Constraints on the deviations from general relativity

From local to cosmological scales

Jean-Philippe UZAN

GR in a nutshell

Underlying hypothesis

Equivalence principle

- Universality of free fall
- Local lorentz invariance
- Local position invariance

$$S_{matter}(\psi, g_{\mu\nu})$$

Dynamics

$$S_{grav} = \frac{c^3}{16\pi G}\int\sqrt{-g_*}\,R_*\,d^4x$$

Relativity

$$\boxed{g_{\mu\nu} = g^*_{\mu\nu}}$$

Equivalence principle and test particles

Action of a test mass:

$$S = -\int mc\sqrt{-g_{\mu\nu}v^\mu v^\nu}\,dt$$

with $v^\mu = dx^\mu/dt$, $u^\mu = dx^\mu/d\tau$

$$\delta S = 0$$

$$a^\mu \equiv u^\nu \nabla_\nu u^\mu = 0$$

(geodesic)

$$g_{00} = -1 + 2\Phi_N/c^2$$

(Newtonian limit)

$$\dot{\mathbf{v}} = \mathbf{a} = -\nabla\Phi_N = \mathbf{g}_N$$

Solar system

Metric theories are usually tested in the PPN formalism

$$\mathrm{d}s^2 = (-1 + 2U + 2(\beta-\gamma)U^2)\mathrm{d}t^2 + (1+2\gamma U)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

Light deflection

$$\Delta\theta = 2(1+\gamma)\frac{GM}{bc^2}$$

Perihelion shift of Mercury

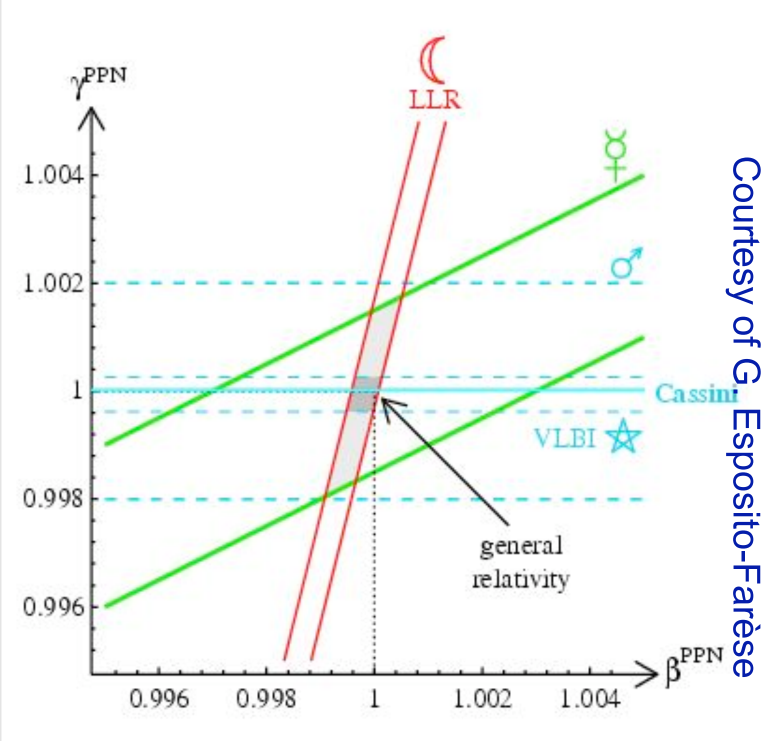
$$\Delta\varphi = \frac{2\pi GM}{a(1-e^2)}(2+2\gamma-\beta)$$

Nordtvedt effect

$$\delta r \sim 13.1(4\beta-\gamma-3)\cos(\omega_0-\omega_s)t \quad (\mathrm{m})$$

Shapiro time delay

$$\delta t \propto (1+\gamma)$$



Courtesy of G. Esposito-Farèse

$$\gamma - 1 = (2.1 \pm 2.3)\times 10^{-5}$$

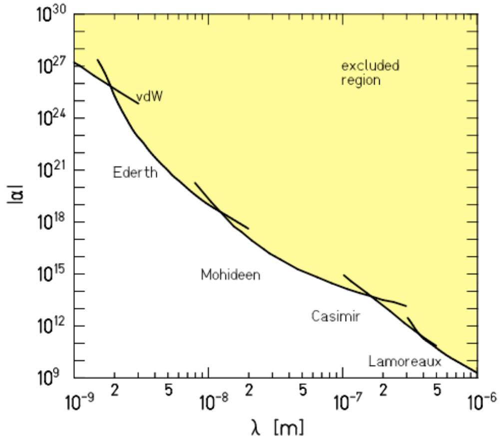
$$|2\gamma-\beta-1| < 3\times 10^{-3}$$

[Will, Liv. Rev. Relat. 2006-3]

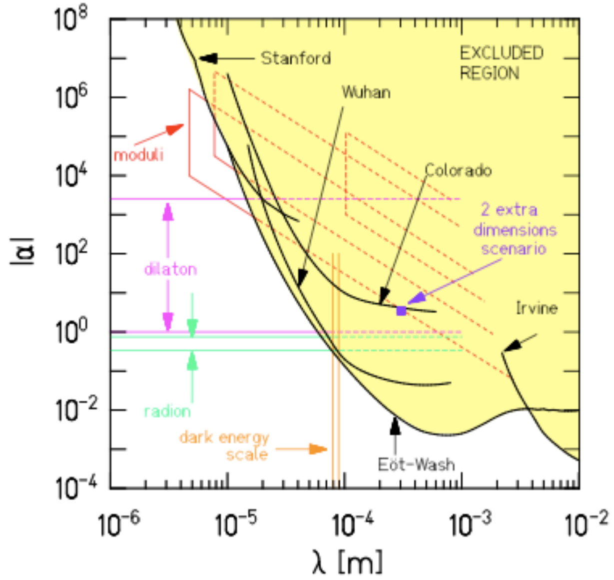
Fifth force

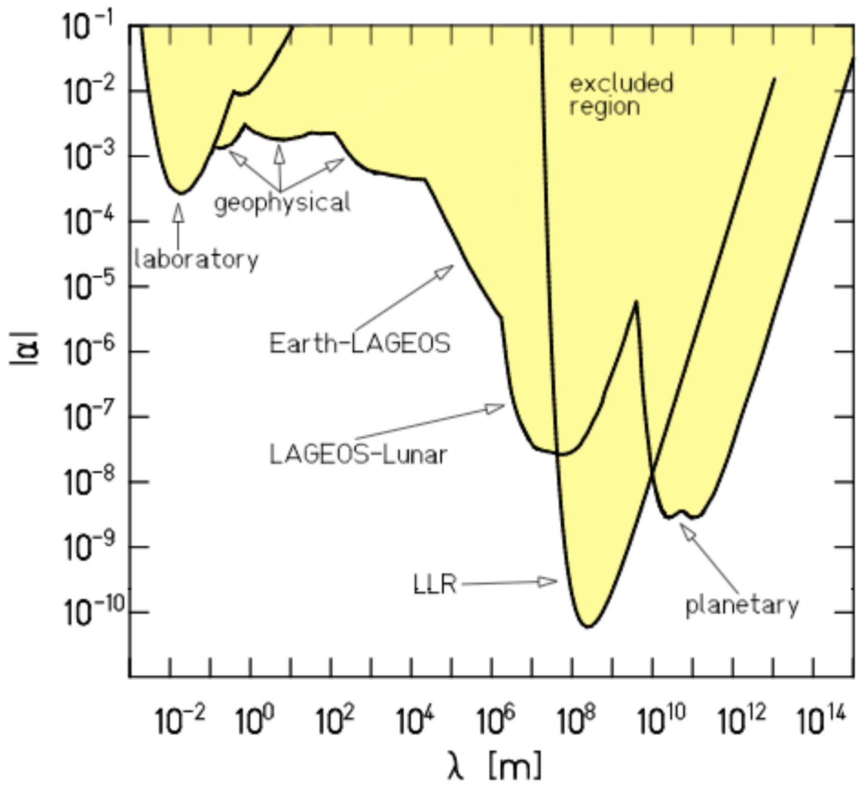
The PPN formalism cannot be applied if the modification of General relativity has a range smaller than the Solar system scale.

Fifth force experiments



Adelberger et al., Ann. Rev. Nucl. Part. Sci., 53 77 (2003)
Adelberger et al., Prog. Part. Nucl. Phys 62, 102 (2009)





Parameter space

Tests of general relativity on astrophysical scales are needed

- *galaxy rotation curves: low acceleration*
- *acceleration: low curvature*

Solar system:

$$\frac{R}{\phi^3} = \frac{c^4}{G^2 M_\odot^2}$$

Cosmology:

$$R = 3H_0^2\{\Omega_m(1+z)^3 + 4\Omega_\Lambda\}$$

Dark energy:

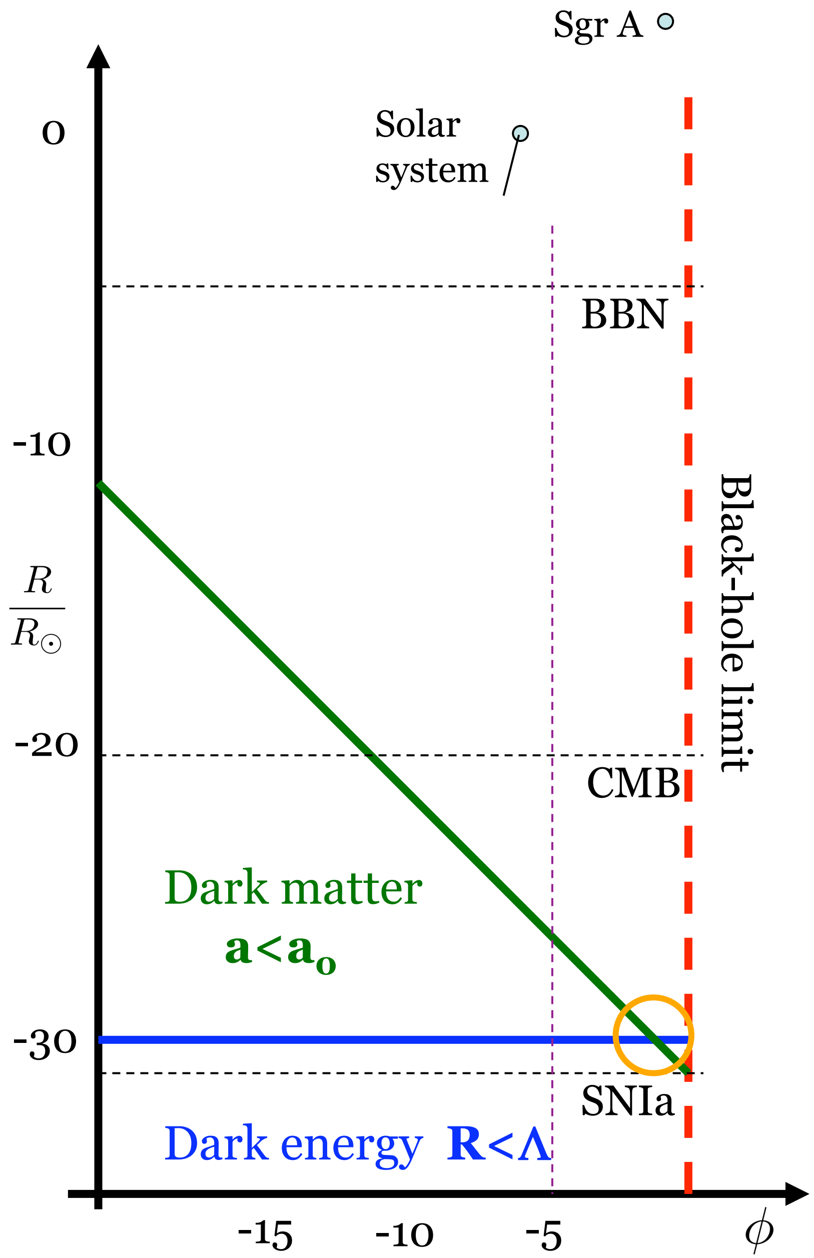
$$R < R_\Lambda = 12H_0^2\Omega_\Lambda$$

Dark matter:

$$a < a_0 \sim 10^{-8}\mathrm{cm.s}^{-2}$$

$$a^2 = \phi R < a_0^2$$

[Psaltis, 0806.1531]



Some theoretical insights
in
modifying general relativity

Modifying GR

The number of modifications are numerous.

I restrict to field theory.

We can require the following constraints:

- Well defined **mathematically**

  full Hamiltonian should be bounded by below

  - no ghost ($E_{kinetic} > 0$)

  - no tachyon ($m^2 > 0$)

  Cauchy problem well-posed

- In agreement with existing **experimental** data

  Solar system & binary pulsar tests

  Lensing by « dark matter » - rotation curve

  Large scale structure – CMB – BBN - …

- Not pure fit of the data!

Example: higher-order gravity…

At quadratic order

$$S_g = \frac{c^3}{16\pi G}\int \left(R + \alpha C^2_{\mu\nu\rho\sigma} + \beta R^2 + \gamma GB\right)\sqrt{-g}\, d^4x$$

- $GB = R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$ does not contribute to the field eqs.
- $\alpha C^2_{\mu\nu\rho\sigma}$ theory contains a ghost [Stelle, PRD**16** (1977) 953]

$$\frac{1}{p^2+\alpha p^4} = \underbrace{\frac{1}{p^2}}_{\text{massless graviton}} - \underbrace{\frac{1}{p^2+\alpha^{-1}}}_{\text{massive degrees of freedom with } m^2=1/\alpha}$$

massless graviton

massive degrees of freedom with $m^2=1/\alpha$
carries negative energy
$\alpha<0$: it is also a tachyon.

- βR^2 equivalent to positive energy massive scalar d.o.f

...and beyond

These considerations can be extended to $f(R,R_{\mu\nu},R_{\mu\nu\alpha\beta})$

[Hindawi et al., PRD**53** (1996) 5597]

Generically contains massive spin-2 ghosts but for f(R)

These models involve higher-order terms of the variables.

the Hamiltonian is then generically non-bounded by below

[Ostrogradsky, 1850]
[Woodard, 0601672]

Argument does not apply for an infinite number of derivative
non-local theories may avoid these arguments

Only allowed models of this class are *f(R), i.e* scalar-tensor theories.

Scalar-tensor theories

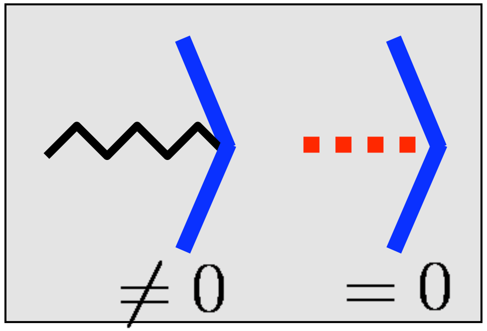
spin 2

spin 0

$$S = \frac{c^3}{16\pi G}\int\sqrt{-g}\,\{R - 2(\partial_\mu\phi)^2 - V(\phi)\}$$
$$+ S_m\{\text{matter}, \tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}\}$$

Maxwell electromagnetism is conformally invariant in d=4

$$S_{em} = \frac{1}{4}\int\sqrt{-\tilde{g}}\,\tilde{g}^{ab}\tilde{g}^{cd}F_{ac}F_{bd}\mathrm{d}^d x$$
$$= \frac{1}{4}\int\sqrt{-g}\,g^{ab}g^{cd}F_{ac}F_{bd}A^{d-4}(\phi)\mathrm{d}^d x$$



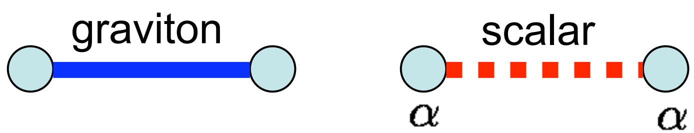
Light deflection is given as in GR

$$\delta\theta = \frac{4GM}{bc^2}$$

What is the difference?

The difference with GR comes from the fact that massive matter feels the scalar field

$$G_{\mathrm{cav}} = G(1+\alpha^2)$$



$$\alpha = \mathrm{d}\ln A/\mathrm{d}\phi$$

Motion of massive bodies determines $G_{\mathrm{cav}}M$ **not** GM.

Thus, in terms of observable quantities, light deflection is given by

$$\delta\theta = \frac{4G_{\mathrm{N}}M}{(1+\alpha^2)bc^2} \leq \frac{4GM}{bc^2}$$

which means

$$M_{\mathrm{lens}} \leq M_{\mathrm{rot}}$$

Cosmological features of ST theories

Close to GR today

assume light scalar field

Can be attracted toward GR during the cosmological evolution.

[Damour, Nordtvedt]

Dilaton can also be a quintessence field

[JPU, PRD 1999]

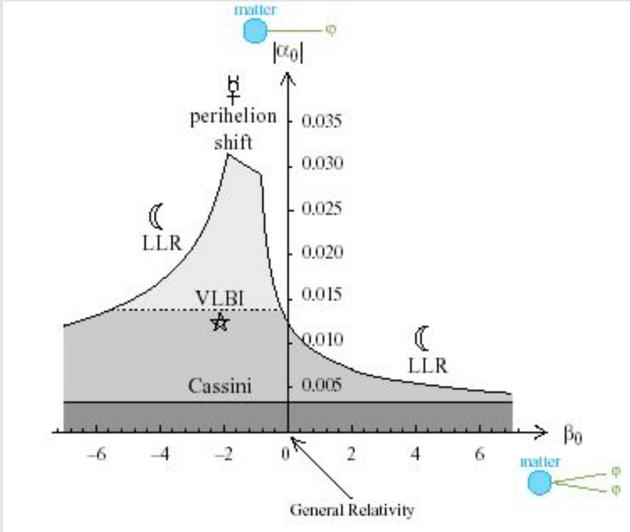
Equation of state today

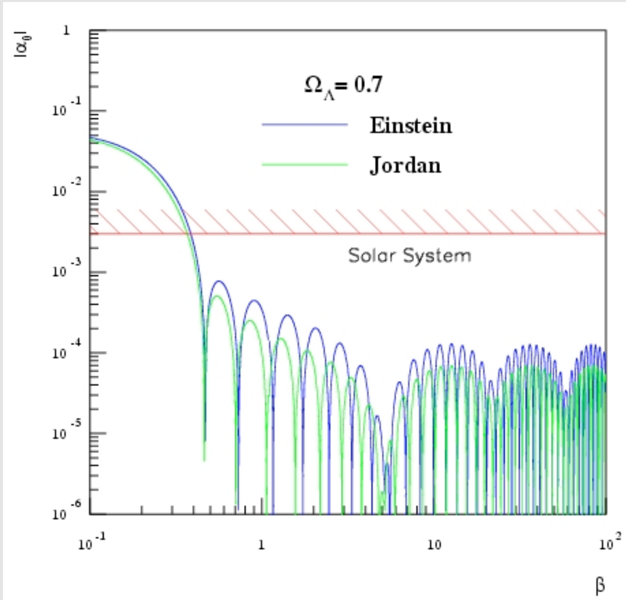
$$3\Omega_{\mathrm{de}0}(w_0+1) \simeq 2(1-\beta_0)\phi_0'^2 - 2\alpha_0\phi_0''$$

[Martin, Schimd, JPU, 0510208]

Cosmological predictions computable
(BBN, CMB, WL,...)

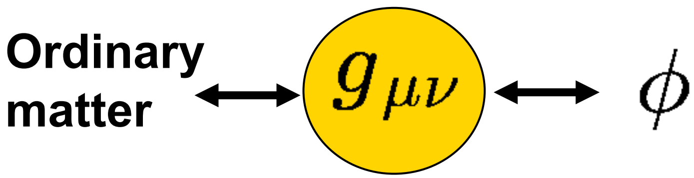
[Schimd et al., 2005; Riazuelo JPU, 2000, Coc et al., 2005]





[Coc et al, 0601299]

Universality classes of extensions

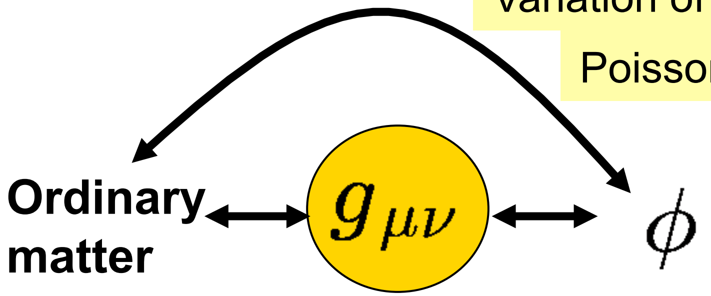


Ex : quintessence, ...

$S_{\rm de}[{\rm de}; g_{\mu\nu}]$

Variation of constants

Poisson equation

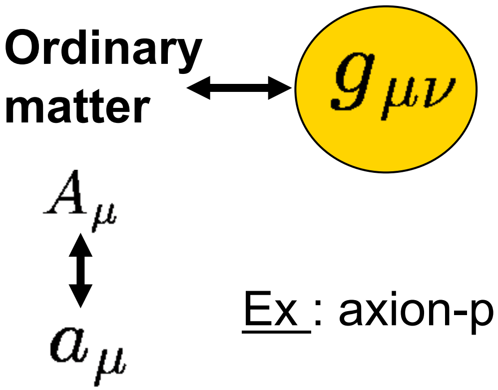


Ex : scalar-tensor, TeVeS ....

$S_\varphi[\varphi; g_{\mu\nu}]$

$S_m[{\rm mat}; g_{\mu\nu}] \rightarrow S_m[{\rm mat}_i; A_i^2(\varphi) g_{\mu\nu}].$

$S_{\rm em}[A_\mu; g_{\mu\nu}] \rightarrow S_{\rm em}[A_\mu, a_\mu; g_{\mu\nu}].$

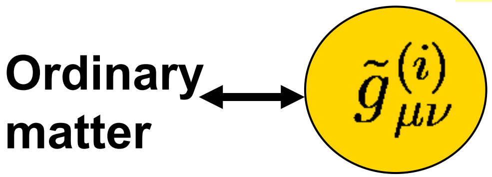


Ex : axion-photon mixing

Distance duality

Variation of constants

Poisson equation



Ex : brane induced gravity
multigravity,...

Always need NEW fields

[JPU, Aghanim, Mellier, PRD 05]
[JPU, GRG 2007]

Equivalence principle and constants

Action of a test mass:

$$S = -\int m_A[\alpha_i] c \sqrt{-g_{\mu\nu} v^\mu v^\nu}\, dt \quad \text{with} \quad \begin{aligned} v^\mu &= dx^\mu/dt \\ u^\mu &= dx^\mu/d\tau \end{aligned}$$

Dependence on some constants

$$\delta S = 0$$

$$a_A^\mu = -\sum_i \left(\underbrace{\boxed{\frac{\partial \ln m_A}{\partial \alpha_i}}}_{f_{A,i}} \frac{\partial \alpha_i}{\partial x^\beta} \right) \left(g^{\beta\mu} + u^\beta u^\mu \right)$$

(NOT a geodesic)

$$g_{00} = -1 + 2\Phi_N/c^2$$

(Newtonian limit)

$$\mathbf{a} = \mathbf{g}_N + \delta \mathbf{a}_A$$

$$\delta \mathbf{a}_A = -c^2 \sum_i f_{A,i} \left(\nabla \alpha_i + \dot{\alpha}_i \frac{\mathbf{v}}{c^2} \right)$$

Anomalous force
Composition dependent

Field theory

If a constant is varying, this implies that it has to be replaced by a dynamical field

This has 2 consequences:

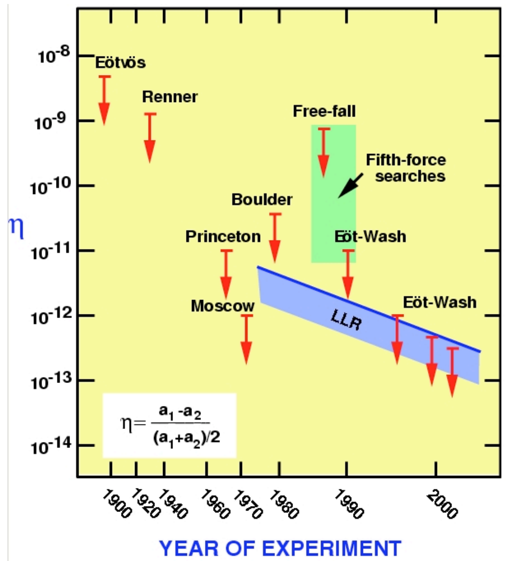
1- the equations derived with this parameter constant will be modified

one cannot just make it vary in the equations

2- the theory will provide an equation of evolution for this new parameter

The field responsible for the time variation of the « constant » is also responsible for a long-range (composition-dependent) interaction

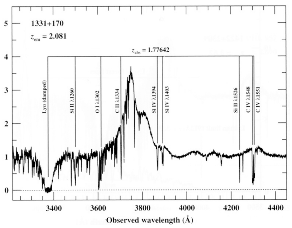
i.e. at the origin of the deviation from General Relativity.

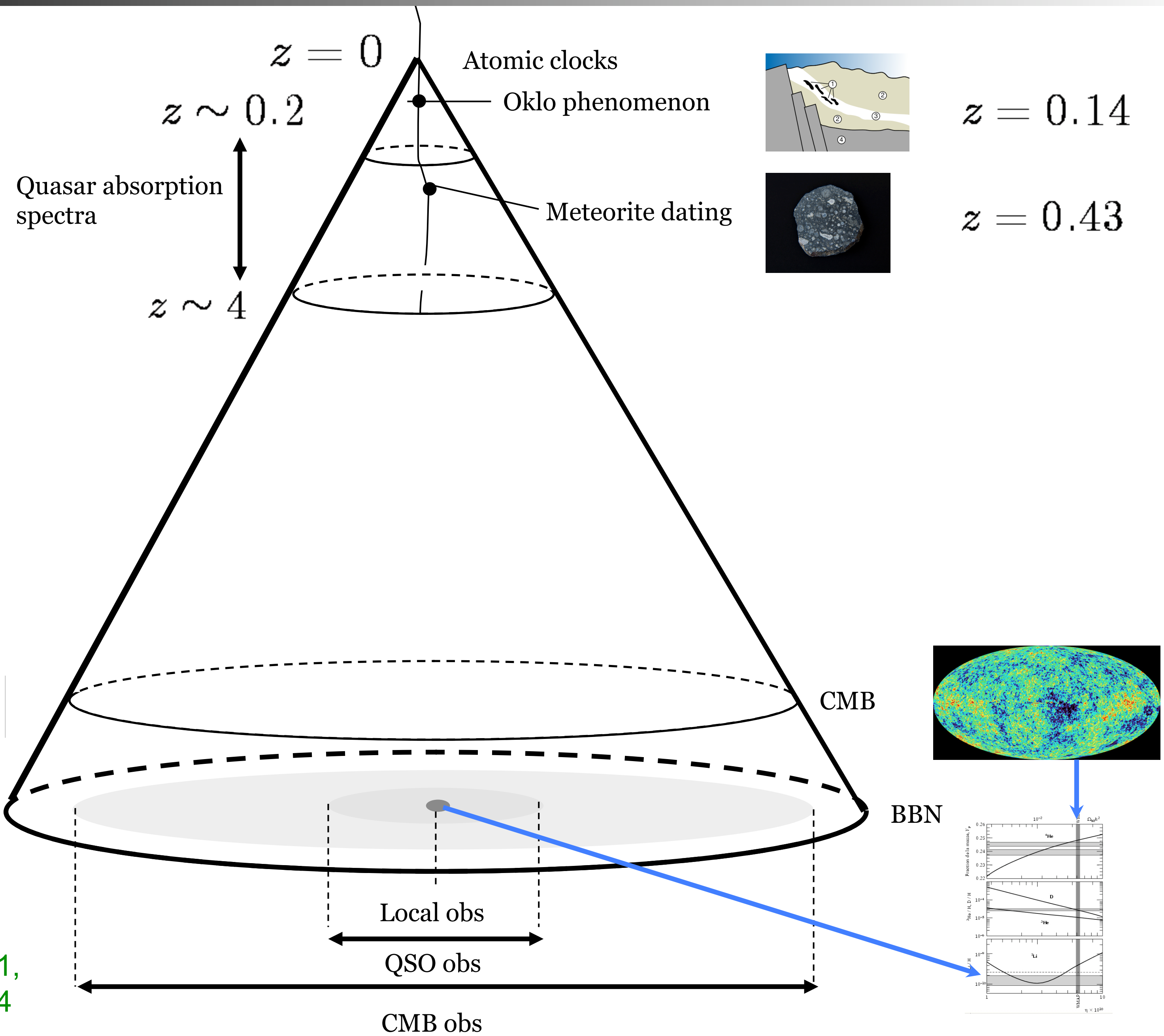


Testing general relativity on astrophysical scales

- Constants
- Field equations

Physical systems

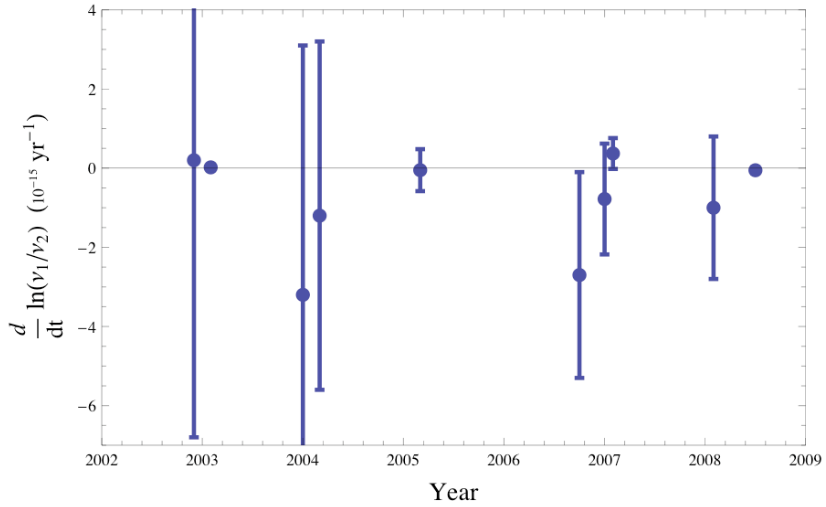




JPU, RMP (2003);
arXiv:0907.3081,
arXiv:1009.5514

Constraints

$$\frac{\mathrm{d}}{\mathrm{d}t}\ln\left(\frac{\nu_{\mathrm{Al}}}{\nu_{\mathrm{Hg}}}\right) = (-5.3 \pm 7.9) \times 10^{-17}\,\mathrm{yr}^{-1}$$

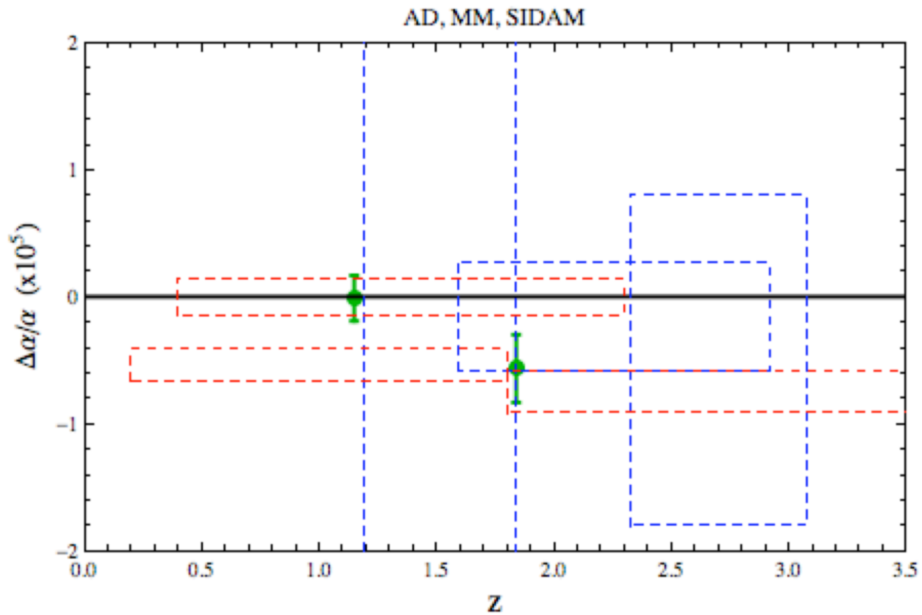


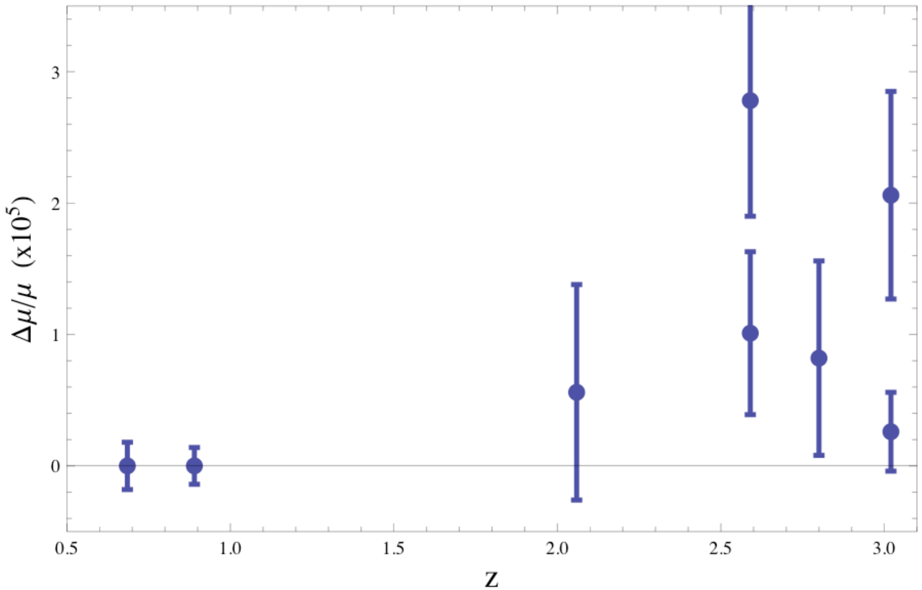
$$(\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}})_{\mathrm{VLT};\,z<1.8} = (-0.06 \pm 0.16) \times 10^{-5}$$

$$(\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}})_{\mathrm{VLT}\;z>1.8} = (+0.61 \pm 0.20) \times 10^{-5}$$

$$(\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}})_{\mathrm{Keck};\,z<1.8} = (-0.54 \pm 0.12) \times 10^{-5}$$

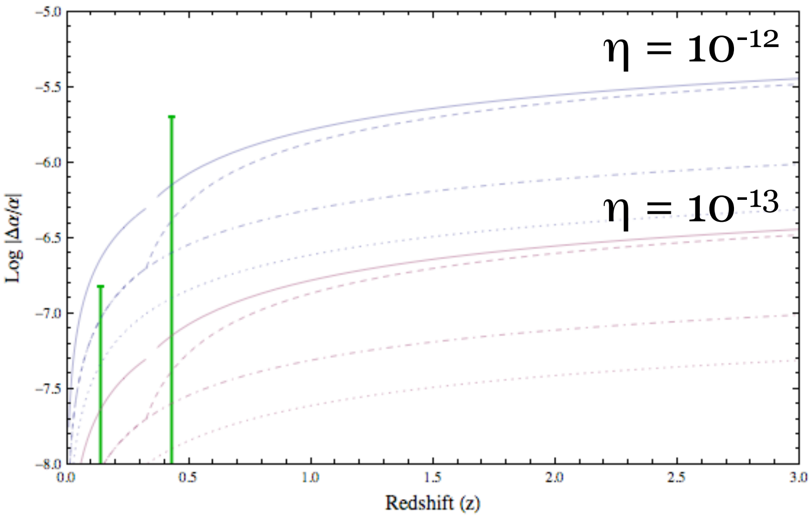
$$(\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}})_{\mathrm{Keck};\,z>1.8} = (-0.74 \pm 0.17) \times 10^{-5}$$





Comparison of constraints

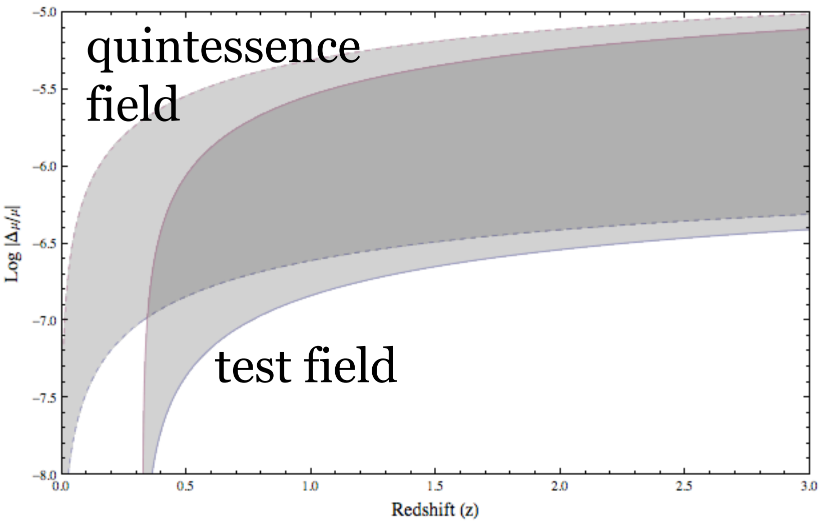
Model-dependent: test-field vs quintessence field / coupling to all matter fields (GUT?)



Example: string dilaton model

Models in agreement with clock constraints

Dilaton field can be either :
- test (not origin of late time accelleration)
- Quintessence field



All constants vary

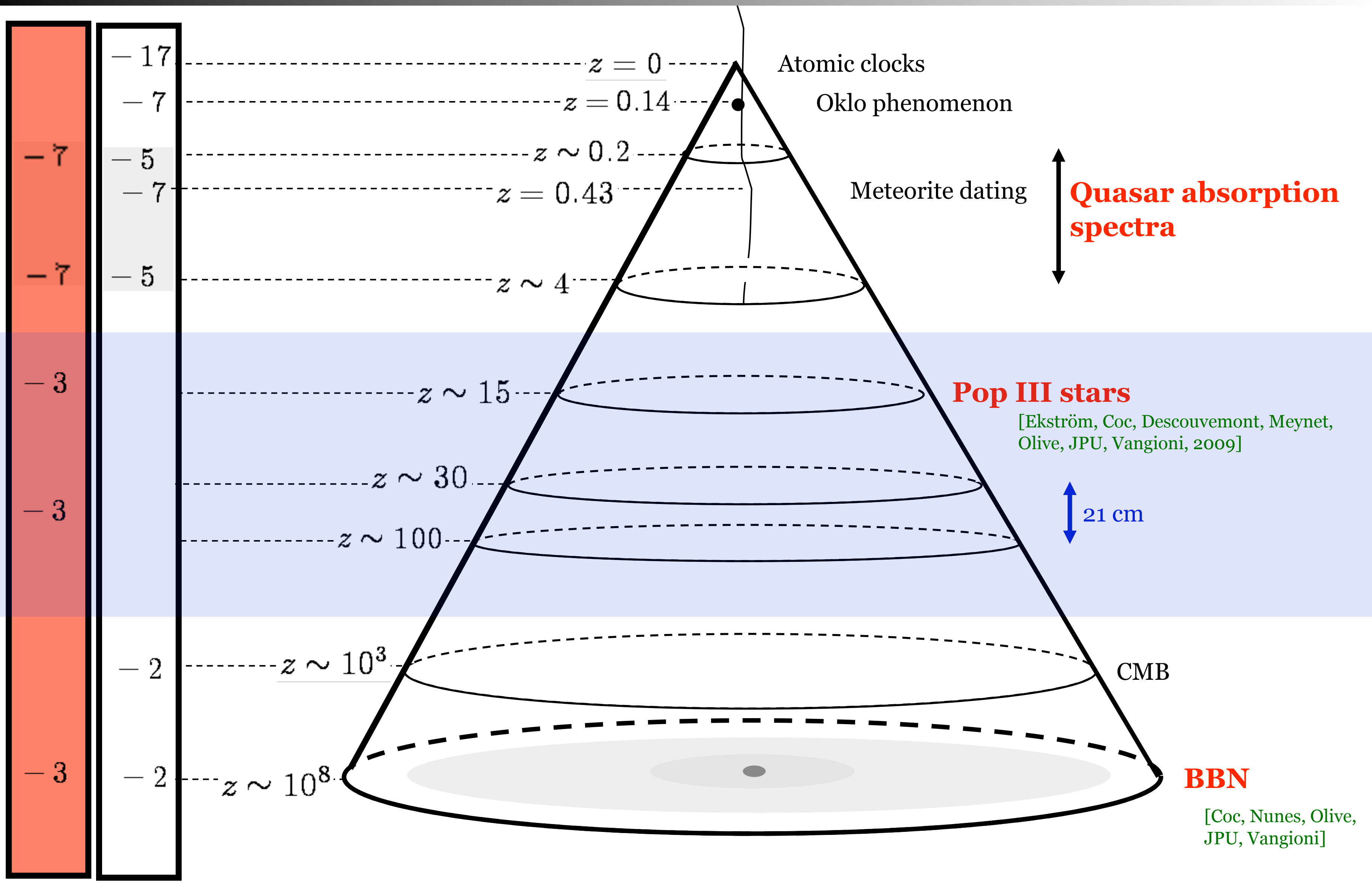
R= 10-200

$\eta = 10^{-13}$

Most sensitive probe depends on the model:
UFF (z=0) / clocks (z=0) / astrophysical

Importance to use different constants

Physical systems: new and future



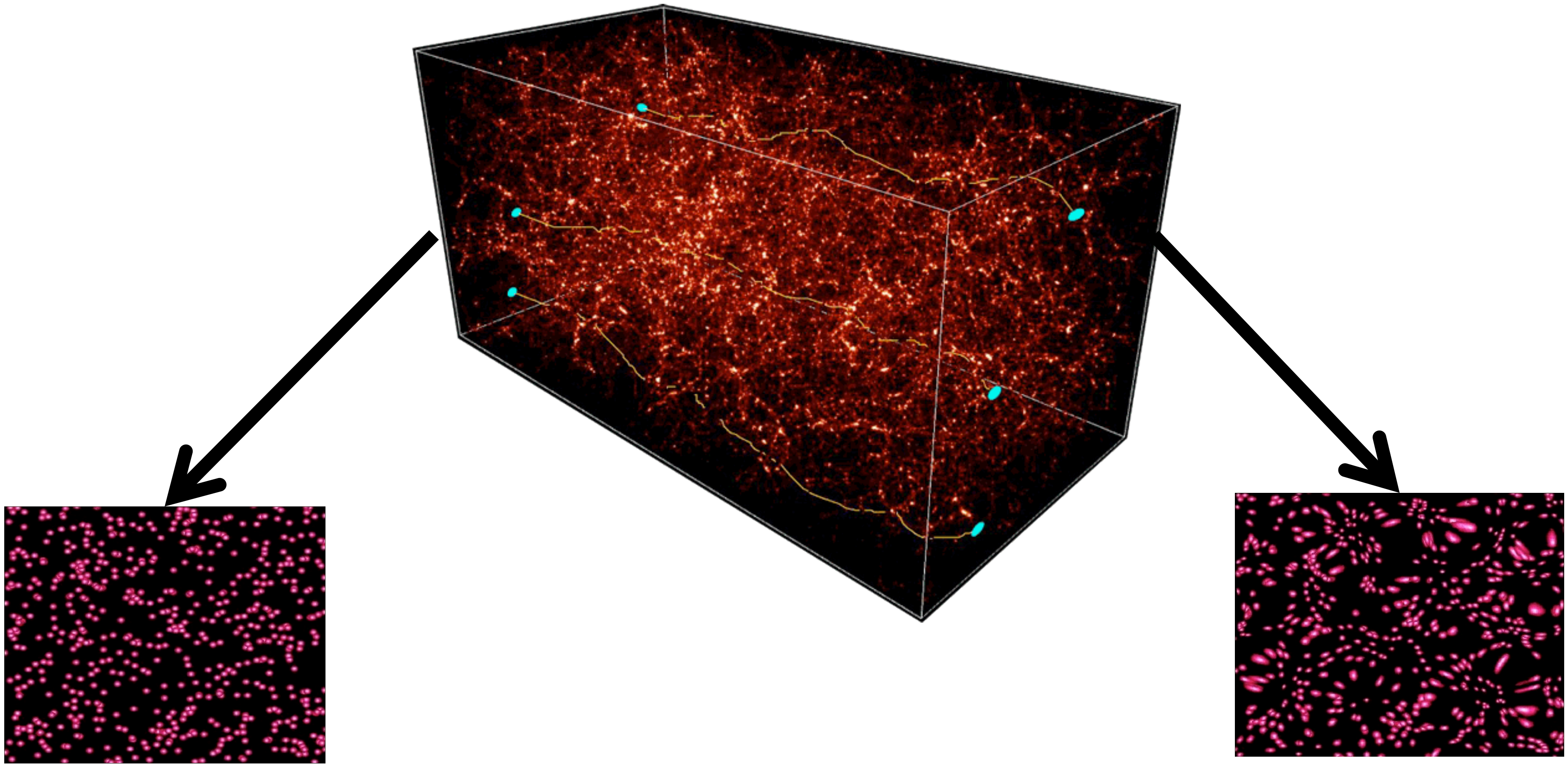
JPU, Liv. Rev. Relat., arXiv:1009.5514

Testing GR on large scales

One needs at least **TWO** independant observables

Large scale structure

[Uzan, Bernardeau (2001)]



Matter distribution

Weak lensing

Structure in ΛCDM

Restricting to low-*z* and sub-Hubble regime

$$\mathrm{d}s^2 = a^2(\eta)[-(1+2\Phi)\mathrm{d}\eta^2 + (1-2\Psi)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j]$$

Background

$$H^2/H_0^2 = \Omega_m^0(1+z)^3 + (1-\Omega_m^0-\Omega_\Lambda^0)(1+z)^2 + \Omega_\Lambda^0$$

Sub-Hubble perturbations

$$\Phi = \Psi$$

$$\Delta\Psi = 4\pi G\rho a^2\delta$$

$$\delta' + \theta = 0$$

$$\theta' + \mathcal{H}\theta = -\Delta\Phi$$

$$\theta \propto -f\delta$$

$$f \propto \Omega_{\mathrm{mat}}^{0.6}$$

This implies the existence of **rigidities** between different quantities

Second rigidity: original idea of 2001

On sub-Hubble scales, in weak field
(typical regime for the large scale structure)

$$\Delta\Phi = 4\pi G\rho a^2\delta$$

<u>Weak lensing</u>

$$\delta\theta = \frac{2}{c^2}\int\nabla_\perp\Phi\,\mathrm{d}\lambda$$

$$\langle\Phi(\theta)\Phi(\theta+n)\rangle$$

Distribution of the gravitational potential

<u>Galaxy catalogues</u>

$$n_{gal}(\mathbf{x})$$

$$\xi(r) = \langle\delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r})\rangle$$

Distribution of the matter

Compatible?

[JPU, Bernardeau (2001)]

Can we construct a post-ΛCDM formalism for the interpretation the large scale structure data?

Post-ΛCDM

Restricting to low-z and sub-Hubble regime

$$\mathrm{d}s^2 = a^2(\eta)[-(1+2\Phi)\mathrm{d}\eta^2 + (1-2\Psi)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j]$$

Background

$$H^2/H_0^2 = \Omega_m^0(1+z)^3 + (1-\Omega_m^0-\Omega_\Lambda^0)(1+z)^2 + \boxed{\Omega_{\mathrm{de}}(z)}$$

Sub-Hubble perturbations

$$\Delta(\Phi-\Psi) = \boxed{\pi_{\mathrm{de}}}$$

$$-k^2\Phi = 4\pi G_N \boxed{F(k,H)}\rho a^2\delta + \boxed{\Delta_{\mathrm{de}}}$$

$$\delta' + \theta = 0$$

$$\theta' + \mathcal{H}\theta = -\Delta\Phi + \boxed{S_{\mathrm{de}}}$$

ΛCDM

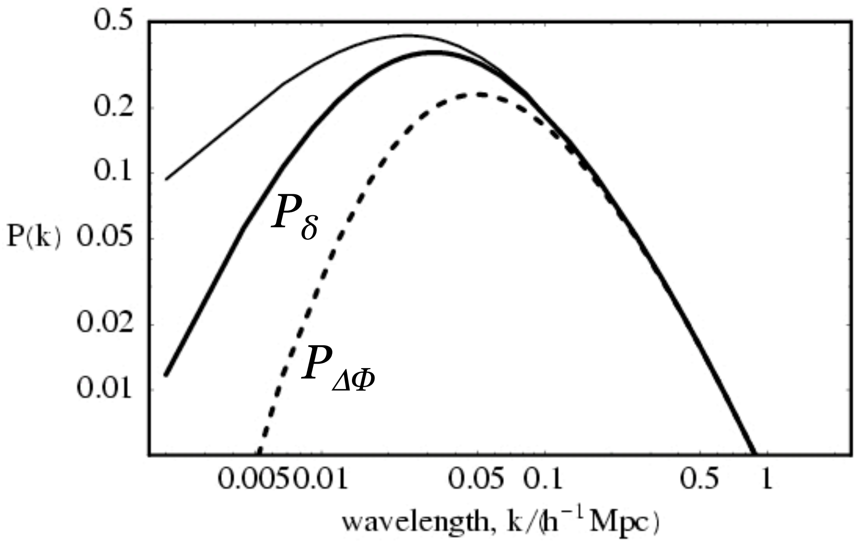
$$(F, \pi_{\mathrm{de}}, \Delta_{\mathrm{de}}, S_{\mathrm{de}}) = (1,0,0,0)$$

[JPU, astro-ph/0605313; arXiv:0908.2243]

Data and tests

DATA	OBSERVABLE
Weak lensing	$\kappa \propto \Delta(\Phi + \Psi)$
Galaxy map	$\delta_g = b\,\delta$
Velocity field	$\theta = \beta\,\delta$
Integrated Sachs-Wolfe	$\Theta_{SW} \propto \dot{\Phi} + \dot{\Psi}$

Various combinations of these variables have been considered



JPU and Bernardeau, Phys. Rev. D **64** (2001)

EUCLID: ESA-class M-phase A

Gravitational waves and bimetric

In models involving 2 metrics (scalar-tensor, TeVeS,...), gravitons and standard matter are coupled to different metrics.

In GR:

photons and gravitons are massless and follow geodesics of the same spacetime

$$\delta T_{\gamma g} = T_\gamma - T_g = 0$$

In bi-metric:

photons and gravitons follow geodesics of two spacetimes

(not in scalar-tensor theories)

$$\delta T_{\gamma g} \neq 0$$

Example:

TeVeS model. Observable=SN1987a

$$\delta T_{\gamma g} = -\,5.3\,\mathrm{days}$$

Kahya & Woodard, 0705.0153

Conclusions

- General Relativity is well-tested in the Solar system

- Tests need to be extended in other regimes
  [large scale / low acceleration / low curvarture /strong field]

-Equivalence principle
  - tested directly in the solar system
  - tested astrophysically via the constancy of constants
  - most sensitive test depends on the model
  - importance of various systems on different time/spatial scales

- Field equations
  - Large scale structure allow to extend PPN-like analysis
  - requires weak lensing / galaxy distribution / velocity fields
  - Euclid

- Multi messanger
  - γ / GW / ν : mutli-metric theories

- Non-linear dynamics
  - CMB (Planck @ $l>2000$) or LSS on small scales (weakly NL regime)