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ST. ANTHONY FALLS HYDRAULIC LABORATORY

Technical Paper No. 51, Series B

Performance of Supercavitating Hydrofoils with Flaps, with Special Reference to Leakage and Optimization of Flap Design

by

R. OBA

Prepared for
OFFICE OF NAVAL RESEARCH
Department of the Navy
Washington, D.C.
under
Contract Nonr 710(24), Task NR 062-052

May 1965
Minneapolis, Minnesota

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ABSTRACT

A modified linearized theory was developed for supercavitating, flapped foils of arbitrary form in which effects of fluid leakage through the flap hinge were considered. It was found that the performance at conditions other than design conditions was improved with the use of trailing edge flaps. The drag-lift ratio may be reduced by a factor of 1/2 to 1/3. The foil performance is dependent on the flap-chord ratio, e, and the flap deflection, performance increasing with small e. Effects produced by leakage through the flap hinge were significant in their influence on velocity distribution and flap effectiveness (increment in lift coefficient per unit change in flap angle). The effect of hinge leakage on the overall performance, i. e., lift, drag, and moment, was small and may be neglected in most cases.

CONTENTS

LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

C_L - lift coefficient

C_D - drag coefficient

C_M - moment coefficient

ΔC_L - increment in lift coefficient due to leakage

ΔC_D - increment in drag coefficient due to leakage

ΔC_M - increment in moment coefficient due to leakage

$\Delta C_{L\,max}$ - maximum lift increment due to flap

ΔC_{LF} - increment in lift coefficient due to flap

$e = 1-x_Q$ - ratio of flap chord to foil chord; $m = \sqrt{x_Q}$

β - flap angle; S - mean slot width of flap hinge

α - incidence angle

Q - volume rate of water leakage through flap hinge

$q = 1 + u - iv$ - flow velocity, where u and v are perturbation velocities

$\Delta u - i\Delta v$ - perturbation velocity only related to hinge leakage

$z = x+iy$ - physical plane

$\bar{z} = \bar{x}+i\bar{y}$ - Riegels' mapping plane

$\zeta = \xi+i\eta$ - mapping plane

$A_n (n = 0, 1, 2, \ldots)$ - hydrofoil form parameter

J - factor introduced to increase the accuracy of linearized solutions up to the higher accuracy of second order (see Ref. [4])

The suffixes s, f, Q, and T signify respectively conditions on the real axis (foil surface or free streamline), free streamline, flap hinge, and trailing edge of the foil. Also, to normalize the present problem, it is assumed that the foil chord and main free flow speed are unity.

PERFORMANCE OF SUPERCAVITATING HYDROFOILS WITH FLAPS, WITH SPECIAL REFERENCE TO LEAKAGE AND OPTIMIZATION OF FLAP DESIGN

I. INTRODUCTION

Due to the development of high speed surface craft, the necessity of obtaining more detailed information relative to the characteristics of low drag lifting surfaces has become apparent. The development of high performance hydrofoil sections is also important in the design of supercavitating machinery, such as supercavitating ship propellers, pumps, and turbines.

During the course of an experimental program to determine the unsteady effects associated with an oscillating trailing edge flap, it was discovered that water issued through the flap hinge into the cavity. This water jet was readily observed in high speed motion pictures taken of the cavity. It was felt that the existence of such leakage through the flap hinge may affect the experimental results. While attempts were being made to reduce the quantity of leakage, a theoretical analysis of the leakage effect for steady flow was also carried out. The results of the analysis are described in the present report; the experimental data on the oscillating flap will be presented in a separate report.

The pioneering theoretical work of Tulin and Burkhart [1]*, and the analyses by Auslaender [2] and Lin [3] are related to the present problem. However, the above authors limited their analysis to the special case of flat plate foils with flaps and also ignored any effect of fluid passing through the flap hinge on the force characteristics. As the pressures at the hinges are nearly equal to the stagnation pressures, a leak at this location may seriously alter the lift and drag forces. Therefore, the author analyzed the steady flow around an arbitrary two-dimensional supercavitating foil equipped with a trailing edge flap as a function of the leakage through the flap hinge. Numerical computations were also made to estimate the performance of several flapped foils. The boundary value problem was solved with sufficient accuracy by means of a modified linearized theory previously used by the author [4, 5].

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*Numbers in brackets refer to the List of References on page 17.

The numerical calculations for flapped flat-plate hydrofoils, neglecting leakage effects, indicate the drag-lift ratio, C_D/C_L, of flapped foils greatly decreases as the flap-chord ratio, e, decreases. In particular, at $e \fallingdotseq 0$, the value is fairly close to that of the Tulin profile [1]. On the other hand, the maximum increment in lift due to a flap $\Delta C_{L\ max}$ is large, even for a fairly small e. For example, $\Delta C_{L\ max} \fallingdotseq 0.5$ or 0.7 for $e = 0.1$ or 0.25, respectively. Therefore, the author recommends that small flap-chord ratios e of the order of 0.1 or less be selected for practical use.

The change in performance with a flap is similar to that observed by changing the camber of foil sections without flap. It is desirable that the optimum flap-chord ratio and flap deflection be investigated further. It was also found that the relatively poor off-design performance of supercavitating foils may be improved with the use of flapped foils. The analysis of the leakage effect has shown that the flap effectiveness $\Delta C_{LF}/\Delta\beta$ might be changed by 20 per cent or more, primarily through a change in the surface pressure distribution in the neighborhood of the hinge. The lift or drag may be decreased or increased by an order of Q, where Q is the volume rate of water leakage through the hinge. It is thus suggested that foil performance could not be improved by water suction at a slot in the foil.

II. BASIC EQUATIONS

To generally discuss the effects of the foil and flap geometry and water leakage through the hinge on the foil performance, and the possibility of improving performance by suction through slots on the foil, the solution was carried out for arbitrary foil shape, hinge location x_Q, flap angle β, and leakage rate Q. The cavitation number was taken to be zero. A definition sketch is shown in Fig. 1. For convenience, it is assumed that the foil chord is unity, the leading edge is the origin of the coordinate system, and the main flow speed on the cavity boundary is unity. These assumptions do not alter the generality of the solution. The flow around the supercavitating, flapped foil is assumed to be a two-dimensional, incompressible, potential flow similar to that considered in many previous studies [1, 3, 6, 7, 8, 9, 10]. The present theory is restricted to an infinite fluid.

The modified linearized theory (second order theory) of Ref. [4] will be used in the solution of the problem.

The flow velocity at any point, $z = x + iy$, is defined as

$$q = 1 + u - iv = 1 + F(z) \tag{1}$$

By Riegels' transformation, the boundary BTMOD of Fig. 1 is mapped onto the $\bar{z} = \bar{x} + i\bar{y}$ plane. The boundary conditions in the $\bar{z}$-plane are given only on the x-axis and are denoted with the subscript s as shown in Fig. 2. If the perturbation velocities u and v are small quantities of first order $O(\epsilon)$, Riegels' factor $\sqrt{1 + (\frac{dy}{dx})^2} = 1 + O(\epsilon^2)$. Then the following relations hold for the perturbation velocities in the z-plane and the $\bar{z}$-plane:

$$u(x) = u(\bar{x}) + O(\epsilon^2),\ v(x) = v(\bar{x}) + O(\epsilon^2) \tag{2}$$

The $\bar{z}$-plane is then mapped onto the lower half ζ-plane shown in Fig. 3 by the mapping function

$$\zeta = -\sqrt{\bar{z}} \tag{3}$$

The present problem is reduced again to a very simple boundary value problem in which the boundary conditions are given only on the real ξ-axis. That is,

$$x_s = \xi_s^2 \tag{4}$$

$$\frac{dx_s}{d\xi_s} = 2\xi_s \tag{4'}$$

$$\left(\frac{dy}{dx}\right)_s = \left(\frac{d\eta}{d\xi}\right)_s = v_s(x) + O(\epsilon^2) \tag{5}$$

The nature of the suction flow around the hinge-slot M will now be considered. The slot width, as well as the flow rate through the slot, is usually very small. Thus, the Reynolds number for the flow through the slot is very small, and the use of potential flow theory does not give meaningful results for this region. However, at some distance from the slot, potential flow theory is valid and the flow should be very similar to that of a point sink of strength Q, because the slot width is usually very small. Thus, in this analysis, the nature of the slot flow is replaced by a point sink of strength Q.

As the Laplace equation is a linear equation, the solution of the problem shown in Fig. 3 can be obtained by adding the solutions of two rather simple boundary value problems: (1) a solution for arbitrary form foils and flaps, arbitrary flap angles β, arbitrary flap positions x_Q, and zero leakage flow rate $Q = 0$, and (2) a solution for a flat plate foil placed parallel to the direction of the main flow and with an arbitrary leakage flow rate Q [see Fig. 4(a) and (b)]. The solution for (1) is the same as that previously reported in [4, 5]. The solution is given as follows:

$$\sqrt{x_s} = \xi_s = \frac{1}{2}(1 - \cos\theta), \quad 0 \geq \theta \geq - \pi$$

$$u_s(x_s) = A_o \cot\frac{\theta}{2} + \sum_1^{\infty} A_n \sin n\theta \tag{6}$$

$$v_s(x_s) = - A_o + \sum_1^{\infty} A_n \cos n\theta$$

$$u_s(x_s) = 0 \quad \text{for} \quad \xi_s > 1, \quad \text{or} \quad \xi_s < 0 \tag{7}$$

From Eqs. (6) and (7), the perturbation velocity $u - iv$ can be considered as the sum of two components: (1) $u_o - iv_o$, for arbitrary form foils and flaps, with a fixed flap angle $\beta = 0$ (design flap angle)*, and (2) $u'_\beta - iv'_\beta$, for flat plate foils and flaps with arbitrary β (see Ref. [1, 2, 3]). Thus,

$$u = u_o + u'_\beta, \quad v = v_o + v'_\beta \tag{8}$$

The additional lift due to flap deflection is usually much smaller than the design lift for $\beta = 0$. As seen in a previous report [4], the performance of non-flapped foils is largely dependent on the shape of the rear half of the foils. Therefore, it may be expected that the foil performance is strongly dependent on the flap geometry (see Figs. 10 and 11).

To find the particular solution for the problem shown in Fig. 4(b), the following auxiliary function $H(\zeta)$ is introduced:

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*These perturbation velocities are given by the previously reported solution.

$$H(\zeta) = \sqrt{\frac{\zeta}{\zeta - 1}} \tag{9}$$

The perturbation velocities in the ζ-plane are written with an upper bar to distinguish from those in the z-plane. It is desired to consider the product of $H(\zeta)$ and the perturbation velocity $\Delta F(\zeta) = \Delta\bar{u}(\zeta) - \Delta\bar{v}(\zeta)$ which is related only to the sink Q in the ζ-plane. Then, the real part of the product $\Delta F(\zeta)\, H(\zeta)$ is known on the whole ξ-axis. That is,

$$\begin{aligned} \operatorname{Re} \Delta F(\xi)\, H(\xi) &= \infty \qquad \text{at} \quad \xi = m \\ &= 0 \qquad \text{elsewhere} \end{aligned} \tag{10}$$

where $\Delta F(1)$ should be zero to first order, at least. It is then possible to express $\Delta F(\zeta)\, H(\zeta)$ as follows:

$$\Delta F(\zeta)\, H(\zeta) = \frac{iQ}{\pi} \frac{1}{\zeta - m} \tag{11}$$

Thus $\Delta F(\xi_s) = \Delta\bar{u}_s - i\Delta\bar{v}_s$ on the real ξ-axis is

$$\begin{aligned} &\Delta\bar{u}_s = 0, \quad \Delta\bar{v}_s = -\frac{Q}{\pi} \frac{\sqrt{(\xi_s - 1)/\xi_s}}{\xi_s - m}, \quad \xi_s > 1 \text{ or } \xi_s < 0 \\ &\Delta\bar{u}_s = \frac{Q}{\pi} \frac{\sqrt{(1 - \xi_s)/\xi_s}}{\xi_s - m}, \quad \bar{v}_s = 0, \quad 1 > \xi_s > 0, \quad \xi_s \neq m \\ &\qquad \bar{v}_s = \infty, \quad \xi_s = m \end{aligned} \tag{12}$$

$\Delta F(\zeta)$ satisfies completely the boundary conditions in the ζ-plane and has the nature of a point sink at $\zeta = m$. It is readily seen in the following equation that Q is equal to the leakage flow rate.

$$\lim_{z \to \infty} \Delta F(z) = -\frac{Q}{2\pi} \left(\frac{1}{z}\right) \tag{13}$$

Therefore, if the leakage flow rates Q are known, the problem has been solved.

The changes in the lift, drag, and moment coefficients ΔC_L, ΔC_D, and ΔC_M respectively, due to the leakage Q, are given by

$$\Delta C_L = -2\,\mathrm{Re}\int_0^1 \Delta u_s(x)dx = -2Q + O(\epsilon^2) \tag{14}$$

$$\begin{aligned} \Delta C_D &= -2\int_0^1 \frac{dy}{dx}\Delta u_s(x)dx - 2\,I_m\int_0^1 \Delta u_s(x)dx \\ &= -2Q\left[-\sqrt{\frac{1-m}{m}} - A_o + \sum_1^\infty A_n Q_n\right] + O(\epsilon^3) \end{aligned} \tag{15}$$

$$\Delta C_M = -2\,\mathrm{Re}\int_0^1 \Delta u_s(x)\; x\; dx = 8Q\,(a^2 + a - \tfrac{1}{2}) + O(\epsilon^2) \tag{16}$$

where

$$\begin{aligned} &Q_1 = \pi(1-a),\ Q_2 = -2\pi a(1-a),\ Q_3 = \pi(1-a)(4a^2 - 1) \\ &Q_4 = -4\pi a(1-a)(2a^2 - 1),\ Q_5 = \pi(1-a)(16a^4 - 12a^2 + 1) \\ &a = 2m - 1 \end{aligned} \tag{15'}$$

Cauchy's principal values of the integrals are to be taken. The lift, drag, and moment coefficients C_L, C_D, and C_M are (see Ref. [4]):

$$C_L = \frac{\pi}{2J}\,(A_o + A_1 - \frac{A_2}{2}) - 2Q \tag{17}$$

$$C_D = \frac{\pi}{2J}\,(A_o + \frac{A_1}{2})^2 - 2Q\left[-\sqrt{\frac{1-m}{m}} - A_o + \sum_1^\infty A_n Q_n\right] \tag{18}$$

$$C_M = \frac{\pi}{32J^2}\left[5A_o + 7A_1 - 7A_2 + 3A_3 - \frac{A_4}{2}\right] + 8Q(a^2 + a - \tfrac{1}{2}) \tag{19}$$

where J is a factor introduced to increase the accuracy of solution up to the higher accuracy of second order (see Ref. [4]).

$$J = \frac{1 + \frac{\pi}{4}\left(A_o + A_1 - \frac{A_2}{2}\right)}{\cos A_o}$$

III. DISCUSSION OF LEAKAGE EFFECTS

A. Leakage Rate

The volume rate of leakage through the slot, Q, can be estimated in terms of the effective head difference, ΔH, as follows:

$$\zeta_e \frac{V_{mean}^2}{2g} = \Delta H \tag{20}$$

$$Q = S\, V_{mean} \tag{21}$$

where V_{mean} is the mean flow velocity in the slot channel of width S, and ζ_e is a loss coefficient which is related to the shape of the slot and to the slot Reynolds number $Re = \frac{SV_{mean}}{\nu}$. The loss coefficient, ζ_e, is assumed to be approximately 3 or 4. The effective head difference, ΔH, is unknown. Suitable data to determine ΔH have not been obtained, as the pressure distribution at the slot is very complicated. It would be desirable to continue the investigation in the future in an effort to more satisfactorily determine the unknown constants. However, for the present purpose, it is sufficient to use very rough values of Q, as Q is usually very small. If it is assumed that ΔH is the stagnation pressure at x_Q for $\beta > 0$, Q is of the order of $S/2$.

B. Influence of Leakage on Flap Effectiveness

The increment in lift coefficient ΔC_{LF} is defined as the difference between the lift coefficients $C_{L\beta}$ for an arbitrary flap angle β and $C_{L\beta = 0}$ for $\beta = 0$, or

$$\Delta C_{LF} = C_{L\beta} - C_{L\beta = 0} \tag{22}$$

ΔC_{LF} is generally much smaller than the total lift coefficient, C_L, and may be 0.1 or less. In the following, a prime sign will be used to distinguish quantities for the $Q = 0$ case, that is, for no leakage. The leak effect factor, E, for ΔC_{LF} then may be expressed as follows:

$$E = \frac{\Delta C_{LF} - \Delta C_{LF'}}{\Delta C_{LF'}} = 2\,\frac{Q_{\beta = 0} - Q_{\beta}}{\Delta C_{LF'}} \tag{23}$$

As an example, a flat plate foil will be considered with a flap-chord ratio $e = 0.33$, an angle of attack $\alpha < 16^{\circ}$, a flap angle $\beta < 5^{\circ}$, and a slot width $S = 0.01$. As the surface velocities at $x = M$ are small quantities of first order for $\alpha < 16^{\circ}$, $Q_{\beta = 0} \ll Q_{\beta}$. Then,

$$E \fallingdotseq -\frac{0.01}{\Delta C_{LF'}} \tag{23'}$$

For $\Delta C_{LF'} = 0.1$, E is -0.10 and for $\Delta C_{LF'} = 0.05$, E is -0.20. It thus appears that the effect of leakage at the flap hinge may not always be negligible.

C. Influence of Leakage on Surface Velocity Distributions

In a discussion of the hydrodynamic characteristics of hydrofoils, special attention should be given to the velocity distribution since the forces, the stability of the boundary layer, and the stability of the cavity are related to the velocity distribution. Therefore, in this section, the change in the velocity distribution due to the leakage, Q, will be examined in detail.

For arbitrary form flapped foils, Fig. 5 shows the surface velocity changes in a dimensionless form, $2u_s/Q$, for various hinge positions $x_Q = 0.25, 0.56, 0.75$. In Fig. 6, as an example, the surface velocity distributions for a flat plate foil with a slot at $x_Q = 0.75$ are shown for various leakage rates $Q = 0.004, 0.02, 0.1$; an incidence angle $\alpha = 10^{\circ}$; and a flap angle $\beta = 0^{\circ}$.

The curves shown in Figs. 5 and 6 indicate the following trends:

1. The surface velocity distribution changes by an order of several times Q. The influence of the leakage is not always restricted to the neighborhood of the slots.
2. The sign of Δu_s changes from positive to negative at the slot location. The magnitude of Δu_s is much greater forward of the slot than aft of the slot. The lift forces thus decrease with Q [see Eq. (14)].

As previously mentioned, good accuracy in computing Δu_s in the neighborhood of the slot cannot be expected.

D. Flow Detachment Angle at Leading Edge

The angle of flow detachment at the leading edge* of the foil is strongly related to the minimum drag-lift ratio and the maximum foil speed, and is also important in the structural design of the foil. To investigate the effect of hinge leakage on this angle, the changes $\Delta y_f/Q$ on the suction-side free streamline forms due to the leakage flow rates were calculated for various slot locations and are presented in Fig. 7. As seen in Fig. 7, the cavity thickness at the leading edge as well as the detachment angle increases with Q. Furthermore, the cavity thickness increases with Q and therefore the minimum angle of attack at which the upper cavity clears the foil surface decreases slightly with Q. These findings suggest that leakage through a slot may be useful in alleviating the problem associated with small flow detachment angle on conventional supercavitating profiles.

E. Influence of Leakage on Lift, Drag, and Moment

To indicate the effect of leakage on the drag, $\frac{\Delta C_D}{Q}$, for various foil shape parameters A_n [$n = 0, 1, 2, \ldots$; see Eqs. (5) and (6)] and hinge positions x_Q, the parameters $\sqrt{\frac{1-m}{m}}$ and Q_n $(n = 1, 2, 3, 4)$ in Eq. (15') are shown in Fig. 8. Leakage effects on the moment, $\Delta C_M/Q$, are shown in Fig. 9.

From Figs. 8 and 9 and Eqs. (14), (15), and (16), it can be seen that the effect of hinge leakage on lift, drag, and moment is proportional to the leakage flow rate Q, specifically:

1. The effect of leakage on the lift, $\Delta C_L/Q$, is -2, and is independent of the foil shape as well as the hinge position x_Q (or flap-chord ratio, e). [See Eq. (17).]
2. The first term, $2\sqrt{\frac{1-m}{m}}$, of the leakage effect on the drag, $\Delta C_D/Q$, is the order of 1 (see Fig. 8). It should also be noted that as the flap

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*The included angle between the suction-side free streamline and the foil surface line at the leading edge.

chord ratio, e, decreases, $\Delta C_D/Q$ also decreases. The term $\sum_1^\infty A_n Q_n$ of $\Delta C_D/Q$ is related to parameters A_n and the hinge positions x_Q (see Fig. 8).

3. The leakage effect on the moment, $\Delta C_M/Q$, is not dependent on the foil shape but is a function of the hinge position x_Q only as shown in Fig. 9. Note that $\Delta C_M/Q = 0$ at $x_Q = 0.466$, which is not always identical to the position of the aerodynamic center.

IV. IMPROVEMENT IN FOIL PERFORMANCE

As seen in Eqs. (17), (18), and (19), the force change attributable to leakage is of the order of the leakage flow rate Q, and this is small when the slot width S is small. Thus the effect of hinge leakage may be neglected in a first order analysis. In this section, the possibility of improving foil performance will be considered.

The flap effectiveness, $\frac{dC_L}{d\beta}$, is a function of the flap angle β and the flap-chord ratio e, but it is independent of the geometry of the foils and the flap, as previously discussed in Section II. The calculated lift coefficient[*], C_L, and the drag-lift ratios, C_D/C_L, for various flap geometries are shown in Fig. 10. Each of the curves is a straight line. The line for $e = 0$ shows the limiting case as $e \rightarrow 0$, and the line for $e = 1.00$ is the same as the performance relationship for a flat plate foil without flaps for arbitrary incidence angles α. It is relatively simple to estimate the foil performance[**] as a function of flap angle β and incidence angle α. For example, for a flap-chord ratio, e, of 0.5, the performance at any given flap angle is shown by the point P. It will be recalled that the flap effects were calculated for $\alpha = 0^\circ$; thus, for an arbitrary angle of attack the performance is given by the point R on the PQ line. Note that PQ is

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[*] Cos A_o in the correction factor J in Eqs. (17) and (18) (see Ref. [4]) is assumed as unity because it is very close to unity and changes only for e close to one.

[**] Strictly speaking, superposition technique is not valid on the drag [see Eq. (18)], and the PQ straight line should be replaced by a PQ" line which is very close to the PQ line.

parallel to OT, and OR' = PR, where the point R' indicates the performance due to α only.

It should be observed in Fig. 10 that the drag-lift ratio for a given C_L improves as the flap-chord ratio decreases. In the previous section it was shown that for small e, $\Delta C_D/Q$ was also small, resulting in low drag, even in the case of leakage through the flap hinge. To further clarify the behavior of foils with small flap-chord ratios, the slopes $d\frac{C_D}{C_L}/dC_L$ of the straight lines in Fig. 10, the flap effectiveness $dC_L/d\beta$, and the maximum lift increments $\Delta C_{L\ max}$ due to the flaps are calculated and are shown in Fig. 11. It is seen that the foil performance is improved for the smaller flap-chord ratios, as $\Delta C_{L\ max}$ is fairly large. Also, for practical purposes, it is desirable to install the flap near the trailing edge where the foil thickness is relatively large. However, the theoretical results* should be subjected to experimental verification, particularly for small values of e, as the theoretical performance may be reduced by boundary layer effects.

The results for e = 1.0 require further discussion. The magnitude of $d\frac{C_D}{C_L}/dC_L$ for e = 1.0 is about three times as large as that for e = 0.25, and furthermore both $dC_L/d\beta$ and $\Delta C_{L\ max}$ are less than their maximum values attained at some other e. Therefore, it is obvious in Fig. 11 that the flat plate foil exhibits rather poor performance characteristics. The same performance characteristics are noted for a cambered foil at an arbitrary angle of attack [see Eq. (6)]. For example, a cambered Tulin profile with performance ratio given by point A in Fig. 10 ($C_L = 0.4$, $\alpha = 0^\circ$, without flap) also experiences a reduction in efficiency as α is increased. This is shown by the line AT' where AT' is parallel to OT. Therefore, the performance of foils without flaps decreases as the angle of attack differs from the design angle, even though good performance is found at the design value. The line AB' (AB' is parallel to A'B) in Fig. 10 shows the performance for a Tulin profile ($C_L = 0.4$, $\alpha = 0^\circ$) with a flap (e = 0.25) at arbitrary flap angles β. It is noted that the performance indicated by the line AB' is much better than that corresponding to the same lift achieved by increasing α as shown by line AT'. The performance is also improved over the flat plate foil

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*The accuracy of the modified linearized theory is apparently adequate (see section VI following).

with $e = 0.25$. It is thus theoretically possible to reduce the drag-lift ratio C_D/C_L by one-half or more. By proper selection of the foil and flap geometries, improvements can be made in the drag-lift ratios as well as the flow detachment angle at the leading edge, as has been previously discussed in Ref. [4] for foils without flaps. It is suggested that more efficient foils than the flat plate foils and flaps should be used in machinery. Also, it is seen in Fig. 10 that the line AB' for the Tulin profile with a flap is higher than the line for the family of Tulin profiles without a flap. Therefore, the performance at a given C_L of a flapped foil with camber is less than the performance of another foil without a flap whose camber is determined by the same value of C_L. However, if the theoretical results previously mentioned for small flap-chord ratios are experimentally verified, some of these shortcomings may be reduced.

V. SHAPE OF UPPER CAVITY WALL FOR FLAPPED FOILS

As previously mentioned, foil performance is strongly dependent on the shape of the upper cavity wall. It has already been shown that flaps can be used to improve the performance of foils operating at conditions other than the design condition; however, it is necessary to determine the location of the cavity streamline. If the cavity streamline shifts downward with flap deflection, the minimum usable angle of attack and the minimum drag-lift ratio will increase in proportion to the shift. Furthermore, if the cavity becomes thicker, a numerical estimate of the distances shifted should be made.

Figures 12 and 13 show the upward shift of Δy_{fT} at the trailing edge of the foil as a function of flap angle β and flap-chord ratio e, and the shape of the upper cavity streamline as a function of x for flap-chord ratios $e = 1.0, 0.75, 0.44, 0.25$.

In Figs. 12 and 13, it is ascertained that with an increase in β the suction-side free streamline shifts upward*, for all flap-chord ratios e, and that the shifted free streamline forms are very similar to each other. Therefore, it may be concluded that the use of flapped foils results in an improvement of off-design performance, as well as a smaller reduction in the minimum drag-lift ratio or the minimum usable incidence angle.

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*The upward shift corresponds to a reduction in the minimum usable incidence angle, and to a reduction in the drag-lift ratio (see Ref. [5]).

VI. ACCURACY OF THE MODIFIED LINEARIZED SOLUTION

Since a large flap angle is likely to be involved in practical applications, it is desirable to check the accuracy of the theory at large β. This will be done by comparing the modified linearized solution (second order solution) with a non-linear exact theory. Lin's exact solution [3] for a flat plate foil with a flap (no hinge leakage) is compared in Fig. 14 with the author's solution. In Fig. 14, the increments $\Delta C_{L\beta}$ in lift due to the flap only are given for various flap-chord ratios, e, and flap angles, β. The accuracy of the modified linearized solution is apparently sufficient for all e and $\beta < 30^\circ$. At larger β, the modified linearized solution overestimates the lift increment, particularly for $e > 0.2$.

VII. CONCLUSIONS

The theoretical results may be summarized as follows:

1. In an attempt to derive hydrofoil sections with improved performance characteristics, an analytical solution was obtained for the flow about a supercavitating hydrofoil with an arbitrary profile and trailing edge flaps. The effects of water leakage through the flap hinge were also considered.
2. The effect of hinge leakage on the velocity distribution, the shape of the upper cavity wall (and thus the flow detachment angle and minimum drag-lift ratio), and the flap effectiveness $dC_L/d\beta$ are significant and cannot be neglected. However, hinge leakage had very little effect on the lift, drag, and moment characteristics, and may be neglected for most practical cases. The results indicate that foil performance cannot be improved by water suction through a slot in the foil.
3. Cavity thickness as well as the flow detachment angle increased considerably due to leakage through the flap hinge. Thus, a possibility is found for alleviating a disadvantage of supercavitating foils, that is, the very small flow detachment angle.
4. It was found that the poor performance associated with a supercavitating foil operating at other than design conditions can be improved with the proper use of trailing edge flaps of small flap-chord ratio e of the order of 0.1 or less. The drag-lift ratios of flapped foils were 1/2 to

1/3 times smaller than those for non-flapped foils. Therefore, with the application of flapped foils to hydraulic machinery in mind, satisfactory efficiency may be obtained even for partial loads.

5. As foil performance can be improved with flapped foils, additional theoretical and experimental research should be conducted to determine the optimum flap geometry.

6. The performance of a flapped foil increases as the flap-chord ratio, e, decreases. For small e, the maximum increment in lift due to the flap, $\Delta C_{L\,max}$, is fairly large, even though the flap effectiveness, $dC_L/d\beta$, is considerably reduced.

   More experimental work is required to verify the theoretical results.

7. The proper use of flapped foils results in some reduction in the minimum drag-lift ratio and the minimum usable incidence angle, as well as a large improvement in performance at off-design conditions.

ACKNOWLEDGMENTS

The author wishes to express his thanks to Professors E. Silberman and C. S. Song for their valuable discussions and to Mr. J. M. Wetzel for his critical reviews. The author also wishes to thank Mr. Chuan Lee, Research Assistant, St. Anthony Falls Hydraulic Laboratory, for his aid in the numerical calculations.

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FIGURES

(1 through 14)

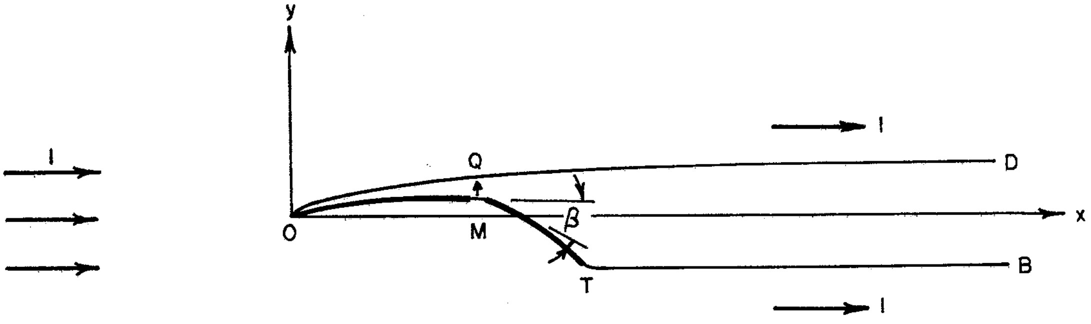


Fig. 1 - Physical Plane, $z = x + iy$

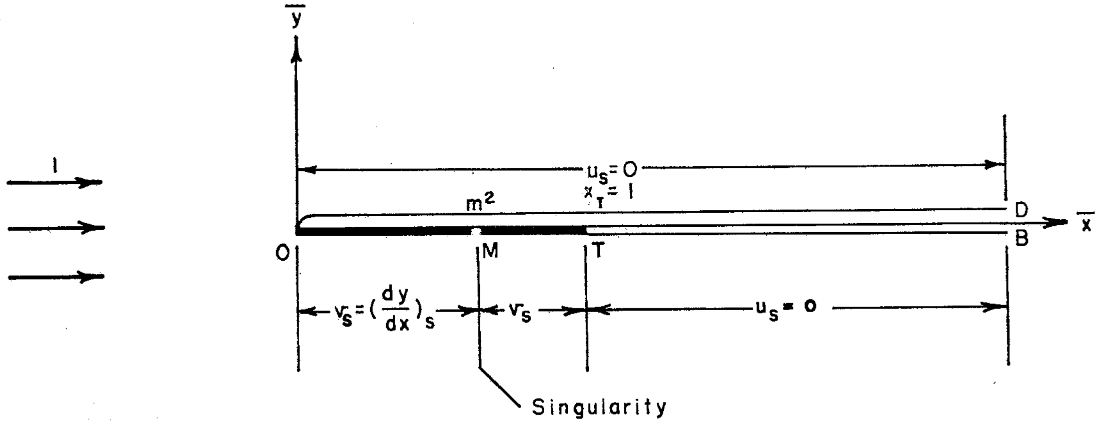


Fig. 2 - Mapping Plane Transformed by Riegels' Transformation, $\bar{z} = \bar{x} + i\bar{y}$

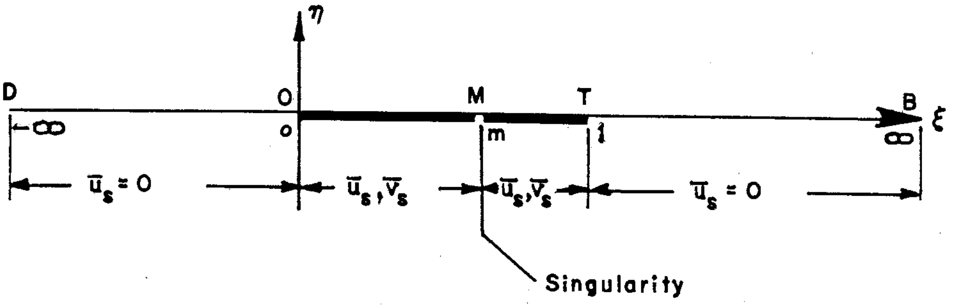
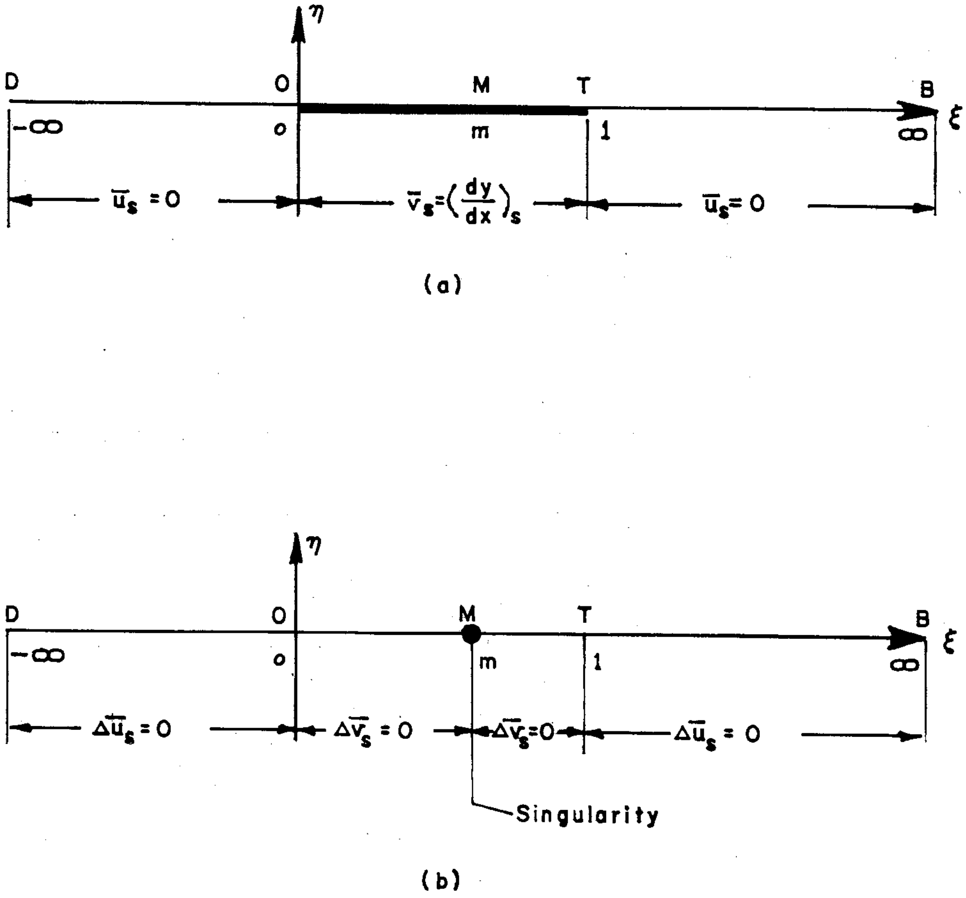


Fig. 3 - Mapping Plane (Lower Half Plane), $\zeta = \xi + i\eta$



(a)

(b)

Fig. 4 - Reduction of General Problem into Two Simple Boundary Value Problems

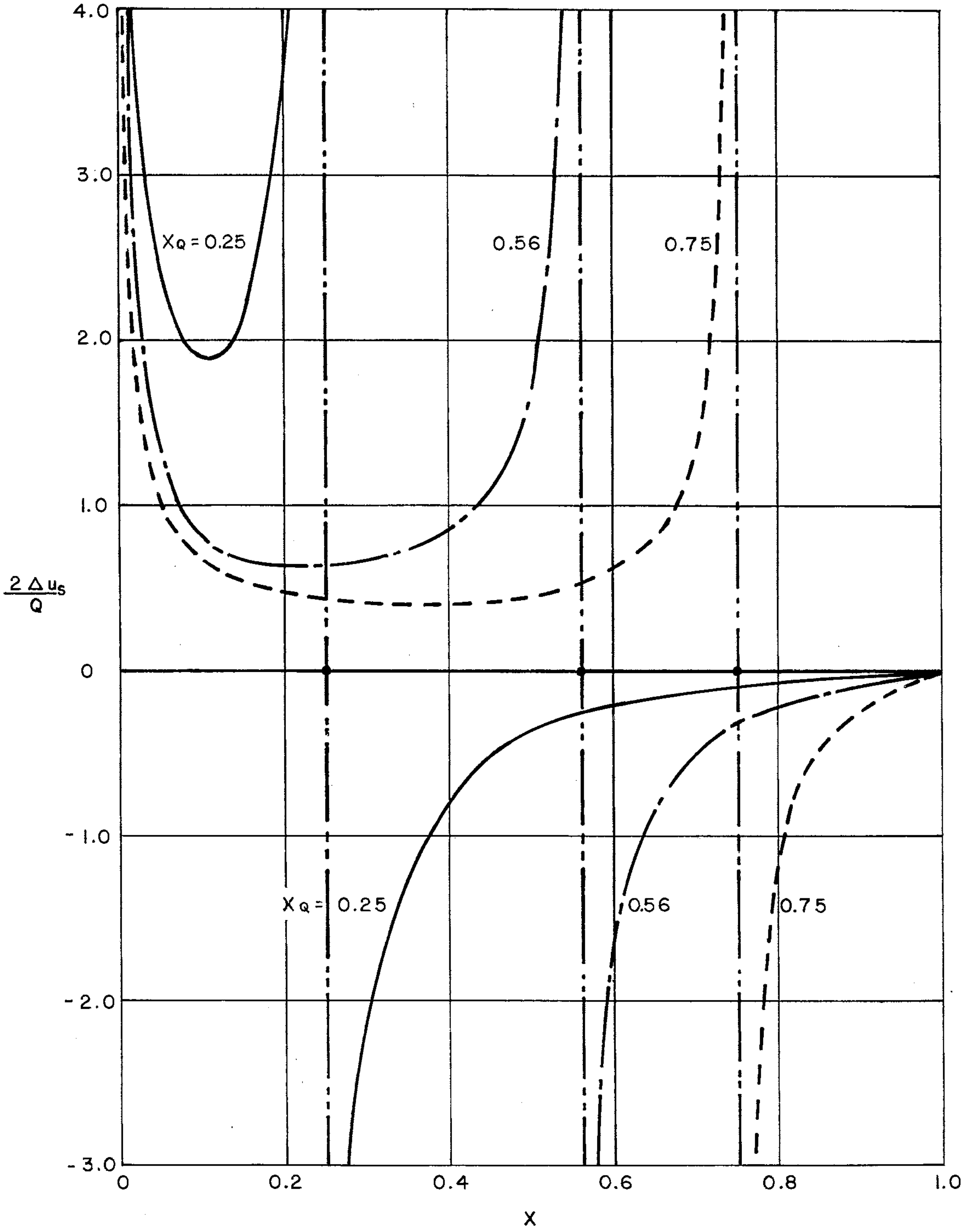


Fig. 5 - Leakage Effects $\Delta u_s/Q$ on Foil Surface Velocity Distribution; x: Distance along Foil Chord

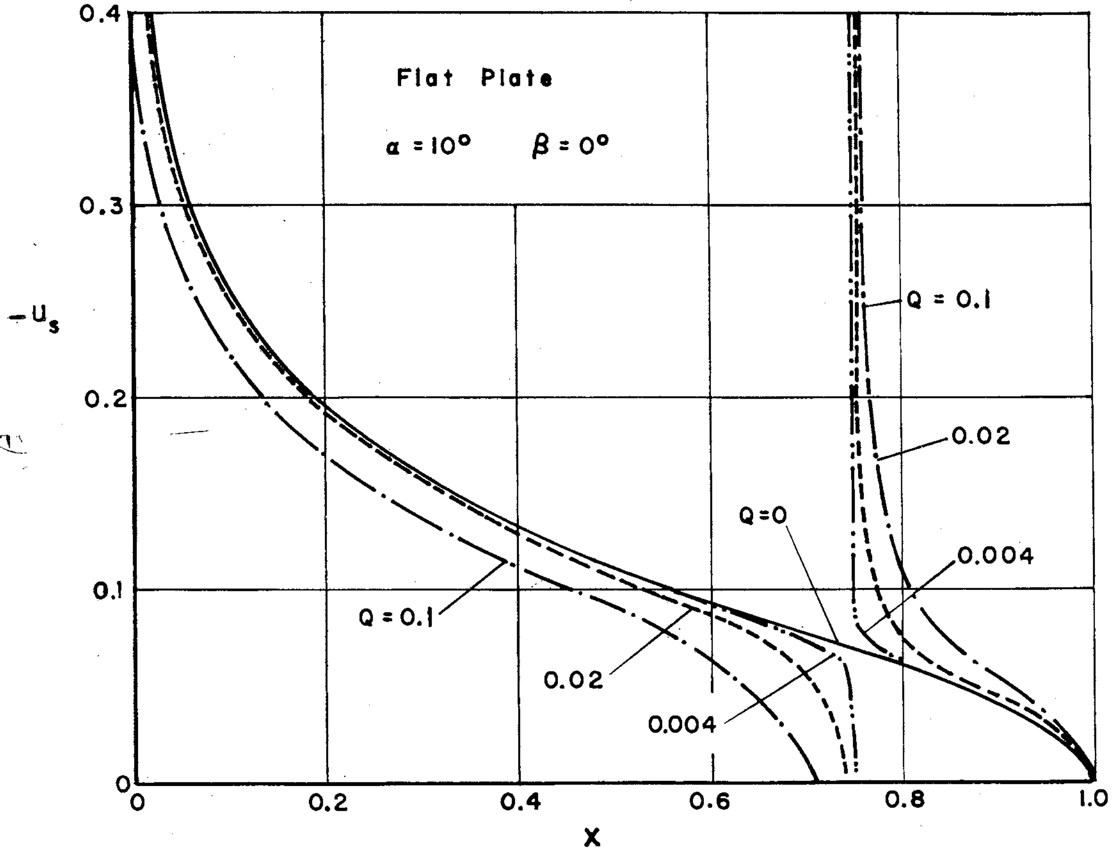


Fig. 6 - Typical Example for Surface Velocity Distribution of Flapped Foil with Leakage, $x_Q = 0.75$

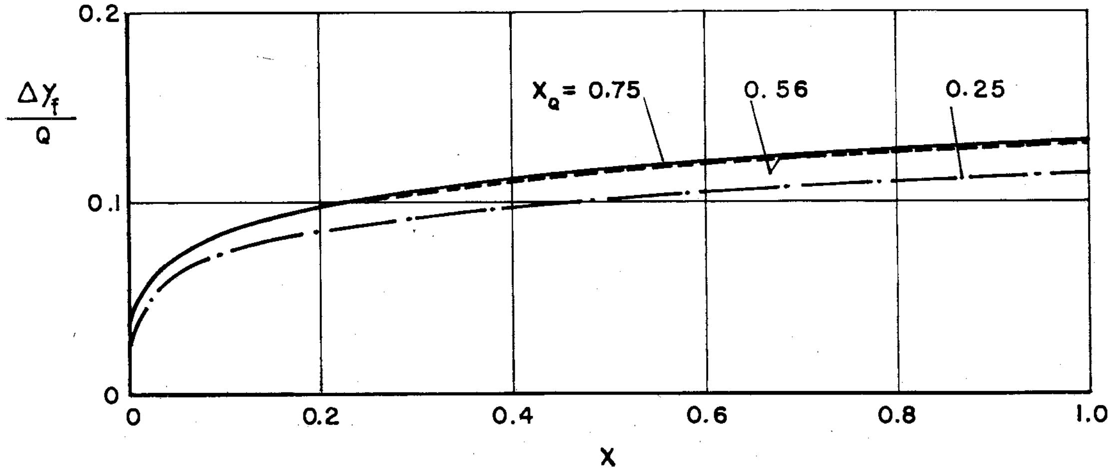


Fig. 7 - Variation of Suction-side Free Streamline Forms $\Delta y_f/Q$ with Leakage Q

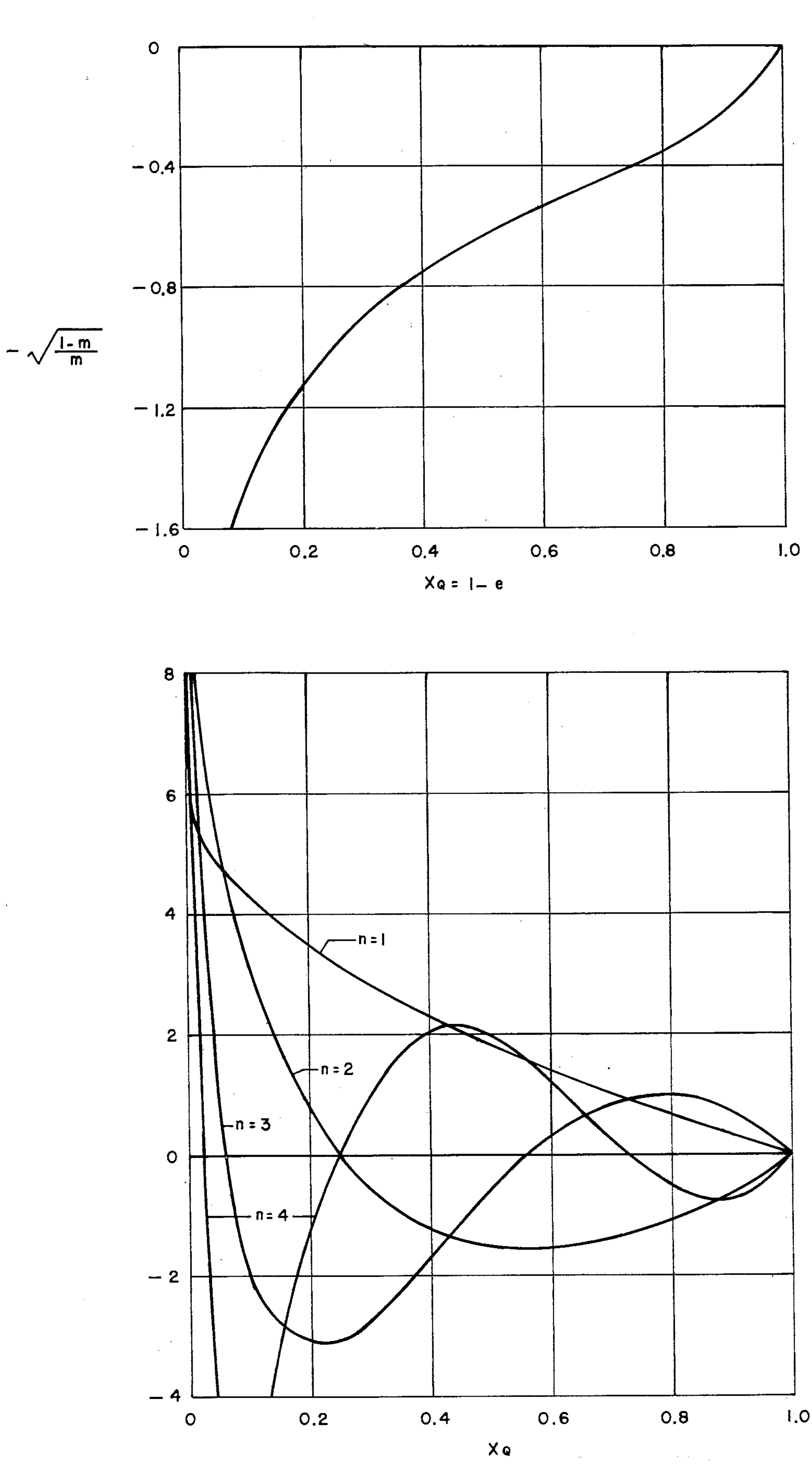


Fig. 8 - Hinge Leakage Parameters, $\sqrt{\frac{1-m}{m}}$, Q_n, for Drag;
where $\Delta C_D = -2Q\left[-\sqrt{\frac{1-m}{m}} - A_o + \sum_1^{\infty} A_n Q_n\right]$

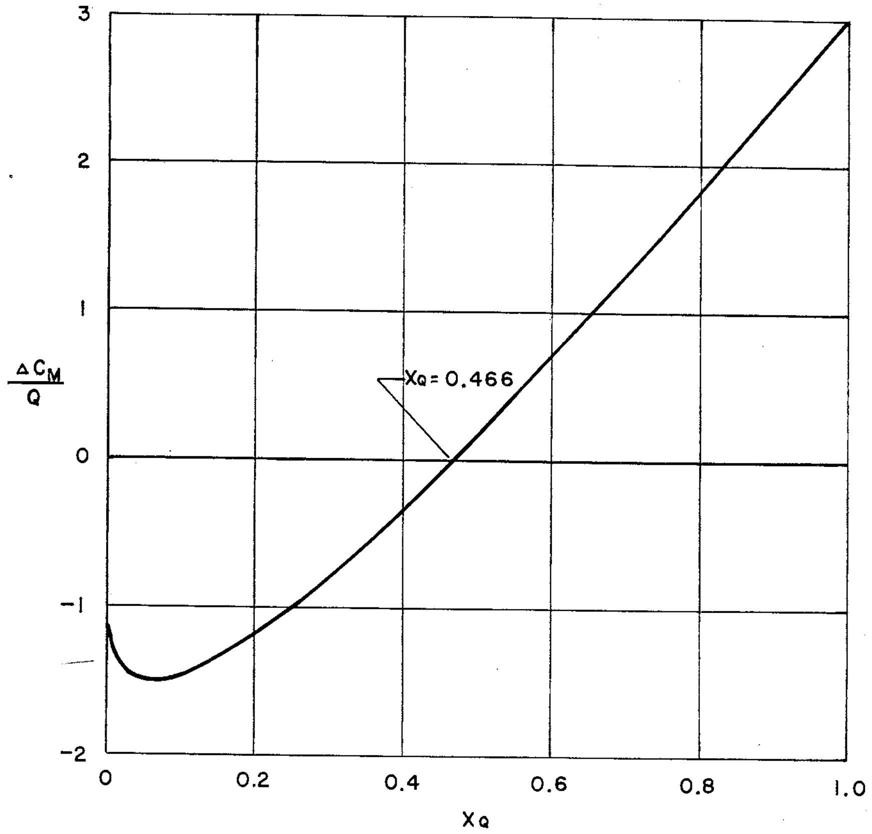


Fig. 9 - Effect of Leakage on Moment $\Delta C_M/Q$

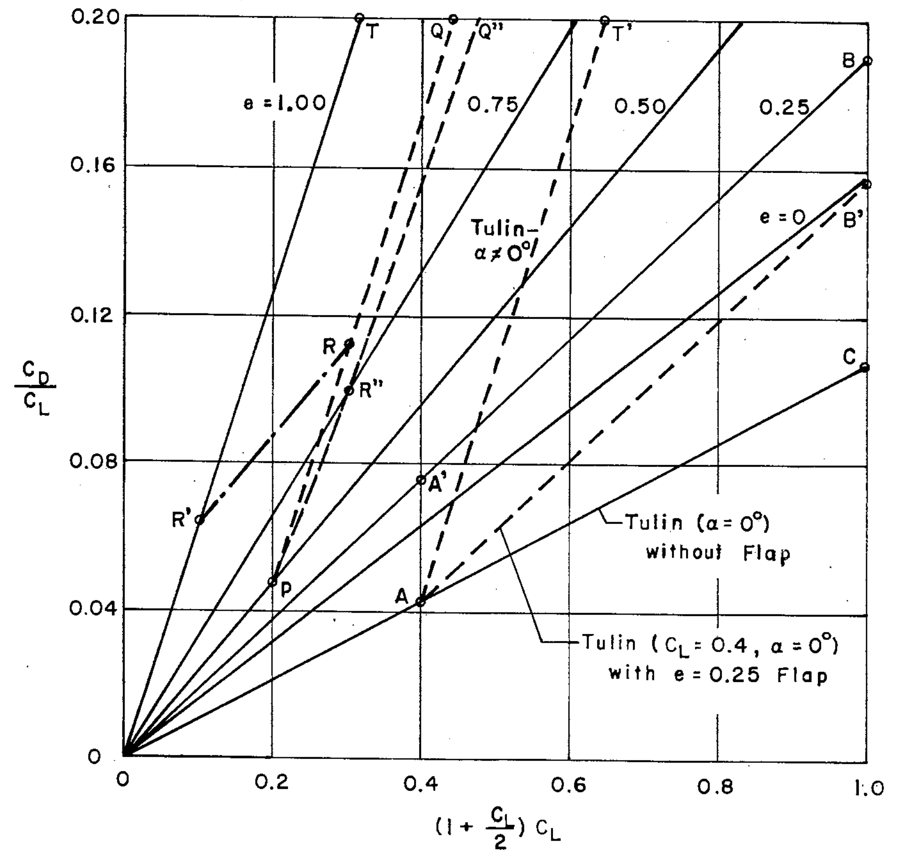


Fig. 10 - Relation between Lift Coefficient C_L and Drag Lift Ratio C_D/C_L, for Various Flapped Foils and Various Foils without Flaps

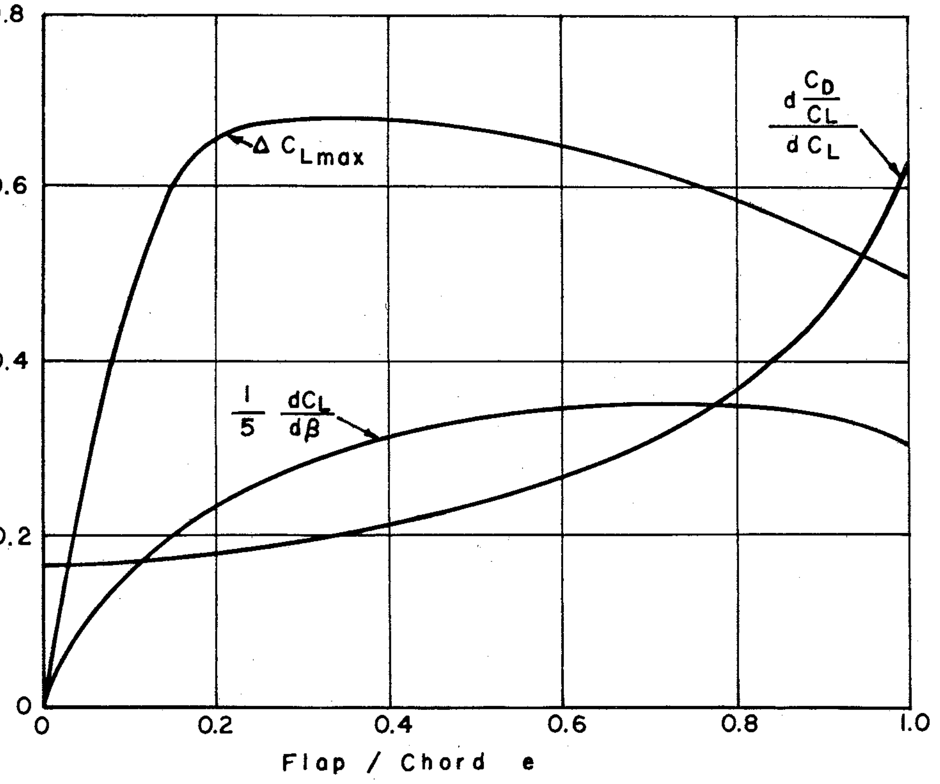


Fig. 11 - Rate of Loss $d\left(\frac{C_D}{C_L}\right)/dC_L$, Flap Effectiveness $dC_L/d\beta$, and Maximum Lift Increments $\Delta C_{L\ max}$ due to Flap

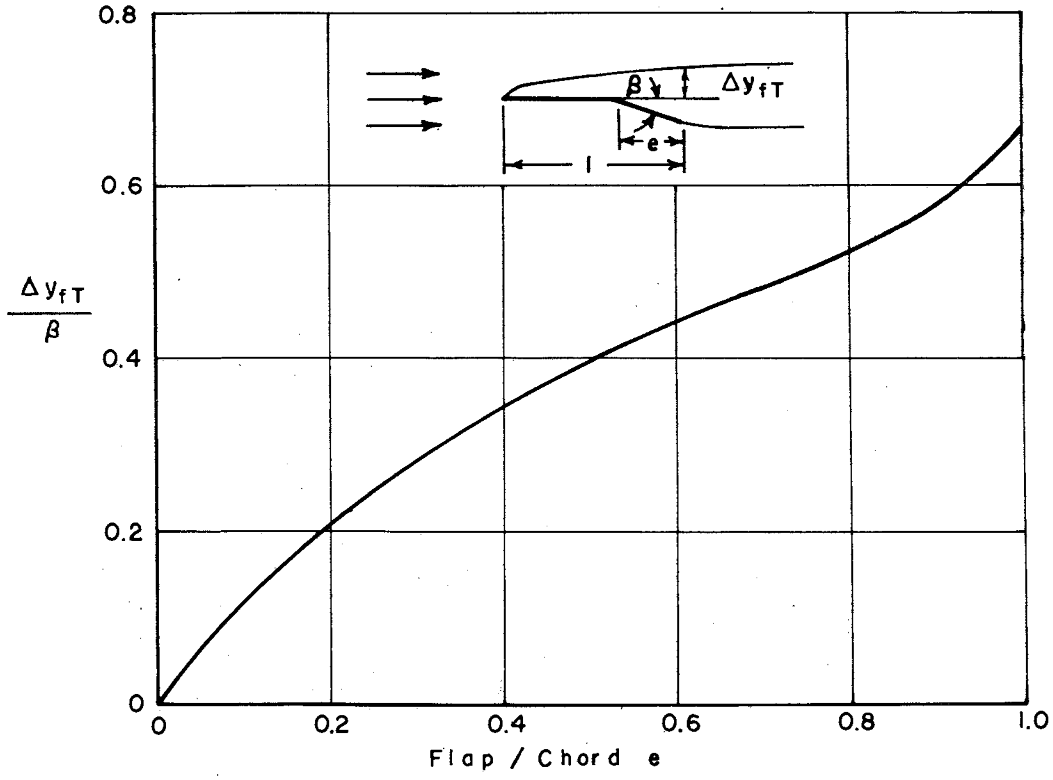


Fig. 12 - Upward Shift in Suction-side Free Streamlines Δy_{fT} due to Flap

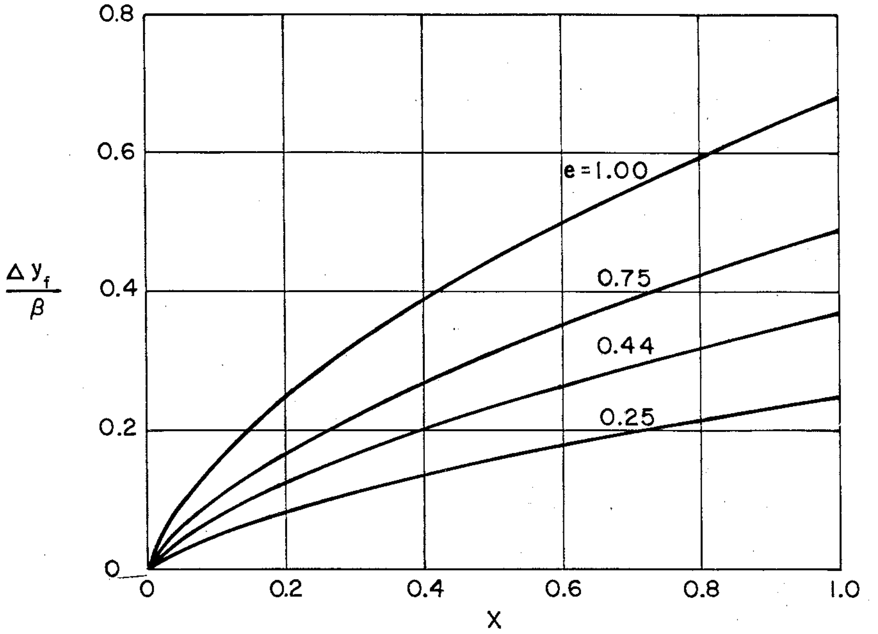


Fig. 13 - Shape of Shifted Suction-side Free Streamlines Δy_f

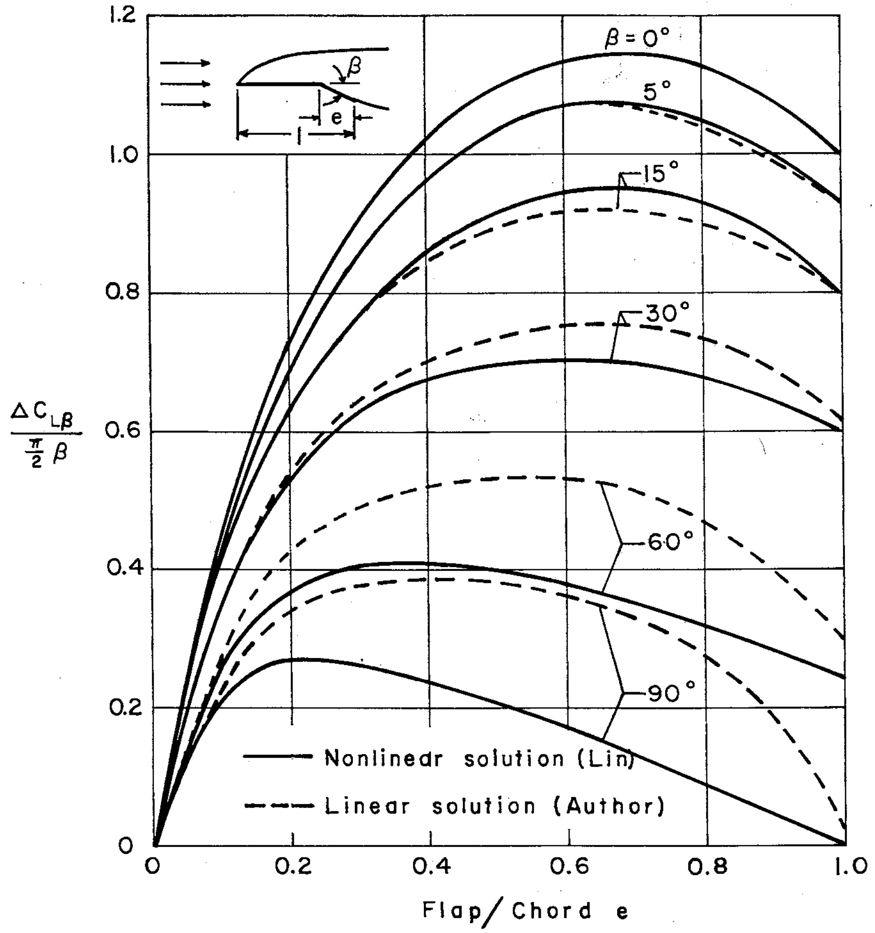


Fig. 14 - Accuracy of the Linearized Solution

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3. REPORT TITLE
PERFORMANCE OF SUPERCAVITATING HYDROFOILS WITH FLAPS, WITH SPECIAL REFERENCE TO LEAKAGE AND OPTIMIZATION OF FLAP DESIGN

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Interim Memorandum

5. AUTHOR(S) *(Last name, first name, initial)*
Ōba, R.

6. REPORT DATE
May 1965

7a. TOTAL NO. OF PAGES
28

7b. NO. OF REFS
11

8a. CONTRACT OR GRANT NO.
Nonr 710(24)

b. PROJECT NO.
NR 062-052

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)
Technical Paper No. 51-B

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A second order theory was developed for supercavitating flapped foils in which hinge leakage effects were also considered. With the use of trailing edge flap, the drag-lift ratio may be reduced by a factor of 1/2 to 1/3. Leakage effects were significant in their influence on velocity distribution and flap effectiveness, but negligibly small in their influence on lift, drag, and moment.

14. KEY WORDS	LINK A		LINK B		LINK C	
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St. Anthony Falls Hydraulic Laboratory

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