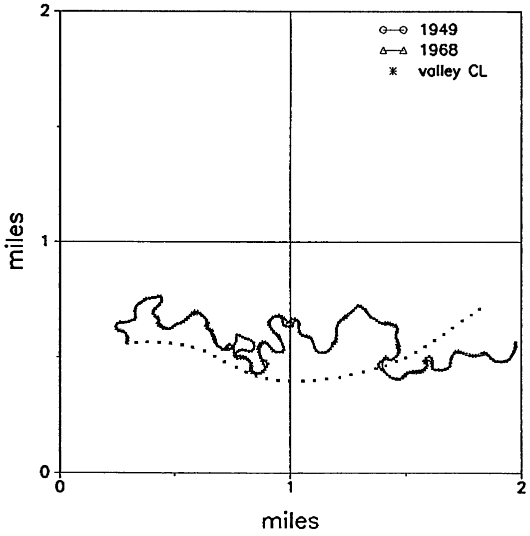
Base map, Kanaranzi Creek, 1967



Aerial photograph, Kanaranzi Creek, 1949



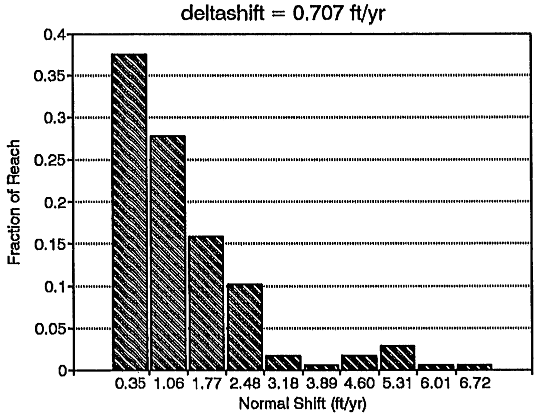
Aerial photograph, Kanaranzi Creek, 1968



Digitized centerlines, Kanaranzi Creek

PROGRAM RESULTS

SHIFT	
Average Normal Shift:	1.34 ft/yr
Standard Deviation of Normal Shift:	0.05
Average Absolute Transverse Shift:	0.90 ft/yr
Average Absolute Longitudinal Shift:	0.80 ft/yr
Average Transverse Shift:	0.03 ft/yr
Average Longitudinal Shift:	0.06 ft/yr
Shift Ratio:	3.03
SINUOSITY	
Average Stream:	1.97
Average Change:	0.00151
AVERAGE CURVATURE (ft^{-1}):	0.00655
AREA WORKED ($\mathrm{ft}^2/\mathrm{ft}/\mathrm{yr}$):	2.66



Shift distribution, Kanaranzi Creek

5 Shift Measurement

Discussion of Lagrangian Versus Wave Shift

There are two conceivable fields of reference which can be used to measure stream shift. These are the wave and the Lagrangian viewpoints.

The wave viewpoint focuses attention on marked points along the stream centerline, and their movement is traced through time with respect to initial position. Thus, it is assumed that the stream centerline moves as a propagating wave.

With the Lagrangian viewpoint, attention is not focused on the movement of each point on the stream centerline. Instead, it is focused on the area surrounding the stream centerline. Centerline movement is measured in the direction normal to the stream bank or, alternatively, the stream centerline.

To better understand the difference between Lagrangian and wave shift, let us consider a simple sine wave undergoing pure translation, with no growth, at a constant wave speed c_w, as shown in Fig. 5: The wave movement is described by the equation,

$$y = y_A \sin(x - c_w t) \tag{5}$$

If the wave speed is described using the wave reference, the horizontal wave speed $\dot{x}$ and the vertical wave speed $\dot{y}$ are, simply,

$$\dot{x} = c_w \tag{6}$$

$$\dot{y} = 0 \tag{7}$$

If the wave speed is described using the Lagrangian reference, however, the components of wave speed are, as follows (Fig. 6):

$$\dot{x} = -\dot{n} \sin\theta \tag{8}$$

$$\dot{y} = \dot{n} \cos\theta \tag{9}$$

Where,

$$\dot{n} = \dot{c}_w \sin\theta \tag{10}$$

Therefore,

$$\dot{x} = \dot{c}_w \sin^2\theta \tag{11}$$

$$\dot{y} = \dot{c}_w \sin\theta \cos\theta \tag{12}$$

Thus, for the simple case of a wave undergoing pure translation, the wave reference is a simpler means of describing the wave movement. If we consider the more general case of a wave undergoing both translation movement and amplitude growth, however, the situation becomes more complicated. Using the wave reference, it is assumed that the meander peaks and inflection points translate and grow to the peaks and inflection points at some later time. The points in between, however, must be interpolated. Indeed, using the wave reference, $\dot{x}$ and $\dot{y}$ cannot be easily determined for more than one wavelength at a time.

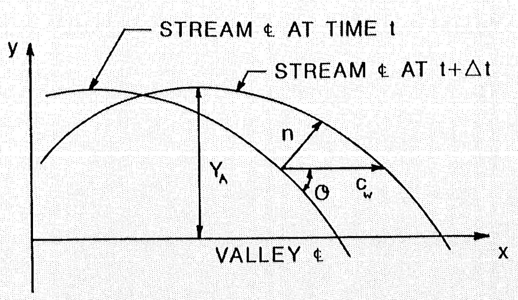


Figure 5: Translating wave (no growth)

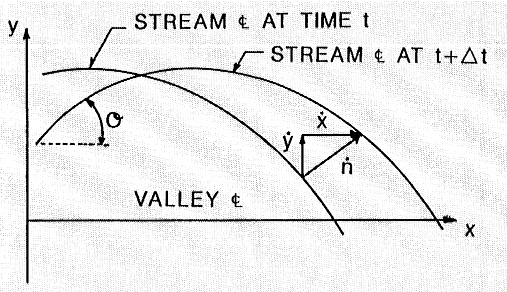


Figure 6: Lagrangian components of pure wave translation

Minnesota River A
at
Nicollet and Blue Earth Counties
T109N-R28W, secs. 29,30,31,32

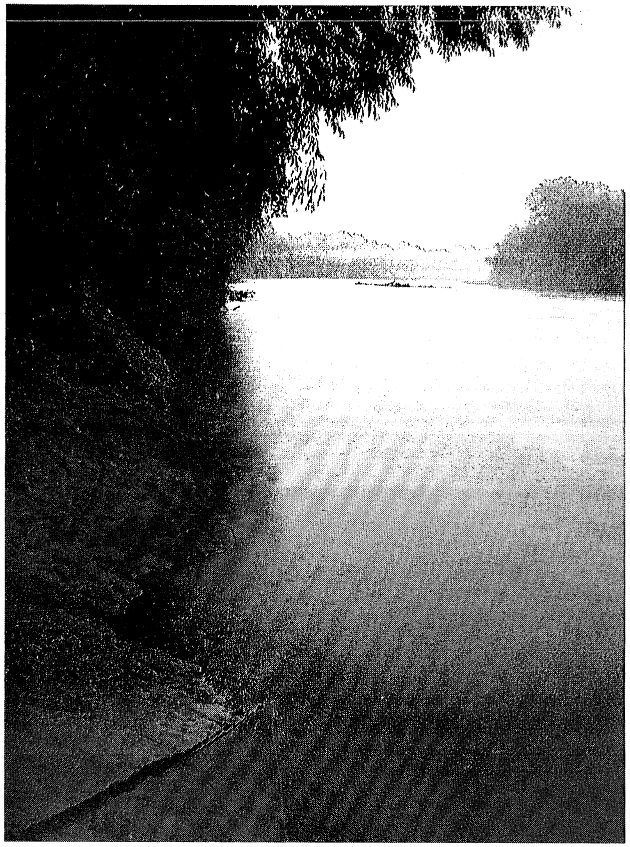
DESCRIPTION: The Minnesota River is a large sand-bed river flowing from western Minnesota to the Mississippi River at Minneapolis.

HYDROLOGIC INFORMATION

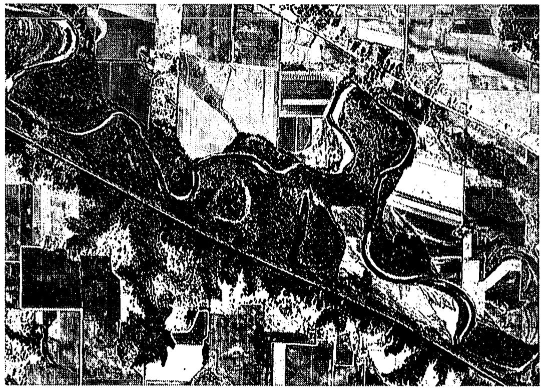
2-Year Storm Discharge:	11084 cfs
Bankfull Stream Width:	140 ft
Stream Depth (Synthesized):	15.4 ft
Stream Slope:	0.00024

GEOMORPHIC INFORMATION

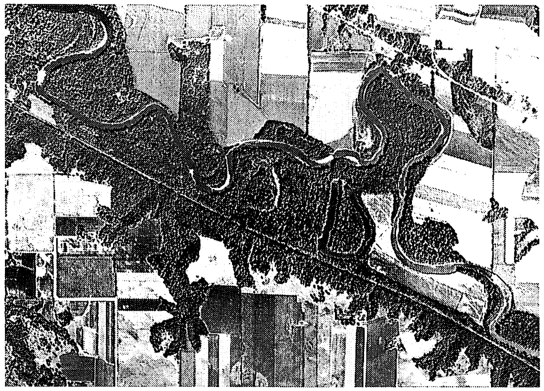
Sediment samples were not obtained for the Minnesota River, but grain size diameter D_{50} = 0.5 mm was estimated based on work conducted by Parker, Garcia, Johannesson, and Okabe (16) on the Minnesota River near Mankato, Minnesota.



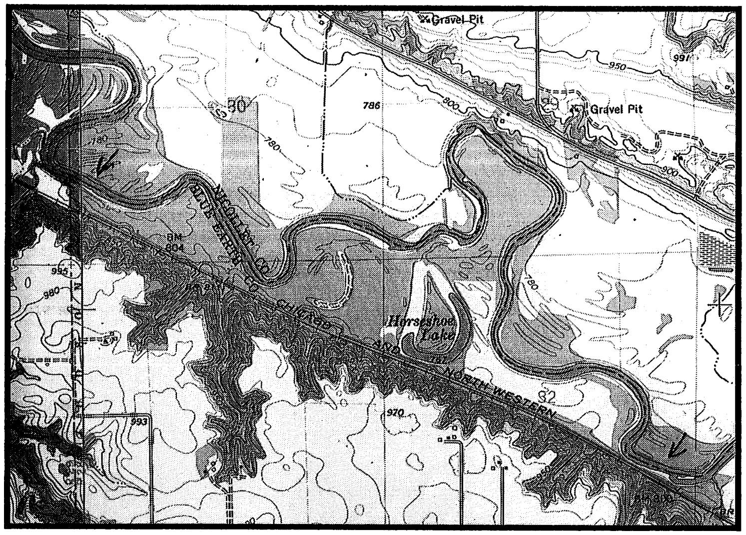
Ground photo of Minnesota River A, 7/26/90



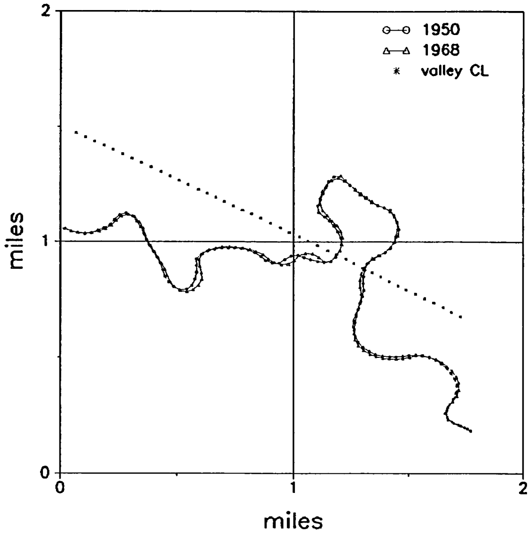
Aerial photograph, Minnesota River A, 1950



Aerial photograph, Minnesota River A, 1968



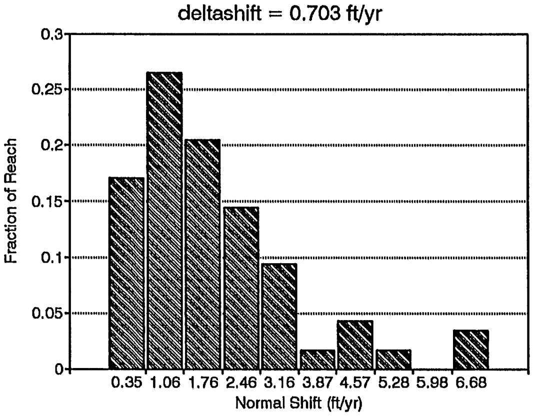
Base map, Minnesota River A, 1974



Digitized centerlines, Minnesota River A

PROGRAM RESULTS

SHIFT	
Average Normal Shift:	2.00 ft/yr
Standard Deviation of Normal Shift:	0.06
Average Absolute Transverse Shift:	1.34 ft/yr
Average Absolute Longitudinal Shift:	1.20 ft/yr
Average Transverse Shift:	-0.73 ft/yr
Average Longitudinal Shift:	0.61 ft/yr
Shift Ratio:	1.22
SINUOSITY	
Average Stream:	1.94
Average Change:	0.00240
AVERAGE CURVATURE (ft^{-1}):	0.00163
AREA WORKED ($ft^2/ft/yr$):	3.86



Shift distribution, Minnesota River A

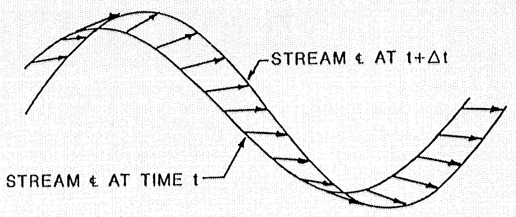


Figure 7: Stream shift with wave reference

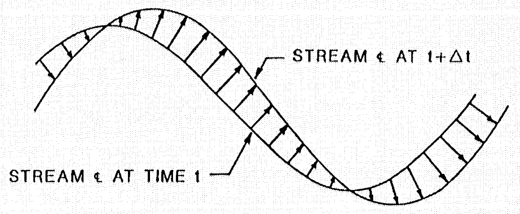


Figure 8: Stream shift with Lagrangian reference

It is seen from Fig. 8, however, that the Lagrangian reference allows $\dot{x}$ and $\dot{y}$ to always be determined for any length of reach, provided that the time interval Δt is small enough for accurate measurement.

Δt is considered small enough when the vector $\vec{n}$, which is perpendicular to the channel centerline at time 1, is also approximately perpendicular to the channel centerline at time 2. If $\vec{n}$ is perpendicular to the wave at time 1, but deviates significantly from perpendicularity at time 2, Δt is too large, and $\dot{x}$ and $\dot{y}$ cannot be accurately measured. This is illustrated in Fig. 9:

How Program MEANDER Measures Stream Shift

The computer program used to measure stream shift does so from a Lagrangian reference. This reference was chosen for several reasons: it can be implemented over an extended stream reach, the shift results are well suited to land planners, and it yields the area worked by the stream as a useful by-product.

As discussed in the previous section, Lagrangian vs. Wave Shift, a primary drawback of measuring shift with a wave reference is that it is difficult to implement over more than one meander wavelength of the stream. This difficulty does not present itself when using the Lagrangian reference. Indeed, the only limiting factor in the length of channel that can be analyzed is the amount of computer memory available. This enables the engineer to account for an extended reach rather than the immediately surrounding reach when designing a structure.

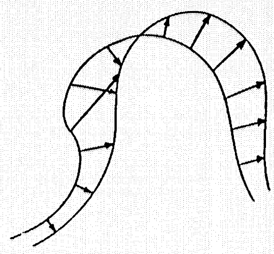


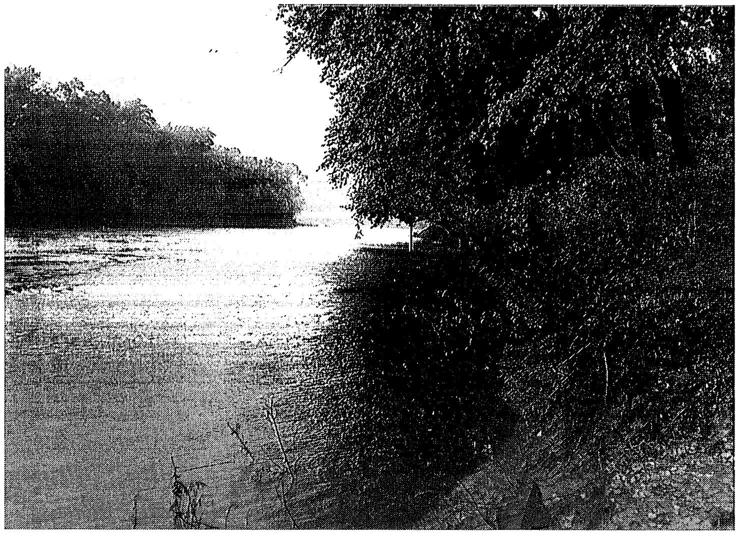
Figure 9: Δt too large for accurate measurement

The Lagrangian reference yields shift data that are useful to those who must design bridges, roads, erosion control devices, and other river or stream structures. When designing such structures, it is useful to know where erosion will occur with respect to the structure, not the river. Using the scheme presented here, the designer will quickly obtain information as to how much area the stream requires for natural shift per unit time.

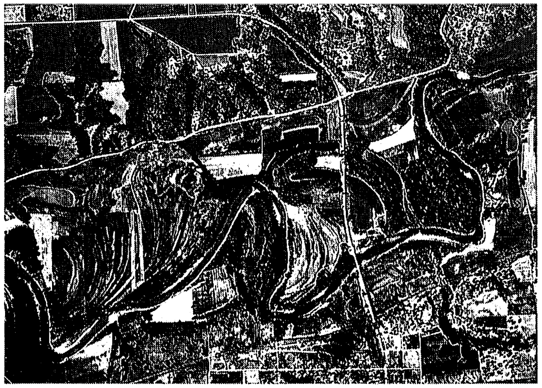
Finally, when using the Lagrangian reference for shift measurement, it is useful to compute the flood plain area worked (eroded or deposited) by a reach per unit time. This quantity is very useful for land planners, who need to know how quickly the stream works the surrounding flood plain. This quantity will be described in greater detail in the next section.

The shift measurement process is easy to visualize. After equally spacing and smoothing cartesian x-y coordinates for the stream centerline at time 1 and at time 2, as well as for the valley centerline, piece wise spline functions are fitted to each centerline. Then, the angular alignment θ of the stream centerline, and the curvature C, is computed for each coordinate (see

Minnesota River B
at
Scott County
T113N-R25W, secs. 1,2
T114N-R25W, secs. 35,36
T114N-R24W, secs 31



Ground photo of Minnesota River B, 7/26/90



Aerial photograph, Minnesota River B, 1964

Aerial photograph, Minnesota River B, 1980

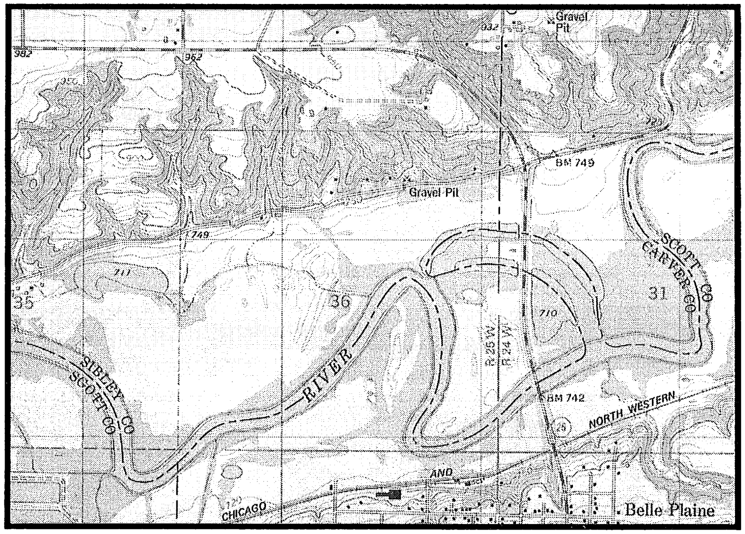
DESCRIPTION: The Minnesota River is a large, sand-bed river flowing from western Minnesota to the Mississippi River at Minneapolis.

HYDROLOGIC INFORMATION

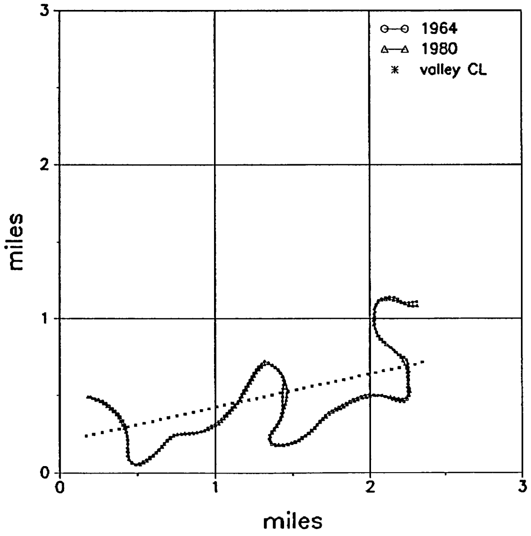
2-Year Storm Discharge:	15092 cfs
Bankfull Stream Width:	290 ft.
Stream Depth (Synthesized):	14.8 ft.
Stream Slope:	0.00010

GEOMORPHIC INFORMATION

Sediment samples were not obtained for the Minnesota River, but grain size diameter D_{50} = 0.5 mm was estimated based on work conducted by Parker, Garcia, Johannesson, and Okabe (16) on the Minnesota River near Mankato, Minnesota.



Base map, Minnesota River B, 1981



Digitized centerlines, Minnesota River B

PROGRAM RESULTS

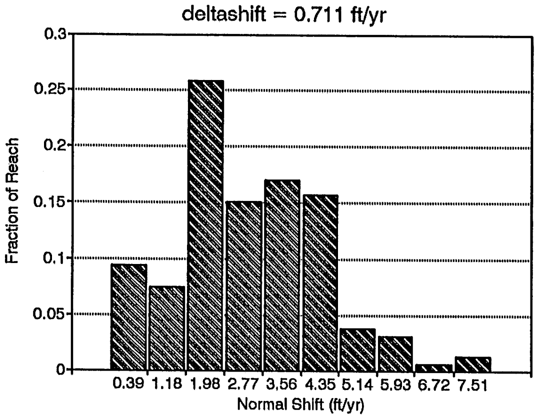
SHIFT

Average Normal Shift:	2.92 ft/yr
Standard Deviation of Normal Shift:	0.04
Average Absolute Transverse Shift:	1.98 ft/yr
Average Absolute Longitudinal Shift:	1.75 ft/yr
Average Transverse Shift:	0.96 ft/yr
Average Longitudinal Shift:	0.47 ft/yr
Shift Ratio:	1.95

SINUOSITY

Average Stream:	1.97
Average Change:	-0.00231

AVERAGE CURVATURE (ft^{-1}):	0.00119
AREA WORKED ($\mathrm{ft}^2/\mathrm{ft}/\mathrm{yr}$):	5.77



Shift distribution, Minnesota River B

Fig. 11 and Eq. 15). The number of coordinates for each reach depends on the smoothness of the channel; a tightly meandering stream requires more coordinates to accurately represent it, than, say a very gently meandering, large river.

The shift measurement process consists of moving from coordinate to coordinate along the stream centerline at time one, while measuring the normal distance between each coordinate and the stream centerline at time two. The normal shift at each coordinate is dissected into valley and cross-valley components, and each component, as well as the normal shift, is integrated over the entire stream reach to yield an average rate.

It is important to remember that, when considering stream shift, the x-y coordinate system is now taken with respect to the valley centerline, which is often gently curved. Then, x corresponds to the down-valley direction, and y corresponds to the cross-valley direction.

Useful Results

Many useful results are obtained through the shift measurement process. These include the stream sinuosity S, sinuosity change per unit time $\dot{S}$, average stream curvature $\bar{C}$, average normal shift $\bar{n}$, average absolute down-valley shift $|\ \bar{\dot{x}}\ |$ and cross-valley shift $|\ \bar{\dot{y}}\ |$, average down-valley shift $\bar{\dot{x}}$ and cross-valley shift $\bar{\dot{y}}$, shift ratio $\dot{y}_{rms}/\bar{\dot{x}}$, where $\dot{y}_{rms}$ is the root mean square of the cross-valley shift rate $\dot{y}$, and area worked by the reach per unit time $\dot{r}$.

Stream sinuosity S provides a measure of the intensity of the meandering of a reach. Sinuosity is defined as the ratio of the stream channel length to the valley length, for the selected reach (Fig. 10):

$$S = \frac{L_s}{L_v} \tag{13}$$

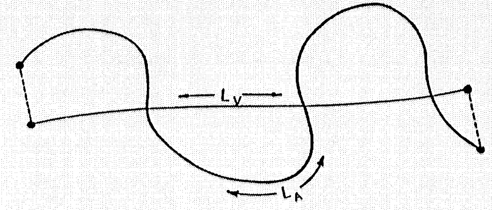


Figure 10: Sinuosity of a stream

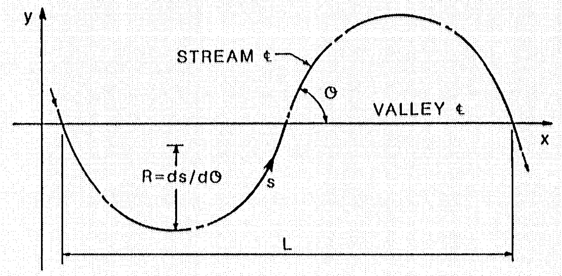


Figure 11: Definition diagram for radius of curvature

After the sinuosity is calculated at time 1 and time 2, the sinuosity change over time is computed, which is a measure of stream meander growth rate.

$$\dot{S} = \frac{S_2 - S_1}{t_2 - t_1} \tag{14}$$

Generally, sinuosity increases over time, and thus the rate will usually be positive. In unusual circumstances, however, it can have a negative value, particularly if a cut-off has recently occurred near the reach of interest. Cutoffs are not included in the shift analyses of any of the reaches included in this study.

Stream curvature is defined by the relation,

$$C = \frac{d\theta}{ds} \tag{15}$$

Curvature is more easily defined, however, in terms of its reciprocal, which is the radius of curvature of the stream. A higher curvature corresponds to tighter meanders (Fig. 11).

Average normal shift $\bar{n}$, which is unsigned, corresponds to the average distance the stream centerline moves normal to itself per unit time. It is perhaps the most basic of any of the derived

Mississippi River A
at
Aitkin County
T52N-R23W, secs. 8,9,17,20

HYDROLOGIC INFORMATION

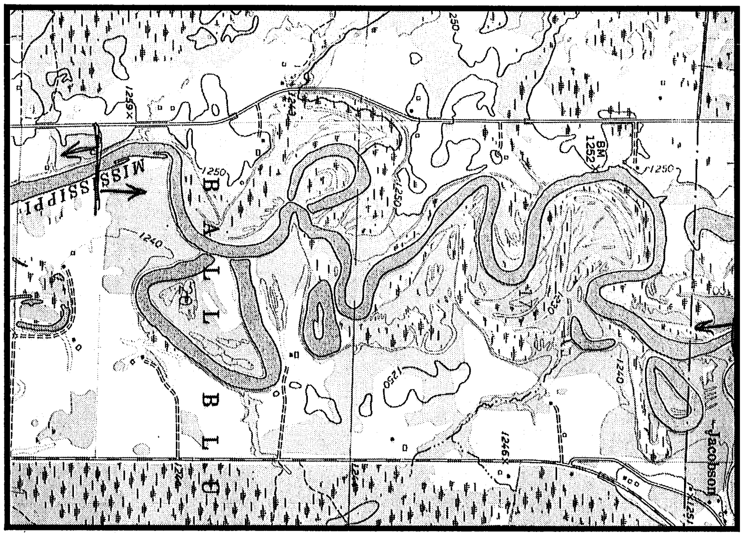
2-Year Storm Discharge:	2580 cfs
Bankfull Stream Width:	185 ft
Stream Depth (Synthesized):	6.2 ft
Stream Slope:	0.00010

GEOMORPHIC INFORMATION

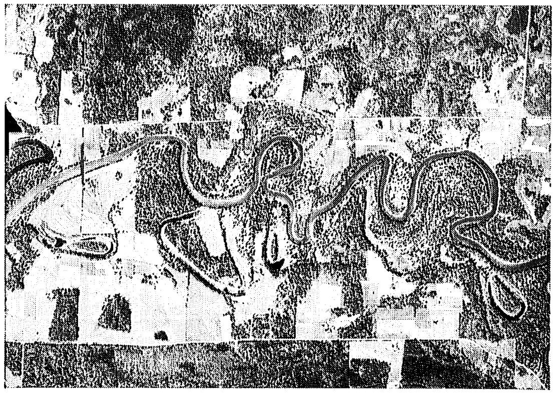
Sediment samples were not obtained for the Mississippi River, but grain size diameter D_{50} = 0.5 mm was estimated from observation.

Ground photo of Mississippi River A, 9/19/90

DESCRIPTION: The Mississippi River is, in this location, highly meandering, with fairly well defined valley walls, about 50-60 feet high.

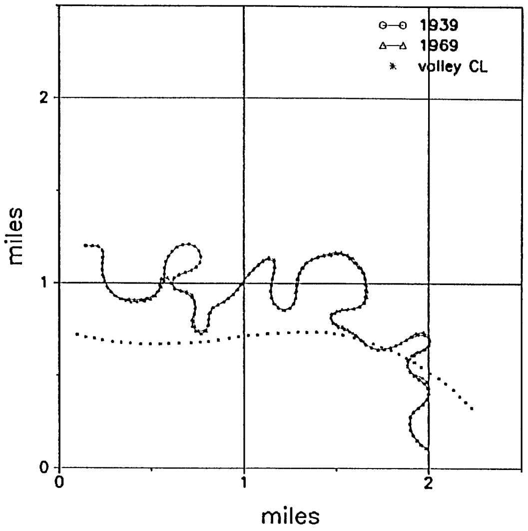


Base map, Mississippi River A, 1970



Aerial photograph, Mississippi River A, 1939

Aerial photograph, Mississippi River A, 1969



Digitized centerlines, Mississippi River A

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                    PROGRAM RESULTS

SHIFT
   Average Normal Shift:                      0.92 ft/yr
   Standard Deviation of Normal Shift:        0.04
   Average Absolute Transverse Shift:         0.64 ft/yr
   Average Absolute Longitudinal Shift:       0.55 ft/yr
   Average Transverse Shift:                  -0.03 ft/yr
   Average Longitudinal Shift:                0.02 ft/yr
   Shift Ratio:                               10.19

SINUOSITY
   Average Stream:                            2.25
   Average Change:                            0.00170

AVERAGE CURVATURE (ft^-1):                    0.00194

AREA WORKED (ft^2/ft/yr):                     2.06
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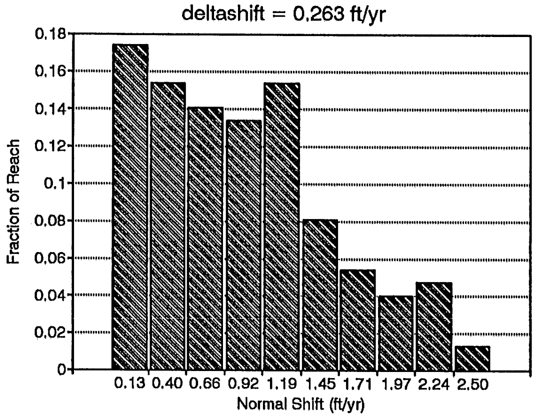


Shift distribution, Mississippi River A

results. It gives the planner a good idea of how much and how fast the stream banks are eroding.

$$\overline{n} = \frac{\Sigma n \Delta s}{\Delta t \Sigma \Delta s} \tag{16}$$

The average normal shift, $\overline{n}$, may be divided into average absolute down-valley shift, $\overline{|\dot{x}|}$, and average absolute cross-valley shift, $\overline{|\dot{y}|}$ to indicate shift direction with respect to the stream valley centerline. The average absolute cross-valley shift is especially useful, because it indicates how much the stream can be expected to shift to either side of the stream centerline. The shifts are computed as follows:

$$\overline{|\dot{x}|} = \frac{\Sigma \mid n \sin\theta \mid \Delta s}{\Delta t \Sigma \Delta s} \tag{17}$$

$$\overline{|\dot{y}|} = \frac{\Sigma \mid n \cos\theta \mid \Delta s}{\Delta t \Sigma \Delta s} \tag{18}$$

where n is the normal shift, as shown in Figure 6.

While the above quantities are absolute (unsigned) values, it is also useful to know the signed, average down-valley shift $\overline{\dot{x}}$ and cross-valley shift $\overline{\dot{y}}$, which can be computed as indicated below:

$$\overline{\dot{x}} = \frac{\Sigma n \sin\theta \Delta s}{\Delta t \Sigma \Delta s} \tag{19}$$

$$\overline{\dot{y}} = \frac{\Sigma n \cos\theta \Delta s}{\Delta t \Sigma \Delta s} \tag{20}$$

One would expect that over a sufficiently long reach of stream, the average cross-valley shift would be very small, because in an ideal case the stream shifts an equal amount in either direction transverse to the stream valley. This expectation would be better realized for long stream reaches. In shorter reaches, lateral shift is usually biased to one side or the other.

The average down-valley shift $\overline{\dot{x}}$ is smaller using the Lagrangian reference than if the wave reference were used, in which case $\overline{\dot{x}}$ would be equal to the translation component of the stream shift. An indication of how much the stream shifts in the cross-valley direction, as compared to the down-valley direction is given by the ratio $\dot{y}_{rms}/\overline{\dot{x}}$, which is the root mean square of the cross-valley shift divided by the average down-valley shift.

After the absolute normal shift has been computed for each stream coordinate, the flood plain area reworked by the stream per unit stream length and time, is computed, as shown in Fig. 12:

The area reworked is simply an integration of the area between the stream centerline at time 1 and at time 2. If $\dot{r}$ denotes the time rate of area reworked per channel length, then

$$\dot{r} = \frac{\Sigma \mid n \mid \Delta s^2}{\Delta t \Sigma \Delta s} \tag{21}$$

The area reworked is useful for land managers, as it allows them to predict how much land surrounding a stream reach will be affected by meandering.

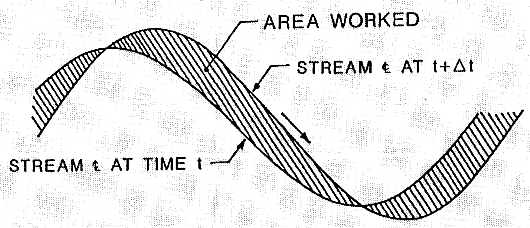


Figure 12: Area reworked by a stream

Error Sources and Caveats

There are several significant sources of error inherent to the shift-measurement process. It must be remembered that the results are reach-specific and time-specific, and that error is introduced through the photography and digitizing process.

It is important that it be known that a given shift rate applies only to the stream reach in question. This holds true especially for smaller streams, which often change rapidly from one point to the next.

Mississippi River B
at
Aitkin County
T52N-R23W, secs. 20,29,30,31,32

HYDROLOGIC INFORMATION

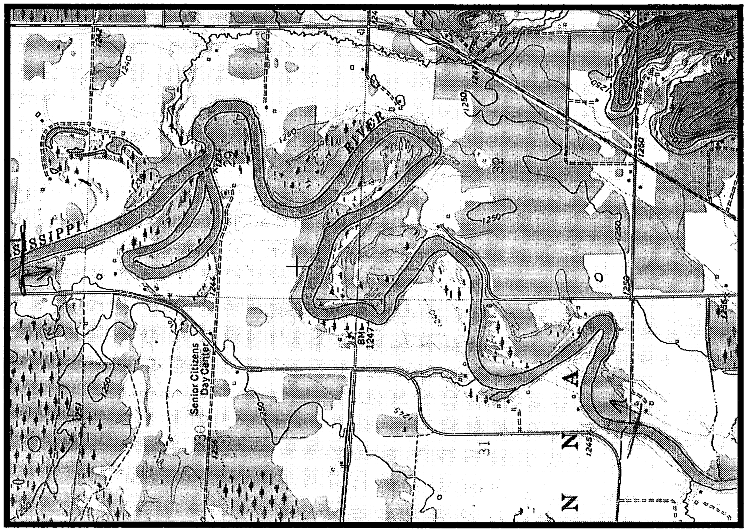
2-Year Storm Discharge:	2580 cfs
Bankfull Stream Width:	220 ft
Stream Depth (Synthesized):	5.6 ft
Stream Slope:	0.00010

GEOMORPHIC INFORMATION

Sediment samples were not obtained for the Mississippi River, but grain size diameter $D_{50} = 0.5$ mm was estimated from observation

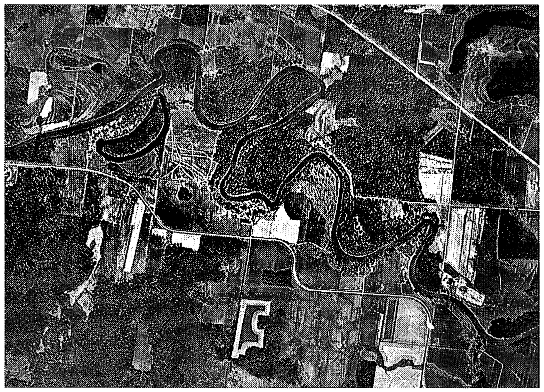
Ground photo of Mississippi River B, 9/19/90

DESCRIPTION: The Mississippi River is, in this location, highly meandering, with fairly well defined valley walls, about 50-60 feet high.

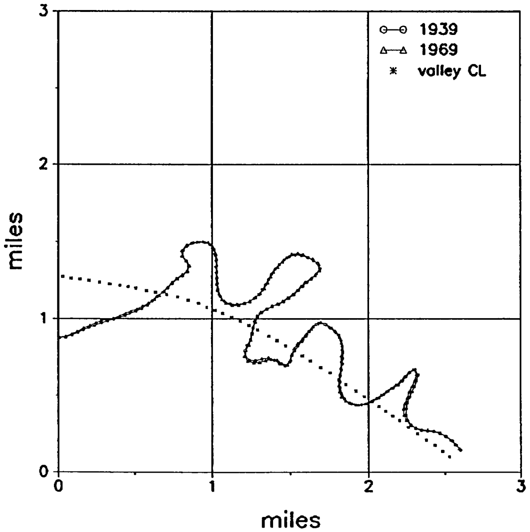


Base map, Mississippi River B, 1970

Aerial photograph, Mississippi River B, 1939



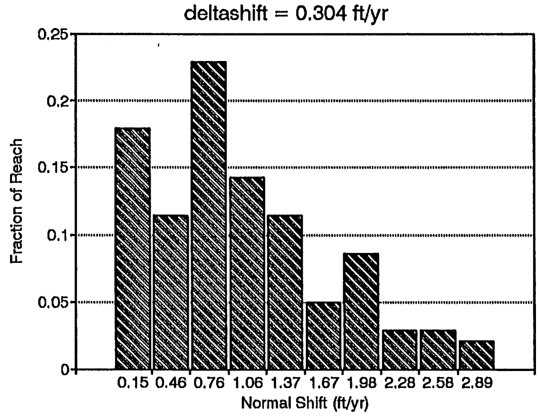
Aerial photograph, Mississippi River B, 1969



Digitized centerlines, Mississippi River B

PROGRAM RESULTS

SHIFT	
Average Normal Shift:	1.02 ft/yr
Standard Deviation of Normal Shift:	0.04
Average Absolute Transverse Shift:	0.63 ft/yr
Average Absolute Longitudinal Shift:	0.68 ft/yr
Average Transverse Shift:	0.26 ft/yr
Average Longitudinal Shift:	-0.40 ft/yr
Shift Ratio:	-0.84
SINUOSITY	
Average Stream:	2.17
Average Change:	0.00067
AVERAGE CURVATURE (ft^{-1}):	0.00141
AREA WORKED ($ft^2/ft/yr$):	2.22



Shift distribution, Mississippi River B

Likewise, the given shift rates apply only during the stated time period. The importance of this was demonstrated during the analysis of the Root River. Because the Root River shifts so fast, the original 21 year time period had to be split into three smaller time periods. During the first time period, the average sinuosity change was 0.00072/yr. During the second time period, it was -0.00113/yr. During the third period, -0.00301/yr. The rate of change varied greatly from one time period to the next, even though no cut-offs occurred during this time. Thus, the importance of confining the results to the time period in question cannot be overstated.

Another frustrating source of error lies within the photographs. Each photograph has a certain amount of distortion which can only be partially compensated for in the enlargement process. The topographic base maps, however, being free of significant distortion, were used to gauge the acceptability of the enlarged photographs. Nevertheless, this source of error cannot be overlooked.

Finally, the digitizing process can produce small errors. Although the digitizer has an advertised resolution of 0.0001 inch, the hands of the user are not likely to be as accurate. Thus, small errors are introduced here. Although the filtering process undoubtedly removes most of the error, it is difficult to estimate how much error remains.

The computer program, which measures the shift, is a potential source of error. However, this is checked within the program, and significant sources of error are removed.

***

(Cont'd from page 5)

Parker, assumes that erosion rates are proportional to the difference between near-bank velocity and reach-averaged velocity. He tests the results of his model by analyzing aerial photos, maps, and flow information for the Des Moines River and the East Nishnabotna River in Iowa.

Various other techniques have been used to predict stream migration rates. Watanabe, Kiyoshi, Murakami, and Hasegawa (17) reproduced flow direction and velocities at peak discharge for an August 1988 flood of the Rumoi River in Japan. They did so by noting the angle of deflection of trees and grass in the flood plain, and relating flow velocity to the force required to drag the trees. By doing so, they deduced that the bank erosion was not dependent on bank vegetation and soil type. They were able to show that flood plain flow contributed to bank erosion when the flow on the flood plain was parallel to flow in the channel.

Hasegawa (6) derived an equation for bank erosion and migration rate, confirming earlier studies by Hasegawa and Ito (7), and Parker (15), which hypothesized that bank erosion is proportional to the near-bank excess velocity. Hasegawa went on to define an erosion index, and related it to the number of blows in a standard penetration test. The erosion index varied inversely with the number of blows. He found two distinct trends, depending on whether the bank material was sandy or clayey.

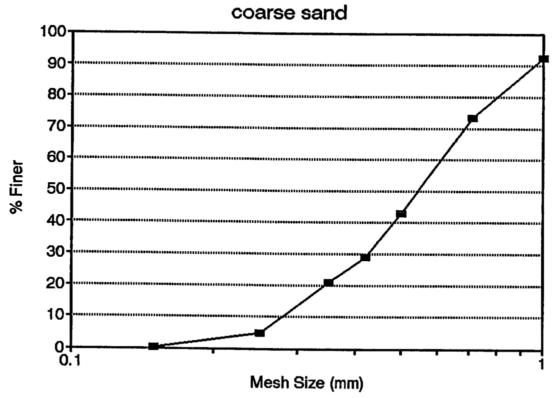
While many methods have been developed to predict stream channel migration, methods of testing the validity of the models have consisted largely of manually measuring stream shift in the field or from aerial maps or photos. The analysis presented here should provide an easier means of validating the accuracy of the predictive models.

Nemadji River
at
Carlton County
T47N-R16W, secs. 27,28,29,32

HYDROLOGIC INFORMATION

2-Year Storm Discharge:	1810 cfs
Bankfull Stream Width:	110 ft
Stream Depth (Synthesized):	2.3 ft
Stream Slope:	0.00180

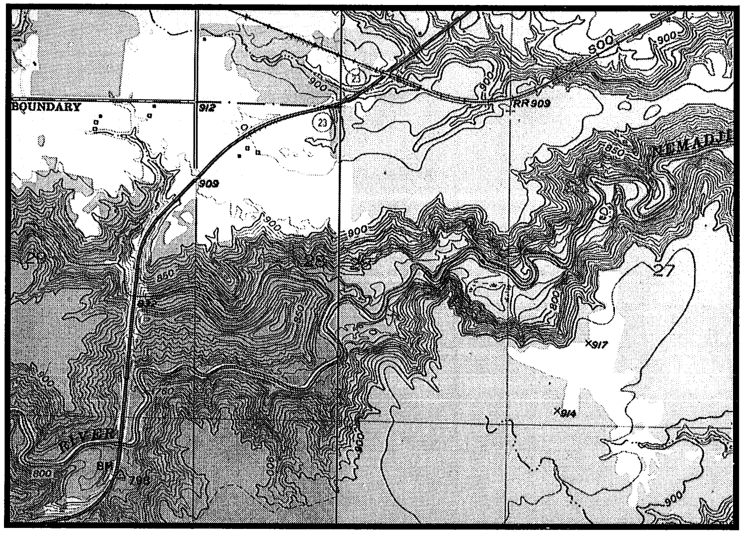
GEOMORPHIC INFORMATION



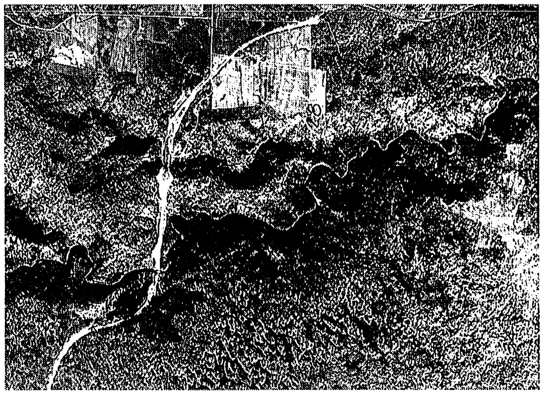
Grain size distribution, Nemadji River

Ground photo of Nemadji River, 8/6/90

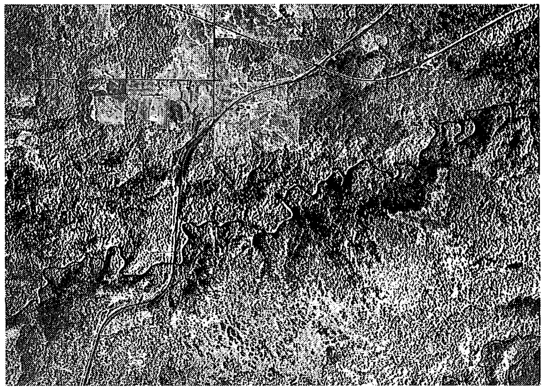
DESCRIPTION: The Nemadji River is a sand-bed stream in northeastern Minnesota. It is deeply incised, with 150 ft. valley walls. Although it doesn't shift very much, it is the leading contributor of sediment (primarily red clay) to Lake Superior.



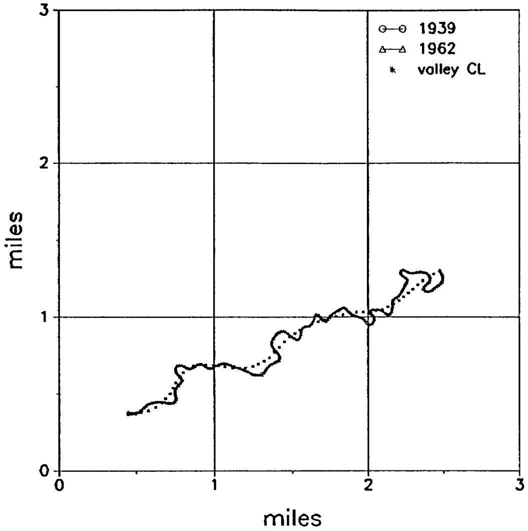
Base map, Nemadji River, 1954



Aerial photograph, Nemadji River, 1939



Aerial photograph, Nemadji River, 1962



Digitized centerlines, Nemadji River

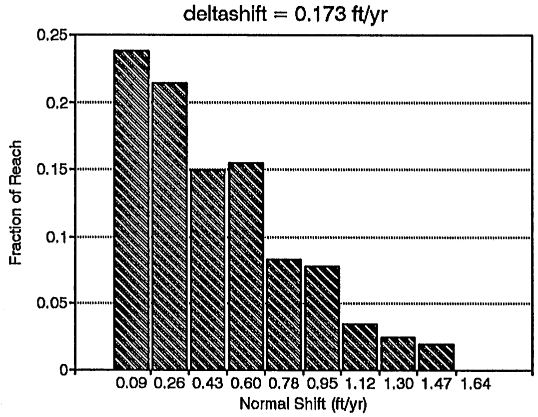
PROGRAM RESULTS

SHIFT
Average Normal Shift: 0.48 ft/yr
Standard Deviation of Normal Shift: 0.01
Average Absolute Transverse Shift: 0.34 ft/yr
Average Absolute Longitudinal Shift: 0.27 ft/yr
Average Transverse Shift: 0.02 ft/yr
Average Longitudinal Shift: 0.12 ft/yr
Shift Ratio: 0.60

SINUOSITY
Average Stream: 1.46
Average Change: 0.00046

AVERAGE CURVATURE (ft^{-1}): 0.00419

AREA WORKED ($ft^2/ft/yr$): 0.70



Shift distribution, Nemadji River

6 Plotted Relationships and Regression Analysis

Having analyzed sixteen Minnesota streams for shift and meander, relationships were sought between several stream parameters and the measured results. Relations were found for internal hydraulic geometry and external meander properties, both dimensioned and dimensionless.

Independent Parameters

The internal stream parameters used were variations of slope S, sinuosity S, two-year storm discharge Q, bankfull width B, and depth H. There are differences in the accuracy with which each parameter was determined for the sixteen streams, and thus the reader should be aware of how each was measured or computed.

Stream slope S was measured from 7.5' topographic maps of the reach considered. There is likely some deviation between the slopes as measured from the topographic maps and the actual stream slopes. The deviation is due to inaccuracies in the maps, and to the fact that, in most cases, the topographic maps are of different dates than the aerial photography used in the studies.

Stream sinuosity S was measured within the computer program MEANDER, and should be very accurate for the reach considered.

The stream discharge Q used in the regressions was the flood that occurs on a two-year interval. According to a study conducted by Nixon (12) the two-year flood discharge corresponds closely to flow over the full bankfull width. Stream discharge Q was estimated for each stream reach using one of two methods, depending on the existence and location of gage stations. Where there existed a gaging station at or near the study site, a Log-Pearson type III frequency analysis was performed on recorded historical data. Where a gage did not exist near the site, discharge was determined by transferring information from a gaged site, using regional regression equations. Stream data contained in the U.S. Geological Survey Water-Resources Investigations Report 87-4170, titled "Techniques for estimating the magnitudes and frequency of floods in Minnesota," were used with both methods.

Bankfull stream width B was estimated directly from a topographic map of the area considered. This was deemed more appropriate than field measurement because the size and condition of many of the rivers made field measurement dangerous or impossible. The measurement method for bankfull stream width is admittedly rough, and this should be remembered when viewing the regressions.

Stream depth was synthesized using the method proposed by Brownlie (5) in 1981 for sand bed streams, or the Keulegan (10) method for gravel bed streams. For this purpose, the two-year flood discharge was used, as well as the stream slope and width, as measured from the topographic maps. Median grain size diameter was determined using grain size data from our own sieve analyses. For streams where we were unable to obtain sediment samples, the diameter was obtained from other sources, as explained in Chapter 2 and in the individual Stream Meander Results.

Dimensionless Parameters

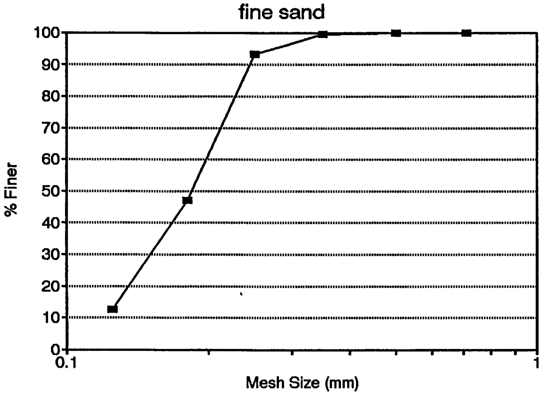
Where appropriate, independent (stream) parameters and dependent parameters (shift results) were made dimensionless, using a length scale D_s, velocity scale $\sqrt{gD_s}$, and time scale $\sqrt{\frac{D_s}{g}}$. The appropriate scale is used to make width B, depth H, flow rate Q, shift rate $\dot{n}$, time rate of change of sinuosity $\dot{S}$, time rate of area reworked per unit stream length, and curvature C dimensionless. The dimensionless terms are defined as follows:

Rice Creek
at
Anoka County
T30N-R24W, sec. 6

HYDROLOGIC INFORMATION

2-Year Storm Discharge:	456 cfs
Bankfull Stream Width:	44 ft
Stream Depth (Synthesized):	1.6 ft
Stream Slope:	0.00175

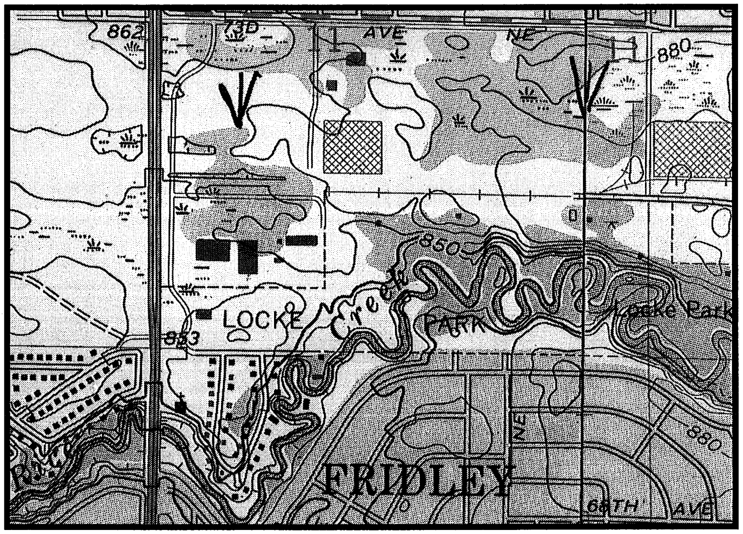
GEOMORPHIC INFORMATION



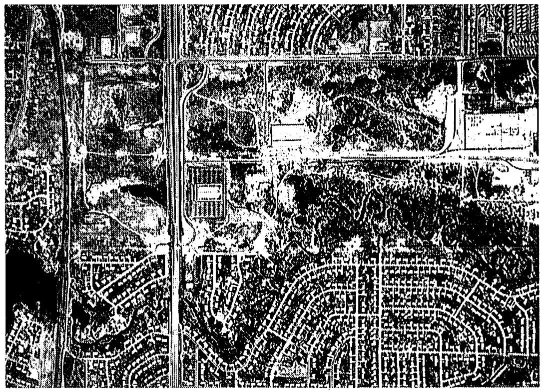
Grain size distribution, Rice Creek

Ground photo of Rice Creek, 4/16/91

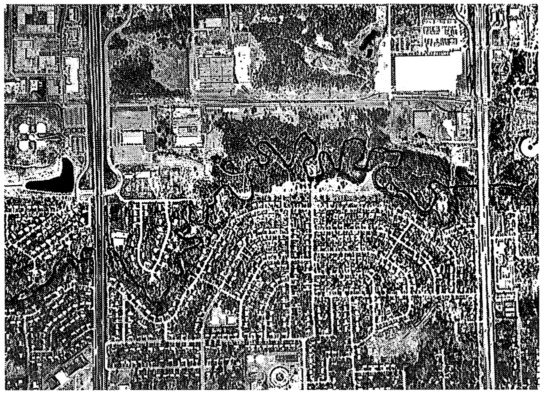
DESCRIPTION: Rice Creek is a small, highly meandering sand-bed stream near Minneapolis.



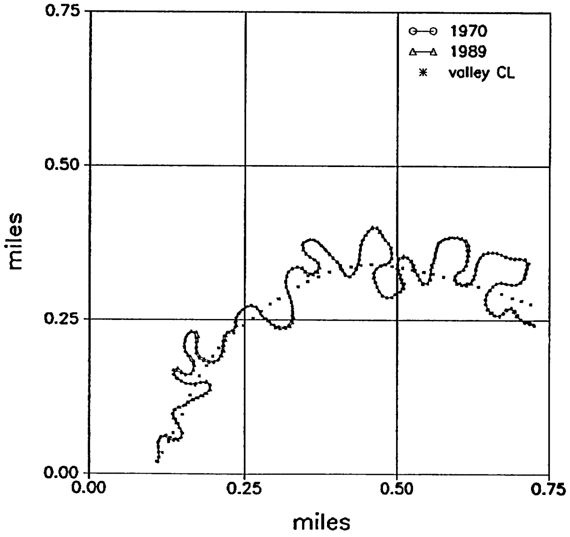
Base map, Rice Creek, 1967



Aerial photograph, Rice Creek, 1970



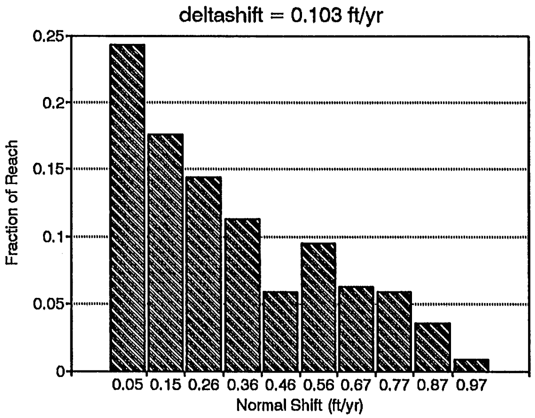
Aerial photograph, Rice Creek, 1989



Digitized centerlines, Rice Creek

PROGRAM RESULTS

SHIFT	
Average Normal Shift:	0.33 ft/yr
Standard Deviation of Normal Shift:	0.03
Average Absolute Transverse Shift:	0.19 ft/yr
Average Absolute Longitudinal Shift:	0.22 ft/yr
Average Transverse Shift:	0.07 ft/yr
Average Longitudinal Shift:	0.01 ft/yr
Shift Ratio:	5.89
SINUOSITY	
Average Stream:	2.61
Average Change:	0.00255
AVERAGE CURVATURE (ft^{-1}):	0.01674
AREA WORKED ($ft^2/ft/yr$):	0.86



Shift distribution, Rice Creek

Independent parameters:

$$B^* = \frac{B}{D_s} \tag{22}$$

$$H^* = \frac{H}{D_s} \tag{23}$$

$$Q^* = QD_s^2\sqrt{gD_s} \tag{24}$$

$$\dot{S}^* = \dot{S}\sqrt{\frac{D_s}{g}} \tag{25}$$

Dependent Parameters:

$$\dot{n}^* = \dot{n}(gD_s)^{-0.5} \tag{26}$$

$$\dot{r}^* = \dot{r}(rD_s)^{-0.5} \tag{27}$$

Each of the stream parameters used, both dimensioned and dimensionless, is also defined in the List of Symbols. It is hoped that by using dimensionless forms of the parameters, sediment properties are incorporated into the regressions.

Each of the regressions plotted has additional information and comments for the user's reference. For each regression, the coefficient of correlation r and the standard error of the estimate s_{yx} is given, to indicate how well the data regressed.

Each dimensioned plot uses each of the 16 stream results. The dimensionless plots, however, exclude the two gravel bed streams because grain size diameter is used to make many of the parameters dimensionless. Occasionally, an outlier is also excluded from a plot, as mentioned in the comments.

Appendix 2 lists each of the regressions which were performed, with the relationship, where there appeared to be one.

Internal Relations - Dimensioned

The internal relations were plotted to see if the hydraulic geometry of the analyzed streams are related. In particular, it was desired to compare our relations to those suggested by Leopold and Maddock (11), who found from empirical evidence that at bankfull conditions, width B and depth H vary with discharge Q as follows:

$$B = aQ^{0.5} \tag{28}$$

$$H = bQ^{0.4} \tag{29}$$

where a and b are constants.

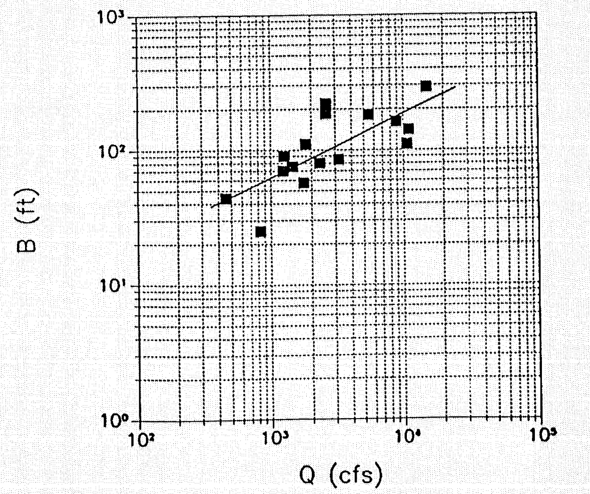


Figure 13: Width vs Discharge

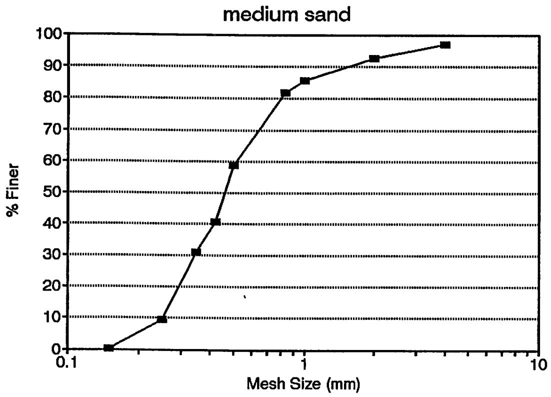
Comments: $B = 2.45Q^{0.47}$, $r = 0.75$, $s_{yx} = 0.19$. There is good agreement between this relation and that theorized by Leopold and Maddock (11).

Root River
at
Houston County
T104N-R7W, secs. 26,34,35

HYDROLOGIC INFORMATION

2-Year Storm Discharge:	10700 cfs
Bankfull Stream Width:	110 ft
Stream Depth (synthesized):	9.5 ft
Stream Slope:	0.00061

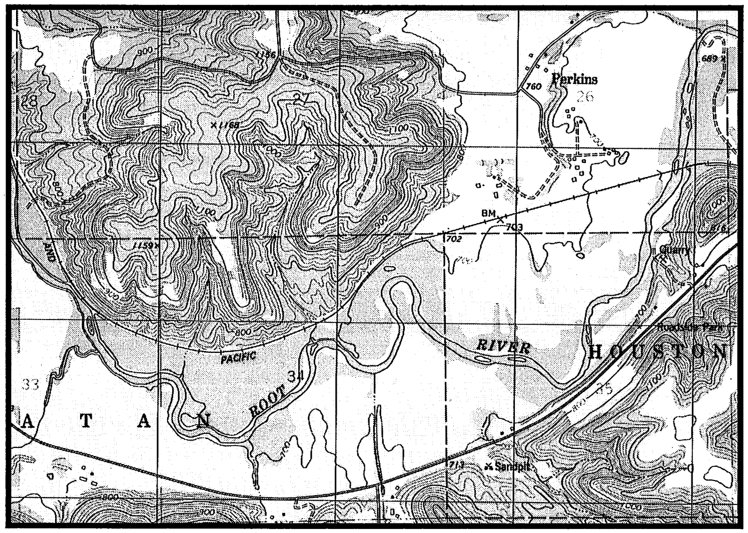
GEOMORPHIC INFORMATION



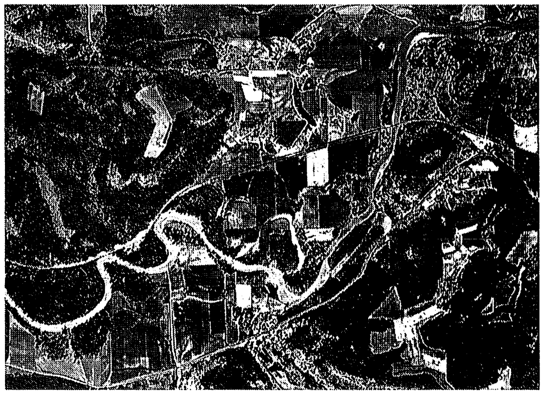
Grain size distribution, Root River

Ground photo of Root River, 6/20/90

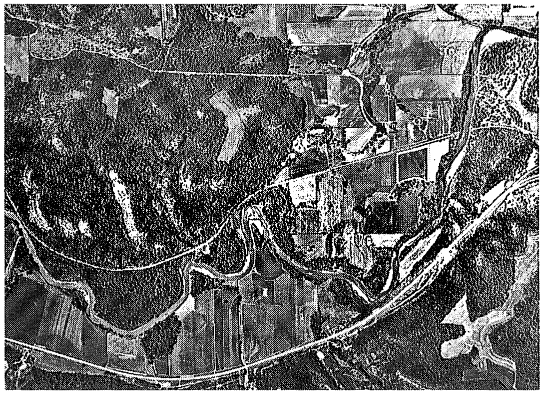
DESCRIPTION: The Root River is a very highly meandering sand-bed stream in southeastern Minnesota. It is prone to rapid shift and avulsion.



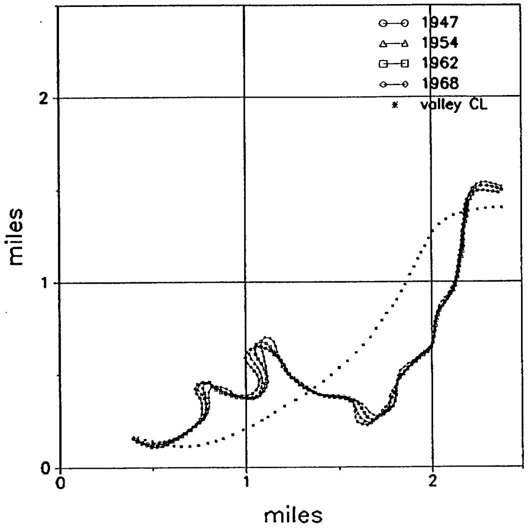
Base map, Root River, 1980



Aerial photograph, Root River, 1947



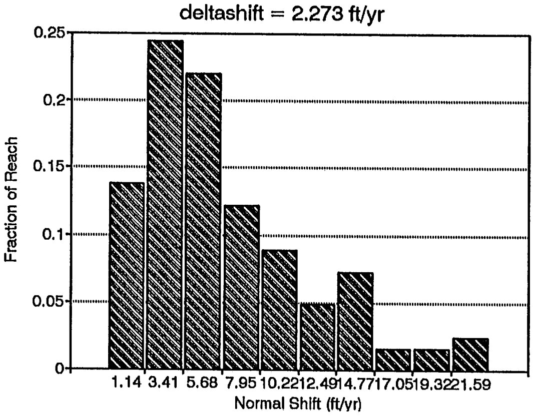
Aerial photograph, Root River, 1968



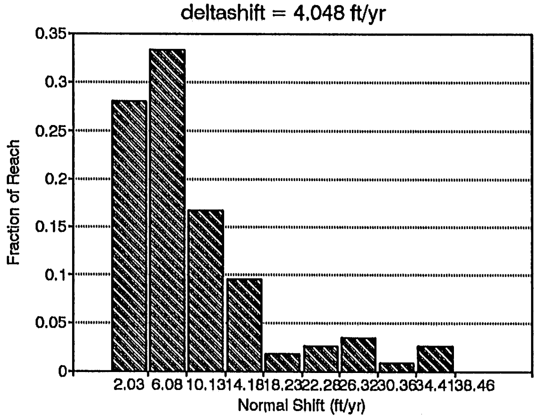
Digitized centerlines, Root River

PROGRAM RESULTS

SHIFT	
Average Normal Shift:	7.30 ft/yr
Standard Deviation of Normal Shift:	0.07
Average Absolute Transverse Shift:	4.67 ft/yr
Average Absolute Longitudinal Shift:	4.49 ft/yr
Average Transverse Shift:	-3.70 ft/yr
Average Longitudinal Shift:	2.37 ft/yr
Shift Ratio:	1.30
SINUOSITY	
Average Stream:	1.44
Average Change:	-0.00105
AVERAGE CURVATURE (ft^{-1}):	0.00160
AREA WORKED ($\mathrm{ft}^2/\mathrm{ft}/\mathrm{yr}$):	10.43



Shift distribution, Root River 1947-1954



Shift distribution, Root River 1962-1968

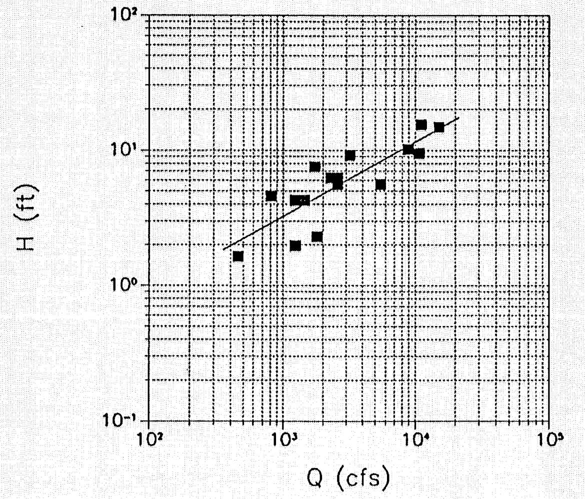


Figure 14: Depth vs Discharge

Comments: $H = 0.074Q^{0.55}$, $r = 0.84$, $s_{yx} = 0.16$. There is some deviation between this relation and that theorized by Leopold and Maddock (11), possibly due to the manner in which channel depth was synthesized. Note the stronger relation than for B vs Q (Fig. 13).

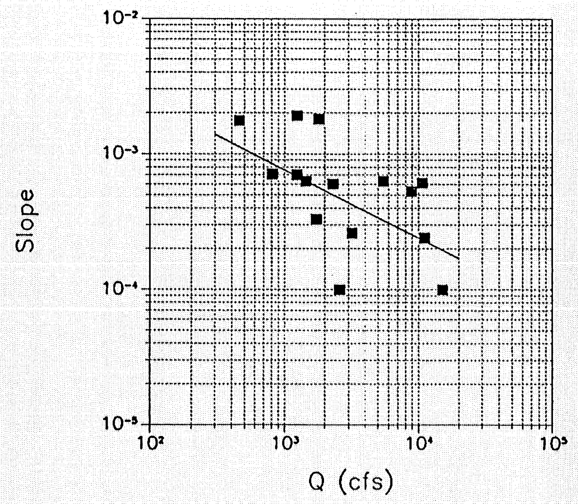


Figure 15: Slope vs Discharge

Comments: (Fig. 15). $S = 0.024Q^{-0.50}$, $r = -0.52$, $s_{yx} = 0.37$. Slope is not as strongly related to discharge as width or depth.

Internal Relations - Dimensionless

The internal relations were also plotted in dimensionless form. By doing so, it can be noted how the introduction of sediment characteristics affects the relations.

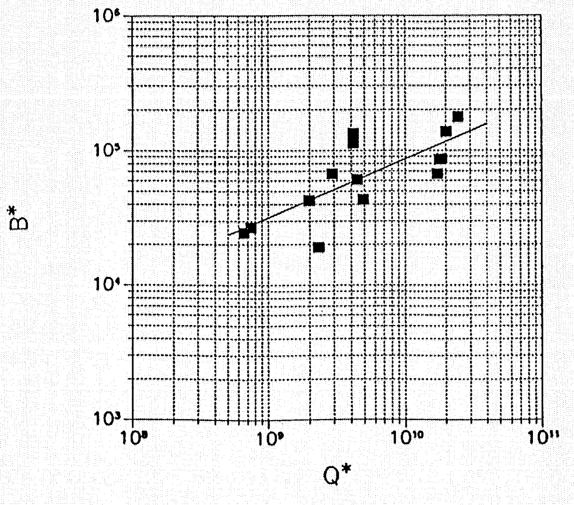


Figure 16: Dimensionless Width vs Dimensionless Discharge

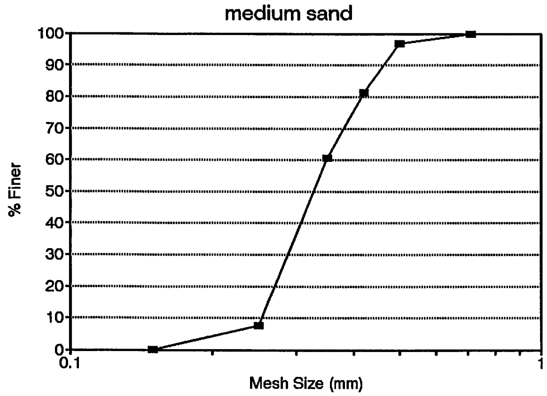
Comments: $B^* = 4.12Q^{*0.43}, r = 0.76, s_{yx} = 0.20$. The exponent has been lowered from 0.47 to 0.43 (see Fig. 13).

Rum River
at
Isanti County
T35N-R25W, secs. 1,12
T36N-R25W, sec. 36

HYDROLOGIC INFORMATION

2-Year Storm Discharge:	3203 cfs
Bankfull Stream Width:	85 ft
Stream Depth (Synthesized):	9.2 ft
Stream Slope:	0.00026

GEOMORPHIC INFORMATION

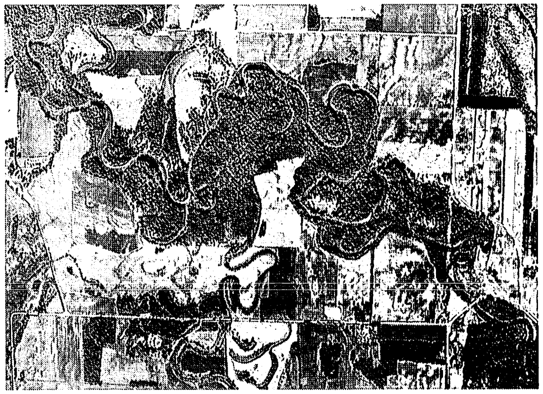


Grain size distribution, Rum River

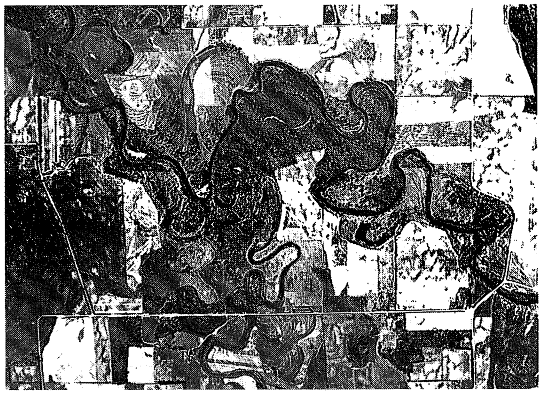
Ground photo of Rum River, 4/12/90

DESCRIPTION: The Rum River is a highly meandering sand-bed stream in eastern Minnesota. It lies in a shallow, ill-defined valley Its sediment is uniform, medium sand.

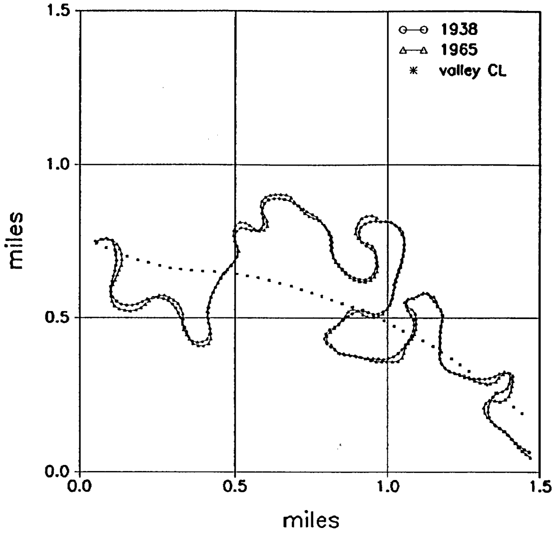
Base map, Rum River, 1961



Aerial photograph, Rum River, 1938



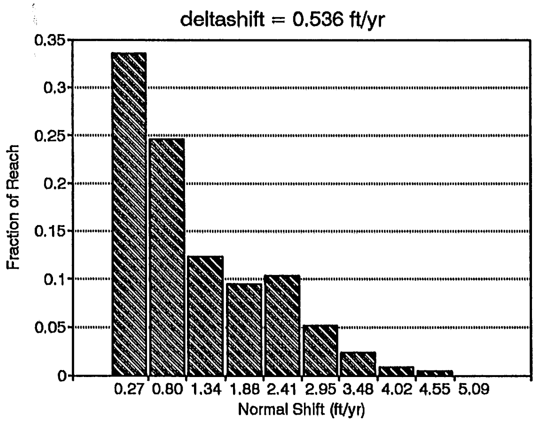
Aerial photograph, Rum River, 1965



Digitized centerlines, Rum River

PROGRAM RESULTS

SHIFT	
Average Normal Shift:	1.18 ft/yr
Standard Deviation of Normal Shift:	0.07
Average Absolute Transverse Shift:	0.79 ft/yr
Average Absolute Longitudinal Shift:	0.72 ft/yr
Average Transverse Shift:	0.06 ft/yr
Average Longitudinal Shift:	-0.23 ft/yr
Shift Ratio:	-1.26
SINUOSITY	
Average Stream:	2.61
Average Change:	0.00801
AVERAGE CURVATURE (ft^{-1}):	0.00386
AREA WORKED ($ft^2/ft/yr$):	3.10



Shift distribution, Rum River

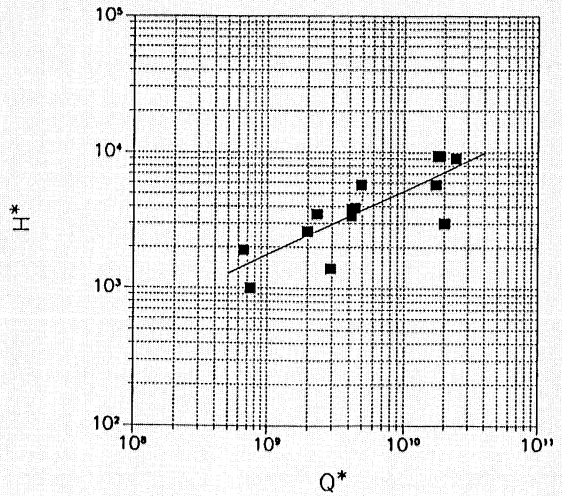


Figure 17: Dimensionless Depth vs Dimensionless Discharge

Comments: $H^* = 0.11Q^{*0.47}, r = 0.82, s_{yx} = 0.18$. The exponent has been lowered from 0.55 to 0.47 (see Fig. 14).

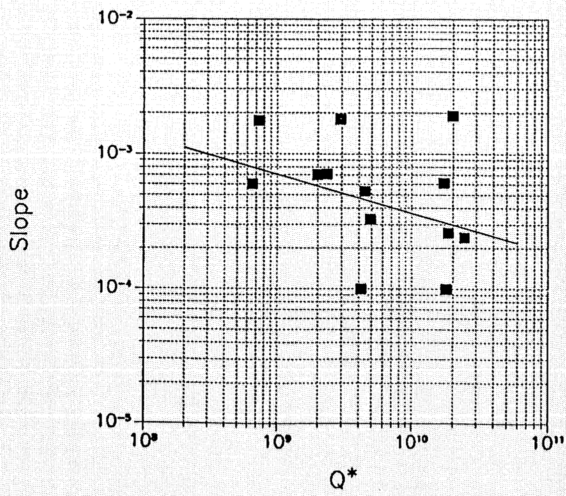


Figure 18: Slope vs Dimensionless Discharge

Comments: $S = 0.26Q^{*-0.29}, r = -0.33$, $s_{yx} = 0.44$. This is a weaker relation than for the dimensioned case (see Fig. 15).

External Relations - Dimensioned

Relationships were sought between the independent stream parameters and the dependent shift parameters. Because similar relations have not been produced elsewhere, comparisons cannot be made with other findings.

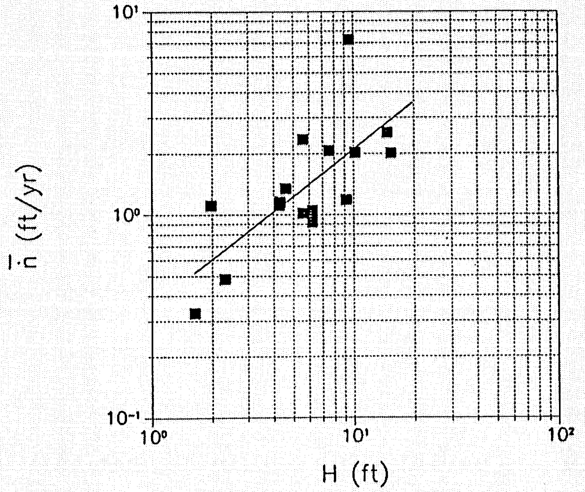


Figure 19: Average Normal Shift Rate vs Depth

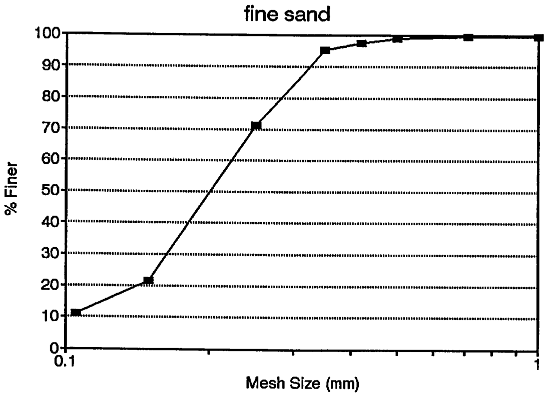
Comments: $\overline{n} = 0.35H^{0.78}, r = 0.73, s_{yx} = 0.18$. It appears that average normal shift rate is more strongly related to stream depth than width.

Wild Rice River A
at
Norman County
T144N-R44W, secs. 16,17,20,21,22

HYDROLOGIC INFORMATION

2-Year Storm Discharge:	1250 cfs
Bankfull Stream Width:	90 ft
Stream Depth (synthesized):	2.0 ft
Stream Slope:	0.00190

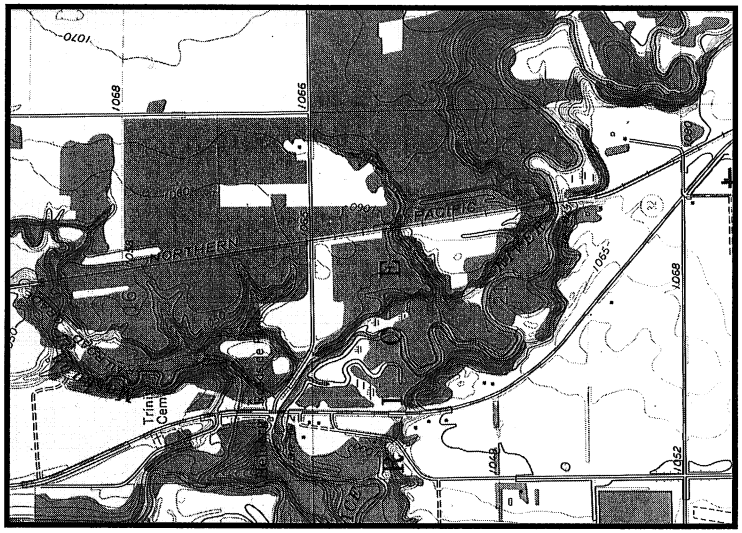
GEOMORPHIC INFORMATION



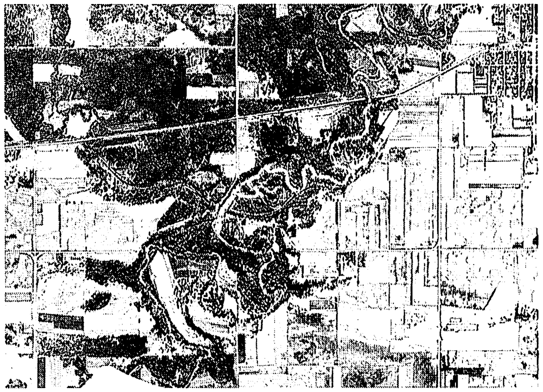
Grain size distribution, Wild Rice River A

Ground photo of Wild Rice River A, 10/12/90

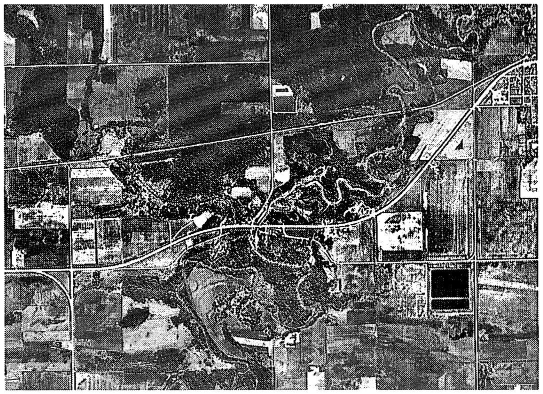
DESCRIPTION: The Wild Rice River is a sand-bed stream with tight, well-defined valley walls approximately 30 feet high.



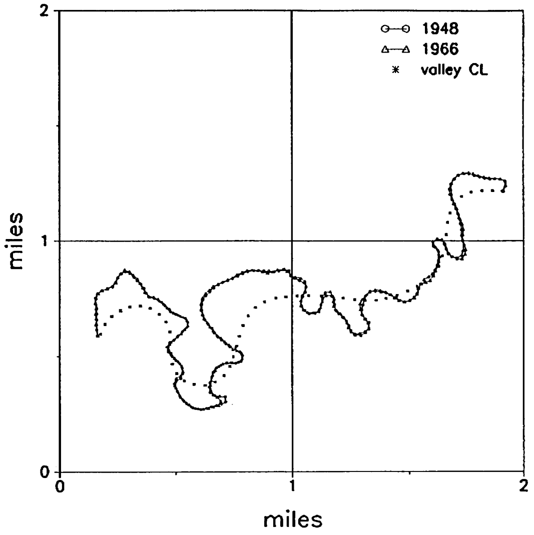
Base map, Wild Rice River A, 1965



Aerial photograph, Wild Rice River A, 1948



Aerial photograph, Wild Rice River A, 1965



Digitized centerlines, Wild Rice River A

PROGRAM RESULTS

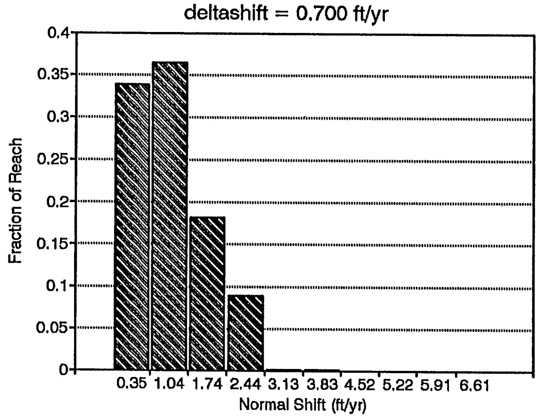
SHIFT

Average Normal Shift:	1.11 ft/yr
Standard Deviation of Normal Shift:	0.03
Average Absolute Transverse Shift:	0.70 ft/yr
Average Absolute Longitudinal Shift:	0.72 ft/yr
Average Transverse Shift:	-0.26 ft/yr
Average Longitudinal Shift:	-0.22 ft/yr
Shift Ratio:	-1.37

SINUOSITY

Average Stream:	1.72
Average Change:	0.00060

AVERAGE CURVATURE (ft^{-1}):	0.00288
AREA WORKED ($ft^2/ft/yr$):	1.91



Shift distribution, Wild Rice River A

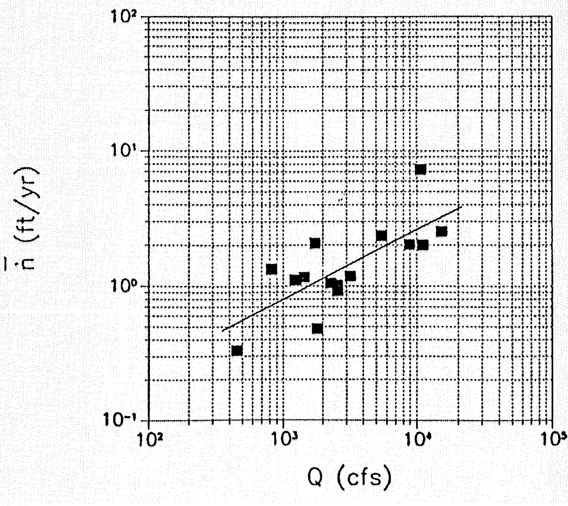


Figure 20: Average Normal Shift Rate vs Discharge

Comments: $\dot{\overline{n}} = 0.022Q^{0.52}, r = 0.75, s_{yx} = 0.21$. The regression indicates that average normal shift rate is more strongly related to discharge than width, but about equally as strongly related to discharge as depth.

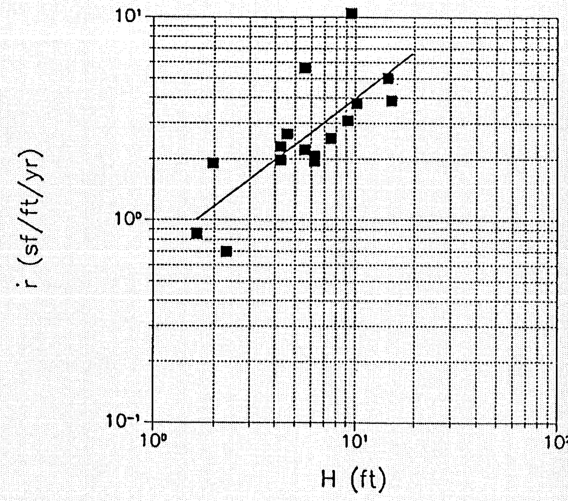


Figure 21: Area Reworked vs Depth

Comments: $\dot{r} = 0.69H^{0.76}, r = 0.76, s_{yx} = 0.19$. As one would expect, this relation is similar to the relation for $\dot{\overline{n}}$ *vs* H (see Fig. 19).

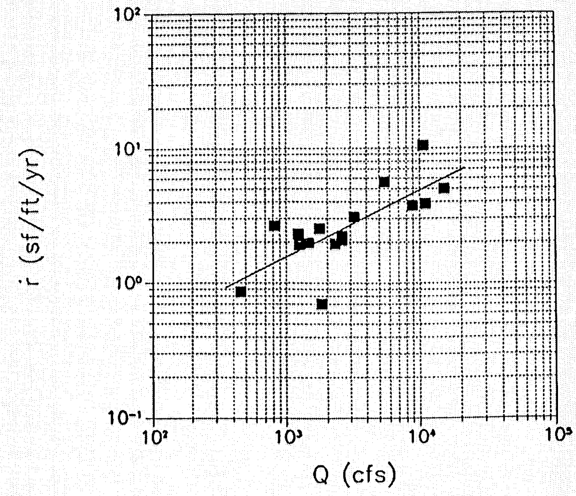


Figure 22: Area Reworked vs Discharge

Comments: $\dot{r} = 0.049Q^{0.50}, r = 0.76, s_{yx} = 0.19$. This relation also is similar to the relation for $\dot{\overline{n}}$ *vs* Q (see Fig. 20).

External Relations - Dimensionless

The external relations were also plotted in dimensionless form. By plotting the parameters in dimensionless form, grain size is introduced as a variable. It is hoped that grain size reflects sediment properties to some extent. While sediment properties can be represented by other measures, such as the standard penetration test used by Hasegawa (6), grain size is a more widely used indicator. Making the plots dimensionless also removes the possibility of misinterpreting results because of the selection of units.

It is interesting to compare the dimensionless and dimensioned plots, and to speculate on differences. This will be attempted in the comments for each plot.

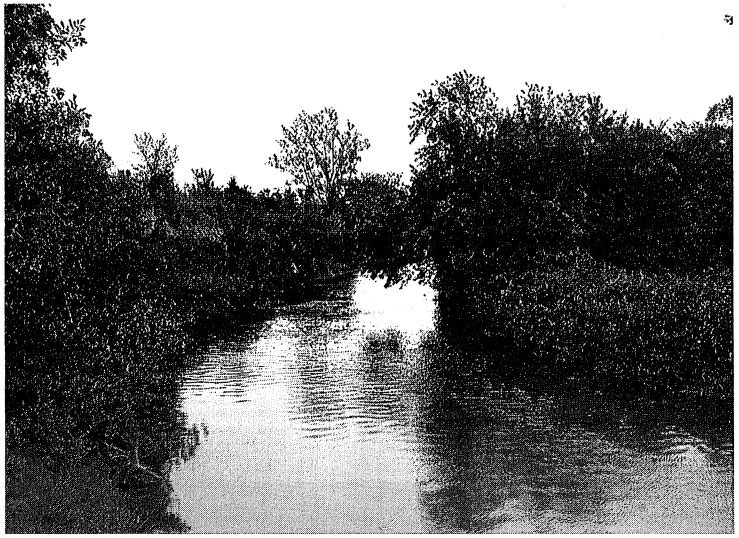
Yellow Medicine River
at
Yellow Medicine County
T114N-R39W, secs. 3,4,5,6,7
T115N-R39W, secs. 33,34

HYDROLOGIC INFORMATION

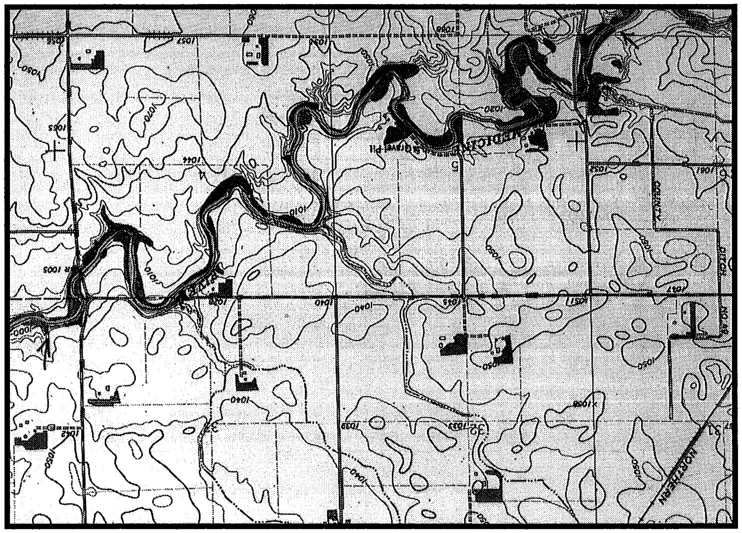
2-Year Storm Discharge:	1230 cfs
Bankfull Stream Width:	70 ft
Stream Depth (Synthesized):	4.3 ft
Stream Slope:	0.00070

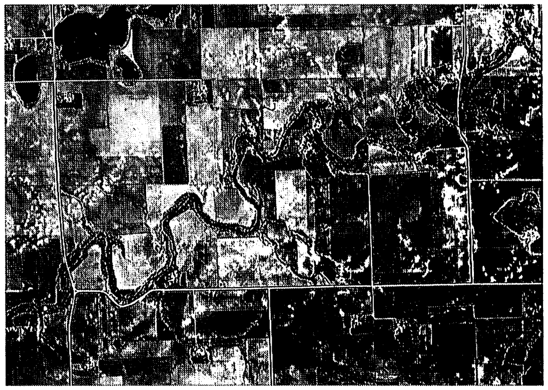
GEOMORPHIC INFORMATION

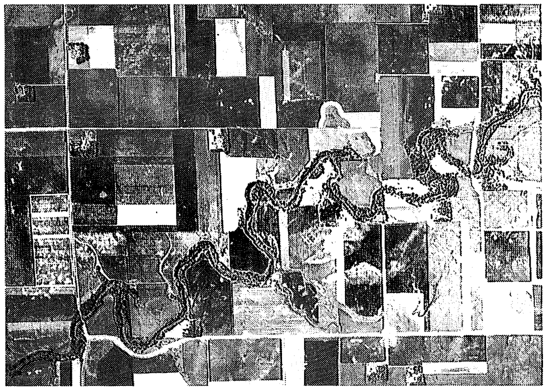
Sediment samples were not obtained for the Yellow Medicine River, but grain size diameter $D_{50} = 0.5$ mm was estimated from observation

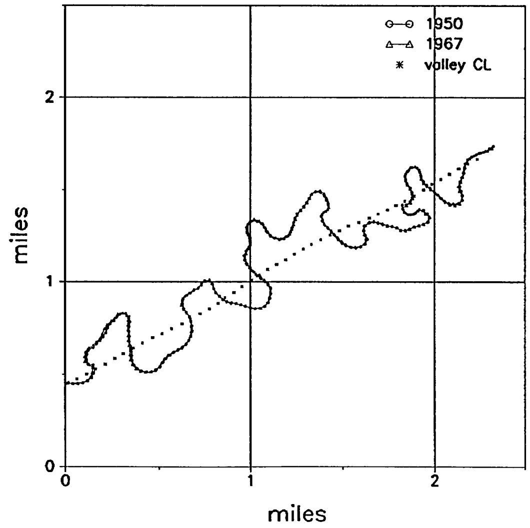

Ground photo of Yellow Medicine River, 7/27/90

DESCRIPTION: The Yellow Medicine River is a sand-bed stream with steep, tight, but shallow valley walls, approximately 20 feet high.


Base map, Yellow Medicine River, 1962


Aerial photograph, Yellow Medicine River, 1950


Aerial photograph, Yellow Medicine River, 1967



Digitized centerlines, Yellow Medicine River

PROGRAM RESULTS

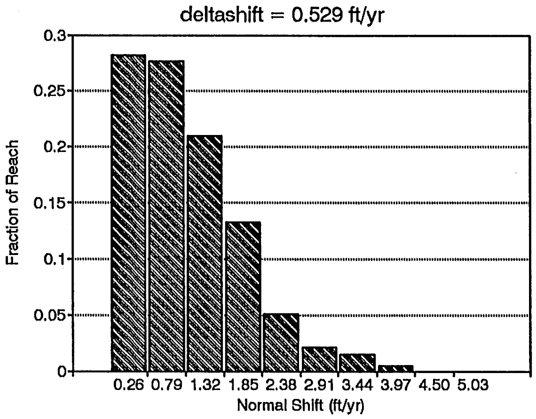
SHIFT

Average Normal Shift:	1.11 ft/yr
Standard Deviation of Normal Shift:	0.02
Average Absolute Transverse Shift:	0.66 ft/yr
Average Absolute Longitudinal Shift:	0.76 ft/yr
Average Transverse Shift:	-0.37 ft/yr
Average Longitudinal Shift:	0.07 ft/yr
Shift Ratio:	5.20

SINUOSITY

Average Stream:	2.07
Average Change:	0.00048

AVERAGE CURVATURE (ft^{-1}):	0.00302
AREA WORKED ($ft^2/ft/yr$):	2.30



Shift distribution, Yellow Medicine River

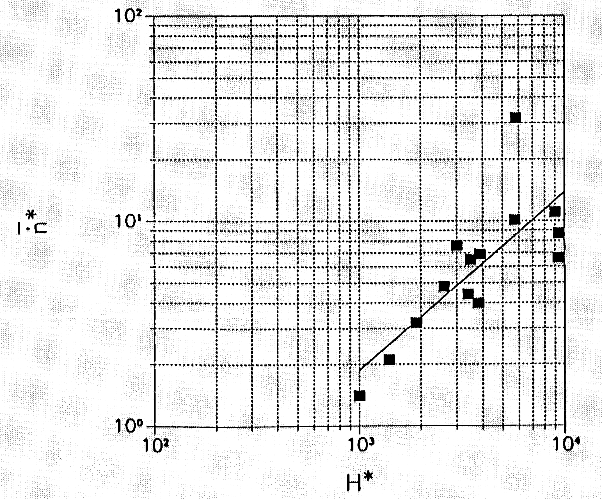


Figure 23: Dimensionless Average Normal Shift vs Dimensionless Depth

Comments: $\dot{n} = 0.049H^{0.86}, r = 0.78, s_{yx} = 0.22$. This relation appears to be similar in strength to the dimensioned case (see Fig. 19).

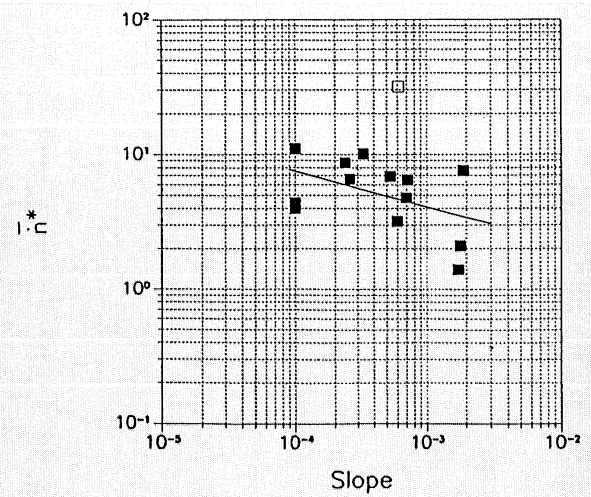


Figure 24: Dimensionless Average Normal Shift vs Slope

Comments: $\overline{\dot{n}}^* = 0.67S^{-0.26}, r = -0.46, s_{yx} = 0.25$. An outlier (Root River) was not included in the regression. Although the relation is somewhat weak, it is stronger than that for the dimensioned case, lending credence to using grain size to make normal shift rates dimensionless.

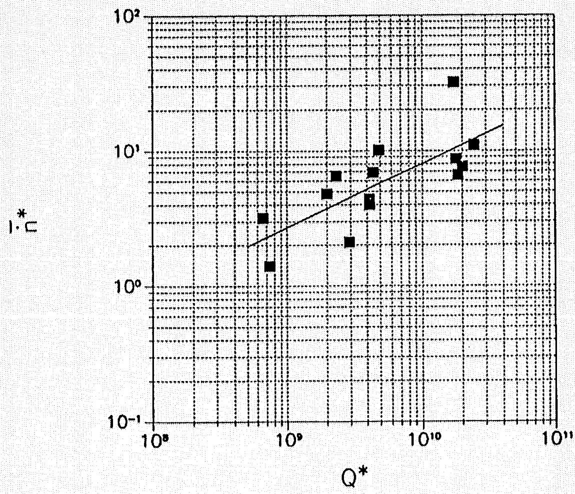


Figure 25: Dimensionless Average Normal Shift vs Dimensionless Discharge

Comments: $\overline{\dot{n}}^* = 0.00017Q^{0.47}, r = 0.74, s_{yx} = 0.23$. This relation is similar to the dimensioned case, with a slightly lower exponent (0.52 for the dimensioned case, see Fig. 20).

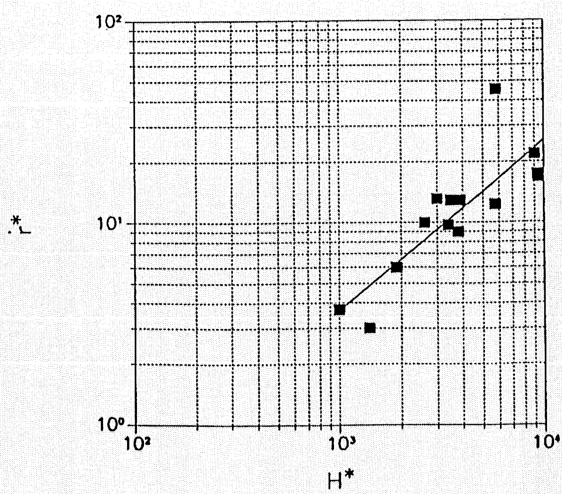


Figure 26: Dimensionless Area Reworked vs Dimensionless Depth

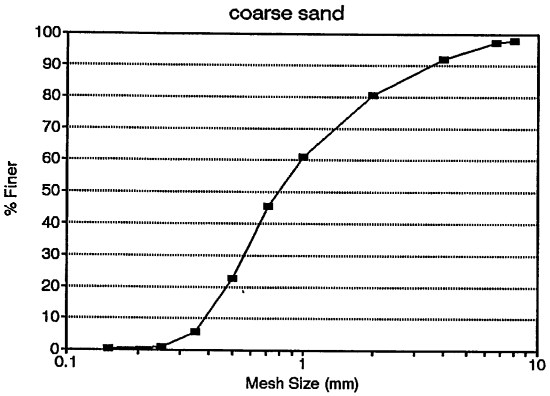
Comments: $\dot{r}^* = 0.011H^{0.84}, r = 0.84, s_{yx} = 0.17$. The exponent is higher than for the dimensioned case (0.76, see Fig. 21), but similar to the relation for dimensionless normal shift rate vs dimensionless depth (see Fig. 23).

Zumbro River
at
Wabasha County
T110N-R10W, secs. 17,18,19,20,21
T110N-R11W, sec. 13

HYDROLOGIC INFORMATION

2-Year Storm Discharge:	8900 cfs
Bankfull Stream Width:	160 ft
Stream Depth (Synthesized):	10.2 ft
Stream Slope:	0.00053

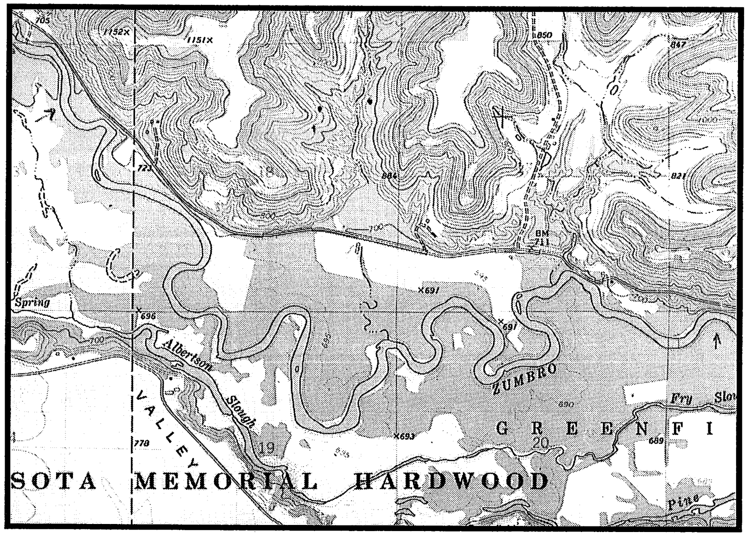
GEOMORPHIC INFORMATION



Grain size distribution, Zumbro River

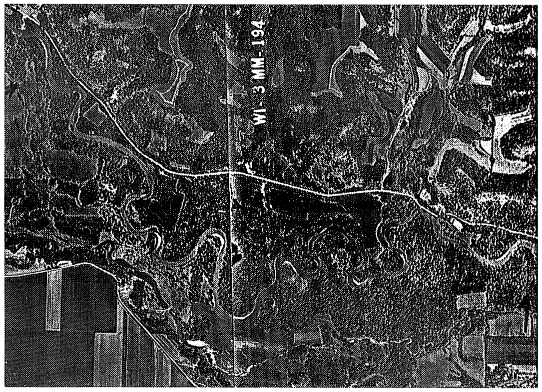
Ground photo of Zumbro River, 6/20/90

DESCRIPTION: The Zumbro River is a sand-bed stream in southeastern Minnesota, flowing eastward into the Mississippi River. It lies in a rather wide valley, with walls of heights ranging from 100 to several hundred feet.

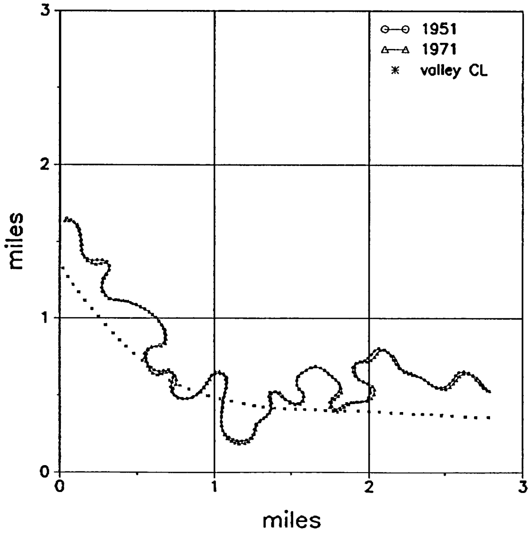


Base map, Zumbro River, 1974

Aerial photograph, Zumbro River, 1951



Aerial photograph, Zumbro River, 1971



Digitized centerlines, Zumbro River

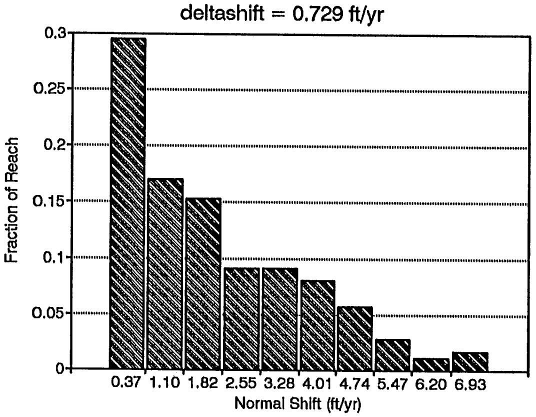
PROGRAM RESULTS

```
SHIFT
    Average Normal Shift:                      2.02 ft/yr
    Standard Deviation of Normal Shift:        0.06
    Average Absolute Transverse Shift:         1.37 ft/yr
    Average Absolute Longitudinal Shift:       1.25 ft/yr
    Average Transverse Shift:                  -0.29 ft/yr
    Average Longitudinal Shift:                0.73 ft/yr
    Shift Ratio:                               0.73

SINUOSITY
    Average Stream:                            1.86
    Average Change:                            0.00241

AVERAGE CURVATURE (ft^-1):                     0.00272

AREA WORKED (ft^2/ft/yr):                      3.75
```



Shift distribution, Zumbro River

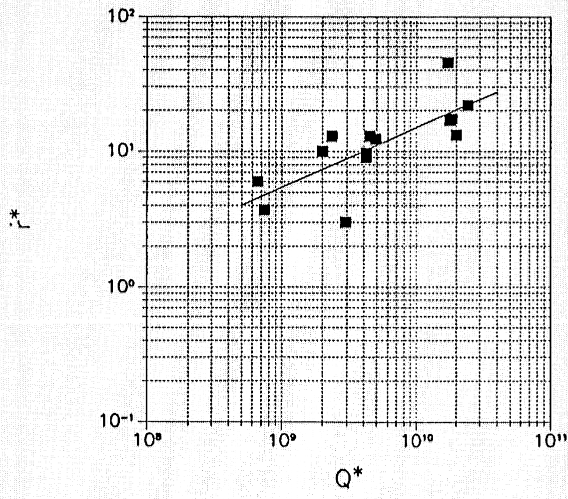


Figure 27: Dimensionless Area Reworked vs Dimensionless Discharge

Comments: $\dot{r}^* = 0.00066 Q^{*0.44}$, $r = 0.76$, $s_{yx} = 0.21$. The relation appears to be similar in strength to the dimensioned case, with a somewhat lower exponent than the dimensioned relation (0.50, see Fig. 22). The exponent is closer in value to that for dimensionless normal shift rate vs discharge (0.47, see Fig. 25).

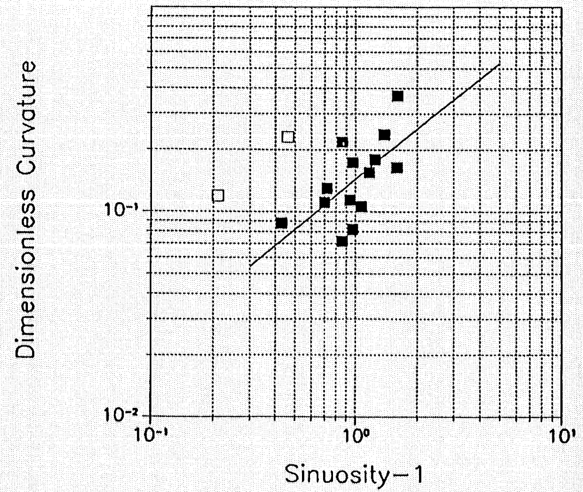


Figure 28: Dimensionless Average Curvature vs Sinuosity-1

Multiple Regressions

Multiple linear regressions were performed for two cases, with stream width B and discharge Q as the independent variables in both cases. Average rate of normal shift $\overline{\dot{n}}$ is the dependent variable for the first case, and rate of area reworked per unit channel length $\dot{r}$ was used for the second case. The results are summarized as follows:

$$\overline{\dot{n}} = 0.038 B^{-0.64} Q^{0.82}, s_{sy} = 0.41 \tag{30}$$

$$\dot{r} = 0.077 B^{-0.49} Q^{0.73}, s_{sy} = 0.40 \tag{31}$$

The relations are poor, and the negative exponents for width are cause for concern, as they should be positive (as for the single regression cases). Perhaps more experimentation can yield better results, but at this point it appears that the single regressions give more reliable equations.

Comments: (Fig. 28). $C^* = 0.14(S-1)^{0.81}$, $r = 0.63$, $s_{yx} = 0.16$. The relation is stronger than for the dimensioned case. This relation indicates that curvature increases with increasing sinuosity, which was suspected.

APPENDIX 1

Stream Parameters for Regression Analyses

Stream Name	Gravel or Sand Bed	D_s (mm)
Big Fork	Gravel	15
Buff. Cr.	Sand	0.4
Cott. R.	Sand	1.0
Hawk Cr.	Gravel	12
Kan. Cr.	Sand	0.4
Minn. R. A	Sand	0.5
Minn. R. B	Sand	0.5
Miss. R. A	Sand	0.5
Miss. R. B	Sand	0.5
Nemadji R.	Sand	0.5
Rice Cr.	Sand	0.5
Root R.	Sand	0.5
Rum R.	Sand	0.3
Wild R. R.	Sand	0.2
Yel. Med. R.	Sand	0.5
Zumbro R.	Sand	0.8

Stream Name	B (ft)	B*	H (ft)	H*	Q (cfs)	Q*
Big Fork	180	3657	5.6	113	5470	1.79E+6
Buff Cr.	57	43432	7.5	5750	1739	4.91E+9
Cott. R.	80	24383	6.2	1900	2296	6.56E+8
Hawk Cr.	75	1905	4.3	108	1437	8.24E+5
Kan. Cr.	25	19049	4.6	3500	826	2.33E+9
Minn. R. A	140	85340	15.4	9400	11084	1.79E+10
Minn. R. B	290	176775	14.8	9000	15092	2.44E+10
Miss. R. A	185	112770	6.2	3800	2580	4.17E+9
Miss. R. B	220	134105	5.6	3400	2580	4.17E+9
Nemadji R.	110	67053	2.3	1400	1810	2.93E+9
Rice Cr.	44	26821	1.6	1000	456	7.38E+8
Root R.	110	67053	9.5	5800	10700	1.73E+10
Rum R.	85	86356	9.2	9333	3203	1.86E+10
Wild R.R.	90	137153	2.0	3000	1250	2.00E+10
Yel. Med.R.	70	42670	4.3	2600	1230	1.99E+9
Zumbro R.	160	60957	10.2	3875	8900	4.45E+9

Stream Name	C (1/ft)	C* (ft/yr)	$\bar{\dot{n}}$	$\bar{\dot{n}}^*$
Big Fork	0.00263	0.237	2.35	1.9
Buff. Cr.	0.00419	0.119	2.07	10.1
Cott. R.	0.00179	0.072	1.05	3.2
Hawk Cr.	0.00295	0.111	1.16	1.0
Kan. Cr.	0.00655	0.082	1.34	6.5
Minn. R. A	0.00163	0.114	2.00	8.7
Minn. R. B	0.00119	0.173	2.92	12.7
Miss. R. A	0.00194	0.179	0.92	4.0
Miss. R. B	0.00141	0.155	1.02	4.4
Nemadji R.	0.00419	0.230	0.48	2.1
Rice Cr.	0.01674	0.368	0.33	1.4
Root R.	0.00160	0.088	7.26	31.6
Rum R.	0.00386	0.164	1.18	6.6
Wild R. R.	0.00288	0.130	1.11	7.6
Yel. Med. R.	0.00302	0.106	1.11	4.8
Zumbro R.	0.00272	0.218	2.02	6.9

Stream Name	S	S	S–1	$\dot{S}$ (1/yr)	$\dot{S}^*$
Big Fork	0.00063	2.39	1.39	–0.00111	–4.3E–5
Buff. Cr.	0.00033	1.21	0.21	0.00171	1.09E–5
Cott. R.	0.00060	1.86	0.86	0.00066	6.66E–6
Hawk Cr.	0.00063	1.70	0.70	–0.00011	–3.8E–6
Kan. Cr.	0.00071	1.97	0.97	0.00151	9.64E–6
Minn. R. A	0.00024	1.94	0.94	0.00240	1.71E–5
Minn. R. B	0.00010	1.97	0.97	–0.00231	–1.60E–5
Miss. R. A	0.00010	2.25	1.25	0.00170	1.21E–5
Miss. R. B	0.00010	2.17	1.17	0.00067	4.78E–6
Nemadji R.	0.00180	1.46	0.46	0.00046	3.28E–6
Rice Cr.	0.00175	2.61	1.61	0.00255	1.82E–5
Root R.	0.00061	1.43	0.43	–0.00105	–7.5E–6
Rum R.	0.00026	2.60	1.60	0.00801	4.43E–5
Wild R. R.	0.00190	1.72	0.72	0.00060	2.71E–6
Yel. Med. R.	0.00070	2.07	1.07	0.00048	3.43E–6
Zumbro R.	0.00053	1.86	0.86	0.00241	2.18E–5

Stream Name	$\dot{r}$ $(1/ft^2/ft)$	$\dot{r}^*$	$\dot{y}_{rms}/\bar{x}$
Big Fork	5.62	4.5	0.95
Buff. Cr.	2.52	12.3	0.88
Cott. R.	1.95	6.0	–1.79
Hawk Cr.	1.97	1.7	0.33
Kan. Cr.	2.65	12.9	3.03
Minn. R. A	3.87	16.8	1.22
Minn. R. B	5.77	25.1	1.95
Miss. R. A	2.07	9.0	10.19
Miss. R. B	2.22	9.7	0.84
Nemadji R.	0.70	3.0	0.60
Rice Cr.	0.86	3.7	5.89
Root R.	10.43	45.4	1.30
Rum R.	3.08	17.3	–1.26
Wild R. R.	1.90	13.1	–1.37
Yel. Med. R.	2.30	10.0	5.20
Zumbro R.	3.75	12.9	0.73

7 Conclusions

This analysis has produced interesting and useful results. Several of the relations produced from the regression analyses appear to be applicable to other streams in the Midwest. While the program MEANDER may be used to analyze a reach directly, the time and expense involved in such an effort make using the relations more attractive.

In particular, the average normal shift rate versus depth and discharge, as well as rate of reworking versus depth and discharge, should prove to be useful to those interested in quantifying stream shift. They are useful both in the dimensioned and dimensionless forms. The dimensionless equations, however, should yield better results because they include a measure of sediment properties. Table 1 summarizes the important relations:

Table 1. Useful Relations for Stream Shift

Dimensioned	Dimensionless
$\bar{\dot{n}} = 0.35H^{0.78}$	$\bar{\dot{n}} = 0.0049H^{*0.86}$
$\bar{\dot{n}} = 0.022Q^{0.52}$	$\bar{\dot{n}} = 0.00017Q^{*0.47}$
$\dot{r} = 0.69H^{0.76}$	$\dot{r}^* = 0.011H^{*0.84}$
$\dot{r} = 0.049Q^{0.50}$	$\dot{r}^* = 0.00066Q^{*0.44}$

Multiple regression relations do not appear promising for use, although only two cases were examined thus far.

Several of the results were surprising, such as the weak relation between shift rate and slope, or the rate of sinuosity change with sinuosity. The latter case may be a consequence of excluding reaches with cut-offs from the analysis. Indeed, none of the reaches included in this study approached a sinuosity of 6, which is the value predicted for cut-off by Beck (1).

While the regression relations can be used directly to compute shift rates and rates of area reworking for a reach of interest, one may also wish to compare that reach with the reaches analyzed in this study. By doing so, the results of the regression relations can be verified to some degree.

The shift distribution graphs contained in Chapter 2 may also prove useful, as they indicate the maximum normal shift that can occur on a particular reach. By comparing a stream of interest with one of the examples outlined in chapter 2, one may be able to estimate the maximum shift rate.

Examples of how one may use the results of this analysis for practical application include estimating maintenance costs for road repair, for the case of a road located along a stream bank, based on estimates of area reworked by the stream. Or, a bridge engineer can calculate how much area to allow for river shift, to lessen the chance of a river undermining bridge supports.

Although the regression equations and stream examples should prove useful, perhaps the most important product of this analysis is the formulation of a data set, which can be studied further and built upon, with the hope that the regression relations can be improved.

The program MEANDER should also prove to be useful as a means of validating the accuracy of models used to predict stream meandering. Whereas in the past most verification was accomplished with manual measurement techniques, MEANDER may be used to verify the models over an extended reach.

Finally, it is hoped that this analysis will contribute toward educating the public that prevention is the key to erosion control. In particular, it is hoped that greater emphasis will be placed upon maintaining a protective vegetative buffer strip along stream edges. It is hoped that, in the future, the program MEANDER can be used to distinguish between rates of shift in forest versus non-forest areas, to provide greater evidence in this regard.

Symbols

Superscripts

*	dimensionless notation
$\cdot$	time rate of change
—	average

Subscripts

A	pertaining to wave amplitude
rms	root–mean–square
s	pertaining to sediment
w	pertaining to wave
50	50% finer than
90	90% finer than

Parameters

a,b,c	constants
B	bankfull stream width
B^*	$B^* = B/D_s$
C	channel planform centerline curvature
c_w	wave speed
D_s	characteristic sediment grain size, set equal to D_{50} for sand–bed streams and D_{90} for gravel–bed streams
f_w	exponential weighting function
g	gravitational acceleration
H	stream depth (est. for bankfull flow)
H^*	$H^* = H/D_s$
L	stream meander wavelength (cartesian)
n	shift distance measured normal to channel centerline
$\dot{n}^*$	$\dot{n}^* = \dot{n}(gD_s)^{-0.5}$
Q	stream discharge at two year flood (estimate of bankfull flow)
Q^*	$Q^* = QD_s^{2.5}g^{0.5}$
r	area reworked per length of channel; coefficient of correlation
$\dot{r}^*$	$\dot{r}^* = \dot{r}(gD_s)^{-0.5}$
s	streamwise coordinate measured along channel centerline
s_{yx}	standard error of the estimate
S	stream slope
$\mathcal{S}$	stream sinuosity
$\dot{\mathcal{S}}$	time rate of change of sinuosity
$\dot{\mathcal{S}}^*$	$\dot{\mathcal{S}}^* = \dot{\mathcal{S}}\sqrt{D_s/g}$
x	east–west coordinate; component of normal shift parallel to valley centerline
y	north–south coordinate; component of normal shift perpendicular to valley centerline
θ	angular alignment of channel centerline relative to down–valley direction
ξ	filter strength parameter
∞	infinity