The Relationship Between the Perceived Risk and Attractiveness of Gambles: A Multidimensional Analysis

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Judgments of perceived risk and attractiveness for a set of 50 two-outcome gambles were obtained from 39 college students. The data were used to test various ordinal properties of the gambles implied by Pollatsek and Tversky's theory of risk and Coombs' Portfolio theory. In addition, the MDPREF multidimensional scaling procedure was used (1) to test the assumption that gambles are perceived and evaluated as multidimensional stimuli; (2) to determine the characteristics of gambles affecting perceived risk and attractiveness; (3) to assess the extent of individual differences in perception of gambles; and (4) to test the implication of Portfolio theory that attractiveness is a function of perceived risk and expected value. The results supported the multidimensional nature of gambles and the implications of Portfolio theory. In the MDPREF analyses large individual differences were found in perceived risk and attractiveness of gambles. Potential uses of multidimensional scaling techniques in further research on individual differences in gambling behavior are proposed and discussed.

The main focus of research on risky decision-making behavior has been to discover a set of simple qualitative or quantitative laws relating relevant psychological variables to observable risky decision-making judgments. Generally, it has been assumed that in a risky decision-making situation an individual maximizes expected utility. Both expected utility theory (EU theory) and its generalized counterpart, subjective expected utility theory (SEU theory; cf. Luce & Suppes, 1965), have, because of their appeal as normative theories, also enjoyed prominence as descriptive theories even though recent experimental research has seriously questioned their validity (Tversky, 1967; Lichtenstein & Slovic, 1971; Lindman, 1971; Coombs & Huang, 1976).

However, the concept of risk itself has been largely ignored as a possible relevant psychological variable in human decision-making. Although decision theorists have paid homage to the concept of risk by dividing decision-making literature into the two categories of "risky" and "riskless" choice situations, risk has played a minor role in decision theory. This has perhaps been due to two constraints—the first historical and the second methodological. Historically, the need for a notion of risk has been precluded by the prominence of EU and SEU theory, beginning with the axiomatization by von Newmann and Morgenstern (1947). Apparent decision-making inconsistencies within individuals as well as differences among individuals could be accounted for, at least theoretically, by appropriately shaping the individuals' utility curves (Friedman & Savage, 1948; Markowitz, 1952). Methodologically, the concept of risk has posed an apparent serious problem since it appears that risk is to some extent idiosyncratic; what

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might appear to be a very risky alternative for one individual might not be risky for another. Hence, early attempts to include the notion of risk in decision theories often did so by operationally defining it (Fisher, 1906; Domar & Musgrave, 1944; Markowitz, 1959).

Two notable exceptions to this treatment of risk in decision-making theories have been proposed by Coombs (1969) and Pollatsek and Tversky (1970). Although the notion of risk is of fundamental importance in Coombs' Portfolio theory, it is left undefined. In contrast risk is itself the object of theoretical investigation in Pollatsek and Tversky's theory.

Portfolio Theory

Portfolio theory (Coombs, 1969; Coombs & Huang, 1970, 1976) can be considered a generalization of EU theory. It assumes that an individual will not only attempt to maximize expected value (EV), but also will optimize the level of risk. It proposes that at any level of EV there exists an ideal level of risk; that is, the individual will exhibit a single-peaked preference function over risk at a given level of EV.

Given three gambles with the same EV ordered in risk by an individual as $A>B>C$, six preference orders are possible. Only two of these are compatible with EU or SEU theory, the *monotone* orders *ABC* and *CBA*. In addition to these two orders, the *folded* orders *BCA* and *BAC* are permissible in Portfolio theory, since *B* may be located closest to the ideal level of risk. The final two *inverted* orders *CAB* and *ACB* are not permissible in either theory, since *B*, the middle gamble in risk, can never be the least preferred.

The results of several experiments by Coombs and Huang (1976) strongly supported Portfolio theory in that their subjects' data exhibited significantly more folded preference orders than inverted orders—thus directly violating EU or SEU theory. Portfolio theory, then, appears to be superior to EU or SEU theories in describing risky decision-making behavior. Its very success, however, points directly to the need for a further understanding of the nature of the concept of risk itself.

Theory of Risk

Pollatsek and Tversky (1970) have proposed an axiomatic theory of risk formulated in terms of a risk system $(S, \bigcirc, \geqslant)$ where S can be interpreted as a set of options or gambles, $\bigcirc$ is a binary addition or convolution relation on the gambles, and $\geqslant$ is a binary risk relation on gambles in S. Given that their axioms are true, Pollatsek and Tversky showed that there exists a real-valued function R defined on S such that for gambles A and B

(1) $A \geqslant B$ if $R(A) > R(B)$
(2) $R(A \bigcirc B) = R(A) + R(B)$
(3) If R' satisfies 1 and 2 above, then $R'(A) = a \cdot R(A)$ for some $a > 0$ (R is a ratio scale)
(4) $R(A) = \alpha \cdot \text{Variance}(A) - (1-\alpha)EV(A)$

If their axiom system is accepted, the theorems imply that risk is a ratio-scaled variable with a unique zero point equal to the "status quo" or, equivalently, to the gamble *0* where one wins or loses nothing. The theories also imply that risk is additive and that it is only a function of the gamble's variance and expected value.

While the fourth implication appears to be too restrictive in describing individual subjects' data (Payne, 1975), the theory is important in that it suggests some interesting testable relations about risk. First, it suggests that for any gamble A, if K is a gamble with constant outcome \$$k$ (i.e., one wins \$$k$ with probability 1), then the gamble $A \bigcirc K$, equal to the new gamble which adds k to all outcomes of A, should have risk equal to $R(A) + R(K)$. Secondly, although appearing to be counterintuitive, it suggests that K cannot have zero risk if A is perceived as being strictly riskier than $A \bigcirc K$ (i.e., $A > A \bigcirc K$), since $R(A \bigcirc K) = R(A)$ only if $R(K) = 0$. This implies, then, that if \$$k > 0$, K should be perceived as less risky than *0* and should have negative risk.

Purpose and Assumptions

The present study was designed to test these two implications concerning the nature of risk as well as to provide a direct test of Portfolio theory; portfolio theory implies that gambles can be represented by points in a two-dimensional space with perceived risk and EV as the dimensions. This study proposes that both gambles and risky-decision alternatives in general are perceived and evaluated as multidimensional stimuli. The idea is not new, since many psychological theories are based on the assumption that stimuli are perceived and evaluated in some dimensionally organized fashion (cf. Carroll, 1972; Nygren & Jones, 1977). Specifically, the present study is based on the assumption that an individual's risk order and preference order for a set of gambles are determined by the location of risk and preference vectors passing through the origin of the multidimensional space. The dimensions of the space represent the psychological attributes of the gambles which are relevant in forming judgments of perceived risk or attractiveness.

If, as Portfolio theory suggests, preference is a function of both risk and EV, then there should be an interesting relationship between multidimensional spaces generated from ratings of risk and attractiveness. If, for an individual, the risk order for a set of gambles can be accounted for by an r-dimensional representation of the gambles, then the preference order should be accounted for by an $r + 1$ dimensional representation of the gambles, the $r + 1$ dimensions being the same r dimensions associated with perceived risk with an added EV dimension. In order to test this implication and to determine the relevant dimensions of the gambles which are used by individuals to make judgments about the perceived risk and attractiveness of gambles, subjects' risk and attractiveness ratings were analyzed via a specific multidimensional scaling model, MDPREF (Carroll, 1972).

MDPREF utilizes subjects' ratings of a set of stimuli in an attempt to generate a multidimensional representation of the stimuli. Individual differences are accounted for in the model by the location of the various subjects' vectors in the space. The projections of the stimuli on the vectors determine the risk (preference) order of the stimuli. The cosines of the angles which the vectors form with the dimensions of the space can be interpreted as reflecting the importance of these dimensions as determiners of perceived risk (preference) among the stimuli. This "vector" model (cf. Carroll, 1972) implies in the case of gambles that the more (or less) of a particular attribute that a gamble has (e.g., amount to win or lose), the more (or less) it will be preferred and the more (or less) risky it will be perceived. Finally, it is important to note that this analysis attempts to determine the relevant dimensions used by individuals in making their judgments; no a priori conceptualizations of risk or attractiveness are imposed on the subjects.

Method

Subjects

The subjects were 44 male students who participated in the experiment as part of an introductory psychology course requirement. Data from five subjects were dropped either because the subjects had failed to follow instructions or because their data were incomplete.

Stimuli

Fifty two-outcome gambles of the form (a¢, ½, b¢) were constructed in which one obtains either outcome a¢ or b¢ with probability ½. They are shown in Table 1 in the abbreviated form (a, b). Four gambles had EV = 0¢, seven had EV = 5¢, 10 had EV = 10¢, 13 had EV = 15¢, and 16 had EV = 20¢. The gambles were constructed in such a way that those with EV = 0¢ could be transformed into the first four gambles with EV = 10¢ by adding the constant gamble (10¢, ½, 10¢). Similarly, the seven gambles with EV = 5¢ could be transformed into the first seven gambles with EV = 15¢; and the 10 gambles with EV = 10¢ could be transformed into the first 10 gambles with EV = 20¢.

Table 1
Gambles Used as Stimuli

EV=0¢ Gamble	EV=5¢ Gamble	EV=10¢ Gamble	EV=15¢ Gamble	EV=20¢ Gamble
1 (0, 0)	5 (4, 6)	12 (10,10)	22 (14,16)	35 (20,20)
2 (-4, 4)	6 (0,10)	13 (6,14)	23 (10,20)	36 (16,24)
3 (-8, 8)	7 (-4,14)	14 (2,18)	24 (6,24)	37 (12,28)
4 (-12,12)	8 (-8,18)	15 (-2,28)	25 (2,28)	38 (8,32)
	9 (-12,22)	16 (-6,26)	26 (-2,32)	39 (4,36)
	10 (-16,26)	17 (-10,30)	27 (-6,36)	40 (0,40)
	11 (-20,30)	18 (-14,34)	28 (-10,40)	41 (-4,44)
		19 (-18,38)	29 (-14,44)	42 (-8,48)
		20 (-22,42)	30 (-18,48)	43 (-12,52)
		21 (-26,46)	31 (-22,52)	44 (-16,56)
			32 (-26,56)	45 (-20,60)
			33 (-30,60)	46 (-24,64)
			34 (-34,64)	47 (-28,68)
				48 (-32,72)
				49 (-36,76)
				50 (-40,80)

Procedure

Each subject was presented with a booklet containing instructions and answer sheets for three different rating tasks. They were then given a subset of five gambles as illustrations of the type of gambles they would be rating; a brief presentation on how to interpret each gamble was then provided. Subjects were told that, following the experiment, each would be allowed to play one gamble. This gamble was to be selected by the experimenter on the basis of their individual ratings.

In Task 1, subjects were instructed to rate the 50 gambles in terms of riskiness. No attempt was made to define risk; subjects were encouraged to make judgments on the basis of what risk meant to them. Each subject was presented with a sheet containing a list of all 50 gambles and was then instructed to (1) search through the list, find the gamble which seemed most risky, and give it a rating of "100"; (2) find the least risky gamble and give it a rating of "1"; and (3) rate the riskiness of the remaining 48 gambles using numbers between 1 and 100. Subjects were allowed as much time as needed and were allowed to make changes or corrections throughout the task. They were instructed that it was entirely permissible to give two or more gambles the same rating if they felt these gambles were equally risky.

Task 2 consisted of making judgments about the differences in risk between a pair of gambles using a scale ranging from "1" (Highly Similar) to "20" (Highly Dissimilar). The set of pairs used was generated from a subset of 20 of the gambles. This task was included to generate pilot data for another study; no further use of the data is reported here.

Task 3 was exactly comparable to Task 1 except that subjects were asked to rate the attractiveness of the same set of 50 gambles in terms of their preference for playing each gamble. The rating procedure was also analogous in that on a new listing of the gambles each subject found his most preferred gamble and rated it "100," found his least preferred gamble and rated it "1," and then rated the remaining 48 gambles.

Half of the subjects made their ratings in order of Tasks 1, 2 and 3; and half made them in the order Tasks 3, 2 and 1.

Results

Risk Ratings

The riskiness ratings from the 39 subjects were rescaled to have a standard deviation of one and an origin of zero for the gamble (0,0). Any gamble having a rescaled risk rating less than zero would have negative risk; that is, it would be less risky than the status quo. In an attempt to examine the theory that some gambles do have negative risk, as Pollatsek and Tversky (1970) suggest, the ratings for the 15 gambles with both outcomes non-negative were examined in detail. Table 2 presents the number of subjects rating each of these gambles as being less risky, equal in risk, or more risky than (0,0). The results indicate that there were large differences among subjects regarding what constitutes risk. A few subjects' data indicated small rating errors in that the two sure-thing gambles, (10,10) and (20,20), were rated as riskier than the status quo. Nevertheless, the majority of subjects rated them as less risky or equal in risk to (0,0). Thirteen subjects rated (10,10) as less risky than the status quo, and 22 rated it equal in risk; 15 subjects rated (20,20) as less risky, and 21 rated it equally risky. An average of 62% of the ratings given to the other 13 non-negative gambles were in the "more risky" category, an average of 24% were in the "less risky" category, and an average of 14% were in the "equal in risk" category. Accordingly, it appears that for most subjects the introduction of variability in a gamble was sufficient to make it more risky than the status quo, even though no actual loss could be sustained. For other subjects, however, as long as no loss can be sustained, it appears that the gamble was perceived as having either no risk or less risk than (0, 0). The final column in Table 2 shows the mean ratings given to the 15 non-negative gambles. Five of the gambles had overall negative mean risk ratings—(4,6), (10,10), (14,16), (20,20), and (16,24).

Table 2
Risk Ratings for Gambles with Nonnegative Outcomes

Gamble	Less Risky than (0,0)	Equal in Risk to (0,0)	More Risky than (0,0)	Mean Rating
(4, 6)	9	6	24	-.012
(0,10)	8	5	26	.231
(10,10)	13	22	4	-.128
(6,14)	8	4	27	.127
(2,18)	8	4	27	.250
(14,16)	12	8	19	-.064
(10,20)	10	6	23	.030
(6,24)	10	5	24	.148
(2,28)	8	4	27	.274
(20,20)	15	21	3	-.180
(16,24)	11	7	21	-.002
(12,28)	10	6	23	.057
(8,32)	9	5	25	.160
(4,36)	10	5	24	.228
(0,40)	7	6	26	.392

These results provided some support for Pollatsek and Tversky's assumption that certain gambles do have negative risk for at least some individuals. On the other hand, Pollatsek and Tversky's implication that risk is additive was not substantiated. Within the set of gambles used, there were 20 (Gamble A) which, when concatenated with the gamble (10,10), form 20 other gambles (Gamble B) that were also in the set. If risk is additive, then $R(B) = (A \bigcirc (10,10))$ should equal $R(A) + R(10,10)$. For example, the risk of (4,36) should equal the sum of the risks of (−6,26) and (10,10).

In an attempt to analyze this implication in detail, the data from the 13 subjects who rated (10,10) as being less risky than (0,0) and the 22 subjects who rated (10,10) as being equal in risk to (0,0) were analyzed separately for the 20 pairs of gambles. The mean absolute discrepancies between the predicted rating $R(A) + R(10,10)$ and observed rating $R(B)$ for the concatenated gambles B were found to be .392 for the 13 subjects and .467 for the 22 subjects. This result indicates that the additivity assumption was substantially violated for both groups. As expected from the theory, the discrepancies for the 13 subjects were generally smaller than for the other group; however, the difference between the means was not significant, ($p > .25$). Moreover, the violations indicated a general superadditive effect. Of the 700 comparisons for predicted and observed ratings (20 pairs for each of the 35 subjects), 540 or 77.1% were in the direction of the predicted value for Gamble B being greater than its observed value.

MDPREF Analysis of Risk Ratings

The MDPREF analysis on the 39 subjects' risk ratings for the 50 gambles indicated that a two-dimensional space was appropriate. The proportion of variance accounted for by these two dimensions was 88.1%. Including a third dimension added virtually nothing; it only accounted for an additional 2.8% of the variance. In addition, the root-mean-square correlation between subjects' risk ratings and the projections of the gambles on the subjects' fitted vectors increased from .88 to .94 to .95 for one-, two-, and three-dimensional spaces respectively. Because MDPREF uses a Euclidean representation, the derived two-dimensional space is invariant to a general class of rotations and translations. Hence, the two dimensions derived in the analysis are not necessarily psychologically unique.

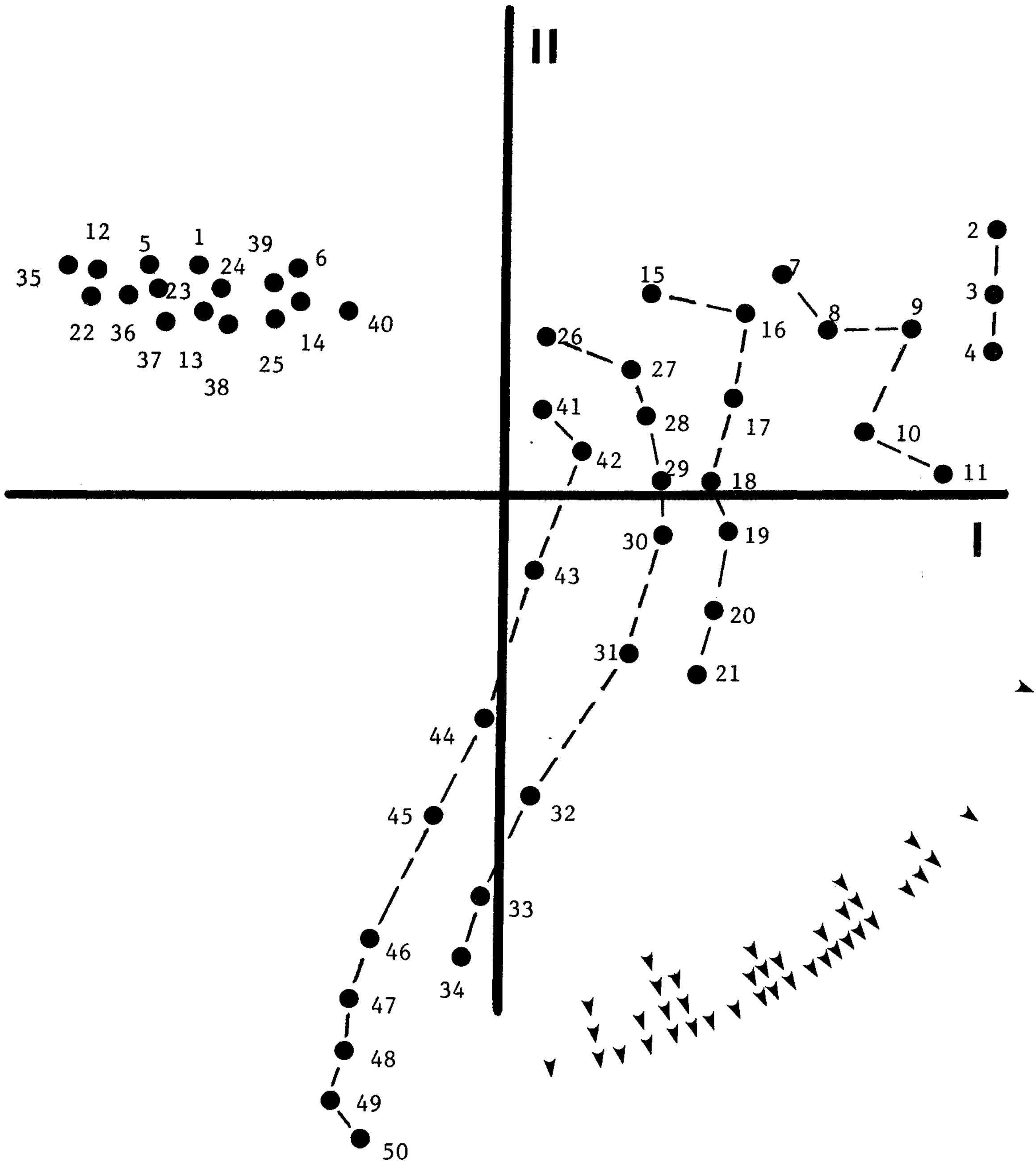
An interpretable dimensional orientation was formed by rotating the derived space about 30°. This normalized two-dimensional space is presented in Figure 1. The gambles are coded by the labels given in Table 1, and those with a negative outcome are connected by lines representing equal levels of EV. The arrows indicate the directions of the 39 subjects' risk vectors.

Dimension 2 was highly correlated with the actual variances of the gambles ($r = -.981$). The projections of the gambles along Dimension 1 indicated an interesting separation; the gambles with non-negative outcomes formed a degenerate set along this dimension. It appears that for most subjects these gambles either were not or could not be meaningfully differentiated with respect to "risk." The gambles with one negative outcome were, however, differentiated by their EV. Hence, Dimension 1 appears to be related to EV but in a non-simple, yet meaningful, way. The subjects' vector directions corresponded to what is expected for a risk scale, yet exhibited a moderate amount of individual differences.

MDPREF Analysis of Attractiveness Ratings

While the two dimensions suggested above are psychologically meaningful, the MDPREF analysis does not guarantee that these are the unique dimensions of the gambles affecting perceived risk. This limitation, however, does not affect the crucial test of Portfolio theory mentioned earlier—that preference is a function of perceived risk and EV. If Portfolio theory accurately describes the means by which preference judgments for gambles are formulated, then the MDPREF analysis based on the attractiveness ratings for these gambles should generate a three-dimensional space with an EV dimension and a two-dimensional subspace

Figure 1
Two-dimensional representation
of the 50 gambles based on risk ratings.



which is equivalent to that derived from the risk judgments.

Two three-dimensional spaces were obtained. The first was a normalized target space; its three dimensions were Dimensions 1 and 2 from the risk analysis and an EV dimension. The representation of the 50 gambles in this target space is shown in Figures 2 and 3 with the filled circles representing the gambles. MDPREF was used to obtain the actual preference space based on the

Figure 2
Dimensions 1 and 2 of the three-dimensional representation of the gambles based on attractiveness ratings. Circles indicate target (predicted) locations of the stimuli and squares indicate actual (observed) locations.

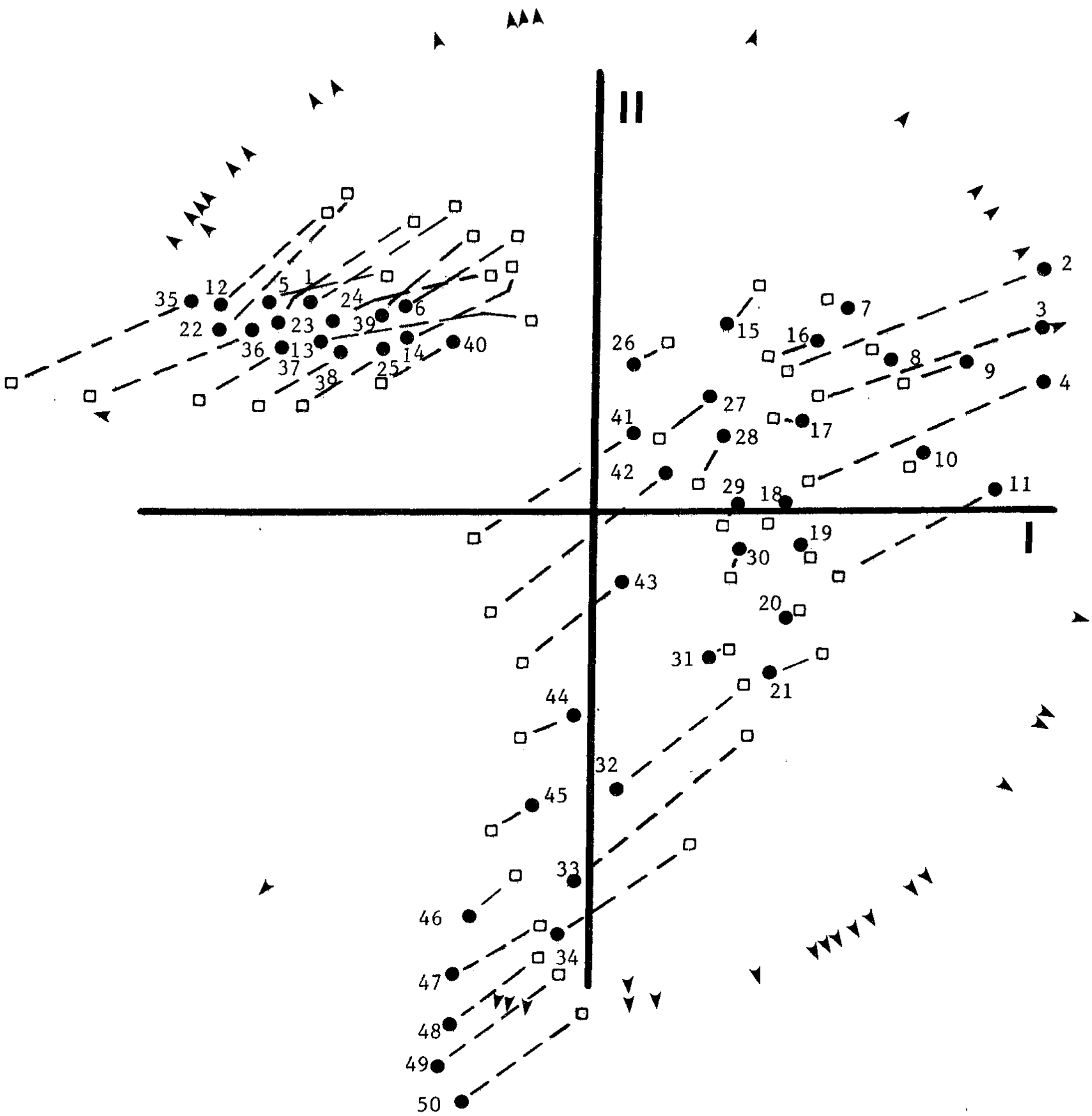
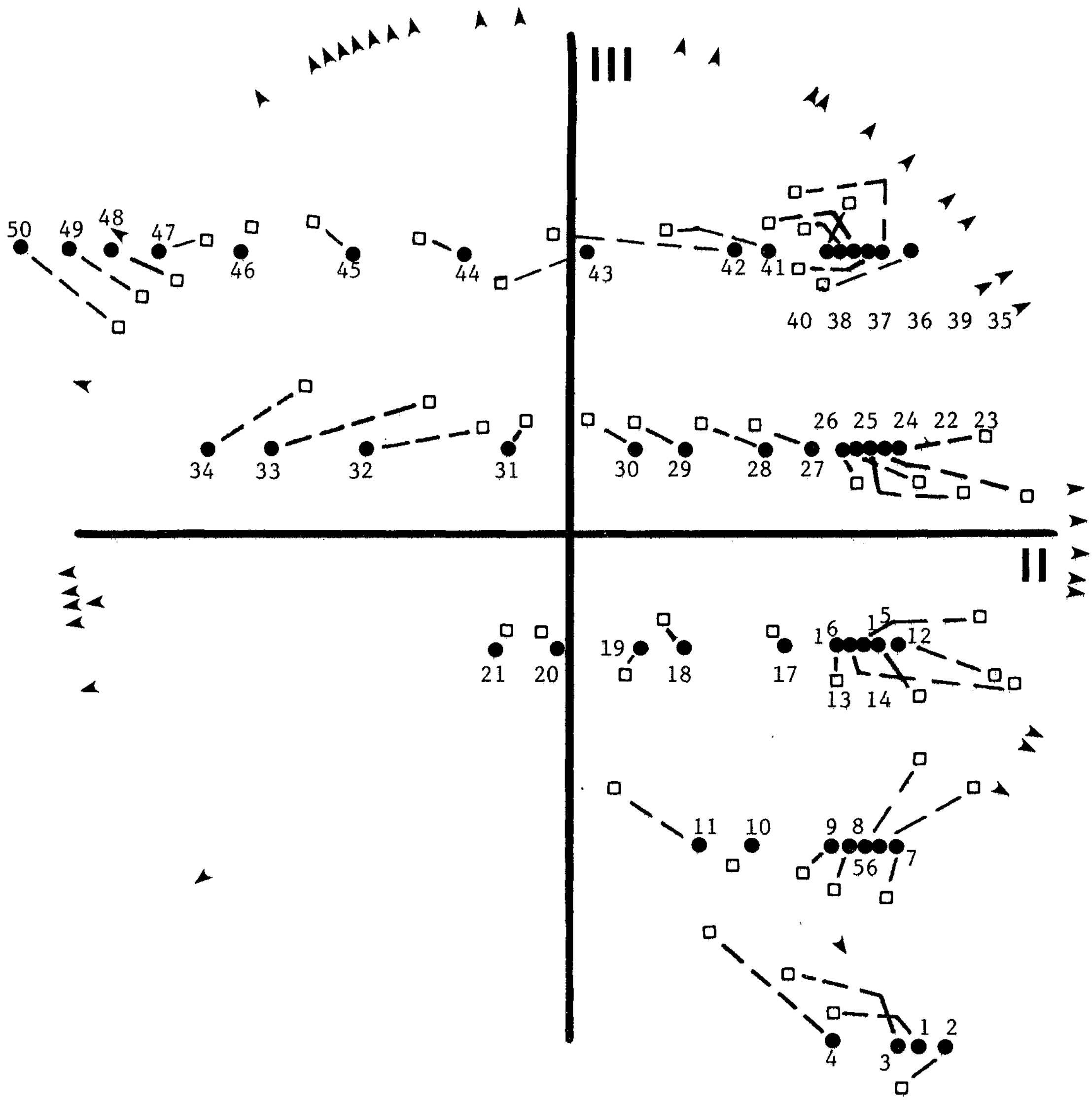


Figure 3
Dimensions 2 and 3 of the three-dimensional representation of the gambles based on attractiveness ratings.



subjects' attractiveness ratings. A three-dimensional space was, in fact, obtained. These three dimensions accounted for 84.0% of the variance in the subject's data. Adding a fourth dimension increased the variance accounted for by only 3.5%. The root mean square correlations between the subjects' attractiveness ratings and their vectors increased from .69 to .88 to .92 to .93 for projections of the gambles in the one-, two-, three-, and four-dimensional spaces respectively.

A general linear regression procedure was used to obtain a transformation matrix, **T**, for rotating the derived three-dimensional solution to maximum congruence with the three-dimensional target space.[1] The locations of the subjects' vectors were then transformed correspondingly by using the appropriate transformation matrix $(\mathbf{T}')^{-1}$. The squares in Figures 2 and 3 represent the derived locations of the gambles in the rotated space. The correspondence between the two independently obtained sets of points (derived vs. target) was very good. Dimensions 1 and 2 corresponded respectively to Dimensions 1 and 2 obtained in the "risk space", and Dimension 3 reflected the EV of the gambles. The correlations between the projections of the pairs of derived and target points for the 50 gambles on Dimensions 1, 2 and 3 were .946, .989, and .864, respectively. These results support the hypothesis that subjects' preferences for these simple two-outcome gambles can be accounted for by a multidimensional representation of the gambles in which the dimensions are EV and those attributes affecting perceived risk.

Figures 2 and 3 also show the locations of the subjects' preference vectors in the space. Although it is difficult to get a true picture of the location of an individual's vector by looking at the 1-2 and 2-3 dimensional planes, it is clear that individual differences are extensive among the 39 subjects. Figure 2, showing the two "risk dimensions," illustrates that the subjects were rather uniformly spread out in terms of some being "high," "moderate," or "low" risk takers.

MDPREF Analyses Based on the 34 Negative-Positive Gambles.

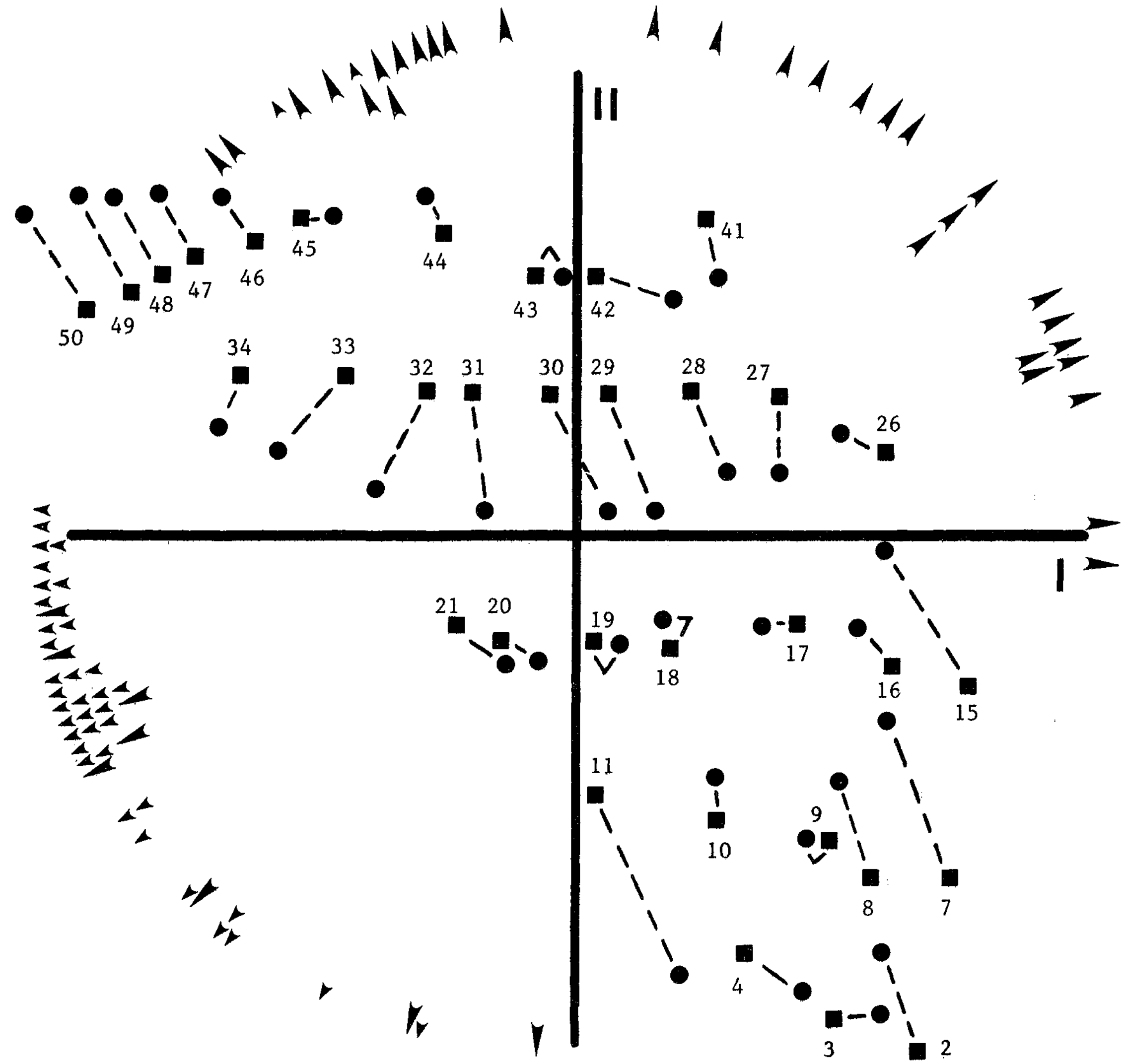
The results from the risk analysis in Figure 1 suggest that had the 16 non-negative gambles not been included, the two-dimensional space might have been characterized both by a variance dimension and by an EV dimension. Similarly, the space based on the attractiveness data might also have been only two-dimensional, since for the remaining 34 gambles, Dimensions 1 and 3 would be essentially identical. To test this hypothesis, a separate MDPREF solution was obtained from the risk and attractiveness data based on ratings for these 34 gambles. In both cases a two-dimensional space was, in fact, obtained. The two-dimensional spaces accounted for 77.4% and 80.1% of the variance in the data for the two respective cases. Each space was rotated (using the same procedure described earlier) to maximum congruence with a two-dimensional target space, the dimensions of which were represented by the variance and EV of the gambles. Figure 4 presents the locations of the 34 gambles in this space based on the risk ratings (circles) and the attractiveness ratings (squares). Again the agreement between corresponding points was very good. The locations of the subjects' risk and attractiveness vectors are represented by small and large arrows respectively. These results are compatible with Portfolio theory in that they suggest that at least for simple gambles of this kind, the risk space and the attractiveness spaces are essentially the same. The exceptions are that variance is weighted more heavily in judging risk and EV is generally more important in judging attractiveness of the gambles.

Ordering Tests of Portfolio Theory

As a final test of Portfolio theory, two additional analyses of the subjects' risk and attractiveness ratings were conducted. Both EU theory and Portfolio theory assume that for any pair of gambles A and B, if A is at least as preferable as B, then $A \bigcirc K$ will be at least as preferable as $B \bigcirc K$. Within the set of 50 gambles there were 78 pairs for which new pairs $A \bigcirc K$ and $B \bigcirc K$ could be obtained by letting K be the constant gambles (10,10) or (20,20). Table 3 presents the

[1]MDPREF is a scalar products model for which the matrix of predicted subjects' ratings (attractiveness in this case) denoted by the matrix $\mathbf{S}$, is equal to the scalar product of the subject vectors (matrix $\mathbf{Y}$) and the stimulus points (matrix $\mathbf{X}$). Specifically, $\mathbf{S} = \mathbf{Y}\cdot\mathbf{X}'$. Hence, a general transformation, $\mathbf{T}$, can be applied to $\mathbf{X}$ to define a "new" space $\mathbf{X}^* = \mathbf{X}\cdot\mathbf{T}$ and "new " vector locations $\mathbf{Y}^* = \mathbf{Y}\cdot(\mathbf{T}')^{-1}$. These two matrices, $\mathbf{X}^*$ and $\mathbf{Y}^*$, form an equally valid representation since $\mathbf{Y}^*\cdot\mathbf{X}^{*\prime} = \mathbf{Y}\cdot(\mathbf{T}')^{-1}\cdot\mathbf{T}'\cdot\mathbf{X}' = \mathbf{Y}\cdot\mathbf{X}' = \mathbf{S}$. The author is indebted to one of the reviewers for pointing out the advantage of using this procedure instead of a special Procrustes procedure.

Figure 4
Two-dimensional representation
of the 34 negative-positive gambles.
Circles represent locations based on risk ratings
and squares represent locations based on attractiveness ratings.



number of pairs for which the preference order was either maintained or reversed. The results support this ordering assumption. For 19 subjects there were no reversals and the mean number over all subjects was 3.51.

For each subject there were a total of 1005 triples of gambles with the same EV which could be formed, 4 at EV = 0¢, 35 at EV = 5¢, 120 at EV = 10¢, 286 at EV = 15¢, and 560 at EV = 20¢. Each triple was ordered in risk according to the ratings given by the subjects. The number of preference orders for the set of triples which were either monotone, folded, or inverted were counted. These frequencies are also reported in Table 3 for each subject. Since no attempt was made in the construction of the gambles to in-

Table 3
Tests of Preference Orders for Concatenated Pairs of Gambles and for Triples of Gambles Ordered in Risk

5 Monotone Ss				17 Most Consistent Ss				17 Least Consistent Ss			
Sub	Pairs: Errors	Triples: Fold	Inv	Sub	Pairs: Errors	Triples: Fold	Inv	Sub	Pairs: Erros	Triples Fold	Inv
1	0	0[a]	0[a]	6	0	90	14	23	2	126	25
2	0	0	0	7	0	66	44	24	2	1	13
3	0	0	0	8	0	31	4	25	3	21	46
4	0	0	0	9	0	62	1	26	3	90	121
5	0	0	0	10	0	287	155	27	3	224	77
Tot	0	0	0	11	0	121	179	28	4	218	246
				12	0	395	88	29	4	96	14
				13	0	140	226	30	5	313	128
				14	0	100	0	31	5	33	53
				15	0	9	5	32	5	12	35
				16	0	191	264	33	7	257	295
				17	0	128	23	34	8	116	427
				18	0	69	23	35	11	127	494
				19	0	0	9	36	12	255	116
				20	1	71	3	37	17	296	190
				21	1	231	46	38	18	19	172
				22	1	118	317	39	25	143	121
				Tot	3	2109	1401	Tot	134	2347	2573

[a]The number of monotone orders for each subject can be found by subtracting the sum of the folded and inverted orders from 1005.

sure that the sets of gambles at each level of EV necessarily bounded every subject's ideal level of risk, no specific number of folded or monotone orders was expected. However, since inverted orders are violations of both EU theory and Portfolio theory, their frequency can be treated as an estimate of the error rate; inverted orders should generally be largest for the most inconsistent subjects. Five subjects were found to be completely monotone over all 1005 triples. The number of inconsistent pair orderings from the previous analysis was used as an estimate of subject reliability for the remaining 34 subjects. These subjects were divided into two groups of 17 as shown in Table 3.

If EU theory is correct, then folded orders are also errors; and their frequency should also be greater for Subjects 23–39 than for Subjects 6–22. Using a Mann-Whitney-Wilcoxon test, the median number of folded orders was not found to differ significantly for the two groups ($z < 1$). The median number of inverted orders was, however, significantly greater for the least consistent subjects ($z = 2.00$, $p < .025$). These results suggest that folded orders have a regularity which is predicted by Portfolio theory; in contrast, inverted orders are errors which are related to subject inconsistency.

Discussion

The basic assumption implicit in the methodology used to test Coombs' Portfolio theory in this study is that gambles are perceived and evaluated as multidimensional stimuli. Two aspects of the results of the MDPREF analyses for both the risk and attractiveness data provided encouraging support for this assumption. First, the goodness-of-fit of the subjects' data as measured by the amount of variance accounted for was very good, being 88.1% for the two-dimensional risk space and 84.0% for the three-dimensional preference space. Secondly, highly interpretable dimensions were found. Perhaps the most encouraging result with respect to Portfolio theory is the confirmation of the prediction that the two-dimensional spatial representation of the gambles derived from the risk ratings should be a subspace of a three-dimensional representation based on the attractiveness ratings. The deviations between the predicted and observed locations of the 50 gambles in Figures 2 and 3 were found to be generally very small, as evidenced by the high correlations between their predicted and observed projections along all three dimensions. In addition, when the MDPREF analyses were based on the subset of 34 negative-positive gambles, the same two-dimensional representation with EV and variance dimensions was obtained.

Coombs (1964) has shown that an analysis of preference data based on a factor analytic procedure will often produce an extra dimension or factor which represents what he called a "social utility" dimension. Although MDPREF uses a scaling procedure which is not strictly based on a factor analysis approach, it might be argued that in the analyses of all 50 gambles, the variance and EV dimensions (Dimensions 2 and 3) are the only "real" dimensions having behavioral significance. Dimension 1 can be considered to be this artifactual "social utility" dimension. The fact, however, that this first dimension was also found in the risk space obtained from a different kind of evaluative judgment suggests that it is probably not an artifact. It appears, rather, to be meaningful in describing how most individuals separated the 16 non-negative gambles from the remaining 34 more "realistic" gambles.

The additional analyses reported in this study are also either compatible with or supportive of Portfolio theory. This is not the case for Pollatsek and Tversky's theory of risk. Perhaps the most powerful implication of the latter theory is that risk is additive. Although the test of this implication in the present study was based on an admittedly small number of observations (n = 20) within each subject's data, the consistent pattern of a strong super-additive effect for all subjects is detrimental to the theory.

One possible argument for these results is that the risk judgments cannot be treated as interval scale data and that a monotonic transformation might be applied to the data in order to make the risk scale additive. Even though this argument seems generally plausible, the data from those subjects rating the gambles (10,10) and (0,0) equal in risk makes an additive scale impossible to achieve. If, as most subjects indicated, $R(-2, 22) > R(8, 32)$, then $R(10, 10)$ cannot equal $R(0, 0)$. Since $(8, 32) = (-2, 22) \circ (10, 10) = (8, 32) \circ (0, 0)$, the additivity of risk postulate implies that $R(-2, 22) + R(10, 10) = R(8, 32) + R(0, 0)$. If an individual says that $R(10, 10) = R(0, 0)$, then this implies that $R(-2, 22) = R(8, 32)$ which contradicts what they indicated. No monotonic transformation of the data can avoid this problem, unless some other concatenation procedure is hypothesized.

A second explanation of these results is suggested both by the distribution of risk ratings given to the 15 non-negative gambles in Table 2 and by the locations of these gambles in the "risk space" (Figure 1). It appears that large individual differences exist in the perceived risk of these gambles with respect to the status quo gamble (0, 0). Figure 1 also suggests that these gambles may be perceived as being uniquely different from those for which an actual loss can be sustained. For most individuals it appears that risk may only make semantic sense if a loss can be sustained. If this is the case, then perhaps the completely non-negative gambles and the negative-positive gambles are not directly comparable in the simple fashion proposed by Pollatsek and Tversky. It might be that risk is approximately additive within each set of gambles but not across sets. Coombs and Bowen (1971), however, found evidence which suggests that this may not be the case.

It is clear from these analyses that perceived risk is both a meaningful and measureable characteristic of gambles and that any descriptive theory of gambling behavior necessitates the inclusion of this concept. The multidimensional scaling methodology employed here has been shown to be a powerful asset for identifying characteristics of perceived risk. One advantageous aspect of this methodology is that no a priori definition of risk or other restrictions are imposed on the subjects in making their ratings. Equally important is the fact that individual differences are not ignored. They are, in fact, easily identified by the locations of the subjects' vectors in the space.

This multidimensional vector representation has interesting potential as a tool for field research. It has potential as an aid in separating and identifying various "types" of gamblers with respect to their degree of risk-taking behavior. The methodology employed in this study also has obvious potential for field research. It could be utilized in research concerned with the correspondence between various personality types and gambling types. Finally, if as suggested by the present study, simple risk and attractiveness ratings are sufficiently rich to yield multidimensional representations of risky decision alternatives, then this methodology offers a simple means of data collection for assessing and identifying individual differences.

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