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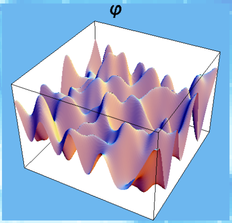
Gravitational Waves from Preheating after Inflation: Overview and Recent Results



GW 2010 - University of Minnesota

Based on:

- JFD, Bergman, Felder, Kofman, Uzan - *PRD(2007)*
- JFD, Felder, Kofman, Navros - *JCAP(2009)*
- JFD - *PRL(2009)*
- JFD, Figueroa, Garcia-Bellido - *PRD(2010)*
- Brax, JFD, Mariadassou - *to appear*

Several other works:

- [Khlebnikov, Tkachev '97]
- [Easther, Lim '06], [Easther, Giblin, Lim '07], [Easther, Giblin, Lim '08]
- [Garcia-Bellido, Figueroa '07], [Garcia-Bellido, Figueroa, Sastre '08]
- [Price, Siemens '08], [Giblin, Price, Siemens '10]
- [Bastero-Gil, Macias-Perez, Santos '10]
- ...

OUTLINE

- Preheating after Inflation
- GW from Preheated Scalar Fields: Methods
- Comparison with Observations for 3 Classes of Inflation Models
- GW from the Non-Perturbative Decay of SUSY Flat Directions
- GW from Preheated Gauge Fields
- Conclusions

Preheating after Inflation

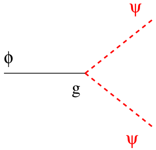
Inflation		**Reheating**		Hot Big Bang
$\phi(t)$: condensate	$\longrightarrow$	Particle-genesis	$\longrightarrow$	Thermal bath
no particle		Thermalization		of particles
$\hookrightarrow$ **CMB, ...**		$\hookrightarrow$ **????**		$\hookrightarrow$ **BBN, ...**

Two kinds of decay channels for the inflaton:

Perturbative:

$\phi(t)$ = collection of independent particles with $\vec{k} = \vec{0}$

$\Rightarrow$ Perturbative decay rate: $\Gamma_{\phi \to \psi\bar{\psi}} = \frac{g^2 \, m_\phi}{8\pi}$



Non-perturbative: coherent and collective effects of $\phi(t) \Rightarrow$ **Preheating**

Explosive production of bosonic particles with $n_k \propto 1/g^2 \gg 1$

Inhomogeneous, non-linear, non-equilibrium dynamics $\Rightarrow$ Lattice Simulations

[Kofman, Linde, Starobinsky], [Shtanov, Traschen, Brandenberger], [Khlebnikov, Tkachev], ...

Example: Tachyonic Preheating after Hybrid Inflation

[Felder, Garcia-Bellido, Greene, Kofman, Linde, Tkachev], ...

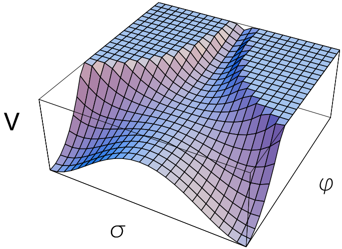
$$V = \frac{\lambda^2}{4}\left(|\sigma|^2 - v^2\right)^2 + \frac{g^2}{2}\phi^2|\sigma|^2 + V_{\rm sr}(\phi)$$

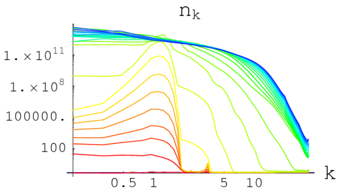
Inflation when $\phi > \phi_c = \sqrt{\lambda}\, v/g$

When $\phi < \phi_c$, (ϕ, σ) roll to minimum $(0, v)$

Fluctuations: $\delta\ddot{\sigma}_k + \left(k^2 + \partial^2 V\right)\delta\sigma_k = 0$

$$m_\sigma^2 = \partial^2 V < 0 \quad \Rightarrow \quad \delta\sigma_k \propto e^{\sqrt{|\partial^2 V| - k^2}}$$

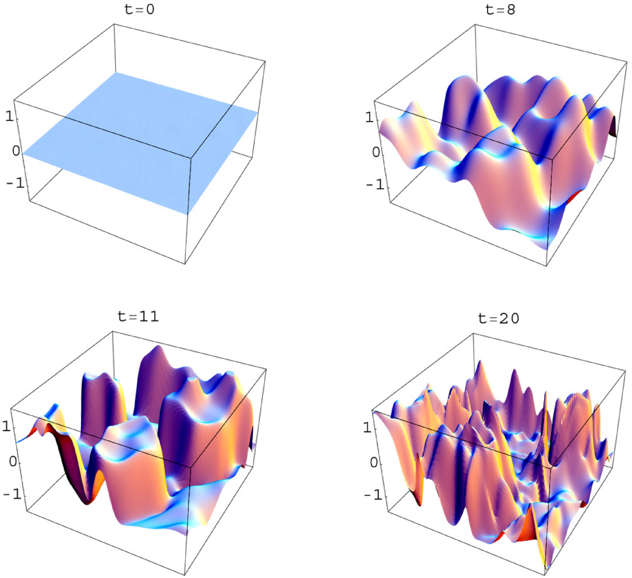




Explosive production of particles with occupation numbers $n_k \sim 1/g^2$.

Followed by long, turbulent-like evolution towards thermal equilibrium.

Spatial profile of the fields in 2-D slices



GW from preheated scalar fields: Methods

[JFD, Bergman, Felder, Kofman, Uzan '07]

Evolve GW on the lattice, together with the scalar fields source

$$\ddot{h}_{ij} + 3\,\frac{\dot{a}}{a}\,\dot{h}_{ij} - \frac{\nabla^2}{a^2}\,h_{ij} = 8\pi G\,\Pi_{ij}^{TT}$$

In Fourier space: $\Pi_{ij}^{TT}(\vec{k}) = \left(P_{il}P_{jm} - \frac{1}{2}\,P_{ij}P_{lm}\right)\,\Pi_{lm}(\vec{k})$ with $P_{ij} = \delta_{ij} - \hat{k}_i\hat{k}_j$

Calculate $\rho_{\rm gw} = \langle \dot{h}_{ij}\,\dot{h}_{ij}\rangle/(32\pi G)$ and GW spectrum at the time of production

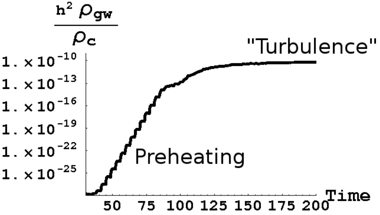
Redshift into spectrum today:

$$\mathbf{h^2\,\Omega_{\rm gw}(f)} = \left(\frac{\mathbf{h^2}}{\rho_{\mathbf{c}}}\,\frac{\mathbf{d}\rho_{\rm gw}}{\mathbf{d\ln f}}\right)_{\mathbf{0}}$$

Depends on evolution of equation of state from preheating up to now

In most models, w jumps quickly towards 1/3 (radiation) at the beginning of preheating **[Podolsky et al '05]**, **[JFD et al '06]**

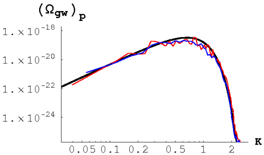
Three Stages of GW Production from Preheating



Linear stage of preheating:

Gaussian random fields and analytical evolution of the source

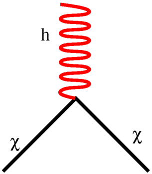
$\Rightarrow$ Check of the lattice results



Turbulence and thermal bath

Scalar fields with $\chi_k(t) \propto \mathrm{Exp}[i\omega_k t]$
GW production *forbidden* by helicity conservation

Different for *massless* vector fields

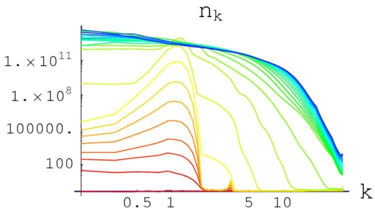


Most GW produced in intermediate non-linear and non-perturbative stage

Spectra of particles produced by preheating strongly peaked around some typical (comoving) momentum k_*

$R_* = a/k_* < H$: Characteristic size of the source inhomogeneities

Can be computed analytically in given model of inflation + preheating



Frequency and amplitude of GW when produced depend essentially on R_*

$\Rightarrow$ Peak frequency and amplitude of GW spectrum today:

$$f_* \approx \frac{1}{(R_* H)_p} \left(\frac{\rho_p^{1/4}}{10^{11}\,\mathrm{GeV}} \right) 10^3\,\mathrm{Hz} \quad , \qquad h^2\,\Omega_{\mathrm{gw}}^* \approx 10^{-6}\,(R_*\,H)_p^2$$

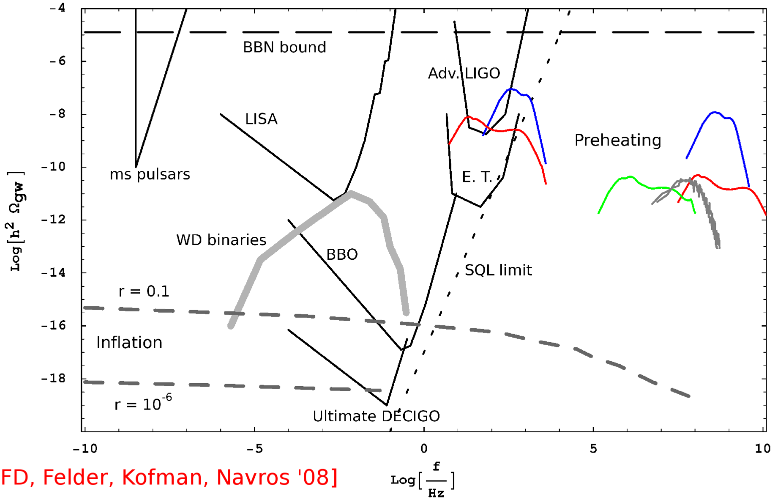
Can be observable if $\rho_p^{1/4} \approx \rho_{\mathrm{infl}}^{1/4} < 10^{11}$ Gev

Complementary to GW from inflation: $h^2\,\Omega_{\mathrm{gw}}^{\mathrm{infl}} \propto \rho_{\mathrm{infl}}/M_{\mathrm{Pl}}^4$

Comparison with Observations in 3 Main Classes of Inflation Models

Preheating after chaotic inflation: $f_* \sim 10^6 - 10^9$ Hz $\quad (\rho_{\rm inf}^{1/4} \sim 10^{15}$ GeV$)$

Preheating after hybrid inflation: GW very sensitive to model parameters. Can be observable by ground-based interferometers



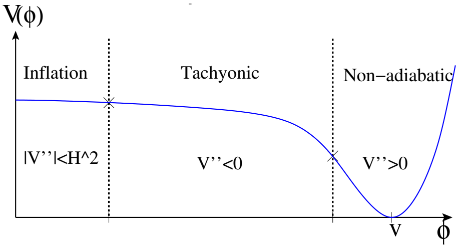
[JFD, Felder, Kofman, Navros '08]

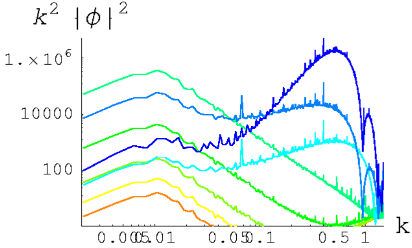
Preheating after Small Field Inflation [Brax, JFD, Mariadassou]

Fluctuations around $\phi(t)$:

$$\delta\ddot{\phi}_k + \left(k^2 + V''(\phi)\right)\,\delta\phi_k = 0$$

$m^2 = V'' < 0$: tachyonic amplification
$\dot{m} \gg m^2$: non-adiabatic production





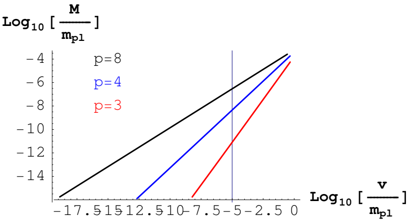
$$k_*^{\rm IR} \sim \sqrt{|V''_{\rm init}|} \sim H$$

$$k_*^{\rm UV} \sim \sqrt{|V''_{\rm min}|} \gg H$$

In these models, inflation can occur at very low energy scales

Ex: **[German, Ross, Sarkar]**

$$V = M^4 \left(1 - \frac{\phi^p}{v^p}\right)^2$$



For $v/m_{\rm Pl} \lesssim 10^{-5}$, tachyonic dominates

$$f_* \approx \frac{M}{10^{10}\,\mathrm{GeV}}\,10^2\mathrm{Hz} \quad , \quad h^2\,\Omega^*_{\rm gw} \approx 10^{-7}$$

Otherwise, non-adiabatic dominates

$$f_* \approx \left(\frac{m_{\rm Pl}}{v}\right)^{\frac{p+2}{2p}} \frac{M}{10^{10}\,\mathrm{GeV}}\,10^3\mathrm{Hz} \quad , \quad h^2\,\Omega^*_{\rm gw} \approx 10^{-8}\left(\frac{v}{m_{\rm Pl}}\right)^{\frac{p+2}{p}}$$

Note: In general, for cosmological GW backgrounds

$E_p = \sqrt{\mathrm{TeV}\, m_{\rm Pl}} \approx 10^{10}$ Gev <≃> $f_0 \sim 10^2$ Hz

Intermediate SUSY-breaking scale — Ground-based detectors

Other Source: Supersymmetric Flat Directions

Combinations of complex scalar fields along which the renormalizable potential is exactly flat in the limit of unbroken SUSY.

Flatness lifted by SUSY-breaking and non-renormalizable terms

$$V = m^2\,|\phi|^2 + \left(\frac{A\,m}{M_P^{n-3}}\,\phi^n + \mathrm{h.c.}\right) + \frac{|\lambda|^2}{M_P^{2n-6}}\,|\phi|^{2n-2} + \ldots \qquad (m \sim \mathrm{TeV})$$

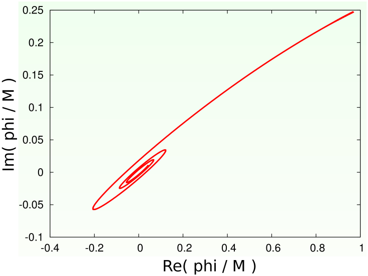
In the early universe:

[Dine, Randall, Thomas '95]

$$V = \left(m^2 - c\,H^2\right)\,|\phi|^2 + \ldots$$

(NB: $V \supset T^2\,|\phi|^2$ for small VEVs)

ϕ can acquire a very large VEV during inflation. Subsequent evolution damped untill $H \sim m$. Then, out-of-phase oscillations of Re(ϕ) and Im(ϕ).



Ex: Affleck-Dine baryogenesis

The Non-Perturbative Decay of Flat Direction Condensates

[Olive, Peloso '06], [Basboll et al '07], [Gümrükçüoğlu et al '08], ...

Model: $V_D = g^2 \left(|\phi_1|^2 - |\phi_2|^2\right)^2$ (D-term potential)

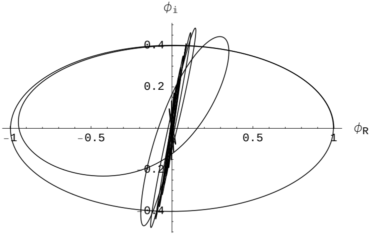
Background: $\phi_1 = \phi_2 = \Phi(t)\, e^{i\,\sigma(t)}$

Fluctuations around this background have the mass matrix:

$$\mathcal{M}^2 = g^2\, \Phi^2(t) \begin{pmatrix} 2\cos^2\sigma(t) & \sin 2\sigma(t) \\ \sin 2\sigma(t) & 2\sin^2\sigma(t) \end{pmatrix}$$

Eigenstates vary non-adiabatically with time $\Rightarrow$ abundant particle production

Once the fluctations have been sufficiently amplified, they backreact on the condensate and convert most of its energy into large inhomogeneities [JFD '09]



Differences w.r.t preheating:

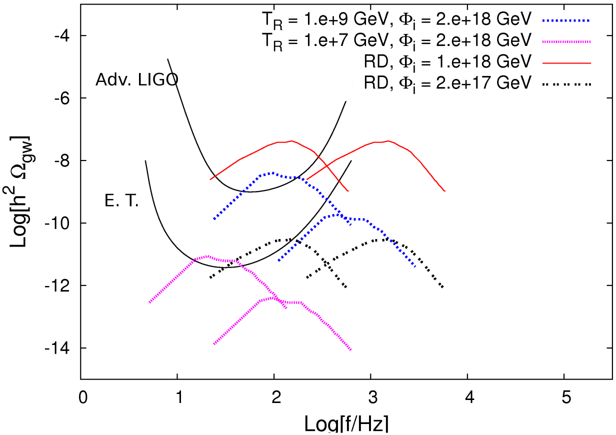
$\hookrightarrow$ $\Omega_{\rm gw} \propto (\rho_{\rm flat}/\rho_{\rm tot})^2$

$\hookrightarrow$ Equation of state and GW spectrum depend also on inflaton sector

GW from the Non-Perturbative Decay of Flat Directions [JFD '09]

Peak frequency and amplitude today:

$$f_* \sim \left(\frac{a_i}{a_r}\right)^{1/4} \sqrt{\frac{m}{\mathrm{TeV}}}\, 5 \times 10^2\,\mathrm{Hz} \quad , \quad h^2\,\Omega^*_{\mathrm{gw}} \sim 10^{-4} \left(\frac{\Phi_i}{M_P}\right)^4 \left(\frac{a_i}{a_r}\right)$$



Main parameters:

Soft SUSY-breaking mass $m \sim TeV$

a_i/a_r: depends on inflaton sector / thermal history of universe

Initial VEV Φ_i of condensate when it starts to oscillate. Needs to be large for GW to be observable (very flat directions)

GW from Abelian Gauge Fields at Preheating [JFD, Figueroa, Garcia-Bellido]

In realistic models, gauge fields also abundantly produced at preheating

Lead to new terms in anisotropic stress sourcing GW (**Ex:** $\Pi_{ij} \supset E_i E_j, B_i B_j$)

May enhance GW production from turbulent evolution towards thermal equilibrium after preheating ; but not in abelian scalar-gauge theories

Out-of-equilibrium gauge fields are also ubiquitous in other sources of GW (**Ex:** phase transitions, local topological defects).

Lattice simulations of gauge theories: (a nightmare...)

$$-\mathcal{L} = (D_\mu X)^* D^\mu X + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \ldots \quad \text{with} \quad D_\mu = \partial_\mu - i\, e\, A_\mu$$

Gauge invariance is lost by naive discretization (**Ex:** $\partial_\mu^+ X = \frac{X(x+\hat{\mu}) - X(x)}{dx^\mu}$).

Links: $U_\mu(x, x+\hat{\mu}) \equiv e^{-ie \int_x^{x+\hat{\mu}} A_\mu dx^\mu} \quad \Rightarrow \quad D_\mu^+ X = \frac{U_\mu(x,x+\hat{\mu})\, X(x+\hat{\mu}) - X(x)}{dx^\mu}$

Derive discrete equations of motion from lattice action invariant under a lattice gauge transformation $\Rightarrow$ Constraint equations follow from dynamical equations that are evolved.

We developed method to compute GW production in scalar-gauge theories on lattice with second order accuracy in lattice spacing and timestep.

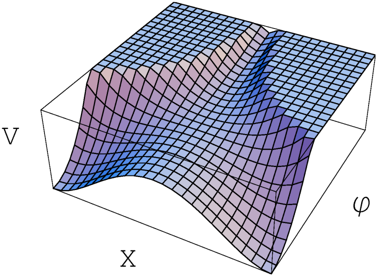
Model: Abelian-Higgs Preheating after Hybrid Inflation

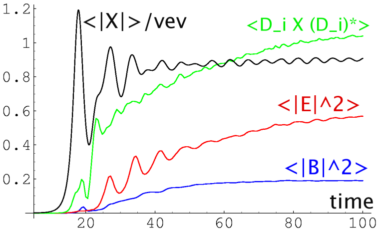
Complex waterfall ("Higgs") field X coupled to $U(1)$ gauge field A_μ

$$V = \frac{\lambda}{4}\left(|X|^2 - v^2\right)^2 + \frac{g^2}{2}\phi^2|X|^2 + V_{\rm sr}(\phi)$$

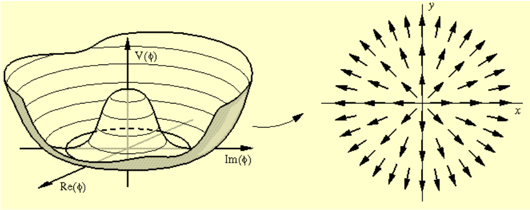
$$D_\mu X = \partial_\mu X - ieA_\mu X$$

As X goes to its VEV after inflation, its fluctuations are amplified by tachyonic preheating. This sources the production of A_μ through its coupling to X.



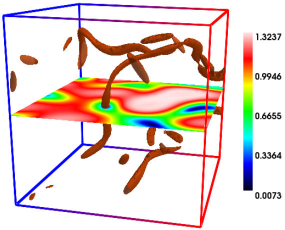


Abelian-Higgs Cosmic Strings during Preheating

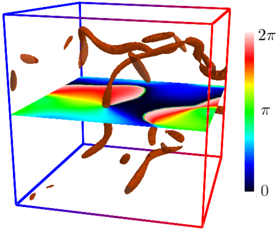


Points where $X = 0$ form line-like configuration with non-zero potential energy A_μ concentrates in those regions to minimize $\rho \supset |\partial_\mu X - ieA_\mu X|^2$

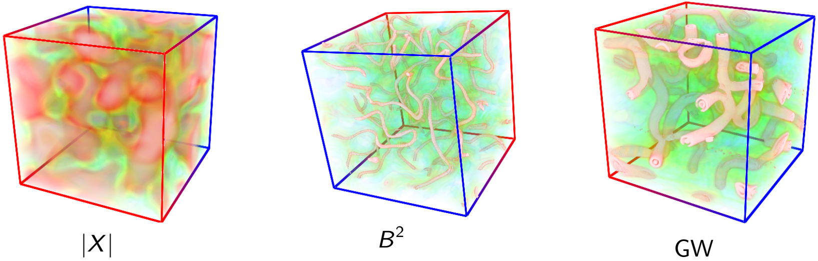
$|X|$-profile around B-strings



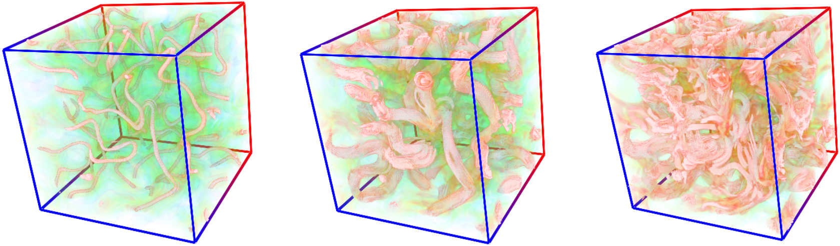
Arg(X)-profile around B-strings



Spatial distributions during symmetry breaking



Spatial distributions of $\vec{B}^2$ at different times



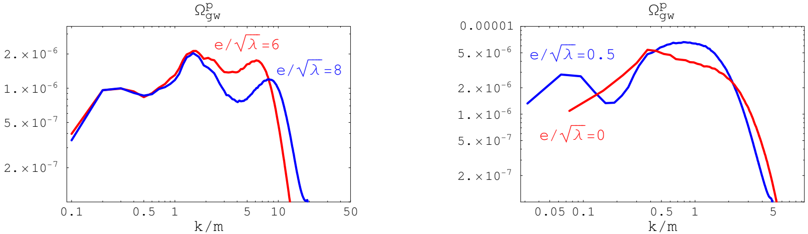
Signatures in the GW Spectra

Three well-distinct characteristic scales:

Length of straight string segments $\gtrsim$ size of X-bubbles (R_*)

Width / interactions of X and A_μ around the strings (m_X and $m_A \gg R_*^{-1}$)

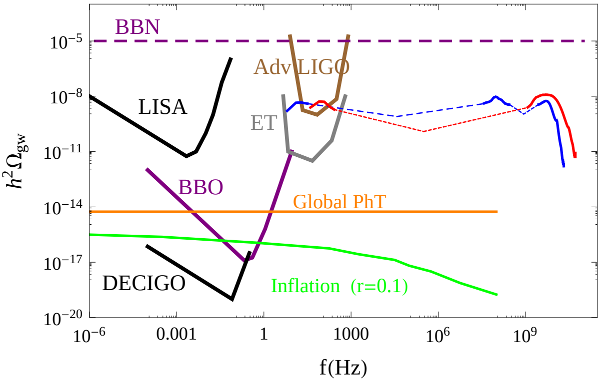
$\Rightarrow$ Three peaks in the GW spectra



$$\text{IR peak:} \quad f_1 \lesssim \begin{cases} \lambda^{\frac{1}{4}}\, V_c^{\frac{1}{3}}\, 10^{11}\,\text{Hz} & \text{for} \quad g^2 \gtrsim \lambda \text{ and } V_c \gtrsim 500\, g^3 \\ \lambda^{\frac{1}{4}}\, g\, 10^{11}\,\text{Hz} & \text{for} \quad g^2 \gtrsim \lambda \text{ and } V_c \lesssim 500\, g^3 \\ \lambda^{\frac{1}{4}}\, \frac{g}{\sqrt{\lambda}}\, 10^{10}\,\text{Hz} & \text{for} \quad g^2 \ll \lambda \end{cases}$$

Middle peak: $f_2 \approx \lambda^{1/4}\, 10^{11}\,\text{Hz}$; UV peak: $f_3 \approx \frac{e}{\sqrt{\lambda}} \lambda^{1/4}\, 10^{11}\,\text{Hz}$

GW Spectrum Today



$$h^2\,\Omega^*_{\rm gw} \sim \begin{cases} 10^{-6}\,V_c^{-2/3}\left(\frac{v}{M_{\rm Pl}}\right)^2 & \text{for} \quad g^2 \gtrsim \lambda \ \text{and} \ V_c \gtrsim 500\,g^3 \\ \frac{10^{-8}}{g^2}\left(\frac{v}{M_{\rm Pl}}\right)^2 & \text{for} \quad g^2 \gtrsim \lambda \ \text{and} \ V_c \lesssim 500\,g^3 \\ 10^{-5}\,\frac{\lambda}{g^2}\left(\frac{v}{M_{\rm Pl}}\right)^2 & \text{for} \quad g^2 \ll \lambda \end{cases}$$

Conclusions and Perspectives

- **There can be several instances in the early universe where scalar field condensates decay in an explosive and highly inhomogeneous way**. This generates a stochastic background of GW that carries unique informations about these high-energy phenomena.
- **For preheating after inflation**, these GW can be observable if inflation occurs at low enough energy scales (complementary to GW from inflation itself). Depending on the model, they can cover a wide range of frequencies and amplitude.
- **For the non-perturbative decay of SUSY flat directions**, these GW fall naturally in the Hz-kHz frequency range where ground interferometers operate. They carry informations on both SUSY (scale of SUSY breaking) and inflation (reheat temperature).
- In both cases, **gauge fields can have important consequences on GW production and leave specific imprints in the GW spectra**
- Non-abelian symmetries ($SU(2) \times U(1)$) necessary to produce **massless gauge fields** (photons) at preheating. $\Rightarrow$ GW production from long, turbulent evolution towards thermal equilibrium after preheating??
- **Non-perturbative decay of gauge flat directions** (Ex: MSSM). **[JFD, Gümrükçüoğlu, Peloso]**
- Applications to **other GW sources: cosmic strings, ...**