THE NATURE OF FLOW IN AN ELBOW

by

St. Anthony Falls Hydraulic Laboratory
University of Minnesota

Project Report No. 5

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Appendix I - The Pitot Cylinder

Appendix II - Errors Resulting from Neglect of the Tube Coefficient
**PREFACE**

Under Contract NObs-34208 between the University of Minnesota and the Bureau of Ships, Department of the Navy, Task Order 3 was submitted on January 24, 1947, by the David Taylor Model Basin. This order called for performance of the work necessary to consummate the study of fluid flow diversion, begun under Task Order 1. Under the latter order, a review of the literature relating to flow diversion was undertaken, and a few preliminary experiments on flow in an elbow were begun. Under the present order, this preliminary work is being furthered by a planned research program, based on the knowledge gained from the review of the existing literature.

This report, the first of a series to be submitted under Task Order 3, covers experimental work on a radius elbow. Other reports on experimental and analytical work on guide vanes in mitre elbows will follow under this order.

This report was written by Edward Silberman. The experimental work was under the supervision of Owen P. Lamb, who was assisted by Feng Hsiao in taking readings, and by Alexander P. Rodionov and June Brevdy in preparing the data. Lettie G. Gudmestad assisted in preparation of the manuscript. The experimental apparatus was originally conceived by Alvin G. Anderson in connection with his work on Task Order 1. Both Mr. Anderson and John F. Ripken, assistant professor of hydraulics, have offered valuable suggestions during the course of the work, and have reviewed the manuscript. Many other members of the Laboratory staff assisted in such details as photography, drafting, and assembling of the experimental apparatus.

The study was prepared under the supervision of Dr. Lorenz G. Straub, Director of the St. Anthony Falls Hydraulic Laboratory.
SYNOPSIS

The available information on fluid flow diversion is limited largely to the experimental values of the bend loss coefficient, and the variation of this coefficient with such factors as radius ratio, aspect ratio, and deflection angle. Some analytical work has also been done in the laminar flow region and on two-dimensional bends of continued curvature. There has been a decided need, however, for a general theory explaining the flow phenomenon and the mechanism of energy loss in a three-dimensional bend of fixed deflection angle. This report describes experiments which were directed toward obtaining such information.

The experiments suggested the formulation of a general theory which fits the trend of the data obtained by many other investigators, as well as that of the data obtained in these experiments. The theory explains the effects of varying radius ratio, aspect ratio, Reynolds number, wall roughness, entrance velocity profile, and deflection angle of the bend. The theory also offers an explanation for the maximum and minimum points in the familiar curve of bend loss coefficient plotted against radius ratio. Additional data are required, however, to give quantitative significance to the theory. The report outlines the research that would be necessary to provide this quantitative significance.

All experimental data obtained are reproduced in the report, so that they will be available for others who may be interested in pursuing the work further.
THE NATURE OF FLOW IN AN ELBOW

I. INTRODUCTION

This report follows Project Report No. 1 of the St. Anthony Falls Hydraulic Laboratory, "Fluid Flow Diversion, A Summary and Bibliography of Literature" [1]*. A review of that report suggests two projects for further research in fluid flow diversion. The first is the development of a general theory explaining the phenomenon of flow in a three-dimensional bend of fixed deflection angle. The second is the development of a method for the design of turning blades, or guide vanes, for mitre bends. This investigation deals with the first project—the development of a general theory for the flow in a bend. The experimental work in connection with this investigation is also preparatory, in the sense of developing instrumentation, apparatus, and technique, to an extensive experimental investigation to be undertaken in connection with the second project.

The flow around a bend produces two interrelated effects. The first is the additional energy loss resulting from the bend. The second is the change in the velocity and pressure distribution downstream from the bend, as compared to that existing upstream from the bend. The additional energy loss in a radius elbow is found to vary with the Reynolds number \( \text{Re} = \frac{VD}{\nu} \), the radius ratio \( R/B \), the aspect ratio \( D/B \), the relative roughness of the surface \( k/D \), the deflection angle \( \alpha \), and the velocity distribution at the entrance to the elbow. In these parameters, \( V \) is the mean velocity in the elbow, \( D \) is the depth, \( B \) is the breadth, \( R \) is the mean radius of curvature of the elbow, \( k \) is the roughness parameter, and \( \nu \) is the kinematic viscosity. The velocity and pressure distribution downstream are dependent on all of the above factors, and in addition, on the location of the measuring point downstream from the elbow; after a radius bend, a flow distance of \( 4D \) or more diameters may be required to reestablish completely the entrance conditions.

* Numbers in brackets refer to the corresponding numbers in the Bibliography.
Because of the many variables and the three-dimensional nature of the flow, analytical methods for analyzing flow in a bend have so far proved impracticable, except when used with simplifying assumptions which severely limit the applicability of the results. Available experimental data yield specific information as to the effect on a given elbow of the separate factors discussed in the previous paragraph, but little of the data is given in such detail that the actual flow process may be analyzed. To obtain the detailed data required to establish a general theory, a completely new set of experiments might be required, involving variation of each of the factors separately; this would be a considerable task.

Nevertheless, it was decided to undertake an experimental program as an approach to the problem of establishing a general theory. To keep the experimental work brief, however, this investigation was limited to a single elbow with fixed radius ratio, aspect ratio, roughness, and deflection. The Reynolds number could be varied over a wide range by using both water and air, by limiting the quantity of flow, and by taking advantage of seasonal temperature changes in the river water supply. The input velocity distribution could also be controlled. A smooth-walled channel with favorable RB was selected to keep the vortices small and thus permit study of their effect on the flow.

II. APPARATUS

The Experimental Elbow

The experimental elbow was fabricated of transparent lucite, with the following principal dimensions:

- Inside cross section ...................... 6 in. square,
- Radius of bend .......... inside ............... 9 in.,
  outside .................... 15 in.,
  mean ....................... 12 in.,
- Deflection angle ........................... 90°,
- Upstream tangent length ..... 21 in. = 3.5 diameters,
- Downstream tangent length .... 80 in. = 13.5 diameters,
- Radius ratio = 2 ...................... Aspect ratio = 1.
A photograph of the elbow in use is shown in Fig. 1. The entrance to the upstream tangent of the elbow was bolted to a plate on the end of a 12-in. diameter supply pipe. Centered in the plate was a 6-in. square hole to match the cross section of the lucite pipe. To control the velocity distribution, provision was made for inserting galvanized mesh between the tangent and the plate. On the opposite side of the plate, inside the 12-in. pipe, bolts were provided for mounting a distributor ring containing fine holes for introducing dye or air bubbles. These bolts were long enough to serve also for mounting a sheet metal bellmouthed entrance or additional lengths of 6-in. square pipe inside the 12-in. diameter pipe. When the additional upstream tangent was used, the bellmouthed entrance was placed at the entrance to the added length. Besides the distributor ring, a 1/4-in. brass pitot tube could be inserted through a gland in the 12-in. pipe wall just above the downstream position of the bellmouthed entrance. This tube with tip pointing downstream was used to introduce dye or air bubbles into the interior of the stream. Details of the elbow are shown in Fig. 2.

The distributor ring and brass tube were connected to a supply tank, which in turn was attached to the Laboratory compressed air system. The tank held various dyes or air, which could be introduced into the distributing system already described. Through suitable rubber tubes and connectors, the dyes or air could also be introduced into any piezometer tap or other hole in the elbow.

Fig. 2 shows the measuring cross sections which were spaced at convenient locations along the elbow and connecting tangents. At first, the downstream tangent was limited in length, terminating at Section 9. After some data were taken, this tangent was increased to the length shown.

Before the drilling of the many holes shown in Figs. 1 and 2, the elbow and tangent walls were marred only by gage lines at the measuring sections. Thus, the walls were kept clear for photographic purposes. Later, 1/32-in. holes were drilled at the corners where gage lines intersected, and through these holes was operated an apparatus for suspending yarns in the elbow to indicate flow direction. Fig. 3 shows this apparatus. The yarns were white, 1 in. long, and were fastened to a black nylon fish line at 1-in. spacing. The fish line was attached to a weight and pulley system which maintained a steady tension and permitted traversing of the yarns. Yarns were
strung horizontally at Sections 3, 5, 7, 8, and 9, and were strung vertically at Sections 5 and 8.

Eventually, the holes shown in Fig. 2 were drilled at each measuring section. The 1/16-in. diameter piezometer holes permitted the reading of wall pressures at 24 points around each section, as well as at some points between measuring sections. The 1/16-in. diameter was maintained for 1/4 in. from the inside wall and the hole was then expanded to 1/4-in. diameter to receive a plug or a plastic tube connector to the gage system. These holes were carefully rounded at the inside edge to about 1/64-in. radius to remove all burrs. At Sections 1A and 15, only four wall piezometer taps were provided, one at the center of each side, and these were connected together to give an average static pressure reading. The reliability of the wall piezometer taps, as constructed, is indicated by pressure readings shown in Fig. 16, Section 5, where piezometers were provided from two perpendicular directions at each corner. (Before the length of the downstream tangent was increased, the spacing of the piezometer holes at each measuring section differed from the spacing shown in Fig. 2. All wall pressure data are plotted at the points where the measurements were actually made.)

Holes of 1/2-in. diameter with removable plugs were provided on the top side of the pipe, just downstream of several measuring sections, as shown in Fig. 2. These were used for inserting a static tube to measure internal pressure, or a pitot tube for checking total head readings. These holes were also used periodically for inserting a thermometer to read fluid temperature. At each measuring section, twenty 1/4-in. holes (five on each wall) were provided for insertion of a pitot cylinder—a device used for measuring total head and direction of flow. Each 1/4-in. hole was provided with a plastic plug which could be inserted when the hole was not in use. All holes and plugs were carefully finished on the inside to prevent disturbance to the flow.

The Test Installation

The 12-in. diameter supply pipe was connected to an air blower of large capacity, and also to the main Laboratory supply channel, drawing water directly from the Mississippi River. This arrangement is shown in Figs. 4 and 5. The regulating valves in the 12-in. line were placed about 40 ft upstream from the elbow. About 35 ft downstream from the valves, a long honeycomb rectified the flow before it entered the bell mouth of the 6-in. pipe.
The flow of water through the elbow was limited to 4.5 cu ft per sec (18 ft per sec mean velocity) by the capacity of the discharge measuring apparatus. The blower limited the flow of air to 1800 cu ft per min (120 ft per sec mean velocity).

The total gravity water head inducing flow in the elbow was about 45 ft; short time variations of 0.1 to 0.2 ft and maximum variations of 0.3 to 0.4 ft in head occurred during a run under presumably constant conditions. The resulting variation in discharge or in pressure readings was not discernible during the course of a run. The centrifugal blower which supplied air to the elbow was driven by a constant-speed induction motor. Although the blower produced a pulsation of very low amplitude which was discernible in pressure readings, there was no measurable variation of the discharge during a run with air.

The downstream tangent to the elbow was connected to a diffusor which led through a vertical pipe Tee to another short length of 12-in. pipe. This pipe led down from the Tee through a tailwater regulating valve to a discharge measuring flume and then into the waste channel. The tailwater regulating valve made it possible to keep the elbow flowing full at low water discharges. The diffusor and the open stand pipe above the Tee limited the working pressures within the lucite duct.

The discharge flume is a type H flume developed by the United States Department of Agriculture, Soil Conservation Service [2]. The discharge weighing tanks at the Laboratory were used to calibrate this flume. It is believed that the discharge measuring error is less than 1 per cent. Air discharges were determined by integration of measured velocities and, once determined, were maintained by reference to the pressure drop across the elbow.

Instrumentation

The instruments used in making readings are pictured in Fig. 6. The static tube and pitot tube shown in Figs. 6d, 6e, and 7 are of standard design, and are matched in length so that from a given mounting hole, static and total head readings could be recorded for the same point in the flow. These tubes were calibrated in a submerged orifice at the same flows as were expected in the experiment. The coefficient of the static tube was 1.00 and
that of the pitot tube was 0.99, based on velocity head. The tubes were inserted into the experimental duct through a cone-shaped rubber diaphragm, reinforced at the apex where it was pierced by the tube. The base of the cone was cemented to a rubber gasket and was held in place by means of a 1/8-in. flat metal plate with a 1-in. diameter hole. This apparatus was fastened by "C" clamps to the duct wall. The tubes were held in position by a clamp attached to a universal joint mounted on a short ring stand. In this manner, the tubes could be rotated to point directly into the three-dimensional curvilinear flow. The arrangement is shown in Fig. 8.

The tube shown in Fig. 6a is termed a "long" pitot cylinder because it completely spans the duct and has physical support at each of the two points where it breaches the duct wall. This is in contrast to the "cantilevered" pitot cylinders, shown in Figs. 6b and 6c, which are supported from the insertion point in only one duct wall. The pitot cylinders were used for making direction readings by balancing the pressures on the two holes, and for making total head readings by pointing one hole directly into the stream. Since static head was not read with this instrument, the angle between the holes was not critical. A report of an investigation of these pitot cylinders is included in this report as Appendix I. In Appendix I, a drawing of the long cylinder is shown in Fig. 2, and the cantilevered cylinders are illustrated in Fig. 3.

When held at the same point in the parallel flow before the elbow, the long pitot cylinder gave the same readings as did the pitot tube which had been previously calibrated. From these identical readings, there was assumed a cylinder coefficient, or tube factor, of 0.99, based on velocity head. However, in comparing the measured discharge with the discharge obtained from integrating velocity measurements in the experimental elbow, the coefficient appeared to be about 0.98 (0.975 is given in Appendix I, but more complete data indicates that 0.98 is more correct). This variation was probably due to a difference in the scale of turbulence between the flow in the elbow and that in the test orifice. A determination made of the coefficients for the two cantilevered cylinders showed them to be smaller by less than 1 per cent than the coefficients for the long cylinder.

The long cylinder and larger cantilevered cylinder were held in the channel by rubber "O" rings set in lucite plates which could be screwed down
at each measuring section. The cylinder could then be pushed in and out through the "0" rings. Fig. 9 shows the direction-measuring equipment for the cylinders: a removable pointer and a protractor on a protractor holder which clamped to the lucite plate. The 90° line on the protractor was made to coincide with the normal to the cross section at each measuring point. This could be done to within 1°, and the protractor could be read to the closest 1°. Engraved on the cylinders was a length scale which was used for determining distance from the wall, and which could be read to the closest 0.05 in. The smaller cantilevered cylinder was held in place in the 1/4-in. holes by a brass bushing and by the micrometer depth gage shown in Fig. 6b. This cylinder was used for total head readings near the wall, and was not used for direction readings. The distance from the wall could be read to the closest 0.001 in.

For pressure readings when water was used in the elbow, two manometers were provided: a 100-in. "U" tube, with a water-air interface, and a 50-in. "U" tube with a water-kerosene interface for reading small pressure differences. Both manometers led to the same outlet which was connected by rubber tubes to the pitot devices; a switching of valves would change the readings from one gage to the other. The manometers could be read to the closest 0.05 in., except at the highest velocities where pulsations caused variations of 0.2 to 0.4 in. The measured velocity heads were between about 10 in. and 100 in. of water, so that readings were accurate to about 1% per cent of the velocity head.

Since the river water used in most of the experiments released considerable dissolved air in the gage lines during the winter season, all lines were carefully designed with steep slopes to facilitate bleeding. In addition, both legs of each manometer were frequently checked against atmospheric pressure to detect the presence of air bubbles. The temperature of the gage fluid was not obtained, and it is possible that this deficiency may have caused some error in the readings with the kerosene gage. To obtain simultaneous photographic pressure readings at a number of points, a multiple manometer board was tried, but had to be abandoned because of the difficulty of keeping all lines free of air bubbles. (As mentioned previously, the flow was essentially constant for a long enough period of time so that no difficulty was occasioned by taking individual readings.)
When air was flowing through the elbow, pressure measurements were made with a calibrated inclined manometer (draft gage), using a gage fluid furnished by the manufacturer. Readings could be made to the closest 0.005 in. on the scale, and the minimum velocity head reading on the scale was .36 in. Thus, the error was less than \( \frac{1}{4} \) per cent of the velocity head in the worst case.

It may be noted here that during about six winter months, the river water used in these experiments is crystal clear and most adaptable for photographic purposes. During the rest of the year, the water is somewhat turbid, principally as a result of the presence of minute plant life and silt. Also during the summer, the lucite becomes coated with mold at low flows, but the duct surfaces may be maintained in a high state of transparency by frequent waxing.

III. EXPERIMENTAL DATA

Visual and Photographic Observations

The first flow observations were made visually and photographically. Fig. 10 is a photograph of the boundary streamlines for a water flow at a Reynolds number of about 250,000* (9.5 ft per sec mean velocity). In the photograph, the dashed lines normal to the duct wall mark the limits of the bend. The photographs cover the duct from just above Section 2 to just beyond Section 9. For making this picture, the interior walls of the lucite elbow were coated with a thin paste of aluminum powder in SAE 30 machine oil. The water was then permitted to flow for several minutes—until the boundary streamlines became stable. While the water continued to flow, the elbow walls were photographed from the outside at various positions. Finally, the negatives obtained were printed by projection (some on curved surfaces) to a uniform scale and mounted in mosaic. A similar series of photographs was made at a Reynolds number of about 140,000, but because of the lower velocity, the boundary streamlines were not as clearly impressed as for the higher flow. In comparing the negatives for these two flows, there was no perceptible

* Reynolds number in this paper is based on a length parameter equal to the width of the elbow—\( \frac{1}{2} \) ft.
difference that could not be attributed to the poor impression of the stream-
lines at the lower flow.

Figs. 11a, 11b, and 12 are photographs showing air bubbles in the
flow of water through the elbow. Fig. 11a shows the pattern of flow along
the inside wall at the bend, and Fig. 11b shows the pattern of these same
bubbles in a horizontal plane at mid-depth at the downstream end of the bend.
Fig. 12 is also taken in a horizontal plane near mid-depth in the bend, and
shows the bubbles being carried around the bend near the center of the stream
at a radius of curvature approximately equal to the mean radius of the bend,
except for a slight deviation near the downstream point of tangency. All
three photographs were made at a Reynolds number of about 250,000. (Although
a photograph was not made of the bubbles near the upstream point of tangency,
these bubbles were seen to curve slightly at all depths before the point of
tangency was reached.)

Another series of photographs was made using yarns to indicate the
direction of flow, in the apparatus which has been previously described.
Fig. 13 is one of this series. Fig. 14 presents mosaics of photographs showing
flow in horizontal, longitudinal sections of the elbow. The yarns are lo­
cated at Sections 3, 5, 7, 8, and 9; \( y \) is the distance from the top boundary
to the yarns, and \( d \) is the depth of the duct (6 in.). Similar photographs
illustrating horizontal and vertical components of flow at several measuring
sections are presented in Fig. 15. In Fig. 15, \( x \) is the distance from the
inside wall to the yarns, and \( b \) is the width of the duct (6 in.). Figs. 14
and 15 represent a water flow at a Reynolds number of about 220,000. A simi­
lar set of photographs was taken at a Reynolds number of about 58,000, the
lowest number at which the yarns would follow the flow. A comparison of the
negatives for the two flows, yarn by yarn, showed identical directions at all
points, with the following exceptions:

Section 3, points d-p and f-p (Fig. 2): The yarns were deflected
about 3° more toward the center of curvature at the lower
Reynolds number than they were for the higher number.

Section 5, point a-p: The yarns were deflected by 20° more toward
mid-depth at the lower Reynolds number than they were for
the higher number. At point a-d', the deflection appeared
the same.
Section 7, points a-w and a-x: The yarns were deflected about 8° further away from the center of curvature for the higher Reynolds number than they were for the lower number. (Yarns were not photographed at the point symmetrical to a-x—a-t.)

Section 7, points d-w, a-y, and d-y: The yarns were deflected about 12° more away from the center of channel curvature for the lower Reynolds number than they were for the higher number. (Yarns were not photographed at the symmetrical points—a-u and d-u.)

Section 8, points a-w, d-w, and f-w: The yarns were deflected about 3° further from the inside wall for the higher Reynolds number than they were for the lower number.

Section 9, points a-w, d-w, and f-w: The yarns were deflected about 8° away from the inside wall at the higher Reynolds number, while the yarns for the lower number were practically parallel to the wall.

The second set of photographs is not published since the prints of the two sets appear so much alike. The differences enumerated above are shown clearly only by a comparison of the negatives by overlaying. The reliability of the results obtained by use of the yarns was indicated in the experiments by retaking four of the high flow photographs at a later time; the photographs were identical with the earlier pictures. The flow directions given by the yarns at a Reynolds number of 220,000 also compare reasonably well with the directions given by the pitot cylinder at a Reynolds number of 225,000.

Several other methods were used to visualize the flow of water in the elbow. A fine nylon fish line was inserted through the measuring holes by means of a hypodermic needle, and the free end of the line appeared to follow the streamlines well. A rather striking demonstration by this means was provided when the line was inserted at Section 3, Line h, to varying depths from the top. When the end of the needle was held close to the wall, the line followed the typical spiral, but when the needle protruded 1/4 to 3/8 in. into the flow, the direction of the line changed rather abruptly and it was carried downstream without spiralling.

Also added to the flow in an attempt to visualize the currents were globules of a mixture of carbon tetrachloride and benzol (with a specific
gravity of one), both clear and with red dye added. These globules, however, were hard to detect with either the camera or the eye. Particles of alumnum powder wetted with alcohol were also inserted into the flow, but were not very satisfactory for observation, since they formed a grey cloud rather than distinct particles. There was no attempt at visual observation of the air flow.

Dye filaments were used for some observations of laminar flow in the elbow. Fluorescein dye was introduced by means of a hypodermic needle which was inserted into the elbow through the piezometer holes at the measuring sections at right angles to the flow. The needle, having a sharpened end, would tend to create turbulent vortices at a fairly low Reynolds number. The investigation began at Re = 1000, where the flow was very definitely laminar at all points.

Dye was added continuously at Section 2, near the center of the duct, while the discharge was gradually increased. The filament of dye remained steady until approximately Re = 2000, at which point it began to waver and break up. To determine whether the curvilinear flow further downstream would be laminar at this Reynolds number, the dye tube was inserted at Section 5 without further changing the discharge. The flow here was also turbulent. Previous experiments [3] had shown the Reynolds critical to be higher in curvilinear flow than in rectilinear flow. The difference in results is probably explained by the fact that the earlier tests were made in established curvilinear flow, while in this case the tests were made in a transition length.

Another notable flow phenomenon illustrated by the dye was a laminar boundary layer which persisted in a very thin layer even to a Reynolds number of between 10,000 and 20,000. At a Reynolds number just above 2000, the laminar boundary layer near the bend followed the typical bend spiral along the top and down the inside, while the flow immediately outside the boundary (where the dye stream wavered and broke up) appeared to follow the duct downstream without spiralling. This flow pattern was observed at both Sections 3 and 5. At higher Reynolds numbers, the laminar boundary layer, as well as some of the turbulent fluid, followed the typical spiral, but there appeared to be a rather abrupt change in direction from the spiralling fluid at the boundary to the remaining fluid in the duct.
Measurements in the Elbow

Tabulated in Table I are the series of runs in the experimental elbow which yielded numerical data.

**TABLE I**

**EXPERIMENTAL DATA**

<table>
<thead>
<tr>
<th>Series No.</th>
<th>Fluid</th>
<th>Mean Velocity (ft per sec)</th>
<th>Re</th>
<th>Data on Figure Nos.*</th>
<th>Remarks</th>
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<tbody>
<tr>
<td>I</td>
<td>Water</td>
<td>7.9</td>
<td>2.25x10^5</td>
<td>16, 17, 18</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>&quot;</td>
<td>7.9</td>
<td>3.99x10^5</td>
<td>19, 20, 21, 22</td>
<td>Differs from I because of temperature difference.</td>
</tr>
<tr>
<td>III</td>
<td>&quot;</td>
<td>16.8</td>
<td>6.26x10^5</td>
<td>23, 24, 25</td>
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<tr>
<td>IV</td>
<td>&quot;</td>
<td>16.8</td>
<td>8.25x10^5</td>
<td>20, 26, 27, 28</td>
<td>Differs from III because of temperature difference.</td>
</tr>
<tr>
<td>V</td>
<td>&quot;</td>
<td>2.5</td>
<td>1.21x10^5</td>
<td>20, 29, 30, 31</td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>Air</td>
<td>41.1</td>
<td>1.193x10^5</td>
<td>35, 36, 37</td>
<td>Approximately same Re as V.</td>
</tr>
<tr>
<td>VII</td>
<td>Water</td>
<td>A 7.9</td>
<td>3.21x10^5</td>
<td>32</td>
<td>Effect of entrance conditions.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B 16.8</td>
<td>6.80x10^5</td>
<td>33</td>
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<tr>
<td></td>
<td></td>
<td>C 8.6</td>
<td>3.0x10^5</td>
<td>34</td>
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* Cross Sections in these figures are to be examined with the observer looking downstream. The duct bends to the left.

For taking data, each measuring cross section was divided into 25 interior 1-in. squares, 20 exterior ½-in. by 1-in. rectangles, and four corner ½-in. squares (see Fig. 2). The aim was to define adequately the flow for each run by obtaining a relative static pressure, a relative total head, and a velocity (both magnitude and direction) at the center of each of these
elementary areas at a sufficient number of sections. Table I indicates the figure number on which may be found data for each series of runs. Each reading is plotted at the point where it was actually made. All data taken are shown in one or the other of these figures, and the absence of data at any measuring point indicates that readings were not made at that point. The Reynolds numbers given in Table I are average values; the actual values varied within 3 per cent of this average, the variation being caused by temperature variations during a series. For Series VI, using air, the Reynolds number was held constant within 1 per cent and the discharge was varied within 3 per cent of the mean to maintain the Reynolds number for varying temperatures.

All pressures in all runs were measured with reference to the average static pressure at Section 1A. The wall piezometers at Section 1A were permanently connected to one leg of the differential manometer. This practice eliminated any local error which might have resulted from the presence of tubes in the flow or from minor changes in flow head. All readings were recorded in inches of water at room temperature (55°F - 80°F), but these data are presented in the diagrams as the dimensionless ratio of the actual reading (in inches) to the velocity head of the mean flow (in feet). The factor, 12, thus introduced, reduced the number of digits necessary at each plotted point.

For each run, static pressures were read at each wall piezometer by connecting the piezometers in turn with the free leg of the differential manometer. In Series I, static pressures were also read at a large number of internal points. From these data, it was determined that internal pressures could be fairly well predicted from wall pressures. Therefore, on all subsequent runs, internal static pressures were read at only a few critical points, and necessary internal pressures were obtained by interpolation from the top and bottom wall pressures.

To obtain an internal static pressure reading, the static tube was mounted in the universal clamp and inserted through the gland previously described. The tube was aligned in the flow by rotation while forcing air out of the static holes and observing the bubbles; when all bubbles appeared alike, it was assumed that the pressure around the holes was uniform and the tube correctly aligned. A head reading with the manometer was then obtained.
Total heads and directions were read by inserting the pitot cylinder to the center of each square and rectangle in turn. On several runs, readings were made only at the bottom or the top half of the channel, since the two halves were found to yield nearly symmetrical results. Direction readings were made first, recording the angle right and left, or up and down, of the normal to the section. The total head readings were then taken in the indicated direction. At each point, a direction reading was made once with the cylinder vertical and once with the cylinder horizontal, in order to obtain both horizontal and vertical components of flow angularity. Because the pitot cylinder could not be inserted along Lines b, n, q, or c' (Fig. 2), directions that would otherwise have been read in those positions were either obtained from the yarns, Figs. 14 and 15, and boundary streamlines, Fig. 10, or were assumed parallel to the wall.

Total heads were reduced to velocity heads by applying measured static heads previously obtained or by applying values interpolated from them. Total and velocity heads were corrected for angularity of flow out of the plane of measurement by dividing the velocity head by the square of the cosine of the angularity (see Appendix I). The corrected total heads are shown on the diagrams. No correction, however, was made for tube coefficient, but this does not seriously affect the relative accuracy of the data presented—see Appendix II.

Although velocities are not basic measured data, the velocity diagrams are presented with the pressure and total head diagrams in order to complete the flow description in one place. All velocity data are given in the dimensionless form, $v/\bar{v}$, where $v$ is a velocity component at a point, and $\bar{v}$ is the mean velocity in the cross section as determined from the flume discharge measurements for water flows, or from integration at one cross section for air flows. The velocity magnitude at each point was determined from the corrected velocity head. (For greater accuracy, these computations were made from the original dimensional data rather than from the plotted dimensionless total heads and static heads.) For plotting purposes, the velocity was reduced to three components by applying the measured direction angles: one longitudinal (normal to the cross section), and two others at right angles to each other within the cross section. Numerical values and contours shown on the velocity diagrams refer to the longitudinal components,
while the resultants of the components within the cross section are shown in both direction and magnitude by the plotted arrows.

Beneath the velocity diagram for each section is shown the mean velocity as obtained by integration of the longitudinal velocity components. It is presented in the dimensionless form, \( \Sigma u\Delta A/\bar{v} \), where \( \Delta A \) is one of the elementary areas shown in Fig. 2, \( u \) is the longitudinal velocity component at the center of that area, and \( \bar{v} \) is as defined above. These values should be particularly reliable in the straight flow, Section 2. Here, as well as at many other sections, the values are larger than unity by a factor of one over the square root of the tube coefficient, within the limits of experimental error. It should be noted that in forming the sum, \( \Sigma u\Delta A \), the flow in the four corner 1/2-in. squares has been neglected. At the same time, the velocity measured at the center of each of the 1/2-in. by 1-in. elementary rectangles is larger than the mean velocity for that area; this may be seen by examining the velocity curves near the boundaries, as shown in Fig. 34. It is assumed that this excess is approximately balanced by neglecting the flow at the corners. (Of course, this assumption can be exactly true at only one Reynolds number.) In Series V, Fig. 31, and in Series VII, Fig. 32, the value of \( \Sigma u\Delta A / \bar{v} \) is less than unity for some sections. This result is evidently in error, and the error is attributed to the failure to observe the temperature of the gage fluid in the kerosene gage, as suggested in Section II of this paper, or to a defect in the method of integration. A rough correction to these data could be made by increasing the value of each velocity component so as to make the value of \( \Sigma u\Delta A / \bar{v} \) equal to unity divided by the square root of the tube coefficient \( (1/\sqrt{0.98}) \).

Beneath the total head diagram for each section, the mean total head for that section is shown. This was obtained by integration \( \Sigma H\Delta A / \Sigma u\Delta A \), where \( H \) is the dimensionless total head at the center of each elementary area, and the other quantities are as defined above. Again the four corners have been neglected in the integration on the same assumption as was made in the case of velocity integration.

Series II, IV, V, and VI are complete series except where data were not taken in the top or bottom half of a section because of symmetry. Series I and III were made before the downstream tangent was increased in length beyond Section 9, and are complete to that point only. Series VII was limited
to checking the effect of entrance conditions. In this series, seven duct widths (42 in.) of additional upstream tangent were first added inside the 12-in. supply pipe, and later a 1/2-in. square mesh screen was introduced between the bellmouthed entrance and the normal upstream tangent. Only a limited amount of data, as shown in the figures, was taken for this series. The data given in Fig. 34 were confined to a region near the wall, and were taken with the smaller cantilevered pitot cylinder.

IV. A DESCRIPTION OF THE FLOW

Flow in the Bend

The data obtained in this investigation provide a rather complete description of the flow in this experimental elbow and of its variation with Reynolds number. The familiar double spiral flow can be seen clearly in the boundary streamlines, Fig. 10. The spiral may also be inferred from the yarns, Figs. 14 and 15, and it can be seen less clearly in the velocity diagrams, Figs. 18, 22, 25, 28, 31, and 37. The experiments using air bubbles and fish line in turbulent flow, and dye filaments in laminar flow all plainly illustrated the spiral.

In this square, smooth duct of favorable R/B, the strength of the spiral is not great and the secondary currents of the spiral can be seen clearly only in the layers near the top and inside boundary. Fig. 38, for instance, presents two composite prints of several pictures of the yarns. In one print are superimposed four photographs taken from the bottom of the duct, and in the other, three photographs taken from the inside, at Section 5 at a Reynolds number of 220,000. In the prints, the yarns which are deflected most are at the lower boundary (y/d = 0), or at the inside (x/b = 0), respectively. Those deflected next most are at y/d = 1/16, or at x/b = 1/12; the next at y/d = 1/8, or at x/b = 1/4; and the yarns which closely follow the duct curvature in the horizontal plane are at y/d = 1/4. A photograph of the yarns in the horizontal plane at y/a = 1/2 shows a pattern identical to that at y/d = 1/4, but it is not included in the composite prints. Likewise, photographs in the vertical plane at x/b = 1/2, 11/12, and 1 show only small differences from that at x/b = 1/4, and are not included in the prints. These prints show that the principal secondary flow is confined to the top
and inside wall, and that the return secondary flow at Section 5 is very small. In fact, in Section V of this paper, it will be shown that the spiral flow must be initially a boundary layer phenomenon.

The experimental data show that the flow streamlines begin to curve before the beginning of channel curvature (between Sections 2 and 3). The streamline curvature commences approximately one duct width upstream from the bend, and is at first the same throughout the depth of the duct. However, by the time the beginning of duct curvature is reached, the boundary streamlines show slightly more curvature than those in the interior. In the bend, the interior streamlines rapidly approach the duct curvature, although the boundary streamlines follow the spiral. Near the end of the bend, the interior streamlines again curve at a radius larger than the duct radius, particularly near the inside wall, and continue to curve perceptibly for about one duct width downstream from the end of the bend. This curvature of the streamlines may be seen in the photographs of the air bubbles, Figs. 11b and 12, and in the photographs of the yarns, Fig. 14.

It is interesting to observe that the flow, at least in the region away from the top and bottom walls, tends to follow a combination of two-dimensional potential flow patterns. Fig. 39 has been prepared to demonstrate this phenomenon. In that figure, longitudinal velocity components from the velocity diagrams, Figs. 18, 22, 25, 28, 31, 32, 33, and 37 have been replotted in the form of horizontal velocity profiles for each section. It is seen that the entrance profile at Section 2 closely represents a parallel, uniform flow at infinity. The velocities in the profiles at Sections 3, 5, and 7, around the bend, are distributed in accordance with the law of potential flow for a free vortex: \( u_r = \text{constant} \), where \( r \) is the radius of curvature of the streamlines, and \( u \) is the longitudinal velocity component. At Section 5, the curvature of the streamlines is identical with that of the channel walls— \( r = 12 \text{ in.} \) at midstream (the curve of \( u_r = \text{constant} \) is displaced from the plotted points to avoid confusion); but at Sections 3 and 7, it is much flatter— \( r = \text{about 24 in.} \) at midstream. Disregarding the low velocity area near the inner wall, the profiles and streamlines at Sections 8 and 9 are also similar to those for a theoretical potential flow leaving a bend. Beyond Section 9, the similarity to potential flow disappears. The energy interchange between streamlines smooths out the velocity distribution, and the boundary layer for parallel flow also begins noticeably to penetrate.
the interior after the flow passes Section 9. The velocity profiles at Section 14 are probably as much like the entrance profiles at Section 2 as is possible; beyond Section 14, the boundary layer will grow until a normal turbulent velocity profile is established.

Separation

In addition to the initial double spiral, a second source of secondary current, resulting from separation, may be seen at the end of the bend, near Section 7. Fig. 11a shows air bubbles following the spiral along the inside wall to the end of the bend. Fig. 11b shows some of the same bubbles in the horizontal plane at mid-depth. These photographs show that the bubbles, and thus the streamlines, leave the inside wall near the end of the bend. This phenomenon can also be observed in the photographs of the yarns, Figs. 14 and 15. In Figs. 39 and 40, longitudinal velocity components have been replotted as vertical and horizontal velocity profiles, respectively. These two figures clearly show that there is a definite surface of separation along which vorticity must be produced.

The various figures also show that separation at Section 7 is confined to a region between approximately the upper and lower quarter of the duct. This confinement may be attributed to the effect of the double spiral. Basically, separation results from the failure of the streamlines of the two-dimensional flow to follow the inner wall of the elbow. Near mid-depth, however, separation is enhanced by the return flow from the spiral, while nearer the top and bottom walls, the spiral itself hinders separation. The trace of the surface of separation in a cross section near the end of the bend, and the resulting vorticity are sketched in the accompanying figure:
The vorticity pattern may be detected in the secondary flow arrows of the velocity diagrams.

The double spiral has yet another effect on the separation at the end of the bend. Figs. 39 and 40 show that fluid is flowing with an appreciable positive velocity in the separated area. In a two-dimensional separation, one would expect no flow or a reversed flow in the separated area. Since no other source of fluid is available, it must be concluded, then, that the spiral is supplying fluid to the separated area. Since this fluid comes from a boundary layer, it has relatively low energy and small velocity. At the same time, the fluid in the rest of the cross section must possess a high velocity in order to satisfy the equation of continuity. This velocity however is not as large as it would be if there were no flow into the separated area.

The separated area near the inside wall of the elbow is gradually extended, and the velocity and energy of the fluid therein are gradually increased by interchange of energy with the adjacent high velocity layers and by influx of fluid from the spiral. The process has been nearly but not entirely completed at the last measuring section, 11 elbow widths downstream from the end of the bend. The double spiral is also still detectable at the last measuring section, but it has been redistributed so that it is negligible on the boundary and spread throughout the interior. The changes in velocity may be traced in the velocity diagrams and profiles, Figs. 18, 22, 25, 28, 31, 32, 33, 37, 39, and 40.

Effect of Reynolds Number and Entrance Profile

Concerning Reynolds number, the remarkable fact is that all these flows are so much alike. There are small, but possibly significant, differences between the photographs of the yarns at \( R = 58,000 \) and \( R = 220,000 \). All of the measured runs, however, appear very much alike—that is, between \( R = 120,000 \) and \( R = 825,000 \).

In Fig. 40, which shows vertical velocity profiles, a slight change in profile may be detected as the Reynolds number changes. The velocity profiles downstream from the bend seem to be less distorted for a high Reynolds number than for a lower Reynolds number. The yarns indicate that the double spiral for the lower Reynolds number deflected the flow more than did the
spiral for a higher number and that, generally, the spiral was relatively stronger for the lower number. Exceptions to this phenomenon occur downstream from the bend (at Sections 7, 8, and 9) at mid-depth, where the flow appears to leave the wall more abruptly at the higher Reynolds number than it did at the lower number. An explanation for these observations is offered in the next section of this paper.

The air flow, at the same Reynolds number as the lowest water flow, appears to produce the same velocity profiles as the water flows.

The effect of entrance velocity profile is shown by the data taken with the altered entrance conditions. The addition of 7 duct widths of entrance length changed the profile at Section 2, as may be seen in Figs. 32, 33, and 34; the longer entrance caused an increase in the velocity in the center of the duct and a decrease in the velocity near the walls, in accordance with the boundary layer theory. The effect is noticeable at Section 8, downstream from the bend (see Fig. 34); there, the absolute velocities along Line f were higher near the top wall and lower in the interior than for a normal entrance. At Section 9 (Fig. 39), the area of low longitudinal velocity in the interior extends further toward the outside wall, and the high velocity along the outside wall is higher than for the previous flows. In other words, the separated area is larger and the velocity gradient across the separation is somewhat greater for this entrance profile.

Energy Loss.

To obtain a total energy curve for each series, average total heads at the several measuring sections around the bend are plotted in Fig. 41 against distance measured along the centerline of the duct. Rather than plotting the average total head, only the average wall pressure drop from the reference section is plotted at Section 15, since internal head readings were not made at this section. As stated in a previous paragraph, it is believed that the entrance profile has been very nearly reestablished at Section 14, and that, therefore, the pressure drop is a satisfactory measure of the energy loss beyond that section. Values of $\lambda$, the loss in a straight, smooth pipe, are also plotted in Fig. 41. These values are taken from the formula $\frac{1}{\lambda} = 2 \log \frac{R}{\lambda} - 0.8$, but check very closely with the pressure drop between the reference section and Section 2 (parallel flow) in the experimental pipe. The $\lambda$ curve is drawn to pass through the value of the
average total energy at Section 2 for each series.

(To reduce the effect of the apparent error in velocities mentioned in Section III of this paper, a corrective factor has been applied to the average total heads at Sections 2, 7, 8, 9, 12, and 14 of Series V and to that at Section 2 of Series VIIA. The corrective factor was obtained by first finding an intermediate factor which would make $E_uA/v$ for each of the above sections equal to the inverse of the square root of the tube coefficient, $1/\sqrt{\gamma}$. This intermediate factor was then squared to obtain the corrective multiplier for the average total head. It has also been assumed in plotting Fig. 41 that the changes in total energy of the air flow are measured by the changes in total head.)

The ordinate of Fig. 41 is $H/\sqrt{g}$, where $H$ is the average total energy in ft of water at a given section, and $v$ is the mean velocity of flow in ft per sec taken from the discharge measurement for the given series. Thus, where the bend loss is $\xi \frac{v^2}{2g}$, the difference between the A line and the corresponding total energy line at any point measures the coefficient $\xi$ for bend loss to that point. At Section 15, the coefficient $\xi$ has the following values:

<table>
<thead>
<tr>
<th>$Re$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.2 \times 10^5$</td>
<td>0.080</td>
</tr>
<tr>
<td>$4.0 \times 10^5$</td>
<td>0.094</td>
</tr>
<tr>
<td>$8.25 \times 10^5$</td>
<td>0.096</td>
</tr>
</tbody>
</table>

The trends of the energy curves are remarkably alike for similar Reynolds numbers and similar discharges, but the experiments showed a marked difference in form between the curves for high and low Reynolds numbers as the flow proceeds around the bend. Near the beginning of the bend, the low flow undergoes a comparatively large loss of head, the medium flow a medium loss, and the high flow a small loss. Near the end of the bend, the trend appears to be reversed, although it is not quite so definite as near the beginning. The total loss is a combination resulting from these two trends, and in this experiment, the loss is higher for the high Reynolds number than for the low one. If the value of $\xi$ for each flow is increased by about .01
in accordance with the remarks in Appendix II, the values of $\xi$ in this experiment will be found to agree well with those obtained by previous investigators [4].

In interpreting the high energy loss near the beginning of curvature, it must be remembered that measurements in these experiments were not made closer to the wall than $\frac{1}{4}$ in. Yet, it has been shown that near the beginning of curvature, the principal spiral currents are that close and closer to the wall. Therefore, much of the kinetic energy contained in the spiral is not included in the measurements at Section 3, and some of it is not included at Section 5. The early drop of the energy curve at low Reynolds numbers then indicates that for these numbers, the unmeasured spiral contains more kinetic energy and is stronger than for the higher numbers. The rise in the energy curve around the bend is explained by the diffusion of the spiral into the interior where it is included in the measurements for later sections.

V. AN ADDITION TO THE THEORY OF FLOW IN BENDS

A theory describing the flow of fluids in bends of established curvature has been formulated previously [4], and this theory has been verified by experiment. The flow in the transition sections at the beginning and at the end of a bend, however, has not been so well described. It is believed that the data obtained in this experiment offer a basis for a theoretical explanation of the flow in these transition lengths. Together with the existing theory, this explanation provides a general theory for the flow in bends.

The Initial Vortex

The commonly accepted explanation for the formation of the bend spiral is that the spiral follows from the unbalance of the centripetal forces of the curving streamlines between the interior of the bend and its boundaries. Such an explanation is not adequate for all of the details observed in this experiment. Instead, there has been developed a somewhat allied explanation, based on the assumption that a fluid will tend to follow the two-dimensional potential flow pattern imposed by its boundaries, except when the tendency
is obstructed by viscous forces near the boundaries. This explanation differs from previous explanations mainly in the point or points of origin of the secondary flow.

In Section IV of this paper, it was concluded that there is a general tendency for the velocity distribution in the vicinity of a bend to follow approximately the distribution for a combination of two-dimensional potential flows. Within a bend, the distribution is that of a potential or free vortex, while approaching or leaving the bend, the distribution changes to that of the appropriate sharp-angled bend. Of course, in these experiments, the input velocity profile was conducive to the maintenance of potential flow. Even with a normal turbulent distribution at entrance, however, there is still a tendency for the flow to follow a combination of potential flow patterns. Fig. 42, which is reproduced from Wattendorf [5], shows the development of the velocity profile in a curved duct with an aspect ratio equal to 18, and a radius ratio of 1.5. Wattendorf's curves show that within the bend, the effect of the turbulent entrance profile is simply to delay the final attainment of the potential distribution; this distribution is attained at about 150° around the bend. Furthermore, Wattendorf's data show that in stabilized, two-dimensional curvature, the potential pattern is maintained and resists the penetration of the boundary layer. The approach to potential flow within the bend is demonstrated by the data of Mockmore [6] for an open channel, and of Robertson [7] and Yarnell [8] for a circular pipe.

Fig. 39, Section 3, mid-depth, and Fig. 42, Section 0°, show graphically that, even at the beginning of duct curvature, there exists the tendency toward a velocity distribution in accordance with the potential vortex. This tendency follows from the curvature imposed by the curved boundaries on the streamlines entering the bend. Fig. 39 also shows the average velocity profiles at 1 in. and at 1/4 in. from the upper boundary at Section 3. These profiles resemble the profile at mid-depth, except that the profile at 1/4 in. tends to remain flat near the inner wall. By its friction, the upper wall has resisted the formation of the higher velocity required by the tendency toward a potential distribution. In considering a boundary layer phenomenon like wall friction, however, 1/4 in. is relatively distant from the wall. It is reasonable to assume that the closer to the upper wall a profile is taken, the less the profile will resemble the potential profile, particu-
larly near the inner wall and near the beginning of curvature. Of course, the same conditions would apply to a region near the bottom wall.

Fig. 39 shows further that in the region between 1 in. from the top and 1 in. from the bottom of the duct, there is a uniformly large increase in velocity near the inside wall as the flow progresses from Section 2, where there is no streamline curvature, to Section 3. There is also a large decrease near the outside wall. Correspondingly, the pressures along the inside wall drop and those along the outside wall increase in this area (see Fig. 20). These changes are all in accordance with the potential flow requirements.

On the other hand, horizontal profiles taken very close to the upper or lower wall would show a relatively small velocity increase near the inner wall as the flow progressed from Section 2 to Section 3, because the top and bottom walls resist the formation of the higher velocities required by a potential flow. Near the outer wall, however, such profiles would show a large decrease of velocity, in accordance with the tendency toward a potential distribution. Correspondingly, the pressures near the top and bottom inside corners would not drop a great deal as the flow progresses between Sections 2 and 3, while the pressure near the outside walls would increase with the decreasing velocity as it does in the interior of the duct. Thus, along the inside wall at Section 3, and even before the flow reaches this area, there would be at the top and bottom an area of pressure relatively higher than in the central portion, but on the outside wall, the pressure would be uniformly high. The same situation would continue to prevail, but on a smaller scale, as long as the curvature continues.

Figs. 16 and 23 present experimental verification of this conclusion. (It was impossible to obtain more data of this nature because of the change in piezometer tap locations made after the completion of Series I and III.) Of course, in Figs. 16 and 23, the pressure is plotted in straight line variation between points a' and d' on the inside wall. The pressure diagrams for the other series, however, show that there is little pressure increase from z to b'. Actually, the curve should look more like the following sketch.
The data show that this considerable curvature occurs only in a pressure curve taken near the inside wall. A curve taken further in the interior of the fluid would be nearly flat as it approached the upper or lower wall (Fig. 16).

Thus, it is seen that in a curved duct, particularly near the beginning of curvature, there exists a pressure gradient from the corners toward mid-depth across the inside wall normal to the longitudinal flow. This pressure gradient induces a corresponding flow component—out of the top and bottom inside corners, across the inner wall toward mid-depth. Close to the upper and lower walls, this component would be quite large, in accordance with the appreciable pressure gradient, and would decrease toward mid-depth. The flow component would also be larger adjacent to the inner wall than it is in the interior. In Fig. 10, inside view, and in Fig. 11, the effect of this component may be seen clearly. It may also be seen in Fig. 15, Section 5, vertical. The latter photograph, or more particularly the vertical view of Fig. 38, shows further that at 1/12 of the width away from the inner wall (1/3 in.), the downward component is almost negligible—the vertical flow component is confined to a narrow boundary. A comparison of the magnitudes of the longitudinal and cross-sectional components in any of the velocity diagrams also shows that the downward component is small in the interior.

A pressure gradient also exists along the radius of curvature of the bend. At least initially, however, this gradient is in balance with the centripetal forces which established it, and there is no resulting flow. (This statement is in opposition to the theory of Mockmore [6].) Since it has already been shown that there is no appreciable gradient across the out-
side wall, the pressure gradient across the inner wall is, then, the only effective gradient of any magnitude induced directly by the curving streamlines.

The induced flow component originating at the top and bottom inside corners tends to reduce the longitudinal velocity component in these corners and to increase the longitudinal velocity component nearer mid-depth. Opposed to these velocity changes, however, is the tendency toward potential flow. Thus, in order to reestablish the potential distribution, there must be an inflow to the corners across the upper and lower walls, and an outflow toward the interior from the inner wall nearer mid-depth. The inflow approaching the corners is squeezed into a layer very close to the upper and lower boundaries, but the outflow from the inner wall may be distributed over a large area (the size of the area depending on the aspect ratio of the bend and the length of the effective transverse pressure gradient). For this reason, there must be a large inward velocity component near the upper and lower walls, and a much smaller outward velocity component in the interior. (In fact, until the radius of curvature of the interior streamlines equals that of the duct walls, there need be no net outward flow since the velocity near the inner wall is continually increasing.) The secondary flow circuit is completed by a small return flow from the interior to the top. These flows are pictured in Fig. 10 (the boundary streamlines), in Figs. 14, 15, and 38 (the photographs of the yarns), and in all of the velocity diagrams. The pronounced flow along the upper boundary was strikingly demonstrated by the use of fish lines, dye filaments, and air bubbles, as described previously. These methods illustrated a rather sharp break between the fluid direction near the upper wall and that a short distance away.

The complete secondary current just described is not confined to a cross section of the duct, nor is it a plane flow. For the square duct used in this experiment, this secondary flow may be idealized, however, as shown in Fig. 43, and will be termed the "initial vortex." As has been shown, its main strength and kinetic energy are concentrated in a boundary layer at the top, bottom, and inside duct walls. Since the secondary currents in this experiment are of such small magnitude, the numerical data do not exactly follow the pattern of Fig. 43, but the trend is evident.
Since the initial vortex is created by the combined action of the streamline curvature and wall resistance, its strength must depend on the radius ratio of the bend, the Reynolds number of the flow, and the wall roughness of the duct. For an infinite radius ratio, there would be no vortex; for a small radius ratio, there must be a strong vortex. Similarly, where there is no wall resistance, there would be no vortex, and where there is a great deal of resistance, there would be a strong vortex. Confirmation of the latter observation is presented in Section IV of this paper, where a comparison was drawn between the photographs of the yarns, taken at two Reynolds numbers in this experiment. For the lower Reynolds number (when \( \lambda \) is greater) the yarns toward the inner half of the duct at the top boundary at Section 3 deflected more toward the inside, indicating more flow into the corner. In the same manner, at Section 5, the yarn at one inside corner was deflected more toward mid-depth at the lower Reynolds number, indicating more flow out of the corner.

On the basis of this theory, the following observations may be made regarding conditions differing from those in the experimental duct: In a duct of circular cross section, there should occur a phenomenon similar to that already described, but the flow at the inside of the bend would have a more uniform vertical component, and the radial outflow would be more concentrated at mid-depth. In a duct of large aspect ratio, the secondary current would be confined to a depth consistent with the effective range of the transverse pressure gradient; rough observation of other experiments indicates that this range is probably of the order of magnitude of two duct widths (for a radius ratio of 2). A normal turbulent velocity profile on entrance would probably give a result similar to that for a circular duct—more uniform vertical pressure gradient and vertical velocity component along the inside wall, and more concentrated radial outflow near mid-depth. (This last conclusion is based on the observations made in Section IV of this paper, regarding the effect of entrance profile.)

The above theory explains a phenomenon observed by Yarnell [8] in his experiments using an elbow bending to the right. When there was a high velocity at the top and a low velocity at the bottom of the approach duct, he noted that there was only one secondary spiral, and that it was counterclockwise as viewed looking downstream. In the Yarnell experiment, the prin-
cipal pressure gradient occurred at the inside bottom corner because at this point, the interior velocity was increasing most rapidly with respect to the velocity at the wall. The familiar single spiral current in a horizontal open channel curve may also be explained readily by the above theory. (On the other hand, the theory would lead one to surmise that a vertical curve in an open channel would exhibit the same double spiral as a closed duct, since the channel side walls would correspond to the top and bottom of the duct. The sense of rotation of the spirals would be such that flow would occur along the side walls toward the center of curvature.) The theory also explains the observations of Hinderks [9], in connection with his experiments on bends in circular pipes. He noted that the spiral currents are not evident along the outer wall until the flow reaches a point past the mid-section of the bend. Along the inner wall, however, he found that converging flow toward the central plane of the bend is apparent for some distance upstream of the bend.

The Spiral

The longitudinal velocity, modified by the initial vortex, produces the familiar bend spiral. This spiral then carries the vorticity into the interior of the fluid where it is gradually distributed. As the flow continues around the bend, vorticity continues to be generated in accordance with the curvature of the streamlines (probably at a lesser rate than at the beginning of the bend) and is added to the vorticity already carried by the stream, thus increasing the spiral strength. For a bend of steady curvature and with favorable aspect ratio, the three-dimensional flow reaches a stable state at a point where the rate of generation of vorticity is just equal to the rate of dissipation through friction. No further change should occur in the streamlines after this point, and the entrance transition should be complete.

In a duct of normal aspect ratio, however, as the existing theory explains, the return spiral currents carry retarded boundary layer fluid into the interior, and as the curvature continues, the thread of maximum velocity is gradually shifted toward the outside. The factors which determine the strength of the return flow from the spiral also determine the rate at which
the maximum velocity will shift to the outside. In a duct where the aspect ratio is great enough so that the spiral currents are ineffective near mid-depth, the maximum velocity remains near the inner wall (see Fig. 42.).

By the time the end of duct curvature has been reached, the appearance of the spiral will depend on its history. For a duct of large aspect ratio, the double spiral will be confined to two separate regions, one near the top and the other near the bottom wall, and each half will look like the top or bottom half of Fig. 43. For a smaller aspect ratio, the return flow from the two spirals will be superimposed, and a more concentrated radial outward flow will result. For a circular duct, as compared to a square duct, or for a normal turbulent velocity profile as compared to a flat profile at entrance, there should also be a more concentrated return flow from the double spiral. For a duct of given central angle, as the radius ratio decreases, the initial vortex should increase in strength, but there should be a decrease in the length of duct over which the spiral may develop. Thus, if the radius ratio is small enough, even in a duct of small aspect ratio, there will be no appreciable return spiral flow at the end of the bend. (This would be the case if, for instance, the channel in this experiment were restricted to $45^\circ$ central angle.)

When the curvature ceases, vorticity is no longer generated at the inside corners. This phenomenon may be seen by close observation of the boundary streamlines, pictured in Fig. 10, inside and top views, beyond Section 7, and by reference to Fig. 15, Section 8, vertical. Beyond Section 7, the spiral is maintained by the vorticity already in the stream, but this vorticity does not penetrate to the corners, and the velocity no longer shows a vertical component at the top and bottom inside corners.

**Conditions at the End of a Bend**

Section IV of this paper presented the fact that a surface of separation was created near the downstream end of the experimental bend. This surface resulted from (1) the normal boundary layer separation of the two-dimensional flow, and (2) from the effect of the double spiral on the two-dimensional separation. The separation is a second source of vorticity in the flow of fluid around a bend. The total strength of the developed vortices will depend on the area of the surface of separation and on the velocity
In a bend in a duct of large aspect ratio, the flow is two-dimensional in a large part of the bend, and separation results for reason (1) only. In this case, two factors determine whether separation will occur at all, and the location of the separation when it does occur. The first determining factor is the pressure gradient along the inner wall as the flow progresses from the curved to the straight duct. This pressure gradient depends, in turn, on the radius ratio; there will be a large adverse pressure gradient unless there is quite a large radius ratio (see Fig. 26, for instance). The second determining factor is a critical Reynolds number, formed possibly from the radius of curvature of the inside wall and from the maximum velocity near the inside wall in established curvature. This Reynolds number, together with a consideration of the initial turbulence in the stream, will determine whether the boundary layer in the established curvature is laminar or turbulent, and therefore, whether the available energy is sufficient to overcome the adverse pressure gradient at the boundary. In a duct of large aspect ratio and large, but not too large, radius ratio, there is probably separation at the end of the bend. This separation would occur early when the flow is below the critical Reynolds number, and it would occur later when the flow is above the critical. Associated with the two-dimensional separation is a steep velocity gradient across the surface of separation. In order to satisfy the equation of continuity, the larger the separated area at the end of the bend, the greater is this velocity gradient. The following sketch shows a cross-sectional view of the two-dimensional separation:
In a duct of small aspect ratio \((D/B = 1\), for instance\), the spiral secondary flow influences the separation. The influence of the spiral may be seen by comparing the figure on p. 18, Section IV, which illustrates the separation in the experimental bend, with the figure in the previous paragraph, which illustrates the separation in a two-dimensional bend. In the case of the smaller aspect ratio, the spiral current reduces separation near the top and bottom walls. If the return flow from the spiral is large enough, the spiral current also increases separation near mid-depth. Furthermore, the spiral inflow to the separated area reduces the velocity gradient across the surface of separation.

To analyze further the effect of the spiral on separation, there will be considered a bend with variable aspect ratio and with other parameters constant. A large aspect ratio may be reduced by decreasing the depth of the duct, increasing the width of the duct, or by a combination of decreasing depth and increasing width. Accordingly, a small reduction in aspect ratio in a bend of large aspect ratio will produce some reduction in the surface of two-dimensional separation, a small reduction in velocity gradient across the surface, or a combination of both. For decreasing depth, the velocity gradient is also reduced by the spiral inflow penetrating farther into the separated area. If the aspect ratio is further reduced by stages, this favorable situation will prevail until the two halves of the spiral in the bend approach each other so closely that their return flows are superimposed. Any further reduction in aspect ratio beyond this point will be characterized by a very strong spiral return flow near mid-depth, increasing the area of separation, and nullifying the reduction in velocity gradient or the reduction in area of separation.

On the other hand, an analysis of the effect of varying wall resistance on a given bend of unity aspect ratio shows that a smooth-walled channel at high Reynolds number would develop a very weak initial vortex and the separation at the end of the bend would be nearly two-dimensional. A slight decrease in Reynolds number, or increase in roughness would increase the initial vortex strength and increase the spiral inflow to the separated area, decreasing the velocity gradient at the surface of separation. A considerable decrease in Reynolds number or increase in roughness, however, would result in a very strong return spiral flow near mid-depth, increasing the area of
separation and nullifying the reduction in velocity gradient caused by spiral inflow to the separated area.

The discussion in the last three paragraphs referred to the separation at the end of a bend. The surface of separation, of course, continues to exist downstream from the end of a bend. Section IV of this paper pointed out that as the flow progresses downstream, the extent of the separated area is increased and the velocity gradient across the surface of separation is decreased by interchange of energy and by spiral inflow to the separated area. Thus, it would seem that the time or length of duct required to eliminate the surface of separation would be less for an initially larger area than for a smaller area. At the same time, it is probably true that the velocity gradient across the surface of separation is relatively large for only a short distance downstream from the end of the bend (about three duct widths in this experiment) and that any vorticity generated beyond that short distance is negligible, regardless of how much farther the surface of separation persists. It should be satisfactory, then, to consider that the vorticity generated at the downstream end of a bend is roughly proportional to the length of the trace of the surface of separation in a cross section near the end of the bend and to the velocity gradient across this trace. These two quantities are determined by the magnitude of the two-dimensional separation and by the strength of the spiral at the end of the bend. Thus, in addition to those factors influencing the two-dimensional separation, the following factors may be considered as influencing the vortex strength generated, since each has an influence on the strength of the spiral at the end of the bend: initial vortex strength, aspect ratio and shape of duct, entrance velocity profile, and deflection angle.

The theory enunciated herein explains the observations made in Section IV of this paper, regarding the differences between the photographs of the yarns taken at a high and at a low Reynolds number downstream from the bend. Recalling that the Reynolds number of one flow was approximately four times that of the other, it may be assumed that the lower flow established a stronger initial vortex, and produced an earlier and greater separation at mid-depth. Thus, in the flow at the higher Reynolds number, at Section 7, near the inside wall of the experimental duct, the separation was just beginning and the yarns showed a larger deflection than they did for the flow at
the lower Reynolds number which had previously separated and was now becoming parallel to the walls. But at the same section, toward midstream, the larger area of separation at the lower Reynolds number caused a deflection of the yarn, while at the higher number, separation did not extend far enough to cause a deflection. Away from mid-depth, the stronger spiral at the lower Reynolds number retarded the separation and caused less deflection. Farther downstream, because the flow at the lower Reynolds number separated early, it was always ahead of the higher flow in returning to the normal parallel flow of the downstream tangent. The lower flow, therefore, showed less deflection farther downstream.

The Mechanism of Energy Loss in a Bend

Wattendorf [5] showed that in two-dimensional flow, the resistance caused by the walls alone was only slightly greater for a curved duct of established curvature than it was for a straight duct. Therefore, in three-dimensional flow in a bend of given deflection angle, the additional loss of energy resulting from the bend must be caused by the generation and dissipation in friction of vorticity. Not all of the energy of the vortices is dissipated in friction; some is recovered in the downstream tangent as shown by Wirt [10] and others. For bends with given lengths of downstream tangent, however, the energy loss may be considered proportional to the vortex strength which is generated. For a given bend, at very low Reynolds numbers (but above the critical) where the wall resistance is great, the initial vortex strength and therefore, the resulting loss, must be comparatively large. With increasing Reynolds numbers, the strength of the vortices and resulting loss decrease, but at a decreasing rate, as is the case with wall resistance. Fig. 41 shows that near the beginning of the experimental bend, where the initial vortex is generated, the apparent rate of energy loss is greater for the lower Reynolds numbers than for the higher numbers—a confirmation of the preceding conclusions. (The apparent rate of loss of energy between Sections 2 and 3 appears in Fig. 41, as explained in Section IV of this paper.)

At the downstream end of a bend, the additional energy loss would depend on the strength of the vortices resulting from separation. In the case of a bend of large aspect ratio, the total energy loss would be determined from a loss from the initial vortex near the top and bottom, plus a
separation loss at the downstream end of the bend. For a bend with constant radius ratio, mass flow, and Reynolds number, the separation loss would slowly decrease as the aspect ratio of the bend is decreased and as the spiral inflow penetrated farther toward mid-depth, but the initial vortex loss would remain constant. However, when the aspect ratio becomes small enough to allow superimposing of the return flow from the upper and lower half spirals, the loss would increase rapidly because of the increased area of separation. With the effective pressure gradient of the initial spiral confined to a depth equal to two duct widths, as has been suggested, an aspect ratio of something greater than 1, perhaps, should be required to overcome the large additional separation loss. (The discussion in this paragraph has referred to total loss which would have to be divided by \( \sqrt{\frac{2g}{v}} \) to obtain \( \xi \). Under the conditions stated, \( \bar{v} \) would decrease with decreasing aspect ratio, so that \( \xi \) would probably increase gradually at first with decreasing aspect ratio, and would then increase more rapidly after the spiral affected the separation. Wirt's data [10] may be examined in this connection.)

In a horizontal bend in an open channel, on the other hand, there is only one spiral and, therefore, the effect of changing aspect ratio should not be as pronounced as it is for a closed duct. For a channel bend, an aspect ratio below 2 should show some increase in separated area because of the crowding of the vortex between the surface and the bottom, but the effect would probably not be important until the aspect ratio became less than one, perhaps. The experimental data of Yen and Howe [11] and of Raju [12], may be examined in this connection.

Near the downstream end of a given bend in a duct of unity aspect ratio, the energy loss should decrease with decreasing Reynolds numbers down to a certain point, in accordance with the previous discussion of vorticity at the downstream end of a bend. This trend is apparent from the data from this experiment, as presented in Fig. 41. Considering the overall energy loss in such a bend, the initial vortex loss is low but the separation loss is high at a high Reynolds number. As the Reynolds number decreases, the separation loss decreases rapidly at first, while the initial vortex loss increases slowly. Thus, at first, the bend loss decreases with decreasing Reynolds numbers. Existing data [14] show that an optimum point is reached
at about $Re = 150,000$ (varying with $R/B$). At lower Reynolds numbers the separation loss decreases only slightly, if at all, while the initial vortex loss increases more rapidly, resulting in an increase in bend loss.

There may now be attempted an explanation for the variation of bend coefficient ($\zeta$) with radius ratio ($R/B$) in a duct of fixed aspect ratio ($D/B = 1$, for example). At small $R/B$, the initial vortex is quite large and the separation, even without assistance from the spiral, is large. The initial vortex and the separation combine to cause a large energy loss. With increasing $R/B$, the initial vortex decreases in strength as does the separation, and the total loss decreases rapidly. At a certain point, the initial vortex becomes small enough so that the return flow from the spiral does not alter the area of separation, but rather only reduces the flow into the separated area. When this point is reached, the trend of energy loss is reversed and there can be noted a slight increase in $\zeta$ with increasing $R/B$. This point will vary with the factors which determine the strength of the return flow from the spiral—roughness, Reynolds number, radius ratio, aspect ratio, shape of cross section, deflection angle, and entrance velocity profile. A duct of greater roughness, for instance, would have a stronger initial vortex and a stronger return spiral flow at any given $R/B$ than a duct of lesser roughness, and should therefore reach its optimum $R/B$ later than does the smoother duct. Published data show that this optimum point lies between $R/B = 2$ and $R/B = 6$, for a channel of $90^\circ$ deflection angle.

As the $R/B$ continues to increase beyond the point of minimum $\zeta$, $\zeta$ also increases somewhat because of the smaller spiral inflow to the separated area. This increase continues until a point is finally reached where the adverse pressure gradient at the inside wall is small enough so that no separation occurs. The loss at this point takes a more or less sudden drop, and thereafter it consists entirely of that loss caused by the initial spiral. Apparently this point occurs in the vicinity of $R/B = 15$ for a deflection angle of $90^\circ$, but again the occurrence of the point depends on such factors as roughness and entrance velocity profile. Beyond this point, the loss drops steadily with increasing $R/B$, until the loss is zero at $R/B = \infty$.

Many other phenomena relating to flow in bends or even in bladed turns may be explained or predicted by this theory. The theory may be applied,
for instance, to a bend with an increased cross section at its central section, obtained by moving the outside wall away from the center of curvature. The enlarged cross section cannot greatly affect the flow in the entrance transition, but at the downstream end of the bend, there is a contraction with possible elimination or reduction of the separation. This, of course, reduces the energy loss. In applying the theory to bladed turns, however, one must also consider the possibility of loss resulting from the surface area of the blades.

It may be argued that any effect of separation at the outside of the bend has been neglected in this theory. Such separation actually occurs in mitre bends or in bends of very small radius ratio. At the outside of a bend, however, the velocities and velocity gradient at separation are small compared to those at the separation at the inside. Furthermore, the vorticity generated at the outside of a bend is rapidly carried away by the high velocities downstream at the outside of the bend, and for this reason has no continuing effect. For these reasons, it is considered that the separation at the outside of a bend is unimportant in explaining the flow process or the mechanism of energy loss.

To state the theory briefly: the character of the flow and the mechanism of loss in a fluid bend is considered to be composed of three parts. The first is the initial vortex and the loss resulting therefrom; the second is the two-dimensional separation at the downstream end of the bend and the loss associated with this separation; and the third is the modification produced by the spiral currents in the separation and separation loss.

VI. SUMMARY

The Proposed Theory

On the basis of these experiments, the following theory is proposed to define the flow in a bend and the associated mechanism of energy loss.

1. In addition to the main through flow, the flow in a bend is characterized by:

   a. An initial vortex which gives the through flow a spiral nature.

   b. A separation at the downstream end of the bend.

   c. The effect of the spiral on the separation.
2. The energy loss in a bend is determined by the combined loss resulting from the dissipation in friction of a part of the energy contained in the initial vortex and in the vortices resulting from separation. The energy loss is proportional to the strengths of the vortices.

3. The strength of the initial vortex, and therefore its associated energy loss, depends principally on the wall roughness of the duct, the Reynolds number of the flow, and the radius ratio of the bend.

4. The total strength of the vortices resulting from separation, and therefore, their associated energy loss is proportional to the area of the surface of separation, and the velocity gradient across this surface.

5. The area of the surface of separation and the corresponding velocity gradient are determined by:

   a. The normal separation in a two-dimensional channel which depends upon the radius ratio of the bend, and a critical Reynolds number (formed possibly from the maximum velocity near the inside wall and the radius of curvature of the bend).

   b. The increase in the area of separation produced by the return flow from the spiral. The strength of the return flow at the end of the bend depends upon the initial vortex strength, the velocity profile at the entrance to the bend, the aspect ratio and shape of the duct, and the deflection angle (or length) of the bend.

   c. The spiral inflow to the separated area which reduces the velocity gradient across the surface of separation. This inflow is proportional to the initial vortex strength and aspect ratio of the bend.

6. Separation at the outside of a bend, when it occurs, is unimportant in explaining the flow process or the mechanism of energy loss.

This theory is not completely verifiable by previously published data, nor by the data obtained in these experiments; nor are any of the data
in disagreement with the theory. If the theory had been available beforehand, these tests would have sought experimental data tending directly to support or alter it. All of the data obtained in these experiments, however, is published in the body of the report, so that those who so desire may make their own interpretation.

Suggested Additional Research

The acceptance of the basic general theory proposed above makes possible the outlining of a research program, the results of which should verify the theory and lend quantitative significance to it. A research program divided into the following three basic parts is suggested:

1. A study of the initial vortex and the resulting spiral. These tests would be largely experimental and should utilize ducts of large aspect ratio and continuous curvature. The study should, however, be confined to the top and bottom portions of the ducts, rather than to the two-dimensional portion. Subjects for study should include the effect on the initial vortex of changing radius ratio, Reynolds number, roughness, entrance profile, and cross-sectional shape, as well as the development of the spiral with deflection angle. In such experiments, the energy loss could be coordinated with the vortex formation.

2. A study of the separation loss at the downstream end of a two-dimensional bend (a bend of large aspect ratio). This study should be both analytical and experimental, and should be based on available theory and data, as well as on any new experimental data obtained. The energy loss in a duct with such a bend could then be coordinated with the vorticity generated. (A correlation of energy loss with vorticity is based on Wattendorf’s [5] finding that there is little increased loss caused by the walls alone in a curved duct.)

3. A study of the effect of the superimposed spiral on the separation and loss studied under (2) above. This would necessarily be an experimental study. Using a two-dimensional duct of large aspect ratio, the spiral should be introduced artificially if practicable, so that the spiral current would be the only variable.
Spiral currents of various strengths and distributions should be used.

The results of these three basic programs, when coordinated, should make it possible to predict the performance of any given flow diversion, whether the diversion be by a pipe elbow, an open channel bend, or a bladed turn.
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DIFFUSOR

ALL SIDES ALIKE

MANOMETER HOLES LOCATION

REINFORCE WALL WITH \( \frac{1}{4} \) X 6 LUCITE STRIP

DEVELOPMENT - OUTSIDE OF BEND

DEVELOPMENT - INSIDE OF BEND

FIGURE 2 ELBOW DETAILS
FIGURE 3. APPARATUS FOR SUSPENDING YARNS IN THE ELBOW TO INDICATE FLOW DIRECTION

(Note that the experimental duct terminates just beyond Section 9 in this photograph.)
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ESTABLISH DIRECTION OF FLOW

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SECTION 5

VERTICAL SECTIONS

HORIZONTAL SECTIONS

\( \text{Re} = 220,000 \)

Figure 15. Flow directions at selected cross sections as obtained from photographs of lines.
SECTION 2

SECTION 3

SECTION 5

SECTION 7

SECTION 8

SECTION 9

FIGURE 16

STATIC PRESSURES
(Pressures in inches divided by velocity head in feet)

Wall Pressures
Internal Pressure Contours
SECTION 2

SECTION 3

SECTION 5

SECTION 7

SECTION 8

SECTION 9

FIGURE 17
TOTAL HEAD CONTours
(Total heads in inches, deviated by velocity head in test)
FIGURE 18

VELOCITY SURFACE

(REC = 1.0 sec. divided by mean velocity)

S E R I E S I

SECTION 7

Reynolds' No. = 2.25 x 10^9 (Water)
Average Velocity = 7.9 fps.
SECTION 2

SECTION 3

SECTION 5

SECTION 7

SERIES III

Reynolds No. = 2.26 \times 10^5 (Water)
Average Velocity = 16.76 fps.

SECTION 8

FIGURE 23

WALL Pressures "\text{in} \times 10^3" units

(Pressures in inches divided by velocity head in feet)
Figure 24
Total head contours

Series II
Section 7
Reynolds No. 626 x 10^5 (W baptism)
Average velocity 16.76 fps

Section 2
$H_2 = 1.062$

Section 3
$H_2 = 1.045$

Section 5
$H_2 = 1.079$

Section 7
$H_2 = 0.954$

Section 8
$H_2 = 0.946$

Section 9
$H_2 = 0.942$

Inside of bend

Outside of bend

Total head is measured in feet, velocity head in feet.
SECTION 2
STATIC PRESSURES
(Pressures in inches divided by the velocity head in feet)

SECTION 2
TOTAL HEAD CONTOURS
(Total heads in inches divided by velocity head in feet)

SECTION 2
VELOCITY CONTOURS
Vel in ft/sec divided by mean vel in ft/sec.

SECTION 9

FIGURE 32
7 DIAMETERS ADDED ENTRANCE LENGTH

SERIES VII
SECTION 9
Reynolds No. = 3.2 x 10^5 (water)
Average Velocity = 7.9 fps.
SECTION 2

STATIC PRESSURES
Pressures in inches divided by the velocity head in feet

SECTION 2

TOTAL HEAD DISTRIBUTION
Total head in inches divided by the velocity head in feet

SECTION 2

VELOCITY DISTRIBUTION
Velocity in f/s divided by mean velocity in f/s

SECTION 9

SERIES VII

Reynolds No. $6.80 \times 10^6$ (water)
Average Velocity = 16.7 f/s

7 DIAMETERS ADD AND ENTRANCE LENGTH

FIGURE 33
FIGURE NO: 34
EFFECT OF ENTRANCE VELOCITY DISTRIBUTION ON DOWNSTREAM DISTRIBUTION

SERIES VII
MEAN VELOCITY = 8.64 FRS.
REYNOLDS NUMBER = 3.32 x 10

SECTION 2-h
SECTION 3-h
SECTION 8-f

NORMAL ENTRANCE
7/8 INCH SQUARE MESH
7/8 INCH TANGENT
FIGURE 41 AVERAGE TOTAL ENERGY BY SECTIONS
FIGURE 42. VELOCITY PROFILES WITH NORMAL TURBULENT ENTRANCE PROFILE

(From Wattendorf [5])
FIGURE 43 INITIAL VORTEX FOR A BEND IN A SQUARE DUCT.
APPENDIX I

THE PITOT CYLINDER
Appendix I is based on experimental work which was performed prior to the work described in the body of this report. To make the information available, it was published as a separate circular of the St. Anthony Falls Hydraulic Laboratory in October, 1947.
INTRODUCTION

In connection with an investigation of flow around bends, the St. Anthony Falls Hydraulic Laboratory made preliminary studies of measuring techniques which included the use of the instrument known as the pitot cylinder. The experiments on this instrument are described in this circular.

The pitot cylinder has been in use for a number of years as a measuring device for determining flow direction and total head in two-dimensional flow. It has also been used for measuring static pressure ([1]). It has been previously described by several other investigators, including Marks ([2]) and Binder and Knapp ([3]).

The cylinders used in these experiments are shown in Figs. 1, 2, and 3. The type of cylinder shown in Figs. 1a and 2, with the pressure holes at the center, will be termed a "long" cylinder, and the type with the pressure holes near the end, shown in Figs. 1b, 1c, and 3, will be termed a "cantilevered" cylinder.

The pitot cylinder as a measuring device has the following features: (1) It can be inserted into a closed, pressurized channel with a comparatively simple gland. (2) It will produce a negligible disturbance in the flow, if the cylinder diameter is kept small compared to the channel cross-section (cylinder diameter 0.1 times the channel breadth or less ([4])). (3) Measurements of both direction and total head at a point in the enclosed fluid can be obtained with the same instrument. (4) The instrument is simple to make.

The cylinders, shown in Figs. 2 and 3, were designed for use in a bend in a six-inch square pipe, with water flowing at a mean velocity of 20 ft per sec. The long cylinder proved adequate for this purpose, but the 1/4 in. cantilevered cylinder vibrated when extending 4 in. or more into water flowing at 20 ft per sec. The cantilevered cylinder was also deficient in that the tip was deflected when extended several inches into high velocity flow so that the angle with the flow was altered. The construction of this tube is faulty in that one pressure hole has a constant *

* Numbers in brackets refer to corresponding numbers in the Bibliography.
FIG. 1-PITOT CYLINDERS
PITOT CYLINDER

SECTION B-B

ENLARGED CENTRAL PORTION OF CYLINDER

FIG.2—LONG PITOT CYLINDER
FIG. 3 - CANTILEVERED PITOT CYLINDERS

(GERRO-BEND IS THE TRADE NAME FOR A LOW MELTING POINT ALLOY)
diameter for only one diameter of depth—two diameters would be required for consistent direction readings (5), and preferably, the two holes should be of the same length. The smaller cantilevered cylinder was designed for obtaining only total head readings near the wall, using a machinist’s micrometric depth gage for accurate positioning.

THE CYLINDER AS A MEASURING DEVICE

Some questions with regard to the method of use and reliability of the pitot cylinder as a measuring device led to the experiments on which this paper is based. The solid curve in Fig. 4 is a part of a typical pressure diagram for a circular cylinder of infinite length in an unbounded parallel flow with the cylinder axis normal to the flow. The Reynolds number is about 41,000 based on the diameter of the cylinder as the length parameter. The abscissa scale in Fig. 4 represents the angle in degrees left or right of the front stagnation point. The ordinate scale is the dimensionless ratio of dynamic pressure, at a point on the cylindrical surface (pressure reading minus static pressure) to stagnation pressure. The shape of the curve depends on such factors as the Reynolds number, the degree of turbulence, and the actual diameter of the cylinder (1) and (4).

The measuring properties of the pitot cylinder are based on the following facts apparent from Fig. 4: (1) The pressure curve is quite flat near the front stagnation point. Thus, for the purpose of measuring total head either of the two holes may be "pointed" into the flow with an error of as much as two to three degrees in either direction without causing an appreciable total head reading error. (2) The curve is symmetrical about the front stagnation point; consequently, if one knows two directions for which the pressure readings are the same, the stagnation point (and the direction of flow) is to be found halfway between these two directions. (3) Between the 35° and 40° points, the curve passes through the points of zero velocity pressure so that the pressure reading yields the static pressure directly; but at these points the curve is very steep and only a small directional change would cause a large pressure change (measurement shows a change of 5 to 10 per cent of the velocity head per degree in this region). The position of the holes in the cylinders is such as to take advantage of this steep gradient to increase
FIG. 4—PRESSURE DISTRIBUTION ON A CIRCULAR CYLINDER
directional sensitivity.

The questions that arise in connection with using the cylinder as a measuring device are these: (1) Can the cylinder be used for reading static pressures? (2) How close to a reference wall piezometer may the cylinder be placed without affecting the piezometer reading? (3) What is the total head coefficient of the pitot cylinder? (4) How close to the tip may the total head reading hole be placed on a cantilevered type cylinder? (5) Will the cylinder be subject to circulation and thus give inaccurate direction readings if placed near a wall, in curvilinear flow, or in flow with a steep lateral velocity gradient? (6) Will the presence of the cylinder, under the conditions of (5) above, alter the basic flow? (7) Can the pitot cylinder be used in three-dimensional flow where its axis is not perpendicular to the flow?

To obtain answers to these questions, some experiments were made with water flowing in a six-inch square lucite pipe containing a radius elbow of one foot mean radius. This equipment was set up for the investigation of flow around bends, and a complete description will be found in a report prepared for that project (6). The pipe provided test sections where cylinders could be inserted at many points around the circumference. Discharge was measured with a calibrated flume and was probably accurate to within less than one per cent. Pressure readings were made on a 100 in. or a 50 in. U-tube differential manometer. The former had a water-air interface and the latter a water-kerosene interface. Both scales were read to the closest 0.01 in., although accuracy was probably no greater than 0.05 in., except at the highest flows where pulsations caused variations of from 0.2 to 0.4 in. At the velocities of the experiments (6 to 20 ft per sec, mean velocity) the pressure readings were thus accurate to within 1/2 of one per cent.

STATIC PRESSURES

A first glance at Fig. 4 would lead one to believe that static pressures are readily obtainable by using the pitot cylinder. This is not the case, however. To obtain a reading with an error of less than one per cent of the velocity head, it would be necessary to point the pressure-reading hole within 0.2 degrees or less of its correct position; the steep slope of the curve at zero dynamic pressure requires this. (A
non-circular cylinder of suitable symmetrical cross section could be designed to permit a larger directional error.) Yet, as previously mentioned, the exact shape of the pressure curve for a circular cylinder is variable, and even in a given experiment the correct location of the hole would change if the Reynolds number changed appreciably. But even when the correct position of the hole is known and is constant in position, it is still necessary for the shop to locate and drill the hole and, for the person taking the readings, to point the hole all within the allowable 0.2°. Further discussion on this point may be found in reference (1).

As an additional source of error, the gage fluid surface near the point of zero dynamic pressure fluctuates considerably and makes it necessary to take average readings. And still another variable is introduced in the static pressure reading by the presence of the cylinder in the enclosed channel. This increases the velocity and decreases the pressure so that an erroneous reading is obtained. That satisfactory static head readings could not be obtained with the pitot cylinder was demonstrated in this experiment when it was found impossible to reproduce static readings at point in the flow not only within one per cent but even within 5 per cent of the velocity head.

In these experiments, static pressures were obtained from wall piezometers and static tubes inserted at critical points in the flow. The static tube was inserted through a special gland (6). All static pressures were referred to a fixed reference wall piezometer, as were the cylinder readings taken later.

SEPARATION OF CYLINDER AND WALL PIEZOMETER

Pressure readings obtained by use of the pitot cylinder, by comparison with a reference wall piezometer through a differential manometer, will be inaccurate if the cylinder is placed too close to the wall piezometer. There is a velocity increase and pressure decrease at the wall which is carried into the reading. Analysis by potential theory using the method of images indicates that at least 16 diameters is required between the cylinder and wall piezometer to produce a pressure reading error of less than one per cent of the velocity pressure. This was substantiated essentially in tests. Total head readings were taken using cylinders of several diameters at a fixed location in a test section and
fixed wall piezometers. Table I contains a sample of the data obtained. The letter "p" is the distance normal to the wall, and "q" is the distance parallel to the wall from the cylinder to the wall piezometer. The columns labeled "Actual Vel. Head" and "Cylinder Reading" were obtained from total head readings taken with a pitot tube and the test cylinder, respectively, at the same point in the flow by subtracting the previously measured static pressure. The pitot tube was inserted through the gland previously mentioned and its stem was sufficiently distant from the reference wall piezometer to introduce no error in the reading.

**TABLE I**

Effect of Wall Piezometer Location on Cylinder Reading

<table>
<thead>
<tr>
<th>Cylinder Diameter</th>
<th>p</th>
<th>q</th>
<th>(\sqrt{p^2 + q^2})</th>
<th>Actual Vel. Head</th>
<th>Cylinder Reading</th>
<th>Difference per cent of Vel. Head</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inches</td>
<td>Diam</td>
<td>Diam</td>
<td>inches</td>
<td>inches</td>
<td>inches</td>
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<td>0</td>
<td>12</td>
<td>22.75</td>
<td>23.23</td>
<td>2.1</td>
</tr>
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<td>0</td>
<td>12</td>
<td>21.43</td>
<td>22.02</td>
<td>2.7</td>
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<td>12</td>
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<td>56.21</td>
<td>2.2</td>
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<td>12</td>
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<td>55.56</td>
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<td>3.65</td>
<td>7.0</td>
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<td>12</td>
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<td>21.32</td>
<td>2.3</td>
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<td>0</td>
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<td>20.83</td>
<td>20.90</td>
<td>0.3</td>
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<td>7/16</td>
<td>7</td>
<td>4(\frac{1}{2})</td>
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<td>20.35</td>
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<td>1.5</td>
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<tr>
<td>1/4</td>
<td>12</td>
<td>8</td>
<td>11(\frac{1}{2})</td>
<td>21.70</td>
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<td>11</td>
<td>19(\frac{1}{2})</td>
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<td>21.75</td>
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</tr>
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</table>
CYLINDER COEFFICIENT

The coefficient of the cylinder was determined by comparison with a calibrated pitot tube of standard design and by integration of flow in a straight section of pipe. The pitot tube was calibrated in a submerged orifice. Its coefficient was found to be 0.99. The pitot tube and long cylinder were then both read at the same points in the flows, where they gave exactly the same readings. (The pitot tube was inserted in the special gland through a plug in the channel wall so that its tip was at the same point as that which the cylinder hole would occupy.) The final design of the 1/4 in. cantilevered cylinder, shown in Fig. 3, when tested in the same manner, gave readings about one per cent less than the pitot tube. The smaller cantilevered cylinder also read less than the pitot tube, but by a difference well within one per cent.

In checking the long cylinder coefficient by integration of flow, comparing the discharge with that measured by the flume, the coefficient appeared to be about 0.975. The difference between this and the orifice calibration is at least partially due to the difference in scale of turbulence in the two flows. The coefficient obtained by integration was constant within one per cent for all flows (Reynolds number from one to $5 \times 10^5$, based on the channel side as length parameter). Since the long cylinder gives the same readings as a standard pitot tube, and its coefficient is constant, such a cylinder may be used without calibration on any normal test arrangement.

HOLE LOCATION ON A CANTILEVERED CYLINDER

For a cantilevered type cylinder, the proximity of the measuring hole to the tip of the cylinder may affect the reading obtained. Fig. 5 is a curve obtained by moving in stages a No. 60 (.040 in. diameter) drill hole away from the hemispherical tip of a 1/4 in. cantilevered cylinder. The ordinate scale is again the dimensionless ratio of dynamic pressure at the measuring point to stagnation pressure and the abscissa scale is the distance from the tip of the cylinder to the center of the hole in cylinder diameters. At these Reynolds numbers, a hole at a distance of 2 diameters from the tip will result in an error of less than one per cent, and this distance has been adopted in the design of the
1. \( \text{Re} = 37,300 \)
2. \( \text{Re} = 20,600 \)

**Figure 5** - End Effect, Cantilevered Cylinder

- \( \frac{3}{8} \)" Long Cyl. (Marks) \( \text{Re} = 27,000 \) (Est)
- \( \frac{1}{4} \)" Long Cyl. \( \text{Re} = 19,000 \)
- \( \frac{1}{4} \)" Long Cyl. \( \text{Re} = 35,000 \)
- \( \frac{1}{4} \)" Cant. Cyl. \( \text{Re} = 19,000 \)
- \( \frac{1}{4} \)" Cant. Cyl. \( \text{Re} = 35,000 \)

**Figure 6** - Effect of Angularity on Stagnation Pressure


Distance from tip, diameters

\( P - P_0 \) vs \( \text{Re} \) vs Distance from Tip, Diameters

1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00

0 1 2 3 4

**FIG. 6 - EFFECT OF ANGULARITY ON STAGNATION PRESSURE**
1/4 in. cylinder shown in Fig. 3. Where it is not necessary to have a short tip distance, more than 2 diameters will result in less error; the smaller cantilevered cylinder, shown in Fig. 3, was constructed with a greater distance.

**OCCURRENCE OF CIRCULATION**

When a cylinder is placed close to a wall, in curvilinear flow, or in flow with a steep lateral velocity gradient, the pressure diagram, Fig. 4, may be distorted asymmetrically, because of the occurrence of circulation. The degree of distortion will affect the accuracy of the direction readings and the circulation will alter the basic flow.

As an experimental check on the occurrence of circulation, measurements of the pressure distribution on the long cylinder were made at several points in the flow. One set of points was selected within four diameters (one in.) from a wall, as close to a wall, laterally, as it was contemplated, placing the cylinder in use. Other sets were selected in curvilinear flow (about 12 in. radius of curvature) and in flow with a steep lateral velocity gradient (a variation from 4.5 f/s to 9 f/s in 3/4 in. with a mean velocity of 8 f/s, and from 12.5 f/s to 20 f/s in 3/4 in. with a mean velocity of 17 f/s).

Experimental points obtained in the curvilinear flow at a Reynolds number of about 42,000 are plotted as solid circles in Fig. 4. Those obtained in the steep lateral velocity gradient, at a Reynolds number of about 46,000, are plotted as open and half solid circles. (The open circles represent a traverse made with one hole, and the half solid circles represent those taken with the other hole of the long cylinder.) Experimental points obtained with the cylinder near the wall at both a high and low velocity and for the other conditions at a low velocity are not plotted. They all lie between the solid curve and the plotted points. These data appear to indicate that within the limits of measuring accuracy of the instrument, and in so far as these conditions are effective in the experiment, there is no effective circulation about the cylinder when it is placed near a wall, in curvilinear flow, or in flow with a steep lateral velocity gradient. This would also indicate that the basic flow pattern is not appreciably altered.
These data are substantiated by visual observation of air bubbles in the flow with and without the cylinder. They are further substantiated by comparison of the measured direction with that indicated by yarns in the flow of other experiments where agreement within 2° at the critical points was obtained at the lower velocities. (Yarns were not used at the higher velocities.)

THREE-DIMENSIONAL FLOW

In three-dimensional flow, the axis of the cylinder will not necessarily be perpendicular to the flow direction. In the basic experiment, for which this equipment is being tested, flows at angles of as much as 20° out of the plane of measurement will occur. (This angle is termed the angularity.) For such flows, readings must be made by inserting the cylinder from two mutually perpendicular directions and describing the flow by its components.

For a cylinder at an angle to the flow, a symmetrical pressure curve similar to Fig. 4, but with a somewhat sharper peak, is obtained. This indicates that direction readings in the plane of measurement will be as accurate in three-dimensional readings as in two-dimensional flow. Total head readings of the cylinder, however, will fall off as the angularity of the flow is increased, as is shown in Fig. 6. The abscissa represents the angularity of the flow and is measured in both directions from the normal. The ordinate is again the dimensionless ratio of dynamic pressure to stagnation pressure (which, in this case, is taken as the dynamic pressure with the tube normal to the flow). (Reynolds number is based on the cylinder diameter.) The solid line was obtained by Marks [2]. His measurements were made on a 3/8 in. diameter cylinder of the long type in an 8 in. air jet. The curve is asymmetrical, probably due to an angularity in the jet or an irregularity in the cylinder. Three experimental points obtained at the St. Anthony Falls Laboratory, using the 1/4 in. long cylinder, are seen to fall on one leg of the curve. The points were obtained by inclining the cylinder at measured angles in a parallel flow.

It is interesting to note that the right hand curve of Fig. 6 fits the cosine square curve of the angularity. Writing h' for the meas-
ured velocity head and $h$ for the true velocity head, we have:

$$h' = h \cos^2 \alpha$$

Also:

$$V - \sqrt{2gh} = \frac{\sqrt{2gh}}{\cos \alpha} \quad \text{or} \quad V \cos \alpha = \sqrt{2gh}$$

Thus, the cylinder measures the component of the velocity normal to the cylinder. In the investigation of flow around bends, some velocity head readings, obtained from two mutually perpendicular directions, have been compared on this basis and show good agreement.

The 1/4 in. cantilevered cylinder does not yield a symmetrical curve when angularity changes from one side of the normal to the other as shown in Fig. 6. The plotted points have already been corrected for angularity of the test stream, but not for deflection of the cylinder; the deflection, however, was very small—probably less than 2° at the high velocity with the cylinder normal to the flow. On the basis of these tests, it is not believed that the cantilevered type cylinder is a satisfactory device for measuring total head in three-dimensional flows where the flow is more than 3° or 4° out of the measuring plane, unless a calibration curve is prepared for the particular instrument.

CONCLUSIONS

The following conclusions are drawn from the investigation:

(1) The long type pitot cylinder is a generally satisfactory device for making velocity and directional flow measurements in a closed channel.

(2) It should not be used for making static pressure readings, however.

(3) The cylinder and the reference wall piezometer should be separated by at least 16 diameters.

(4) The total head coefficient is the same as that for a standard pitot tube under the same conditions, but (5) when the measurement is made with the flow at an angle to the normal to the cylinder, the measured velocity head must be divided by the cosine square of the angularity to obtain the true velocity head.

(6) At least within the limits mentioned previously, direction readings are not appreciably affected by the proximity of a wall, curvilinear flow, or a steep lateral velocity gradient, nor (7) is the basic flow altered by the presence of the cylinder in these circumstances.

(8) The cantilevered cylinder is also reliable within certain
limits. (9) The holes should be 2 diameters or more from the tip and 
(10) if the cylinder is to be used in three-dimensional flow, a total head 
calibration curve should be prepared for the individual instrument.

(11) The instrument should be of such sectional area as to pre-
vent excessive vibration and deflection (which would produce additional 
angularity), and (12) the pressure holes should be carefully designed. Oth-
erwise, the same precautions and conditions as for the long cylinder 
should prevail.

These instruments will be used with some regularity in future 
work at the St. Anthony Falls Hydraulic Laboratory, and more data will be 
added to that presented when it becomes available.
The Pitot Cylinder

BIBLIOGRAPHY


APPENDIX II

ERRORS RESULTING FROM NEGLECT OF THE TUBE COEFFICIENT

In Figs. 16-37, inclusive, the tube coefficient has been neglected in presenting total energy and velocity data. The average value of the tube coefficient is 0.98. This value was determined by comparing the discharge obtained from the integration of the velocities at Section 2 with the measured discharge from the H type flume.

To correct the velocity data given, it is necessary only to multiply any velocity by the square root of the tube coefficient (0.99).

The correction to be applied to the total head values may be analyzed as follows:

\[ E = f(E' - \frac{p}{w}) + \frac{p}{w} \]

where \( E \) is the true total head in feet of water at a point; \( E' \) is the actual head reading obtained with the tube; \( \frac{p}{w} \) is the static head reading in feet of water, and \( f \) is the tube coefficient. This may be rewritten:

\[ E = fE' + \frac{p}{w} (1 - f) = fE' \left[ 1 + \frac{\frac{p}{w}(1 - f)}{fE'} \right] \quad (1) \]

The true total energy is therefore equal to the actual reading multiplied by the tube coefficient, except for a corrective term, \( \frac{p}{w}(1 - f) \). The error introduced by neglecting this term is:

\[ \text{Error} = \frac{\frac{p}{w}(1 - f)}{fE'} = \frac{\frac{p}{w}(0.02)}{0.98 E'} = \frac{1}{49} \frac{p}{w} \quad (2) \]

The experimental data indicate that the maximum value of the ratio \( \frac{\frac{p}{w}}{E'} \) is of the order of magnitude of \( \frac{1}{2.5} \). Hence the maximum \( \frac{\frac{p}{w}}{E'} \) results in a total head about 0.8 per cent too large if the only correction applied is the tube coefficient. Equation (1) becomes:

\[ E = fE' (1 - 0.008). \]

The ratio \( \frac{\frac{p}{w}}{E'} \) reaches its maximum value at only a few points in the cross sections downstream from the elbow and is frequently much smaller.
The average error in obtaining the average energy over a cross section must be less than 0.8 per cent, not more than $\frac{1}{2}$ per cent, let us say, in the worst case. At Sections 2, 3, 5, and 7, especially at Section 2, the ratio $\frac{E_1}{E'}$ is quite small for the most part, so the error can be considered absolutely negligible at those sections. With a negligible error at Section 2, and a plus $\frac{1}{2}$ per cent error at Section 14, there is a certain error introduced into the computed head loss for the bend. Since the $\frac{1}{2}$ per cent is based on the total head, the resulting error in head loss is likely to be something greater than $\frac{1}{2}$ per cent, but certainly not more than 1 per cent, of the velocity head. Thus, the values of $E_1$ as measured should be increased somewhat for each flow, probably something less than $0.01 \frac{v^2}{2g}$.