An Application of Singular Value Decomposition to the Factor Analysis of MMPI Items

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Several measurement problems were identified in the literature concerning the fidelity with which the Minnesota Multiphasic Personality Inventory (MMPI) assesses psychopathology. A straightforward solution to some of these problems is to develop an orthogonal basis in the MMPI; however, there are 550 items, and this is a cumbersome task even for modern computers. The method of alternating least squares was employed to yield a singular value decomposition of these measures on 682 prison inmates. Unsystematic or sample-specific error variance was minimized through a two-stage least squares split thirds replication design. The relative explanatory power of models of psychopathology based on external, internal, naive, and construct-oriented measurement strategies is discussed.

Most of the MMPI clinical scales were developed by the empirical approach, with items selected and keyed on the basis of their ability to distinguish diagnostic groups. There are numerous problems with the empirical approach: (1) the measures developed tend to be heterogeneous; (2) the measures overlap, since the prediction is usually bivariate and the criteria are usually correlated; (3) spurious item keying is introduced by using small samples for item selection, and this is further exacerbated by not cross-validating the item analysis; (4) there is little or no theoretical basis for scale keys and interpretations; (5) the reliability and validity of the criterion measures is usually suspect; (6) the contrasted groups procedure implies dichotomous measurement, but the scores are usually interpreted as representing an underlying continuum; and (7) there is no attempt to control for stylistic variance (e.g., Bentler, Jackson, & Messick, 1971; Block, 1965, p. 117-129; Edwards, 1970; Goldberg, 1974; Holden & Jackson, 1979, 1981; Horn, Wanberg, & Appel, 1973; Jackson, 1970, 1971, 1973, 1979; Jackson & Messick, 1958, 1961; Jackson & Paunonen, 1980; Norman, 1972; Nunnally, 1978; Rogers, 1971; Voyce & Jackson, 1977; Wiggins, 1973).

The purpose of the present study was to identify relatively homogeneous, independent dimensions in the MMPI item pool through factor analytic procedures and to develop scales from this factor analysis with item analytic procedures. Spurious correlations between scales due to heterogeneity, redundancy of item content, and item overlap can be ameliorated through the construction of scales by item factor analytic procedures, but the problem of stylistic responding is more fundamental in that it cannot be easily resolved by a posteriori methods. Stylistic responding is a lurking variable that is best controlled by prior item development and selection (Jackson, 1967; Voyce & Jackson, 1977).
Method

Subjects and Procedure

Subjects were 682 male prison inmates from the Canadian Penitentiary System. Subjects were administered the booklet form of the MMPI (cf. Dahlstrom, Welsh, & Dahlstrom, 1972) in the first few months of their incarceration. They responded on machine scoreable answer sheets which did not allow for "?" responses. Those included in the sample had no missing data.

Method of Analysis

The 16 repeated items were deleted from the data matrix so that there was an equal number of first and second occurrences of these items deleted. The current thinking in numerical analysis is that the most appropriate method for computing principal components is the singular value decomposition algorithm (cf. Chambers, 1977; Kennedy & Gentle, 1980). This procedure does not require the computation of the matrix of phi coefficients, since the data matrix can be decomposed directly, and with appropriate preliminary scaling yields the same solution as factor analyzing the matrix of phi coefficients.

In the present study the singular value decomposition was accomplished by the PARAFAC statistical package developed by Harshman (cf. Carroll & Arabie, 1980; Harshman, 1970; Harshman & Berenbaum, 1981; Harshman & Lundy, 1981). Unlike most orthogonal decomposition algorithms, PARAFAC has the advantage of extracting the number of dimensions requested rather than extracting all dimensions. This is particularly useful in factor analyses of large data matrices where an \( n \)-dimensional solution can be hypothesized that is substantially less than the number of variables. PARAFAC is an iterative procedure designed for three-mode factor analysis or metric multidimensional scaling. The algorithm iterates simultaneously for all dimensions using alternating least squares (cf. Kroonenberg & De Leeuw, 1980; Wold, 1966) and converges to an \( n \)-dimensional least squares fit of the data matrix. For two-mode data, when the factors for both modes (subjects and variables) are constrained to be orthogonal, alternating least squares algorithms converge on the singular value decomposition orientation.

To ensure that a robust solution had been obtained, the sample was split into thirds and separate decompositions were done in each of the three samples. Thus, one data matrix of \( 228 \times 550 \) and two data matrices of \( 227 \times 550 \) were decomposed into the singular value decomposition orientation with the PARAFAC procedure. The dimensionality of the orthonormal basis in the item space was determined by decomposing three independently generated random data matrices of size \( 228 \times 550 \). The intersection of the eigenvalues from the random data sets and the real data sets was used as an upper bound for the dimensionality of the 550 MMPI items (cf. Horn, 1965).

The structure common to the three samples was determined through Meredith's (1964) "method two" technique for factorial invariance. Please (1973) has provided a proof for Meredith's solution in the context of maximum likelihood factor analysis. This replication design increases the confidence in the generalizability of the results and reduces the likelihood of, and minimizes to some extent, unsystematic or sample-specific error variance.

Results

The eigenvalues from the real and random data sets crossed at 15 dimensions (Figure 1). This indicates an upper bound of 15 dimensions for the MMPI item space. The plot of eigenvalues for the three random data sets are essentially collinear. The random eigenvalue distributions are distinctly separate from the eigenvalues for the three real data sets up to six dimensions. At the seventh dimension the random and real eigenvalue plots approach equality, suggesting a strong six-dimensional subspace underlying the 550 MMPI items. The six-dimen-
sional solution is consistent with the number of dimensions that would be retained through the application of Cattell's scree test (Cattell, 1966; Cattell & Vogelman, 1977), since at six dimensions the slopes of the real eigenvalue distributions approach zero and are equivalent to random or chance eigenvalue distributions. The decision in favor of the six-dimensional solution is further supported by the fact that after an orthogonal procrustes rotation (Schönemann, 1966) of the three real data sets to the common underlying structure, the coefficients of congruence (cf. Mulaik, 1972) were all in excess of .80 for all six dimensions, for all three samples. Also, when more than six dimensions were rotated to simple structure, poorly defined factors appeared suggesting that too many factors had been extracted.

Six dimensions were rotated orthogonally to a varimax criterion (Kaiser, 1958). The loadings of the MMPI clinical scales on these dimensions were computed through a factor extension procedure (Dwyer, 1937) and are presented in Table 1 along with the correlations with the scales de-
Table 1
Loadings of the MMPI Clinical Scales on Six MMPI Item Factors
Rotated to Varimax Criterion, and Correlations With Six Scales
Derived By Item Analytic Procedures

<table>
<thead>
<tr>
<th>Factor Scale</th>
<th>L</th>
<th>F</th>
<th>K</th>
<th>HS</th>
<th>D</th>
<th>HY</th>
<th>PD</th>
<th>MF</th>
<th>PA</th>
<th>PT</th>
<th>SC</th>
<th>MA</th>
<th>SI</th>
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<tr>
<td>Factor 1</td>
<td>-.40</td>
<td>.40</td>
<td>-.65</td>
<td>.36</td>
<td>.59</td>
<td>.01</td>
<td>.52</td>
<td>.32</td>
<td>.34</td>
<td>.79</td>
<td>.61</td>
<td>.17</td>
<td>.82</td>
</tr>
<tr>
<td>Scale 1</td>
<td>-.42</td>
<td>.58</td>
<td>-.78</td>
<td>.55</td>
<td>.65</td>
<td>.09</td>
<td>.58</td>
<td>.28</td>
<td>.46</td>
<td>.90</td>
<td>.76</td>
<td>.37</td>
<td>.82</td>
</tr>
<tr>
<td>Factor II</td>
<td>-.01</td>
<td>.33</td>
<td>-.23</td>
<td>.73</td>
<td>.45</td>
<td>.66</td>
<td>.38</td>
<td>.19</td>
<td>.48</td>
<td>.39</td>
<td>.42</td>
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<td>.14</td>
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<td>.58</td>
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<td>.34</td>
<td>.04</td>
<td>.22</td>
<td>.24</td>
<td>-.16</td>
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<td>.11</td>
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<td>-.09</td>
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<td>.54</td>
<td>-.16</td>
<td>.03</td>
<td>.23</td>
<td>-.32</td>
<td>.05</td>
<td>-.04</td>
<td>.36</td>
<td>-.31</td>
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<td>.01</td>
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<td>Factor IV</td>
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<td>-.03</td>
<td>-.30</td>
<td>-.09</td>
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<td>.12</td>
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<td>Scale 4</td>
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<td>.37</td>
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<td>-.22</td>
<td>.36</td>
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<td>.08</td>
<td>.52</td>
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<td>.60</td>
<td>.40</td>
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<td>Factor V</td>
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<td>-.05</td>
<td>-.06</td>
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<td>.11</td>
<td>.00</td>
<td>-.05</td>
<td>.08</td>
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<tr>
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<td>.07</td>
<td>.03</td>
<td>.08</td>
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<td>.10</td>
<td>-.08</td>
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<td>.10</td>
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<td>.11</td>
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<td>.15</td>
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<tr>
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<td>.66</td>
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<td>.24</td>
<td>.44</td>
<td>.09</td>
<td>.69</td>
<td>.75</td>
<td>.89</td>
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The loadings of Wiggin's (1966) content scales were also computed and are reported in Table 2 along with the correlations with the scales derived from these factors by item analytic procedures.

The interpretation of the factors was based on the varimax loadings, the extension loadings and correlations in Tables 1 and 2, factor analyses of the entire MMPI item pool with psychiatric patients (Johnson, Butcher, & Null, 1981) and college students (Lushene, 1977; Stewart, 1974), factor and cluster analyses of MMPI item subsets (cf. Dahlstrom, Welsh, & Dahlstrom, 1975), and factor analyses of items on MMPI scales (cf. Dahlstrom et al., 1975). The factors derived in this study can be tentatively labelled as follows: I—general maladjustment; II—somatic complaints; III—impulse expression; IV—cynical outlook; V—religiosity; and VI—severe maladjustment.

The one highest and the one lowest salient loading for each of the six MMPI item factors are presented in Table 3. The number of items with factor loadings greater than or equal to .30 in absolute value were 145, 49, 35, 38, 28, and 92, respectively, for Factors I through VI. All factors except Factor V were bipolar, although for the bipolar factors only one pole was well defined for each factor. A good many of the MMPI clinical scales and Wiggin's (1966) content scales have salient loadings on each of the six factors, and the interpretations are quite clear. However, the interpretations are equivocal with respect to a response style versus content interpretation. That all factors are essentially unipolar reflects the general nature of the MMPI item pool and invokes an interpretation based on stylistic responding. The largest unrotated factor score (the largest source of variance, see Figure 1) is highly laden with a desirability component. The Jackson and Messick (1961) extreme undesirability scale (DYS) and extreme desirability scale (DY1) correlated .96 and -.68, respectively, with the first unrotated factor score. The social desirability scale values for the MMPI items (Messick & Jackson, 1961) correlated -.74 with the loadings for the first unrotated factor. Also, Factor VI has an extension loading of .71 from the F scale of the MMPI. This indicates that Factor
Table 2
Loadings of Wiggins' (1966) Content Scales on Six MMPI Item Factors Rotated to Varimax Criterion, and Correlations With Six Scales Derived By Item Analytic Procedures

<table>
<thead>
<tr>
<th>Factor Scale</th>
<th>REL</th>
<th>SOC</th>
<th>DEP</th>
<th>MOR</th>
<th>AUT</th>
<th>PHO</th>
<th>HOS</th>
<th>ORG</th>
<th>PSY</th>
<th>FAM</th>
<th>HYP</th>
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<th>FEM</th>
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<td>.53</td>
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<td>.04</td>
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<td>.76</td>
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<td>.94</td>
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<td>.60</td>
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<td>Factor 2</td>
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<td>.17</td>
<td>.11</td>
<td>.37</td>
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<td>.35</td>
<td>.82</td>
<td>.04</td>
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<td>-.17</td>
<td>-.37</td>
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<td>-.09</td>
<td>-.24</td>
<td>-.52</td>
<td>-.04</td>
<td>.04</td>
</tr>
<tr>
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<td>-.02</td>
<td>-.33</td>
<td>-.39</td>
<td>-.60</td>
<td>-.22</td>
<td>-.68</td>
<td>-.16</td>
<td>-.31</td>
<td>-.34</td>
<td>-.64</td>
<td>-.19</td>
<td>.07</td>
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<tr>
<td>Factor IV</td>
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<td>.08</td>
<td>.13</td>
<td>.67</td>
<td>-.10</td>
<td>.41</td>
<td>.17</td>
<td>.25</td>
<td>.08</td>
<td>.35</td>
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<tr>
<td>Scale 4</td>
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<td>.33</td>
<td>.52</td>
<td>.51</td>
<td>.94</td>
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<td>.51</td>
<td>.39</td>
<td>.57</td>
<td>.34</td>
<td>-.10</td>
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<tr>
<td>Factor V</td>
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<td>-.28</td>
<td>-.01</td>
<td>.04</td>
<td>-.09</td>
<td>.16</td>
<td>-.03</td>
<td>-.05</td>
<td>.17</td>
<td>-.04</td>
<td>.22</td>
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<tr>
<td>Scale 5</td>
<td>.95</td>
<td>-.06</td>
<td>.05</td>
<td>.03</td>
<td>-.14</td>
<td>.17</td>
<td>-.08</td>
<td>.11</td>
<td>.26</td>
<td>-.04</td>
<td>.05</td>
<td>.07</td>
<td>.40</td>
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<td>Scale 6</td>
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<td>.60</td>
<td>.46</td>
<td>.63</td>
<td>.64</td>
<td>.78</td>
<td>.90</td>
<td>.41</td>
<td>.56</td>
<td>.59</td>
<td>.21</td>
</tr>
</tbody>
</table>

Note. REL = Religious Fundamentalism, SOC = Social Adjustment, DEP = Depression, MOR = Poor Morale, AUT = Authority Conflict, PHO = Phobias, HOS = Manifest Hostility, ORG = Organic Symptoms, PSY = Psychoticism, HYP = Hypomania, HEA = Poor Health, FEM = Feminine Interests.

VI may be either interpreted as severe psychopathology, as random or careless responding, or some combination of the two (cf. Sechrest & Jackson, 1963).

None of the MMPI clinical scales have any salient loadings on the religiosity factor. There is a serious question about the appropriateness of the inclusion of items dealing with religion on personality questionnaires. The religious items on the MMPI decrease rapport through apparent lack of face validity and perceived invasion of privacy. This suggests that clinicians or researchers that are only scoring the standard clinical scales could delete the religious items from the test administration with little or no loss of generality.

The factor reliabilities were not computed directly, since factor reliability is attenuated by loadings that are trivial (Gorsuch, 1980). Also the major purpose of factor analysis of an item pool is to indicate dimensions that may then be measured more succinctly by constructing new scales. But even if it is desired to retain the MMPI item pool, the factor results may be used to construct new scales. This can be accomplished by using the factor results as a starting point for an item analysis, since there is a close correspondence between item factor analysis and item analysis (cf. Henrysson, 1971).

In the present study items were selected and keyed on scales that had factor loadings of at least .30 and corrected item-total correlations of at least .30. Then, items were selected that had higher correlations with the scale on which they belonged (relevant scale) than with any other scale (irrelevant scale). In some instances these relevant-irrelevant item-total correlations differed only at the second decimal. To ensure that the scales were minimally redundant, all relevant-irrelevant correlations were tested for equality at the .05 level with a modification of Hotteling's T (cf. Steiger, 1980) and items were selected that statistically had a higher item-total correlation with the relevant scale than with any irrelevant scale. This is a fairly conservative procedure, given the large sample size in this study, and is sufficient for an item analysis conducted on one sample. With new samples a more stringent criterion for redundancy, such as the minimum redundancy item analysis algorithm...
Table 3
Highest and Lowest Salient Loadings For Each of the Six MMPI Item Factors

<table>
<thead>
<tr>
<th>Item Number Loading</th>
<th>Item</th>
</tr>
</thead>
<tbody>
<tr>
<td>487 .62</td>
<td>I feel like giving up quickly when things go wrong.</td>
</tr>
<tr>
<td>57 -.53</td>
<td>I am a good mixer.</td>
</tr>
<tr>
<td>72 .44</td>
<td>I am troubled by discomfort in the pit of my stomach every few days or oftener.</td>
</tr>
<tr>
<td>243 -.48</td>
<td>I have few or no pains.</td>
</tr>
<tr>
<td>453 .33</td>
<td>When I was a child I did not care to be a member of a crowd or a gang.</td>
</tr>
<tr>
<td>181 -.48</td>
<td>When I get bored I like to stir up some excitement.</td>
</tr>
<tr>
<td>507 .48</td>
<td>I have frequently worked under people who seem to have things arranged so that they get credit for good work but are able to pass off mistakes onto to those under them.</td>
</tr>
<tr>
<td>377 -.36</td>
<td>At parties I am more likely to sit by myself or just one other person than to join in with the crowd.</td>
</tr>
<tr>
<td>258 .52</td>
<td>I believe there is a God.</td>
</tr>
<tr>
<td>546 .30</td>
<td>I love to read about history.</td>
</tr>
<tr>
<td>512 .53</td>
<td>I dislike a bath.</td>
</tr>
<tr>
<td>187 -.33</td>
<td>My hands have not become clumsy or awkward.</td>
</tr>
</tbody>
</table>

proposed by Neill and Jackson (1976), will be employed.

In order to evaluate the role of desirability responding, the equality of the item-total correlation with the relevant scale was tested against the item-total correlation with the extreme undesirability scale of Jackson and Messick (1961). For Scales 1 and 6 all but a few items had an equivalent item-total correlation with desirability. It was thus not possible to select items for Scales 1 and 6 on the basis of their differentiating specific content from general desirability. For Scales 2 through 5 only a small proportion of the items had equivalent item total cor-
relations with desirability. This smaller proportion of items contaminated with a general factor permitted their elimination.

The correlations of these scales with the rotated factor scores were .93, .73, .84, .71, .73, and .78; and the coefficient alpha reliabilities of these scales were .95, .84, .83, .85, .86, and .95. Thus, the item analysis was quite successful in producing a homogeneous set of scales that putatively assess the content in the factor scores.

Discussion

Factor analysis of an item pool represents the internal approach to personality scale construction, as distinguished from the use of external criterion groups characterizing the construction of most of the MMPI clinical and validity scales (cf. Koss, 1979). The internal approach to scale construction, in contrast to the external approach, ensures a reasonably high degree of scale homogeneity and independence. The internal approach, however, is similar to the naive approach (cf. Ashton & Goldberg, 1973; Goldberg, 1978; Hase & Goldberg, 1967; Jackson, 1975) in that content is not differentiated from style, as it is in the construct-oriented strategy of test construction (cf. Jackson, 1970, 1971, 1973, 1979; Jackson & Paunonen, 1980).

In personality inventories constructed by the empirical, internal, or naive approaches the largest unrotated factor has been interpreted as primarily stylistic in nature. This interpretation has not achieved unanimity (Block, 1965; Rorer, 1965), but the presence of this large, pervasive general factor has never been in dispute. Simple structure rotation, unfortunately, distributes this intrusive, pervasive method variance over all dimensions.

A possible remedy to this problem, given the fixed nature of the MMPI item pool, is to partial out the variance associated with the first dimension (Paulhus, 1981). This will minimize the proportion of stylistic variance on the remaining dimensions. However, in personality inventories constructed by the empirical approach, stylistic processes are inevitably confounded with content in determining responses (Bramble & Wiley, 1974; Voyce & Jackson, 1977). Eliminating the first unrotated item factor would discard most of the reliable variance in the MMPI. Therefore, it would be best to control for stylistic variance by a priori methods. This control of stylistic variance should be part of a test blueprint which clearly distinguishes between content and style, and between generalized and more differentiated dimensions of psychopathology.

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**Acknowledgments**

The authors thank Richard A. Harshman for his comments during the preparation of this paper. Portions of this paper were presented at the annual meeting of the North American Branch of the Classification Society, Toronto, Ontario, June 1981. This study was supported by the Social Sciences and Humanities Research Council of Canada, Grant No. 410-80-0576-X1, and by the Solicitor General of Canada. The views in this paper are those of the authors and do not necessarily reflect the views or policies of the Solicitor General of Canada.

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