

ESSAYS ON PORTFOLIO CHOICE OVER THE LIFE
CYCLE

A DISSERTATION SUBMITTED TO THE FACULTY OF THE
GRADUATE SCHOOL OF THE UNIVERSITY OF MINNESOTA

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

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AUGUST 2010

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Acknowledgements

I am grateful to my advisor Narayana Kocherlakota for his continuous guidance, invaluable support, help and patience. I thank Christopher Phelan, Roger Feldman and Fatih Guvenen for their helpful comments and suggestions on my work and for their service on my exam committees. I received helpful comments on my work from Kim Sau Chung, Hui He, Greg Kaplan, Katya Kartashova, Karen Kopecky, Zheng Liu, Minjung Park, Fabrizio Perri, Jose-Victor Rios-Rull, Martin Schneider, Jan Werner, and the members of the Workshop in Public Economics and Policy at the University of Minnesota. I also thank participants of the 2009 Quantitative Society for Pensions and Saving at Utah State University, as well as seminar participants of the 2009 Midwest Macroeconomics Meetings. Chapter 1 is coauthored with Hui He.

To my parents, Qiqing Guo and Xiaojin Shi

Abstract

Chapter 1 firstly documents that in the U.S., the stock market participation rate over the life-cycle decreases as people age. This fact, however, can not be captured by standard model where smooth expected utility function predicts the decision maker stay in the stock market, given positive equity premium and independence between older people's non-asset income and stock return. This paper successfully replicates this fact by introducing Knightian uncertainty into a dynamic model with choices on medical care and consumption, as well as saving and portfolio. We assume the agents in the model have ambiguity towards the correlations between risky stock return and risky medical price inflation. In this environment, retirees quit the stock market under reasonable range of ambiguity towards the correlation. The key mechanism is that: The agents do not hold positive amount of stocks since they worry stocks co-vary positively with the non-asset income minus the health expenses. Similarly, they do not short stocks as they also worry that stocks and their de facto non-asset income may co-move negatively.

Chapter 2 investigates the nature of riskiness in borrowing costs and its implication for households' portfolio choice over the life cycle. This paper firstly documents a negative correlation between borrowing cost and stock return as well as a wedge between saving and borrowing rates. This fact is then incorporated into life cycle portfolio choice model as key ingredient to study households' risky asset, safe asset and debt holdings. The model can deliver that the households' mean assets holdings are non zero in all three categories: loans, safe bonds and risky assets. And it shows as relative risk aversion increases, the asset holdings among all categories become more realistic. This improves the life cycle models with tight borrowing cost, where loan holdings are zero, which is inconsistent with data; and it improves from the model with loose borrowing constraint and risk free borrowing cost as low as saving rate, where agents invest too much in stocks. Yet the model can't deliver realistic asset holding with low constant relative risk aversion.

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Chapter 1

Health and Portfolio Choice in Retirement: The Impact of Ambiguity

1.1 Introduction

The older people hold disproportional amount of wealth. And health risk is the primary non-financial risk that older people are facing. In public policy, the fresh Health Care Bill can change the distribution of health expenditure that older people are exposed to and might so change their financial asset demand. Despite the grave importance of health risk's impact on older people's financial decision, relatively little has been done to help us understand on these questions: how does health affect people's financial decision, and through what channel?

This paper answers the above questions by first documenting evidence on older people's portfolio choice and health status. We find evidence that suggests that relatively more healthy people are more likely to take financial risks by investing in stocks, and that relatively more healthy people are more likely to invest large share of their wealth into stocks than unhealthy people. If we focus on the participation margin, we observe that even among households with significant amount of financial wealth, the participation rate in stock market starts to drop at age 60. This fact suggests that some older people are divesting their stock holdings completely to safe assets. And the dropout happens mostly to unhealthy and not wealthy group.

Previous literature on life cycle portfolio choice, however, failed to reconcile this dropout. Not only in the environment without health risk, such as (?) a Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2006), and Davis, Kubler, and Willen (2006), but also in the environment with health risk, such as Yogo (2009), the literature has predicted 100% participation rate for retirees. The reason for the failure is based on the assumption that health expenditure risk is orthogonal to financial risk. With this assumption, and with positive equity premium, the decision maker in those economy will always have incentives to invest in stocks when their non asset income risk has been resolved. Intuitively, given the fact that stocks are good deals, and with background risk orthogonal to stock risk, the first dollar they save should actually go to stock account. With such strong incentive to participate in stock market as retirees, the frictions introduced to these models can not help to explain dropout either. More specifically, These literature fails to explain, 1) Why health matters in portfolio choice problem; and 2) Why older people with significant wealth are not investing stocks. For instance, Vissing-Jorgensen tries to introduce different

cost structures such as on-going participation cost or transaction cost to account for nonparticipation. However, there isn't a sound cost structure yet to explain why some people with positive wealth are not investing in stocks. And this cost structures have no potential to account for different portfolio composition by people with different health status.

This paper develops a theoretical model to explain the drop out in the stock market among older people. We use a model of ambiguity with maxmin utility, where the ambiguity is towards the correlation between risky stock return and risky relative medical price inflation. Due to the ambiguity, agent is not sure the exact correlation between his de facto income and stock return. Because of maxmin utility, agent might worry that the income is very much like stock, and thus will not long in stocks. In his mind, the following scenario is his nightmare: that when he gets sick, is also the same time when stock return is low. And this worry can prevent him from longing stocks. Also, the agent is worried that after he shorts in stock, if he gets sick, needing money to hospital bills, is also the time the stock price is high and have to purchase stocks back to clear the short position. The worries from both extreme cases thus prevent the agent from either longing in stocks or shorting. And thus we have non-participants when agent is aged.

This paper conducts quantitative exercise by using the CRRA utility function and using historical correlations between stock return and medical inflation as the set of agents' belie. The quantitative exercise predicts a reasonable threshold on real medical expenditure, above which the households might choose to quit the stock market completely. This quantitative exercise can be extended to conduct welfare analysis given the content of recent Health Care Bill.

This paper finds a novel connection between health risk and financial choice. And the model can be addressed to different policy evaluations and predictions. As baby boomers are getting retired, the age structure in the U.S. will be changed significantly. According to National Center for Health Statistics, the current share of people who are older than 65 is around 13.0%, and it will be increased to 20.0% in 2030. With the share of people older than 75 increased from 6.1% to 9.2% in 2030, and to 11.4% in 2040. In the meantime, the older people hold unproportionate amount of wealth. On the other hand, as to current debate in health reform, it's interesting to ask the following question, as change in public health policy, such as medicare happens, how much will that affect risky asset holding in senior people? And the answers to those questions can be found by applying this paper to accommodate those policy or demographic changes.

1.2 Empirical Evidence

The participation pattern is observed from SCF data. The descriptions of the data set is in the following paragraph. We restrict our samples to those without privately held business, farms, professional practices, limited partnerships or any other types of partnerships. This sub-sample as described in the following paragraph, accounts for a large and stable size of U.S. population, which is around 86%. The reason to exclude private business owner is that as the nature of riskiness of their business is unknown, it's then hard to distinguish the contribution of their background risk to participation pattern.

The Survey of Consumer Finances (SCF) data is a triennial survey data designed

to provide detailed information on the financial status of the U.S. households. The information include details of households' wealth or balance sheet, current income and part of future income, namely, pensions. The survey is sponsored by the Board of governors of the Federal Reserve System in cooperation with Internal Revenue Service of the Department of Treasury. And since 1992, data have been collected by the National Organization for Research at the University of Chicago (NORC).

In terms of its quality and characteristics, the SCF data has the following advantages over other data set:

First, the SCF data has a good coverage of US households. Thanks to a dual-frame sample design, the SCF data has one subset with a good coverage of characteristics such as age, education, income and wealth level (e.x., there are 3,007 cases the this set in 2004 survey); while in the other subset it oversamples relatively wealthy households drawn from a list of records offered by IRS (there are 1,515 cases in 2004 survey). By doing so it helps to enhance accuracy of statistics with respect to asset allocation, since wealthy people tend to more likely to hold and to hold more financial assets. Proper weights are then applied to both samples in order to make estimations right for the whole population.

Second, the SCF data covers a complete breakdown of all categories for household wealth. It covers both liquid and illiquid assets. And for both categories, it further breaks down with more details.

Third, for some assets or pensions, the SCF data disentangle the asset holding to individual levels. Since the survey asks questions to individuals of household, such as head, spouse, and other member of the household, they survey can potentially shed light on intra-household diversification problem.

Besides, SCF has high accuracy. The survey applies computer assisted interviewing program to enhance quality of survey data. There usually exists high frequency of "don't know" responses to questions asking about value of assets. The computer program, however, uses a host of ways to extract information and to avoid "don't know" responses. For example, it offers range cards to those who are uncomfortable saying exact number to specify range. And it also uses series of questions in a decision tree to clarify the range more specifically.

The above advantages make SCF a unique and good data set to studies on household portfolio decisions.

Next we will describe some key variables used in this paper, the questions and variables will refer to 2004 survey. Throughout the paper age refers to that of the head of a primary economic unit (PEU). Where PEU is the household unit in the SCF data and it's comprised of a core single individual or couple in a household and all others who are financially dependent on that individual or couple. And head refers to the single core in individual in a PEU without a core couple or the male in a PEU with mixed-sex couple or the older individual in a PEU with same-sex couple.

The measure of stock investments includes¹⁰:

1. Full investments in directly held stocks. At question Q872, it asks "What is the total market value of this (publicly traded) stock?", and the value is stored as variable X3915.
2. Full investments in stock mutual funds. At question Q834A1, it asks "What is the total market value of all of the stock mutual funds that you have?", and the value is

¹⁰As time progresses, the survey asks more detailed questions on stock investment through different financial vehicles, and thus the measurement covers more different categories. The following list is the measurement from 2004 survey.

stored as variable X3822.

3. Half investments in combination funds. At question Q834A5, it asks “What is the total market value of all of the combination funds that you have?”, and the value is stored as variable X3830, half of which is counted for stock investment.

4. Full investments in other mutual funds. At question Q834A6, it asks “What is the total market value of all of these other funds that you have?”, and the value is stored as variable X7787.

5. Stock investments in IRA/Keogh account of each member of households. At questions Q1705A1B1-Q1705A1B4/Q1705A2B1-Q1705A2B4/Q1705A3B1-Q1705A3B4, they ask “How much is in your Roth IRA account(s)?”, “How much is in your roll-over IRA account(s)?”, “How much is in your regular or other IRA account(s)?”, and “How much is in your Keogh account(s)?”, respectively, to heads/spouses/other member of households, and their answers are stored as variables X6551-X6554/X6559-X6562/X6567-X6570 for heads /spouses/other member of households. At questions Q787A1 and Q790A1/Q787A2 and Q790A2/Q787A3 and Q790A3, they ask “How is the money in (this/these) account(s) invested?” and “About what percent is in stocks?”, and the answers are stored as variables X6555-X6556/X6563-X6564/X6571-X6572 for heads/spouses/other member of households. The percentage times the total value are accounted for stock investment.

6. Stock investments in annuities, more specifically, in variable annuities. At question Q911A1, it asks “How much would you receive if you cashed in these annuities?”, and the values are stored in variable X6577. At questions Q921A1 and Q1734A1, they ask “How is the money in (both types of/these) annuities invested?” and “About what percent is in stocks?”, respectively. And answers are stored as variables X6551 and

X6552. The value of stock investments are calculated as value times corresponding percentage.

7. Stock investments in trust or managed investment accounts. At question Q911A2, it asks “How much would you receive if you cashed in these (trust or managed investment) accounts?”, and values are stored in variable X6587. At questions Q921A2 and Q1734A2, they ask “How is the money in these accounts invested?” and “About what percent is in stocks?”, respectively. And answers are stored in variables X6591 and X6592. The value of stock investments are calculated as value times corresponding percentage in stocks.

8. Stock investments in pensions for head and spouse from current main jobs. Only accounts similar to 401(k) or 403(b) will be counted, and the corresponding amount and percentage in stocks are used to calculate for amounts in stocks.

9. Stock investments in pensions other than IRAs or Keogh, from which households are receiving benefits. At questions Q1661A1-A6, they ask “Is this pension currently an account plan, such as a 401(k), where you could take the whole balance as one payment if you wanted to?” for different pension accounts, values are stored in variables X6461/66/71/76/81/86, correspondingly. At questions Q1663A1-A6, they ask “What is the current balance in this account?”, values are stored in variables X6462/67/72/77/82/87. At questions Q1665A1-A6 and Q1726A1-A6, they ask “How is the account invested? Is it all in stocks, all in interest earning assets, is it split between these, or something else?”, and “About what percent is in stocks?”, values are stored in variables X6933/37/41/45/49/53 and X6934/38/42/46/50/54, respectively. The value of such investment is counted if the account is qualified as account plan, and then using its value times percentage in stocks.

10. Stock investments in other pensions not mentioned above. At questions Q1336A1-A6, they ask “How much is in the account now?” and values are stored in variables X5604/12/20/28/36/44 for different such accounts. And at questions Q1643A1-A6 and Q1731A1-A6, they ask “How is the money in this account invested?” and “About what percent is in stocks?”, respectively. The amounts of investments are summation of all such accounts’ value in stocks.

Follow Campbell (2006), among others, we first assume there is no cohort effect, and then construct the participation pattern over the life cycle by control for time effect. The figure 1.2 shows several cross sectional views for different waves, where we further control the sample such that the household wealth level is greater than \$30,000 (2007 U.S. dollars).

While figure 1.2 illustrates the regressions of participation after grouping all six waves from 1989-2004 and uses another threshold (wealth greater than \$10,000), it shows the same pattern, which is the participation stays high and relatively stable, and decreases around age 60.

The wealth threshold is robust. We conduct robustness check by using other levels such as \$30,000 and \$20,000, and the decrease pattern and the age at which participation rate drops don’t change. We conduct still another robustness check by looking into participation over age and relative wealth level within that age group. Figure 1.2 illustrates the regressions of participation rates against age group and wealth groups of different quartiles within that age.

The figure suggests that the drop in participation mainly happens within asset level in the middle (quartile 2 and 3). Interestingly, the average portfolio share for each group drop as people aged, but the average portfolio share conditional on people

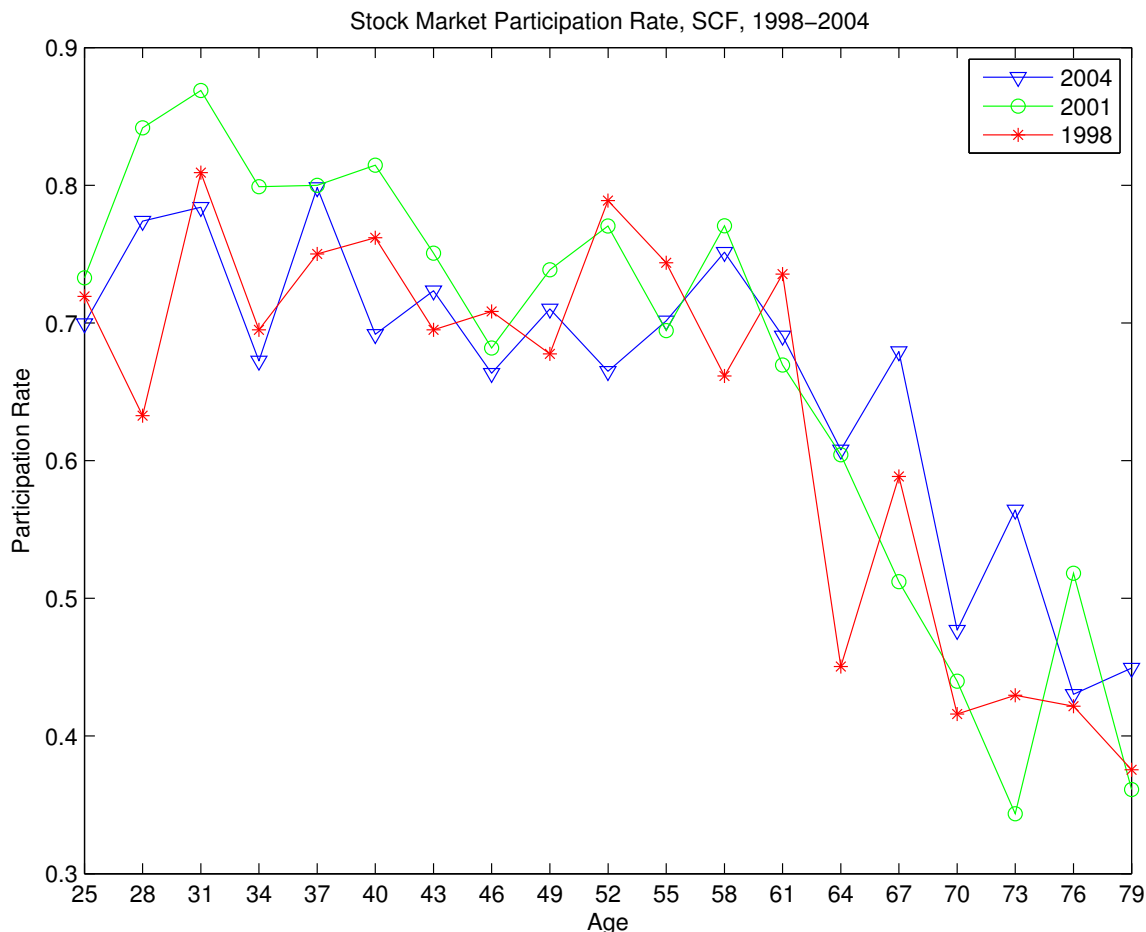


Figure 1.1: Cross section view

are stock market participants don't drop. These patterns can be shown in figure 1.2 and 1.2

This implies that there might exist some factor that hits some group of people most, and affect their portfolio choice so much that those agents decrease their portfolio share to zero. While this factor seems to not affect on-going participants that much that their portfolio share is not affected.

And health is a good candidate for this factor. As figure 1.2 suggests, people with bad health are more likely to drop the stock market. And health status affects most the relative poor people (wealth quartile group 2).

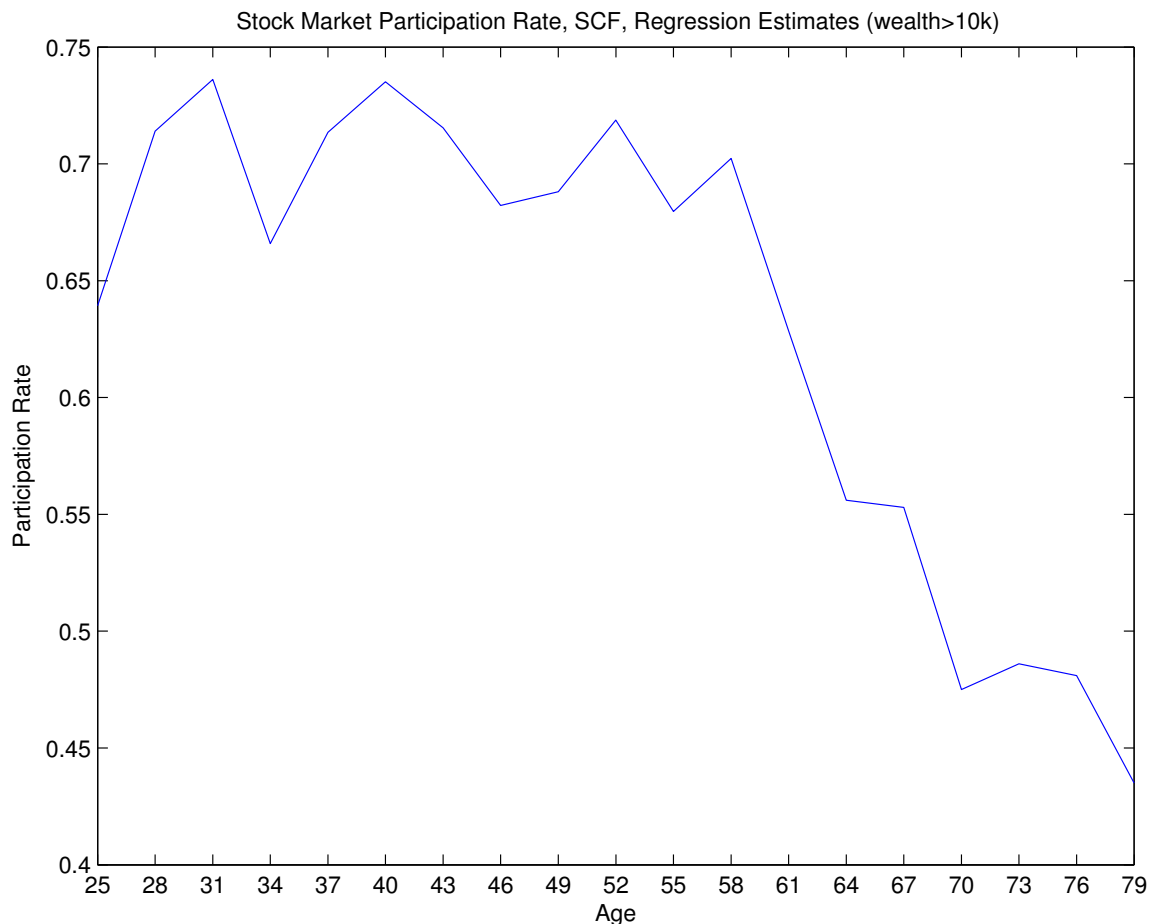


Figure 1.2: Regression controlling time, with wealth > \$10000

As to comovement between stock return and medical care inflation, we first construct real stock return and relative medical care inflation, and then look at the correlation between this two sequences.

To get both the real stock return and relative medical care inflation, we need price index to calculate the inflation. And we use annual CPI-U of U.S. city average and all items for this purpose. The CPI, or Consumer Price Index, is collected, calculated and published by Bureau of Labor Statistics. CPI measures the average change over time in prices paid for a market basket of consumer goods and services. The CPI-U sets this basket of goods and services to reflect spending pattern of all urban consumers,

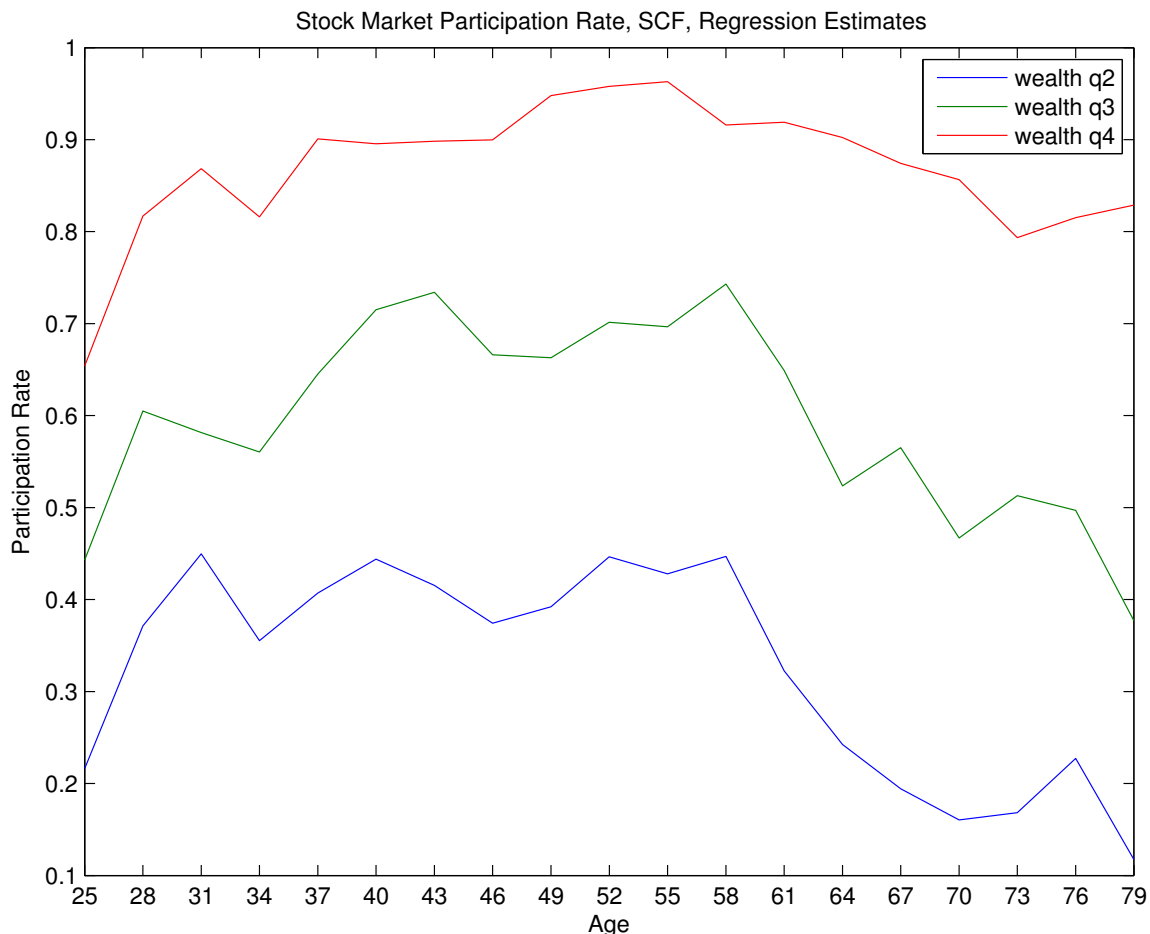


Figure 1.3: Regression controlling time, for asset quartile 2,3,4

which accounts for 87% of total population.

The advantages of using CPI over other price index include, firstly, the social security benefit payments are adjusted according to change in CPI; and secondly, the CPI captures more out-of-pocket fashion expenses than other price index. We use CPI medical care to calculate firstly medical inflation and then relative medical inflation. The medical care accounts for 6.23% of the basket of goods. Medical care includes two major categories, one is medical care commodities and the other is medical care services. We calculate the medical care inflation rate for post war period (1947-2006) and then derive the relative medical care inflation rate by using medical care inflation



Figure 1.4: Regression of unconditional portfolio share, for asset quartile 2,3,4

subtracted by general inflation. Table 1 summarizes the statistics of post war relative medical care inflation and stock return. And Table 2 summarizes the strata and relative importance of components of medical care CPI.

In a sensitivity analysis, for a sub-sample size ranged from 10 to 59 years, the original sample is broken into different blocks composed of consecutive years. For example, there are 2 possible blocks for a sub-sample as long as 59, while there are 31 possible blocks for a sub-sample having 20 consecutive years. The correlations are then calculated for these sub-samples. Table 3-5 list some statistics from this sensitivity analysis.



Figure 1.5: Regression of portfolio share conditional for participants, for asset quartile 2,3,4

1.3 Model

In this section, we will first present a model that is with medical care service, consumption and portfolio choices and that the agent has expected utility. We will show that this model can't robustly deliver nonparticipation in stock market, this conclusion holds no matter what assumptions we have towards the distribution of medical care inflation and stock return.

And then we will present a model deviating from the previous one by assuming the agent has ambiguity towards the distribution of medical care inflation and stock

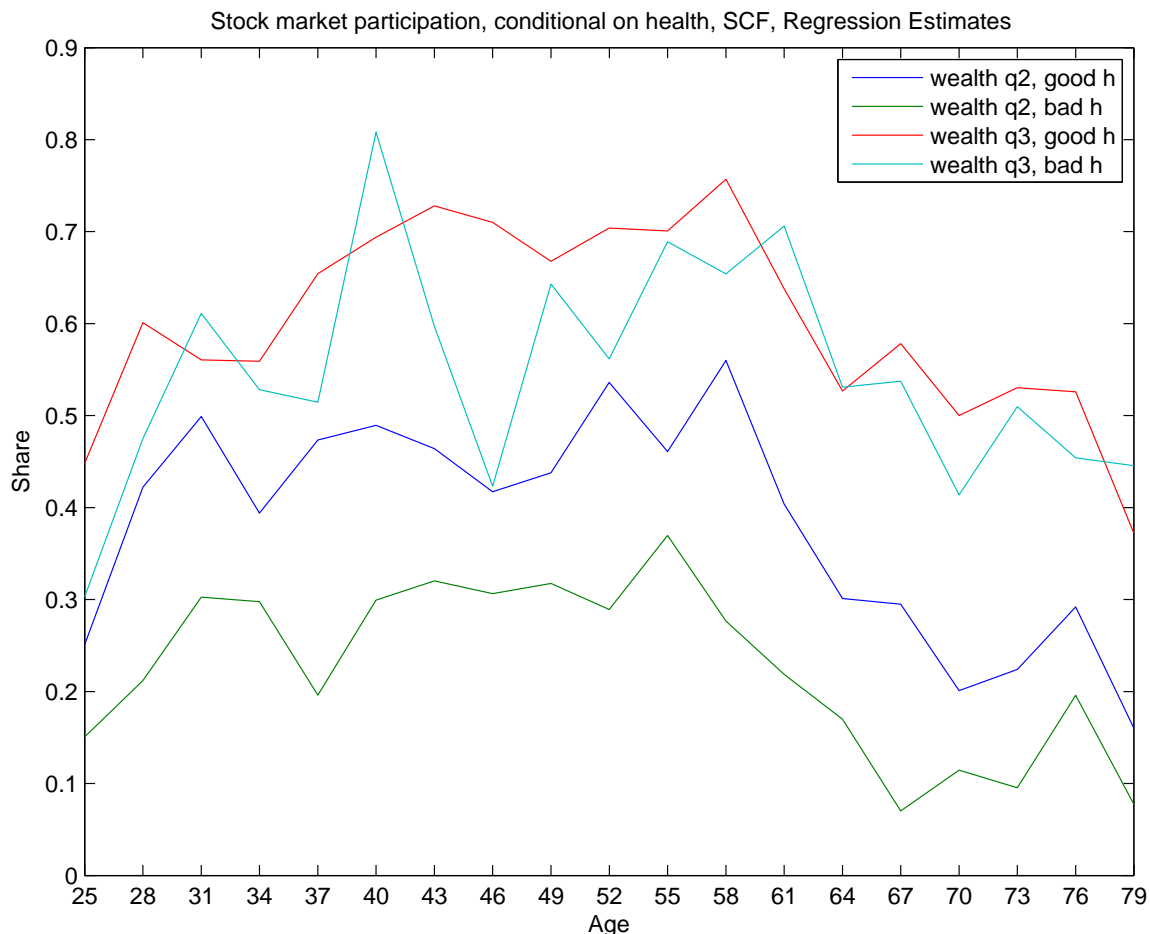


Figure 1.6: Regression of participation controlled health status, for asset quartile 2,3

return. More specifically, the ambiguity is towards the correlation with medical care inflation and stock return. Or in the other way, the set of priors that agent has towards the distributions of joint distributions of medical inflation and real stock return is isomorphic to a set of value of correlations between risky medical care inflation and stock return. In this environment, we present a simplest example where medical expenditure is collapsed so as to illustrate the intuition that how non-participation can take place.

The key intuition for non-participation is summarized as the following. With multi-prior and maxmin utility function, for each action the agent chooses, he picks

Table 1.1: The statistics of relative medical inflation and stock return (1947-2006)

	Sample Means	Sample Stdev	Correlation with Stock (p value)
Stock	0.0869	0.1542	1
Medical Inf	0.0179	0.0185	0.1257 (0.3385)
MCC Inf	-0.0072	0.0304	0.2551 (0.0491)
MCS Inf	0.0154	0.0262	0.0612 (0.6425)

the distribution associated with that action so that it gives the lowest expected utility. If he contemplates to invest positive amount in stock market, the distribution used to calculate the marginal benefit is the one that medical care inflation and real stock return have the most positive correlation. And his marginal benefit might be less than his opportunity cost, namely, investing in safe assets. On the other hand, if he contemplates to invest negative amount in stock market, or to short the stock market, the distribution used to calculate the marginal benefit is the one that medical care inflation and real stock return have the most negative correlation. If the set of priors that the agent bear in mind include both strong negative and positive correlations between medical price inflation and real stock return, the agent will find neither invest in stocks or short stocks are beneficial and thus stay out of the stocks market.

The analysis focuses on senior people who have retired already. The labor supply decision is thus absent from current analysis. And the households are thus not subject to non-asset income risk, as social security income is adjusted annually according to CPI-U of U.S. city average and all items.

1.3.1 The model without ambiguity

In the model, time is discrete, the agent starts at retirement age, which is normalized as 0, and the age can go from 0 up to T . After T the survived agent will die for sure. The agent seeks to maximize his or her expected lifetime utility. The

individual's flow utility depends on current consumption, C , health status, h , and medical care service, m . The flow utility is given by $u(C, h, m)$.

Both the consumption and medical expenditure spaces are $[0, \infty)$. And space of health status are normalized to be $[\underline{h}, 1]$, where $0 < \underline{h} < 1$. And the higher h indicates higher better status.

And the utility has the property that, u is increasing in all arguments. That is to say, with same level of health status and consumption level, the more the medical service an agent receives, the betteroff is he. Besides, we have: $u_{CC}, u_{hh}, u_{mm} < 0$

The individual faces several sources of exogenous uncertainties.

1) Survival uncertainty. The survival probability s_t^{t+1} from period t to $t + 1$ is determined by function $s(t, h_t, m_t)$, $\forall t \in \mathcal{T} - \{T\}$, where $\mathcal{T} = \{0, 1, 2, \dots, T\}$. The survival probability is assumed to have the following property: $s(t, h_t, m_t)$ is non-increasing in t , and non-decreasing in both h and m . That's to say, with the same health status and same level of medical service, the younger the agent is, the more likely he will survive. Similarly, with same age and health status, the more medical service the agent receives, the more likely he will survive. And last, with same age and enjoy same level of medical service, the healthier the agent is, the more likely he will survive.

2) Stock return uncertainty. The real return on stock investment is I.I.D. over time.

3) Medical price uncertainty. The relative medical care price is risky. We assume also that the medical price is I.I.D. over time.

The joint distribution of $(\tilde{R}_t, \tilde{P}_t) \forall t \in \mathcal{T} - \{0\}$ is $Q(\tilde{R}_t, \tilde{P}_t)$.

The evolution of the health status follows that:

$$h_{t+1} = H(t, h_t, m_t), \quad \forall t \in \mathcal{T} - \{T\} \quad (1.3.1)$$

The timing in each period is that, the survival probability resolves in the beginning of each period, if the agent survives, then agent learns the realization of stock return and medical price inflation for that period, he then receives his non-asset income y . The agent afterwards chooses consumption, medical care service, safe and risky assets to save.

The problem can be summarized by a dynamic programming problem:

$$V(t, h_t, B_t, S_t; R_t, P_t) = \max_{C_t, m_t, B_{t+1}, S_{t+1}} u(C_t, h_t, m_t) + s_t^{t+1} \beta E_t \left[V(t+1, h_{t+1}, B_{t+1}, S_{t+1}; \tilde{R}_{t+1}, \tilde{P}_{t+1}) \right] \quad (1.3.2)$$

s.t.

$$C_t + P_t m_t = y + R_f B_t + R_t S_t$$

$$h_{t+1} = H(t, h_t, m_t)$$

$$s_t^{t+1} = s(t, h_t, m_t)$$

$$C_t, m_t \geq 0$$

, where the expectation is taken with respect to the distribution on the uncertainty of next period stock return \tilde{R}_{t+1} , and medical price inflation P_{t+1} . And in this problem, t refers to age, h_t is health status in the beginning of current period, B_t and S_t are safe assets and stock holding for the current period, and R_t, P_t are realizations of current

period stock return and medical care price. y is agent's real non-asset income, which is constant over time for the same agent. And R_f refers to risk free rate, and is also constant over time.

This model can endogenize medical expenses¹. The model has the feature of “investment motive” for health expenditure. But unlike Grossman (1972), the investment motive is not through the channel of labor supply, but through the following channels: higher health expenditure delivers 1) higher current utility, 2) higher survival probability, and thus higher expected (on survival probability) continuation value and 3) better next period health and thus higher next period utility.

1.3.2 A two-period model

We present a two-period model to illustrate the difficulty to deliver non-participation in stock market using the above model. In the beginning of each period, the agent learns his health status, realized stock return and medical care price. The agent then chooses consumption, C and medical expenditure, m ; and in the first period, he chooses bond and stock holdings, B and S . Assume that the stock price in second period is $\tilde{R} = \bar{R} + \varepsilon_1$, where \bar{R} refers to gross expected stock of stock, and ε_1 is a shock with mean zero and variance σ_1^2 . And second period medical price is $\tilde{P} = P_0(I_M + \varepsilon_2)$, where ε_2 is a shock with mean zero and variance σ_2^2 .

The maximization problem of the agent is:

$$V(0, h_0, B_0, S_0; R_0, P_0) = \max_{C_0, C_1, B, S, m_0, m_1} u(C_0, h_0, m_0) + s_0^1 \beta E_0 [u(C_1, h_1, m_1)] \quad (1.3.3)$$

¹which deviates from, say, Palumbo (1999), and De Nardi et al (2006) among others.

s.t.

$$C_0 + P_0 m_0 + B + S = RB_0 + R_0 S_0 + y$$

$$C_1 + \tilde{P} m_1 = R_f B + \tilde{R} S + y$$

$$C_0, C_1, m_0, m_1 \geq 0$$

$$s_0^1 = s(0, h_0, m_0)$$

$$h_1 = H(0, h_0, m_0)$$

To illustrate the investment incentive, it's better to look at the first order condition for medical expenditures at the first period. We have:

$$u_{m_0} + \frac{\partial s_0^1}{\partial m_0} \beta E_0 [u(C_1, h_1, m_1)] + s_0^1 \beta E_0 \left[u(C_1, h_1, m_1) \frac{\partial h_1}{\partial m_0} \right] = u_{C_0} P_0 \quad (1.3.4)$$

The right hand side is the marginal cost of spending in health at period 0, and the left terms are the benefits, which have three components. The first component is the direct benefit from flow utility, which says, with the same health status, extra medical expenditure will deliver marginal utility on medical care. The second term results from higher survival probability by spending more in health in current period. From this term, it is clear that not only marginal utility but the level of utility matters. And the last term is the betteroff from realization of next period health status by increasing current health expenditure.

Given such environment, Lemma 1 shed some light on stock market participation for senior people.

Lemma 1 *Assume that stock return is independent with medical care inflation, and that equity premium is positive, and $u_{cc} < 0, u_{mm} < 0, u_{mc} \geq 0$, then $S > 0$.*

Proof: It's helpful to illustrate it by assuming there are finite states of uncertainty. And the intuition can be easily extended to infinite many states case. Let's assume there are M_1 number of distinctive states for ε_1 and M_2 for ε_2 . As ε_1 and ε_2 are independent, $\pi(\varepsilon_1, \varepsilon_2) = \pi(\varepsilon_1) \cdot \pi(\varepsilon_2)$ for any values of ε_1 and ε_2 , where $\pi(\varepsilon_1, \varepsilon_2)$ denotes the joint probability for $(\varepsilon_1, \varepsilon_2)$ and $\pi(\varepsilon_i)$ denotes the probability for the event that ε_i happens.

The agent's problem can be so rewritten as:

$$\max_{C_0, C_1(\varepsilon_1, \varepsilon_2), B, S, m_0, m_1(\varepsilon_1, \varepsilon_2)} u(C_0, h_0, m_0) + s_0^1(h_0, m_0) \beta \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2))$$

$$C_0 + P_0 m_0 + B + S = R_f B_0 + R_0 S_0 + y \quad (\lambda_0) \quad (1.3.5)$$

$$C_1(\varepsilon_1, \varepsilon_2) + \tilde{P}(\varepsilon_2) m_1(\varepsilon_1, \varepsilon_2) = R_f B + \tilde{R}(\varepsilon_1) S + y \quad (\lambda(\varepsilon_1, \varepsilon_2)) \quad (1.3.6)$$

$$C_0, C_1(\varepsilon_1, \varepsilon_2), m_0, m_1(\varepsilon_1, \varepsilon_2) \geq 0$$

, where λ 's are Lagrange multipliers to budget constraints. Assume that optimal consumptions and medical care expenditures are interior solutions, and we so ignore the slackness conditions.

First order necessary conditions are:

$$u_c(C_0, h_0, m_0) = \lambda_0 \quad (C_0) \quad (1.3.7)$$

$$\beta s(h_0, m_0) \pi(\varepsilon_1, \varepsilon_2) u_c(C_1, h_1, m_1) = \lambda(\varepsilon_1, \varepsilon_2) \quad (C_1(\varepsilon_1, \varepsilon_2)) \quad (1.3.8)$$

$$\lambda_0 = \sum_{(\varepsilon_1, \varepsilon_2)} R_f \lambda((\varepsilon_1, \varepsilon_2)) \quad (B) \quad (1.3.9)$$

$$\text{or } u_c(C_0, h_0, m_0) = \beta s(h_0, m_0) \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1, h_1, m_1) R_f$$

$$\lambda_0 = \sum_{(\varepsilon_1, \varepsilon_2)} \tilde{R}(\varepsilon_1) \lambda((\varepsilon_1, \varepsilon_2)) \quad (S) \quad (1.3.10)$$

$$\text{or } u_c(C_0, h_0, m_0) = \beta s(h_0, m_0) \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1, h_1, m_1) \tilde{R}(\varepsilon_1)$$

$$\begin{aligned} u_m(C_0, h_0, m_0) + \beta \frac{\partial s(h_0, m_0)}{m_0} \mathbf{E}[u(C_1, h_1, m_1)] + \beta s(h_0, m_0) \mathbf{E}[u_h(C_1, h_1, m_1) \frac{\partial h_1}{\partial m_0}] \\ = P_0 u_c(C_0, h_0, m_0) \quad (m_0) \end{aligned} \quad (1.3.11)$$

$$u_m(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)) = u_c(C_1, h_1, m_1) \tilde{P}(\varepsilon_2) \quad (m_1(\varepsilon_1, \varepsilon_2)) \quad (1.3.12)$$

together with the budget constraints, and they are totalled $5 + 3M_1M_2$ equations, which will be solved unknowns: $C_0, m_0, B, S, C_1(\varepsilon_1, \varepsilon_2), m_1(\varepsilon_1, \varepsilon_2), \lambda_0, \lambda(\varepsilon_1, \varepsilon_2)$, totalled at $5 + 3M_1M_2$ as well. Although it's been assumed that the optimal consumption and medical care are interior solutions, as it turns out below that, it's not necessarily needed for the proof.

Now suppose at optimal $S = 0$. And consider a deviation from this plan by moving some marginal safe assets (or borrow if necessary) by amount of Δ to stock

account, the marginal benefit minus marginal cost by doing so is:

$$\begin{aligned}
 & -\beta s_0^1 \Delta \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1, h_1, m_1) R_f + \beta s_0^1 \Delta \sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1, h_1, m_1) \tilde{R}(\varepsilon_1) \\
 & = \beta s_0^1 \Delta \left[\sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)) (\tilde{R}(\varepsilon_1) - R_f) \right] \quad (1.3.13)
 \end{aligned}$$

where, evaluated at $S = 0$, $C_1(\varepsilon_1, \varepsilon_2) = y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon_1, \varepsilon_2)$. From the first order condition for $m_1(\varepsilon_1, \varepsilon_2)$, or equation (1.3.12), we have:

$$u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)) = u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)) \tilde{P}(\varepsilon_2) \quad (1.3.14)$$

We want to show that if $S = 0$, $m_1(\varepsilon_1, \varepsilon_2) = m_1(\varepsilon_1', \varepsilon_2)$ for any $\varepsilon_1, \varepsilon_1'$. Without loss of generality, we will discuss the case for, say, $\varepsilon_1^H, \varepsilon_1^L$ with $\varepsilon_1^H > \varepsilon_1^L$, with $m_1(\varepsilon_1^H, \varepsilon_2) > m_1(\varepsilon_1^L, \varepsilon_2)$. Apply equation (1.3.12), we have conditions:

$$\begin{aligned}
 u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon_1^H, \varepsilon_2), h_1, m_1) & = u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon_1^H, \varepsilon_2), h_1, m_1) \tilde{P}(\varepsilon_2) \\
 u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon_1^L, \varepsilon_2), h_1, m_1) & = u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon_1^L, \varepsilon_2), h_1, m_1) \tilde{P}(\varepsilon_2)
 \end{aligned}$$

which implies:

$$\begin{aligned}
 & \frac{u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon_1^H, \varepsilon_2), h_1, m_1(\varepsilon_1^H, \varepsilon_2))}{u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon_1^H, \varepsilon_2), h_1, m_1(\varepsilon_1^H, \varepsilon_2))} = \\
 & \frac{u_m(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon_1^L, \varepsilon_2), h_1, m_1(\varepsilon_1^L, \varepsilon_2))}{u_c(y + R_f B - \tilde{P}(\varepsilon_2) m_1(\varepsilon_1^L, \varepsilon_2), h_1, m_1(\varepsilon_1^L, \varepsilon_2))} \quad (1.3.15)
 \end{aligned}$$

However, as we also have:

$$\begin{aligned} u_c(y + R_f B - \tilde{P}(\varepsilon_2)m_1(\varepsilon_1^H, \varepsilon_2), h_1, m_1(\varepsilon_1^H, \varepsilon_2)) &> u_c(y + R_f B - \tilde{P}(\varepsilon_2)m_1(\varepsilon_1^L, \varepsilon_2), h_1, m_1(\varepsilon_1^H, \varepsilon_2)) \\ &\geq u_c(y + R_f B - \tilde{P}(\varepsilon_2)m_1(\varepsilon_1^L, \varepsilon_2), h_1, m_1(\varepsilon_1^L, \varepsilon_2)) \end{aligned}$$

, where the first inequality comes from $u_{cc} < 0$, and the second inequality is justified by $u_{cm} \geq 0$, and as:

$$\begin{aligned} u_m(y + R_f B - \tilde{P}(\varepsilon_2)m_1(\varepsilon_1^H, \varepsilon_2), h_1, m_1(\varepsilon_1^H, \varepsilon_2)) &< u_m(y + R_f B - \tilde{P}(\varepsilon_2)m_1(\varepsilon_1^H, \varepsilon_2), h_1, m_1(\varepsilon_1^L, \varepsilon_2)) \\ &\leq u_m(y + R_f B - \tilde{P}(\varepsilon_2)m_1(\varepsilon_1^L, \varepsilon_2), h_1, m_1(\varepsilon_1^L, \varepsilon_2)) \end{aligned}$$

, where the first inequality comes from $u_{mm} < 0$, and the second inequality is again based on the fact that $u_{cm} \geq 0$. The equality of (1.3.15) can never hold.

Henceforth, we have that if $S = 0$, then $m_1(\varepsilon_1, \varepsilon_2) = m_1(\varepsilon_1', \varepsilon_2)$ for any $\varepsilon_1, \varepsilon_1'$.

And from the first order condition of m_1 or (1.3.12), we have also if $S = 0$, then $C_1(\varepsilon_1, \varepsilon_2) = C_1(\varepsilon_1', \varepsilon_2)$ for any $\varepsilon_1, \varepsilon_1'$, which in turn implies that the marginal gain by moving from saving to stocks from (1.3.13) is:

$$\begin{aligned} &\beta s_0^1 \Delta \left[\sum_{(\varepsilon_1, \varepsilon_2)} \pi(\varepsilon_1, \varepsilon_2) u_c(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)) (\tilde{R}(\varepsilon_1) - R_f) \right] \\ &= \beta s_0^1 \Delta \left[\sum_{\varepsilon_1} \sum_{\varepsilon_2} \pi(\varepsilon_1) \pi(\varepsilon_2) u_c(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)) (\tilde{R}(\varepsilon_1) - R_f) \right] \\ &= \beta s_0^1 \Delta \left[\sum_{\varepsilon_1} \pi(\varepsilon_1) (\tilde{R}(\varepsilon_1) - R_f) \right] \left[\sum_{\varepsilon_2} \pi(\varepsilon_2) u_c(C_1(\varepsilon_1, \varepsilon_2), h_1, m_1(\varepsilon_1, \varepsilon_2)) \right] \\ &\hspace{20em} > 0 \quad (1.3.16) \end{aligned}$$

if $\Delta > 0$, where the last equation comes from the fact that $C_1(\varepsilon_1, \varepsilon_2) = C_1(\varepsilon'_1, \varepsilon_2)$ for any $\varepsilon_1, \varepsilon'_1$, and that $m_1(\varepsilon_1, \varepsilon_2) = m_1(\varepsilon'_1, \varepsilon_2)$ for any $\varepsilon_1, \varepsilon'_1$. And the last inequality comes from the facts that: 1) expected equity premium is positive, or $\sum_{\varepsilon_1} \pi(\varepsilon_1)(\tilde{R}(\varepsilon_1) - R_f) > 0$ and 2) $u_c > 0$.

Based on the above argument, the agent actually has incentive to deviate from the proposed optimal plan where $S = 0$ by increasing his investment in stocks. This implies that the stock market participation rate should be 100%. Q.E.D.

This intuition can easily be extended to multi-period model. Lemma 2 is the extension of Lemma 1 to multi-period case.

Lemma 2 *Assume that stock return is independent with medical care inflation, and that equity premium is positive, and $u_{cc} < 0, u_{mm} < 0, u_{mc} \geq 0$, then $S_{t+1} > 0, \forall t \in \mathcal{T} - \{T\}$.*

Lemma 2 reinforces the prediction from life cycle model with portfolio choice problem but abstract from health service decision, that the participation rate is 100% for retirees whose non-asset income is not co-vary with stock return.

When \tilde{R}, \tilde{P} are not independent, however, the nonparticipation in stock market can be shown to be a non-robust result. In a two-period model, an agent enters the economy characterized by her initial health status, h_0 , her initial bond holding, B_0 , her initial stock holding, S_0 , and her constant defined benefit non asset income, y . Without loss of generality, we assume that for any distinctive value (h_0, B_0, S_0, y) , the “natural” measure of such agent is equal to 0. The following Lemma 3 verifies the non-robustness in a two-period model where \tilde{R}, \tilde{P} are not independent, but follows

some certain distribution.

Lemma 3 *For an economy where initial P_0 and R_0 are realized, assume that any specific type of agent has measure 0, and that for any open set of initial parameters, the measure is positive. We also assume that $(\tilde{R}_1, \tilde{P}_1)$ follows some certain distribution, then the set of (h_0, B_0, S_0, y) such that: $S(h_0, B_0, S_0, y) = 0$ has measure 0.*

We next consider a model where health expenditure is instead exogenous. Agent makes consumption, saving and investment decision in this two-period model. The agent can invest in both risk free bonds and stocks. In period zero, the agent holds initial wealth W_0 , a pool of his bond holding, stocks holding and labor income. He decides how much to consume, to invest in bonds and stocks. In period 1, the agent receives risky labor income \tilde{Y} and returns from his investment. The gross return to the stocks is \tilde{R} , that for bonds is R . Both \tilde{R} and \tilde{Y} are random variables defined on probability space (Ω, \mathcal{F}, P) .

The agent has a set of priors \mathcal{P} that over (Ω, \mathcal{F}) . The implicit assumption for multiple priors is that the agent is not able to form a single prior for the joint performance of his de facto income and stock return. We assume that the ambiguity is only towards the correlation between income and stock return.¹ We assume the reference

¹Gilboa and Schmeidler's (1989) work does not impose any restrictions on structure of \mathcal{P} . Some work, including Kogan and Wang (2002), Cao, Wang and Zhang (2005), and Miao (2004), among others, use entropy criterion to define \mathcal{P} , which can deliver analytical and intuitive results. Their work assume that the ambiguity is only towards the mean of stock return, we will show later that this way of modeling can't deliver the pattern of quitting from stock market as we observe from data.

distribution \mathbb{P} for joint performance of stock and income is:

$$(\tilde{R}, \tilde{Y}) \stackrel{\mathbb{P}}{\sim} \mathbb{N} \left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \right) \quad (1.3.17)$$

And the set of priors is

$$\mathbb{P}(\mathbb{P}) = \{ \mathbb{Q} | (\tilde{R}, \tilde{Y}) \stackrel{\mathbb{Q}}{\sim} \mathbb{N} \left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} + v \\ \sigma_{12} + v & \sigma_2^2 \end{pmatrix} \right) : v \in [\underline{v}, \bar{v}] \} \quad (1.3.18)$$

And we assume that $0 \in [\underline{v}, \bar{v}]$.

The agent has multi-prior utility as in Gilboa and Schmeidler (1989). We assume the utility function is CARA. Thus the objective function is

$$u(C_0) + \beta \min_{Q \in \mathbb{P}} [u(C_1)] \quad (1.3.19)$$

where $u(c) = -\frac{1}{\theta} \exp(-\theta c)$, where $\theta > 0$. Thus we can summarize the agent's problem in two-period setting as follows:

$$\max_{B_0, S_0} u(C_0) + \beta \min_{Q \in \mathbb{P}} [u(C_1)] \quad (1.3.20)$$

s.t. :

$$C_0 + B_0 + S_0 = W_0 \quad (1.3.21)$$

$$C_1 = \tilde{Y} + RB_0 + \tilde{R}S_0 \quad (1.3.22)$$

To solve the model, we need first solve the inside minimization problem, and this is equivalent to pick a proper value of disturbance on correlation. Under the

assumption that a generic distribution is normal and that utility function is CARA, we can rewrite the objective function depending on the sign of stock holdings and then solve it accordingly.

Case 1: if $S_0 > 0$, the objective function becomes:

$$\max_{B_0, S_0} \frac{1}{\theta} \exp[-\theta(W_0 - B_0 - S_0)] - \frac{\beta}{\theta} \exp\left[-\theta RB_0 - \theta S_0 \mu_1 - \theta \mu_2 + \frac{\theta^2}{2} (S_0^2 \sigma_1^2 + 2S_0(\sigma_{12} + \bar{v}) + \sigma_2^2)\right] \quad (1.3.23)$$

Solving this for S_0 we can get

$$S_0 = \frac{\mu_1 - R - \theta(\sigma_{12} + \bar{v})}{\theta \sigma_1^2} \quad (1.3.24)$$

where $\bar{v} < \frac{\mu_1 - R}{\theta} - \sigma_{12}$.

The intuition is that, due to the ambiguity towards the correlation, the equity premium is discounted by risk coefficient times the correlation. And the investor longs in stocks only when the discounted equity premium is still positive under the “worst” scenario, where “worst” means when stocks and income are most like each other. If the stocks and income are so much like each other that at some point, the agent will quit the stock market to avoid risks.

Similarly, under case 2, if $S_0 < 0$, we get

$$S_0 = \frac{\mu_1 - R - \theta(\sigma_{12} + \underline{v})}{\theta \sigma_1^2} \quad (1.3.25)$$

where $\underline{v} > \frac{\mu_1 - R}{\theta} - \sigma_{12}$.

Similar intuition applies here. The investor shorts stocks only when in the “best” scenario the discounted equity premium is still negative. And here “best” scenario is when stocks and income are very dislike each other. If stocks and income are too dislike each other, the investor would not have incentive to hedge against his income uncertainty by short selling stocks.

The last and most important case is when agent is not participating in stocks. We have the necessary condition for non-participation:

$$\underline{v} \leq \frac{\mu_1 - R}{\theta} - \sigma_{12} \leq \bar{v} \quad (1.3.26)$$

The agent is not longing since he’s worried that his income will be too much like a stock. His nightmare is that the stock return will be low when his non-asset income is also low. Meanwhile, due to the ambiguity, the agent is also worried that his income will be too dislike a stock. He is concerned with the following scenario: he shorts in stocks, and his income hits a low shock. However, at the same time, the stock return is high, meaning he will be hurt by buying stocks with higher prices to pay back his short selling. The multiple prior utility captures these two direction worries at the same time, and therefore deliver the non-participation.

This two period model offers enough intuitions about why we need ambiguity towards the correlation to solve the puzzle. In the following, we will show other alternative specifications do not work.

No ambiguity Assume for now there is no ambiguity in the model. We have a single prior of joint distribution between stock return and non-asset income then all agents share the same single belief about the joint distribution of stock return and

non-asset income. Then it is easy to show that the stock holding is

$$S_0 = \frac{\mu_1 - R - \theta\sigma_{12}}{\theta\sigma_1^2} \quad (1.3.27)$$

Given expected equity premium is positive, we have $S_0 > 0$. Nobody will ever quit the stock market.

Ambiguity towards the mean of non-asset income Alternatively, if we assume that the ambiguity is towards the mean of the last period non-asset income \tilde{Y} , the set of priors changes to

$$\mathbb{P}(\mathbb{P}) = \{ \mathbb{Q} | (\tilde{R}, \tilde{Y}) \stackrel{\mathbb{Q}}{\sim} \mathbb{N} \left(\begin{pmatrix} \mu_1 \\ \mu_2 - v \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \right) : v \in [\underline{v}, \bar{v}] \}.$$

We can show that it would change the consumption-savings (bond holding) decision. However, it would not change the stock participation decision since the mean of non-asset income μ_2 never enters into the equation of stocking holding as in (13).

Ambiguity towards the mean of stock return Last, if we assume that the ambiguity is towards the mean of the last period stock return \tilde{R} , the set of priors now is as follows

$$\mathbb{P}(\mathbb{P}) = \{ \mathbb{Q} | (\tilde{R}, \tilde{Y}) \stackrel{\mathbb{Q}}{\sim} \mathbb{N} \left(\begin{pmatrix} \mu_1 - v \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \right) : v \in [\underline{v}, \bar{v}] \}.$$

In this case, the optimal stock holding is given by

$$S_0 = \begin{cases} \frac{(\mu_1 - \bar{v}) - R - \theta\sigma_{12}}{\theta\sigma_1^2} & \bar{v} < \mu_1 - R - \theta\sigma_{12} \\ 0 & \underline{v} \leq \mu_1 - R - \theta\sigma_{12} \leq \bar{v} \\ \frac{(\mu_1 - \underline{v}) - R - \theta\sigma_{12}}{\theta\sigma_1^2} & \underline{v} > \mu_1 - R - \theta\sigma_{12} \end{cases}$$

We do generate non-participation region depending on the dispersion of ambiguity. However, in order to explain why *older* people quit the stock market, which is the puzzle we are interested, we have to assume the ambiguity about the mean of risky stock return becomes larger when people are aging. We find it is not convincing. In fact, when we consider people can learn through experience, we would expect the opposite. Besides, it says nothing about the data fact that health status is a significant predictor of stock market participation.

1.4 Dynamic Model with Ambiguity

This section develops a dynamic model with ambiguity about the joint distribution of stock return and medical care inflation. More specifically, as in the previous section, the ambiguity is towards the correlation between stock return and medical care inflation.

To write the model into dynamic programming form, we have:

$$V(W, h, P_m, t) = u(C, h, m') + \beta s(h, m', t) \min_{Q \in \mathcal{Q}} E_Q [V(W', h', P'_m, t + 1)] \quad (1.4.1)$$

, where W is wealth, and is composed of 1) current non-asset income y , for retiree

its social security income, 2) financial wealth, including investments in safe assets and risky assets; h denotes health status; P_m is realized medical care inflation; and t refers to age of the agent. And Q is the joint distribution of risky or stock return and medical care inflation, with \mathcal{Q} denoting the set of such distributions, which the agent has ambiguous of the true underlying distribution.

More specifically, we assume that the set of joint (marginal) distributions on next period risky factors are independent. By that we mean, suppose Q_t are generic distributions from \mathcal{Q}_t , then Q_t and $Q_{t'}$ are independent as long as $t \neq t'$.

1.4.1 Dynamic consistency

With multi-prior and maxmin utility, the independence axiom breaks down, and generally there is dynamic inconsistency problem. However, with assuming the joint distribution of stock return and medical care inflation is independent over time, and there is no learning from the agent, we can restore dynamic consistency property by applying results from Epstein-Schneider (2003).

Domain

Time is discrete and goes over $\mathcal{T} = \{0, 1, \dots, T\}$. The state space is Ω , where $\Omega = X_1 \cdot X_2 \cdots X_T$. And information structure is represented by the filtration $\{\mathcal{F}_t\}_0^T$ with the following properties:

- \mathcal{F}_t is a σ -algebra generated by a finite partition of Ω , $\forall t \in \mathcal{T}$
- $\mathcal{F}_0 = \sigma(\Omega)$
- $\mathcal{F}_t \subseteq \mathcal{F}_{t+1} \quad \forall t \in \mathcal{T} \setminus \{T\}$

$$\cdot \mathcal{F}_T = 2^\Omega$$

, with $\mathcal{F}_t(\omega)$ denoting the partition component containing ω .

Y is a measurable function from measurable space (Ω, \mathcal{F}_T) to (X, \mathcal{B}) , where \mathcal{B} is the Borel σ -algebra on X . When a probability measure P is attached to the measurable space (Ω, \mathcal{F}_T) , $Y(P)$ is a random variable from $(\Omega, \mathcal{F}_T, P)$ to (X, \mathcal{B}) , and Q denotes the distribution of the random variable $Y(P)$, which is a probability measure induced from $Y(P)$. The set of all probability measures on (Ω, \mathcal{F}_T) is \mathcal{P} and the set of all distributions induced from \mathcal{P} is denoted by \mathcal{Q} .

The agent's belief about riskiness in the economy is characterized by a set of priors, \mathcal{P}_0 , which is a subset of \mathcal{P} . The corresponding set of distribution measures is \mathcal{Q}_0 , which is induced from \mathcal{P}_0 , meaning for any $Q \in \mathcal{Q}_0$, $\exists P \in \mathcal{P}_0$ such that Q is the distribution induced from P . There is no ambiguity if $\forall P_1, P_2 \in \mathcal{P}_0$, $P_1 = P_2$, a.s..

Given \mathcal{T} , Ω , \mathcal{P}_0 , and $\{\mathcal{F}_t\}_0^T$, we can define marginal distribution and one-step forward distribution as the following. For any $P \in \mathcal{P}_0$, $P_t(\omega) = p(\cdot | \mathcal{F}_t)(\omega)$, and P_t^{+1} is the restriction of P_t to \mathcal{F}_{t+1} . The sets of marginal priors and one-step-forward priors are:

$$\mathcal{P}_t(\omega) = \{P_t(\omega) : P \in \mathcal{P}_0\} \tag{1.4.2}$$

$$\mathcal{P}_t^{+1}(\omega) = \{P_t^{+1}(\omega) : P \in \mathcal{P}_0\} \tag{1.4.3}$$

Alternatively we can introduce one-step-forward priors first. For each t and ω , we have set of conditional one-step-forward measures:

$$\widehat{\mathcal{P}}_t^{+1}(\omega) = \{\widehat{P}_t^{+1}(\omega)\} \tag{1.4.4}$$

, where $\widehat{P}_t^{+1}(\omega) : \Omega \rightsquigarrow \Delta(\Omega, \mathcal{F}_{t+1})$ is \mathcal{F}_t -measurable, $\Delta(\Omega, \mathcal{F}_{t+1})$ is the set of probability measures on $(\Omega, \mathcal{F}_{t+1})$.

Based on one-step-forward set of priors, we can define unconditional set of priors:

$$\widehat{\mathcal{P}}_0(\omega) = \left\{ \int_{\Omega} \int_{\Omega} \cdots \int_{\Omega} d\widehat{P}_{T-1}^{+1} \cdots d\widehat{P}_1^{+1} d\widehat{P}_0^{+1}(\omega), \text{ for some } \widehat{P}_0^{+1} \in \widehat{\mathcal{P}}_0^{+1}, \dots, \widehat{P}_{T-1}^{+1} \in \widehat{\mathcal{P}}_{T-1}^{+1} \right\} \quad (1.4.5)$$

Consistencies

Lemma 4 *Suppose $T = 2$, $\Omega = X_1 \cdot X_2$, $\mathcal{F}_1 = \sigma(X_1)$, $\mathcal{F}_2 = \sigma(X_1, X_2)$, and we define $\mathcal{Q}_0 \equiv \{Q_0 | Q_0(x_1, x_2) = Q_0^{+1}(x_1)Q_1^{+1}(x_2|x_1), \text{ for some } Q_1 \in \mathcal{Q}_0^{+1} \text{ and } Q_1^{+1} \in \mathcal{Q}_1^{+1}\}$, and we define:*

$$\widehat{\mathcal{Q}}_{1,\{x_1\}} \equiv \left\{ \widehat{Q}_{1,\{x_1\}} | \widehat{Q}_{1,\{x_1\}} = Q_0(\cdot|x_1), \text{ for some } Q_0 \in \mathcal{Q}_0 \right\} \quad (1.4.6)$$

, then we have $\widehat{\mathcal{Q}}_{1,\{x_1\}} = \mathcal{Q}_1^{+1}(\cdot|x_1)$.

Proof: Pick any $Q_{1,\{x_1\}} \in \widehat{\mathcal{Q}}_{1,\{x_1\}}$, we have $Q_{1,\{x_1\}} = Q_0(\cdot|x_1)$ for some $Q_0 \in \widehat{\mathcal{Q}}_0$.

By construction of $\widehat{\mathcal{Q}}_0$, $Q_0(x_1, x_2) = Q_0^{+1}(x_1)Q_1^{+1}(x_2|x_1)$, for some $Q_1 \in \mathcal{Q}_0^{+1}$ and $Q_1^{+1} \in \mathcal{Q}_1^{+1}$, so $Q_0(\cdot|x_1) = Q_1^{+1}(\cdot|x_1)$.

Since $Q_1^{+1}(\cdot|x_1) \in \mathcal{Q}_1^{+1}(\cdot|x_1)$, we have $Q_0(\cdot|x_1) \in \mathcal{Q}_1^{+1}(\cdot|x_1)$ as well. Thus $\widehat{\mathcal{Q}}_{1,\{x_1\}} \subseteq \mathcal{Q}_1^{+1}(\cdot|x_1)$

Pick any $Q_1^{+1}(\cdot|x_1) \in \mathcal{Q}_1^{+1}(\cdot|x_1)$, and pick any $Q_0^{+1} \in \mathcal{Q}_0^{+1}$, define $Q_0 = Q_0^{+1}Q_1^{+1}$, by definition of \mathcal{Q}_0 , this particular Q_0 is an element of \mathcal{Q}_0 . So $Q_0(\cdot|x_1)$ is an element

of $\widehat{\mathcal{Q}}_{1,\{x_1\}}$ by definition of $\widehat{\mathcal{Q}}_{1,\{x_1\}}$. Q.E.D.

The Lemma can be easily extended to any finite periods. What the Lemma guarantees that the way we construct the set of unconditional priors is probability-measure-wise consistent. While the next Lemma tells that using the same way to construct the set of unconditional priors, we have dynamic consistency.

Lemma 5 *Suppose $T = 2$, $\Omega = X_1 \cdot X_2$, $\mathcal{F}_1 = \sigma(X_1)$, $\mathcal{F}_2 = \sigma(X_1, X_2)$, and $\mathcal{Q}_0 \equiv \{Q_0 | Q_0(x_1, x_2) = Q_0^{+1}(x_1)Q_1^{+1}(x_2|x_1)$, for some $Q_1 \in \mathcal{Q}_0^{+1}$ and $Q_1^{+1} \in \mathcal{Q}_1^{+1}\}$, we have \mathcal{Q}_0 satisfy rectangularity condition.*

Proof: Notice that originally, the rectangularity conditions says that for any set of unconditional priors, it's $\{\mathcal{F}_t\}$ -rectangular if for all t, ω , we have:

$$\mathcal{P}_t(\omega) = \left\{ \int_{\Omega} P_{t+1}(\omega') dm : \text{for some } P_{t+1}(\omega') \in \mathcal{P}_{t+1}(\omega'), \forall \omega', \text{ and } m \in \mathcal{P}_t^{t+1}(\omega) \right\}$$

And when $T = 2$, it's trivial to check that it's true when $t = 0$ and for all ω by the definition of \mathcal{Q}_0 . And as the $P_2(\omega)$ are degenerated distribution, it's easy to prove that equality holds for rectangularity when $t = 1$.

1.4.2 An example in three period model

To qualitatively capture the stock market participation pattern over the life cycle, we extend two period model to a three period model. The three periods are 0,1,2.

We assume an agent has income profile (W, aW, bW) over three periods. Parameters a, b are designed to approximate real life income profile, where $a > 1$ can approximate the peak of mid-aged income and bW can capture the social security benefits,

which depends on agent's previous income. The income profile is certain. The agent faces the health expenditure shock at the last period. The expenditure is a random variable denoted by \tilde{H} . In each period, the agent decides how much to consume and to invest in bonds and stocks. The returns on stocks are uncertain. And there is ambiguity towards the joint distribution on last period stock return and health expenditure shock. We assume that the above joint distribution is independent of first period stock return. These assumptions guarantee the regularity condition hold. (See Epstein and Schneider (2003)²). Henceforth, we can use backward induction to solve the maximization problem.

The three period problem can be summarized as³:

$$\begin{aligned} & \max_{B_0, S_0, B_1, S_1} u(C_0) + \beta E_1 [u(C_1)] + \beta^2 \min_{Q \in \mathbb{P}} [u(C_2)] \\ & \text{s.t. :} \\ & C_0 + B_0 + S_0 = W \\ & C_1 + B_1 + S_1 = aW + RB_0 + \tilde{R}_1 S_0 \\ & C_2 = (bW - \tilde{H}) + RB_1 + \tilde{R}_2 S_1 \quad (1.4.7) \end{aligned}$$

Since we use backward induction to solve this problem, we can use the results

²Here the one-step-ahead belief as defined in Epstein and Schneider (2003) is a collection of joint distributions on stock return and health shock in period 2. This collection is independent of the realization of period 1 stock return due to the assumption of independence, which, in turn, guarantees the rectangularity condition satisfied. We thus have the legitimacy to use the one-stop-ahead belief rather than belief at time 0 when we define the utility function.

³ The utility function is still CARA.

from the two period model and determine the stock holding at time 1 as follows:

$$S_1 = \begin{cases} \frac{\mu_1 - R - \theta(\sigma_{12} + \bar{v})}{\theta\sigma_1^2} & \bar{v} < \frac{\mu_1 - R}{\theta} - \sigma_{12} \\ 0 & \underline{v} \leq \frac{\mu_1 - R}{\theta} - \sigma_{12} \leq \bar{v} \\ \frac{\mu_1 - R - \theta(\sigma_{12} + \underline{v})}{\theta\sigma_1^2} & \underline{v} > \frac{\mu_1 - R}{\theta} - \sigma_{12} \end{cases} \quad (1.4.8)$$

where non-asset income \tilde{Y} is equal to $bW - \tilde{H}$. And we assume that the set of joint distributions on \tilde{Y} and \tilde{R}_2 is similar to that in definition 1.3.17.

The bond holding, in case when $S_1 > 0$, is:

$$B_1 = \frac{\ln \beta R}{\theta(1+R)} + \frac{W_0 - \mu_2 - (1 + \mu_1)S_1}{1+R} + \frac{1}{1+R} \frac{\theta\sigma_2^2}{2} + \frac{1}{1+R} \frac{\theta S_1^2 \sigma_1^2}{2} + \frac{\theta S_1(\sigma_{12} + \bar{v})}{1+R} \quad (1.4.9)$$

where the first term is the savings caused by impatience. It's positive if interest rate dominates time discount factor ($\beta R > 1$). The second term is the difference between tomorrow's expected income and today's wealth. Here $W_0 = aW + RB_0 + R_1S_0$ is the total wealth received at the beginning of period 2, where R_1 is the realization of period 1 stock return. The third term is the precautionary savings due to uncertainty in income. Similarly, the fourth term is the precautionary savings caused by uncertainty in stock return. The last term is precaution saving for the co-movement between stock and income.

For the case when $S_1 < 0$, the B_1 formula will be modified accordingly to the last term, which is changed to $\frac{\theta S_1(\sigma_{12} + \underline{v})}{1+R}$. When $S_1 = 0$, the bond holding is:

$$B_1 = \frac{\ln \beta R}{\theta(1+R)} + \frac{W_0 - \mu_2}{1+R} + \frac{1}{1+R} \frac{\theta\sigma_2^2}{2} \quad (1.4.10)$$

obviously, it is absent of the two precautionary savings terms due to stock holding uncertainty and ambiguity towards correlation between stock return and de facto income.

Using backward induction, in period 0, it's easy to show that everyone holds stock as follows

$$S_0 = \frac{1 + R}{R} \frac{\mu_1 - R}{\theta \sigma_1^2} \quad (1.4.11)$$

In fact, the time 0 stock holding is bigger than zero for every agent, therefore, the model predicts that the stock market participation rate in period 0 is 1, however, due to the existence of ambiguity, someone will quit the stock market in period 1. Hence we deliver the decreasing stock market participation for older people in this model. It's also worth pointing out that S_0 is larger than S_1 , therefore we generate the decreasing stock share for older people over the life cycle.

1.4.3 Who Quit and Who Stay in Stock Market

From the discussion above, we found that in this model the key component to determine non-participation is the range of band on ambiguity towards the correlation between stock return and non-asset income.

The non-participation condition in 1.4.10 is that:

$$\underline{v} \leq \frac{\mu_1 - R}{\theta} - \sigma_{12} \leq \bar{v}$$

where $\sigma_{12} = \text{Cov}_P(\tilde{Y}, \tilde{R})$. Here P is the reference distribution as defined in (5). We also have $\tilde{Y} = bW - \tilde{H}$. Where \tilde{H} is the random health expenditure. We can rewrite

$\tilde{H} = W\tilde{h}$, i.e., \tilde{h} is a share of health expenditure over initial period labor income W .

Henceforth, the correlation for reference distribution can be rewritten as:

$$\sigma_{12} = W[E(\tilde{h})E(\tilde{R}) - E(\tilde{h}\tilde{R})]$$

and the non-participation condition can be rewritten as:

$$\underline{v} \leq \frac{\mu_1 - R}{\theta} + W[E(\tilde{h}\tilde{R}) - E(\tilde{h})E(\tilde{R})] \leq \bar{v} \quad (1.4.12)$$

First we assume people have same knowledge of μ and R . When people share same parameters of risk aversion, ambiguity level $E(\tilde{h}\tilde{R}) - E(\tilde{h})E(\tilde{R})$, the higher is W , the less likely that people will stop participating. If $E(\tilde{h}\tilde{R}) - E(\tilde{h})E(\tilde{R}) > 0$, the agent will likely to long if W is big enough. And they will short when W is big enough and $E(\tilde{h}\tilde{R}) - E(\tilde{h})E(\tilde{R}) < 0$.

If we assume people have same parameters except for ambiguity level, then those with smaller band of ambiguity will less likely stop participating.

Similarly, if people have same parameters as others except for risk aversion θ , then the less risk averse the agent is, the more likely he will long stocks.

1.5 Conclusion

This paper documents a life cycle pattern of stock market participation rate. We find that the stock market participation rate decreases dramatically after age 60. Why do older people quit the stock market?

We provide an answer based on the observation that older people face much stronger health risk than young and the empirical evidence that shows the health risk is indeed a powerful predictor of stock participation (Rosen and Wu 2004). In our model, older people face health expenditure shock. In addition, they are uncertain about the correlation between stock return and de facto nonfinancial income which is negatively affected by their health expenditure. Under a multi-prior utility framework, we show that there exists a range of ambiguity level towards this correlation over which investors neither long or short sell a stock. Heterogeneity among older people on the ambiguity towards the correlation between risky stock return and uncertain health expenditure thus guarantees some people will fall into this range. Therefore they will choose not to participate in the stock market.

Chapter 2

Risky Borrowing Cost And Portfolio Choice Over The Life Cycle

2.1 Introduction

This paper investigates the nature of riskiness in borrowing costs and its implication for households' portfolio choice over the life cycle. The borrowing costs are the interest rates at which household pay back their bank loans. In reality, households can not only make deposit into safe bonds and invest in risky stocks, but also can take loans. And households in the U.S. hold these three assets at the same time.

The borrowing costs are important incentives for households to make financial decisions. It defines the discounted total wealth and essentially, borrowing capacity. It

also affects the opportunity cost to hold stock, especially when considering a leveraged position.

Given the importance of borrowing costs on understanding portfolio choice over the life cycle, it's worth noticing the nature of borrowing costs in the real world. And they have the following features, they are: 1) not infinite, 2) not as low as saving rate, borrowing costs are generally higher than saving rate 3) negatively correlated with stock return, 4) not homogeneous across households and 5) not linear with the size of loan. These observations, however, have been violated in the literature. And such violations have made models fail to predict important empirical facts.

This fact, however, has been missing from the predictions of literature. Life cycle models using tight borrowing constraint can deliver realistic equity holdings, but they fail to predict loan holding by essentially assuming infinite borrowing cost. Some other literature assumes that households **can** borrow at the same rate as at which they save, which is against observation 2). In these models, the prediction is that households will borrow aggressively, meaning they will explore the borrowing limit, and use borrowed money to invest in equity. In the mean time, the investment in safe bonds is too little. This failure actually demonstrates another facet of the equity premium puzzle in that given high asset return, households hold too many stocks by contrast to the data. We also notice that in such models, the total wealth should be much bigger than the case with borrowing constraint and that gives intuition why households would invest relatively more into stocks.

Davis, Kubler and Willen (2006) noticed that the borrowing costs are generally bigger than saving rates, i.e., there is a wedge between borrowing and saving rates. They use this wedge and incorporate it into a life cycle model with portfolio choice

to look at the debt and equity holding. Although the results improve previous literature, it still predicts much more equity holding compared to the data. In the benchmark model with relative risk aversion as 2, with households being able to access the credit market, the model predicts that the bond holding is 0 over the life cycle, and that equity holding is very much levered. And these findings are inconsistent with the data.

In this paper, the facts 1)-3) about the nature of borrowing costs are incorporated into a life cycle portfolio choice problem. It delivers that the average holdings of safe bonds are non zero in the benchmark model with relative risk aversion as low as 2. In the benchmark model, the equity demand is higher than that those in Davis, Kubler and Willen (2006), while the demand is dampened when the relative risk aversion is increased.

The intuition can be explained as following. The negative correlation between borrowing cost and stock return widens the wedge between the two risky factors. On one hand, the borrowing cost is likely to be low when stock return is likely to be high (the good state), which gives investor some more incentive to hold equity; and on the other, it's also likely that the moment stock return being low is also the time that borrowing cost is high (the bad state), which dampens the incentive to hold stocks. When relative risk aversion is low, households' dislike the bad state is overcome by the gain from good state, thus the equity holding is even bigger. As the relative risk aversion increases, households' dislike the volatility between bad and good states overwhelms the gain from good state. Another way to explain the mechanism is by usage of intuition from background risk literature. Given negative correlation between stock return and borrowing cost, it gives positive correlation between loan payment

and stocks. Thus from the diversification point of view, as relative risk aversion increase, bank payment crowds out equity demand and henceforth the equity demand decreases.

In terms of relationship to the literature, this paper is in the line of Merton (1969) and Samuelson (1969) among others in the discussion of life cycle portfolio choice. Heaton and Lucas (2000a, 2000b) pay attention to background risk and implication for the portfolio choice over the life cycle. The intuition there is similar but not exactly the same as shown in this paper. They point out that the nature of the background risk plays an important role in determining the equity demand. If the background risk makes the households' income similar to stock, then the equity demand will be reduced by existence of such background risk. This effect, as Heaton and Lucas point out, can be verified by looking at business owners' portfolio choice. Gomes and Michaelides (2005), Cocco, Gomes, and Maenhout (2005) points out that if background risk, such as labor income is orthogonal to the equity return risk, making labor income a substitute for riskless asset holding, the optimal equity holding should not be affected by such background risk. Actually, they point out that when households retire, the demand for equity decrease. I.e, their equity demand was not dampened purely due to the existence of labor income shock during the time when they work. Jagannathan and Kocherlakota (1996) shows that the background risk affects older people's portfolio choice.

This paper is more related to Davis, Kubler and Willen (2006) and Willen-Kubler (2006) which sheds light on the borrowing costs' implication on life cycle portfolio choice. As mentioned before, this paper deviates from their work in that it documents the fact of negative correlation and incorporates that into a life cycle portfolio choice

model. Which delivers non zero bond holdings even when relative risk aversion is as low as 2. This paper also deviates from the literature which discusses the impact of changing credit line, such as Mateos-Planas and Rios-Rull (2007). In their paper, the credit line is endogenously given and changes over time, while the interest rates are fixed.

In the credit spread literature, it has been well documented and broadly discussed about the nature and implication of borrowing costs that firms face. Among the most recent development in this literature, it includes Gomes and Schmid (2010) and Chen (2010), where they study relationship between firms' financial decisions such as issuing bonds and credit spreads. This paper can be seen as counterpart of that literature in that it studies similar question: households' financial decisions under similar condition: risky borrowing wedge (spread).

2.2 The Facts

I start with the facts on households' portfolio choice over the life cycle. I look at SCF data¹. I estimate the age effects on equity share, debt share and safe assets share over total wealth. And I use 6 waves of SCF data: 1992-2004. For each wave, each age group, the mean equity/safe asset/debt shares over total wealth are calculated. I then run regression of the mean shares over age category as well as time/cohort effect. The coefficients before age groups are reported in figures 1-3.

The borrowing costs that U.S. households are facing are heterogeneous according to terms of loan, such as size and duration of the loan, adjustability of the rates as

¹Chapter 1 has more detail.

Table 2.1: The statistics of annual bank prime loan rate and stock return (1949-2006)

	Sample Means	Sample Stdev	Correlation with Stock (p value)
Stock	9.0225	16.0235	1
Bank Prime Loan Rate	7.0831	3.4506	-0.1866 (0.1986)

well as economic condition of households. To investigate the time series property between borrowing costs and stock return, we take the bank prime loan rate as the average borrowing cost. The prime rate is the interest rate commercial banks charge their most creditworthy customers, which are usually corporations. The prime rate, as reported by the Wall Street Journal's bank survey, is among the most widely used benchmark in setting home equity lines of credit and credit card rates. It is in turn based on the fed funds rate, which is set by the Federal Reserve. We use the monthly prime bank loan rate released by the Fed as approximation of average borrowing cost that U.S. households face. To find out its relationship with stock return, we look at the time series correlation, where stock return is defined as S&P 500.

The following graph demonstrates the movement of annual stock return and bank prime loan rate. And the following table summarizes the key statistics of annual stock return and bank prime loan rate.

As given later when I lay out the model, the borrowing cost is assumed such that its realization is unknown until when the loan is repaid. And this paragraph is dedicated to justifying this assumption. I can assume the borrowing cost as an approximation of credit card rate. As Durkin (2000) among other papers point out that, a large portion (74% in 1995) of U.S. households have at least one credit card, and (44%) keep a positive balance after the most recent payment. Given the availability and usage of credit cards, I believe taking credit card rates as an approximation of the general rates would be a good approximation. And the variable credit cards rates are

based their on the U.S. prime rate. For example, the APR on a credit card might be the prime rate plus 13%. That means, if the prime rate is 3.25%, the current APR on that variable rate card would be 16.25%. On the other hand, even for some bank loans which nominal rates are predetermined when the loan is taken, it's still subject to inflation risk².

The parallel relationship between prime rate and actual borrowing costs gives me justification why I use the empirical correlation between prime rate and stock return as an approximation of the population correlation between borrowing cost and stock return.

In calibration, I also assume the population standard deviation of borrowing cost that U.S. households faces mimics the empirical standard deviation of prime rate. I argue that this assumption can also be justified by the parallel nature between prime rate and stock return.

The investigation of why there exists such negative correlation between borrowing costs and stock return is beyond the scope of this paper. But a possible explanation is given here. There is a possible negative correlation between stock return and the unemployment risk. That's to say, when the economy is in good state, it's also very likely that households face small unemployment risk, and thus the default risk is small as well. Henceforth, the borrowing cost in this state might fall as well. Similarly, during recession, it's also very likely that unemployment rate is high, and thus banks may charge a higher risk as they face higher default risk at this time.

²The author thanks Chris Phelan to point this out.

2.3 The Model and Basic Intuition

2.3.1 Two Period Model

The basic intuition can be illustrated in a two-period consumption-portfolio choice model where the agent faces risky stock return and risky borrowing costs. The agent has initial financial asset W_0 to allocate among consumption, risk-free saving and stock. He can also take out loan from the bank. The borrowing rate and stock return are realized at the beginning of the second period. That's to say, in the initial period, the agent is unaware of the borrowing rate at which he pays back the loan. In the beginning of second period, the agent receives a non-stochastic non-asset income y_1 , he pays back loan, and he receives risk-free saving and stock investment. He uses all the asset to consume during the second period.

The two-period budget constraints are:

$$C_0 + B + S = W_0 + L \quad (2.3.1)$$

$$C_1 = y_1 + R_f B + \tilde{R}_S S - \tilde{R}_L L \quad (2.3.2)$$

, with no-short-sell constraints:

$$L \geq 0, \quad B \geq 0, \quad S \geq 0 \quad (2.3.3)$$

as well as:

$$C_0 \geq 0, \quad C_1 \geq 0 \quad (2.3.4)$$

To look at the intuition that why and how risky borrowing cost can affect agent's consumption-portfolio choice, we can look at the contingent inter-temporal budget constraint. It's obvious that when $\tilde{R}_L \equiv \infty$, or there is a borrowing constraint then $L = 0$. While when \tilde{R}_L is not infinite, the borrowing constraints can be rewritten in the inter-temporal form as:

$$C_0 + \frac{C_1}{\tilde{R}_L} = W_0 + \frac{y_1}{\tilde{R}_L} + \left(\frac{R_f}{\tilde{R}_L} - 1 \right) B + \left(\frac{\tilde{R}_S}{\tilde{R}_L} - 1 \right) S \quad (2.3.5)$$

The expected initial wealth can be defined as the expectation of the right hand side of the inter-temporal budget constraint. When the borrowing rate is non-stochastic, as long as the expected return on stock is higher than borrowing cost, the expected wealth is higher if the agent invests more in stocks. On the other hand, if the borrowing cost R_L is non-stochastic and is bigger than expected stock return, the more investment in stock using loans, the less the expected wealth is. Moreover, even if R_L is stochastic and independent to stock return, the previous insight still holds that only the difference between expected values of borrowing cost and stock return matters for the expected wealth. While if R_L is stochastic and countercyclical to stock return, the stock investment has less contribution to the total wealth compared to the case if they are independent. The reason is that when stock return is low, it's the same time that borrowing cost is high, this will make the marginal contribution to wealth from stock investment, $\frac{\tilde{R}_S}{\tilde{R}_L} - 1$ low.

I next compared with two different specifications where one has constant borrowing cost and its borrowing rate is equal to the expected asset return, while in the other specification, the borrowing cost is risky, with negative correlation to the stock

return, and with same mean value as mean equity return.

In the specification where the borrowing cost is risky and has negative correlation with stock return, the gap for the wealth between good and bad state is widen. When it's in good state, the high risky asset return \tilde{R}_S and low borrowing cost \tilde{R}_L happens together and it makes the “good” state even better. Similarly, it also makes the bad state even worse compared to the specification without riskiness in borrowing cost. According to the calculation in full fledged life cycle model, it happens that when risk aversion is low, the good state being better effect dominates bad state being worse, and it enhances the equity and debt holding; While when the risk aversion is increased, the equity and debt demand are dampened, as the diversification incentives dominates the “good state being even better” effect.

2.3.2 The Life Cycle Model

The fully-fledged model has this intuition imbedded. A household works and receives stochastic labor income. It consumes, and it also invests in stock, risk free asset and takes loan. The labor income is assumed to be independent to both risky stock return and risky borrowing cost.

The log income y_t is assumed to follow the following process:

$$\begin{aligned} y_t &= p_t + \varepsilon_t \\ p_t &= g_t + p_{t-1} + \eta_t \end{aligned} \tag{2.3.6}$$

, where p_t denotes age-dependent permanent income component, g_t is deterministic income growth factor, ε_t is transitory shock, and η_t is the shock to the permanent

income component with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, $\eta_t \sim N(0, \sigma_\eta^2)$ and ε_t being independent to η_t .

After retirement age, the household receives a fraction of its income during working age. In each year, households first receive their non-asset income, either labor income during working age or social security income during retirement age. Then they realize the risky returns: stock return and bank loan. They decide how much to consume and to invest. They have three financial assets to invest in: loan, safe asset and risky asset. So they choose how much loan to take, without knowing the rate to pay back at that moment, how much investment in safe asset and how much in risky asset. They can't short in risk free asset and risky asset, nor can they short in loan, i.e. they can't lend their saving at the borrowing cost they face. Households are not allowed to die with positive debt. The debt limit is set to be multiple times annual income.

Households choose life time consumption and investment plan to maximize life time expected utility:

$$\sum_{t=0}^T E_0 \left[\beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \right] \quad (2.3.7)$$

subject to budget constraints and no-short-sale constraints:

$$C_t + B_t + S_t = y_t - R_L L_{t-1} + R_f B_{t-1} + R_S S_{t-1} + L_t \quad (2.3.8)$$

$$C_t \geq 0, \quad L_t \geq 0, \quad B_t \geq 0, \quad S_t \geq 0 \quad (2.3.9)$$

2.3.3 Calibration

I set relative risk aversion $\gamma = 2$ as benchmark specification. Time discount factor is set as 0.933. I assume the annual risk free return is 2%, and the expected return on equity is 8%, with standard deviation of stock return is 15%.

Household enters at age 21, they retire at 65 and die at 80 in the model. When they work, their labor income is uncertainty. In the benchmark model and throughout the experiments, the correlation between non asset income shock and stock return is set equal to zero. So is the correlation between non asset income shock and borrowing cost. Following Gourinchas and Parker (2002), I make the deterministic component life cycle income profile approximated by a fifth-order polynomial of head of household's age. This was estimated according to CEX data on log family income. The standard deviation of permanent shock is 12%, that of transitory shock is 15%.

Following Davis, Kubler and Willen (2006), I assume that the expected borrowing cost is also equal to 8%. While in the benchmark model, the borrowing cost is risk and it has negative correlation with stock return. The correlation coefficient is set to be equal to -0.2 , as observed from the empirical correlation between prime rate and stock return.

When households are retired, their retirement income is determined in the following way: Step 1), sort the average value of labor income during working age in descendent order; Step 2), take the average of the highest 30 years of the sorted average working age income, denote it by D_{30} ; Step 3), take the ratio of D_{30} to the last working year average income, denote it by d_{30} ; Step 4), use d_{30} multiply by the realization of last working year income as the retirement basis b_{30} ; Step 5), multiply b_{30} by replacement rate, which is equal to 80%. The above method including parameters such as 30 years and replacement rate 80% are set so that the average income profile mimic the data.

The realization of income and asset return are discretized by the method of Tauchen and Hussey (1991).

The computational strategy is discussed in more details in Appendix. After calculating the decision rule, I simulate the economy by 10000 samples. The average asset holdings are calculated according to stored decision rule, and they are reported in the next section.

2.3.4 Computation Strategy

The computation strategy in the full-fledged life cycle model with consumption-portfolio choice follow that used in Davis-Kubler-Willen (2006) and that in Willen-Kubler (2006). The computation uses backward induction policy-function iteration. And since there are three assets: loan, risk free asset and risky asset, to choose, and the no-short-sales constraints frequently bind, it uses Kuhn-Tucker equations instead of Kuhn-Tucker conditions to solve the portfolio choice problem.

The computation starts from the last period, where the solution is trivial in that the asset holdings are all equal to zero, and the agent consumes everything. Given the policy at the last period, we solve for the policy function at the second to last period for each state variable(s). And so on.

The core problem is thus a two-period optimization problem. Suppose the optimal policy has been pinned down from time $t + 1$ to the end period T . Now at time t , with state variable ω with non-asset income y_t , and wealth W_t , the optimal policy need to specify the loan, safe asset and risky asset holdings at this time t and this state ω , while the optimal policy has already been known for every state ω' at time $t + 1$. The optimal policy for all the periods (the global problem) need to solve the

time t to $t + 1$ (the local problem) as well. The local problem at t is specified as:

$$\max_{C_t(\omega), L_t(\omega), B_t(\omega), S_t(\omega)} \frac{C_t(\omega)^{1-\gamma}}{1-\gamma} + \beta \mathbf{E}_t \left[\frac{C_{t+1}^{1-\gamma}}{1-\gamma} \right] \quad (2.3.10)$$

$$\text{s.t. } C_t(\omega) = W_t(\omega) + y_t(\omega) + L_t(\omega) - B_t(\omega) - S_t(\omega)$$

$$C_{t+1}(\omega'|\omega) = W_{t+1}(\omega'|\omega) + y_{t+1}(\omega'|\omega) + L_{t+1}(\omega'|\omega) - B_{t+1}(\omega'|\omega) - S_{t+1}(\omega'|\omega)$$

$$W_{t+1}(\omega'|\omega) = -L_t(\omega)R_L(\omega'|\omega) + B_t(\omega)R_f + S_t(\omega)R_S(\omega'|\omega) \quad , \forall \omega' \in \Omega = \{\omega_1, \dots, \omega_N\}$$

$$\text{and } L_t(\omega) \geq 0, B_t(\omega) \geq 0, S_t(\omega) \geq 0.$$

, where $\Omega = \{\omega_1, \dots, \omega_N\}$ is the set of all possible states for each date and state.

$L_{t+1}(\omega'|\omega)$, $B_{t+1}(\omega'|\omega)$, and $S_{t+1}(\omega'|\omega)$ are policy function at time $t + 1$, given then the state is ω' . And it's known at time t .

Taking advantage of homotheticity of CRRA utility function, we can rewrite the problem as:

$$\max_{c_t(\omega), l_t(\omega), b_t(\omega), s_t(\omega)} \frac{c_t(\omega)^{1-\gamma}}{1-\gamma} + \beta \mathbf{E}_t \left[\frac{c_{t+1}^{1-\gamma}}{1-\gamma} \right] \quad (2.3.11)$$

$$\text{s.t. } c_t(\omega) = w_t(\omega) + l_t(\omega) - b_t(\omega) - s_t(\omega)$$

$$c_{t+1}(\omega'|\omega) = \frac{y_{t+1}(\omega'|\omega)}{y_t(\omega)} [w_{t+1}(\omega'|\omega) + l_{t+1}(\omega'|\omega) - b_{t+1}(\omega'|\omega) - s_{t+1}(\omega'|\omega)]$$

$$w_{t+1}(\omega'|\omega) = \frac{y_{t+1}(\omega'|\omega) - l_t(\omega)y_t(\omega)R_L(\omega'|\omega) + b_t(\omega)y_t(\omega)R_f + s_t(\omega)y_t(\omega)R_S(\omega'|\omega)}{y_{t+1}(\omega'|\omega)}$$

$$\forall \omega' \in \Omega = \{\omega_1, \dots, \omega_N\}$$

$$\text{and } l_t(\omega) \geq 0, b_t(\omega) \geq 0, s_t(\omega) \geq 0.$$

, where all choice variables are normalized by current non-asset income:

$$\begin{aligned} c_t(\omega) &= \frac{C_t(\omega)}{y_t(\omega)}, & l_t(\omega) &= \frac{L_t(\omega)}{y_t(\omega)} \\ b_t(\omega) &= \frac{B_t(\omega)}{y_t(\omega)}, & s_t(\omega) &= \frac{S_t(\omega)}{y_t(\omega)} \\ c_{t+1}(\omega'|\omega) &= \frac{C_{t+1}(\omega'|\omega)}{y_t(\omega)} \end{aligned}$$

After we solve the normalized single state variable maximization problem (2.3.11), we can backup the solution in the original problem (2.3.10) by having:

$$C_t(\omega) = c_t(\omega)y_t(\omega), \quad L_t(\omega) = l_t(\omega)y_t(\omega), \quad B_t(\omega) = b_t(\omega)y_t(\omega), \quad S_t(\omega) = s_t(\omega)y_t(\omega)$$

When we solve the normalized single state variable two-period maximization problem, it's necessary and sufficient to solve the Kuhn-Tucker conditions, which essentially is a set of equations and inequalities including complimentary conditions and slackness conditions. Based on Garcia and Zangwill (1981, pp 65-68) and following Davis-Kubler-Willen (2006) and that in Willen-Kubler (2006), a change of variables method is applied to transform inequalities in Kuhn-Tucker conditions into a set of only equations.

To begin the reformulation, denote k as a natural number and define:

$$x^+ = [\max(0, x)]^k \tag{2.3.12}$$

$$x^- = [\max(0, -x)]^k \tag{2.3.13}$$

, thus it's always true that:

$$x^+ \geq 0, \quad x^- \geq 0, \quad x^+ x^- = 0 \quad (2.3.14)$$

It's worth to notice that both x^+ and x^- are $(k - 1)$ -continuously differentiable.

For the two-period problem:

$$\max_{C_0, C_1(\cdot), L, B, S} u(C_0) + \beta E[u(C_1)] \quad (2.3.15)$$

$$\text{s.t. } C_0 = W_0 + L - B - S$$

$$C_1(\omega) = y_1 - R_L(\omega)L + R_f B + R_S(\omega)S, \quad \forall \omega \in \Omega$$

$$C_0 \geq 0, \quad C_1 \geq 0, \quad L \geq 0, \quad B \geq 0, \quad S \geq 0$$

It's Kuhn-Tucker conditions are (assuming optimal consumption is positive and

budget constraints bind):

$$u'(C_0) - \lambda_0 = 0 \quad (2.3.16)$$

$$\beta u'(C_1(\omega)) \pi(\omega) - \lambda_1(\omega) = 0 \quad (2.3.17)$$

$$\lambda_0 - \beta \sum_{\omega \in \Omega} [\lambda_1(\omega) R_L(\omega)] + \lambda_L = 0 \quad (2.3.18)$$

$$\lambda_0 - \beta \sum_{\omega \in \Omega} [\lambda_1(\omega) R_f] - \lambda_B = 0 \quad (2.3.19)$$

$$\lambda_0 - \beta \sum_{\omega \in \Omega} [\lambda_1(\omega) R_S(\omega)] - \lambda_S = 0 \quad (2.3.20)$$

$$C_0 = W_0 + L - B - S \quad (2.3.21)$$

$$C_1(\omega) = y_1 - R_L(\omega)L + R_f B + R_S(\omega)S \quad (2.3.22)$$

$$\lambda_L L = 0 \quad (2.3.23)$$

$$\lambda_L \geq 0 \quad (2.3.24)$$

$$L \geq 0 \quad (2.3.25)$$

$$\lambda_B B = 0 \quad (2.3.26)$$

$$\lambda_B \geq 0 \quad (2.3.27)$$

$$B \geq 0 \quad (2.3.28)$$

$$\lambda_S S = 0 \quad (2.3.29)$$

$$\lambda_S \geq 0 \quad (2.3.30)$$

$$S \geq 0 \quad (2.3.31)$$

While the Kuhn-Tucker equations are:

$$u'(C_0) - \lambda_0 = 0 \quad (2.3.32)$$

$$\beta u'(C_1(\omega)) \pi(\omega) - \lambda_1(\omega) = 0 \quad (2.3.33)$$

$$\lambda_0 - \beta \sum_{\omega \in \Omega} [\lambda_1(\omega) R_L(\omega)] + \lambda_L^+ = 0 \quad (2.3.34)$$

$$\lambda_0 - \beta \sum_{\omega \in \Omega} [\lambda_1(\omega) R_f] - \lambda_B^+ = 0 \quad (2.3.35)$$

$$\lambda_0 - \beta \sum_{\omega \in \Omega} [\lambda_1(\omega) R_S(\omega)] - \lambda_S^+ = 0 \quad (2.3.36)$$

$$C_0 = W_0 + L - B - S \quad (2.3.37)$$

$$C_1(\omega) = y_1 - R_L(\omega)L + R_f B + R_S(\omega)S \quad (2.3.38)$$

$$\lambda_L^- - L = 0 \quad (2.3.39)$$

$$\lambda_B^- - B = 0 \quad (2.3.40)$$

$$\lambda_S^- - S = 0 \quad (2.3.41)$$

It's easy to show that the Kuhn-Tucker equations are equivalent to the Kuhn-Tucker conditions. Moreover, if $(C_0^*, C_1^*, L^*, B^*, S^*, \lambda_0^*, \lambda_1^*(\omega), \lambda_L^*, \lambda_B^*, \lambda_S^*)$ solves the Kuhn-Tucker equations, then $(C_0^*, C_1^*, L^*, B^*, S^*, \lambda_0^*, \lambda_1^*(\omega), \lambda_L^{**}, \lambda_B^{**}, \lambda_S^{**})$ solves the Kuhn-Tucker conditions. And there is:

$$\lambda_L^{**} = (\lambda_L^*)^+ \quad (2.3.42)$$

$$\lambda_B^{**} = (\lambda_B^*)^+ \quad (2.3.43)$$

$$\lambda_S^{**} = (\lambda_S^*)^+ \quad (2.3.44)$$

On the other hand, if $(C_0^*, C_1^*, L^*, B^*, S^*, \lambda_0^*, \lambda_1^*(\omega), \lambda_L^{**}, \lambda_B^{**}, \lambda_S^{**})$ solves the Kuhn-

Tucker conditions, then $(C_0^*, C_1^*, L^*, B^*, S^*, \lambda_0^*, \lambda_1^*(\omega), \lambda_L^*, \lambda_B^*, \lambda_S^*)$ solves the Kuhn-Tucker equations with:

$$\lambda_L^* = \begin{cases} (\lambda_L^{**})^{\frac{1}{k}} & \text{if } L^* = 0 \\ -(L^*)^{\frac{1}{k}} & \text{if } L^* > 0 \end{cases} \quad (2.3.45)$$

$$\lambda_B^* = \begin{cases} (\lambda_B^{**})^{\frac{1}{k}} & \text{if } B^* = 0 \\ -(B^*)^{\frac{1}{k}} & \text{if } B^* > 0 \end{cases} \quad (2.3.46)$$

$$\lambda_S^* = \begin{cases} (\lambda_S^{**})^{\frac{1}{k}} & \text{if } S^* = 0 \\ -(S^*)^{\frac{1}{k}} & \text{if } S^* > 0 \end{cases} \quad (2.3.47)$$

To pick the proper k , we can make the Kuhn-Tucker equations very smooth in that the left-hand side of the Kuhn-Tucker equations can be differentiable of order \mathcal{C}^{k-1} .

2.4 Asset Holdings

I start with the benchmark specification. I'll then report how asset holdings are affected by 1) alternative borrowing cost regime, namely, without including the negative correlation between borrowing cost and stock return, and 2) relative risk aversion.

Figure 2 reports mean asset holding, average income and consumption, as well as wealth in benchmark model. As to asset holdings in benchmark model, mean equity holding peaks around age 60, and it then decreases quickly. The mean bond holding however, picks up slowly from age 40, and it increases after retirement. The mean borrowing keeps increase till age 50, and it the decreases since then. Figure 3 reports safe asset participation rate over the life cycle. There is no participation

till age 34. After that, the participation is 100%. This is somehow consistent with financial adviser's suggestion: when worker is young, larger portion of investment should be put in risky asset to enjoy the equity premium as the loss can be recovered by labor income. Figure 4 reports risky asset participation rate over the life cycle. It's eventually 100% participation rate over time. It's not surprising given big positive equity premium and uncorrelated income shock and stock return.

As to asset share over the life cycle. Under the specification without correlation, the mean of bond share is almost always equal to 0 over the life cycle with CRRA = 2 or 3. While under the specification with negative correlation, the mean bond share can be positive. When without riskiness in borrowing cost and CRRA is small, the marginal gain to invest in bond is too small compared to that of taking loan and investing in stock.

When switching to the specification where there is no correlation between stock return and borrowing cost, the mean asset holdings are reported in figure 5. As it tells, the mean equity holding has about the same peak age, but shifts lower from the specification with correlation. The bond holding is 0. And the participation is of course 0 over the life cycle. The risky asset participation is lower before age 30, and it picks up quickly to 100% participation rate after age 42. Households totally quit the loan market after age 43. This implies all investments are in stocks. This can be explained as following: as households expect that non asset income risk will be resolved, they are more willing to hold stocks.

We next consider the impact of relative risk aversion. In the experiment, I use the risky borrowing cost and relative risk aversion as 3. The mean asset holding is reported in figure 7, while the asset participation has the exact shape as in bench-

mark specification. From the mean asset holding, it says the equity demand and loan demand are smaller compared to the case where $CRRA=3$. While it's still slightly bigger than that in the case where $CRRA=2$ and without correlation between borrowing cost and stock return. The equity and loan demand are smaller as agents are more risk averse, and lower asset holding can reduce the volatility of future consumption. While the incentive to enjoy "good" states still prevails so that it has bigger equity and loan holding than the case without considering correlation between borrowing cost and stock return.

2.5 Conclusion

This paper investigates the nature of borrowing cost and studies its implication for portfolio choice over the life cycle. It documents a negative correlation between risky borrowing cost and uncertain stock return. This fact is incorporated into life cycle portfolio choice model as key ingredient as well as a wedge between expected borrowing cost and risk free rate. In the model, households can take out loan, invest in safe bonds and risky stock. And in the realistic calibration, this paper contributes to the portfolio choice literature by delivering the average asset holding in all categories are non zero, even when relative risk aversion is as low as 2. Though the equity is even higher compared to the case without including negative correlation between borrowing cost and asset return. As risk aversion increases, the average equity demand is reduced, so is the average debt holding. Compared to the specification without correlation between stock return and borrowing cost, the "good state" is even better under the specification with negative correlation incorporated, since when "good

state” happens, both stock return is high and loan payment is low. When relative risk aversion is low, the gain from “good state” seems to dominate the extra volatility induced by difference in wedges between borrowing cost and stock return across good and bad state. When relative risk aversion increases, however, the equity demand is dampened as the incentive to diversify the risk prevails. As stock return comoves negatively with borrowing cost, the loan payment (negative term) thus have positive correlation with stock, or it looks like stock. And to diversify the risk, households should reduce the equity holding.

This paper has the potential to deliver more understanding of anomaly in households’ portfolio choice, such as low equity holdings in the face of high equity premium. It can be extended into future research such as heterogeneity in risky borrowing costs and its implication for the portfolio choice problem among households. For instance, it may help to explain the heterogenous equity holdings among skilled and unskilled workers. It’s well known that equity holdings are significantly different between skilled and unskilled workers even after controlling other demographic and economic factors such as family composition, income and wealth. Namely, workers with college degree are more likely to hold stocks, and conditional on participating, they hold significantly more stocks compared to those with at most high school diploma.

And I believe that this heterogeneity in asset holding among skilled and unskilled workers might be explained by heterogenous borrowing costs. It’s well known that skilled and unskilled workers face different unemployment risks, which thus implies different default risks among skilled and unskilled workers, and henceforth different borrowing costs. This gives skilled workers higher borrowing capacity and low opportunity cost even when the realization of their incomes are the same as those of some

unskilled workers.

On the other hand, I expect this project to be extended with housing asset. In reality, the purchase of housing assets requires taking out sizable loan and the ownership of housing asset may affect the future borrowing cost as well. To include this realistic interaction into life cycle model may help to explain a more broad and complete picture of asset composition over the life cycle.

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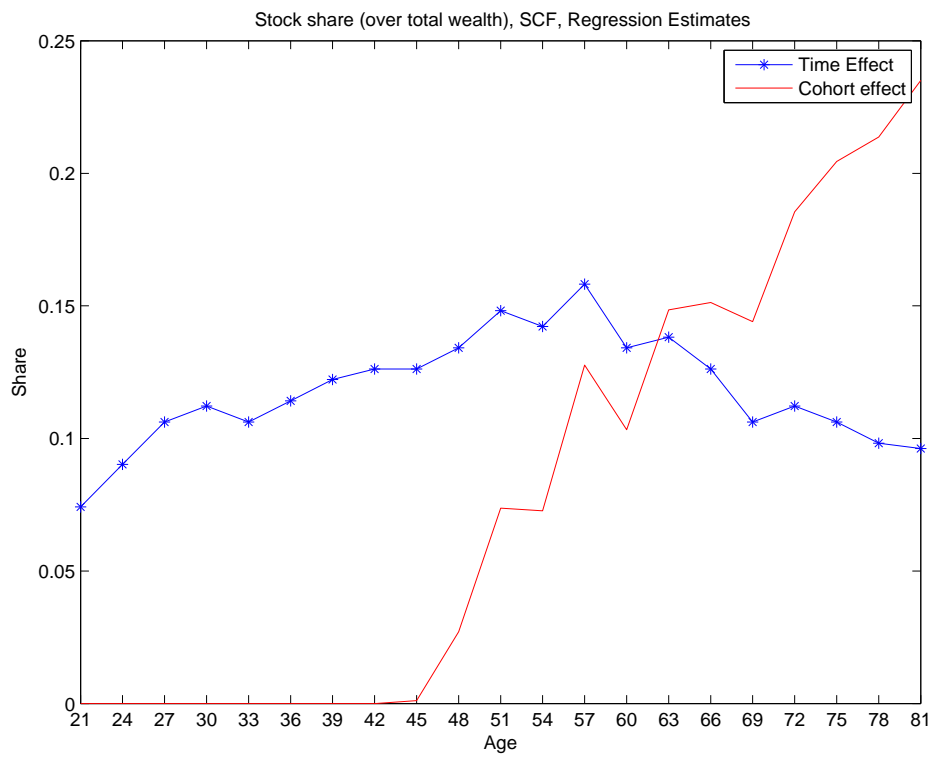


Figure 2.1: Equity over total wealth over the Life Cycle (SCF: 1992-2004)

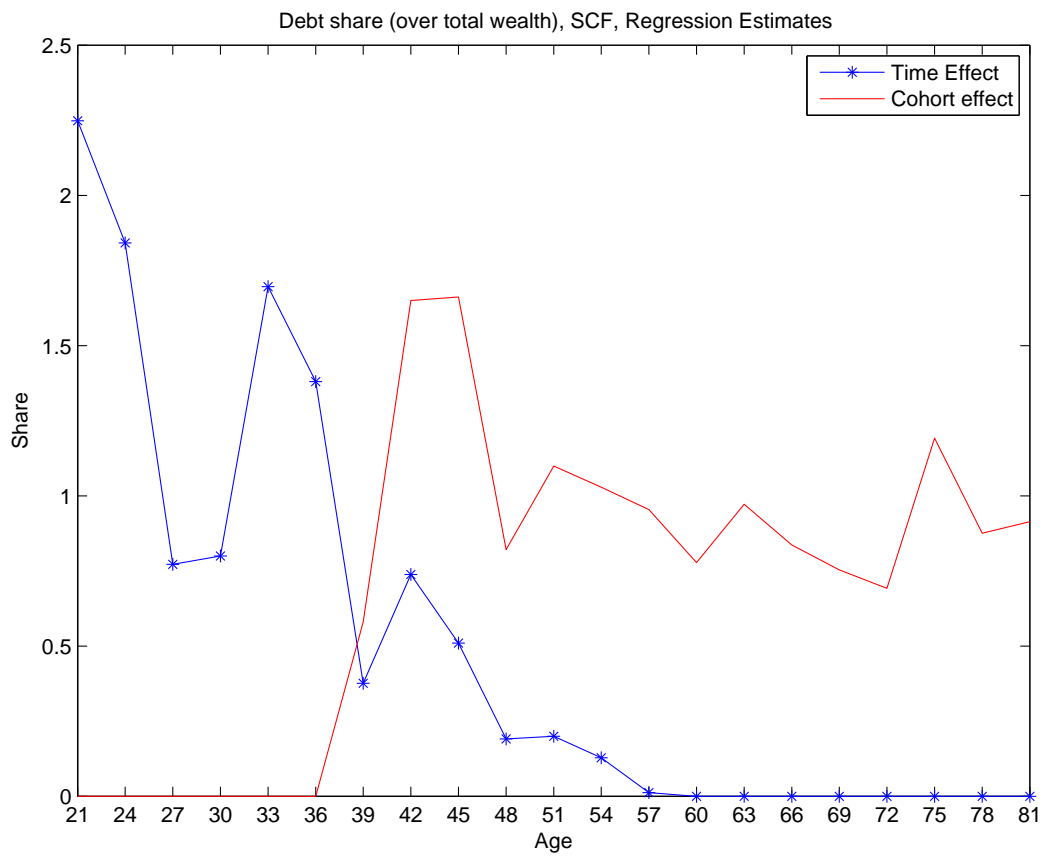


Figure 2.2: Debt over total wealth over the Life Cycle (SCF: 1992-2004)

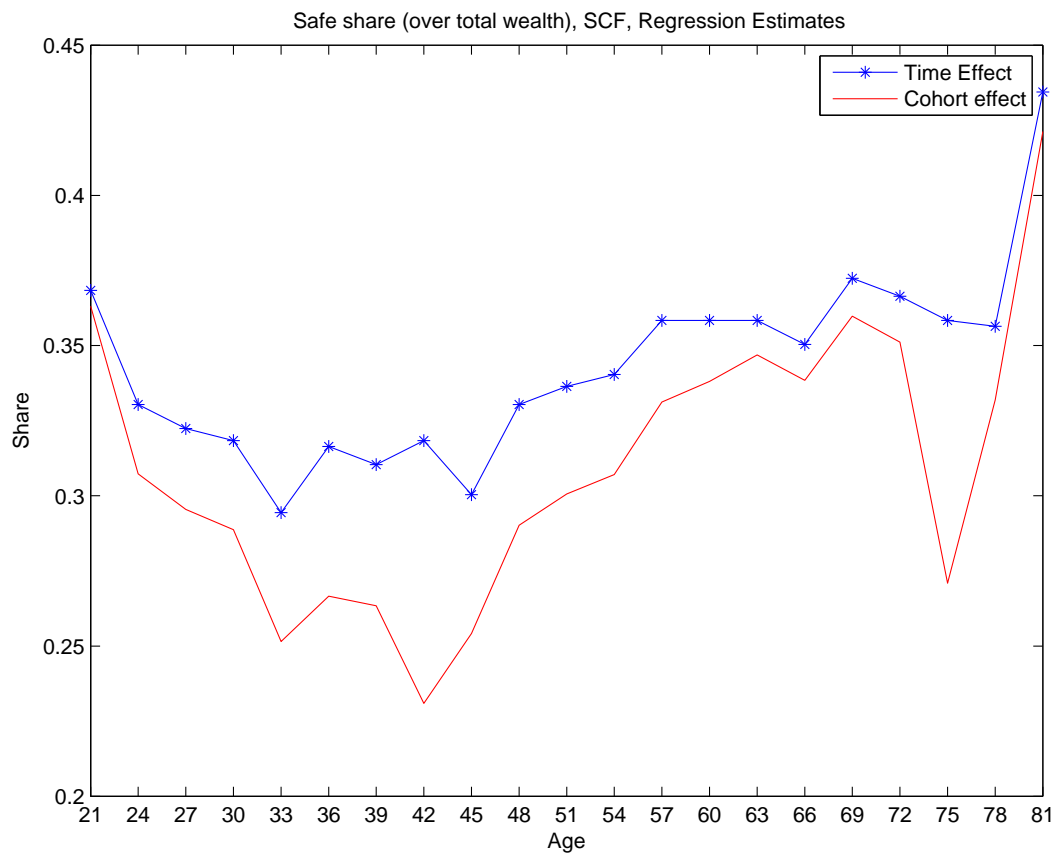


Figure 2.3: Equity over total wealth over the Life Cycle (SCF: 1992-2004)

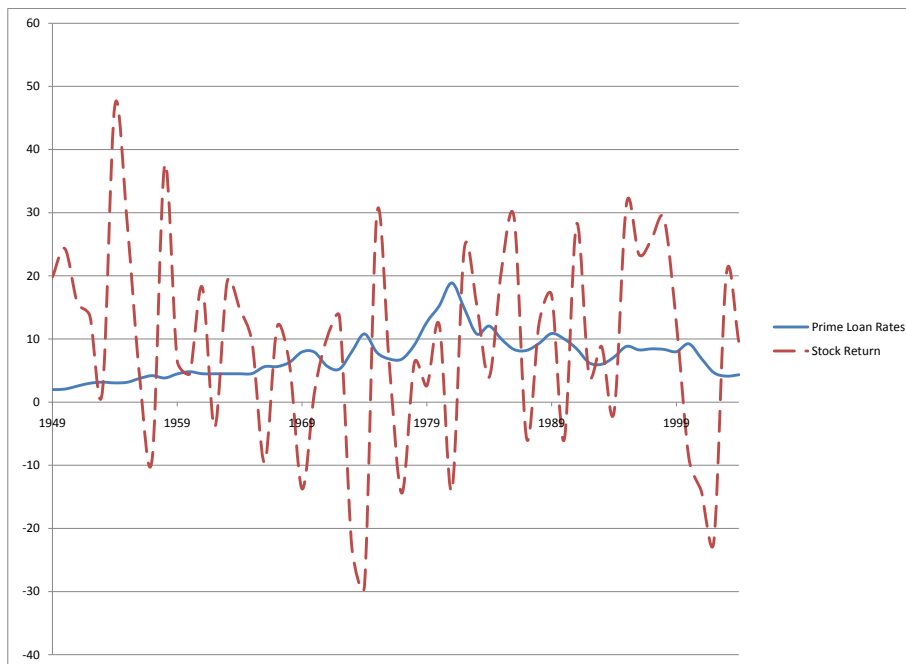
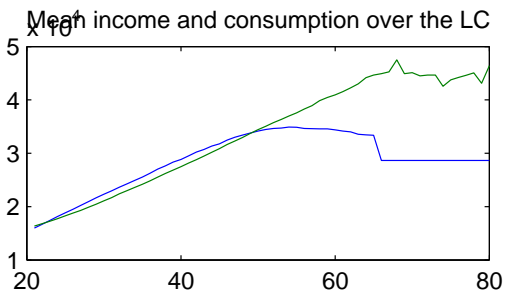
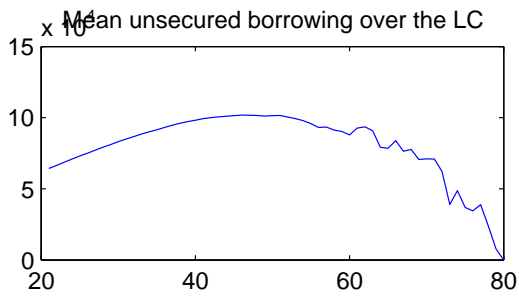
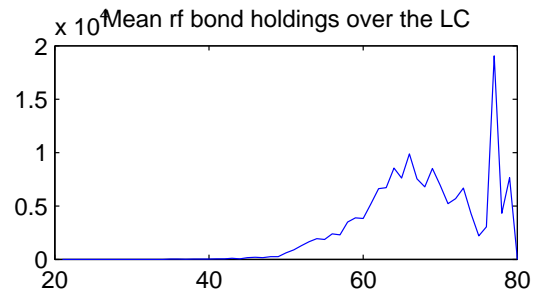
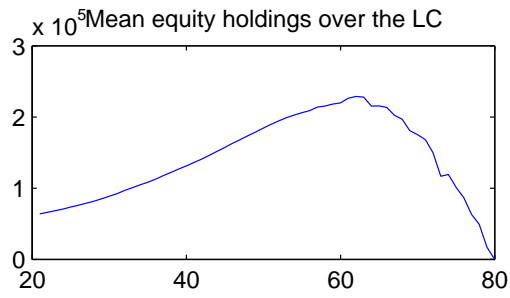
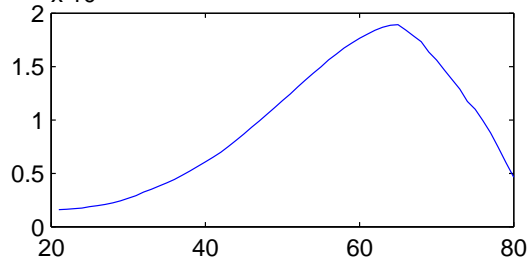


Figure 2.4: Co-movement of annual bank prime loan rate and stock return (1949-2006)



Mean beginning of period Cash-on-hand holdings over the LC



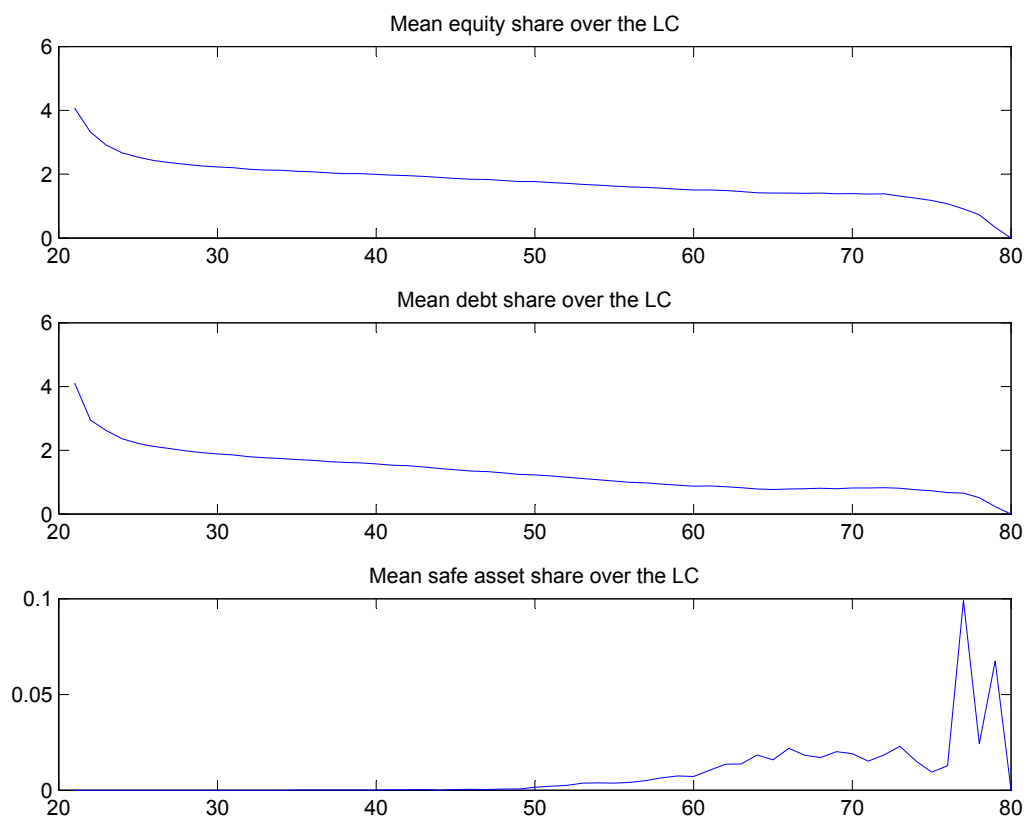


Figure 2.6: Mean Asset shares, CRRA= 2

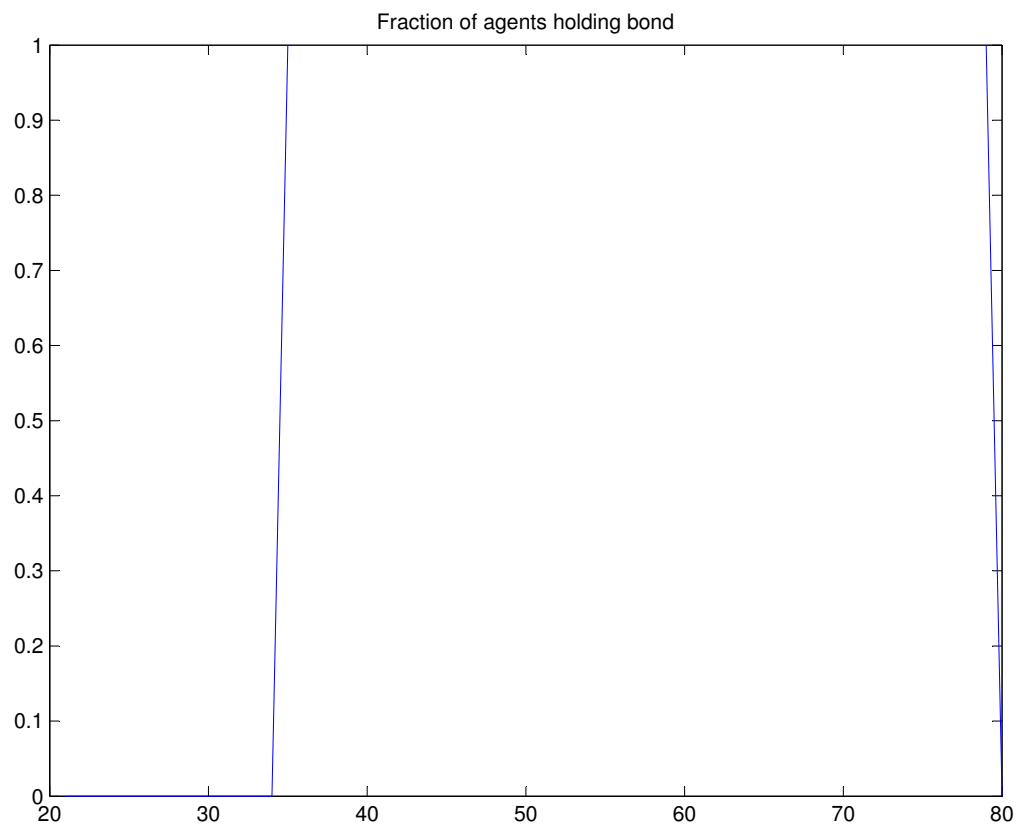


Figure 2.7: Safe Asset Participation, CRRA= 2

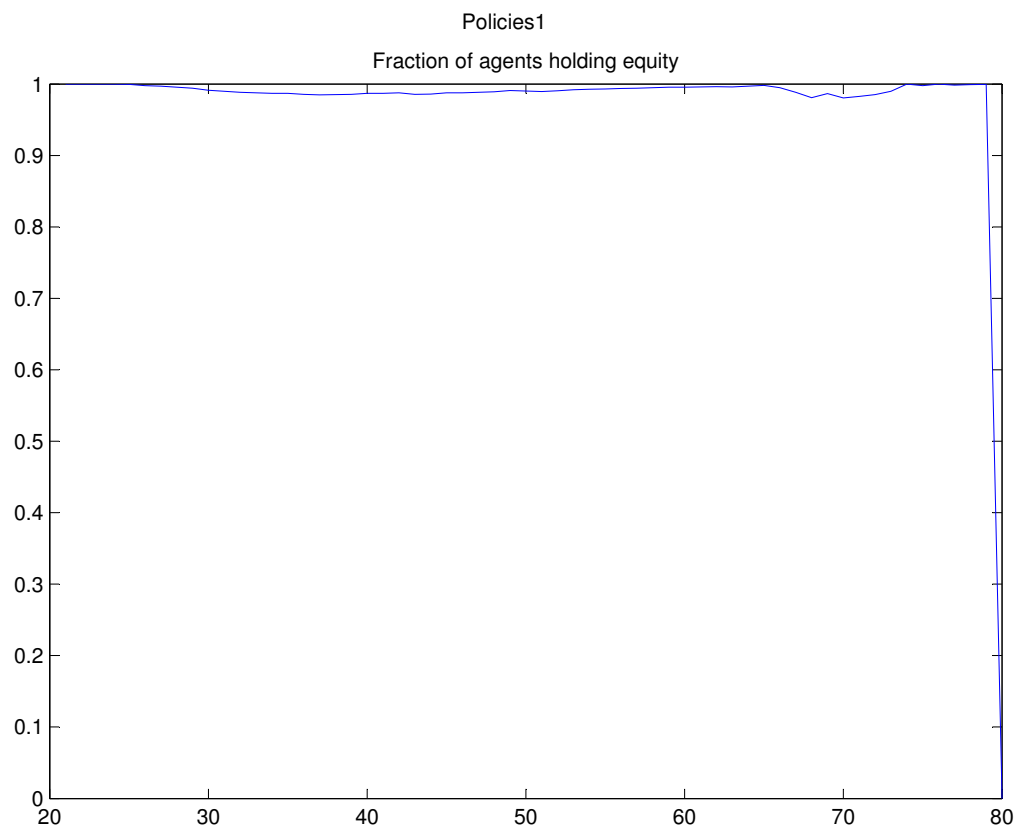


Figure 2.8: Risky Asset Participation, CRRA= 2

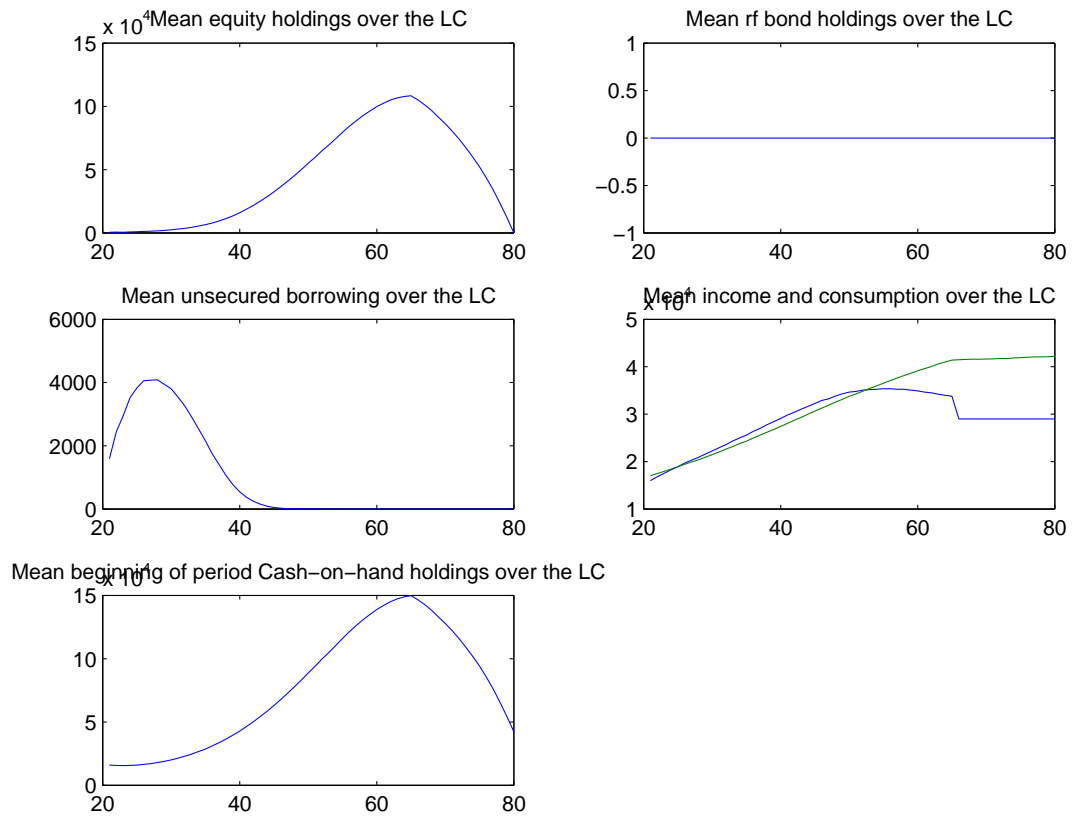


Figure 2.9: Mean Asset Holdings, Mean Income-Consumption and Asset, CRRA= 2, without correlation

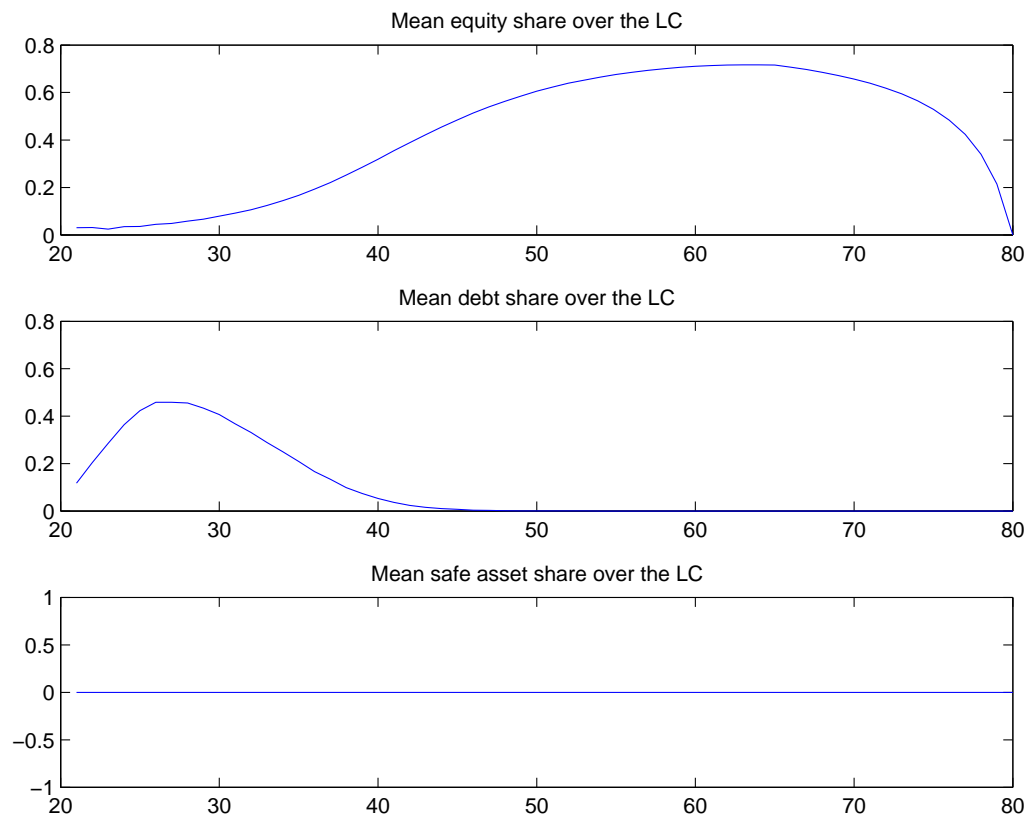


Figure 2.10: Mean Asset shares, CRRA= 2, without correlation

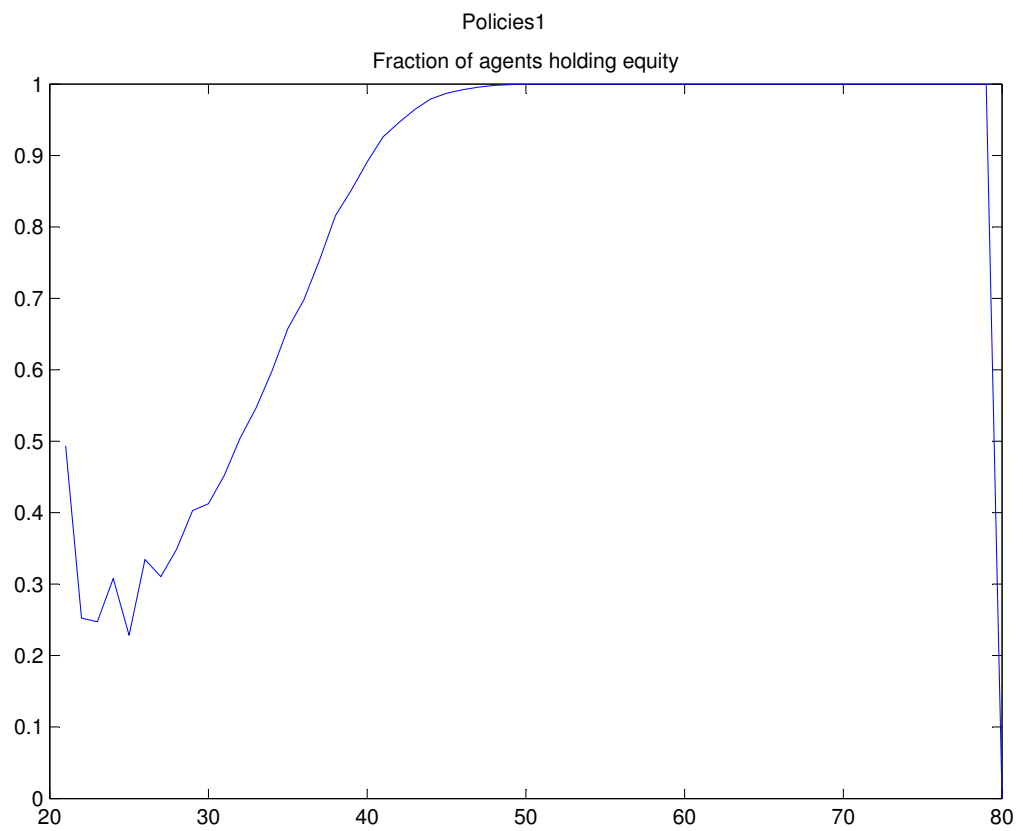


Figure 2.11: Risky Asset Participation, CRRA= 2, without correlation

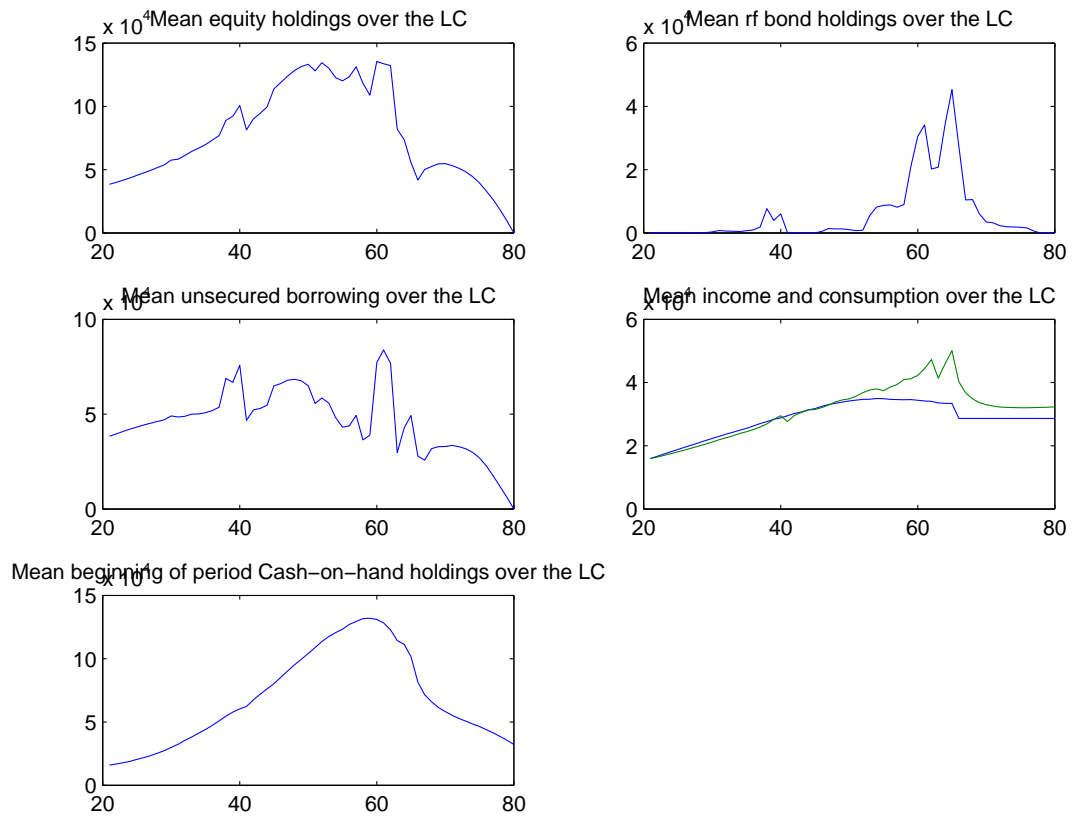


Figure 2.12: Mean Asset Holdings, Mean Income-Consumption and Asset, CRRA= 3

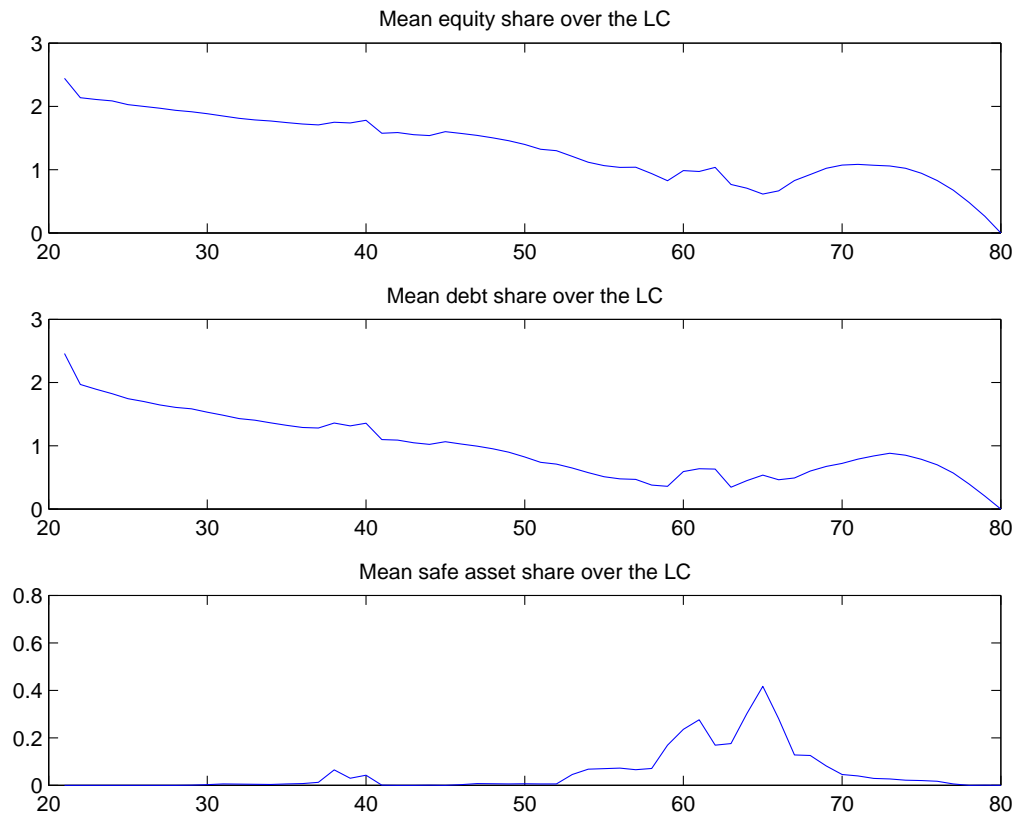


Figure 2.13: Mean Asset shares, CRRA= 3

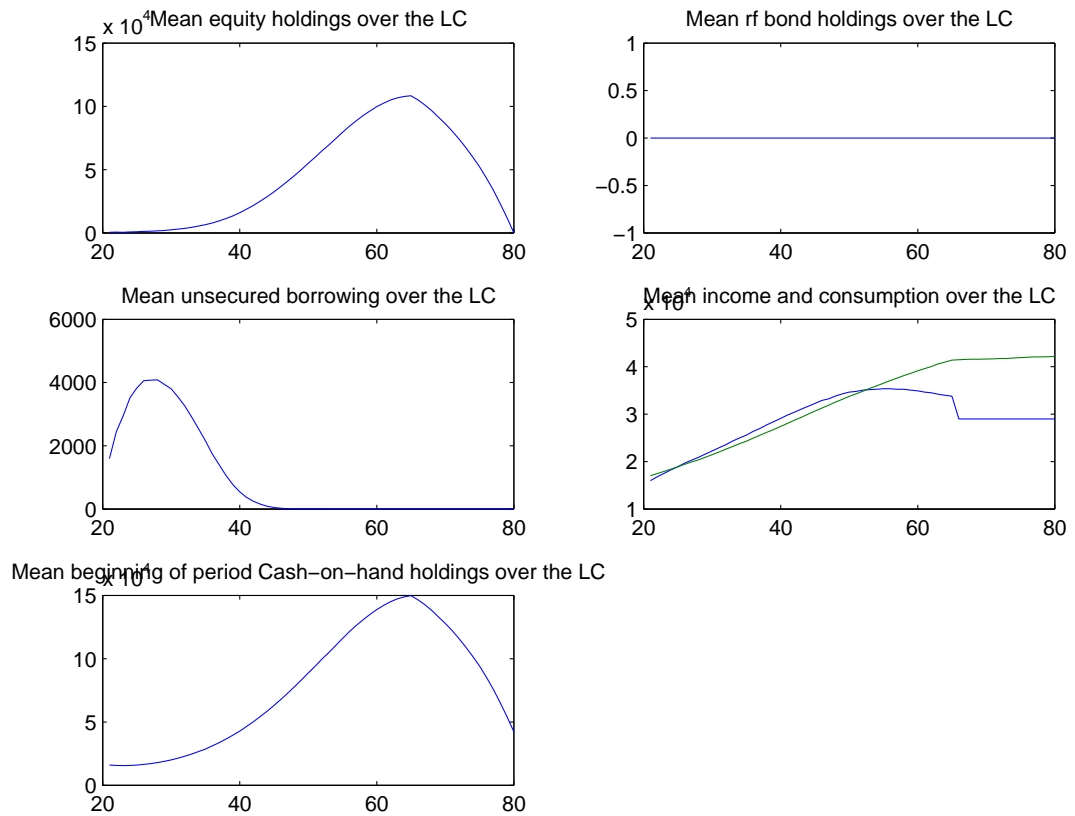


Figure 2.14: Mean Asset Holdings, Mean Income-Consumption and Asset, CRRA= 3, without correlation

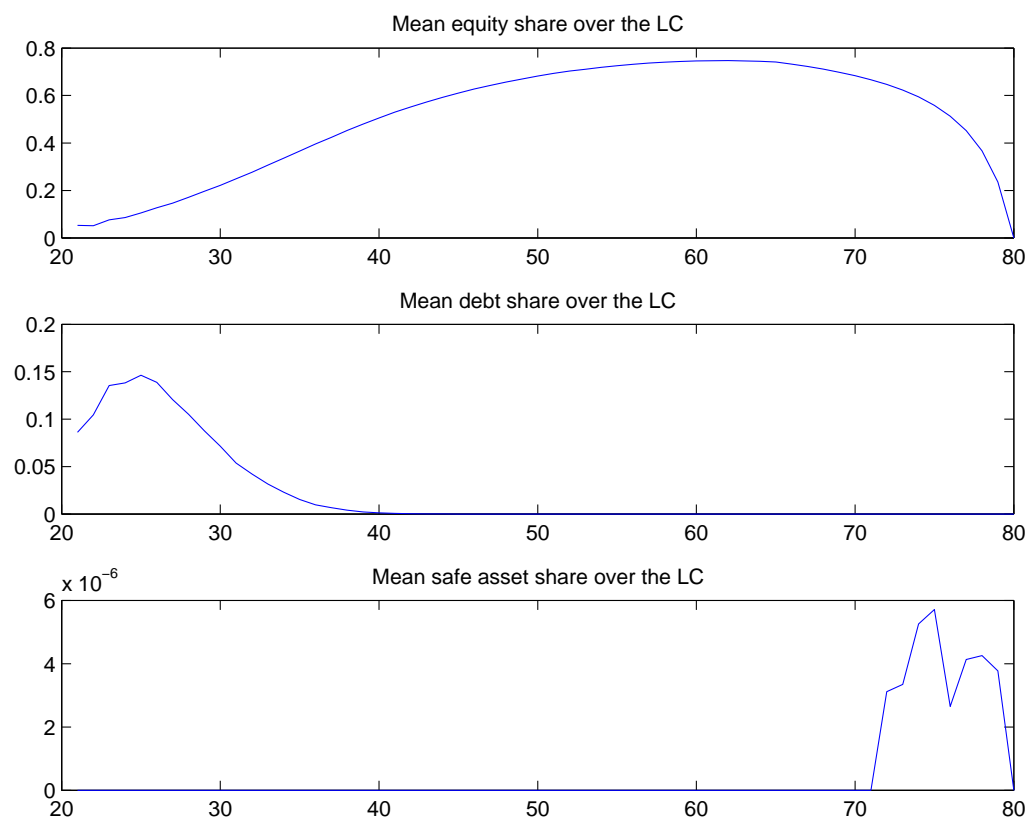


Figure 2.15: Mean Asset shares, CRRA= 3, without correlation

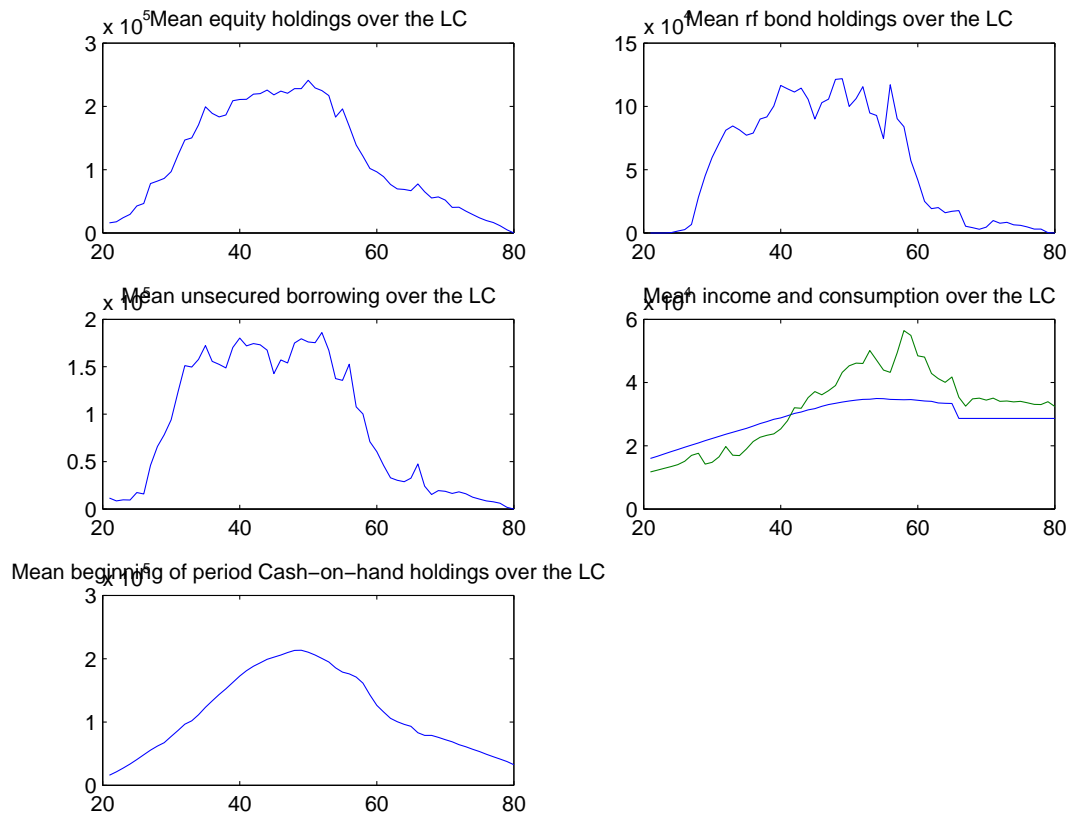


Figure 2.16: Mean Asset Holdings, Mean Income-Consumption and Asset, CRRA= 6

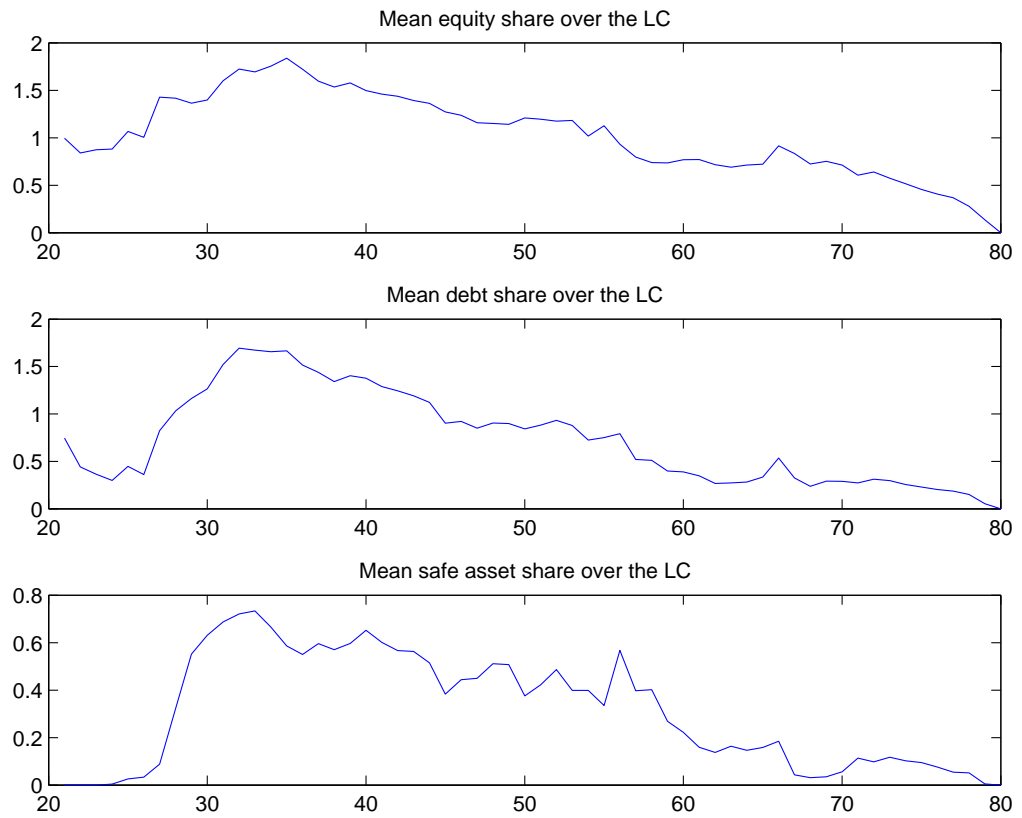


Figure 2.17: Mean Asset shares, CRRA= 6