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Technical Paper No. 38, Series B

Unsteady, Symmetrical, Supercavitating Flows Past a Thin Wedge in a Solid Wall Channel

by

C. S. SONG and F. Y. TSAI

Prepared for
OFFICE OF NAVAL RESEARCH
Department of the Navy
Washington, D.C.
Contract Nonr 710(24), Task NR 062-052

June 1962
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A B S T R A C T

Problems of symmetrical two-dimensional supercavitating flow about a thin wedge in a finite fluid with two parallel solid boundaries are solved by means of a linearized method utilizing the complex acceleration potential. The solution contains no singularity and, as a result, pressure is finite everywhere.

It is shown that the term indicating the effect of cavity pressure change on the drag which existed in the case of the flows with free boundaries is identical to zero when the boundaries are solid. It is also concluded that, in steady flow cases, the accuracy of the solutions using the linearized acceleration potential method is comparable to that using the linearized velocity potential method. In fact, the two methods give identical solutions when cavities are infinitely long.

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I. INTRODUCTION

Since there is a practical limitation to the size of a water tunnel to be used for experiments, it is sometimes very desirable to know the extent of boundary effects on experimental data. For two-dimensional tests, the boundary may consist of two free surfaces, two solid walls, or a free surface and a solid wall. In the case of steady supercavitating flows, some work on the free-surface and solid wall effects is available in the literature [1, 2, 3, 4]*.

A problem of symmetrical, unsteady, supercavitating flow about a thin wedge in a jet of finite width was solved and reported by the first author [5]. The flow was solved in terms of the acceleration potential and by means of a linearized method. It was demonstrated that a singularity-free solution exists and that the previously required closure condition could be eliminated. Basically, there is little difference between the free-boundary problem and the solid-boundary problem. The known quantity on the free surface is the pressure which is equivalent to the real part of the complex acceleration potential, whereas the known quantity on the solid surface is the vertical velocity component which is related to the imaginary part of the complex acceleration potential. The present paper, dealing with the effect of solid boundaries, is a supplement to the previous one [5] and follows the same method.

This research has been supported by the Office of Naval Research of the U. S. Department of the Navy under Contract Nonr 710(24), Task NR 062-052. The authors wish to express their gratitude to Professor Edward Silberman, who has critically reviewed the present work, and to Messrs. John Kolodnycki and Paul Edstrom for their help with numerical computation. The manuscript was prepared for printing by Marjorie Olson and Carol Takyi under the general supervision of Loyal Johnson.

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*Numbers in brackets refer to the List of References on p. 13.

II. THE ACCELERATION POTENTIAL AND THE BOUNDARY CONDITIONS

It is known that in flow starting from rest under conservative forces in inviscid and incompressible fluid, the curl of the acceleration is zero, and thus there exists an acceleration potential. For a gravitation-free two-dimensional field, the Euler equation of motion is

$$\vec{a} = \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \text{grad})\, \vec{q} = -\frac{1}{\rho}\, \text{grad}\ p \qquad (1)$$

where $\vec{q}$ is the velocity and p is the pressure. By choosing a suitable reference speed, q_c, and a suitable reference pressure, P, the Euler equation of motion may also be written as

$$\vec{a} = q_c^{\,2}\ \text{grad}\ \phi \qquad (2)$$

where

$$\phi = \frac{P - p}{\rho\, q_c^{\,2}} = \text{acceleration potential} \qquad (3)$$

Taking the divergence of Eq. (2) and applying the equation of continuity $\nabla \cdot q = 0$ and the condition of irrotationality $\nabla \times \vec{q} = 0$, it may be shown that

$$\nabla^2 \phi = q_c^{-2} \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 \right] \qquad (4)$$

where u and v are the horizontal and the vertical components respectively of the velocity $\vec{q}$.

If the flow is uniform, Eq. (4) becomes the Laplace equation and acceleration potential is a harmonic function. Moreover, since the right-hand side of Eq. (4) involves only square terms of velocity derivatives, the acceleration potential satisfies the Laplace equation up to the first order of smallness, even for a non-uniform flow field if the latter is produced by a small disturbance to the undisturbed, uniform flow. Therefore, for a slightly distrubed flow, there exists a complex acceleration potential

$$F(z, t) = \phi(x, y, t) + i\psi(x, y, t) \qquad (5)$$

which is an analytic function of the space variable $z = x + iy$. Here, the imaginary part of the complex acceleration potential is defined so that the Cauchy-Riemann equations

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \qquad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \tag{6}$$

are satisfied.

For a steady motion, Eq. (1) may be integrated to yield the Bernoulli equation from which the speed on the cavity boundary q_c may be computed as

$$q_c = U\sqrt{1 + \sigma} \tag{7}$$

where σ is the cavitation number

$$\sigma = \frac{P - p_c}{1/2 \; \rho \; U^2} \tag{8}$$

U is the speed at infinity and P and p_c are the pressures at infinity and at the cavity boundary respectively.

For the type of problems considered in this paper it is assumed that flow at infinity is steady. Unsteady flow is caused by the perturbation velocity $\dot{A} = dA/dt$ of the wedge with respect to the coordinates fixed with respect to the flow at infinity. As indicated in Fig. 1(a), the wedge is placed along the centerline of the stream and the perturbation velocity $\dot{A}$ is parallel to the main stream direction so that the flow is symmetrical with respect to the x-axis. The three quantities U, P, and p_c are assumed to be constants and, hence, the speed q_c and the cavitation number σ defined by Eqs. (7) and (8) are also constants. It should be observed that q_c is, in general, not equal to the speed at a point on the cavity boundary, but simply a convenient speed chosen to be the reference speed. It should also be noted that $\dot{A}$ should be a small quantity, but the displacement A and the acceleration $\ddot{A}$ need not be small.

Using q_c as the reference speed, the velocity vector $\overrightarrow{q}$ may be written in terms of the perturbation velocity components (u', v') as

$$\overrightarrow{q} = q_c \, (1 + u', v') \tag{9}$$

and

$$u' << 1, \ v' << 1$$

However, for the sake of briefness the superscript will be omitted hereafter. Neglecting the higher order terms of the perturbation speeds, the equation of motion (2) is linearized to

$$\phi_x = \psi_y = q_c^{-1} u_t + u_x, \ \phi_y = -\psi_x = q_c^{-1} v_t + v_x \tag{10}$$

The unsteady supercavitating flow problem now is to find an analytic function $F(z, t)$ which is finite everywhere in the region bounded by the solid walls and the closed curve, which consists of the wedge and the cavity satisfying the following boundary conditions:

(1) On the solid walls $v = 0$

$$\therefore \psi = 0 \tag{11}$$

(2) On the cavity boundary, $p = p_c$, hence

$$\phi = \frac{\sigma}{2(1 + \sigma)} \tag{12}$$

(3) The equation of the moving wedge surface may be written as

$$h(x, t) = \pm \gamma[A(t) + x] \tag{13}$$

where h is the ordinate and γ is the half wedge angle. The y-component of the nondimensional velocity of the wedge surface is, up to the first order approximation,

$$v = q_c^{-1} h_t + h_x = \pm \gamma(1 + \dot{A} q_c^{-1}) \tag{14}$$

The second equation of (10) may be integrated, with the help of Eq. (14), and yields

$$\psi = \pm \gamma[1 + 2\dot{A} q_c^{-1} + \ddot{A} q_c^{-2} x + \Psi(t)] = \psi_1 \tag{15}$$

where $\Psi(t)$ is an unknown function to be determined later. Equation (15) is the boundary condition to be satisfied on the wedge surface.

(4) At upstream infinity, $x = -\infty$, $p = P$, hence,

$$\phi = 0 \tag{16}$$

To determine the unknown function in Eq. (15) an additional condition is required. The condition is furnished by considering the continuity of the velocity field [6]. Taking into account the fact that the vertical velocity component at $x = -\infty$ is zero, the general solution of the second equation of (10) is

$$v = -\int_{-\infty}^{x} \psi_x\left(z,\, y,\, t - \frac{x}{q_c} + \frac{\tau}{q_c}\right) d\tau \tag{17}$$

By choosing a point (x, y) on the wedge surface where v is given by the motion of the wedge, Eqs. (14) and (17) furnish the required condition. As usual in linearized theory, the second and third boundary conditions are applied on the x-axis. The linearized z-plane is shown in Fig. 1(b).

III. SOLUTION

A. The Conformal Transformations

Since the flow is symmetric with respect to the x-axis, only the upper half of the flow region needs to be considered. On the line of symmetry, the vertical velocity component is identical to zero and, hence, $\psi = 0$. In other words, the line of symmetry may be regarded as a solid wall.

The upper half of the flow region is transformed into the upper half of a ζ'-plane by the following Schwarz-Christoffel transformation:

$$z = \frac{b}{2\pi} \log \zeta' - A \tag{18}$$

where b is the channel width normalized with the wedge chord. It is more convenient to work on a new plane, the ζ-plane, given by the following linear transformation:

$$\zeta = x_o(\zeta' - 1) \tag{19}$$

This transformation transforms the point at $-\infty$ to $(-x_o, 0)$, the nose of the wedge to $(0, 0)$ and the tail of the cavity to $(\ell', 0)$. Here x_o and ℓ' are given by

$$x_o = (e^{\frac{2\pi}{b}} - 1)^{-1} \tag{20}$$

$$\ell' = x_o(e^{\frac{2\pi\ell}{b}} - 1) \tag{21}$$

and ℓ is the cavity length. The problem now becomes one of finding $F(\zeta, t)$ which will satisfy the boundary conditions sketched in Fig. 2.

B. Method of Solution

A general method of solution for $F(\zeta, t)$ which is essentially due to Carleman and Munk may be applied here. Detailed description of the method is given in Reference [5] and Reference [7]. The homogeneous solution which is finite for finite value of ζ is

$$H(\zeta) = \sqrt{(\zeta - 1)\ (\zeta - \ell')} \tag{22}$$

The formal solution can then be written as

$$F(\zeta) = \frac{1}{\pi} H(\zeta) \left[\int_0^1 \frac{\psi 1(\tau, t) d\tau}{H(\tau)\ (\tau - \zeta)} + \frac{\sigma}{2(1 + \sigma)} \int_1^{\ell'} \frac{d\tau}{(\tau - \zeta) \sqrt{(\tau - 1)(\ell' - \tau)}} \right] \tag{23}$$

Here $\psi_1(\tau, t)$ is given by

$$\psi_1(\tau, t) = -\gamma[1 + \frac{2\dot{A}}{q_c} + \frac{b\ddot{A}}{2\pi q_c^2} \log(1 + \tau/x_o) + \Psi(t)] \tag{15a}$$

which is derived from Eq. (15).

For the case when the unsteady motion is due to an oscillatory motion of the body or when the cavity is infinitely long, the function $\Psi(t)$ is equal to $-\frac{\dot{A}}{q_c}$. The detailed calculation of $\Psi(t)$ is shown in Appendix A of Reference [5].

It can be readily shown that the acceleration potential given by Eq. (23) satisfies all boundary conditions except the condition at infinity ($x = -\infty$ or $\zeta = -x_o$). To satisfy the condition it is further required that $F(-x_o) = 0$. Since $H(-x_o) \neq 0$, it follows that

$$\int_0^1 \frac{\psi_1(\tau, t)d\tau}{H(\tau)(\tau + x_o)} + \frac{\sigma}{2(1+\sigma)} \int_1^{\ell'} \frac{d\tau}{(\tau + \dot{x}_o)\sqrt{(\tau - 1)(\ell' - \tau)}} = 0 \qquad (24)$$

Performing the necessary integrations with the help of Eq. (15a) it follows that

$$\frac{\sigma}{1+\sigma} = \frac{2\gamma}{\pi}\left(1 + \frac{2\dot{A}}{q_c} + \Psi\right)\log \frac{\left[\sqrt{\ell'(x_o + 1)} + \sqrt{x_o + \ell'}\ \right]^2}{x_o(\ell' - 1)}$$

$$+\frac{\gamma b\ddot{A}\, I}{\pi^2 q_c^2}\sqrt{(x_o + \ell')(x_o + 1)} \qquad (24a)$$

where

$$I = \int_0^1 \frac{\log(1 + \tau/x_o)d\tau}{(\tau + x_o)\sqrt{(\tau - 1)(\ell' - \tau)}}$$

For the steady flow case the equation is reduced to

$$\frac{\sigma}{1+\sigma} = \frac{4\gamma}{\pi}\log\frac{\sqrt{\ell'(x_o + 1)} + \sqrt{x_o + \ell'}}{\sqrt{x_o(\ell' - 1)}} \qquad (25)$$

When the cavity becomes infinitely long, $\ell' \longrightarrow \infty$ and the blockage cavitation number is given by

$$\frac{\sigma_\infty}{1+\sigma_\infty} = \frac{2\gamma}{\pi}\log\frac{(\sqrt{x_o+1}+1)^2}{x_o} = \frac{4\gamma}{\pi}\cosh^{-1}\exp\frac{\pi}{b} \qquad (26)$$

Equation (26) agrees with the result given by Cohen and Gilbert [2]. For the infinite fluid case, $x_o \rightarrow \infty$ and Eq. (24a) leads to

$$\frac{\sigma}{1+\sigma} = \frac{4\gamma}{\pi}\left(1+\frac{2\dot{A}}{q_c}+\Psi\right)\log\frac{\sqrt{\ell}+1}{\sqrt{\ell-1}} + \frac{2\gamma\ddot{A}}{\pi q_c^{\,2}}\left[\sqrt{\ell}+(\ell+1)\log\frac{\sqrt{\ell}+1}{\sqrt{\ell-1}}\right] \qquad (27)$$

which agrees with the result given in Reference [5]. The functional relationship between the steady cavity length, cavitation number, and the channel width given by Eq. (25) is shown graphically in Fig. 3. Also included in Fig. 3 is the free-jet result given in Reference [5].

The drag coefficient is defined as

$$C_D = \frac{\text{Drag}}{1/2\ \rho\ U^2} = \frac{4\gamma}{\rho U^2}\int_{-A}^{1-A}(p-p_c)dx = 2\gamma\sigma - 4\gamma(1+\sigma)\int_{-A}^{1-A}\phi(x)dx \qquad (28)$$

On the ζ-plane it reads

$$C_D = 2\gamma\sigma - \frac{2}{\pi}\gamma b(1+\sigma)\int_0^1 \operatorname{Re} F(\xi)\,\frac{d\xi}{\xi+x_o} \qquad (28a)$$

Substituting Eq. (23) into Eq. (28a), it follows that

$$C_D = C_{D_0} + C_{D_1} + C_{D_2} \qquad (29)$$

where

$$C_{D_0} = \frac{8\gamma^2}{\pi}\left(1+\frac{2\dot{A}}{q_c}+\Psi\right)(1+\sigma)\ L_o \qquad (30)$$

$$C_{D_1} = \frac{8\gamma^2 \ddot{A}}{\pi U^2} L_1 \tag{31}$$

$$C_{D_2} = 2\gamma\sigma L_2 \tag{32}$$

and

$$L_o = \frac{b}{4\pi} \operatorname{Re} \int_0^1 \frac{\sqrt{(\xi - 1)(\xi - \ell')}\,d\xi}{\xi + x_o} \int_0^1 \frac{d\tau}{(\tau - \xi)\sqrt{(\tau - 1)(\tau - \ell')}} \tag{33}$$

$$L_1 = \frac{b^2}{8\pi^2} \operatorname{Re} \int_0^1 \frac{\sqrt{(\xi - 1)(\xi - \ell')}}{\xi + x_o}\, d\xi \int_0^1 \frac{\log\,(1 + \tau/x_o)\,d\tau}{(\tau - \xi)\sqrt{(\tau - 1)(\tau - \ell')}}$$

$$L_2 = 1 - \frac{b}{2\pi^2} \operatorname{Re} \int_0^1 \frac{\sqrt{(\xi - 1)(\xi - \ell')}}{\xi + x_o}\, d\xi \int_1^{\ell'} \frac{d\tau}{(\tau - \xi)\sqrt{(\tau - 1)(\ell' - \tau)}} \tag{35}$$

The first integral, L_o, can be evaluated by the method shown in the Appendix. The result is

$$L_o = \frac{b}{2\pi} \left\{ \left[\log \frac{\sqrt{x_o(\ell' - 1)}}{\sqrt{(x_o + 1)\ell'} - \sqrt{x_o + \ell'}}\right]^2 - \left[\log \frac{\sqrt{\ell' - 1}}{\sqrt{\ell'} - 1}\right]^2 \right\} \tag{33a}$$

As a limit when $b \rightarrow \infty$ it can be shown that

$$\lim_{b \rightarrow \infty} L_o = \ell \log \frac{\sqrt{\ell} + 1}{\sqrt{\ell} - 1} \tag{33b}$$

and when $\ell \rightarrow \infty$

$$\lim_{\ell \rightarrow \infty} L_o = \frac{b}{2\pi} \left[\log \frac{1 + \sqrt{x_o + 1}}{\sqrt{x_o}}\right]^2 \tag{33c}$$

Equation (33b) agrees with the result given in Reference [5] and Eq. (33c) agrees with Cohen and Tu's result [3]. The second integral, L_1, cannot be integrated in a closed form. Since $\ell' \geq \ell > 1$, it will be expanded into a power series in $1/\ell'$ as follows:

$$L_1 = L_1(\infty) + \frac{b^2}{8\pi^2 \ell'} \left[\sqrt{1 + x_o} \log \frac{1 + \sqrt{1 + x_o}}{\sqrt{x_o}} - 1\right] I_o + O(\ell'^{-2}) \tag{34a}$$

where O means order of magnitude and

$$L_1(\infty) = \lim_{\ell \to \infty} L_1 = \frac{b^2}{8\pi^2} \operatorname{Re} \int_0^1 \frac{\sqrt{1 - \xi}\, d\xi}{\xi + x_o} \int_0^1 \frac{\log(1 + \tau/x_o) d\tau}{(\tau - \xi)\sqrt{1 - \tau}} \tag{36}$$

$$I_o = \int_0^1 \frac{\log(1 + \tau/x_o) d\tau}{\sqrt{1 - \tau}} \tag{37}$$

Equation (36) may be integrated once with respect to ξ and the result is reduced to

$$L_1(\infty) = \frac{b^2}{4\pi^2} \left[\frac{\sqrt{1 + x_o}}{x_o} \log \frac{\sqrt{1 + x_o} + 1}{\sqrt{x_o}} I_1 - I_2 \right] \tag{36a}$$

where

$$I_1 = \int_0^1 \frac{\log(1 + \tau/x_o) d\tau}{(1 + \tau/x_o)\sqrt{1 - \tau}} \tag{38}$$

$$I_2 = 1/4 \int_0^1 \frac{\log^2(1 + \tau/x_o) d\tau}{\tau \sqrt{1 - \tau}} \tag{39}$$

It can be shown that when $b \to \infty$, L_1 is reduced to

$$\underset{b \to \infty}{L_1} = \sqrt{\ell} \log \frac{\sqrt{\ell + 1}}{\sqrt{\ell - 1}} + \frac{1}{4} \left[\sqrt{\ell} - (\ell - 1) \log \frac{\sqrt{\ell + 1}}{\sqrt{\ell - 1}} \right]^2 \qquad (34b)$$

which agrees with the result given in Reference [5].

Finally, by straight-forward integrations, it can be shown that

$$L_2 = 0 \qquad (40)$$

It should be noted that the corresponding function for the free-jet problem [5] is not always zero but, rather, a function of jet width and cavity length. Therefore, it is clear that the existence of the function L_2 is strictly a free-surface phenomenon.

To show the effect of the solid walls on the steady and the added mass parts of the drag coefficient, L_0 and $L_1(\infty)$ for an infinite cavity as given by Eq. (33c) and (36a) are plotted in Fig. 4. It is seen that the existence of the solid walls tends to increase both the steady and the added mass parts of the drag coefficient whereas the existence of free surfaces (shown by dotted lines) tends to reduce the corresponding terms. Equation (33a), which shows the effect of cavity length and channel width on the steady part of the drag coefficient, is plotted in Fig. 5. To show the effect of cavity length on the added mass term of the drag coefficient, the second term of the righthand side of Eq. (34a), which is approximately equal to $L_1 - L_1(\infty)$ when $\ell \gg 1$, is plotted in Fig. 6. It is readily seen that the added mass effect is larger when the cavity is shorter, just as in the case of the free jet [5].

IV. CONCLUSIONS

This presentation is another demonstration of how a singularity-free solution may be obtained, without using the closure condition, in a perturbation theory. The problem treated here is an unsteady version of Cohen and Gilbert's problem solved in 1957. The present result is comparable with the result of Cohen and Gilbert and with the solution obtained by the first author of a similar problem in a free jet. Following are some other important conclusions which may be drawn from the present study:

1. The acceleration potential method may be used for steady as well as unsteady flow problems.

2. When the linearized acceleration potential method is applied to the steady flow problems, the accuracy of the solutions is comparable to that obtained by the linearized velocity potential method.

3. Since most of the existing linear solutions admit a singularity at the tail of the cavity, the solutions are naturally slightly different from the solutions using present methods. As could be expected, for infinitely long cavities the solutions using the two methods become identical.

4. Whereas the existence of free surfaces reduces the drag, the existence of a solid boundary tends to increase the drag.

5. The effect of cavity pressure change on drag, which shows up through the function L_2, is characteristic of the existence of free surfaces. That is, L_2 is identical to zero when there is no free boundary.

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F I G U R E S
(1 through 6)

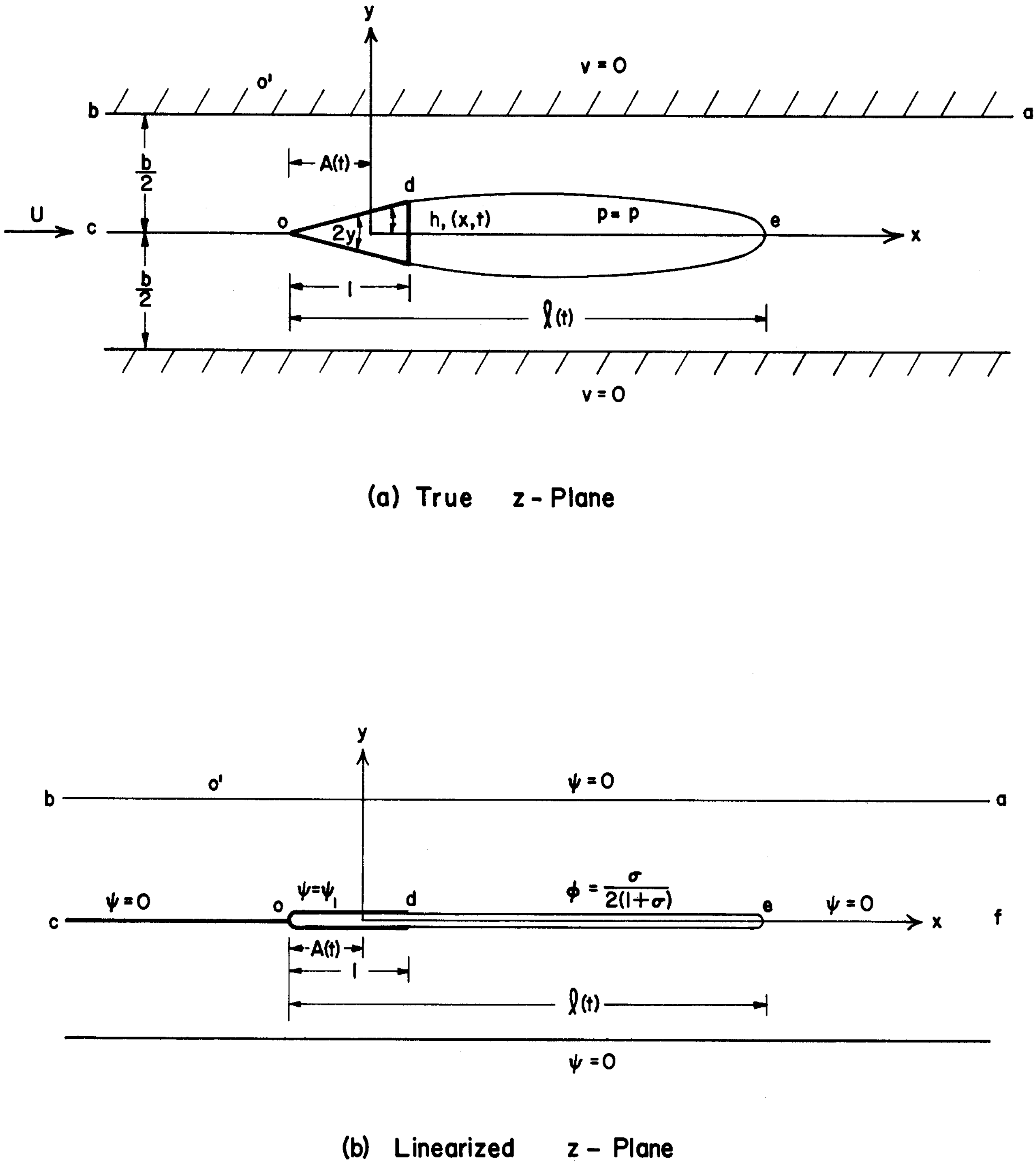
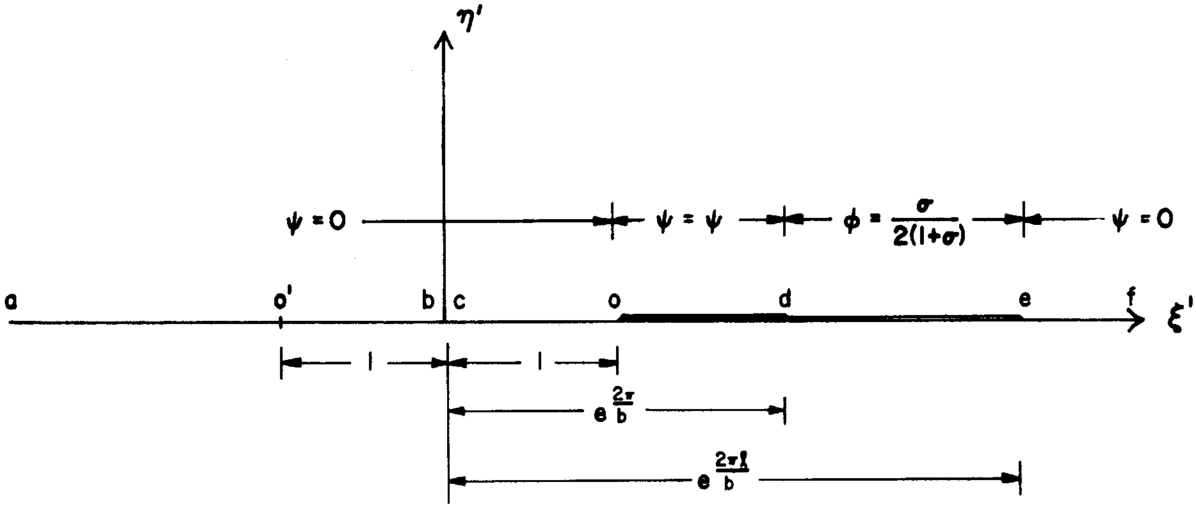
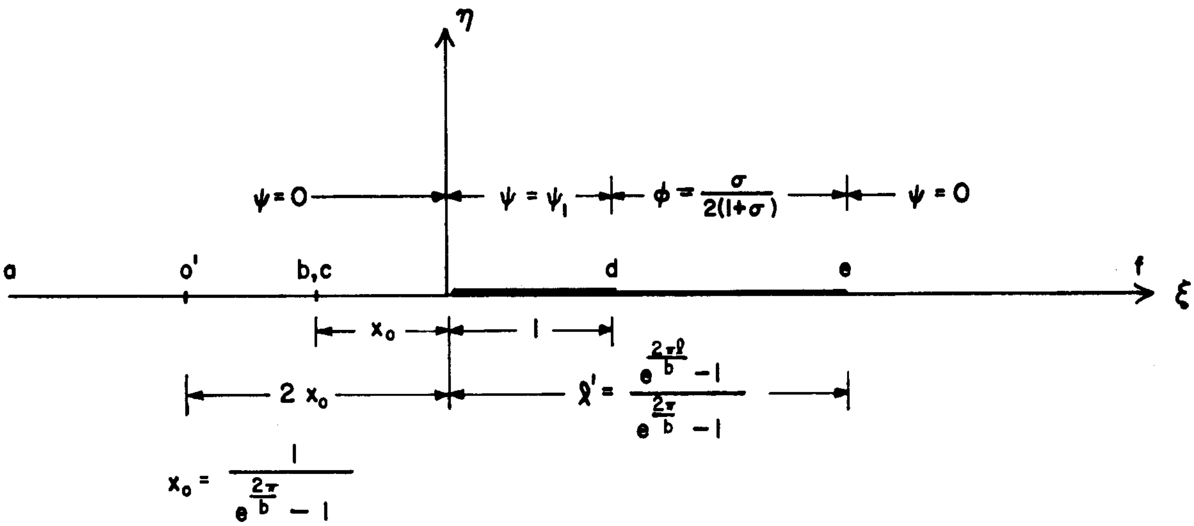


Fig. 1 - Flow Configuration and Linearized Boundary Conditions



(a) ζ' – Plane



(b) ζ – Plane

Fig. 2 – The Conformal Transformations

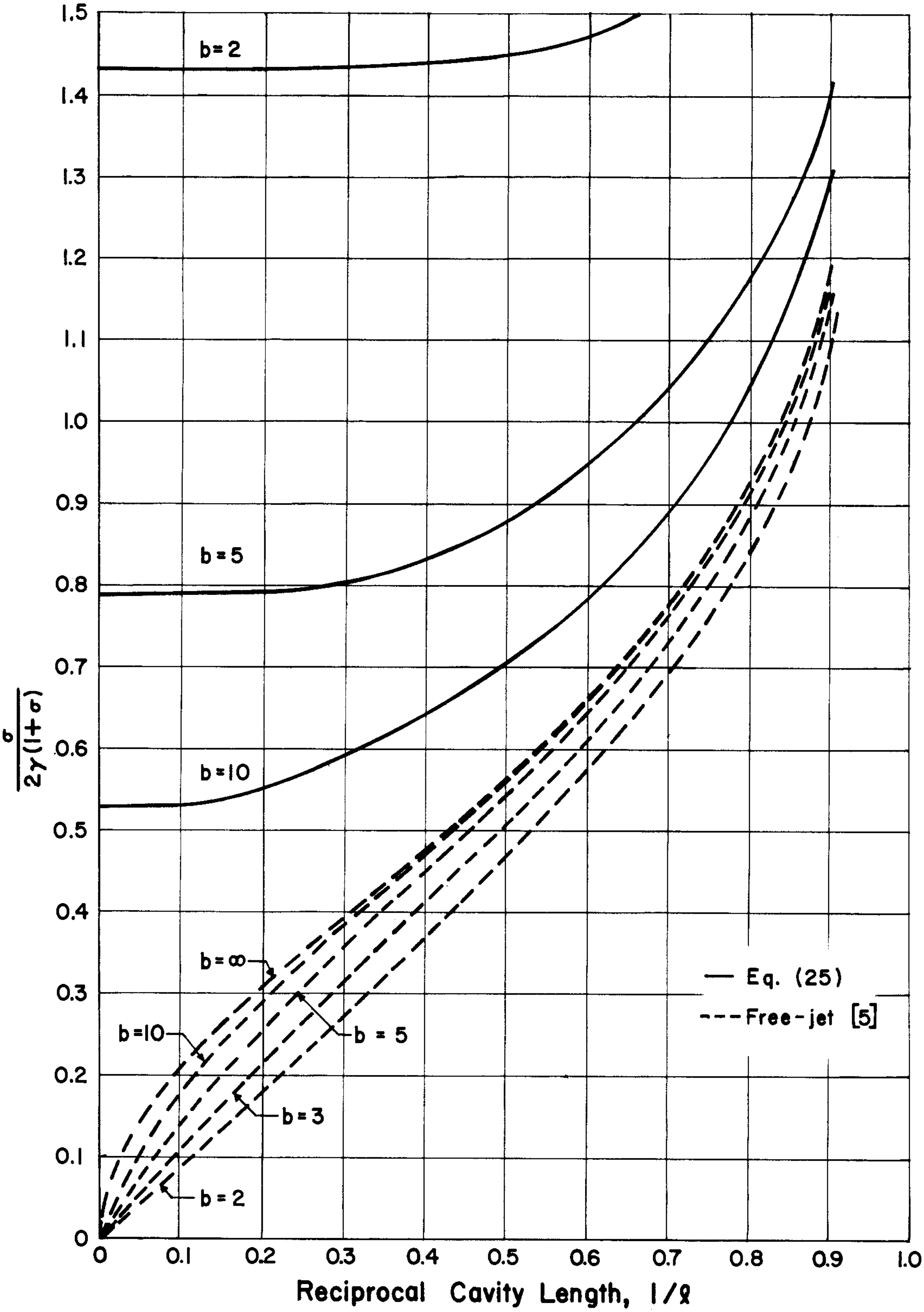


Fig. 3 - Steady Cavity Length as a Function of Cavitation Number

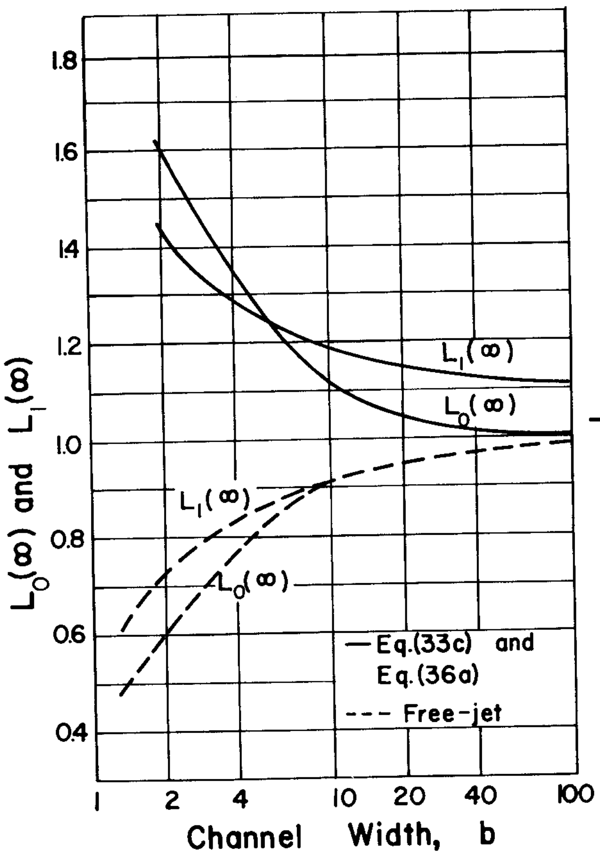


Fig. 4 - The Effect of Solid Walls on the Drag Coefficient for the Infinite-Cavity Case

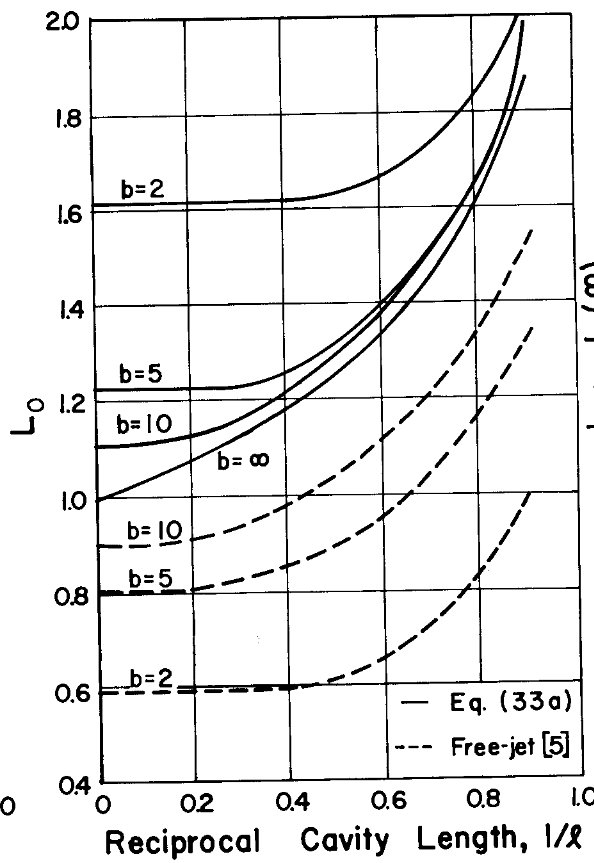


Fig. 5 - The Effect of Cavity Length and Channel Width on the Steady Part of the Drag Coefficient

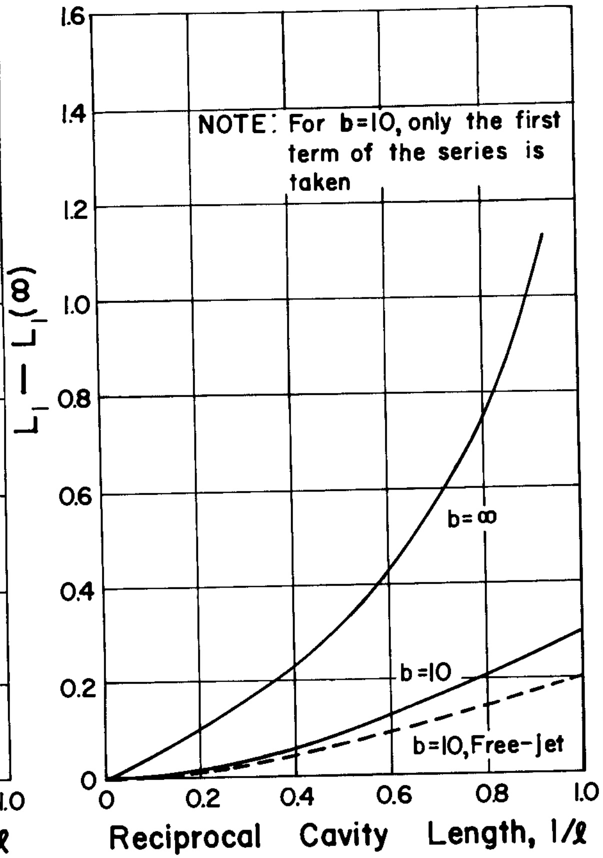


Fig. 6 - The Effect of Cavity Length and Channel Width on the Added Mass Part of the Drag Coefficient

A P P E N D I X

A P P E N D I X

A. Evaluation of L_0

By changing the order of integration Eq. (33) may also be written as

$$L_0 = \frac{b}{4\pi} \operatorname{Re} \int_0^1 \frac{d\tau}{\sqrt{(\tau - 1)(\tau - \ell')}} \int_0^1 \frac{\sqrt{(\xi - 1)(\xi - \ell')}\, d\xi}{(\xi + x_o)(\tau - \xi)} \tag{A-1}$$

By repeating the method of partial fractions, it can be shown that

$$\frac{\sqrt{(1 - \xi)(\ell' - \xi)}}{(\xi + x_o)(\tau - \xi)} = \frac{\sqrt{\ell' - \xi}}{\sqrt{1 - \xi}\,(\xi + x_o)} + \frac{1 - \tau}{\sqrt{(1 - \xi)(\ell' - \xi)}\,(\xi + x_o)}$$

$$+ \frac{(1 - \tau)(\ell' - \tau)}{\sqrt{(1 - \xi)(\ell' - \xi)}\,(\tau + x_o)(\xi + x_o)} + \frac{(1 - \tau)(\ell' - \tau)}{\sqrt{(1 - \xi)(\ell' - \xi)}\,(\tau + x_o)(\tau - \xi)}$$

and Eq. (A-1) is reduced to

$$L_0 = \frac{b}{4\pi} \operatorname{Re} \left[\int_0^1 \frac{d\tau}{\sqrt{(1 - \tau)(\ell' - \tau)}} \int_0^1 \sqrt{\frac{\ell' - \xi}{1 - \xi}}\, \frac{d\xi}{\xi + x_o} \right.$$

$$+ \int_0^1 \frac{1 - \tau}{\ell' - \tau}\, d\tau \int_0^1 \frac{d\xi}{\sqrt{(1 - \xi)(\ell' - \xi)}\,(\xi + x_o)}$$

$$+ \int_0^1 \frac{\sqrt{(1 - \tau)(\ell' - \tau)}\, d\tau}{\tau + x_o} \int_0^1 \frac{d\xi}{\sqrt{(1 - \xi)(\ell' - \xi)}\,(\xi + x_o)}$$

$$- \int_0^1 \frac{d\,\xi}{\sqrt{(1 - \xi)(\ell' - \xi)}} \int_0^1 \frac{\sqrt{(1 - \tau)(\ell' - \tau)}\, d\tau}{(\tau + x_o)(\xi - \tau)} \Big] \qquad \text{(A-2)}$$

Since the last term of Eq. (A-2) is also L_0, it follows that

$$L_0 = \frac{b}{8\pi} \Big[\int_0^1 \frac{d\,\tau}{(1 - \tau)(\ell' - \tau)} \int_0^1 \sqrt{\frac{\ell' - \xi}{1 - \xi}}$$

$$+ \int_0^1 \sqrt{\frac{1 - \tau}{\ell' - \tau}}\, d\tau \int_0^1 \frac{d\,\xi}{\sqrt{(1 - \xi)(\ell' - \xi)}\,(\xi + x_o)}$$

$$+ \int_0^1 \frac{\sqrt{(1 - \tau)(\ell' - \tau)}\, d\,\tau}{\tau + x_o} \int_0^1 \frac{d\,\xi}{\sqrt{(1 - \xi)(\ell' - \xi)}\,(\xi + x_o)} \Big]$$

Performing the integrations the following result is obtained:

$$L_0 = \frac{b}{2\pi} \left\{ \Big[\log \frac{\sqrt{x_o(\ell' - 1)}}{\sqrt{(x_o + 1)\ell'} - \sqrt{x_o + \ell'}} \Big]^2 - \Big[\log \frac{\sqrt{\ell' - 1}}{\sqrt{\ell'} - 1} \Big]^2 \right\} \qquad \text{(A-3)}$$

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