MEASUREMENT OF A COST FUNCTION FOR US AIRLINES:
RESTRICTED AND UNRESTRICTED TRANSLOG MODELS
WITH ENERGY COST PERTURBATIONS

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Abstract

This thesis continues and expands several themes from previous studies of commercial airline cost functions. A well specified industrial cost function reveals characteristics about the market players, such as economies of scale and the cost elasticities with respect to operational styles. These parameters are updated, the experimental design reworked and new analysis is given to describe the spectrum of choices facing airline firms in recent times.

I first construct a cost function for recent data using methods similar to Caves, Christensen and Tretheway (1984). As an energy intensive business, the US airline industry has seen its energy cost share rise and fall over time with potentially destabilizing effects. The model in this paper allows the energy cost share to interact with other variables and illuminate what factors may exacerbate cost sensitivity to energy prices. It was found that fuel cost shares tend to be higher with older equipment, smaller fleet sizes, and to be increasing in aircraft size and seating density.

The translog results include a positive cost of older aircraft designs, suggesting that airlines with poorer access to capital may suffer a cost disadvantage, particularly during a fuel spike. The model does not reject constant returns to scale (CRS) for fleet expansion, or IRS in aircraft size.
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I. Introduction / General Problem Description

The cost function is historically important because it can reveal to us nearly everything about the production technology (Chambers, 1988). By examining and modeling public data, we can explain how airline firms convert inputs into outputs. We can then identify whether the efficiency of firms appears to increase with scale. The effects of various external shocks – elevating the price of oil, for example – can be seen in the model outputs.

The airline industry of the U.S. collects and publicly reports a wide range of data, allowing a focused analysis of their cost functions. The U.S. industry can be seen as a laboratory for industrial behavior in a fairly “natural” setting. A competitive “settling out” process has been in progress since deregulation in 1978, or arguably for even longer. Understanding the cost functions for airlines, or their revenue functions, can yield helpful insights into transportation policy and/or general industrial forecasting. The most remarkable elements of this industry, perhaps inspiring past researchers, are (1) the contestability and commodification of players, implying efficiency; and (2) centrally collected data whose accuracy is overseen by a regulator.

A cost function allows us to view the cost effects of aircraft size, average length of haul, and seating capacity as matters of general principle within the industry. The framework employed in this thesis initially examines prior literature. Several conceptual and tactical changes are proposed and implemented. Earlier work suggests a cost function can exist. The distinctions between indirect and direct costs; returns to scale; returns to scope; and fixed effects modeling are analyzed here with respect to earlier studies.
II. Literature Review

II.A Rationale of the Model: Contestability

The validity of a nationwide cost model has been approached with practicality by a number of authors. While we can measure firm operations, and fix identity labels to their data, it is unclear whether the coefficients have a direct application in the industry. Previous authors have implied that firms are subject to a common set of institutions, markets, technology and the laws of physics. It is important to assert that competition does prevail, rather than some arbitrary uncompetitive space in which costs are not necessarily relevant, such as expense preference behavior in the regulated, as mentioned by Sickles (1986). It is most often claimed that the markets are contestable between the players whose costs we are attempting to fit into the common model.

A perfectly contestable market is defined as “one into which entry is completely free, from which exit is costless, in which entrants and incumbents compete on completely symmetric terms, and entry is not impeded by fear of retaliatory price alterations” (Baumol, Panzar, 1982). Can the industry in question satisfy this definition? The verdicts from various authors have been mixed (See Appendix I.A). I assume that the production in this study takes place in a contestable fashion. Hence, a general set of cost elasticities (and cross-elasticities) are obtained and reported from all firms, although the cost intercepts are unique by firm. Adoption of contestable markets is an essential ingredient to a cost study.

II.B Cost Model Specification Literature – Cost Model Theory

A theoretically valid cost function must exhibit certain properties (Varian, 1984). In particular, the cost function should be:

• non-decreasing in input factor prices and output quantities;
• homogeneous of degree one in factor prices;
• concave in factor prices; and
• continuously differentiable (ideally, twice differentiable) in prices.
Also, there should be zero fixed costs, and fulfillment of Shephard’s Lemma, such that:

\[
\frac{dT C(w, y)}{d w_i} = x_i
\]  

(1)

with \(w_i\) input prices and \(x_i\) input quantities.

The properties of a valid cost function are discussed in Chambers, 1984 (See Appendix I.B).

II.C Cost Model Specification Literature – Engineering Based / Micro Models

Swan (2006) builds a simulation of generic airline production costs at the micro level. This idea suggests a way to achieve the aim of the present work: to model actual airline cost data at the micro level. The operational cost function for aircraft is described by Swan as a model based solely on seat counts and flight distances. Swan’s model takes the form:

\[
\ln(cost) = A + B \ln(seats) + C \ln(distance)
\]  

(2)

Other factors could be considered endogenous – and assuming CRS in fleet size, scale could be irrelevant as well. While we lack sufficient data resolution to use this model exactly, it is important to recognize that disaggregation has its benefits.

Aircraft ownership costs (and maintenance costs) are very real, but may reasonably be discarded in group analysis. Swan argues that costs may equilibrate and be discarded, as “the driver of used airplane values is the need to establish a cost position along the cost frontier... the cost frontier itself is designated by the price of newly manufactured airplanes.” This cost frontier could be construed as an operational indifference curve, over which profitability is equal; thus, equipment costs could drop out.¹

¹ The assumption here by Swan is that a sufficiently liquid exchange market exists for used aircraft. It follows that the marginal profitability for such aircraft should be equal among firms, equalizing their lease values.
Morrison (1984) looks more deeply at the relationship between aircraft capital cost and the inherent efficiency of an aircraft’s design. Uniform physical principles may underlie a significant portion of airline cost functions. The laws of physics, with their limitations of speed, reliability and fuel efficiency, affect aircraft operators uniformly in such a model, with respect to the equipment they choose. Firm equipment factor demands have a point of interface in the aircraft exchange market. As prices prevail along a cost frontier, differences that we observe can be assigned to coefficients in a fixed-factor or fixed-effects model.

![Figure 1. Capital Operating Efficiency vs. Cost](courtesy Brookings Papers)

In Morrison’s graphic, a frontier exists between capital costs and operating costs per mile (capital cost of aircraft versus its energy efficiency and maintenance costs). If such a frontier is monotonic, and exchange markets are liquid, then each firm chooses aircraft at its preferred location along that frontier, assuming all players are efficient. Like firms would choose like equipment, at like prices. An airline might wish to buy a more expensive aircraft with lower fuel consumption to provide a cushion against cost volatility of future fuel price spikes. Concerns about capital shortages, meanwhile, may encourage hoarding of cash and depress demand for new equipment temporarily.
These firm heterogeneity scenarios are each precluded by Morrison’s assumption of a single frontier. Using the liquidity of aircraft purchasing and leasing, we are left with an operational cost solely determined by output, exclusive of aircraft cost, which is also the method suggested by Swan. This would perhaps rationalize new aircraft purchases at the firm level, a key point for Swan’s employer, the Boeing Corporation, (as such purchases could be described as cost neutral).

The feature that distinguishes the “engineering-based” approach to econometric analysis is the analysis of a specific group of machines whose production process can be usefully understood by a single unit of production data. This disaggregated analysis comes closer to a “mechanical” functional form that is amenable to forecasting or static interpretation. To report only on aggregates of machines, while it can be interpreted observationally, has limited practicality in decision-making. Practical validity is enhanced if we have an accurate model of the micro production choices, as they exist, because this clears away aggregation bias. That issue is discussed directly in II.G, and the proposed remedy in III.C.

II.D Cost Model Specification Literature – Panel Data of Firm Operations

Caves, Christensen and Tretheway (1984) (hereafter CCT) is the main precursor of this study. CCT use scale and density coefficients to obtain the returns to scale (RTS) and returns to density (RTD). Their treatment of production “density” (or, increased product per geographical area) gives a fine substitute for scale itself within a geographic system. CCT use the first-order coefficients of a translog for their cost model (omitting the second-order components of the translog):

\[
\ln[\text{Total Cost}] = \beta_0 + \beta_1 \ln[\text{Aircraft Miles}] + \beta_2 \ln[\text{points served}] + \\
\beta_3 \ln[\text{mean flight distance}] + \beta_4 [\text{load factor}] + \beta_5 [\text{labor price}] + \\
\beta_6 [\text{fuel price}] + \beta_7 [\text{capital-materials price}] + \beta_8 [\text{capacity}] + \\
\beta_9 [\text{firm identities}].
\]
Expressed symbolically, CCT did a first-order restricted translog regression:

\[
\ln[\text{Total Cost}] = \alpha_0 + \alpha_Y \ln Y + \sum_i \beta_i \ln W_i + \sum_i \phi_i \ln Z_i + \sum_T \alpha_T + \sum_F \alpha_F
\]  

(4)

Where \( Y \) is output, \( W_i \) are input prices, and the \( Z_i \) are operational control variables that describe airline characteristics, or production styles. The coefficients on \( \ln Z_i \) can give us cost elasticities of these characteristics.

CCT also performed both a full translog model:

\[
\ln[\text{Total Cost}] = \\
\alpha_0 + \alpha_Y \ln Y + \sum_i \beta_i \ln W_i + \sum_i \phi_i \ln Z_i + \frac{1}{2} \delta_{yy} (\ln Y)^2 \\
+ \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln W_i \ln W_j + \frac{1}{2} \sum_i \sum_j \psi_{ij} \ln Z_i \ln Z_j \\
+ \sum_i \rho_{yi} \ln Y \ln W_i + \sum_i \mu_{yi} \ln Y \ln Z_i + \sum_i \sum_{ij} \lambda_{ij} \ln W_i \ln Z_j + \sum_T \alpha_T + \sum_F \alpha_F
\]  

(5)

Here, \( \text{Aircraft Miles} \) is the output \( Y \) variable. This scale variable’s coefficient, in a regression, tells us how unit costs would be influenced by simply repeating the activities of production more times. This gives us an indication of whether an industry becomes more efficient as firm size increases, i.e., whether it experiences positive “economies of scale.” In CCT, seats and average flight distance (\( \text{Stage} \)) are held constant while \( \text{Aircraft Miles} \), the scale variable, grows or shrinks. Effectively, this becomes departure count,\(^2\) the logical building block of “scale.” The partial derivative of cost with respect to \( \text{Available Seat Miles (ASMs)} \) is also found here by using \( \text{Aircraft Miles} \) as a proxy for departures.

\(^2\) Supposing the length of haul remains the same, a doubling in \( \text{aircraft miles} \) would by definition be synonymous with a doubling of \( \text{departures} \). This may help readers come to terms with \( \text{aircraft miles} \) being used as a marker for firm size; it works because we are holding other factors equal.
CCT estimate the above coefficients both as a full translog model, and a simplified model in which only the first-order coefficients are included. CCT report that “the coefficients for the simplified translog form... are remarkably similar to the first-order coefficients from the basic translog model.” This suggests that second-order coefficients were either insignificant, or counterbalanced each other.

The interpretations of key coefficients for CCT were as follows:

- **Aircraft Miles**, Cost elasticity of density, identical flights, fixed network;
- **Points served**, the cost impact of spreading resources over an additional service point;
- **Flight distance**, cost of impact spreading the aircraft miles over fewer, longer flights.
- **\( \beta_1 + \beta_2 \)**, customary elasticity of scale, allowing geographic area to expand.

II.E Cost Model Literature – Returns to Scale, Density, Scope

Positive economies of scope suggest that, even in a contestable market, firms will produce multiple products (or expand their networks) thanks to cross-product economies (Baumol, 1982). Without economies of scope, firms would not enter multiple cities or pursue hub networks. Most earlier papers have observed economies of density (decreasing unit costs as output increases in a given geographical area). They have also observed economies of scope, at least in the sense that one firm offering two products (or two travel routes) will be more efficient than two such firms covering that ground separately.

CCT find constant returns to scale in firm operations of the US airline industry. Economies of scope should be assumed in our environment; as said by Baumol, “Economies of scope are necessary for the existence of multiproduct competitive firms” (Baumol, 1982).
Gillen uses the formulation

$$RTS_{cale} = \left( \frac{d\ln C}{d\ln \text{Point Served}} + \frac{d\ln C}{d\ln Y} \right)^{-1}, \quad (6)$$

where values above unity signify positive “returns” to scale. The reciprocal, cost elasticity of scale, would be less than unity. From their log-linear model, they find RTS of 1.35 ($t$-stat = 2.1), meaning that RTS was significantly above unity, when firm dummies were included. With firm dummies omitted, there were no significant observed economies of scale. Canadian airline companies show increasing returns to density, when airline identities are included in regression, but no difference from unity if they are not included (Gillen, 1985).

Caves and Tretheway, writing in the period of the initial post-deregulation shakeout following 1978, believed that low costs of entry would facilitate equilibrium slowly. Their expectation was for any clear differences in technical efficiency among firms to abate over time, “as the regulatory era recedes into history,” replaced by the apparently more thriving competitive market of today. Returns to scale would favor industrial consolidation, something that has in fact occurred dramatically since the CCT 1984 paper.

<table>
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<tr>
<th></th>
<th>SCALE</th>
<th>DENSITY</th>
<th>FUEL PRICE</th>
<th>Years</th>
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<td>Gillen (1985)$^3$</td>
<td>.741 ($t = 2.1$)</td>
<td>.568 ($t = 4.8$)</td>
<td>.04 ($t = 2.3$)</td>
<td>1964-1981</td>
</tr>
<tr>
<td>Wei (2003)</td>
<td>.811 ($t = 24.3$)$^4$</td>
<td>N/A</td>
<td>.240 ($t = 7.21$)</td>
<td>1987-1998</td>
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Table 1. Prior Literature First Order Approximated Cost Elasticities of Scale, Density, Fuel Price

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$^3$ Gillen uses Canadian data. Log-linear model. The rest are U.S. only.

$^4$ This is returns to seat count (machine scale) within single aircraft types

$^5$ With local concavity imposed; for fleets.

$^6$ While these are from full translog models, in which interactions matter, the first order coefficients (without interactions) can signify the cost elasticities at the sample means of the data (Wei, 2003).
Positive economies of scope, if found, imply that airline hubs will emerge, according to simulation literature on the topic (Hendricks et al., 1999). Hub-spoke networks can be “a deterrent” to the entry of smaller carriers, due to structural revenue advantages (in essence, the revenue premium obtained by scope, as customers would prefer firms with greater network points, affording ease of shopping). In competitive markets, the increased cost of larger scope would apparently be justified by that revenue premium. While comparative network costs are beyond the topic of their study, Hendricks says that a least-cost hub will be located where origin and destination distances are minimized across the cumulative traffic spectrum. This, too, explains how multiple hubs can arise in opposition to one another. Significantly, contestability and cost advantages are not simply analyzed at the flight leg level. Instead, airline costs represent networks that may compete from differing geographical perches (heterogeneity, as allowed by Berry, 1992). In competition, Hendricks finds the industry prone to monopoly due to economies of density. “Every city-pair market is effectively served by only one carrier, the one with a length advantage [across its transfer hub]…” Also, they note that while one hub carrier will tend to dominate, “nonhub networks raise average costs of service but may allow the carriers to price less aggressively,” meaning they can expect another type of revenue premium.

Table 1 shows a compendium of previous airline cost studies in which returns to scale and density (and cost elasticity of fuel price) are obtained. While these may be expected to change over time, and particularly as regulatory or technological characteristics varied, a general CRS or light IRS characteristic is visible. CCT did not reject CRS in translog.

II.F Cost Model Specification Literature – Concavity Considerations

The concavity of a cost function is one of the key conditions of its validity (Varian, 1984; Chambers, 1988). Concavity checks of the first order log-log cost function can be done directly by taking the second derivatives of the raw (non-logged) cost function. With the full translog-style equation however, it is noted that a concave function is
always log concave. Therefore, by retaining logs, we can at least allow the possibility to reject (if the function is not log concave). Improvement on that basis can be measured.

A more complete way to check concavity of a translog style function is to check the Allen-Uzawa matrix of partial cost elasticities. Allen-partial matrix concavity implies Hessian matrix concavity (Featherstone, 2007). For the $i^{th}$ cross price elasticity at each observation, we have $\eta_{ij} = S_j + \frac{b_{ij}}{S_i}$, where $S_i$ is the cost share of input factor $i$. The equation also works where $i=j$ to get own-price Allen-Uzawa cost partial elasticities, from which the partials matrix can be made. This is done in the project to diagnose any concavity problems (See Appendix II, Codebase).

A process has been developed to impose cost function concavity, if needed, in projects such as CCT and the present one (Ryan and Wales, 2000). This procedure was used recently by Chua, Kew and Yong, 2005; (hereafter, CKY). Earlier methods to impose global concavity will negate the flexibility of the translog form. Instead, CKY and Ryan / Wales impose local concavity. To do this, a particular data observation is designated the “normalization point.” All its price, scale and state of nature variables are scaled to equal one; this ensures that the cost Hessian is negative semidefinite for that observation. By selecting the normalization point carefully, many or all of the observations’ violations of concavity can be remedied. CKY detail several examples of cost studies whose results are questionable or reversed, following this procedure. The CKY method of imposing local concavity maintains the translog model’s flexibility.

Using data for 10 airlines over the period 1994 to 2001, CKY present an enhancement and update of the earlier CCT cost estimation technique. CCT’s cost function violates concavity at approximately half its observations. CKY were able to rehabilitate those data. They present their new study, which includes a translog cost function. They re-estimate the CCT cost function after imposing concavity, finding “material differences in scale economies” after doing so. Note that the CCT and CKY papers both use aggregate corporate data. In this thesis, the concavity check recommended by CKY is used with consideration for their normalization remedy.
Further, modeling all normalizations and choosing the best one is a feasible extension that should prove simple and useful. Other concavity normalizations can impact flexibility of the cost function.

II.G Cost Model Specification Literature – Aggregation Considerations

The second recent innovation subsequent to the CCT study is a method to disaggregate data when necessary. A recent translog cost estimation study provides a logical way to disaggregate operational numbers from firms into fleets (Basso and Jara-Diaz, 2005). It is noted that aggregate outputs $y_h$ are implicit functions of fleet outputs $Y$. Looking at operational metrics, Basso and Jara-diaz use ton-kilometres ($TK$) and average length of haul (ALH) in $Y$. Then,

$$TK(Y) = \sum_{ij} y_{ij} \cdot d_{ij} \quad \text{[where } d_{ij} \text{ is distance of haul]} \quad (7)$$

and

$$ALH(Y) = \frac{\sum_{ij} y_{ij} \cdot d_{ij}}{\sum_{ij} y_{ij}}, \quad \text{[as opposed to } \left(\sum_{ij} y_{ij} \cdot d_{ij}/\sum_{ij} y_{ij}\right) \cdot \text{deps}_{type} \sum_{i=1}^{\text{deps}} \text{.}] \quad (8)$$

Basso and Jara-diaz explain that the above applies “where $d_{ij}$ is the distance travelled by flow $y_{ij}$ between origin $i$ and destination $j$. Therefore, even if the true (disaggregated) product vectors $Y^i$, $Y^d$ and $Y^D$ were unknown, SC [economies of scale] could still be calculated correctly if the corresponding aggregates $\mathcal{F}(Y^A)$, $\mathcal{F}(Y^B)$ and $\mathcal{F}(Y^D)$ were known, and an estimated cost function $\tilde{C}(\tilde{Y},PS)$ was available...”\(^7\) This is a method of rationalizing the distribution of aggregate metrics such as average length of haul by weighting it by product, in this case the ton-kilometer (the passenger-mile could work equivalently well, if we need a method to distribute such costs in a passenger study).

\(^7\) Basso and Jara-Diaz (2005), p. 32
In similar recognition of aggregation problems, another recent study uses disaggregated, fleet level data\(^8\) to construct a translog operations cost model from Form 41 data (Wei and Hansen, 2003). This was done to study aircraft capital costs and the demand for certain aircraft types. Their use of Form 41’s unaggregated direct operating cost data tables indicates some curiosity about fleet-level cost analysis, something expanded in the present project. I use the same data source tables as Wei and Hansen, together with a dis-aggregation technique similar to Basso, as will be described in the Model section for indirect costs.

While Form 41 includes detailed *direct* operating costs for all fleet types separately, data for *indirect* costs are not available by fleet. Indirect, or overhead, costs are only available at the firm level. To deal with this, Wei and Hansen confine their study to *direct* operating costs only, of fleets. But a full study of returns to scale (RTS) within the fleet context would require a total cost specification. By combining the methods of prior researchers, this paper makes a special arrangement to pursue a disaggregated model, by synthetically disaggregating the *indirect* costs, and achieving a total cost measurement for the fleets, which is the dependent variable to be studied here.

\(^8\)“Fleet level data” refers to a data source that specifically breaks out operational numbers by aircraft type within the firms. For example, it will give quarterly departures and gallons of fuel burned by AA MD-80s, 737s, etc. Otherwise, data is aggregated at the firm level (i.e., American Airlines as a whole).
III. Model Specification

III.A Model Overview

The first goal of this thesis is to build an effective base total cost model similar to prior literature. The CCT econometric model introduced earlier is still an effective framework if applied legitimately (as authors have done in various ways). The translog cost model (such as Eq. 5) is a well-tested practical method to approximate an unknown cost function with multiple input prices, operational styles, a time vector and firm dummy variables, nonlinearities, and interactions among the variables.

Ideally, that cost model would apply to fleets individually, to the extent that separate cost data sources exist. In this project, we have the right data ingredients to construct a cost study using the CCT framework at the fleet level, rather than the firm level. This allows analysis of fuel price effects on costs, as it impacts individual fleets within firms.

Fixed-effects model building is probably necessary, presuming firm dummy variables are significant. A fixed effects model estimates coefficients for the variables, controlled for the time period and the entity (airline) reporting the costs. The model would then become more targeted, specific to each airline identity. By having fixed identities, the interpretation of coefficients (cost elasticities) refers to the experiences of individual firms whose parameters have changed over time, within the sample set. (“Within,” as opposed to “between” firms).

Fleet-level direct operating costs are reported directly in the data. The assignment of indirect costs to flight operations is an inherently subjective endeavor that I must perform, because I lack the private data on fleet allocations of overhead costs. A researcher can perform this allocation nearly as well as the firms themselves, since we possess the firm-wide overhead cost data. These indirect costs such as advertising,
insurance and corporate real estate are here weighted by *aircraft miles*. Thus, we have a source of *Total Cost* data for individual fleets\(^9\):

\[
\textit{Total Cost} = (\textit{Direct Costs} + \textit{Pro-Rated Indirect Costs})
\] \hspace{1cm} (9)

This project uses Eq. 5 with the slight difference that *Year* is included among the \(Z_i\) as a trend variable, rather than as dummy variables.

Because of the homogeneity of degree one restriction in input prices, (the \(W_i\)), we impose the following restrictions on (Eq. 5):

\[
\sum_i \beta_i = 1 \quad \sum_i \gamma_{ij} = 0; \quad \sum_i \rho_{ij} = 0;
\] \hspace{1cm} (10)

I estimate the first-order translog functions using SAS PROC SYSLIN; the translog equation is estimated using PROC MODEL, iterated SUR with restrictions entered imposed on the primal translog cost function.

Because the full translog cost function alone is prone to multicollinearity, it is standard practice to estimate it together with its dual expenditure-share functions (Ray, 1982). Two of the three inputs whose prices I include (*Oil Price* and *Pilot Wages*) have share equations included; the other share equation, *Capital-Materials Price*, need not be included because it is defined by the remaining two. The three equations (*Total Cost* and two of its input expenditure shares) are estimated together in an iterated Zellner seemingly unrelated regression (SUR) as a system of linear equations with correlated

---

\(^9\) Another method might have been to weight overhead costs by *Available Seat Miles (ASMs)*, which might be useful in while accounting things like catering, which are sensitive to passenger count. Such accounting assignments are to some extent an arbitrary decision. Please see Appendix for the derivation of this Total Cost model from *ASMs*. 

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errors. The estimated parameters on the variables should be the maximum likelihood estimator (MLE) of their true values (CCT, 1984).

Expenditure shares are calculated as follows from the above translog cost primal:

\[ S_i = \frac{W_i X_i}{TC} = \left[ \frac{\delta TC}{\delta w_F} \right] \frac{w_F}{TC} = \frac{\delta \ln TC}{\delta \ln W_i} = \beta_i + \sum_j \gamma_{ij} \ln W_j + \rho_i \ln Y + \sum_j \lambda_{ij} \ln Z_j. \] (11)

A translog function is a second order Taylor series approximation of an unknown “true” cost function. Unlike with first-order translog, the elasticities of substitution between factors and/or production characteristics can vary in a so-called “flexible” functional form such as translog. First-order translog assumes the production technology is homothetic\(^{10}\), while translog does not. Still, this is not to say translog entirely captures the behavior of complex functions. For example, a generated CES function has been shown to be poorly approximated by translog.

In this notation, the intercept, firm dummies, output, input prices, and production characteristics are entered exactly as above. We also have symmetry since \( \gamma_{ij} = \gamma_{ji} \). In the SAS code for this project, the symmetric variables were combined to conserve degrees of freedom. Given that this project uses time series data, severe multicollinearity may be a concern. Estimating the cost function alone as a single equation model would typically be vulnerable to multicollinearity (Ray, 1982). To avoid it, I exploit the features of duality to estimate the flexible cost function together with its input share equations, in a Zellner Seemingly Unrelated Regression (SUR) technique. Singularity is avoided by including only two of the three share equations in the model (fuel and pilots).

Unlike CCT 1984, I include time as a continuous variable so that time can be interacted with other items of interest (i.e., here I include it as one of the \( Z_i \)). Otherwise, the model is very similar to that earlier study, computationally. But keep in mind that the simultaneous study here of firm-level and fleet-level economies of scale is a major conceptual adjustment to their model.

---

\(^{10}\) Generally, \( f(x) \) is homothetic if and only if it can be written as \( f(x) = \Phi(g(x)) \), where function \( g(\cdot) \) is homogenous of degree 1, and \( \Phi \) is some transformation of the reals.
Including factor share equations with the above will improve efficiency (Ray, 1982) by estimating the full dual system of equations, both cost and share equations together.

III.B Model Key Point – Returns to Scope and/or Scale

This model, like CCT, gives cost elasticities with respect to scale, density, or breadth/scope of service (the network points). Scale equals greater density over greater breadth (so those two separate coefficients can be summed to find RTS). I use the economies of network scope as described by Basso and Jara-Diaz to identify both RTS and RTD, as described in Table 2:

<table>
<thead>
<tr>
<th>Within An Aircraft,</th>
<th>For Fleets of Aircraft, (of Fleet data)</th>
<th>For Airline Firms, (of Firm data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E SCALE = ( \beta \text{SEATS} + \beta \text{CABIN AREA} )</td>
<td>E SCALE = ( \beta \text{MILES} + \beta \text{POINTS SERVED} )</td>
<td>E SCALE = ( \beta \text{FIRM ASM} + \beta \text{POINTS SERVED} )</td>
</tr>
<tr>
<td>E DENSITY = ( \beta \text{SEATS} )</td>
<td>E DENSITY = ( \beta \text{FLEET MILES} )</td>
<td>E DENSITY = ( \beta \text{FIRM AVAIL SEAT MILES} )</td>
</tr>
<tr>
<td>E SCOPE = ( \beta \text{CABIN AREA} )</td>
<td>E SCOPE = ( \beta \text{FLEET POINTS SERVED} )</td>
<td>E SCOPE = ( \beta \text{FIRM POINTS SERVED} )</td>
</tr>
</tbody>
</table>

Table 2. Cost Elasticies of Scale

III.C Model Key Point – An Effort to Model Individual Flights, through Quarterly fleet Aggregates

The Hicks Composite Commodity Theorem states that, “if the prices of a group of goods move in parallel, then that group of goods can be treated as a single good.” This implies that it is desirable to model costs below the firm level. It is unlikely that firm-wide statistics will have the same cost elasticities of operational parameters, scale, and so on, as for its component fleets. Therefore, the firm-wide production process ought not to be treated as a single production function. Instead, using disaggregated data at the fleet
level is more desirable. Meaningfully, equipment-specific data will ensure that the cost function does not mis-state the fleet level cost elasticities, which could occur without warning in the context of conventional firm-level aggregation, whose metrics are most often averaged by departure count. Please see more discussion of this in Appendix I.C.

I use *typical flights* which are the average departure-weighted journey for each fleet. Fleets usually are directed to fairly narrow bands of operations, to capitalize on the relative strengths of each aircraft type. The mean distance per flight should have low variation within these fleets – certainly a lower variation than when attempting firm-wide analysis. By using quarterly data, a researcher is partway to an engineering-based cost model. Operationally similar flying should have similar costs, regardless of which particular cities are involved. This is because the laws of physics acting upon the aircraft are uniform, airports contestable, and a parameter for airway congestion can be included in the model.

Consider this hypothetical example in Table 3:

<table>
<thead>
<tr>
<th>Aircraft Type</th>
<th>Seats</th>
<th>Stage</th>
<th>Quarterly Departures</th>
<th>Quarterly Operating Cost</th>
<th>Cost per Available Seat Mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>A319</td>
<td>120</td>
<td>375 mi</td>
<td>900</td>
<td>$8 million</td>
<td>$0.198</td>
</tr>
<tr>
<td>A319</td>
<td>120</td>
<td>1,500 mi</td>
<td>360</td>
<td>$9 million</td>
<td>$0.139</td>
</tr>
</tbody>
</table>

Table 3. Example of Assignment-Driven Cost Efficiency Differential

This sort of data display can be used to isolate the frontier of operational feasibility. Firms that operate in a particular way would be expected to have particular costs by virtue of their schedule, and its interactions with other factors. The efficient frontier of production can best be found by examining fleet-level data. The vastly different Cost per Available Seat Mile (*CASM*) in the above pair could obscure the idea that both came from the same cost function. They may be equally *efficient* in terms of some underlying physical model. Their schedules largely dictate the average speed at which the fleets operate, which directly results in “outputs” such as *ASMs* that a model would expect can be produced by such fleets. We must take care that the costs are modeled realistically, noting that each fleet has a particular operational cost frontier.
The LHS of our regression will be the total cost of a subfleet’s operations quarterly. This is reminiscent of a micro model of individual flights, in a Morrison/Swan engineering sense, as if the subfleet is iterating many identical average flights. Unfortunately, in the typical heterogeneous flight schedule, this assumption of flight uniformity is a gross simplification that is nonetheless required, since the cost data themselves are aggregates to some extent. It is hoped that mean values such as length of haul, divided by departures, will sufficiently model the micro flights from fleet data. Then the fleet average operational metrics could approximate the metrics of individual flights, upon which the laws of physics are likely to exert a role in terms of fuel and time requirements, and therefore cost.

This is a second-best method compared to actual flight modeling. We do not have data points for each of the roughly 10 million commercial passenger flights in the USA annually. The dataset compares fleet level data for several hundred fleets over the period 2000-2007 (comprising 2,660 usable observation points).
IV. Data and Empirical Strategy

IV.A Data Overview

This project uses US DOT Form 41 data for the years 2000-2007, an eight-year panel of operational, financial, equipment and macro environment data. This is a large data source with high presumed reliability and completeness. Its metrics include quarterly data for each aircraft subfleet (such as Boeing 737-300, Airbus A319, etc) for each airline included in the sample. The sample includes most of the major players that comprise the US commercial market, as shown in Table 4.

<table>
<thead>
<tr>
<th>Legacy Carriers</th>
<th>Alaska</th>
<th>Continental</th>
<th>Northwest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alaska</td>
<td>Continental</td>
<td>Northwest</td>
<td></td>
</tr>
<tr>
<td>Aloha</td>
<td>Delta</td>
<td>United</td>
<td></td>
</tr>
<tr>
<td>American</td>
<td>Hawaiian</td>
<td>US Airways</td>
<td></td>
</tr>
<tr>
<td>America West</td>
<td>Midwest</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Major Low-Cost Carriers</th>
<th>AirTran</th>
<th>JetBlue</th>
<th>Southwest</th>
</tr>
</thead>
<tbody>
<tr>
<td>AirTran</td>
<td>JetBlue</td>
<td>Southwest</td>
<td></td>
</tr>
<tr>
<td>ATA</td>
<td>Frontier</td>
<td>Spirit</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>“Regional” Airlines</th>
<th>American Eagle</th>
<th>Pinnacle</th>
<th>SkyWest</th>
</tr>
</thead>
<tbody>
<tr>
<td>(&lt;100 Seats / aircraft)</td>
<td>American Eagle</td>
<td>Pinnacle</td>
<td>SkyWest</td>
</tr>
<tr>
<td>Comair</td>
<td>Mesa</td>
<td>Air Wisconsin</td>
<td></td>
</tr>
<tr>
<td>ExpressJet</td>
<td>Mesaba</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Airlines in Sample, by Category

This sample is diverse in equipment types, fleet counts, and styles of utilization. The data source is time series panel data with about 8000 observations, of which about 2500 are complete. The most commonly missing variable in dataset completeness has been network size (points served), for which certain values could not be obtained. Other values were manually corrected or deleted by the author if obviously false. Some included erratic numbers in connection with extremely small operational levels.

The Form 41 data source includes fleet specific direct operating costs (DC) such as pilot wages, fuel, etc, but does not include indirect costs (IC) like advertising and
headquarters salaries. Indirect costs are only available at the aggregate corporate level (summing the fleets for each firm.) While corporate level Total Cost data are available on Form 41 Schedule P-12, subfleet level data must be simulated based on a linkage assuming that overhead costs (= TC – Direct Costs) are distributed uniformly across the firm’s produced units. For now, we say overhead costs are distributed evenly across Aircraft Miles. This allows us to have realistic total operating costs (TC) for each fleet, in addition to the detailed operational data.

A representative data observation is described in Table 5, and the data are summarized in Table 6.

<table>
<thead>
<tr>
<th>Airline</th>
<th>Aircraft Type</th>
<th>Quarter</th>
<th>Departures</th>
<th>Seats Available</th>
<th>Avg Length of Haul</th>
<th>Seat Capacity</th>
<th>Available Seat Miles</th>
<th>Gallons of Fuel</th>
<th>Crew Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Airlines</td>
<td>MD-80</td>
<td>Q2 2003</td>
<td>112,681</td>
<td>17,352,000</td>
<td>856 mi</td>
<td>129</td>
<td>12.94 Billion</td>
<td>260.5 Million</td>
<td>$150.5 Million</td>
</tr>
</tbody>
</table>

Table 5. A Representative Data Observation from the Sample

<table>
<thead>
<tr>
<th>N=2660</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Weighted Mean (ASMs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FleetMi</td>
<td>16,249,217</td>
<td>16,466,421</td>
<td>7,944</td>
<td>114,559,245</td>
<td>31,684,496</td>
</tr>
<tr>
<td>ACPoins</td>
<td>34.444</td>
<td>24.651</td>
<td>2</td>
<td>132</td>
<td>43,037</td>
</tr>
<tr>
<td>ACSeats</td>
<td>158.019</td>
<td>76.622</td>
<td>30</td>
<td>430</td>
<td>178.210</td>
</tr>
<tr>
<td>ACCabinArea</td>
<td>124.168</td>
<td>80.460</td>
<td>27.195</td>
<td>380</td>
<td>143.502</td>
</tr>
<tr>
<td>ACFlightDist</td>
<td>1357.54</td>
<td>1122.62</td>
<td>6570.79</td>
<td>1605.851</td>
<td></td>
</tr>
<tr>
<td>ACPIlotWage</td>
<td>481.70</td>
<td>348.37</td>
<td>2.01</td>
<td>519.923</td>
<td></td>
</tr>
<tr>
<td>FuelPrice</td>
<td>36.16</td>
<td>12.867</td>
<td>18.494</td>
<td>70.325</td>
<td>36.522</td>
</tr>
<tr>
<td>CapMPrice</td>
<td>5293.75</td>
<td>2717.54</td>
<td>863.15</td>
<td>50530.4</td>
<td>5816.6</td>
</tr>
<tr>
<td>TechAge</td>
<td>14.668</td>
<td>8.702</td>
<td>1</td>
<td>40</td>
<td>15.205</td>
</tr>
<tr>
<td>FirmASMs</td>
<td>20355210153</td>
<td>14072344362</td>
<td>75669086</td>
<td>45920502143</td>
<td>27411965363</td>
</tr>
<tr>
<td>FirmPoints</td>
<td>111.67</td>
<td>41.00</td>
<td>5</td>
<td>163</td>
<td>121.12</td>
</tr>
<tr>
<td>YearCount</td>
<td>7.44</td>
<td>2.24</td>
<td>4</td>
<td>11</td>
<td>7.478</td>
</tr>
</tbody>
</table>

Table 6. Data summary, with Arithmetic and Weighted Means

The dataset was assembled in Microsoft Excel using raw data from the US Department of Transportation Bureau of Transportation Statistics online sources (www.bts.gov). Panels were merged into quarterly, fleet specific numbers for operations.
and direct costs of operation from Form 41 T2, T100. Lastly, the data were fed into SAS 9.1.3 for further data manipulation and econometrics processing.

IV.B  Model Regression Variables

For both the first-order translog and translog cost models, the dependent variable was a calculation of Total Operating Costs. This was the sum of itemized direct costs and synthetically disaggregated indirect costs, by fleet, in 1997 dollars.

The 13 explanatory variables are:

- Fleet Aircraft Miles
- Fleet Network Points
- Seat Capacity
- Cabin Area
- Mean Flight Distance
- Cockpit Wage
- Oil Price
- Capital, Materials Price
- Equipment Design Age
- Firm Production Quantity
- Firm Network Size
- Year

(Firm identity.)

Please see Appendix II.E: Variable Definitions for detailed explanations of the variables.

Cockpit wage is included in the regressions because, as noted by Wei and Hansen (2003), “pilots flying larger aircraft are paid more than those flying smaller aircraft, and these ‘diseconomies’ of pilot cost offset the ‘technical’ economies of aircraft size…” Therefore, I explicitly itemize the cost of the pilot factor in each fleet, from the item “Pilots and Copilots” of quarterly operational costs on Form 41. This allows us to treat pilot wages as essentially exogenous by entering the data. This removes bias from the coefficients on operational choices (aircraft types and statistics), removing potential bias of the estimators that were hiding pilot wage.

Is it reasonable to treat pilot wages as an exogenous correction of the model, but aircraft least rates as implicit? The rationale is the liquidity of aircraft, compared to the pools of airline pilots, which are fundamentally illiquid due primarily to unionization. But aircraft cannot unionize. From the data, similar airlines display stark wage
disparities, both over time within firms, and among firms during constant time. This persistent illiquidity of factors is presumed not to exist with the capital stock.

A key data omission to mention again is aircraft capital costs. I seek to omit aircraft costs by assuming that depreciation and rent parameters are economic equals.

\[ Rent = Amortization = Economic Depreciation of Equipment. \]  

While maintenance costs are explicitly available in Form 41, the purchase, ownership opportunity costs, or lease arrangements are not especially amenable to direct comparison. This becomes a serious problem if equipment cost items are only partially specified in the model, leading to problems with the adding-up restriction across many diverse firms with different equipment procurement or repair behaviors. Confusing the bookkeeping, a firm could try to maximize depreciation booked on such assets for tax purposes, or engage in accounting logic that deviates from economic realities. Rent may, consequently, be a problematic variable in our cost function. By instead focusing on operational parameters, as in the Morrison and Swan theory, we build equipment costs into the intercept of the equation. Or, in our case, it could form an amorphous “ready aircraft” commodity that we can build into Capital-Materials pricing. Firm identity variables might capture any remaining special nature of aircraft cost. This is based on the assumption of a liquid secondhand market for aircraft and continuous operating decisions on repairs vs. replacement of new aircraft.
V. Estimation Results

The cost model implies an industry-wide cost model and, therefore, an industry-wide production function. The industry’s cost and production data are put into a model. The model yields estimated coefficients for the cost elasticities of various factors and styles of production. The degree of statistical confirmation of these estimates is reflected in the closeness of fitment of cost outcome estimation to real cost data – i.e., the $R^2$ for the estimated linear regressions.

By estimating a disaggregated model, the hope is that the most appropriate estimate takes place, which represents firm costs with respect to the type of machines employed. These machines represent, over time, variable decision-management units that might be subject to actual decisions or variations of production based on variables (such as seats per aircraft) dealt with here. If more seats are installed in an aircraft, this thesis claims to model the cost elasticity – without, and then with second-order interactions – of that type of decision upon existing fleets. Therefore, the empirical model is historically meaningful, and also has something to say about the effects might be of actions taken by market actors. So, the model could reasonably be used for simulation.

The restricted model is:

\[
\ln[\text{Total Cost}] = \\
\beta_0 + \beta_1\ln[\text{Fleet Aircraft Miles}] + \beta_3\ln[\text{Fleet points served}] + \beta_3[\text{Seat Capacity}] + \\
\beta_4\ln[\text{Cabin Area}] + \beta_5[\text{Mean Flight Distance}] + \beta_6[\text{Cockpit Wage}] + \\
\beta_7[\text{Oil Price}] + \beta_8[\text{Capital-Materials Price}] + \beta_9[\text{Design Age}] + \beta_{10}[\text{Firm Quantity}] + \\
\beta_{11}[\text{Firm Scope}] + \beta_{12}[\text{Year}] + \beta[\text{firm identities}].
\]
The less restricted, translog-style model is:

\[
\ln[\text{Total Cost}] = \alpha_0 + \gamma Y + \sum_i \beta_i \ln W_i + \sum_i \phi_i \ln Z_i + \frac{1}{2} \delta_{yy} (\ln Y)^2 + \\
\frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln W_i \ln W_j + \frac{1}{2} \sum_i \sum_j \psi_{ij} \ln Z_i \ln Z_j + \\
\sum_i \rho_i \ln Y \ln W_i + \sum_i \mu_i \ln Y \ln Z_i + \sum_i \sum_j \lambda_{ij} \ln W_i \ln Z_j + \sum_i \alpha_i \text{Firms}
\]

(plus restrictions in Eq. 10)

V.A First-order, Restricted Specification

<table>
<thead>
<tr>
<th>Variable</th>
<th>(\beta)</th>
<th>S Error</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-2.794</td>
<td>.4156</td>
<td>-6.72</td>
</tr>
<tr>
<td>Log ACMiles</td>
<td>.8610</td>
<td>.0069</td>
<td>124.89</td>
</tr>
<tr>
<td>Log ACPPointsSrv</td>
<td>.1562</td>
<td>.0103</td>
<td>15.16</td>
</tr>
<tr>
<td>Log ACSeats</td>
<td>-.5126</td>
<td>.0370</td>
<td>-13.86</td>
</tr>
<tr>
<td>Log ACCabinArea</td>
<td>.5944</td>
<td>.0328</td>
<td>18.15</td>
</tr>
<tr>
<td>Log ACFlightDist</td>
<td>-.0994</td>
<td>.0130</td>
<td>7.64</td>
</tr>
<tr>
<td>Log ACPilotWage</td>
<td>.0575</td>
<td>.0084</td>
<td>6.84</td>
</tr>
<tr>
<td>Log OilSpot</td>
<td>.2037</td>
<td>.0128</td>
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</tr>
<tr>
<td>Log C-MPrice</td>
<td>.7387</td>
<td>.0147</td>
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</tr>
<tr>
<td>ACtechAge</td>
<td>.0043</td>
<td>.0004</td>
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</tr>
<tr>
<td>Log FirmASMs</td>
<td>.0169</td>
<td>.0193</td>
<td>0.88</td>
</tr>
<tr>
<td>Log FirmPointsSrv</td>
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<td>.0190</td>
<td>0.03</td>
</tr>
<tr>
<td>YearCount</td>
<td>-.0009</td>
<td>.0023</td>
<td>-0.39</td>
</tr>
</tbody>
</table>

Fixed Effects Firm Dummies (JetBlue as base)

<table>
<thead>
<tr>
<th>Firm</th>
<th>(\beta)</th>
<th>(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>-.1458</td>
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</tr>
<tr>
<td>DL</td>
<td>-.1365</td>
<td>-2.61</td>
</tr>
<tr>
<td>NW</td>
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<td>-2.43</td>
</tr>
<tr>
<td>WN</td>
<td>.0139</td>
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<tr>
<td>US</td>
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<td>-.35</td>
</tr>
<tr>
<td>UA</td>
<td>-.1023</td>
<td>-1.90</td>
</tr>
<tr>
<td>CO</td>
<td>-.1400</td>
<td>-2.95</td>
</tr>
<tr>
<td>HP</td>
<td>-.0364</td>
<td>-1.01</td>
</tr>
<tr>
<td>TZ</td>
<td>.3429</td>
<td>9.61</td>
</tr>
<tr>
<td>B6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FL</td>
<td>-.0047</td>
<td>-0.13</td>
</tr>
<tr>
<td>F9</td>
<td>-.1118</td>
<td>-1.26</td>
</tr>
<tr>
<td>YV</td>
<td>-.0957</td>
<td>-2.55</td>
</tr>
<tr>
<td>XE</td>
<td>-.1479</td>
<td>-2.71</td>
</tr>
<tr>
<td>XJ</td>
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<td>-2.98</td>
</tr>
<tr>
<td>ZW</td>
<td>-.1667</td>
<td>-2.66</td>
</tr>
<tr>
<td>OO</td>
<td>-.1979</td>
<td>-4.65</td>
</tr>
<tr>
<td>OH</td>
<td>.3390</td>
<td>6.07</td>
</tr>
<tr>
<td>MQ</td>
<td>-.1201</td>
<td>-2.24</td>
</tr>
<tr>
<td>YX</td>
<td>-.1606</td>
<td>-3.51</td>
</tr>
</tbody>
</table>

Table 7. Subfleet First Order Total Cost Regression, 2000-2007 (1997 dollars):

The first order translog model appears to be a good specification (according to Figure 3) and enjoys easy interpretability. The coefficients represent fleet-level cost
elasticities. Holding all else equal, this regression allows us to interpret the economies of density and scope, cost effects of increasing aircraft size, seat density, or oil costs and cockpit crew wages. While prohibiting interactions among the coefficients may be a bit simplistic, this regression does give some clear results that can be reported.

As shown in Table 8, cost elasticity of scale is approximately one (CRS is not rejected). In practical terms, this means that a firm could expand a particular fleet with constant unit costs. So, this means that firms have exploited available economies of scale, and attained efficient scale. This is not unexpected, considering the contestability of markets asserted.

The cost elasticity of aircraft scale is a bit new, but also well worth examination. The physical size of aircraft should engender greater efficiency, at least to a point. The coefficient on cabin area suggests that total costs increase with cabin area with an elasticity of approximately 0.594 (SE 0.33). This is so after controlling for seat count and operational style, including length of haul. Hence, aircraft have increasing returns to scale. See Table 8 (below) for this breakdown.

Returns to scale in terms of the firm are measured to be insignificant. The coefficient on FirmASMs and FirmPointsSrv suggest that attributes of the overall firm do not directly impact costs accrued to the fleets, which is my topic of concern. Therefore the firm size, irrespective of fleets (which themselves have sizes), appears to carry little or no predictive power over costs. We might say that there are no economies, positive or negative, associated with firm size after specifying fleet size. That’s surprising, because the firms’ fixed costs are allocated into the fleet costs, so we would expect economies of firm size to be visible. Also, no time trend is visible (all cost numbers are inflation-adjusted by CPI-Urban).
First Order Restricted Fixed Effects: Residuals

Figure 2. First Order Restricted Fixed Effects: Residuals

Fixed Effects Restricted Model:
Cost Elasticities of Scale and Scope

<table>
<thead>
<tr>
<th></th>
<th>Coeffs</th>
<th>SE</th>
</tr>
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<tbody>
<tr>
<td>Fleet Miles</td>
<td>0.861</td>
<td>.070</td>
</tr>
<tr>
<td>Fleet Scope</td>
<td>0.156</td>
<td>.010</td>
</tr>
<tr>
<td>=&gt; Fleet Scale</td>
<td>=&gt; 1.017</td>
<td>=&gt; .080</td>
</tr>
<tr>
<td>Aircraft Seats</td>
<td>-.513</td>
<td>.037</td>
</tr>
<tr>
<td>Aircraft Scope</td>
<td>.594</td>
<td>.033</td>
</tr>
<tr>
<td>=&gt; Aircraft Scale(^{11})</td>
<td>=&gt; 0.082</td>
<td>=&gt; .070</td>
</tr>
</tbody>
</table>

Table 8. Cost Elasticities of Scale and Scope

Note: "**Fleet Scale**" refers to economies from more or wider operations of a given fleet; "**Aircraft Scale**" refers to larger or smaller aircraft.

\(^{11}\) This appears to be a victim of model endogeneity. While increased seat count may be associated with less costly operators, any great increase in seat count should bring higher costs. Evidently, Cabin Area accounts for fleet scale while Seat Count is contingent upon it. The two do not interact properly for a scale measurement.
### V.B Translog Style Regression Results

**Equation**

<table>
<thead>
<tr>
<th>Equation</th>
<th>N</th>
<th>DF Model</th>
<th>DF Error</th>
<th>SSE</th>
<th>Root MSE</th>
<th>R-Square</th>
<th>D-Watson</th>
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<tr>
<td>Log_TC</td>
<td>2660</td>
<td>95.78</td>
<td>2564</td>
<td>29.2993</td>
<td>0.1069</td>
<td>0.9899</td>
<td>0.9508</td>
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<tr>
<td>ShareFuel</td>
<td>5.611</td>
<td>2564</td>
<td>1.6670</td>
<td>0.0251</td>
<td>0.8429</td>
<td>0.4728</td>
<td></td>
</tr>
<tr>
<td>SharePilot</td>
<td>5.611</td>
<td>2564</td>
<td>0.5977</td>
<td>0.0150</td>
<td>0.7244</td>
<td>0.8730</td>
<td></td>
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**log TotalCost =**

<table>
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<tr>
<th>Variable</th>
<th>β</th>
<th>S Error</th>
<th>t</th>
<th>Variable</th>
<th>β</th>
<th>S Error</th>
<th>t</th>
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<tr>
<td>Intercept</td>
<td>3.322</td>
<td>2.423</td>
<td>1.37</td>
<td>(3) FuelPrice</td>
<td>0.269</td>
<td>0.014</td>
<td>18.77</td>
</tr>
<tr>
<td>(1) Log ACMiles</td>
<td>-1.043</td>
<td>0.122</td>
<td>-8.53</td>
<td>(8) Log C-MPrice</td>
<td>0.492</td>
<td>0.015</td>
<td>32.07</td>
</tr>
<tr>
<td>(2) Log ACMiles</td>
<td>2.477</td>
<td>0.181</td>
<td>13.70</td>
<td>(9) AC TechAge</td>
<td>0.005</td>
<td>0.012</td>
<td>0.41</td>
</tr>
<tr>
<td>(3) Log ACMiles</td>
<td>-6.768</td>
<td>0.751</td>
<td>-9.02</td>
<td>(10) Log FirmASMs</td>
<td>0.158</td>
<td>0.251</td>
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<tr>
<td>(4) Log AC CabinArea</td>
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<td>0.770</td>
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<td>(11) Log FirmPtsSrv</td>
<td>-640</td>
<td>0.362</td>
<td>-1.77</td>
</tr>
<tr>
<td>(5) Log AC FlightDist</td>
<td>0.146</td>
<td>0.305</td>
<td>0.48</td>
<td>(12) Year Count</td>
<td>-0.953</td>
<td>0.030</td>
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<td>(6) Log AC PilotWage</td>
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<td>30.61</td>
<td>(Log AC Miles)^2</td>
<td>0.122</td>
<td>0.006</td>
<td>19.57</td>
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<tr>
<td>(7) Log Fuel Price</td>
<td>0.269</td>
<td>0.014</td>
<td>18.77</td>
<td>(Log AC PointsSrv)^2</td>
<td>0.266</td>
<td>0.021</td>
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<tr>
<td>(8) Log C-M Price</td>
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<td>0.015</td>
<td>32.07</td>
<td>(Log AC Seats)^2</td>
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<td>(9) AC Tech Age</td>
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<td>0.012</td>
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<td>(ACCabin Area)^2</td>
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<td>(Log AC Flight Dist)^2</td>
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<td>0.044</td>
<td>-1.15</td>
</tr>
<tr>
<td>(11) Log Firm PtsSrv</td>
<td>-640</td>
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<td>-1.77</td>
<td>(Log AC Pilot Wage)^2</td>
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<td>0.001</td>
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<tr>
<td>(12) Year Count</td>
<td>-0.953</td>
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<td>-1.75</td>
<td>(Log AC Price)^2</td>
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<td>(ACTech Age)^2</td>
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<td>0.000</td>
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<td>(Log Firm ASMs)^2</td>
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<tr>
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<td>0.071</td>
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<tr>
<td>(4)^2</td>
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<td>0.001</td>
<td>-10.84</td>
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<td>7.01</td>
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<tr>
<td>(5)^2</td>
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<td>(5)^3</td>
<td>0.046</td>
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<tr>
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<td>-0.002</td>
<td>0.001</td>
<td>-3.31</td>
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<tr>
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<td>0.000</td>
<td>11.98</td>
<td>(7)^3</td>
<td>0.100</td>
<td>0.014</td>
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<td>(8)^2</td>
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<td>0.006</td>
<td>0.001</td>
<td>6.98</td>
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<tr>
<td>(9)^2</td>
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<td>0.001</td>
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<td>(10)^2</td>
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<td>6.98</td>
<td>(11)^3</td>
<td>0.006</td>
<td>0.001</td>
<td>6.98</td>
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<td>(12)^2</td>
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<td>0.001</td>
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<td>0.006</td>
<td>0.001</td>
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</tr>
<tr>
<td>(15)^2</td>
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<td>0.022</td>
<td>-4.55</td>
<td>(15)^3</td>
<td>-0.009</td>
<td>0.003</td>
<td>-2.79</td>
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Table 9 Continued: Subfleet Translog Style Total Cost Regression, 2000-2007 (1997 dollars)

<table>
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<tr>
<th>Firm</th>
<th>$\beta$</th>
<th>$t$</th>
<th>Firm</th>
<th>$\beta$</th>
<th>$t$</th>
<th>Firm</th>
<th>$\beta$</th>
<th>$t$</th>
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</thead>
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<tr>
<td>AA</td>
<td>0.134</td>
<td>2.36</td>
<td>TZ</td>
<td>-0.019</td>
<td>-0.63</td>
<td>OH</td>
<td>0.284</td>
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<td>DL</td>
<td>0.149</td>
<td>2.83</td>
<td>FL</td>
<td>-0.009</td>
<td>0.14</td>
<td>MQ</td>
<td>-0.088</td>
<td>-1.49</td>
</tr>
<tr>
<td>US</td>
<td>0.141</td>
<td>3.90</td>
<td>F9</td>
<td>-0.108</td>
<td>-1.93</td>
<td>AS</td>
<td>0.013</td>
<td>0.43</td>
</tr>
<tr>
<td>UA</td>
<td>0.177</td>
<td>3.34</td>
<td>YV</td>
<td>-0.175</td>
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<td>FL</td>
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</tr>
<tr>
<td>CO</td>
<td>0.128</td>
<td>2.92</td>
<td>XE</td>
<td>-0.108</td>
<td>-1.93</td>
<td>YX</td>
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<td>-1.49</td>
</tr>
<tr>
<td>HP</td>
<td>0.034</td>
<td>1.19</td>
<td>XJ</td>
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<td>NW</td>
<td>0.116</td>
<td>2.54</td>
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<tr>
<td>B6 (base)</td>
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<td>1.19</td>
<td>OO</td>
<td>-0.224</td>
<td>-4.85</td>
<td>WN</td>
<td>-0.096</td>
<td>-2.37</td>
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</tbody>
</table>

Fig 3. Unrestricted Translog Style Regression: Residuals

Translog style regression results (in Table 9) can be at first difficult to interpret since second-order effects (particularly with output) may obscure the first-order effects of the coefficients. For example, the inclusion of quadratics decomposes the first order interpretability we had in the first-order translog function. While CCT reported near-exact matches between first-order translog and first-order coefficients from the full translog regressions, it does not occur here.
The most interesting aspects of the translog results occur in observing the sign and interpretation of cross-elasticities in Table 9. For example, the cost share of fuel was positively correlated with older aircraft designs. Knowing this can lead to more realistic analysis and simulation.

The interaction of output (*AC Miles*) with inputs or characteristics also yielded interesting information. For bigger fleet outputs, the cost efficiency has been improving over time. With a coefficient of .006 (.002), as each year will have passed, growth of *AC Miles* will have had a cost elasticity closer to unity. The cost economy available from growing a fleet has diminished over time.

**V.C  Returns to Scope, Scale**

*  **i. fleets**

The initial first-order translog total cost model suggests that fleets have constant returns to fleet scale (*AircraftMi*), with cost elasticity of density (SE) of 0.861 (.007), and cost elasticity of fleet network size [scope] .156 (.010). Cost elasticity of total scale may be unity, and CRS cannot be rejected. The interpretation is, it appears that most firms operate their fleet at sizes large enough that further cost economies of scale are beyond reach.

As expected, the coefficient on mean *Flight Distance* is negative; as a flight distance doubles, costs will fall by 7.6%. This is because *Aircraft Miles* have remained fixed, and as such, longer flights will be cheaper, with fewer departures, higher average speeds, and better productivity, as this model understands it.

Evidently, costs also increase with the design age of the fleets. Total costs rise by 0.43% (SE .0004) for each year older an aircraft fleet’s mechanical design may be (these data were compiled by the author). The *YearCount* dummy (measuring annual changes in cost efficiency) is insignificant in itself in the TL style model, ignoring the issue of fleet design aging.
ii. Firms

In the context of the fleet costs, the model does question whether firm total output quantity (ASMs) or firm geographic scope (PointsSrv, for the whole company) affect fleet costs. It appears that firm output quantity has no impact on a fleet’s costs.

The economy of scope (additional airports served) suggests it is quite cheap for firms to expand a given fleet’s geographical “footprint,” with an elasticity of only 0.156 (SE .01) for adding new network points while maintaining service quantity. This suggests an existing fleet could serve an additional city, or group of cities, rather cheaply. This, in turn, aids the argument that the overall market is contestable.

iii. Aircraft

With respect to physical aircraft size, IRS are seen. The coefficient on ACCabinArea suggests that as we double aircraft interior space, costs rise by 59%. This suggests positive economies of scale.

While the coefficient on ACSeats appears to suggest more seats result in actual cheaper flying, this is only an artifact of the context of fixed cabin size. This is probably because higher density seating configurations may be correlated with generally lower-cost firms, somehow missed despite our control for firm identities. Correspondingly, if a fleet within a firm were higher density, there may be unobserved traits about such a fleet that make it cheaper to operate (perhaps being an experimental subsidiary airline such as Metrojet, Song or TED, whose operations are included in this study under their corporate parents). This might explain, in part, why we see costs apparently falling as seat counts rise. An alternative model, with seating density in place of count, also produced this.

V.D Energy Cost Share and Response to Perturbation

The sum of cost shares of Pilot Wages, Oil and Capital-Materials were constrained to unity in the regression using SUR in PROC SYSLIN (for the restricted model), and
later in PROC MODEL (for the full TL style model). Their computed cost shares are 5.8%, 20.4% and 73.9% respectively. It should be noted again that the first-order translog functional form does not allow these cost shares to vary across the diversity of our sample, or even across time. A more flexible cost function specification allow them to move within their constraints.

The unrestricted translog style results show that energy cost share varies negatively with output scope and density; negatively with cabin area; and positively with flight distance. This suggests that larger aircraft have lower energy cost shares, which seems logical.

In an unexpected energy shock, the returns to “green technology” rise would more steeply than anticipated, providing a windfall for those who invested in efficient machinery vs. their own forecasts. Conversely, those with the oldest technology will face an unexpected competitive problem. Typically, in a clearing market with risk neutral entities, we would expect that:

\[ \Sigma(\text{costs new / green technology}) = \Sigma(\text{costs of old technology}); \]
\[ \mathbb{E}[\text{marginal rent on new aircraft}] = \mathbb{E}[\text{added fuel cost in old aircraft}]. \]

Yet, we can see from our translog style results that apparently old machinery does not unduly penalize their operators. As oil prices rise, the coefficient on the interaction between log \((\text{Oil Price})\) and \text{Tech Age} was not significant in this study. Total costs of the airline are not exacerbated by the presence of older equipment, or minimized by the use of new equipment. Unexpected shifts in fuel cost seem to maintain the pure cost equivalence I assumed between new and old aircraft. However, it is worth remembering that older aircraft are more costly to run, according to the unrestricted model’s \text{TechAge} coefficient.

The movements of oil prices are exogenous to the airline industry. But rolling expectations of oil prices may guide fleet procurement. From the data and the cost function measured here, we can describe what total costs \textit{would have been} for the sample group of firms, between 2000-2007, at various fuel prices. This does not include
feasibility conditions or capacity readjustments. A trend toward energy efficiency can be seen, as, although output has remained relatively constant, fuel burn has fallen and, hence, the industry is more robust to an oil price spike in 2007 than it was in 2000. The cost function is undefined at oil price = $0, which is why the curve appears to join the origin. This graphic (Fig. 4) uses the translog style cost function from this project to extrapolate various costs for each observation in the data. The projected costs are then summed across the industry to get a total.

Similar to literature such as Thompson (2006), we can find energy cost share by Shephard’s Lemma:

\[
FUEL = \frac{d(TOTAL\_COST)}{d(FUEL\_PRICE)};
\]

\[
d\ln(Total\ Cost)\bigg|\frac{d\ln(Fuel\ Price)}{d\ln(Total\ Cost)}\bigg| = d\frac{TC}{FP} \cdot \frac{FP}{TC} = \epsilon(Cost : Fuel\ Price);
\]

It follows that:

\[
Share\_Energy = \frac{d\ln(TOTAL\_COST)}{d\ln(FUEL\_PRICE)}.
\]

(14)

For the first-order translog result in V.A, the fixed effects model result gives a cost / fuel price elasticity of 0.2037 (SE .0128). If oil prices increase 10%, we would expect overall operating costs to increase by 2.04%, for each fleet.

For the translog style function in V.B, fuel share could be computed as follows:

\[
\frac{d\ln C}{d\ln w_i} = \beta_i + 2\beta_i(\ln w_i) + \sum_{j=1}^{k} \beta_{i j} \ln w_j.
\]

(15)
Using (14), we can plug in the estimated translog coefficients (5% sig. threshold),

$$\frac{\partial \ln TC}{\partial \ln W_F} = S_F =$$

\[.113 + .113 \cdot (\ln_{OilSpot}) - .005 \cdot (\ln_{ACMiles}) + -.046 \cdot (\ln_{PointsSrv}) + 0 \cdot (\ln_{Seats}) -0.228 \cdot (\ln_{CabinArea}) +.104 \cdot (\ln_{Stage}) + -.007 \cdot (\ln_{CockpitWage}) - .027 \cdot (\ln_{CM_TPrice}) + 0 \cdot (TechAge) + 0 \cdot (\ln_{FirmASMs}) + 0 \cdot (\ln_{FirmPoints}) - 0 \cdot (YearCount)\]

As has been noted by other authors, in the full translog model, each fleet of each firm had its own factor price elasticity. These could be aggregated by common output (ASMs) and then reported as industry-wide metrics (or as empirical means). This is done in Table 10.

Empirical means are reported as compared to first-order restricted translog results. Here, “empirical mean” of the full translog is meant as the cost elasticity of the input price, output level, or state of nature variable.

The derivative of log_TotalCost with respect to each variable can be seen by reusing equation (16) and weighting the resulting cost elasticity by the ASM production in that fleet, to get an aggregate industry statistic (an empirical mean). Figure 4 shows a useful projection of total industry costs as a function of oil prices, using the empirical model.
<table>
<thead>
<tr>
<th></th>
<th>First-Order, Restricted Model</th>
<th>Full TL Style Sample Mean Elasticities</th>
<th>Full TL Style Empirical, ASM-weighted Elasticities</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(coeff)</td>
<td>(SE)</td>
<td></td>
</tr>
<tr>
<td>Log _ACMiles</td>
<td>0.861</td>
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<td>0.972</td>
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<td>Log _ACPointsSrv</td>
<td>0.156</td>
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<td>Log ACSeats</td>
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<td>Log FirmASMs</td>
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<td>YearCount</td>
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</table>


**Translog Industry Cost Extrapolation:**

**Oil Price Shock**

Figure 4. Sample Group Simulated Total Costs, Varying with Oil Price / Bbl
VI. Conclusion

A Modified Empirical Framework

This thesis has framed and measured a fleet-level cost model of production machines, as used by firms in the US airline industry. This project focuses primarily on the benefits of careful treatment of aggregation. Whereas other studies have used firm-level data only, or fleet-level data with indirect costs only, this work uses enjoys fleet-level data in a total-cost context, partially thanks to a synthetic disaggregation routine. This could be convenient for industrial simulation, because consumers pay total costs.

By placing all production (large and small aircraft, companies, and/or fleet sizes) over a panel of time periods, and putting them in one unified model, this project deploys the available analytical tools to model the industry’s costs as they are affected by operational decisions. These decisions have effects across three levels of aggregation: (1) growth or shrinkage of an individual fleet, (2) the total firm as a whole; or (3) the nature of aircraft themselves, with respect to size, number of seats, and design age. These issues were all specified in the restricted and flexible TL models, and results were obtained.

In summary, the study finds that fleet economies of scale are not demonstrably different from CRS. However, there are economies of density, which has implications for regulators. Firm size, given the magnitude of fleet production, has no cost impact. Finally, bigger aircraft enjoy economies of scale, while increased density of seating is apparently associated with lower-cost producers, even while controlling for firm identity.

Aggregation framework

This thesis examines economies of scale across three simultaneous levels of aggregation. Regarding fleets, each airline company uses a number of different aircraft types, as reported in the data. By modeling fleets (squads of like aircraft) as decision-management units (DMUs), aggregation bias is reduced. The practical relevance is also improved; a company manager or regulator, rather than contemplating a uniform expansion of overall output, will more often consider increasing or decreasing each fleet’s activities independently.
The cost function being modeled describes a hypothetical fleet of aircraft with parameters allowing typical situations – numbers of seats, total aircraft miles, number of airports, etc. The results should therefore be a forecast of total costs given reasonable inputs that are readily known at the outset of any planning process. These are used for a preliminary industry cost projection across fuel cost perturbations (Fig. 4).

When allowing flexibility of network size, CRS appeared to prevail at the fleet level. However, holding network size constant, economies of density were seen, in terms of additional aircraft miles. The fact this coefficient is being mitigated over time suggests economies of density may be fading away. However, at the aircraft-size level, continuing economies of airframe size were seen, as aircraft get larger. Firm-level returns to scale or density, often the subject of studies such as this, appeared largely irrelevant, after first accounting for the fleets.

Future research might find a compatible revenue function that can interface and provide a more complete industrial model with an econometric basis, to engage in further energy shock simulations.
REFERENCES


Appendix I: Remarks

Appendix I.A: Contestability

An update of research into airline contestable markets assumptions is probably of interest, given that researchers continue to study the industry building upon this result. When a researcher detects economies of service density, the entire premise of contestability (and therefore the premise of one cost function for the industry) becomes questionable. Some further work is needed to determine what is implied by economies of density and scale, in the context of hubs and strategic pricing in the industry. The reader should cautiously interpret the results of a cost function study in terms of its exact meaning within the existing market structure.

Contestability suffers, for example, when barriers to entry exist, behind which an incumbent can rent-seek. Airport presence by one carrier (departures per day) may serve as a barrier to entry for an opposing firm (Berry, 1992). Berry finds that “airport presence is correlated with entry decisions into airline city pair markets and… different firms appear to be heterogeneously suited to serve different markets.” Even with only one carrier present, however, credible threats of entry may serve to keep prices near cost.

An “aggressive pricing” strategy could award the entire grid to one carrier, especially if the carrier has advantageous amounts of capital for pricing defense. The illegality of predatory pricing should limit an incumbent’s powers of strategic retaliation. If, thanks to such laws, carriers did not strategically pursue fare war attrition, then oligopolies are possible. Regardless, the Hendricks logic will devolve into multiple (or single) national or regional hub-spoke networks. Apparently still unexplored would be an equilibrium with contestability between a hub system and “cherry picking” behavior by new entrants.

Excess capacity (when it exists) also aids contestability market-wide in a network industry, since the excess capacity “can be offered at any location” (Hamilton and
Thisse, 1993). As a second-best action, time-misallocated equipment can be deployed to extract whatever revenue exists above variable costs. We can take a degree of satisfaction that an oligopoly of networks will still provide significant contestability.

While each of the specific elements of perfect contestability (the credible threat) is not always met, each is present in some form. While it has been said that “theoretical conditions for perfect contestability certainly are not met in airline markets” (Hurdle, 1989), over time, due to iterative positioning, these time-specific configurations should dissipate toward equilibrium. Surviving carriers’ competitiveness will have been verified by their survival, or a rival would have overlain their routes. In total, this is enough verification to accept contestability for the market in the present work.

Appendix I.B: Aggregation

In this thesis, a method of cost disaggregation is required, because the data set does not include Total Costs assigned to individual fleets. Instead, we have:

<table>
<thead>
<tr>
<th>Aggregation</th>
<th>Data Source Resolution (quarterly)</th>
<th>Aggregate Figures Normally Weighted By:</th>
<th>Aggregate Figures C.C.T. Compatible Reweights:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Costs</td>
<td>Subfleet Totals</td>
<td>Departures</td>
<td>N/A; Full Data</td>
</tr>
<tr>
<td>Indirect Costs</td>
<td>Firm Totals</td>
<td>Departures</td>
<td>AircraftMi</td>
</tr>
<tr>
<td>Revenue</td>
<td>Firm Totals</td>
<td>Departures</td>
<td>Avail Seat Miles</td>
</tr>
</tbody>
</table>

Table 11. Data Source State of Aggregation

Since Total Costs = Direct Operating Costs + Indirect Costs, for each fleet, we must reconstruct those Indirect Costs for each fleet using a routine that is logical with respect to the Composite Commodity Theorem. Here, indirect costs are assigned to the individual fleets according to the Aircraft Miles traveled by each fleet. By doing so, we are able to construct a reasonable Total Cost metric which can form the LHS of the cost function(s) we aim to study.
We can use this logic to assign aggregate firm costs to particular subfleets for which we have no subfleet level cost data (such as with overhead costs). AircraftMi are logical as a weighting basis for overhead costs because costs such as advertising, financial analysis, etc are made more complex on a per departure basis. However, longer departures consume more money in part due to catering, somewhat elevated need for advertising, ad valorem ticket sales costs that may vary with price. If we were to include these ASM-derived overhead costs (the ones we synthetically disaggregated) in an OLS regression at the fleet level, we can then compute total costs for all the subfleets, making the best use of our disaggregated direct costs data source.

Few previous airline cost models have evaluated fleet types without the problems of firm-level aggregation. Convenient data sources often present statistics that are aggregated at the firm level. Most analysts simply use these data. However, as noted by previous authors, “If the aggregate functions... are to be meaningful for economic analysis, the aggregate marginal productivities and aggregate marginal costs should equate, when they exist, to the rates at which the corresponding microfunction changes partially with respect to the components of the aggregate variable involved.” (Shephard, 1970). Aggregation prior to statistical analysis of the operations could result in mis-measured cost functions, unless the author is extremely careful about what is meant by the coefficients. Aggregated regressions would appear to analyze “aggregate” machines with no specific interpretation of outputs. In reality, airlines use multiple types of equipment that may respond differently to changes in cost components like wages or fuel prices.

Theoretically, as shown by the Composite Commodity Theorem, cost and production rates for individual classes of machinery should be contained in their own special observations, directly comparing machine vs. machine to elucidate the mechanical truths of transport machinery, rather than guess at such processes through firm data. While estimation of costs based on corporate aggregates (by departures, say) will give some kind of result, the conceptual basis for their use is unclear. What is a firm’s length of haul? The precise definition might more usefully be given in terms of the smaller machines that must exist in the firm. In sales, the “firm” as machine should be aggregated in terms of its sale unit product, the available seat mile. Aggregations (or,
alternatively, *disaggregations* or *accounting cost assignments*) could be made on the basis of product. Or, the assignment here based on *Aircraft Miles* makes the claim that this is a sensible method of *cost disaggregation* or *cost assignment*.

Fortunately, there seems to be a shortcut to “rebuild” corporate level figures so they will reveal the disaggregated cost function. It is the Basso method mentioned above. By weighting the corporate data (*ACMiles, Seat Count, Length of Haul*) by *Aircraft Miles*, suddenly our corporate statistics could reveal the original cost function by OLS analysis. Thus, the *product* weighted method of aggregation (or disaggregation, because this experiment can be bidirectional) appears to reveal the functional coefficients of fleets of machines without distortion. Assuming this, we could allocate fleet-wide costs on an *Aircraft* wide basis without fear of distorting our coefficients beyond theoretical limits.

Regarding aggregation, Wei and Hansen (2003) deserve praise for approaching a cost function for individual aircraft types. In Swan’s case, each individual *flight* is framed theoretically on its own, the supreme example of a disaggregated model. In CCT, and others, entire firms operations are measured and compared, as totals. But it could be argued that interpretations based on *Length of Haul*, from aggregated statistics, typically will be distorted by an overreliance on departure count. Thus, interpretations of whole-firm data lack the interface with aerodynamic drag, economies of scale in the size of jets, etc that, we argue, would unveil fundamental production and cost functions more visible at the subfleet level.

**Appendix I.C: Variable Definitions**

**Aircraft Miles**¹² – Miles traveled by that aircraft fleet in that quarter.

**Available Seat Miles** – This is an industry standard measure of firm output. It is used here as a control variable for firm scale, when examining the fleets. A positive coefficient means that, for fleets, there is a positive expense associated with a firm being larger.

¹² Note: we cannot include both *ACHours* and *ACMiles* in the same regression because they are largely collinear scale variables. Instead, we can use the parameter *ACSpeed* as a proxy for the total *Hours* spent per mile. This allows us to examine the otherwise inexplicable speed variations, all else equal, which are indicative of congestion. Less congestion should mean faster speeds, holding stage length and other parameters constant. In interpretation, this can later be re-converted into delay minutes.
**Seat Capacity** – Typically, the number of seats on each aircraft of the fleet. If the fleet is uniform, it is an exact figure. If the fleet is irregular, with different seating configurations, this equals (sellable seats / departures).

**Size of Aircraft cabin floors, in m^2** – Floor area can more directly measure the physical exertion of the production, as larger aircraft are moved through the air. Ideally, floor area and seat count can simultaneously identify the cost of flying physically larger aircraft, and the seating density within an aircraft, as separate parameters. Seating density (seats / m^2) varies among our sample, apparently in response to differing market demands on various flight profiles, and perhaps firm identities.13

<table>
<thead>
<tr>
<th></th>
<th>Midwest Airlines Boeing 717</th>
<th>Southwest Airlines 737-300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seat Count</td>
<td>88</td>
<td>137</td>
</tr>
<tr>
<td>Cabin Area (m^2)</td>
<td>73.41</td>
<td>78.30</td>
</tr>
<tr>
<td>Seat Density</td>
<td>1.20 seats / m^2</td>
<td>1.60 seats / m^2</td>
</tr>
<tr>
<td>Product Information</td>
<td>“All First Class”</td>
<td>All Coach</td>
</tr>
</tbody>
</table>

Table 12. Seating Density Example

**Stage Length** – This is average length of haul. For the subfleets, it equals $\frac{\text{Total Aircraft Miles}}{\text{Total Aircraft Departures}}$.

Corporate totals customarily use the same calculation, but in aggregate, this can lead to interpretive confusion as mentioned. Therefore it is best for corporate Stage Length to be defined as, for example,$\sum_{i=1}^{\text{types}} \frac{\text{stage} \times \text{ASMs}}{\text{ASMs All Types}}$.

If done that way, stage will represent the typical length of flight for a given product sold by the airline. Fortunately we have all the disaggregated Stage data so this is not necessary.

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13 This can signify the internal scope of aircraft. Typically, by including only seat count, the cost of the internal density of seats is conflated with the cost of the floor area upon which those seats sit. Both sum to be are components of aircraft Scale. When isolated, unique coefficients for each can be obtained. An increase of seats, on one hand, represents higher density, holding floor area constant. This does not in itself guarantee a larger aircraft. Scope would be represented by floor area alone. The sum of these two parameters represents the valid Returns to Scale in aircraft size. By omitting floor area, previous studies have been vulnerable to biased scale coefficients (typically on “seats”), with bias arising from assumed constant density.
To interpret a coefficient on *Stage*, we must think within the ceterus paribus framework of the model. *Stage* refers to the cost of operating an equal quantity of output with that aircraft type, in flights composed of shorter or longer hauls. Reducing the *stage* would typically drive greater costs because more flights must be performed to reach the same total output. At the limit, flying many one-mile flights would be very expensive, while longer flights would enjoy greater speed and superior unit efficiency (lower costs, holding unit output constant). So the cost coefficient would probably negative.

**Age of aircraft types** – This is used as an instrument for aircraft lease and repair costs. Equals the number of years since aircraft type was introduced to commercial passenger service, in years. It is also proxy for technological quality, or freshness of the design. Rather than build date, this is when the design was first put into service. Data compiled by author.\(^{14}\)

The coefficient on *Tech Age* might be expected to be zero. Older aircraft are cheaper to own, but burn more fuel, which introduces a cost volatility. A clearing and liquid market for aircraft would suggest uniform operating costs (*Rent + Maintenance + Fuel*) regardless of age, given that lease payments or proper amortization should embody the penalties of age and/or disrepair.

**Points Served** – Each sub-fleet serves a number of cities during each year. Taking a May schedule as typical for each year 2000-2007, I referred to the firms’ weekly schedules to count the cities served by each fleet. This allows later calculations of returns to scale, and returns to density, respectively, of those sub-fleets. Ideally, the proper measurement of points served for a fleet is

\[ \sum (Int \times Days) / Days, \]

to give us the average number of points served, per day, for each fleet in a period. The network data here are annual numbers. This is not ideal, but it should be usable.

Points served is used both for the fleets, and their firms. Thus, cost elasticity of firm network size and fleet network size are examined independently in the cost regressions.

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\(^{14}\) Older aircraft designs generally burn more fuel. As compensation, the capital rent they earn is depressed. During an oil price spike, older equipment can be expected to be abnormally costly (and likely unprofitable) to operate. Baumol notes regarding input factors, “in a contestable market, any industry structure must be efficient,” suggesting that cheaper equipment will be used to kill competitors and/or inevitably be bid up in price, such that no equipment will be unreasonably cheap. In the translog specification, the interaction of *fuel price* with *tech age* should capture an expected positive cost elasticity. Aircraft leases cannot be renegotiated at short intervals, so during fuel spikes, we expect to observe cost penalties associated with older, higher fuel burn equipment.

The age of technology is measured as a subtraction of the year of service introduction of that aircraft type, from the period year. This helps to account for widely varying aircraft ages – particularly among fleets. Aircraft of a certain design vintage will have similar parameters from their design – regardless of manufacture date. Equipment characteristics in this model are therefore a combination of their design’s age, floor area and seating capacity. If older-design aircraft do imply higher costs, this might be because of illiquidity or market irregularities. Given risk aversion, operation of older aircraft actually should be cheaper. Exceptions to this may include sales promotions by manufacturers, economies of being well-capitalized firms with ready access to new jets, or so-called “maintenance honeymoons” in which new jets incur near zero maintenance costs, enhancing their operational costs temporarily.
Delay Time – This variable was not used because it reduced interpretability of other coefficients. However, the concept is worth further study. Ceterus paribus extra hours are time expended apparently not producing any output gains; perhaps due to taxi, ground or air traffic delays. Minutes per departure refers to excess time per departure holding Stage constant ceterus paribus, indicative of flow problems or airway congestion.\textsuperscript{15}

Morrison (1984) tells us that higher aircraft speeds are typically associated with higher costs and greater aerodynamic drag. But there is a contrary argument in the analysis of routine operations. Higher operational speeds – as opposed to design speeds – may signify the lack of congestion and holding time, speed being less wasteful of time-sensitive resources: pilots, monthly leases. In some cases, better network speed may allow an additional roundtrip per day for an aircraft. This would lead to cheaper unit outputs.

Utilization – Not used in this study. The number of hours per day an aircraft is used. Firms with greater utilization may be able to produce more output for less cost. However, the marginal revenue may not justify this.

YearCount – Number of years since 1996. So 1997 = 1, 1998 = 2, etc. Integers. The YearCount variable accounts for technological (or productivity) change. We have already normalized all dollar amounts to 1997 constant dollars using CPI-urban metrics. Computer systems, labor saving devices or scheduling methods may improve over time. We would expect this coefficient to be negative, as costs decrease thanks to better productivity. By building CPI neutrality into our costs, we can measure technological improvement for the industry in direct relation to the wider economy: \( \frac{\delta C(\text{airlines})}{\delta C(\text{CPI basket})} \).

US Form 41 direct operating costs, itemized. 2000-2007. This is the source of reported pilot wages, direct flight operational costs, including those paid for fuel, aircraft retention and/or repairs, etc.

Cockpit Wage – Reported pilot pay and benefits for each firm, by sub-fleet, divided by the number of revenue block hours performed by the aircraft. These are equipment and not worker-specific (although typically, there would be 2 to 3 pilots onboard).

Spot Price of Oil and Jet-Fuel Refinement, 2000-2007 –Includes mean oil prices per barrel, quarterly (St. Louis Fed). Oil refinement premiums for Jet-A fuel as reported, annually by the Air Transport Association. Oil is used alone in the regressions.

Capital – Materials Price – This provides the remainder of the cost share not covered by pilot wages and fuel expenditures. Theoretically, it is therefore clear that the three factor shares should sum to one.

\textsuperscript{15} By holding miles constant but letting hours vary, excess time becomes “visible.” We could also specify the cost of extra minutes per flight departure. While it might appear that including Aircraft Hours in the regression would afford the same opportunity, this would be collinear and distracting with our scale variable, Aircraft Miles and our desire to keep the “RTD\_NETWORK” isolated within that one variable.
Average Federal Funds Rate, by quarter – This was used in trial regressions. This, or a time-lag thereof, might control for varying costs of financing due to macroeconomic factors.


Appendix II. SAS Codebook

SAS Regression for First-order translog Cost function with HOD 1 imposed:

```
proc syslin data=trial8 sur;
model log_TC = log_ACMiles log_PointsSrv log_Seats log_CabinArea log_Stage log_CockpitWage log_FuelPricePaid log_CM_TPrice TechAge1 log_FirmASMs log_FirmOpsPointsSrv YearCount

AA DL NW US UA CO HP TZ B6 FL F9 WN YV XE XJ ZW OO OH YX AS;
restrict log_CockpitWage + log_FuelPricePaid + log_CM_TPrice = 1;
run;
```

SAS Regression for Translog Cost function with HOD 1 imposed:

```
title1 'Translog Airline Total Cost';
title2 'Symmetric Model';
proc model data=trial8 outresid trial44;
endogenous ShareF ShareP log_TC;

parms intercept
r1 r2 r3 r4 r5 r6 r7 r8 r9 r10 r11 r12
r0101 r0102 r0103 r0104 r0105 r0106 r0107 r0108 r0109 r0110 r0111 r0112
r0202 r0203 r0204 r0205 r0206 r0207 r0208 r0209 r0210 r0211 r0212
r0303 r0304 r0305 r0306 r0307 r0308 r0309 r0310 r0311 r0312
r0404 r0405 r0406 r0407 r0408 r0409 r0410 r0411 r0412
r0505 r0506 r0507 r0508 r0509 r0510 r0511 r0512
r0606 r0607 r0608 r0609 r0610 r0611 r0612
```

46
log_TC = intercept + (r1*log_ACMiles) + (r2*log_PointsSrv) + (r3*log_Seats) + 
(r4*log_CabinArea) + (r5*log_Stage) + (r6*log_CockpitWage) + (r7*log_OilSpot) + 
(r8*log_CM_TPrice) + (r9*TechAge1) + (r10*log_FirmASMs) + 
(r11*log_FirmOpsPointsSrv) + (r12*YearCount) + 
(r0101*t1) + (r0202*t2) + (r0303*t3) + (r0404*t4) + (r0505*t5) + (r0606*t6) + 
(r0707*t7) + (r0808*t8) + (r0909*t9) + (r1010*t10) + (r1111*t11) + (r1212*t12) + 
(r0102*c0102) + (r0103*c0103) + (r0104*c0104) + (r0105*c0105) + (r0106*c0106) + 
(r0107*c0107) + (r0108*c0108) + (r0109*c0109) + (r0110*c0110) + 
(r0111*c0111) + (r0203*c0203) + (r0204*c0204) + (r0205*c0205) + 
(r0206*c0206) + (r0207*c0207) + (r0208*c0208) + (r0209*c0209) + 
(r0210*c0210) + (r0211*c0211) + (r0212*c0212) + (r0304*c0304) + 
(r0305*c0305) + (r0306*c0306) + (r0307*c0307) + (r0308*c0308) + 
(r0309*c0309) + (r0310*c0310) + (r0311*c0311) + (r0312*c0312) + (r0405*c0405) + 
(r0406*c0406) + (r0407*c0407) + (r0408*c0408) + (r0409*c0409) + (r0410*c0410) + 
(r0411*c0411) + (r0412*c0412) + (r0506*c0506) + (r0507*c0507) + (r0508*c0508) + 
(r0509*c0509) + (r0510*c0510) + (r0511*c0511) + (r0512*c0512) + (r0607*c0607) + 
(r0608*c0608) + (r0609*c0609) + (r0610*c0610) + (r0611*c0611) + (r0612*c0612) + 
(r0708*c0708) + (r0709*c0709) + (r0710*c0710) + (r0711*c0711) + (r0712*c0712) + 
(r0809*c0809) + (r0810*c0810) + (r0811*c0811) + (r0812*c0812) + (r0910*c0910) + 
(r0911*c0911) + (r0912*c0912) + (r1011*c1011) + (r1012*c1012) + (r1112*c1112) + 
(rAA*AA) + (rDL*DL) + (rNW* NW) + (rUS* US) + (rUA* UA) + (rCO* CO) + (rHP* HP) + 
(rTZ*T2) + (rFL* FL) + (rF9* F9) + (rWN* WN) + (rYW* YW) + (rXE* XE) + (rXJ* XJ) + 
(rZW* ZW) + (rOO*OO) + (rOH* OH) + (rMQ*MQ) + (rAS* AS) + (rXY* XY); 

ShareF = (r7 + (r0707*v7) + (r0107*v1) + (r0207*v2) + (r0307*v3) + (r0407*v4) + 
(r0507*v5) + (r0607*v6) + (r0708*v8) + (r0909*v9) + 
(r0110*v11) + (r0712*v12)); 
ShareP = (r6 + (r0606*v6) + (r0106*v1) + (r0206*v2) + (r0306*v3) + (r0406*v4) + 
(r0506*v5) + (r0607*v7) + (r0608*v8) + (r0609*v9) + (r0610*v10) + (r0611*v11) + 
(r0612*v12)); 

fit log_TC ShareF ShareP 
/ outsused = smatrix itsur dw; 
run;quit;
Concavity:

\[
\begin{align*}
au_{0606} &= -1 + \text{ShareP} + (rt6/\text{ShareP}); \\
au_{0607} &= -1 + \text{ShareF} + (rc0607/\text{ShareP}); \\
au_{0608} &= -1 + \text{ShareCM} + (rc0608/\text{ShareP}); \\
au_{0706} &= -1 + \text{ShareP} + (rc0607/\text{ShareF}); \\
au_{0707} &= -1 + \text{ShareF} + (rt7 / \text{ShareF}); \\
au_{0708} &= -1 + \text{ShareCM} + (rc0708/ \text{ShareF}); \\
au_{0806} &= -1 + \text{ShareP} + (rc0608/\text{ShareCM}); \\
au_{0807} &= -1 + \text{ShareF} + (rc0708/ \text{ShareCM}); \\
au_{0808} &= -1 + \text{ShareCM} + (rt8 / \text{ShareCM});
\end{align*}
\]

**proc iml;**

use trial10;
read all var {OBS,
au0606, au0607, au0608,
au0706, au0707, au0708,
au0806, au0807, au0808}
into hess;
start doit;
do i=1 to 2660;
call execute
("dt",left(char(i)),"=det(hess[i,2:4]//hess[i,5:7]//hess[i,8:10]);");
end;
finish doit;
run doit;

start doit;
call execute ("create determ from dt1;");
do i=1 to 2660;
call execute ("append from dt",left(char(i)),";");
end;
finish doit;
run doit;
print dt1;
quit;