Experimental Verification of Heat/Mass Transfer Analogy in Turbulent Separated Flow behind a Backward Facing Step

A THESIS
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

Rajat Mittal

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Master of Science

Dec, 2010
Acknowledgements

I am deeply indebted to Dr. Richard J. Goldstein, my adviser, for his kind guidance, encouragement and support throughout the course of my research. Working with him has been an honor. I am thankful to him for the freedom that he gave me in my research and cannot express my gratitude in words for the understanding he showed in my tough times.

I am also thankful to Dr. Terrence W. Simon for his suggestions and advice during the work. Interaction with him has increased the depth and breadth of my knowledge.

I would also like to thank the extremely co-operative and knowledgeable staff in the ME and EE machine shops. Pat Nelson, especially, helped me tremendously during the setup and troubleshooting phases of the experiment. Mark Erickson in the Tool Crib was always there to provide me with the necessary paraphernalia.

I am thankful to my research group members, Dr. Kaustubh Kulkarni, Dr. Kalyanjit Ghosh, Dan Keena, Federico Fassio, Matt Stinson and Rishabh Srivastava without whom my research would not have been enjoyable. They kept me socially active through interesting discussions and gatherings.

I would also like to thank my friends, Govind, Pushkar, Hullas, Pranav and Tanuj who made my stay in Minneapolis comfortable. They have been like a second family to me away from my home. The fact that I was able to bank on them for the smallest of the things took a lot of pressure off and helped me concentrate on my research work.
I cannot thank my parents and sister back in India enough for their constant encouragement throughout the work. Extended telephonic conversations with them in tough times infused me with more enthusiasm and determination to go ahead and succeed in the endeavors I have undertaken.
Dedication

Dedicated to my parents, Dr. Rajesh K. Mittal and Mrs. Rajni Mittal and my sister Ms. Rimi Mittal
Abstract

Heat and mass transfer from a surface to a stream of fluid are governed by Fourier’s law and Fick’s law respectively. These are mathematical manifestations of the process of diffusion. In the realm of transport processes, the mathematical equations describing the two phenomena can become analogous under certain assumptions and boundary conditions.

From an engineering perspective, it is difficult to measure heat transfer coefficients in separated flows because of high spatial thermal gradients and the intrusive nature of the various techniques. The analogous mass transfer measurement using the naphthalene sublimation technique, on the other hand, overcomes these challenges and presents significant advantages of speed, economy, better resolution and accuracy over its thermal counter-part. However, the diffusion rates of naphthalene and heat into a stream of air are different. So, the physical and mathematical similarity between the two processes can be utilized effectively only when the analogy factor \( F = \frac{Nu}{Sh} \) is determined.

This study investigates the heat/mass transfer analogy in a turbulent separated flow behind a backward facing step. The heat \( (Nu) \) and mass \( (Sh) \) transfer measurements were made using the thermal boundary layer technique and the naphthalene sublimation technique respectively under identical flow conditions. Analogous boundary conditions of constant temperature and constant concentration were imposed on the active surface in the study which is the recirculation-reattachment region behind the backward facing step.
The Nu and Sh values thus obtained were used to calculate the analogy factor, F. The analogy factor was found out to be 0.692 which is within 3% of the empirical value of 0.677 ($Nu \propto Pr^{1/3}$) as predicted by a previous study.
# Contents

**List of Tables** ix  
**List of Figures** x  
**Nomenclature** xii  

## 1 Introduction  
1.1 Motivation of Present Research .................................................. 2  
1.2 Fundamental Theory ....................................................................... 3  
1.3 Flow over a Backward Facing Step ................................................... 5  
1.4 Heat Mass Transfer Analogy - Literature Review ............................... 6  
1.5 Expected results for Analogy in Turbulent Separated Flows ............... 7  
1.5.1 Laminar Boundary Layer ................................................................. 7  
1.5.2 Turbulent Boundary Layer ............................................................... 9  
1.5.3 Separated Flows ........................................................................... 9  
1.6 Objectives and Scope of Present Work ............................................. 10  

## 2 Experimental Apparatus  
2.1 Wind Tunnel .................................................................................. 12  
2.2 Test Section and Backward Facing Step.......................................... 13  
2.3 Three Axis Measurement Systems .................................................... 13  
2.4 Hotwire ............................................................................................ 13  
2.5 Heat Transfer Plate .......................................................................... 14  
2.6 Temperature Control System ............................................................ 14  
2.7 Thermocouple Probe ...................................................................... 15
7.3 Correlation for Maximum Nu, Sh ................................. 75
  7.3.1 Correlation Proposition ........................................ 75
  7.3.2 Comparison of Proposed Correlation with Literature ............ 76

8 Conclusion 83

Appendix A. Sample Calculations 85
  A.1 Fluid Mechanics Calculations ...................................... 85
  A.2 Determination of Nu from Heat Transfer Experiments ............. 85
  A.3 Determination of Sh from Mass Transfer Experiments .......... 87
    A.3.1 Property Evaluation ......................................... 87
    A.3.2 Mass Transfer Calculations ................................. 89
  A.4 Analogy Factor Calculations ..................................... 90

Appendix B. Uncertainty Analysis 91
  B.1 Uncertainty Analysis of Heat Transfer Experiment ................ 91
    B.1.1 Uncertainty in Heat Transfer Coefficient .................. 91
    B.1.2 Uncertainty in Nu ......................................... 93
  B.2 Uncertainty Analysis of Mass Transfer Experiment ............... 94
    B.2.1 Uncertainty Analysis of $h_m$ ............................. 94
    B.2.2 Uncertainty in Sh ......................................... 98
  B.3 Uncertainty in Analogy Factor .................................. 100

References 101
List of Tables

3.1 Basic Properties of Naphthalene (Kudchadker et al. (1978)[1]) . . . . . . 30
4.1 Spanwise variation of boundary layer characteristics, x=-12.7 mm, $u_\infty \approx$

12.5m/s . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 44
4.2 Spanwise variation of boundary layer characteristics, x=-12.7 mm, $u_\infty \approx$

17.5m/s . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 44
4.3 Flow details prior to boundary layer separation, z=0 $u_\infty \approx 12.46m/s$ . . 45
4.4 Flow details prior to boundary layer separation, z=0 $u_\infty \approx 17.47m/s$ . . 45
5.1 List of Mass Transfer Experiments . . . . . . . . . . . . . . . . . . . . . . 54
5.2 Location of Minimum and Maximum Sh numbers . . . . . . . . . . . . . . . . 56
6.1 List of Heat Transfer Experiments . . . . . . . . . . . . . . . . . . . . . . 65
6.2 Location of Minimum and Maximum Sh numbers . . . . . . . . . . . . . . . . 67
7.1 Comparison With Literature . . . . . . . . . . . . . . . . . . . . . . . . . . 77
A.1 Variables measured during the heat transfer experiment . . . . . . . . . . . . 86
A.2 Variables measured during the mass transfer experiment . . . . . . . . . . . . 87
# List of Figures

1.1 General features of flow separation over a Backward Facing Step . . . . 11  
2.1 Suction Type Wind Tunnel (Kulkarni (2008) [2]) (a) Schematic Diagram  
(b) Speed-Inverter Frequency Characteristics . . . . . . . . . . . . . . . . . . . . 17  
2.2 Schematic Diagram of the Plexiglas Components . . . . . . . . . . . . . . . 18  
2.3 Schematic Diagram of Test Section and the Assembled Setup . . . . . . . . . 19  
2.4 Hot Wire Probe (TSI-1218-T1.5) (Ghosh (2009) [3]) . . . . . . . . . . . . . . . 20  
2.5 Schematic Diagram of Heat Transfer Plate (Kulkarni (2008) [2]) . . . . . . . . 21  
2.6 Schematic Diagram of Thermocouple Probe . . . . . . . . . . . . . . . . . . . . 22  
2.7 Schematic Diagram of (a) Mass Transfer Plate (Kulkarni (2008) [2]) and  
(b) Plexiglass bracket . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 23  
2.8 Schematic Diagram of Surface Scanning System for Mass Transfer Ex-  
periment (Kulkarni (2008) [2]) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 24  
2.9 Author with the Experimental Setup . . . . . . . . . . . . . . . . . . . . . . 25  
3.1 Hot Wire Calibration, Sep 13, 2010 . . . . . . . . . . . . . . . . . . . . . . . . . . . . 36  
3.2 Schematic Diagram of Thermocouple Calibration Facility, Heat Transfer  
Lab (Kulkarni (2008) [2]) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 37  
3.3 Thermocouple Probe Calibration, Feb 1, 2010 . . . . . . . . . . . . . . . . . . . . 38  
3.4 Schematic Diagram of Wall Finding Technique for Thermocouple Probe  
(Kulkarni (2008) [2]) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 39  
3.5 LVDT Calibration, Displacement Vs Voltage, Mar 21, 2009 . . . . . . . . 40  
3.6 Mock Sublimation Test, Apr 2, 2009 . . . . . . . . . . . . . . . . . . . . . . . . . . . . 41  
3.7 Natural Convection Correction, Aug 27, 2009 (a) Spatially Averaged  
Depth Change Vs Time (b) Sublimation Rate Vs Time . . . . . . . . . . . . . . . . . . . 42
4.1 Velocity profile at x=-12.7 mm for two different free stream speeds (a) $u_\infty \approx 12.5m/s$ (b) $u_\infty \approx 17.5m/s$

4.2 Velocity Profile in Wall Coordinates (x=-12.7 mm, z=0) (a) $u_\infty \approx 12.5m/s$

4.3 Velocity Fluctuations in Wall Coordinates (x=-12.7 mm, z=0) (a) $u_\infty \approx 12.5m/s$

4.4 Velocity Profile at six different streamwise locations, $u_\infty \approx 12.5m/s$,

4.5 Velocity Profile at six different streamwise locations, $u_\infty \approx 17.5m/s$,

4.6 Determination of Virtual Origin for the two speeds (a) $u_\infty \approx 12.5m/s$, n=5.36  (b) $u_\infty \approx 17.5m/s$, n=5.40

5.1 Inconsistent Sh data at $u_\infty \approx 12.5m/s$, NOT USED

5.2 Sh variation over the napthalene surface at $u_\infty \approx 12.5m/s$ (Exp M1)

5.3 Sh variation over the napthalene surface at $u_\infty \approx 12.5m/s$ (Exp M2)

5.4 Sh variation over the napthalene surface at $u_\infty \approx 12.5m/s$ (Exp M3)

5.5 Sh variation over the napthalene surface at $u_\infty \approx 17.5m/s$ (Exp M4)

5.6 Sh variation over the napthalene surface at $u_\infty \approx 17.5m/s$ (Exp M5)

5.7 Sh variation over the napthalene surface at $u_\infty \approx 17.5m/s$ (Exp M6)

5.8 Streamwise variation of Sh Vs x/H at two speeds

6.1 Temperature Profiles at different Streamwise Locations for (a) $u_\infty \approx 12.5m/s$

6.2 Collapse of temperature profiles for both the speeds

6.3 Sample Temperature Profile at x=3.76 H or 75.2 mm

6.4 Streamwise variation of Nu Vs x/H at two speeds

7.1 Reynolds Number Scaling for (a) Sh, m=0.650 (b) Nu, m=0.642

7.2 Heat/Mass Transfer Analogy (a) $u_\infty=12.46$ m/s, n=0.310 (b) $u_\infty=17.47$

7.3 Normalised Nu and Sh for the two speeds $u_\infty=12.46$ m/s $u_\infty=17.47$

7.4 Heat/Mass Transfer Analogy-1

7.5 Heat/Mass Transfer Analogy-2
Nomenclature

English Symbols

$A$  Hotwire Calibration Constant
$B$  Hotwire Calibration Constant
$C_f$  Skin Friction Coefficient
$C_n$  Nozzle Coefficient of Hotwire Calibration Apparatus
$D_{ch}$  Chamber Diameter in Hotwire Calibration Apparatus [m]
$D_{na}$  Diffusivity of Naphthalene into air $[m^2/s]$
$d$  Nozzle Diameter in Hotwire Calibration Apparatus [m]
$E$  Temperature Compensated Hotwire Voltage Output [V]
$e$  Hotwire Voltage Output [V]
$F$  Heat / Mass Transfer Analogy Factor (Nu/Sh)
$g$  Gravitational Acceleration
$H$  Step Height [20 mm]
$h$  Convective Heat Transfer Coefficient $[W/m^2 - K]$
$h_{Hg}$  Height of Mercury Column [m]
$h_m$  Convective Mass Transfer Coefficient $[m/s]$
$k_{air}$  Thermal Conductivity of air $[W/m-K]$
$M_n$  Molecular Weight of Naphthalene [Kg/mol]
$m$  Naphthalene Sublimation rate (convective contribution) [$kg/s$]
$Nu$  Nusselt Number
$Nu_{MAX}$  Maximum Nusselt Number
$P_{atm}$  Atmospheric Pressure [Pa]
$P_{Ch}$  Chamber Static Pressure in Hotwire Calibration Apparatus [Pa]
$P_{\text{dyn}}$ Dynamic Pressure [Pa]
$P_{st}$ Static Pressure [Pa]
$P_{v,w}$ Naphthalene Vapor Pressure at Wall [Pa]
$Pr$ Prandtl Number
$Pr_t$ Turbulent Prandtl Number ($\epsilon/\epsilon_H$)
$q_w$ Heat Flux in Normal Direction at the wall [W/m$^2$]
$R_{\text{air}}$ Gas Constant for Air [J/Kg – K]
$Re_H$ Reynolds Number based on Step Height, H
$Re_{Ch}$ Reynolds Number based on Chamber Diameter ($Re_{Ch} = U_{Ch}D_{Ch}/\nu$)
$Sc$ Schmidt Number
$Sc_t$ Turbulent Schmidt Number ($\epsilon/\epsilon_M$)
$Sh$ Sherwood Number
$Sh_{\text{MAX}}$ Maximum Sherwood Number
$T$ Temperature [$^\circ C$]
$T_{Ch}$ Chamber Temperature [$^\circ C$]
$T_{\infty}$ Freestream Temperature [$^\circ C$]
$T_{\text{REF}}$ Reference Temperature for Hotwire Calibration [25 $^\circ C$]
$T_{w}$ Wall Temperature [$^\circ C$]
$T_{n,w}$ Naphthalene Wall Temperature [$^\circ C$]
$T_{\text{wire}}$ Hotwire Temperature [250 $^\circ C$]
$t$ Time [s]
$\delta\tau$ Time Interval for Forced Convection Mass Transfer Experiment [s]
$\Delta\tau_k$ Time Interval for Natural Convection of Naphthalene [s]
$U_{Ch}$ Chamber Velocity in Hotwire Calibration Apparatus [m/s]
$U^+$ Streamwise Velocity in Wall Coordinates
$U_n$ Nozzle Velocity in Hotwire Calibration Apparatus [m/s]
$u_{\infty}$ Free stream velocity [m/s]
$u_\tau$ Friction Velocity [m/s]
$u$ Velocity in x-direction [m/s]
$u'$ Velocity fluctuation in x-direction [m/s]
$w$ Spanwise Width of Test Plate [m]
$x$ Streamwise Coordinate [mm], $x = 0$ at the edge of the step
\( x_r \)  Reattachment Point [mm]
\( x_{\text{max}} \)  Location of maximum Nu (Sh)[mm]
\( \hat{x}_i \)  Non-Dimensional Coordinates
\( Y \)  Compressibility Factor in Hotwire Calibration Apparatus
\( Y^+ \)  Normal Distance in Wall Coordinates
\( Y_{\text{shift}} \)  Uncertainty in Wall Location for Hotwire Measurements [m]
\( y \)  Normal Coordinate [mm], \( y = 0 \) along the plate
\( \delta y \)  Naphthalene Sublimation Depth [m]
\( \delta y_{\text{nc}} \)  Naphthalene Sublimation Depth due to Natural Convection [m]
\( \delta y_{(i,j)_{\text{cor}}} \)  Corrected Naphthalene Sublimation Depth [m]
\( z \)  Spanwise Coordinate [mm], \( z = 0 \) along the center of the plate

**Greek Symbols**

\( \alpha \)  Thermal diffusivity \([m^2/s]\)
\( \gamma \)  Ratio of Specific Heats of air
\( \delta_{99} \)  Momentum Boundary Layer Thickness [m]
\( \delta_1 \)  Momentum Boundary Layer Displacement Thickness [m]
\( \delta_2 \)  Momentum Boundary Layer Momentum Thickness [m]
\( \epsilon \)  Turbulent Momentum Diffusivity \((-\overline{u'v'}/(\partial u/\partial y)) \) \([m^2/s]\)
\( \epsilon_H \)  Turbulent Thermal Diffusivity \((-\overline{T'v'}/(\partial T/\partial y)) \) \([m^2/s]\)
\( \epsilon_M \)  Turbulent Mass Diffusivity \((-\overline{w'v'}/(\partial w/\partial y)) \) \([m^2/s]\)
\( \eta \)  Similarity Variable
\( \mu \)  Dynamic Viscosity \([Kg/m-s]\)
\( \nu \)  Kinematic Viscosity \((\mu/\rho) \) \([m^2/s]\)
\( \omega \)  Mass fraction in binary diffusion system
\( \overline{\omega} \)  Time Averaged Mass fraction in binary diffusion system
\( \omega_\infty \)  Freestream mass fraction
\( \omega_w \)  Mass Fraction at the naphthalene surface
\( \Phi \)  Nondimensional Mass Fraction in binary diffusion system
\( (\omega - \omega_\infty)/(\omega_w - \omega_\infty) \)
\( \rho_{\text{air}} \)  Air density \([Kg/m^3]\)
$\rho_{Ch}$ Chamber Air density in Hotwire Calibration Apparatus [Kg/m$^3$]

$\rho_{Hg}$ Density of Mercury [Kg/m$^3$]

$\rho_{v,w}$ Naphthalene Vapor Density at the wall [Kg/m$^3$]

$\rho_{v,\infty}$ Naphthalene Vapor Density in free stream air flow [0 Kg/m$^3$]

$\rho_s$ Solid Naphthalene Density [Kg/m$^3$]

$\theta$ Nondimensional temperature ($(T - T_\infty)/(T_w - T_\infty)$)
Chapter 1

Introduction

Separated flows are of fundamental importance because of their significant effects on heat, mass and momentum transfer. They are frequently encountered in various engineering applications. Some examples include microelectronic circuit boards, combustors, heat exchangers, axial and centrifugal compressor blades and gas turbine blades. Separated flows thus merit a detailed study of thermo-fluidic transport.

In some applications such as design of turbine blades and wing airfoils very high spatial resolution of heat transfer coefficients is necessary. However, the complex nature of the separated flow, often encountered on these geometries, in the recirculation and the reattachment region makes the heat transfer measurement process difficult and less accurate. Fortunately, heat and mass transfer over a solid surface are both diffusion processes, governed by Fourier’s and Fick’s laws, respectively, and can become analogous under certain sets of assumptions and boundary conditions. Nusselt and Schmidt [4] had discovered this similarity between heat and mass transfer and hence suggested the transformation of a problem in one domain to the other.

Thus, it follows from the above argument to transform this problem of thermal transport in separated flows into an analogous domain of mass transfer, if similar measurements for mass transfer can be carried out more easily with better accuracy. This chapter describes some advantages/ motivations of this transformation. Also presented are the underlying theory behind the heat - mass transfer transformation, the physical
background of the transport equations and the objective and the scope of the present work.

1.1 Motivation of Present Research

The transformation of the heat transfer problem into an analogous mass transfer situation using the naphthalene sublimation technique is viable from both engineering and economical perspective because the technique has some significant advantages over direct heat transfer measurements as illustrated in Goldstein and Cho (1995)[5]. In brief:

1. Errors in heat transfer measurements as a result of wall conduction and surface radiation are eliminated.

2. Large gradients can be captured in mass transfer measurements. This is impossible in direct heat transfer measurements due to wall conduction.

3. Since naphthalene can be cast in many shapes, measurements can be made for a variety of engineering geometries.

4. Most heat transfer techniques are intrusive. Those which are not, have high uncertainty. Naphthalene sublimation technique on the other hand is both non-intrusive and highly accurate.

5. High spatial resolution measurements are possible without a significant increase in the time invested.

While both the heat transfer to air and mass transfer from naphthalene surface are diffusion processes they differ in the rates of transport. Thus, though, we expect the heat transfer coefficients to follow a trend similar to mass transfer coefficients’, the values would not be the same. They are related by a factor called the heat/mass transfer analogy factor. The success of the aforementioned transformation with all its significant advantages, hinges on the existence and determination of an analogy factor that can transform the transport coefficients from the mass transfer to the heat transfer domain. The analogy factor, which can either be a simple constant, as in case of laminar flow
over a flat plate, or a complicated function of flow parameters \([6]\), has been verified for laminar and turbulent flows over a flat plate by Kulkarni (2008) \([2]\). However, the analogy is yet to be verified for separated flows. Subsequently, the goal of the present study is to experimentally prove the existence of an analogy and obtain an analogy factor for separated flows.

As stated already, separation can occur in various engineering geometries and applications. Thus there is a need to choose a geometry that is easily set up in a laboratory and is representative of all the aspects of flow separation. Owing to its geometrical simplicity and ability to capture the basic features of separated flows: separation, reattachment, recirculation and development of free-shear layers, the flow over a backward facing step (BFS) is chosen to investigate the heat and mass transfer analogy in separated flows. In addition, most of the engineering flows are high speed turbulent flows. Thus the scope of the present work is restricted to turbulent flows.

In summary, the goal of the present study is to verify and determine experimentally the heat/mass transfer analogy factor in turbulent separated flow over a backward facing step.

### 1.2 Fundamental Theory

The non dimensionalized heat transfer equation (Eqn 1.1) and mass transfer equation (Eqn 1.2) are similar but for the differences between \(Pr\) and \(Sc\) and \(Pr_t\) and \(Sc_t\), where \(Pr_t = \epsilon/\epsilon_H\) and \(Sc_t = \epsilon/\epsilon_M\), \(\epsilon, \epsilon_H, \epsilon_M\) being the turbulent momentum, heat and mass diffusivities respectively. Hence, under certain experimental conditions if \(Pr = Sc\) and \(Pr_t = Sc_t\) then the equations become identical and non dimensional temperature, \(\theta\) (Eqn 1.3) and non dimensional concentration, \(\phi\) (Eqn 1.3) have the same variation in time and space (Eqn 1.4) provided the initial and boundary conditions are also identical. This is the basic heat/mass transfer analogy which stems from the similarity of the two governing equations.

\[
\frac{D\theta}{Dt} = \frac{1}{RePr} \frac{\partial}{\partial \hat{x}_i} \left( \left(1 + \frac{\epsilon}{\nu Pr_t} \right) \frac{\partial \theta}{\partial \hat{x}_i} \right) \quad (1.1)
\]
\[
\frac{D\phi}{Dt} = \frac{1}{ReSc} \frac{\partial}{\partial \hat{x}_i} \left( \left( 1 + \frac{\epsilon}{\nu Sc} \right) \frac{\partial \phi}{\partial \hat{x}_i} \right) \tag{1.2}
\]

\[
\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{\omega - \omega_\infty}{\omega_w - \omega_\infty}, \tag{1.3}
\]

\[
Pr = Sc \Rightarrow Nu = Sh \tag{1.4}
\]

According to Simpson and Field (1972)[7], experimental measurements indicate that turbulent heat and mass diffusivities are equal for simple flow cases. Hence \(Pr_t = Sc_t\). The only parameters, then differentiating the two processes are the \(Pr\) and \(Sc\) values. Thus in order to be able to exploit this similarity between the two processes there exits a need to extend this analogy for practical cases of different \(Pr\) and \(Sc\). As discussed by Eckert et al. (2001) [8] this can be achieved if the following relations hold true:

1. The flow field is independent of \(Pr\) and \(Sc\). In other words while flow affects the thermal and mass transport the opposite dependence is not true i.e the variation in \(Pr\) and \(Sc\) does not have an affect on the flow characteristics.

2. The functional relation between the temperature field and the independent variables that describes the temperature field should also describe the concentration field if the independent variable \(Pr\) is changed to \(Sc\) in the same functional relation. This can be indicated as follows :

\[
Nu = f(Re, Pr, \hat{x}_i), \quad Sh = f(Re, Sc, \hat{x}_i) \tag{1.5}
\]

where the function 'f' in the above realtions is the same.

3. Gas turbine blades operate at high Reynolds numbers, \(Re_C\). Their boundary layers transport, therefore, information with high approximation in downstream direction only and Nusselt numbers are functions of \(Re_x\) and \(Pr\) when \(Pr\) is of order one. The chord length, \(C\), can be introduced as a dummy variable.

\[
Nu_x = f(Re_x, Pr) = f(Re_C \cdot \frac{x}{C}, Pr) \tag{1.6}
\]
From Eqs 1.5 and 1.6 we can thus write:

\[ Sh_x = f(Re_x, Sc) = f(Re_C \cdot \frac{x}{C}, Sc) \]  

(1.7)

Thus from the above description of an extended analogy we can deduce the following:

\[ Pr \neq Sc \Rightarrow Nu = F \cdot Sh \]  

(1.8)

where \( F \) is the analogy factor.

### 1.3 Flow over a Backward Facing Step

The flow behind a backward facing step is fairly complex and presents an opportunity to understand the characteristics of separated flow. Fig 1.1 illustrates the basic features of flow over a backward facing step.

Eaton and Johnston (1981)[9] and Simpson (1981)[10] present excellent reviews of flow separation behind a backward facing step. The boundary layer separates at the edge of the step where a shear layer develops downstream of the point of separation. As the shear layer moves downstream it grows rapidly due to successive pairing of spanwise vortices (large eddy structures) and curves sharply downward to eventually impinge on the wall.

These spanwise vortices move upstream and downstream intermittently as observed by McGuinness(1978)[11]. There is, however, disagreement on this observation as Bradshaw and Wong (1972)[12] argue that the impinging eddies are torn away into two parts at impaction to be washed away both upstream and downstream of reattachment. Kasagi et al. (1977)[13] showed by means of smoke-wire method that the impinging structures become fully three dimensional upstream of reattachment. The impinging shear layer is subjected to the effects of stabilizing curvature, adverse pressure gradient and strong interaction with the wall in the reattachment zone which cause a rapid decay of the turbulent stresses in the region. The flow in this zone is very unsteady and turbulent as very large eddy structures of the order of the step height pass this through this
region. In addition the reattachment length also fluctuates as has been confirmed by Abbot and Kline (1962)\textsuperscript{14}, Kim et al. (1978)\textsuperscript{15} and Eaton and Johnston (1980)\textsuperscript{16}. For a turbulent boundary layer at separation, the reattachment length has been observed to be 6-8 step heights.

The region between the free shear layer and the wall is the recirculation region which is the domain of interest in this study. The recirculating flow region cannot be characterized as a dead air zone. Substantial backflow velocity of around 20\% of the freestream velocity and skin friction coefficient as high as $C_f = -0.0012$ (based on freestream velocity) have been measured (Chandrasuda (1975)\textsuperscript{17} and Eaton and Johnston (1980)\textsuperscript{16}). Aung and Goldstein (1971) \textsuperscript{18} also confirmed this fact through heat transfer measurements by demonstrating a sizeable temperature drop across the wall boundary layer thus contradicting the notion of constant enthalpy fluid sandwiched between the wall and the free shear layer.

Downstream of reattachment the Reynolds stresses continue to decay rapidly and a new boundary layer starts developing. Bradshaw and Wong (1972)\textsuperscript{12} and Smyth (1979)\textsuperscript{19} demonstrated that the large vortical structures, washed downstream of reattachment, persist in the outer part of the reattached shear layer up to 50 step heights downstream of reattachment.

1.4 Heat Mass Transfer Analogy - Literature Review

There have been only a few studies to date to verify the existence of heat/mass transfer analogy. Sakamoto and Simon (2000) \textsuperscript{20} used TEXSTAN simulations to calculate an analogy factor for GE90 and CF6 turbine blades for laminar flows. They found the analogy factor to be $F = 0.677$ for flow over a constant temperature wall. Pachhapur (2000) \textsuperscript{21} made heat transfer measurements on a turbine blade and compared them with earlier mass transfer measurements. He concluded that the analogy factor ranges between 0.49 and 0.64.

Han (2008)\textsuperscript{22, 23} were the first studies to explore the existence of heat/mass transfer analogy experimentally. He demonstrated the existence of a Colburn heat/mass transfer
analogy for flow over turbine blades and endwalls. Kulkarni (2008) [2] experimentally verified the existence of Colburn analogy in laminar and turbulent boundary layers over a flat plate. For laminar and turbulent flows (for the range of Re explored in the study), the analogy factor was found to be \((\frac{Pr}{Sc})^{1/3} = 0.677\) and 0.667 respectively.

A constant temperature boundary condition for the heat transfer experiments and an analogous constant concentration boundary condition for the mass transfer experiments were employed in both these studies. Waldron (2004) [24] made some heat and mass transfer measurements in turbulent flow downstream of a backward facing step. This project is a continuation of his work though, a different experimental setup has been used.

1.5 Expected results for Analogy in Turbulent Separated Flows

1.5.1 Laminar Boundary Layer

With the boundary layer assumptions, the continuity and momentum equations for the steady, incompressible flow of a constant property fluid can be written as presented in Eqns (1.9) and (1.10)

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1.9)
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P_{st}}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial P_{st}}{\partial y} = 0 \quad (1.10)
\]

The energy and the two component species transport equation are shown in Eqns (1.11) and (1.12).

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (1.11)
\]

\[
u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = D_{na} \frac{\partial^2 \omega}{\partial y^2} \quad (1.12)
\]

It must be noted that these two equations are decoupled from the momentum equations by virtue of the assumption of the constant property fluid. Thus, the temperature field and the concentration field shall have the an analogous spatial variation if analogous
boundary conditions are imposed. Consider the following case of constant temperature,

\[ T = T_w \] (constant concentration, \( \omega = \omega_w \)) boundary conditions:

\[
\begin{align*}
    u &= 0 \quad @ \quad y = 0 \\
    u &= U_\infty \quad @ \quad y \to \infty \\
    T &= T_w \quad @ \quad y = 0 \\
    T &= T_\infty \quad @ \quad y \to \infty \\
    \omega &= \omega_w \quad @ \quad y = 0 \\
    \omega &= \omega_\infty = 0 \quad @ \quad y \to \infty
\end{align*}
\]

From the similarity solution for a zero pressure gradient boundary layer we have:

\[
\frac{C_f}{2} = 0.332 \frac{Re^{0.5}}{x}
\]

From Kays and Crawford [25] we have:

\[
Nu = 0.332 Re^{1/2} Pr^{1/3}, \quad Pr > 0.6
\]

Expecting an analogous behavior for the mass transfer we can say:

\[
Sh = 0.332 Re^{1/2} Sc^{1/3}, \quad Sc > 0.6
\]

The Colburn analogy factor, \( F \) can then be calculated as shown in Eqn 1.19

\[
F = \frac{Nu}{Sh} = \frac{0.332 Re^{1/2} Pr^{1/3}}{0.332 Re^{1/2} Sc^{1/3}} = \left( \frac{Pr}{Sc} \right)^{1/3}
\]

For naphthalene sublimation into air at room temperature (Pr=0.707), Sc = 2.28 which results in \( F = 0.677 \).
1.5.2 Turbulent Boundary Layer

Employing the two dimensional boundary layer assumptions, the Reynolds time averaged transport equations for the turbulent incompressible flow of a constant property fluid can be presented in Eqns 1.20 - 1.23.

\[
\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{1.20}
\]

\[
\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho} \frac{\partial \bar{P}_{st}}{\partial x} + \frac{\partial}{\partial y} \left[ (\nu + \epsilon) \frac{\partial \bar{u}}{\partial y} \right] , \quad \frac{\partial \bar{P}_{st}}{\partial y} = 0 \tag{1.21}
\]

\[
\bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} = (\alpha + \epsilon_H) \frac{\partial^2 \bar{T}}{\partial y^2} \tag{1.22}
\]

\[
\bar{u} \frac{\partial \bar{\omega}}{\partial x} + \bar{v} \frac{\partial \bar{\omega}}{\partial y} = (D_{na} + \epsilon_M) \frac{\partial^2 \bar{\omega}}{\partial y^2} \tag{1.23}
\]

For the boundary conditions as described in Eqns 1.14-1.16 from Kays and Crawford [25] we have:

\[
Nu = \frac{0.0296 Re_x^{4/5} Pr}{1 + 1.48 Re_x^{-1/10} Pr^{-1/6} (Pr - 1)} \tag{1.24}
\]

\[
Sh = \frac{0.0296 Re_x^{4/5} Sc}{1 + 1.48 Re_x^{-1/10} Sc^{-1/6} (Sc - 1)} \tag{1.25}
\]

From Eqns 1.24 and 1.25 we can deduce the analogy factor, F as presented in Eqn 1.26.

\[
F = \frac{Nu}{Sh} = \left( \frac{Pr}{Sc} \right) \frac{1 + 1.48 Re_x^{-1/10} Sc^{-1/6} (Sc - 1)}{1 + 1.48 Re_x^{-1/10} Pr^{-1/6} (Pr - 1)} \tag{1.26}
\]

1.5.3 Separated Flows

Several heat and mass transfer studies have been undertaken to understand the transport phenomena in the recirculation region behind a backward facing step. To the author’s knowledge there are still no correlations available to describe the heat/mass transfer distribution behind a backward facing step for turbulent flow.
A.I. Leont’ev et al. (1984)[26] presented a correlation for evaluating the maximum heat transfer level behind the boundary layer separation point

\[ \frac{h_{x_r}}{k_{air}} = 0.0803 \left( \frac{U_\infty x_r}{\nu} \right)^{0.72} P_r^{0.43} \]  

(1.27)

Based on many studies Ota and Nishiyama (1987)[27] presented a universal correlation, Eqn. 1.28, to predict the maximum heat/mass transfer coefficient in separated flows.

\[ \frac{h_{x_r}}{k_{air}} = 0.192 \left( \frac{U_\infty x_r}{\nu} \right)^{0.6656} P_r^{1/3} \]  

(1.28)

An interesting aspect of these two correlation is their markedly different dependence on Prandtl number. Based on the correlations given by A.I. Leont’ev et al. (1984)[26] and Ota and Nishiyama (1987)[27] together with the Colburn analogy, the analogy factor in turbulent separated flows can be predicted as follows:

\[ \frac{N_u}{Sh} = \left( \frac{P_r}{Sc} \right)^{0.43} = 0.604 \]  

(1.29)

\[ \frac{N_u}{Sh} = \left( \frac{P_r}{Sc} \right)^{1/3} = 0.677 \]  

(1.30)

Although these correlations are applicable only for the maxima, the analogy factor can be calculated based on the assumption that the Nu (Sh) dependence on Pr (Sc) would not vary with the streamwise coordinate. This assumption shall be checked in Chapter 7.

### 1.6 Objectives and Scope of Present Work

The present work deals with finding the analogy factor for turbulent separated flows behind a backward facing step and comparing it with the theoretical benchmarks available in the literature as indicated in the previous section. Naphthalene sublimation shall be used to conduct the mass transfer experiments and the standard thermocouple traverse technique shall be used in the flow over a constant temperature plate to obtain the heat transfer coefficients.
Figure 1.1: General features of flow separation over a Backward Facing Step
Chapter 2

Experimental Apparatus

This chapter describes in detail the apparatus used for the experimental verification of heat/mass transfer analogy in the present study behind a backward facing step.

2.1 Wind Tunnel

Placed in the Heat Transfer Laboratory of the Mechanical Engineering Department of the University of Minnesota, the wind tunnel used for this study is an open circuit, suction type wind tunnel. A schematic diagram of the same is shown in Fig 2.1.

Air is sucked in through a plenum chamber and passes through a coarse filter. It then travels through a long diffuser before passing through a fine filter. The guide vanes downstream of the fine filter then turn the air 180 degrees. It then passes through a series of screens and flow straighteners before being accelerated to the test section through a nozzle of 15:1 contraction ratio. After the test section it passes through a diffuser before entering the centrifugal fan which expels the air outside the building via an exit diffuser.

The fan is driven by a 7.5 HP electric motor (Toshiba BY754FLF2USH) and is controlled by a transistor inverter type variable frequency controller (Toshiba VFS9-2055PL). The resolution of the motor frequency is 0.1 Hz. The speed in the test section without any obstruction has a linear dependence on the motor frequency (Fig 2.1) as
demonstrated by Kulkarni (2008)\cite{2}.

2.2 Test Section and Backward Facing Step

The wind tunnel test section consists of a steel structure holding Plexiglass walls (19 mm in thickness) to form a rectangular duct of 2500 mm in length, 300 mm in height and 600 mm width. The bottom wall is replaced by a ramp that starts flush with the bell mouth of the wind tunnel and then thickens to a height of 20 mm, 177 mm downstream of the bell mouth. The step is located further 150 mm downstream of this ramp. Also, pasted on the step 10 mm downstream of the bell mouth, is a 1 mm diameter trip wire to facilitate transition to turbulence. Detailed schematics of the various plexiglass components and the complete assembly as it sits in the test section shown in Figs 2.2 and 2.3. As can be seen from the figure the expansion ratio is \( \frac{300}{300-20} = 1.0714 \).

2.3 Three Axis Measurement Systems

In order to characterize the velocity and temperature profiles using boundary layer probes, a three axis measurement system designed by Waldron(2004)\cite{24} is used. It consists of three Uni-slide glide rails with 1 mm pitch lead screws mounted on a rigid frame of Uni-struts just at the top of the test section outside the windtunnel. The three uni-slides present three translational degrees of freedom in streamwise direction (X), wall normal direction (Y) and spanwise direction (Z). The boundary layer probes can be mounted on the Y axis and enter the test section through the slots cut on the top plexiglas plate. Each axis is individually controlled by a SLO-SYN motor with a resolution of 200 steps per revolution. This gives a minimum step size \( \frac{1}{200} = 0.005 \) mm = 5 \( \mu \)m in each direction which is small enough to explore the thin boundary layers near the impaction region.

2.4 Hotwire

The velocity field upstream of the step is quantified using a TSI boundary layer single hotwire probe (TSI 1218-T1.5). The probe has a 1.27 mm long platinum coated
tungsten wire of diameter 3.8 \( \mu m \) (l/d=333). The hotwire is used in the constant temperature mode (CTA), the operating and reference temperatures being 250\(^\circ\)C and 25\(^\circ\)C respectively. The signal from the hotwire is transmitted to IFA-100 (Intelligent Flow Analyzer) for signal processing. A schematic of the hotwire probe used in this study is shown in Fig 2.4.

### 2.5 Heat Transfer Plate

The heat transfer plate designed by Lee (2002)\[28\] is used in this study (Fig 2.5). It consists of an aluminum top plate (228.6 mm (9 in), 152.4 mm (6 in) and 12.7 mm (1/2 in) thickness) and a phenolic insulator base plate. Ten wire wound silicone rubber heaters manufactured by Minco (Minco part nos. HR 5248, HR 5334), are sandwiched between these two plates. Also, 0.5 mm below the top surface, thirty three (\( \phi 0.254 \) mm (30 AWG)) E-type (Chromel - Constantan) thermocouples are mounted along the periphery of these heaters in a 11X3 fashion to monitor the temperature of the surface. The high conductivity of aluminum (237 W/m-K) helps to mitigate any temperature gradients and assists in maintaining the plate at a uniform temperature.

All measurements are taken along the centerline of the heat transfer plate to avoid any three dimensional effects due to surface conduction to adjacent Plexiglas.

### 2.6 Temperature Control System

In order to achieve a constant temperature boundary condition for the heat transfer plate, a temperature control system is used. The temperature control infrastructure consists of a 128 channels DC power supply rack built by Han (2004)\[29\], a 6 channel HP power supply (HP 66104A, 0-60 V, 2.5 A) and a Linux workstation. The four heaters next to the leading edge of the plate are connected to the 128 channels DC power supply rack and the other six heaters are connected to the HP power supply. This arrangement was builtis necessary because of the current carrying limit of the power lines. A programmable operational amplifier is used to control the power across every channel/heater. A PI control algorithm developed by Han (2004)\[29\] in conjunction with
thirty three thermocouples (sensors) inside the plate serves the purpose of maintaining the plate at a constant temperature. The thermocouple signals act as a feedback for the PI control system which then alters the power setting on each heater to bring the plate closer to the preset temperature value. With proper choice of the control constants it is possible to maintain the plate within $\pm 0.2^\circ C$ at steady state. The program that runs on the Linux workstation in addition to controlling the heater power and PI control system also generates a time log of the temperatures inside the plate and the power settings of the heater.

2.7 Thermocouple Probe

The constantan and chromel wires ($\phi 76\mu m$ each) threaded through two capillary tubes ($\phi 1mm$) are butt welded at the lowest point of the arc as shown in Fig 2.6. These capillary tubes are fixed to a 4.8 mm OD and 3mm ID steel support. Also the two capillary tubes are connected by a plastic needle to provide rigidity to the structure and reduce vibrations of the probe.

2.8 Mass Transfer Plate and Plexiglass Bracket

The mass transfer plate (Fig 2.7) used in this study was designed by Lee (2002)[28]. It is actually an assembly of two different plates. While the one houses the naphthalene cast, the other one is solely for the casting procedure. The plate that houses the naphthalene cast has a 2.54 mm depression flanked by a 3 mm polished rim which acts as a container to hold the molten naphthalene during the casting. The mirror polished surface of the other plate sits flush with the rim of the housing plate to ensure that the naphthalene does not leak during the casting. The two plates are machined to the same dimensions and are fastened using eight bolts.

Both the plates have a reversible liquid crystal tape pasted on one of the surfaces which is used to monitor the temperature during the casting. The rim on the housing plate also defines a reference plane with respect to which the sublimation depth of the naphthalene is read by the LVDT scanner. There is a slight indentation on one of
the rims that is used as the origin for the scanning purposes. Also, in the housing plate are two holes through the casting area where two (φ 0.254 mm (30AWG)) E-type thermocouples sit very close to the naphthalene surface. They are used to monitor the temperature of the naphthalene cast during the experiment.

Though both the plates have the same area exposed to the flow, they differ in the heights of the sections exposed. The slot in the wind tunnel bottom plate is machined to house the heat transfer plate flush with the bottom of the wind tunnel. In order to achieve the same for the mass transfer plate a plexiglass bracket was built which would sit on the periphery of the mass transfer plate and impart it a desired thickness so that it comes flush with the bottom of the wind tunnel as well. In order to prevent any drift going into the windtunnel because of added clearances between the plate and the bracket and the wind tunnel bottom and the plexiglas bracket, o-rings were placed on both sides of the bracket.

2.9 Naphthalene Surface Scanning System

The naphthalene surface scanning system (Fig 2.9) used in this study consists of a LVDT probe mounted on a carriage that can travel in two perpendicular directions (streamwise and spanwise) parallel to the plane of the naphthalene surface. The two axes consist of lead screws driven by SLO-SYN MN-63 stepper motors controlled by NF-90 motion controller. While one pair of lead screws moves the heavy table top in the streamwise direction the other pair moves the guide rail with the LVDT probe (Schaevitz PCA 220-010) mounted on it in the spanwise direction. The mass transfer plate is fixed to the heavy table top using various fixtures and brackets to ensure there is no movement of the plate with respect to the table. The height of the LVDT probe can be manually adjusted such that the elevation of the entire plate is within the displacement range of the probe. The output signal of the LVDT probe is filtered for electrical noise using a signal conditioner (Schaevitz CAS - 025) to produce a DC voltage output.
Figure 2.1: Suction Type Wind Tunnel (Kulkarni (2008)[2]) (a) Schematic Diagram (b) Speed-Inverter Frequency Characteristics

Velocity = 0.294*Frequency - 0.543

Free Stream Velocity (m/s)

Inverter Frequency (Hz)
Screws to fix the pieces to the wind tunnel steel structure

The two plexiglas pieces are fastened together through these screws

Trip Wire
Dia 1 mm

Figure 2.2: Schematic Diagram of the Plexiglas Components
Figure 2.3: Schematic Diagram of Test Section and the Assembled Setup
Figure 2.4: Hot Wire Probe (TSI-1218-T1.5) (Ghosh (2009))
Figure 2.5: Schematic Diagram of Heat Transfer Plate (Kulkarni (2008))
Figure 2.6: Schematic Diagram of Thermocouple Probe
Figure 2.7: Schematic Diagram of (a) Mass Transfer Plate (Kulkarni (2008)[2]) and (b) Plexiglass bracket.
Figure 2.8: Schematic Diagram of Surface Scanning System for Mass Transfer Experiment (Kulkarni (2008)[2])
Figure 2.9: Author with the Experimental Setup
Chapter 3

Procedure

This chapter describes in detail the calibration and data acquisition procedures used in the present study. The heat transfer measurements were carried out using the wall normal thermocouple probe traverse technique. For mass transfer measurements naphthalene sublimation techniques was used. The velocity field upstream of the step was measured with a single hot wire anemometer.

3.1 Hotwire Calibration and Velocity Measurement

3.1.1 Calibration

The hotwire is calibrated in a jet facility developed by Wilson (1970)[30]. For low speeds (0-7 m/s) the hotwire is placed inside the chamber and for higher speeds (0-20 m/s) in the core of the jet from the exit nozzle.

The temperature of the air inside the apparatus is measured using a resistive thermal device (RTD). The difference between the chamber static pressure and the atmospheric pressure is measured using a Baratron pressure transducer in conjunction with MKS 670 signal conditioner for different chamber velocities. The density of the air inside the setup can then be calculated using the ideal gas law. The relationships between different variables as obtained by Wilson (1970)[30] are given in Eqns (3.1) - (3.9).

\[ D_{ch} = 25.564 \text{mm}, \quad d = 8.128 \text{mm} \]  

(3.1)
\[ \rho_{ch} = \frac{P_{ch}}{RT_{ch}} \quad Re_{ch} = \frac{\rho_{ch} U_{ch} D_{ch}}{\mu} \] (3.2)

\[ C_n = 1.000 \pm 0.002 \] (3.3)

For \( 4600 < Re_{ch} < 23000 \)

\[ C_{ch} = 1.1518 - 4.4131e^{-6} Re_{ch} + 1.7252e^{-11} Re_{ch}^2 + 3.7937e^{-15} Re_{ch}^3 \] (3.4)

\[ Y = \left[ \frac{1}{1 - \left( \frac{P_{atm}}{P_{ch}} \right)^{2/\gamma} \left( \frac{d}{D_{ch}} \right)^{4} \left( \frac{\gamma - 1}{\gamma} \left( 1 - \frac{P_{atm}}{P_{ch}} \right)^{\frac{\gamma - 1}{\gamma - 1}} \right) + \frac{\gamma - 1}{\gamma - 1}} \right]^{1/2} \] (3.5)

\[ U_{ch} = C_{ch} \left( \frac{d}{D_{ch}} \right)^2 \left( \frac{P_{atm}}{P_{ch}} \right)^{1/\gamma} \left( \frac{\gamma - 1}{\gamma} \right) Y \left[ \frac{2(P_{ch} - P_{atm})}{\rho_{ch}} \right]^{1/2} \] (3.6)

\[ U_n = C_n Y \left( \frac{2(P_{ch} - P_{atm})}{\rho_{ch}} \right)^{1/2} \] (3.7)

The temperature compensated calibration curve has been represented in Fig.??.

\[ T_{wire} = 250^\circ C \quad T_{REF} = 25^\circ C \] (3.8)

\[ E^2 = e^2 \left( \frac{T_{wire} - T_{REF}}{T_{wire} - T_{ch}} \right) = A + BU_{ch}^n \] (3.9)

### 3.1.2 Velocity Measurement

The calibrated hotwire is used to measure the velocity profile upstream of the step. The hotwire probe is brought close to the plexiglas tunnel wall until the support of the probe appears to be in contact with the surface. Since there is no magnification involved it is very difficult to ensure that the support is actually in contact. This introduces an uncertainty in the location of the wall \( Y_{shift} \) which is used as a parameter (varied within \(-0.001 \text{ mm} \) to \( 0.001 \text{ mm} \), negative value indicates a possibility of bending the support) in the least square methods of Clauser technique. Once the hotwire probe seems to be close enough to the surface, the probe is then moved away from the wall in small
steps and the entire boundary layer is traversed. The hot wire can then be moved to a different streamwise location upstream of the step and the same procedure can be repeated to get the boundary layer profile.

### 3.2 Thermocouple Calibration and Temperature Measurement

#### 3.2.1 Calibration

The thermocouple probe was calibrated in the Thermocouple Calibration Facility housed in the Heat Transfer Laboratory, ME 225 at the University of Minnesota-Twin Cities. The thermocouple probe was immersed in a bath of water whose temperature was precisely controlled. Once the bath had achieved the preset temperature value the exact value was recorded using a calibrated platinum resistance thermometer (PRT). Also, was recorded the voltage signal from the thermocouple probe. After having repeated the same process for eight different temperature values from 10-55°C a fourth order polynomial was fitted to relate the voltage output by the thermocouple to the temperature recorded by the PRT. This calibration equation was then used subsequently to ascertain the temperature value from the voltage signal of the thermocouple probe.

#### 3.2.2 Thermal Boundary Layer Measurement Technique

The heat transfer coefficients can be measured from the temperature profiles obtained by traversing the thermocouple probe inside the thermal boundary layer and by the application of Fourier’s law as given in Eqn 3.10. The accuracy of the thermal boundary layer measurement depends on two crucial factors - the resolution of the thermal boundary layer and the precise location of the wall / heated surface.

\[
h = \frac{q_w}{T_w - T_\infty} = \frac{-k_{air} \partial T}{T_w - T_\infty} \left| \frac{\partial T}{\partial y} \right|_{wall}
\]  

(3.10)

As already mentioned the Y-axis uni-slide has the smallest step size of 5 μm which is sufficient to resolve the thinnest thermal boundary layers near the impaction region. In order to locate the heated surface accurately a wall finding circuit was established
as shown in Fig (2.14). This circuit consists of a 4.7 kΩ current limiting resistor in series with one of the wires of the E-type thermocouple. The two ends of a power supply are connected to the plate (heated surface) and the current limiting resistor. The thermocouple probe descends in small steps of 5 µm. When the probe comes in contact with the plate the circuit is established and a voltage jump (from 1.5 mV to 0.3 V) is recorded. This point is labeled as y=0 or the wall location. Once the wall has been located the wall finding circuit is turned off and the thermocouple ascends traversing the entire thermal boundary layer and the free shear layer up to the step height level.

The temperature profile thus obtained can be used to compute the heat transfer coefficients and the $Nu$ numbers which is sensitive to the choice of the points used to determine the slope of the temperature profile in Eqn 3.10.

### 3.3 Naphthalene Sublimation Technique - Description and Qualification tests

#### 3.3.1 Description

The naphthalene sublimation technique was used in this study to obtain the Sh numbers behind a backward facing step. A detailed discussion of this method can be obtained from Goldstein and Cho (1995)[5]. Table 3.1 lists some basic properties of naphthalene used in this experiment.

The basic steps of the method can be explained in brief as follows:

1. A smooth flat solid naphthalene surface is cast by pouring in the molten naphthalene in the mold of the mass transfer plate discussed in 2.1.3. The procedure to obtain a high quality naphthalene surface is very crucial as it dictates the accuracy of the mass transfer results. It is desirable to have a smooth white non-glossy surface without any surface or under surface cracks. There are various factors that affect the finish of a cast such as mold size, its temperature, molten naphthalene temperature and ambient conditions. Kulkarni (2008)[2] after rigorous experimentation came up with the following values.

   Temperature of the molten naphthalene - 160 C
<table>
<thead>
<tr>
<th>Chemical Formula</th>
<th>( C_{10}H_8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molecular Weight</td>
<td>128.17</td>
</tr>
<tr>
<td>Melting Point</td>
<td>80.2(^\circ)C</td>
</tr>
<tr>
<td>Freezing Point (in air at 1.01325 bar)</td>
<td>80.285(^\circ)C</td>
</tr>
<tr>
<td>Triple Point</td>
<td>80.28(^\circ)C</td>
</tr>
<tr>
<td>Normal Boiling Point (in air at 1.01325 bar)</td>
<td>217.993(^\circ)C</td>
</tr>
<tr>
<td>Critical Temperature</td>
<td>472.5(^\circ)C</td>
</tr>
<tr>
<td>Critical Pressure</td>
<td>40.51 bar</td>
</tr>
<tr>
<td>Critical Volume</td>
<td>(4.13 \times 10^{-4} ) ( m^3/mol )</td>
</tr>
<tr>
<td>Density of Solid at 20(^\circ)C</td>
<td>1175.0 ( Kg/m^3 )</td>
</tr>
<tr>
<td>From Dean (1973)</td>
<td>1145.0 ( Kg/m^3 )</td>
</tr>
<tr>
<td>From Dean (1987)</td>
<td>1162.0 ( Kg/m^3 )</td>
</tr>
<tr>
<td>Density of Liquid at 1.01325 bar</td>
<td></td>
</tr>
<tr>
<td>At 80.28(^\circ)C</td>
<td>0.978 ( Kg/m^3 )</td>
</tr>
<tr>
<td>At 120(^\circ)C</td>
<td>0.946 ( Kg/m^3 )</td>
</tr>
<tr>
<td>At 130(^\circ)C</td>
<td>0.938 ( Kg/m^3 )</td>
</tr>
<tr>
<td>Thermal Conductivity at 25(^\circ)C</td>
<td>0.333 ( W/m-K )</td>
</tr>
<tr>
<td>Enthalpy of Sublimation</td>
<td>70.36 ( KJ/mol )</td>
</tr>
</tbody>
</table>

Table 3.1: Basic Properties of Naphthalene (Kudchadker et al. (1978)[1])

Temperature of the mold - 110 F
Volume of molten napthalene to be used - 150 ml

It was found that following these guidelines repeatedly produced casts of desirable quality. A detailed discussion of the casting procedure can be found in Kulkarni (2008)[2].

2. A surface scanning system together with a linear variable differential transformer (LVDT) (Schaevitz PCA-220-010) probe is used to scan the naphthalene solid surface. The LVDT probe tracks the surface depth at discrete points according to a predefined grid. An indentation on one of the rims of the mass transfer plate serves as the origin for the x-y plane and the rim level itself serves as a base for the height/depth against which the depth/height level at different points can be measured. The temperature of the ambient and the plate are recorded during the process. The scan takes twenty minutes.
3. In the meantime the wind tunnel is started to let it achieve thermal equilibrium with the surroundings. Also ice baths are prepared so that the temperature of the naphthalene surface could be recorded during the experiment.

4. Once the scan is complete the plate is placed inside a sealed plastic container saturated with naphthalene vapor to avoid any sublimation during transportation to the other room in which the wind tunnel is placed. The wind tunnel is shut off and the mass transfer plate is installed in the wind-tunnel.

5. The wind tunnel is then started at a preset speed and the plate is exposed to forced convection. The time of exposure can be varied and depends on the speed of the flow. Typically, a sublimation depth of 60-80 micron is desirable. The sublimation depth should be large enough so that the uncertainties due to natural convection are negligible and small enough so as not to change the profile of the surface and hence the flow and the transport coefficients. The naphthalene surface temperature is monitored using the thermocouples embedded in the cast. Also, recorded are the ambient air temperature inside and outside the wind tunnel, the atmospheric and static pressure differential inside the wind tunnel and the dynamic pressure of the free stream. These measurements are then used to calculate the speed of the flow through the following equations.

\[ P_{atm} = \rho_{Hg} \cdot g \cdot h_{Hg} \quad (3.11) \]

The static pressure inside the wind tunnel can then be calculated from Eqn (3.12) as \( \Delta P \) is measured.

\[ P_{st} = P_{atm} - \Delta P \quad (3.12) \]

Also, the density of the air is calculated using the ideal gas law as shown in the Eqn (3.13)

\[ \rho_{air} = \frac{P_{st}}{R_{air}T} \quad (3.13) \]

6. After the wind-tunnel has run for the allotted time (30-40 mins depending on the speed of the flow) the mass transfer plate is taken out from the wind tunnel and
then placed again into the sealed plastic container to be transported back to the room with the scanning table.

7. The plate is then brought to the scanning room and mounted in the exact same way as the previous scan. The LVDT probe then moves over the surface at the exact same grid points and then measure the new surface depth/heights with respect to the rim surface. The same temperatures as in 2 are recorded again. The Sh numbers at various locations can then be obtained from the depth sublimated, the time of run and naphthalene properties.

8. The last step is to process the data to ascertain the mass transfer coefficients \((h_m)\) at various locations. During the scan the data is written in a way such that each row represents various spanwise locations at a particular streamwise location depending on the row number. The first and the last entry in each row represent the locations on the rims. A straight line is fit between these values and it is with respect to this fit that the voltage difference is calculated for all the interior points. Since the rims are non-sublimating surfaces there won’t be any change in their heights before and after the experiment and hence they can be used as a reference for individual scans against which the surface levels at various points can be measured. Thus the voltage difference at the same point before and after the run (after accounting for the difference with respect to rim voltage) is a measure of sublimation depth. The calibration curve of LVDT as described in section (2.2.3.3) is used for this conversion.

The mass transfer coefficient can be defined as in Eqn 3.14:

\[
h_m = \frac{\dot{m}}{\rho_{v,w} - \rho_{v,\infty}} = \frac{\rho_s \cdot \delta y/\delta \tau}{\rho_{v,w}}
\]  

(3.14)

where \(\rho_s\) is the density of the solid naphthalene and \(\delta y/\delta \tau\) is the rate of sublimation as calculated from the sublimation depth \(\delta y\) and time of run \(\delta \tau\) in the wind tunnel, \(\rho_{v,w}\) is the vapor density at the surface and \(\rho_{v,\infty} = 0\) for no naphthalene in the free stream.

The vapor density of the naphthalene can be calculated using the ideal gas law (Eqn 3.15) provided the vapor pressure and temperature are known.
Also the saturation vapor pressure and temperature for naphthalene have been expressed in a close form relation by Ambrose et al. (1975)\textsuperscript{[31]}. The uncertainty in this relation is $\approx 3.8\%$.

$$T_{n,w} \cdot \log P_{\nu,w} = \frac{a_0}{2} + \sum_{i=1}^{3} a_i E_i(\chi), \quad 230K < T_{n,w} < 344K$$

$$a_0 = 301.6247, \quad \chi = (T_{n,w} - 287)/57$$

$$a_1 = 791.4937, \quad E_1(\chi) = \chi$$

$$a_2 = -8.2536, \quad E_2(\chi) = 2\chi^2 - 1$$

$$a_3 = 0.4042, \quad E_3(\chi) = 4\chi^3 - 3\chi$$

Once the mass transfer coefficient has been evaluated $Sh$ numbers can be calculated using Eqn 3.17:

$$Sh = \frac{h_m H}{D_{na}}$$

where $H$ represents the step height (2 cm) and $D_{na}$ is the binary diffusion coefficient of naphthalene into air.

The diffusion coefficient of naphthalene is very sensitive to temperature and pressure variation can be calculated using Eqn 3.19, an average of the individual relations given by Cho (1989)\textsuperscript{[32]} and Chen and Wung (1996)\textsuperscript{[33]}.

$$D_{na}^{298.16} = 6.81 \times 10^{-6}, m^2/s$$

$$D_{na} = D_{na}^{298.16} \cdot \left[\frac{T_{n,w}}{298.16}\right]^{1.93} \left[\frac{1.015 \times 10^5}{P_{atm}}\right], m^2/s$$
3.3.2 LVDT Calibration

As already mentioned a LVDT probe is used to quantify the depth change in the naphthalene surface before and after the experiment in the wind tunnel. The output of a LVDT is a voltage signal which needs to be related to its displacement. The linear relationship between the two variables is obtained via calibration. Eleven gage blocks in the size range of 2.69 mm (106 mil) to 3.20 mm (126 mil) are placed one by one under the LVDT probe to cause different displacements. The gage blocks are handled using latex gloves to avoid any thermal expansion due to body heat and cleaned using a cotton swab dipped in alcohol. The voltage signal of the LVDT is recorded and then the least squares method is used to obtain the a linear fit between the displacement and the voltage signal of the LVDT. Once the calibration curve has been obtained it is adjusted so that 0V corresponds to zero displacement. The calibration curve is shown in Fig 3.5. The net uncertainty in the displacement from the calibration is ±0.1610µm which is ∼0.3% of the typically observed sublimation depths of ∼50µm.

3.3.3 Mock Sublimation

The repeatability in mounting the mass transfer plate is of utmost significance as it is desired that the LVDT probe scans the exact same point on the naphthalene surface before and after the exposure to forced convection in the wind tunnel. Hence it is important to quantify the uncertainty in mounting the plate on the scanning table. A mock sublimation test is run wherein a polished aluminum plate is mounted on the scanning table, scanned once, unmounted, mounted back and then scanned again. The data is then crunched in the exact same fashion as for a mass transfer experiment. Since the metallic surface represents a non sublimating surface the difference in the two sets of measurements indicates the uncertainty due to misalignment of the plate in the two runs. The distribution of this fictitious depth change is gaussian as can be seen from the Fig 3.6. The uncertainty in misalignment is found to be ±0.24µm which is ∼0.5% of the typically observed sublimation depths of ∼50µm.
3.3.4 Natural Convection Correction

During a typical mass transfer experiment the naphthalene plate is exposed to natural convection as well. The natural convection loss is quantitatively significant during the surface scanning process, marginal during the mounting of the plate inside the wind tunnel and negligible when carrying the plate inside the sealed container. In order to account for the loss during the scanning process a 12- hr natural convection test was carried out. The plate was mounted on the surface scanning system and scans were done after every two hours to record the amount of naphthalene sublimated. An average sublimation rate of $1.1 \mu m/hour$ was observed. With this rate, an average of $0.7 \mu m$ of naphthalene is lost due to natural convection which is $\sim 1.5\%$ of the typically observed sublimation depths of $\sim 50 \mu m$.

The natural convection rate is strongly dependent on the naphthalene surface temperature and the ambient temperature. The natural convection rate is appropriately adjusted before the correction is applied to the experiment. The total time of exposure $\Delta \tau_{nc}$ to natural convection is calculated and is then multiplied by the adjusted natural convection rate to account for the change in sublimation depth due to natural convection. (Eqn (3.20))

$$\Delta y_{nc} = \frac{\delta y}{\delta t} \cdot \Delta \tau_{nc} \quad (3.20)$$

The depth change thus obtained is then subtracted from the total depth change to give the depth change due to forced convection inside the wind tunnel. (Eqn (3.21))

$$\Delta y_{i,j,corr} = \Delta y_{i,j,meas} - \Delta y_{nc} \quad (3.21)$$
Figure 3.1: Hot Wire Calibration, Sep 13, 2010
Figure 3.2: Schematic Diagram of Thermocouple Calibration Facility, Heat Transfer Lab (Kulkarni (2008) [2])
Figure 3.3: Thermocouple Probe Calibration, Feb 1, 2010
Figure 3.4: Schematic Diagram of Wall Finding Technique for Thermocouple Probe (Kulkarni (2008)[2])
Figure 3.5: LVDT Calibration, Displacement Vs Voltage, Mar 21, 2009
Figure 3.6: Mock Sublimation Test, Apr 2, 2009
Figure 3.7: Natural Convection Correction, Aug 27, 2009 (a) Spatially Averaged Depth Change Vs Time (b) Sublimation Rate Vs Time
Chapter 4

Velocity Measurements

As discussed in section 3.1.2 the velocity measurements were made using the hotwire probe upstream of the step to qualify the boundary layer prior to flow separation at the edge of the step.

4.1 Boundary Layer Characteristics

Fig 4.1 shows the velocity profiles at $x=-12.7$ mm and spanwise locations of $z/H = -3, 0, 3$ for the two freestream speeds of $u_\infty \approx 12.5$ m/s and $u_\infty \approx 17.5$ m/s. It can be seen that the flow is slightly accelerated just outside the boundary layer because of the contraction in the test section due to the ramp-up to the step. The extreme $z/H$ values of -3 and 3 ensure that the entire plate is spanned. From the plots it can be observed that the flow is essentially two dimensional prior to separation. In this study the maximum speed outside the boundary layer was used as a reference speed for quantification of flow conditions. Some characteristic details of the boundary layers at $x=-12.7$ mm have been delineated in Tables 4.1 and 4.2 for the two freestream speeds.

For a direct comparison with the law of the wall the velocity profile must be plotted in wall coordinates ($u^+$ vs $Y^+$). In order to obtain the wall coordinates, two quantities - friction velocity, $(u_\tau = \sqrt{\tau/\rho})$ and precise wall normal locations of velocity measurement - are required. Due to the uncertainty in wall location a displacement correction, $Y_{shift} \sim 100\mu m$ was introduced. A shooting technique was used to estimate the value
of $Y_{shift}$. The recorded location of measurement, $y$ was then corrected to $y + Y_{shift}$. To obtain $u_{r}$, Clauser technique was used. The values of $u_{r}$ were varied between the typically observed values of 0.4-1.5.

Table 4.2: Spanwise variation of boundary layer characteristics, $x$=-12.7 mm, $u_{\infty} \approx 17.5m/s$

<table>
<thead>
<tr>
<th>$z/H$</th>
<th>$\delta_1$ (mm)</th>
<th>$\delta_2$ (mm)</th>
<th>Shape Factor $\frac{\delta_1}{\delta_2}$</th>
<th>$\frac{\delta_2}{H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>1.09</td>
<td>0.85</td>
<td>1.29</td>
<td>0.043</td>
</tr>
<tr>
<td>0</td>
<td>1.21</td>
<td>0.91</td>
<td>1.33</td>
<td>0.046</td>
</tr>
<tr>
<td>3</td>
<td>1.15</td>
<td>0.85</td>
<td>1.35</td>
<td>0.043</td>
</tr>
</tbody>
</table>

Once $u_{r}$ and $Y_{shift}$ were known the velocity profile can be plotted in wall coordinates as shown in Fig 4.2. The plots show the velocity profiles measured at $x$=-12.7 mm, $z$=0 for both the speeds. It can be observed that the profiles at both the speeds collapse well onto the Gersten and Herwig profile (Schlichting and Gersten (2004) [34]) as is expected for a turbulent boundary layer. Since the flow is accelerated the wake regions appear diminished in both the plots. For a further reassurance the velocity fluctuations were plotted in wall coordinates for both the speeds as shown in Fig 4.3. The typical turbulent profiles show peak of $u' = 2.5$ at $Y' = 15$ (Purtell et al. (1997) [35] and Marusic et al. (1998) [36]). The plots agree well with the expected trend.
4.2 Determination of Virtual Origin of Turbulent Boundary Layer

Additional velocity profile measurements were made along the central plane of the test section (z=0), as summarized in Tables 4.3 and 4.4 to track the development of the boundary layer prior to flow separation. For both the speeds considered in this study, the boundary layer momentum thickness, $\delta_2$ at $x = -12.7\, \text{mm}$ is found out to be $\sim 0.90\, \text{mm}$ which is $\sim 0.045H$ and the freestream turbulence intensity is $\sim 0.4\%$. In addition, it can be observed from the Tables 4.3 and 4.4 that the shape factor for both the flow conditions is $\sim 1.3$ which compares well with the expected value of 1.29 for a fully developed turbulent boundary layer.

<table>
<thead>
<tr>
<th>x (mm)</th>
<th>$\delta_1$ (mm)</th>
<th>$\delta_2$ (mm)</th>
<th>Shape Factor</th>
<th>$\frac{\delta_1}{\delta_2}$</th>
<th>$\frac{\delta_2}{H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-12.70</td>
<td>1.17</td>
<td>0.90</td>
<td>1.30</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>-32.70</td>
<td>1.14</td>
<td>0.88</td>
<td>1.30</td>
<td>0.044</td>
<td></td>
</tr>
<tr>
<td>-52.70</td>
<td>1.11</td>
<td>0.85</td>
<td>1.31</td>
<td>0.043</td>
<td></td>
</tr>
<tr>
<td>-72.70</td>
<td>1.11</td>
<td>0.83</td>
<td>1.34</td>
<td>0.041</td>
<td></td>
</tr>
<tr>
<td>-90.70</td>
<td>1.02</td>
<td>0.77</td>
<td>1.32</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td>-102.70</td>
<td>0.97</td>
<td>0.72</td>
<td>1.34</td>
<td>0.036</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: Flow details prior to boundary layer separation, $z=0$ $u_\infty \approx 12.46\, \text{m/s}$

<table>
<thead>
<tr>
<th>x (mm)</th>
<th>$\delta_1$ (mm)</th>
<th>$\delta_2$ (mm)</th>
<th>Shape Factor</th>
<th>$\frac{\delta_1}{\delta_2}$</th>
<th>$\frac{\delta_2}{H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-12.70</td>
<td>1.19</td>
<td>0.90</td>
<td>1.32</td>
<td>0.045</td>
<td></td>
</tr>
<tr>
<td>-32.70</td>
<td>1.15</td>
<td>0.88</td>
<td>1.30</td>
<td>0.044</td>
<td></td>
</tr>
<tr>
<td>-52.70</td>
<td>1.09</td>
<td>0.84</td>
<td>1.30</td>
<td>0.042</td>
<td></td>
</tr>
<tr>
<td>-72.70</td>
<td>1.01</td>
<td>0.78</td>
<td>1.29</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td>-90.70</td>
<td>0.99</td>
<td>0.76</td>
<td>1.30</td>
<td>0.038</td>
<td></td>
</tr>
<tr>
<td>-102.70</td>
<td>1.01</td>
<td>0.74</td>
<td>1.36</td>
<td>0.037</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4: Flow details prior to boundary layer separation, $z=0$ $u_\infty \approx 17.47\, \text{m/s}$

Also, from the Tables 4.3 and 4.4 it is possible to determine the virtual origin of the turbulent boundary layer. The virtual origin is defined as the point where the turbulent
boundary layer would have originated had there been no transition from laminar to turbulence. A turbulent boundary layer is expected to follow a relation of the kind as shown in Eqn [4.1]

\[ \frac{u}{u_\infty} = \left( \frac{y}{\delta} \right)^{1/n} \]  

(Eq. 4.1)

Employing the definitions of displacement and momentum thickness, we can extend the Eqn 4.1 to have the following relations:

\[ \frac{u}{u_\infty} = k_1 \left( \frac{y}{\delta_1} \right)^{1/n} \]
\[ \frac{u}{u_\infty} = k_2 \left( \frac{y}{\delta_2} \right)^{1/n} \]  

(Eq. 4.2)

The value of ‘n’ can be obtained by plotting the velocity profile in log-log scale and determining the slope of the linear region. The details of the technique can be found out in Ghosh 2009 [3]. The activity was performed for both the speeds as shown in Figs 4.4 and 4.5 ‘n’ values of 5.36 and 5.40 were obtained for \( u_\infty \approx 12.5m/s \) and \( u_\infty \approx 17.5m/s \) respectively.

The variation of \( \delta_1 \) and \( \delta_2 \) is proportional to \( x^{(n+1)/(n+3)} \) [3]. Utilizing this fact we can track the virtual origin as is depicted in Fig 4.6 for the two speeds. It was found that the virtual origin for the turbulent boundary layer is located at \( x=-21.51H \) and \( x=-18.64H \) for \( u_\infty \approx 12.5m/s \) and \( u_\infty \approx 17.5m/s \) respectively. In addition the extrapolated values of \( \delta_2 \) at the step edge, \( x=0 \), could be found out as 0.92mm (0.046H) and 0.91mm (0.046H) for \( u_\infty \approx 12.5m/s \) and \( u_\infty \approx 17.5m/s \) respectively.
Figure 4.1: Velocity profile at x=-12.7 mm for two different free stream speeds (a) $u_\infty \approx 12.5\, m/s$ (b) $u_\infty \approx 17.5\, m/s$
Figure 4.2: Velocity Profile in Wall Coordinates (x=-12.7 mm, z=0) (a) $u_\infty \approx 12.5 m/s$
(b) $u_\infty \approx 17.5 m/s$
Figure 4.3: Velocity Fluctuations in Wall Coordinates ($x=-12.7$ mm, $z=0$) (a) $u_\infty \approx 12.5\text{m/s}$ (b) $u_\infty \approx 17.5\text{m/s}$
Figure 4.4: Velocity Profile at six different streamwise locations, $u_\infty \approx 12.5 \text{m/s}$, $n=5.36$
Figure 4.5: Velocity Profile at six different streamwise locations, $u_\infty \approx 17.5m/s$, $n=5.4$
Figure 4.6: Determination of Virtual Origin for the two speeds (a) $u_\infty \approx 12.5 m/s$, $n=5.36$ (b) $u_\infty \approx 17.5 m/s$, $n=5.40$
Chapter 5

Mass Transfer Measurements

5.1 Summary of Experiments

As section 2.6 states a plexiglas bracket with o-rings was used in conjunction with the mass transfer plate for the naphthalene sublimation experiments. This was necessary to seal the test section completely. Earlier mass transfer experiments had not yielded consistent data. Fig 5.1 shows three (of ~ 50) such mass transfer experiments done prior to using the plexiglas bracket with o-rings. One important observation from the plot was the fact that the variability in the Sh is confined only to the recirculation region. Downstream of reattachment the data are in excellent agreement. Also, it can be observed that the point of peak Sh varies a little (~ 0.3H) across the three runs.

This led us to believe that there might be a leakage in the wind tunnel. Since the same slot was used to house both heat and mass transfer plates, it is possible that there is some gap between the plate and the windtunnel bottom. In addition, since it is a suction type wind tunnel we expect a drift of air to get into the wind tunnel. This drift would have a considerable effect on the recirculation region because of the low momentum fluid there. Injection of extra fluid with x-momentum would increase the Sh and would move the reattachment point upstream ([37] and [38]).

This speculations was confirmed when the smoke from an incense stick was used to detect the leak in the wind tunnel bottom right where the plexiglas plate at the step
mates with the leading edge of the mass-transfer plate. In order to ensure that the leak is sealed different things were tried and multiple experiments were done across these trials. Some things that were tried are listed below:

1. Scotch tape along the bottom edge of the mass transfer plate
2. Duct tape along the bottom edge of the mass transfer plate
3. Neoprene between the mass transfer plate and the plexiglas.

None of these yielded consistent data. Eventually a plexiglass bracket with o-rings was tried. The following table presents a summary of the experiments done with the final configuration, which resulted in repeatable data.

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Date Performed</th>
<th>Free Stream Speed (m/s)</th>
<th>Re$_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>12/05/2009</td>
<td>12.5</td>
<td>16076</td>
</tr>
<tr>
<td>M2</td>
<td>12/06/2009</td>
<td>12.5</td>
<td>16231</td>
</tr>
<tr>
<td>M3</td>
<td>12/14/2009</td>
<td>12.5</td>
<td>16197</td>
</tr>
<tr>
<td>M4</td>
<td>12/18/2009</td>
<td>17.5</td>
<td>22365</td>
</tr>
<tr>
<td>M5</td>
<td>12/30/2009</td>
<td>17.5</td>
<td>22260</td>
</tr>
<tr>
<td>M6</td>
<td>12/31/2009</td>
<td>17.5</td>
<td>22651</td>
</tr>
</tbody>
</table>

Table 5.1: List of Mass Transfer Experiments

5.2 Key Characteristics of Sh variation over the Naphthalene Surface

5.2.1 Spanwise Characteristics

The variation of Sherwood number (Sh) over the naphthalene surface has been shown in Figs 5.2a - 5.7a for the two speeds. It can be seen that for $u_\infty \approx 12.5m/s$ there are two peaks in the Sh profile, separated by a distance of $\sim 3H$ in the spanwise direction in the recirculation region. However for $u_\infty \approx 17.5m/s$ only one peak is observed. This can be better realized by looking at the spanwise variation of Sh at different streamwise locations as represented in Figs 5.2b - 5.7b. The difference between the maximum and
the minimum Sh values in the reattachment region (x=5H-7H) was found out to be
\(~ 12.28\%\) and \(~ 11.55\%\) of the spanwise average (average of middle one-fourth of the plate) of Sh for the speeds of 12.5 m/s and 17.5 m/s, respectively. This spanwise variation in the Sh in the recirculation region is attributable to the three dimensionality of the flow caused by the presence of streamwise vortex structures, confirmed by Furuichi and Kumada (2002)\[39\]. These three dimensional structures have also been confirmed in flow over cavities by Maull and East (1963)\[40\]. The authors have proposed a correlation for the structure size based on the cavity depth and chord (length in the streamwise direction).

5.2.2 Streamwise Characteristics

The middle one fourth (-.8H to 0.8H) of the plate was considered as the two dimensional zone and the corresponding Sh numbers were averaged to obtain a unique value at each stream wise location. This variation of the Sherwood number (Sh) vs the streamwise coordinate (x) non dimensionised by the step-heigh (H) have been shown in Fig 5.8a for free stream speeds of 12.5 m/s and 17.5 m/s. The separated shear layer impacts somewhere close to the point of maximum Sh. Both upstream and downstream of the reattachment a concentration boundary layer starts developing. As the flow progresses farther from the impaction point the boundary layer thickens and hence the Sh number decreases. However upstream of the reattachment the boundary layer separates again against the vertical wall of the step, somewhat analogous to separation over a rearward facing step, leading to another recirculation bubble close to the step.

At this point of separation, though instead of impinging, the fluid is pulled away from the wall and hence corresponds to a minima in the mass transfer. Also as expected at higher speeds a higher value of Sh number is obtained. The location and values of maximum and minimum Sh are reported in Table 5.2 for all the mass transfer runs. It must be noted that these values were obtained by using spline interpolation between the data points. Thus it can be concluded from this table that the secondary recirculation bubble just adjacent to the step is \( \approx 0.5H \) long and primary recirculation zone extends around \( \approx 5.6H \) downstream of the step. The locations of minima and maxima have a standard deviation of \(~ 0.1H\) across the experiments. The average minimum Sh observed at speeds of 12.5 m/s and 17.5 m/s are 30.93 and 40.28, respectively, and have
Table 5.2: Location of Minimum and Maximum Sh numbers

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>$Re_H$</th>
<th>Location of min Sh ($x/H$)</th>
<th>Minimum Sh</th>
<th>Location of max Sh ($x/H$)</th>
<th>Maximum Sh</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>16076</td>
<td>0.48</td>
<td>31.33</td>
<td>5.62</td>
<td>102.71</td>
</tr>
<tr>
<td>M2</td>
<td>16231</td>
<td>0.56</td>
<td>32.40</td>
<td>5.62</td>
<td>101.35</td>
</tr>
<tr>
<td>M3</td>
<td>16197</td>
<td>0.51</td>
<td>29.06</td>
<td>5.58</td>
<td>97.61</td>
</tr>
<tr>
<td>M4</td>
<td>22365</td>
<td>0.68</td>
<td>38.48</td>
<td>5.76</td>
<td>122.93</td>
</tr>
<tr>
<td>M5</td>
<td>22260</td>
<td>0.51</td>
<td>40.86</td>
<td>5.48</td>
<td>124.05</td>
</tr>
<tr>
<td>M6</td>
<td>22651</td>
<td>0.76</td>
<td>41.50</td>
<td>5.40</td>
<td>124.47</td>
</tr>
</tbody>
</table>

a standard deviation of 5.6% and 3.9%, respectively. The correspoding maximum values are 100.56 and 123.82 with standard deviations of 2.6% and 0.6%, respectively.

Fig. 5.8b shows all the six data sets collapsing onto a single curve when Sh numbers in every profile are non dimensionalized by the maximum Sh value of the corresponding profile. This indicates a self similarity in the profiles with respect to the speed of the approaching flow.
Figure 5.1: Inconsistent Sh data at $u_\infty \approx 12.5 \text{m/s}$, NOT USED
Figure 5.2: Sh variation over the naphthalene surface at $u_\infty \approx 12.5 m/s$ (Exp M1)
Figure 5.3: Sh variation over the napthalene surface at $u_\infty \approx 12.5m/s$ (Exp M2)
Figure 5.4: Sh variation over the naphthalene surface at $u_\infty \approx 12.5 m/s$ (Exp M3)
Figure 5.5: Sh variation over the naphthalene surface at $u_\infty \approx 17.5\text{m/s}$ (Exp M4)
Figure 5.6: Sh variation over the napthlene surface at $u_\infty \approx 17.5 m/s$ (Exp M5)
Figure 5.7: Sh variation over the napthalene surface at $u_\infty \approx 17.5m/s$ (Exp M6)
Figure 5.8: Streamwise variation of ShVs x/H at two speeds
Chapter 6

Heat Transfer Measurements

6.1 Summary of Experiments

The following table presents a summary of heat transfer experiments.

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Date Performed</th>
<th>Free Stream Speed (m/s)</th>
<th>$Re_H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>05/18/2010</td>
<td>12.5</td>
<td>16188</td>
</tr>
<tr>
<td>H2</td>
<td>05/25/2010</td>
<td>12.5</td>
<td>16276</td>
</tr>
<tr>
<td>H3</td>
<td>05/20/2010</td>
<td>17.5</td>
<td>22512</td>
</tr>
<tr>
<td>H4</td>
<td>05/21/2010</td>
<td>17.5</td>
<td>22809</td>
</tr>
</tbody>
</table>

Table 6.1: List of Heat Transfer Experiments

6.2 Temperature Profile Characteristics

Fig.6.1 shows the non dimensionalised temperature profiles plotted for the two speeds. The temperature profiles at both the speeds show excellent repeatability except for the region around $x=4.16H$ for $u_\infty \approx 17.5$ m/s. It is evident from the profiles that there are two zones of temperature drop - the wall thermal boundary layer and the free shear layer that originates from the edge of step. Progressing downstream from the step the temperature drop across the free shear layer keeps diminishing, being highest at around 35-40% closest to the step. This is because the convective mixing in the recirculation
region and the free shear layer smoothens out the temperature gradients. The same observation was made by Aung and Goldstein (1971) [18], Vogel and Eaton (1985) [41] and Terekhov et al. (2003) [42]. Terekhov et al. (2003) [42] also showed that there is a local minima in the temperature profile in the region where the fluid is turned between the two recirculation regions. This local minima is caused by the vortex transfer of heated fluid away from the wall and can be observed at a streamwise location of less than 2H in the present study. In addition, it can be observed from Fig 6.2 that the non-dimensionalised temperature profiles for both the speeds seem to overlay. Also, from the nature of temperature profiles at both the speeds, it can be ascertained that the point of reattachment is somewhere in between 5.8H and 6.8H. The temperature profile at x=5.8H depicts an inflection point which is not observed in the temperature profile at x=6.8H indicating that the two temperature drop zones have merged and the region of convective mixing (the primary recirculation bubble) has disappeared. Thus we can conclude that the point of maximum heat transfer for this study is upstream of the point of reattachment. Sparrow et al. (1987) [43] found that the reattachment point can be either upstream or downstream of the point of maximum heat/mass transfer. The relative location depends on the interplay between two factors - the magnitude of the wall adjacent streamwise velocity and the wall normal velocity. There is a local flow acceleration (increase in wall adjacent velocity) as the flow moves upstream because the streamlines squish together. Also, the transverse component of velocity tends to decrease in magnitude. Thus the point of peak transport coefficient occurs where the combination of the two effects is maximized. For long separation bubbles, it is believed that the wall adjacent streamwise velocity is the dominating factor and hence \( x_{\text{max}} < x_r \).

From the above discussion it is clear that the zone governing the heat transfer in the reattachment region is the near wall conduction layer region. Hence a higher resolution was used near the wall while acquiring temperature data. Fig. 6.3 shows a sample temperature profile obtained using the thermocouple traverse technique. The first part of the profile which shows a nearly constant temperature zone is the part when the thermocouple rests on the wall. The first significant temperature drop is taken to the inception of the conduction/linear region close to the wall. The details of the method used to obtain the Nu values can be obtained from Qiu et al. (1995) [44]. A minimum of 5
points were taken to determine the slope of the temperature profile in the linear region.

### 6.3 Nu variation over the Streamwise Coordinate

Following the technique listed in Qiu et al. (1995)\[44\], the Nusselt (Nu) numbers from the temperature profiles can be calculated at different streamwise locations as mentioned in section 3.2.2. Fig 6.4 show a plot of Nu vs non dimensionalised streamwise distance from the step (x/H) for two speeds respectively. It can be observed that the nature of variation of Nu is the same as is for Sh. This is because an analogous mechanism governs the heat transfer.

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>Date Performed</th>
<th>Location of min Nu (x/H)</th>
<th>Minimum Nu</th>
<th>Location of max Nu (x/H)</th>
<th>Maximum Nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>16188</td>
<td>0.68</td>
<td>17.87</td>
<td>5.48</td>
<td>67.70</td>
</tr>
<tr>
<td>H2</td>
<td>16276</td>
<td>0.74</td>
<td>16.09</td>
<td>5.32</td>
<td>69.79</td>
</tr>
<tr>
<td>H3</td>
<td>22512</td>
<td>0.48</td>
<td>24.49</td>
<td>5.17</td>
<td>83.97</td>
</tr>
<tr>
<td>H4</td>
<td>22809</td>
<td>0.43</td>
<td>21.78</td>
<td>5.72</td>
<td>84.30</td>
</tr>
</tbody>
</table>

Table 6.2: Location of Minimum and Maximum Sh numbers

The separated shear layer impacts downstream of the point of maximum Nu. Upstream of the impaction a temperature boundary layer starts developing. Adams, Johnston and Eaton (1984)\[45\] have shown that this near wall layer below the recirculation region develops as a laminar sublayer. As the flow moves upstream the boundary layer thickens which corresponds to a decrease in the Nu. Further upstream this near wall layer separates against the vertical wall of the step, somewhat analogous to separation over a rearward facing step, leading to another recirculation bubble close to the step. At this point of separation, though instead of impinging, the fluid is pulled away from the wall and hence corresponds to a point of minima in the heat transfer profile. Downstream of reattachment the sublayer grows and thickens and thus accounts for the reduction in the Nu values there. In the reattachment region the Nu values are governed by the turbulence intensity of the approaching shear layer. Vogel and Eaton (1985)\[41\]
corroborate this by demonstrating a good correlation between the variations of Stanton number and skin friction fluctuations.

In addition some key characteristics of the Nu profiles are delineated in the Table 6.2. To obtain the minima for Nu, a spline interpolation was used. However, to ascertain the location and value of the maxima a third order polynomial was fit through the points in the vicinity of peak Nu. A spline interpolation would not have given physical values due to the scatter in the Nu data at the reattachment. The mean locations of Nu minima and maxima across the four experiments are found to be at 0.58H and 5.42H respectively with corresponding standard deviations of ∼ 0.15H and ∼ 0.24H. In addition the average minimum Nu observed at speeds of 12.5 m/s and 17.5 m/s are 16.98 and 23.13, respectively, and have standard deviations of 7.4% and 8.3%, respectively. The corresponding maximum values are 68.74 and 84.13 with standard deviations of 2.15% and 0.3%, respectively. More variability in the location of maximum and minimum Nu as compared to the corresponding values for the Sh are attributable to the extremely thin boundary layers in the recirculation region which make the heat transfer coefficient and hence Nu measurements complex.
Figure 6.1: Temperature Profiles at different Streamwise Locations for (a) $u_\infty \approx 12.5$ m/s and (b) $u_\infty \approx 17.5$ m/s.
Figure 6.2: Collapse of temperature profiles for both the speeds
Figure 6.3: Sample Temperature Profile at x=3.76 H or 75.2 mm
Figure 6.4: Streamwise variation of Nusselt (Nu) Vs x/H at two speeds
Chapter 7

Heat/Mass Transfer Analogy

7.1 Analogy Factor Calculation Method

Once the heat and mass transfer data has been obtained it is intended to develop a correlation of the form

\[ Nu = C \cdot Re^m \cdot Pr^n \]  \hspace{1cm} (7.1)

\[ Sh = C \cdot Re^m \cdot Sc^n \]  \hspace{1cm} (7.2)

The job at hand is to determine the values of m and n. The analogy factor can then be obtained as \((\frac{Pr}{Sc})^n\). In order to obtain the analogy factor the following methodology was undertaken.

1. Obtain 'm': Both mass transfer and heat transfer data were used separately to obtain 'm' in the constitutive relation. At each streamwise location \(Sh/Re^m\) was calculated for all the mass transfer profiles. The range of these values was then divided by the mean to get a percentage scatter at every location. This percentage scatter was then averaged to obtain one representative percentage scatter value for all locations across all the mass transfer profiles. The value of 'm' which minimized this percentage scatter was then chosen. The same procedure was followed with the Nu data. The collapsed heat and mass transfer profiles can be seen in Fig 7.1.
It must be noted that the two sets of data collapse for slightly different values of m - 0.650 for mass transfer and 0.642 for heat transfer. The ratio \( \left( \tilde{R}e_{12.46}/\tilde{R}e_{17.47} \right)^m \) differs by less than 0.5% for the two values of ‘m’.

2. Obtain ‘n’ : Two values of ‘n’ were calculated - one at each freestream speed. A fifth order polynomial fit was used to interpolate Nu values for the locations where Sh number was known from mass transfer experiments. At these locations, \( Sh/Sc^n \) and \( Nu/Pr^n \) were calculated for all five (three - mass transfer and two - heat transfer) profiles at \( u_\infty=12.46 \) m/s. An average percentage scatter was then obtained for all the locations across all these five profiles in a manner similar to the one in step 1. The value of ‘n’ which minimized the mean percentage scatter across these five profiles was chosen as the representative value of ‘n’ for \( u_\infty=12.46 \) m/s. A similar activity was performed for the other five profiles at \( u_\infty=17.47 \) m/s. The ‘n’ values thus obtained are 0.310 and 0.319 for freestream speeds of 12.46 m/s and 17.47 m/s respectively.

The analogy factor, F, can be calculated for the two speeds as :

\[
F = \frac{Nu}{Sh} = \left( \frac{Pr}{Sc} \right)^{0.310} = 0.696 \tag{7.3}
\]

\[
F = \frac{Nu}{Sh} = \left( \frac{Pr}{Sc} \right)^{0.319} = 0.689 \tag{7.4}
\]

The two values are within 2% of each other. The mean F value for this study is 0.692. The collapsed data for the two speeds can be seen in Fig 7.2.

### 7.2 Comparison with Literature

It was argued in section 1.5 that the maximum values of Nu and Sh could be used to estimate the analogy factor provided the dependence on Pr and Sc does not vary with location. This can be proved as follows.

Let us say that

\[
\frac{Nu}{Nu_{max}} = G(x, Re_{x_c}, Pr)
\]
\[ \frac{Sh}{Sh_{\text{max}}} = H(x, Re_{x_r}, Sc) \quad (7.5) \]

Also, if

\[ \frac{Nu}{Nu_{\text{max}}} = \frac{Sh}{Sh_{\text{max}}} \]

\[ \Rightarrow G(x, Re, Pr) = H(x, Re_{x_r}, Sc) \]

\[ \Rightarrow G(x, Re, Pr) = H(x, Re_{x_r}, Sc) = K(x, Re_{x_r}) \quad (7.6) \]

as \( G \) and \( H \) cannot be equal unless they are functions on \( x \) and \( Re \) only. In addition, for a particular speed, \( Re = C \), a constant. Thus:

\[ \frac{Nu}{Nu_{\text{max}}} = \frac{Sh}{Sh_{\text{max}}} = K(x, Re_{x_r}) = K(x, C) = Z(x) \quad (7.7) \]

\[ \Rightarrow \frac{Nu}{Sh} = \frac{Nu_{\text{max}}}{Sh_{\text{max}}} \quad (7.8) \]

Fig 7.3 shows that \( \frac{Nu}{Nu_{\text{max}}} = \frac{Sh}{Sh_{\text{max}}} \) for the two speeds and hence the correlations proposed by A.I. Leont’ev et al. [26] and Ota and Nishiyama [27] can be used for comparison with the present study. As per section 1.5 the analogy factors predicted by Leont’ev et al. [26] and Ota and Nishiyama’s [27] correlations are 0.604 and 0.677 which differ by 13% and 2% respectively from the value of analogy factor obtained from this study. The collapsed data with the values obtained from this study can be seen in Fig 7.4 for the two speeds. In addition a comparison with literature has been shown in Fig 7.5.

### 7.3 Correlation for Maximum \( Nu, Sh \)

#### 7.3.1 Correlation Proposition

From the data obtained in this study the correlation in Eqn 7.10 is proposed to estimate the maximum values of \( Nu \) and \( Sh \).
\[ Nu_{\text{max}} = 0.142 R_e^{0.65} P_r^{0.315} \]
\[ Sh_{\text{max}} = 0.142 R_e^{0.65} S_c^{0.315} \]  
(7.9)  
(7.10)

There are some important observations key points about this correlation:

1. Note the scaling of 0.65 on Re. Earlier it was mentioned that Nu and Sh scale with \( R_e^{0.642} \) and \( R_e^{0.65} \) respectively. There is no physical argument which could underscore the difference in the powers. The experimental uncertainty can lead to a slightly different scaling. In the vicinity of the location of peak Nu the thermal boundary layer is expected to be extremely thin leading to greater uncertainty in Nu. Hence the Sh scaling of \( R_e^{0.65} \) is used in the correlation.

2. Note the scaling of 0.315 on Pr (Sc). This is slightly different from the value of 1/3, exponent of Pr(Sc) in the correlation used by Ota and Nishiyama\[27\]. As argued by Adams et al.(1984)\[45\], the near wall layer below the recirculation region develops as a laminar sublayer. This sublayer thickens as the flow moves upstream until it separates against the vertical wall of the step, somewhat analogous to separation over a rearward facing step, leading to another recirculation bubble close to the step. Sparrow et al. \[43\] argued that the magnitude of the wall adjacent velocity upstream of reattachment point increases first, as can be visualized by the squish of the elliptical streamlines in the recirculation region and then decreases. Thus though an exponent of Pr (Sc) close to 1/3 is expected, the exact value of 1/3 is a bit too far fetched.

3. The fact that this correlation is based on the step height based Reynolds number underscores the importance of it as there is no need to quantify the reattachment length before a value of peak Nu (Sh) could be estimated as is required in correlations proposed by Ota and Nishiyama\[27\] and A.I.Leont’ev et al. \[26\].

### 7.3.2 Comparison of Proposed Correlation with Literature

Ota and Nishiyama \[27\] had assumed the coincidence of point of reattachment and the location of peak Nu, Sh. As has been argued by Sparrow et al. \[43\] the two locations can
be different. For thoroughness, the values from the present study have been compared with the two correlations using both $x_{\text{max}}$ and $x_r$ as the characteristic lengths. While $x_{\text{max}} \sim 5.6H$ as is known from the experiments, a value for the reattachment length, $x_r$ must be estimated to facilitate a comparison between the results of this study and the correlations proposed by Ota and Nishiyama\cite{27} and A.I.Leont’ev et al. \cite{26}.

Seung Koo et al. (2000)\cite{46} performed flow visualization for turbulent flow over a backward facing step. They used a freestream speed of 12m/s and a step height of 25 mm. From their experiments the time averaged reattachment length was found out to be 6.7H, well within the expected range of 6H-8H. Due to a similar freestream speed of 12 m/s used by them, the reattachment length for this study is estimated to be 6.7H. For the other set the same value of reattachment length is used since the locations of peak Nu and peak Sh do not vary across the two speeds and hence the Sh and Nu profiles at the two speeds are expected to be geometrically similar. Table 7.1 shows the comparison of the peak Nu and Sh values at the two speeds.

<table>
<thead>
<tr>
<th>Nu or Sh</th>
<th>$Re$</th>
<th>Present Study</th>
<th>Ota and Nishiyama, $x_r$ \cite{27}</th>
<th>Ota and Nishiyama, $x_{\text{max}}$ \cite{27}</th>
<th>A.I.Leont’ev et al., $x_r$ \cite{26}</th>
<th>A.I.Leont’ev et al., $x_{\text{max}}$ \cite{26}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nu</td>
<td>16168</td>
<td>69.26</td>
<td>56.92</td>
<td>60.42</td>
<td>43.54</td>
<td>45.78</td>
</tr>
<tr>
<td>Sh</td>
<td>16168</td>
<td>100.51</td>
<td>84.07</td>
<td>89.27</td>
<td>72.04</td>
<td>75.74</td>
</tr>
<tr>
<td>Nu</td>
<td>22425</td>
<td>85.67</td>
<td>70.37</td>
<td>75.11</td>
<td>55.11</td>
<td>57.94</td>
</tr>
<tr>
<td>Sh</td>
<td>22425</td>
<td>123.88</td>
<td>104.5</td>
<td>110.97</td>
<td>91.17</td>
<td>95.87</td>
</tr>
</tbody>
</table>

Table 7.1: Comparison With Literature

The values predicted by Ota and Nishiyama\cite{27} are closer to the values of the present study. However, there is still a difference of $\approx 16.48\%$ in the two sets when $x_r$ is used as the characteristic length. The difference $\sim 31.13\%$ is bigger when compared with the correlation from A.I.Leont’ev et al. \cite{26}. The differences however, shrink to $\sim 11.50\%$ and $\sim 27.74\%$ respectively when $x_{\text{max}}$ is used as the characteristic length in the two correlations.
Figure 7.1: Reynolds Number Scaling for (a) Sh, m=0.650 (b) Nu, m=0.642
Figure 7.2: Heat/Mass Transfer Analogy (a) \( u_\infty = 12.46 \text{ m/s, } n = 0.310 \) (b) \( u_\infty = 17.47 \text{ m/s, } n = 0.319 \)
Figure 7.3: Normalised Nu and Sh for the two speeds (a) $u_\infty = 12.46$ m/s (b) $u_\infty = 17.47$ m/s
Figure 7.4: Heat/Mass Transfer Analogy-1

\[ \frac{Nu}{(Re^{0.650} Pr^{0.315})}, \frac{Sh}{(Re^{0.650} Sc^{0.315})} \]

Red: Mass Transfer Runs (Sh)
Blue: Heat Transfer Runs (Nu)
Analogy Factor (F)

Analogy factor variation, $u_\infty \approx 12.5$ m/s
Analogy factor variation, $u_\infty \approx 17.5$ m/s
Analogy Factor from the residual minimization
Ota and Nishiyama (F=0.677)
A.I.Leontev at al (F=0.604)

Figure 7.5: Heat/Mass Transfer Analogy-2
Chapter 8

Conclusion

Heat and mass transfer measurements were done in the recirculation and the reattachment region of the turbulent flow over a backward facing step using the thermal boundary layer measurement technique and napthalene sublimation technique respectively. A constant temperature boundary condition and an analogous constant concentration boundary condition were imposed for the two sets of measurements. Nu and Sh numbers thus obtained, were used to calculate the analogy factor for turbulent flow over a backward facing step. The analogy factor from the study was compared with the expected values from the literature.

The following conclusions can be made from this study:

1. For the flow conditions investigated in this study, two recirculation bubbles form in the separated flow region as evident from the presence of minima in the Nu and Sh profiles.

2. The location of peak Nu and Sh is found to be ≈5.6H downstream of the step.

3. There are two zones of temperature drop in the temperature profiles in the recirculation region - wall thermal boundary layer and the shear layer. The temperature drop across the shear layer diminishes downstream of the step being highest at around 35-40% closest to the step. Thus, most of the thermal resistance to heat transfer is located in the wall thermal boundary layer.
4. There is appreciable variation in the Sh in the spanwise direction near the reattachment region which is indicative of the three dimensional vortex structures in this region. Moreover, for $u_\infty = 12.46 \text{m/s}$, this variation seems to have a periodic character with the two Sh peaks separated by a distance of $\approx 3H$. While the periodic behavior may still be there for the other speed, it could not be ascertained from the present experiments as only one peak in the Sh profile could be observed at this speed. This might be due a greater periodic length at this speed.

5. The Sh and Nu values scale with $Re_H^{0.650}$ and $Re_H^{0.642}$ respectively.

6. The analogy factor, F, was found out to be 0.696 and 0.689 for $u_\infty = 12.46 \text{m/s}$ and $u_\infty = 17.47 \text{m/s}$, respectively, which corresponds to ‘n’ values of 0.310 and 0.319. The overall analogy factor can be estimated to be the mean of the two values which yields F=0.692 and a corresponding ‘n’ value of 0.315. The analogy factor obtained from this study compares well with the value of 0.677 (n=1/3) (a difference of 2 %) as predicted by Ota and Nishiyama [27]. This proves the existence of heat/mass transfer analogy for turbulent flow separation over a backward facing step.

7. A new correlation for the maximum Nu(Sh) values has been proposed based on Re (based on step height). The values predicted by Ota and Nishiyama [27] are $\approx 11.50\%$ lower than the values predicted by this correlation. Since, there is no reattachment length involved in the correlation, a maximum value of Nu (Sh) can be estimated solely on the basis of experimental parameters.
Appendix A

Sample Calculations

The sample calculations presented in this chapter are divided into three sections - (1) Determination of $Re_H$ (2) Determination of $Nu$ from heat transfer experiment (3) Determination of $Sh$ from masss transfer experiment (4) Determination of Analogy factor($F$).

A.1 Fluid Mechanics Calculations

From the hotwire calibration equation the freestream speed just outside the boundary layer was calculated to be 12.46 m/s. Based on this speed and the step height the Re number can be calculated as shown in Eqn. (A.1).

$$Re_X = \frac{\rho U_{\infty}H}{\mu}$$

$$= \frac{1.1700 \times 12.46 \times 20 \times 10^{-3}}{18.13692 \times 10^{-6}}$$

$$= 1.6076 \times 10^4$$  \hspace{1cm} (A.1)

A.2 Determination of $Nu$ from Heat Transfer Experiments

The calculations showed here are for the heat transfer experiment of May 18, 2010. The Nu number has been determined for the sample temperature profile at $x=3.76H$ or $x=75.2$ mm plotted in Fig. 6.3. Table A.1 delineates some values that shall be used to calculate the Nu number.
<table>
<thead>
<tr>
<th>Thermocouple voltage for $T_\infty$</th>
<th>$V$</th>
<th>1.354 $mV$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature profile slope at the wall $\frac{\partial T}{\partial y}</td>
<td>_{wall}$</td>
<td>$-33.94 \times 10^3$ K/m</td>
</tr>
<tr>
<td>Extrapolated Wall Temperature $T_w$</td>
<td>35.21 $^\circ C$</td>
<td></td>
</tr>
</tbody>
</table>

Table A.1: Variables measured during the heat transfer experiment

Freestream temperature was measured using the thermocouple probe and was calculated as shown in Eqn A.2

$$T = 0.1006 + 17.0569V - 0.2027V^2 - 0.0156V^3 + 0.0033V^4$$

$$T_\infty = 0.1006 + 17.0569 \times (1.354) - 0.2027 \times (1.354)^2 - 0.0156 \times (1.354)^3 + 0.0033 \times (1.354)^4$$

$$= 22.81^\circ C$$ \hspace{1cm} (A.2)

Thermal conductivity can be calculated using the Eqn A.3

$$k = 0.0001 \times T + 0.0243$$

$$k_{air@T_w} = 0.0001 \times (35.22) + 0.0243 = 0.0278 W/m - k$$

$$k_{film} = 0.0001 \times \left( \frac{22.80 + 35.22}{2} \right) + 0.0243 = 0.0272 W/m - k$$ \hspace{1cm} (A.3)

The heat flux was calculated using the following relation:

$$q_w = -k_{air@T_w} \frac{\partial T}{\partial y}|_{wall}$$

$$= -(0.0278) \times (-33.94 \times 10^3)$$

$$= 943.50 W/m^2$$ \hspace{1cm} (A.4)

Once the wall flux is calculated the heat transfer coefficient $h$ can be determined from the following equation:

$$h = \frac{q_w}{T_w - T_\infty}$$

$$= \frac{943.50}{35.22 - 22.80}$$

$$= 75.97 W/m^2 - K$$ \hspace{1cm} (A.5)
Eventually Nu can be calculated using the following relation:

\[
N_u = \frac{hH}{k_{film}} = \frac{75.97 \times 0.02}{0.0272} = 55.85
\] (A.6)

A.3 Determination of \( Sh \) from Mass Transfer Experiments

The Table A.2 presents the values of some variables measured during the mass transfer experiment of Dec 5, 2009.

| Naphthalene Surface Temperature | \( T_{n,w} \) | 17.47 °C |
| Wind Tunnel Temperature | \( T_\infty \) | 18.66 °C |
| Ambient Temperature | \( T_{amb} \) | 16.19 °C |
| Height of Mercury column | \( h_{Hg} \) | 738.7 mm |
| Height of Water column | \( h_{H_2O} \) | 12.80 mm |
| Wind tunnel run time | \( \delta \tau \) | 3630 s |
| Sublimation depth | \( \delta y_{corr} \) | 23.88 µm |

Table A.2: Variables measured during the mass transfer experiment

The values are calculated at the location \( x=74.6 \) mm and \( z=0.11 \) mm.

A.3.1 Property Evaluation

1. Density of Mercury - The density of mercury is obtained using Eqn(A.7)

\[
\rho_{Hg} = (13.595516 - 2.47403 \times 10^{-3}T_{amb} + 4.6753 \times 10^{-7}T_{amb}^2) \times 1000 = 13556.65 Kg/m^3
\] (A.7)

2. Static Pressure - The atmospheric pressure, \( P_{atm} \) and \( \Delta P \) are calculated as shown in Eq.(A.8) and Eq.(A.10) respectively. The static pressure can then be calculated
using Eq.(A.11). The value of $\Delta P$ is measured using a manometer.

$$P_{\text{atm}} = \rho_{\text{Hg}} \cdot g \cdot h_{\text{Hg}}$$
$$= (13556.65) \times (9.81) \times (738.7 \times 10^{-3})$$
$$= 98237.45 \text{N/m}^2 \quad (\text{A.8})$$

In order to calculate $\Delta P$, we need $\rho_{\text{H}_2\text{O}}$ from Eqn (A.9)

$$\rho_{\text{H}_2\text{O}} = 999.839 + 8.48774 \times 10^{-2}T_{\text{amb}} - 1.05628 \times 10^{-2}T_{\text{amb}}^2 + 1.055 \times 10^{-4}T_{\text{amb}}^3$$
$$= 998.96698 \text{kg/m}^3 \quad (\text{A.9})$$

$$\Delta P = 2 \cdot \rho_{\text{H}_2\text{O}} \cdot g \cdot h_{\text{H}_2\text{O}}$$
$$= 2 \times (998.96698) \times (9.81) \times (12.80 \times 10^{-3})$$
$$= 251.58 \text{N/m}^2 \quad (\text{A.10})$$

$$P_{\text{st}} = P_{\text{atm}} - \Delta P$$
$$= 98237.45 - 251.58$$
$$= 97985.87 \text{N/m}^2 \quad (\text{A.11})$$

3. Density of air - The density of air is calculated using the ideal gas equation using the static pressure and temperature inside the test section as shown in Eq.(A.12)

$$\rho_{\text{air}} = \frac{P_{\text{st}}}{R_{\text{air}}T_{\infty}}$$
$$= \frac{97985.87}{287 \times (273.15 + 18.66)}$$
$$= 1.1700 \text{kg/m}^3 \quad (\text{A.12})$$

4. Dynamic viscosity of air - The dynamic viscosity of air is calculated using Eq.(A.13) with $T_{\infty}$ expressed in $K$.

$$\mu = 1.805 \times 10^{-5} + 4.8 \times 10^{-8}(T_{\infty} - 290)$$
$$= 1.805 \times 10^{-5} + 4.8 \times 10^{-8}((273.15 + 18.66) - 290)$$
$$= 18.13692 \times 10^{-6} \text{Kg/ms} \quad (\text{A.13})$$
5. Diffusivity of Naphthalene vapor - Naphthalene vapor diffusivity is calculated using Eq.(A.14) with the naphthalene surface temperature \(T_{n,w} = 17.47^\circ C\) and atmospheric pressure \(P_{static} = 97985.87 N/m^2\) measured during the experiment.

\[
D_{na} = D_{na}^{298.16} \cdot \left[ \frac{T_{n,w}}{298.16} \right]^{1.93} \left[ \frac{1.013 \times 10^5}{P_{st}} \right] \\
= 6.81 \times 10^{-6} \left[ \frac{290.62}{298.16} \right]^{1.93} \left[ \frac{1.013 \times 10^5}{97985.87} \right] \\
= 6.70 \times 10^{-6} m^2/s \quad (A.14)
\]

6. Naphthalene vapor pressure - The naphthalene vapor pressure is calculated using Eq.(3.16) and \(T_{n,w} = 17.47^\circ C\) and is found to be \(P_{v,w} = 5.25 Pa\).

7. Naphthalene vapor density - The naphthalene vapor density is obtained using ideal gas law and is shown in Eq.(A.15).

\[
\rho_{v,w} = \frac{M_n P_{v,w}}{RT_{n,w}} \\
= \frac{128.17 \times 5.25}{8314.3 \times 290.62} \\
= 2.78 \times 10^{-4} Kg/m^3 \quad (A.15)
\]

### A.3.2 Mass Transfer Calculations

The corrected sublimation depth at the considered location \((x = 74.6 \text{ mm}, z = 0.11 \text{ mm})\) is calculated as 23.55\(\mu\)m and the wind tunnel run time for the experiment is 3630s. Using these values, the mass transfer rate (Eq.(A.16)), local mass transfer coefficient (Eq.(A.17)) and Sh (Eq.(A.18)) are obtained.

\[
\dot{m} = \rho_s \frac{\delta y_{corr}}{\delta \tau} \\
= 1162 \times \frac{23.88 \times 10^{-6}}{3630} \\
= 7.644 \times 10^{-6} Kg/m^2 - s \quad (A.16)
\]

\[
h_m = \frac{\dot{m}}{\rho_{v,w}} \\
= \frac{7.644 \times 10^{-6}}{2.78 \times 10^{-4}} \\
= 2.75 \times 10^{-2} m^2/s \quad (A.17)
\]
\[ Sh = \frac{h_m H}{D_{na}} \]
\[ = \frac{2.75 \times 10^{-2} \times 2 \times 10^{-2}}{6.69 \times 10^{-6}} \]
\[ = 82.10 \]
\hspace{1cm} (A.18)

### A.4 Analogy Factor Calculations

The analogy factor is calculated as shown in Eq. (A.19).

\[ F = \frac{Nu}{Sh} \]
\[ = \frac{55.85}{82.10} \]
\[ = 0.680 \]
\hspace{1cm} (A.19)
Appendix B

Uncertainty Analysis

The uncertainty analysis presented in this chapter is based on the method of single sample experiments as described in Kline and McClintock (1953)[47] and are expressed at 95% confidence level. The uncertainty analysis is divided into three sections - (1) Determination of $Nu$ from heat transfer experiment (2) Determination of $Sh$ from mass transfer experiment and (3) Determination of Analogy Factor (F).

B.1 Uncertainty Analysis of Heat Transfer Experiment

This section is subdivided into uncertainty analysis of heat transfer coefficient $h$ and $Nu$.

B.1.1 Uncertainty in Heat Transfer Coefficient

The convective heat transfer coefficient is obtained using Eq[A.5] and its uncertainty can be calculated as shown in Eq.(B.1).

\[
\frac{\Delta h}{h} = \left[ \left( \frac{\Delta q_w}{q_w} \right)^2 + \left( \frac{\Delta T_w}{T_w - T_\infty} \right)^2 + \left( \frac{\Delta T_\infty}{T_w - T_\infty} \right)^2 \right]^{1/2} \quad \text{(B.1)}
\]

1. The convective heat flux is calculated using Eq[A.4] and its uncertainty can be calculated using Eqn [B.2]

\[
\frac{\Delta q_w}{q_w} = \left[ \left( \frac{\Delta k_{air@T_w}}{k_{air@T_w}} \right)^2 + \left( \frac{\Delta \frac{\partial T}{\partial y}}{\frac{\partial T}{\partial y}} \right)^2 \right]^{1/2} \quad \text{(B.2)}
\]
• The thermal conductivity of air was calculated using Eq A.3 and the uncertainty can be calculated by the Eqn B.3

\[
\Delta k_{\text{air}wT_w} = \left[ (S E E_{\text{Line fit}}^2 + (0.0001 \times \Delta T_w)^2 \right]^{1/2}
\]

\[
= \left[ (7.76 \times 10^{-18})^2 + (0.0001 \times 0.1349)^2 \right]^{1/2}
\]

\[
= 1.349 \times 10^{-5} W/m - K \quad \text{(B.3)}
\]

\(\Delta T_w\) has been calculated from Eq B.4 described below.

• The uncertainty in the slope \((\sigma_{\frac{\partial T}{\partial y}})\) can be calculated using Eq B.4

\[
\frac{\Delta}{\delta y}T = \frac{N(Sy_i T_i) - (Sy_i)(\Sigma T_i)}{\Delta}
\]

\[
\Delta = N (\Sigma y_i^2) - (\Sigma y_i)^2
\]

\[
\sigma^2_{T_w} = \frac{\sigma^2_{\Sigma y_i^2}}{\Delta}
\]

\[
\Delta \sigma_{\frac{\partial T}{\partial y}} = \frac{\sigma^2_{\Sigma y_i^2} \Delta}{N}
\]

Using the relations in B.4 we have:

\[
\Delta T_w = 0.1349^\circ C, \quad \Delta \frac{\partial T}{\partial y} = \sigma_{\frac{\partial T}{\partial y}} = 1.48 \times 10^3 K/m \quad \text{(B.5)}
\]

Hence the uncertainty in \(q_w\) from Eq B.3 is

\[
\frac{\Delta q_w}{q_w} = \left[ \left( \frac{1.349 \times 10^{-5}}{0.0278} \right)^2 + \left( \frac{1.48 \times 10^3}{33.94 \times 10^3} \right)^2 \right]^{1/2}
\]

\[
= \left[ (4.85 \times 10^{-4})^2 + (0.0436)^2 \right]^{1/2}
\]

\[
= 4.36\% \quad \text{(B.6)}
\]

2. The uncertainty in extrapolated wall temperature is found to be 0.1349K from Eq. B.4.

3. The uncertainty in the freestream temperature can be calculated from the Eq (B.7)

\[
\Delta T_\infty = [(\Delta T_{\text{calib}})^2 + (\Delta T_{\text{volt}})^2 + (\Delta T_{\text{explo}})^2]^{1/2}
\]

• Thermocouple calibration uncertainty, is found to be \(\Delta T_{\text{calib}} = 0.0613K\) ((Fig 3.3(a))).
• The uncertainty of voltmeter is 1µV which translates into $\Delta T_{\text{volt}} = 0.017K$
  from the calibration equation. (Fig 3.3(a))

• The uncertainty in the naphthalene surface temperature is found out to be $\Delta T_{\text{expt}} = 0.094K$
  for the experiment.

Thus from Eqn [B.7] we have

$$\Delta T_\infty = [(0.06)^2 + (0.017)^2 + (0.094)^2]^{1/2} = 0.1124K$$ (B.8)

The combined uncertainty in $h$ can be calculated as shown in Eq. [B.9].

$$\frac{\Delta h}{h} = \left[\left(\frac{0.0436}{12.42}\right)^2 + \left(\frac{1.349}{12.42}\right)^2 + \left(\frac{0.1124}{12.42}\right)^2\right]^{1/2} = 4.57\%$$ (B.9)

### B.1.2 Uncertainty in Nu

The Nu can be obtained from $h$ using Eq. (B.10) and its uncertainty can be calculated using Eq. (B.11).

$$Nu = \frac{hH}{k_{film}}$$ (B.10)

$$\frac{\Delta Nu}{Nu} = \left[\left(\frac{\Delta h}{h}\right)^2 + \left(\frac{\Delta H}{H}\right)^2 + \left(\frac{\Delta k_{film}}{k_{film}}\right)^2\right]^{1/2}$$ (B.11)

1. The uncertainty in $h$ is found to be $4.57\%$ from Eq. [B.9].

2. The uncertainty in the step height, $H$ can be assumed to be $0\%$.

3. The uncertainty in $k_{air}$ can be calculated from Eqn [B.12]

$$\Delta k_{film} = [(S E E_{Line fit}^2 + (0.0001 \times \Delta T_{film})^2)^{1/2}$$ (B.12)

• Also the uncertainty in $T_{film}$ can be calculated as in Eq [B.13]

$$\Delta T_{film} = \left[\left(\frac{\Delta T_{\text{w}}}{2}\right)^2 + \left(\frac{\Delta T_{\infty}}{2}\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{0.1349}{2}\right)^2 + \left(\frac{0.1124}{2}\right)^2\right]^{1/2}$$

$$= 0.0878K$$ (B.13)
From Eq. (B.12) we have:

\[ \Delta k_{\text{film}} = \left( (S E E_{\text{Line fit}}^2 + (0.0001 \times \Delta T_{\text{film}})^2 \right)^{1/2} \]
\[ = \left( (7.76 \times 10^{-18})^2 + (0.0001 \times 0.0878)^2 \right)^{1/2} \]
\[ = 8.78 \times 10^{-6} \text{W/m} - {K} \]  

(B.14)

The combined uncertainty in Nu can be calculated using these three uncertainties as shown in Eq. (B.15).

\[ \Delta N_u = \left( \left( 0.0457 \right)^2 + (0)^2 + \left( \frac{8.78 \times 10^{-6}}{0.0272} \right)^2 \right)^{1/2} \]
\[ = 4.57\% \]  

(B.15)

### B.2 Uncertainty Analysis of Mass Transfer Experiment

This section is subdivided into two parts - uncertainty analysis of \( h_m \) and Sh.

#### B.2.1 Uncertainty Analysis of \( h_m \)

The mass transfer coefficient is calculated using Eq. (3.14) and its uncertainty is calculated using Eq. (B.16).

\[ \frac{\Delta h_m}{h_m} = \left( \left( \frac{\Delta \dot{m}}{\dot{m}} \right)^2 + \left( \frac{\Delta \rho_v}{\rho_v} \right)^2 \right)^{1/2} \]  

(B.16)

1. The sublimation mass transfer rate is obtained using Eq. (3.14) and the uncertainty in \( \dot{m} \) can be calculated using Eq. (B.17).

\[ \frac{\Delta \dot{m}}{\dot{m}} = \left[ \left( \frac{\Delta \rho_s}{\rho_s} \right)^2 + \left( \frac{\Delta \delta y_{\text{corr}}}{\delta y_{\text{corr}}} \right)^2 + \left( \frac{\Delta \delta \tau}{\delta \tau} \right)^2 \right]^{1/2} \]  

(B.17)

- The uncertainty in density of solid naphthalene is estimated to be \( \Delta \rho_s = 30Kg/m^3 \) from Tab. (3.1).
- The uncertainty in corrected sublimation depth change has four main components - calibration uncertainty of LVDT, uncertainty in LVDT voltage measurement, uncertainty due to alignment of naphthalene mass transfer
plate and uncertainty in correcting the measurements for natural sublimation. Equation (B.18) represents the procedure to obtain the net uncertainty in corrected sublimation depth change.

\[
\frac{\Delta \delta y_{corr}}{\delta y_{corr}} = \left[ \left( \frac{\Delta \delta y}{\delta y} \right)_{\text{calib}}^2 + \left( \frac{\Delta \delta y}{\delta y} \right)_{\text{volt}}^2 + \left( \frac{\Delta \delta y}{\delta y} \right)_{\text{align}}^2 + \left( \frac{\Delta \delta y}{\delta y} \right)_{natsub}^2 \right]^{1/2}
\] (B.18)

- The LVDT calibration uncertainty is found to be 0.1610 \(\mu m\) from Fig. (3.5). A typical sublimation depth of 23.88 \(\mu m\) resulted in 60 min and 30 seconds experimental time is considered here for calculation purposes.
- The voltmeter uncertainty is given to be 1 \(\mu V\) which translates into \(19.08 \times 10^{-6} \mu m\) using the LVDT calibration.
- The mock sublimation test (Fig.3.6) shows that the uncertainty due to misalignment of the naphthalene sublimation plate is 0.209 \(\mu m\). The uncertainty due to backlash error is negligible in comparison to the misalignment uncertainty.
- The average rate for natural sublimation of naphthalene is found to be 1.1 \(\mu m/m^2\) – hour (Fig.3.7) and the uncertainty in this measurement due to temperature variation is estimated to be 3% and translates into 0.036 \(\mu m/hour\) which can be adjusted to a value of 0.036 \(\mu m/m^2\) sublimation depth for a 3630 s experiment.

• These four uncertainties are combined to calculate the uncertainty in corrected sublimation depth change as shown in Eq. (B.19).

\[
\frac{\Delta \delta y_{corr}}{\delta y_{corr}} = \frac{0.1610}{23.88}^2 + \frac{19.08 \times 10^{-6}}{23.88}^2 + \frac{0.209}{23.88}^2 + \frac{0.036}{23.88}^2 = 1.12\%
\] (B.19)

• The uncertainty in time measurement can be subdivided into two categories - uncertainty due to accuracy of stop watch and uncertainty due to transients resulting due to starting and stopping of the wind tunnel. The uncertainty is calculated using Eq. (B.20).

\[
\frac{\Delta \delta \tau}{\delta \tau} = \left[ \left( \frac{\Delta \delta \tau}{\delta \tau} \right)_{\text{stopwatch}}^2 + \left( \frac{\Delta \delta \tau}{\delta \tau} \right)_{\text{transient}}^2 \right]^{1/2}
\] (B.20)
The uncertainty in time measurement using the stopwatch is estimated to be 1s. Typical experimental run time is \( \sim 60 \) min. A value of 3630 s is used for calculation purposes.

The transient time due to wind tunnel startup and shutdown is estimated to be 10 seconds.

The uncertainty in time is calculated using Eq. (B.21).

\[
\frac{\Delta \delta \tau}{\delta \tau} = \left[ \left( \frac{1}{3630} \right)^2 + \left( \frac{10}{3630} \right)^2 \right]^{1/2} = 0.28\%
\]

(B.21)

Thus, the uncertainty in sublimation mass transfer can be calculated combining Eqs. (B.19) and (B.21) into Eq. (B.17) as shown in Eq. (B.22).

\[
\frac{\Delta \dot{m}}{\dot{m}} = \left[ (0.0258)^2 + (0.0112)^2 + (2.80 \times 10^{-3})^2 \right]^{1/2} = 2.83\%
\]

(B.22)

2. The naphthalene vapor density is calculated using Eq. (A.15) and its uncertainty can be calculated using Eq. (B.23) after assuming no uncertainty in molecular weight of naphthalene and universal gas constant.

\[
\frac{\Delta \rho_{v,w}}{\rho_{v,w}} = \left[ \left( \frac{\Delta P_{v,w}}{P_{v,w}} \right)^2 + \left( \frac{\Delta T_{n,w}}{T_{n,w}} \right)^2 \right]^{1/2} \]

(B.23)

The partial pressure determined from Ambrose’s correlation (Eq. 3.16) has two sources of uncertainty - the uncertainty of correlation curve fit and uncertainty due to naphthalene surface temperature. This combined uncertainty can be calculated using Eq. (B.24)

\[
\frac{\Delta P_{v,w}}{P_{v,w}} = \left[ \left( \frac{\Delta P_{v,w}}{P_{v,w}} \right)^{\text{correlation}} + \left( \frac{\partial P_{v,w}}{\partial T_{n,w}} \frac{\Delta T_{n,w}}{P_{v,w}} \right)^2 \right]^{1/2} = \left[ \left( \frac{\Delta P_{v,w}}{P_{v,w}} \right)^{\text{correlation}} + (29.26 \frac{\Delta T_{n,w}}{T_{n,w}})^2 \right]^{1/2} \]

(B.24)

The uncertainty due to Ambrose correlation is found to be 3.77%
The uncertainty in the naphthalene surface temperature can be calculated from the Eq. (B.25)

\[ \Delta T_{n,w} = \left[ (\Delta T_{\text{calib}})^2 + (\Delta T_{\text{volt}})^2 + (\Delta T_{\text{expt}})^2 \right]^{1/2} \]  

(B.25)

* Thermocouple calibration uncertainty, is found to be \( \Delta T_{\text{calib}} = 9.4 \times 10^{-3} K \) ((Fig 2.17, Kaustubh(2008)).

* The uncertainty of voltmeter is 1\( \mu \)V which translates into \( \Delta T_{\text{volt}} = 0.016K \) from the calibration equation.((Fig 2.17, Kaustubh(2008))

* The uncertainty in the naphthalene surface temperature is found out to be \( \Delta T_{\text{expt}} = 0.09K \) for the experiment.

Thus from Eqn B.25 we have

\[ \Delta T_{n,w} = \left[ (9.4 \times 10^{-3})^2 + (0.016)^2 + (0.09)^2 \right]^{1/2} \]

\[ = 0.092K \]  

(B.26)

The uncertainty in partial pressure can be calculated as shown in Eq. (B.27).

\[ \frac{\Delta P_{v,w}}{P_{v,w}} = \left[ (0.0377)^2 + \left( \frac{29.26}{290.62} \right)^2 \right]^{1/2} \]

\[ = 3.88\% \]  

(B.27)

The uncertainty in naphthalene surface temperature is found to be \( \Delta T_{n,w} = 0.092K \).

Thus, the uncertainty in naphthalene vapor density is calculated using Eq. (B.28).

\[ \frac{\Delta \rho_{v,w}}{\rho_{v,w}} = \left[ (0.0388)^2 + \left( \frac{0.092}{290.62} \right)^2 \right]^{1/2} \]

\[ = 3.99\% \]  

(B.28)

Combining Eq.s (B.22) and (B.28) into Eq. (B.16), the uncertainty is calculated as shown in Eq. (B.29).

\[ \frac{\Delta h_m}{h_m} = \left[ (0.0283)^2 + (0.0399)^2 \right]^{1/2} \]

\[ = 4.89\% \]  

(B.29)
B.2.2 Uncertainty in Sh

The $Sh$ can be obtained from $h_m$ using Eq. (B.30) and its uncertainty can be calculated using Eq. (B.31).

$$Sh = \frac{h_m H}{D_{na}} \tag{B.30}$$

$$\frac{\Delta Sh}{Sh} = \left[ \left( \frac{\Delta h_m}{h_m} \right)^2 + \left( \frac{\Delta H}{H} \right)^2 + \left( \frac{\Delta D_{na}}{D_{na}} \right)^2 \right]^{1/2} \tag{B.31}$$

1. The uncertainty in $h_m$ is found to be $4.89\%$ from Eq. (B.29).

2. The uncertainty in the step height is taken to be $0\%$.

3. The diffusivity of napthalene into air is calculated using Eq. (A.14) and its uncertainty is calculated using Eq. (B.32).

$$\frac{\Delta D_{na}}{D_{na}} = \left[ \left( \frac{\Delta D_{na}}{D_{na}} \right)^2_{correlation} + \left( \frac{1.93 \Delta T_{n, w}}{T_{n, w}} \right)^2 + \left( \frac{\Delta P_{st}}{P_{st}} \right)^2 \right]^{1/2} \tag{B.32}$$

- The correlation uncertainty is found to be $5.08\%$.
- The uncertainty in naphthalene surface temperature is found to be $0.092$ K.
- The uncertainty in static pressure is calculated using Eq. (B.33).

$$\frac{\Delta P_{st}}{P_{st}} = \left[ \left( \frac{\Delta P_{atm}}{P_{atm}} \right)^2 + \left( \frac{\Delta(\Delta P)}{\Delta P} \right)^2 \right]^{1/2} \tag{B.33}$$

- Atmospheric pressure is measured using a barometer and the uncertainty is calculated as shown in Eq. (B.34).

$$\frac{\Delta P_{atm}}{P_{atm}} = \left[ \left( \frac{\Delta H_{Hg}}{H_{Hg}} \right)^2 + \left( \frac{\Delta \rho_{Hg}}{\rho_{Hg}} \right)^2 + \left( \frac{\Delta g}{g} \right)^2 \right]^{1/2} \tag{B.34}$$

* The uncertainty in measuring the height of mercury column is estimated to be $0.05$ mm in a nominal value of $738.7$ mm.

* The uncertainty in mercury density due to air temperature is estimated from Eq. (A.7) and is shown in Eq. (B.35).

$$\Delta \rho_{Hg} = (-2.47403 + 2 \times 4.6753 \times 10^{-4} T_{amb}) \Delta T_{amb} \tag{B.35}$$
For $T_{amb} = 16.19^\circ C$ the $\Delta T_{amb}$ can be determined in the same manner as in Eq. (B.25) and is shown in Eq. (B.36). $\Delta T_{calib}$ and $\Delta T_{volt}$ have the same values and $\Delta T_{expt} = 0.104 K$.

$$\Delta T_{amb} = \sqrt{(9.4 \times 10^{-3})^2 + (0.016)^2 + (0.104)^2} = 0.106 K$$ (B.36)

Substituting the value of $\Delta T_{amb}$ in Eqn. B.35, $\Delta \rho_{Hg} = -0.2606 kg/m^3$.

- Hence from Eq. (B.35) we have:

$$\frac{\Delta P_{atm}}{P_{atm}} = \left[ \left( \frac{0.05}{738.7} \right)^2 + \left( \frac{-0.2606}{13556.65} \right)^2 + \left( \frac{0}{9.81} \right)^2 \right]^{1/2} = 7.036 \times 10^{-3}\%$$ (B.37)

- The uncertainty in $\Delta P$ can be estimated using the Eq (B.38)

$$\frac{\Delta(\Delta P)}{\Delta P} = \left[ \left( \frac{\Delta \rho_{H_2O}}{\rho_{H_2O}} \right)^2 + \left( \frac{\Delta h}{h} \right)^2 + \left( \frac{\Delta g}{g} \right)^2 \right]^{1/2}$$ (B.38)

* $\rho_{H_2O}$ can be calculated from the Eq (A.9) and corresponding uncertainty from Eq (B.39)

$$\Delta \rho_{H_2O} = (8.48774 \times 10^{-2} + 2 \times 1.05628 \times 10^{-2}T_{amb} + 3 \times 1.055 \times 10^{-4}T_{amb}^2)\Delta T_{amb} = 0.054 kg/m^3$$ (B.39)

- The uncertainty in measuring the height of mercury column is estimated to be 0.01 mm in a nominal value of 12.80 mm.
- The uncertainty in $g$ is assumed to be 0%.

From Eq. (B.38) we have:

$$\frac{\Delta(\Delta P)}{\Delta P} = \left[ \left( \frac{0.054}{998.967} \right)^2 + \left( \frac{0.01}{12.80} \right)^2 + \left( \frac{0}{9.81} \right)^2 \right]^{1/2} = 7.83 \times 10^{-2}\%$$ (B.40)
Thus the net uncertainty in static pressure is $7.83 \times 10^{-2}\%$.

$$\frac{\Delta P_{st}}{P_{st}} = \left[ (7.036 \times 10^{-3})^2 + (7.83 \times 10^{-2})^2 \right]^{1/2}$$

$$= 7.86 \times 10^{-2}\% \quad (B.41)$$

4. These three uncertainties can be combined to calculate the uncertainty in diffusivity of naphthalene as shown in Eq. (B.42).

$$\frac{\Delta D_{na}}{D_{na}} = \left[ (0.0508)^2 + \left( \frac{1.93 \times 0.092}{290.66} \right)^2 + (7.86 \times 10^{-4})^2 \right]^{1/2}$$

$$= 5.08\% \quad (B.42)$$

The combined uncertainty in $Sh$ can be calculated using Eqns. (B.29 and (B.42) as shown in Eq. (B.43).

$$\frac{\Delta Sh}{Sh} = \left[ (0.0489)^2 + (0)^2 + (0.0508)^2 \right]^{1/2}$$

$$= 7.05\% \quad (B.43)$$

**B.3 Uncertainty in Analogy Factor**

The uncertainties in $Nu$ and $Sh$ are found to be $4.57\%$ (Eq. B.15) and $7.05\%$ (Eq. B.43) respectively. The uncertainty in analogy factor can be calculated as shown in Eq. (B.44).

$$\frac{\Delta F}{F} = \left[ \left( \frac{\Delta N_u}{N_u} \right)^2 + \left( \frac{\Delta Sh}{Sh} \right)^2 \right]^{1/2}$$

$$= \left[ (0.0457)^2 + (0.0705)^2 \right]^{1/2}$$

$$= 8.40\% \quad (B.44)$$
References


