GRAVITATIONAL WAVES FROM FIRST ORDER PHASE TRANSITIONS

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CC, R. Durrer and X. Siemens, arXiv:1007.1218
CC, R. Durrer and G. Servant, asXiv:0909.0622
CC, R. Durrer and E. Fenu, arXiv:0906.4976
CC, R. Durrer, T. Konstadin and G. Servant, arXiv:0901.1661
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CC and R. Durrer, astro-ph/0603476

Gravitational waves

- Once emitted, propagate without interaction: direct probe of physical processes in the early universe
- first order phase transitions are a source of GW
- temperature of the phase transition: characteristic frequency
- strength of the phase transition: characteristic amplitude
- construction of an analytical model of the GW source in terms of a few free parameters
- evaluation of the GW signal: amplitude and shape of the spectrum
- signal potentially interesting for LISA, PTA

GW from first order phase transitions

- universe expands and temperature decreases : PT
- nature of PT depends on the particle theory model
- if it is first order it can lead to the production of GW

(Hogan '83, Witten '84, Hogan '86...)

(Turner et al '92, Kosowsky et al '92, Kosowsky and Turner '93, Kamionkowski et al '94, Kosowsky et al '02, Dolgov et al '02...)

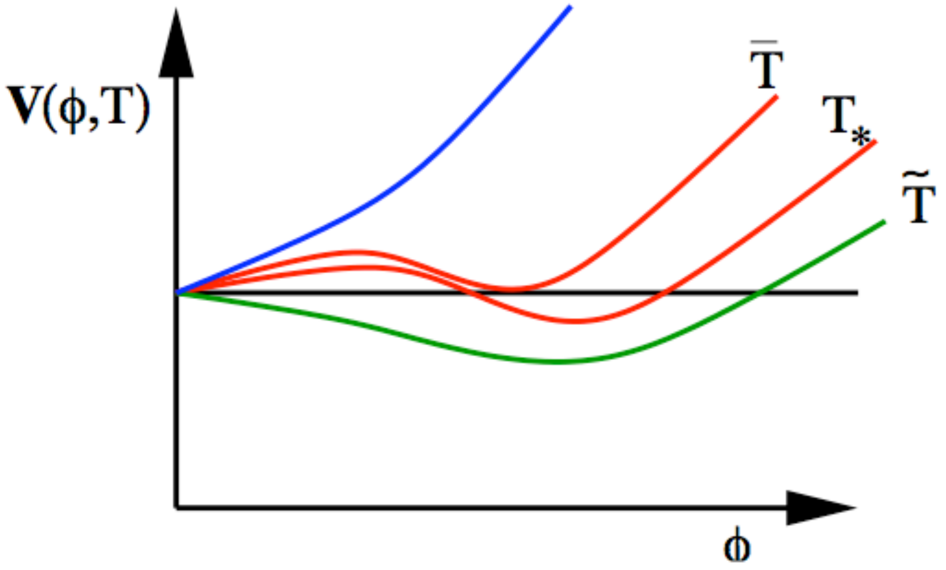
- EWPT : beyond the standard model (baryogenesis)

(Apreda et al '01, Nicolis '04, Grojean et al '05, Huber and Konstandin '08, Kahniashvili et al '09, Kehayias and Profumo '09, Chung and Long '10...)

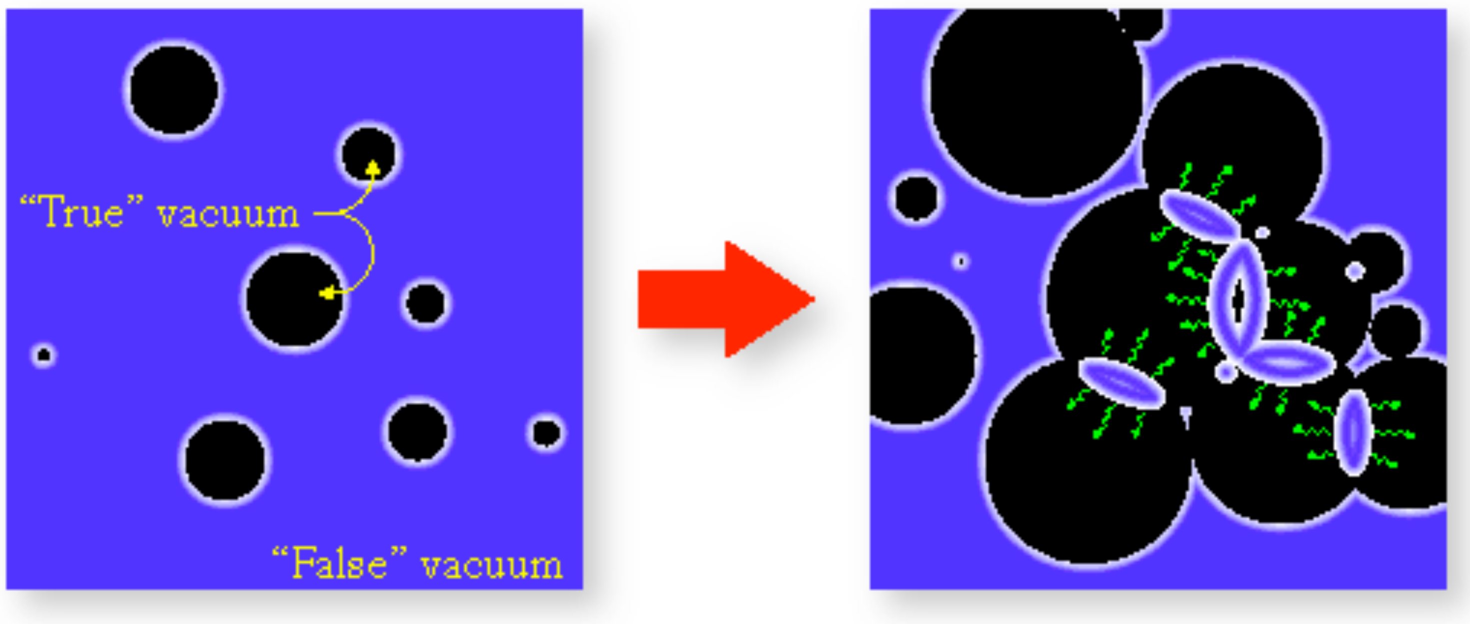
- QCDPT : if lepton asymmetry is large (Schwarz and Stuke '09)

GW from first order phase transitions

potential barrier separates
true and false vacua



quantum tunneling across the barrier :
nucleation of bubbles of true vacuum



- collisions of bubble walls
- MHD turbulence in the primordial fluid
- primordial magnetic fields

GW from PT : characteristic frequency

GW generation processes related to size of the bubbles
towards the end of the PT

characteristic wavenumber of
causal source of GW : $k_* \geq \mathcal{H}_*$ (cosmological horizon)

$$k_* = 1.6 \cdot 10^{-7}\,\epsilon^{-1} \left(\frac{T_*}{1\,\mathrm{GeV}}\right) \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \mathrm{Hz}$$

$\epsilon \leq 1$

dynamics of the
source

temperature (energy density) of the
universe at the source time

(standard thermal history)

GW from PT : characteristic frequency

GW generation processes related to size of the bubbles towards the end of the PT

characteristic wavenumber of causal source of GW : $k_* \geq \mathcal{H}_*$ (cosmological horizon)

$$k_* = 1.6 \cdot 10^{-7} \boxed{\epsilon^{-1}} \left(\frac{T_*}{1\,\mathrm{GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}} \mathrm{Hz}$$

β^{-1}	duration of the PT
$R_* = v_b\, \beta^{-1}$	size of bubbles at collision
$v_b \leq 1$	speed of bubble walls

GW from PT : characteristic frequency

$$\boxed{\epsilon \simeq \mathcal{H}_* \, \beta^{-1} \, , \; \mathcal{H}_* \, R_*}$$

- corresponding to the source characteristic time or scale depending on the source properties: space and time correlations

- value : $$\frac{\beta}{\mathcal{H}_*} \sim 4 \ln \left(\frac{m_{\rm Pl}}{T_*} \right)$$ → $$\epsilon \simeq \; 0.01$$

(Hogan '83)

EWPT

$$\boxed{k_{100\,\rm GeV} \sim 10^{-3} {\rm Hz} \quad {\rm LISA}}$$

QCDPT

$$\boxed{k_{100\,\rm MeV} \sim 10^{-7} {\rm Hz} \quad {\rm pulsars}}$$

$$(\;\; k_{10^7\,\rm GeV} \sim 100 \, {\rm Hz} \; {\rm LIGO} \,)$$

GW from PT : scaling of the characteristic amplitude

energy density of GWs: $\rho_G \sim \dfrac{\dot{h}^2}{8\pi G}$

$\delta G_{ij} = 8\pi G\, T_{ij}$

$\beta^2 h \sim 8\pi G\, T$

characteristic time of evolution

tensor perturbation

energy momentum tensor

$$\dot{h} \sim \frac{8\pi G\, T}{\beta}$$

$$\boxed{\Omega_{\mathrm{GW}} \sim \Omega_{\mathrm{rad}} \left(\frac{\mathcal{H}_*}{\beta}\right)^2 (\Omega_{\mathrm{s}}^*)^2}$$

radiation parameter

DURATION of the source with respect to Hubble time

RELATIVE ENERGY DENSITY available in the (radiation-like) source for the GW generation

$$T \sim \rho_*\, \Omega_{\mathrm{s}}^*$$

GW from PT : scaling of the characteristic amplitude

energy density of GWs: $$\rho_G \sim \frac{\dot{h}^2}{8\pi G}$$

$$\delta G_{ij} = 8\pi G\, T_{ij} \qquad \beta^2 h \sim 8\pi G\, T \qquad \dot{h} \sim \frac{8\pi G\, T}{\beta}$$

β^2: characteristic time of evolution

h: tensor perturbation

T: energy momentum tensor

$$\boxed{\Omega_{\rm GW} \sim \Omega_{\rm rad} \left(\frac{\mathcal{H}_*}{\beta}\right)^2 (\Omega_{\rm s}^*)^2}$$

$\Omega_{\rm rad}$: 10^{-5}

$\left(\frac{\mathcal{H}_*}{\beta}\right)^2$: 10^{-4}

$(\Omega_{\rm s}^*)^2$: 10^{-2} for strongly first order PT

amplitude OK for LISA and future PTA

GW from PT : scaling of the characteristic amplitude

$T \sim \rho_* \, \Omega_{\rm s}^*$ relative energy density in the source

- bubble collisions : kinetic energy of bubble walls $\Omega_{\rm s}^* \sim \dfrac{\rho_{\rm kin}}{\rho_{\rm vac} + \rho_{\rm rad}}$
- MHD turbulence : kinetic energy of chaotic fluid motions and magnetic field energy density $\Omega_{\rm s}^* \sim \langle v_f^2 \rangle \sim \langle b^2 \rangle$ (equipartition)

1. from the particle theory model know strength and friction $\alpha = \dfrac{\rho_{\rm vac}}{\rho_{\rm rad}}$
2. hydrodynamics of bubble growth at late times determine parameters $v_b \, , \; \rho_{\rm kin} \, , \; v_f$
3. simple example: Jouguet detonations

$$\alpha = \frac{1}{3} \, , \;\; v_b = 0.87 \, , \;\; v_f = \frac{1}{\sqrt{3}} \quad \Longrightarrow \quad \boxed{\Omega_{\rm s}^* \simeq 0.1}$$

Analytical evaluation of the GW spectrum

GW power spectrum: $\Omega_{\rm GW} = \frac{\langle \dot{h}_{ij}\dot{h}_{ij}\rangle}{8\pi G\, a^2\, \rho_c} = \int \frac{{\rm d}k}{k}\frac{{\rm d}\Omega_{\rm GW}}{{\rm d}\ln k}$

$$\frac{d\Omega_{\rm GW}}{d\ln k} \propto k^3 \int_{t_{\rm in}}^{t_{\rm fin}} \frac{dt_1}{t_1} \int_{t_{\rm in}}^{t_{\rm fin}} \frac{dt_2}{t_2} \cos[k(t_1 - t_2)]\, \Pi(k, t_1, t_2)$$

source: anisotropic stress power spectrum at unequal time

$$\langle \Pi_{ij}(\mathbf{k}, t_1)\Pi^*_{ij}(\mathbf{q}, t_2)\rangle = \delta(\mathbf{k} - \mathbf{q})\Pi(k, t_1, t_2)$$

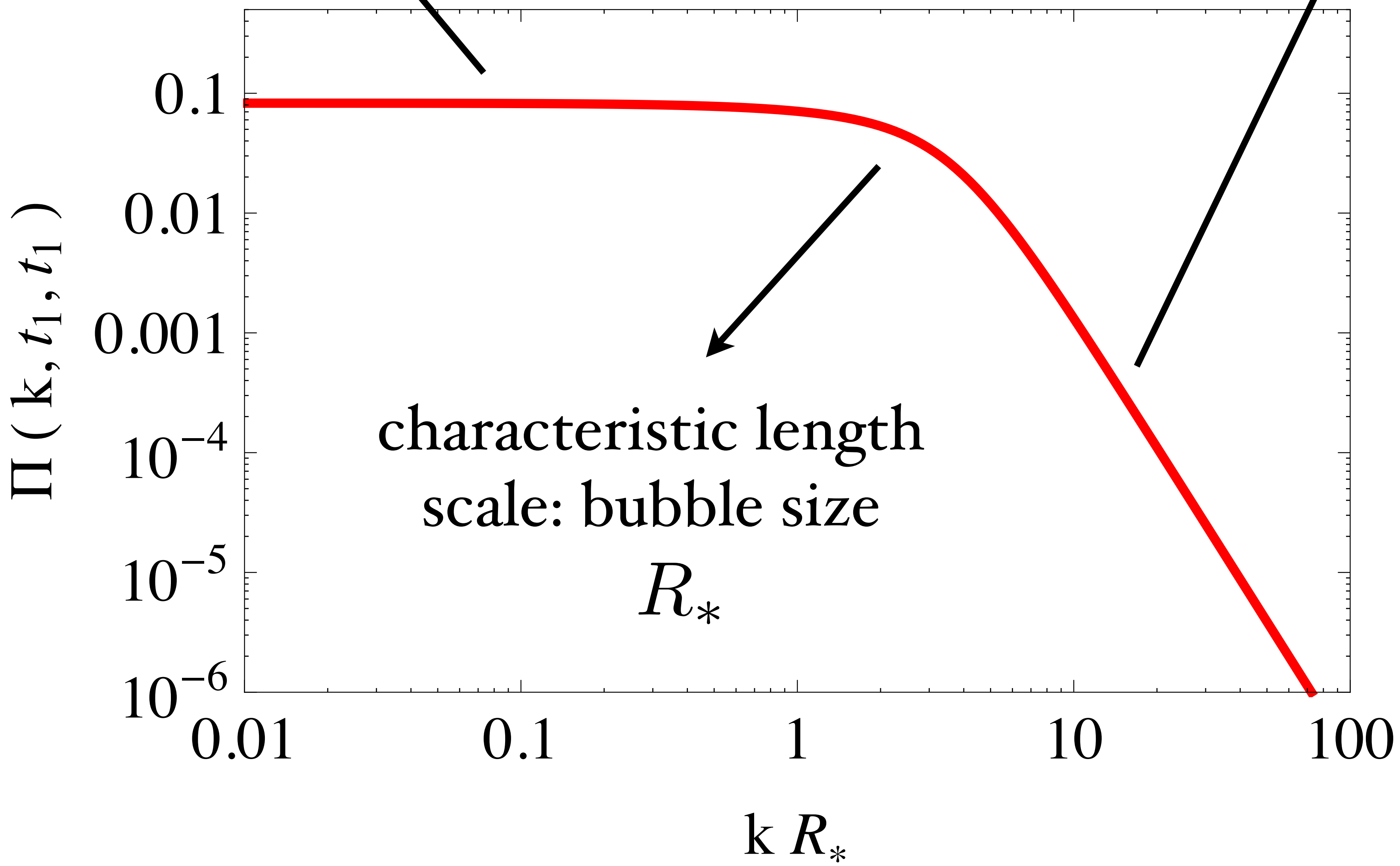
analytical model of the stochastic source for bubble collisions and MHD turbulence

1. space correlation structure (at equal times)
2. time correlation structure
3. overall time evolution

Spatial correlation of the anisotropic stress

bubbles and MHD, causal processes with typical length scale: bubble size

flat: spatially
uncorrelated,
causality



characteristic length
scale: bubble size
R_*

slope depending on
source power
spectrum

k^{-4} bubbles

$k^{-11/3}$

Kolmogorov
turbulence

Temporal correlation of the anisotropic stress

BUBBLES : completely coherent

- different collision events are uncorrelated in time
- single collision event is coherent : time evolution deterministic

$$\Pi(k,t_1,t_2)=\sqrt{\Pi(k,t_1,t_1)}\sqrt{\Pi(k,t_2,t_2)}$$

- GW spectrum becomes the square of the time Fourier transform of the source : peak at the characteristic time of the source

$$k_* \simeq \beta \qquad\qquad (\beta < R_*^{-1})$$

- the source lasts for a short time compared to the Hubble time

$$\beta^{-1} \simeq 0.01\,\mathcal{H}_*^{-1}$$

Temporal correlation of the anisotropic stress

MHD TURBULENCE : decorrelating in time

- motions decorrelate with eddy turnover time
- decorrelation time depends on eddy size

$$\tau_\ell \simeq \frac{\ell}{v_\ell}$$

correlated for $|t_1 - t_2| < \dfrac{1}{k}$

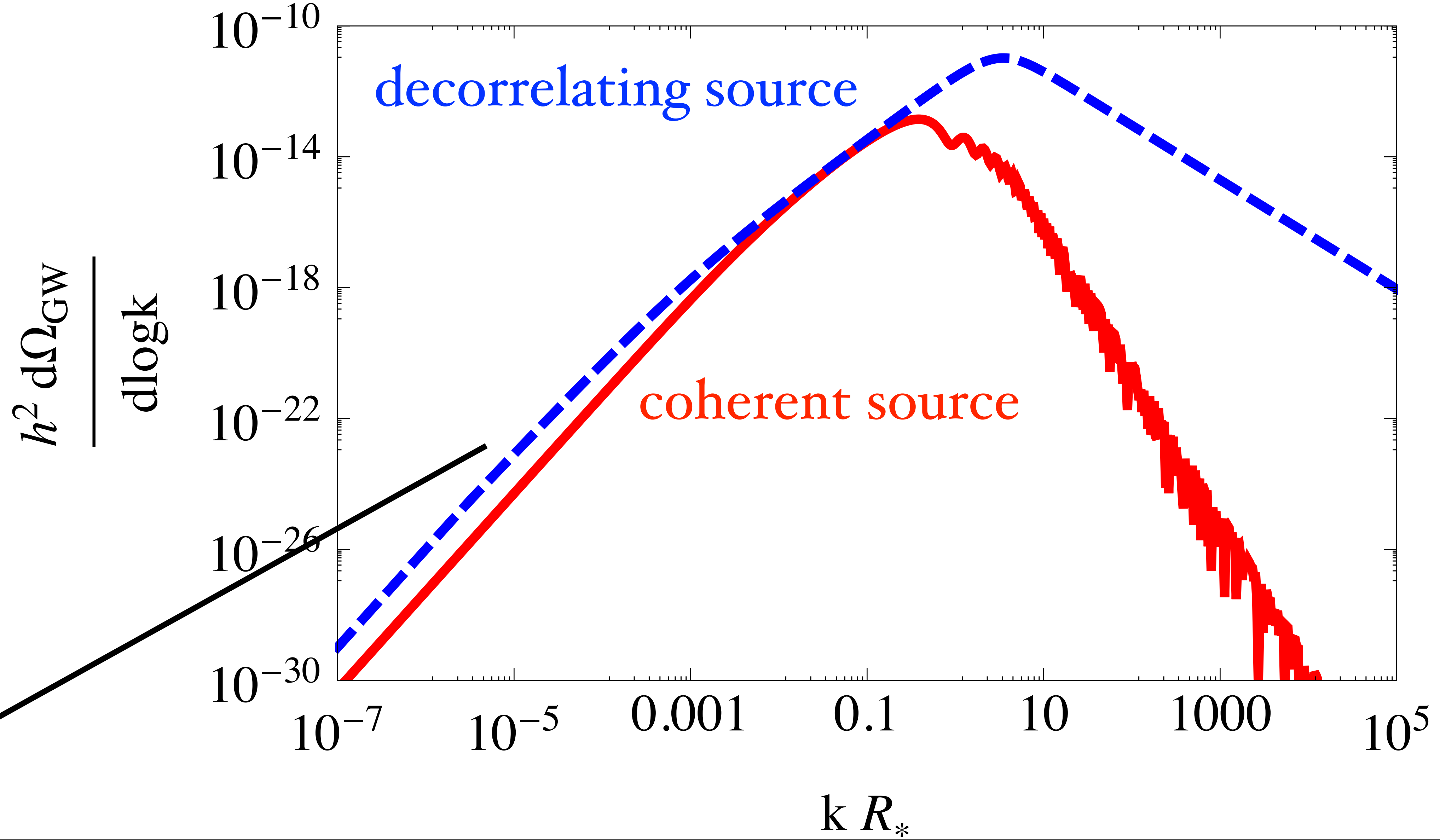
$$\Pi(k, t_1, t_2) = \{\Pi(k, t_1, t_1)\Theta[t_1 - t_2]\Theta[1 - k(t_1 - t_2)] + t_1 \leftrightarrow t_2\}$$

- no temporal Fourier transform: peak at the spatial correlation scale

$$k_* \simeq R_*^{-1}$$

- the source lasts for a long time compared to the Hubble time: determined by the decay of the turbulent motions, not very efficient because of low viscosity of the primordial fluid

Characteristic shape of the GW power spectrum



low frequency
slope: causality of
the source

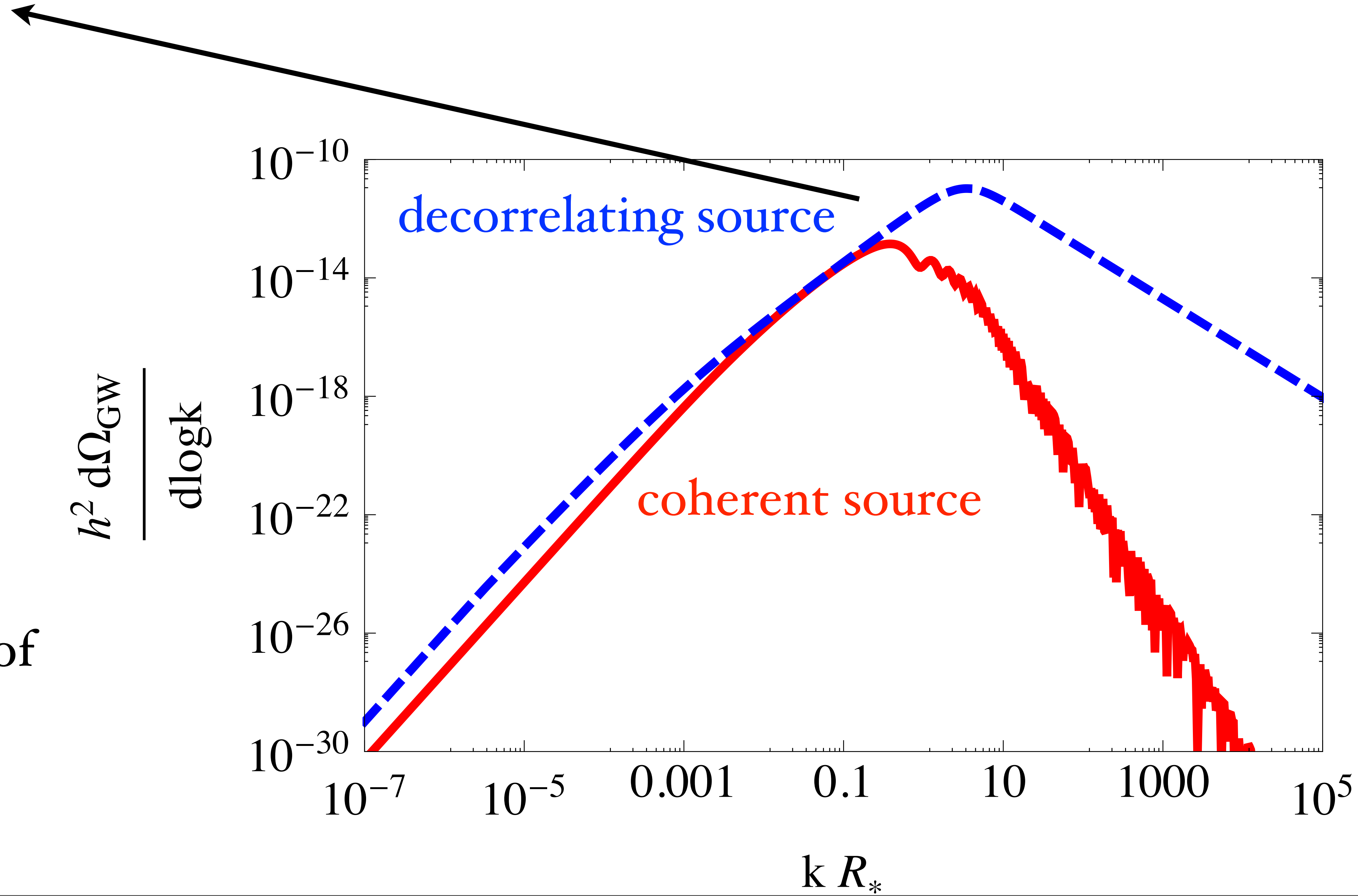
k^3

Characteristic shape of the GW power spectrum

peak position:

$k_* \sim \beta \quad k_* \sim R_*^{-1}$

$R_* = v_b/\beta$



low frequency slope: causality of the source

k^3

Characteristic shape of the GW power spectrum

peak position:

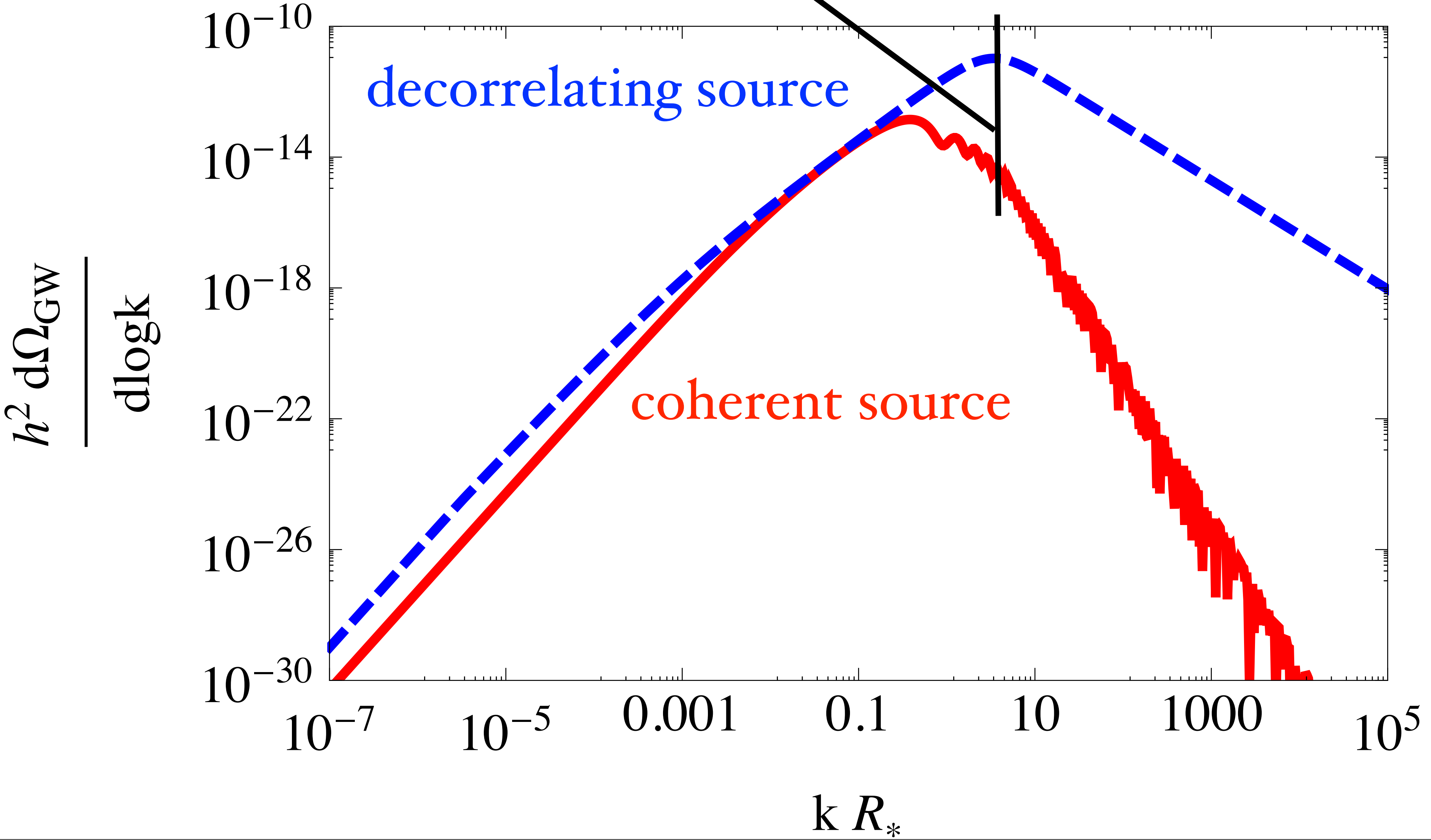
$k_* \sim \beta \quad k_* \sim R_*^{-1}$

$R_* = v_b/\beta$

coherent source:

feature at

$k_* \sim R_*^{-1}$



low frequency slope: causality of the source

k^3

Characteristic shape of the GW power spectrum

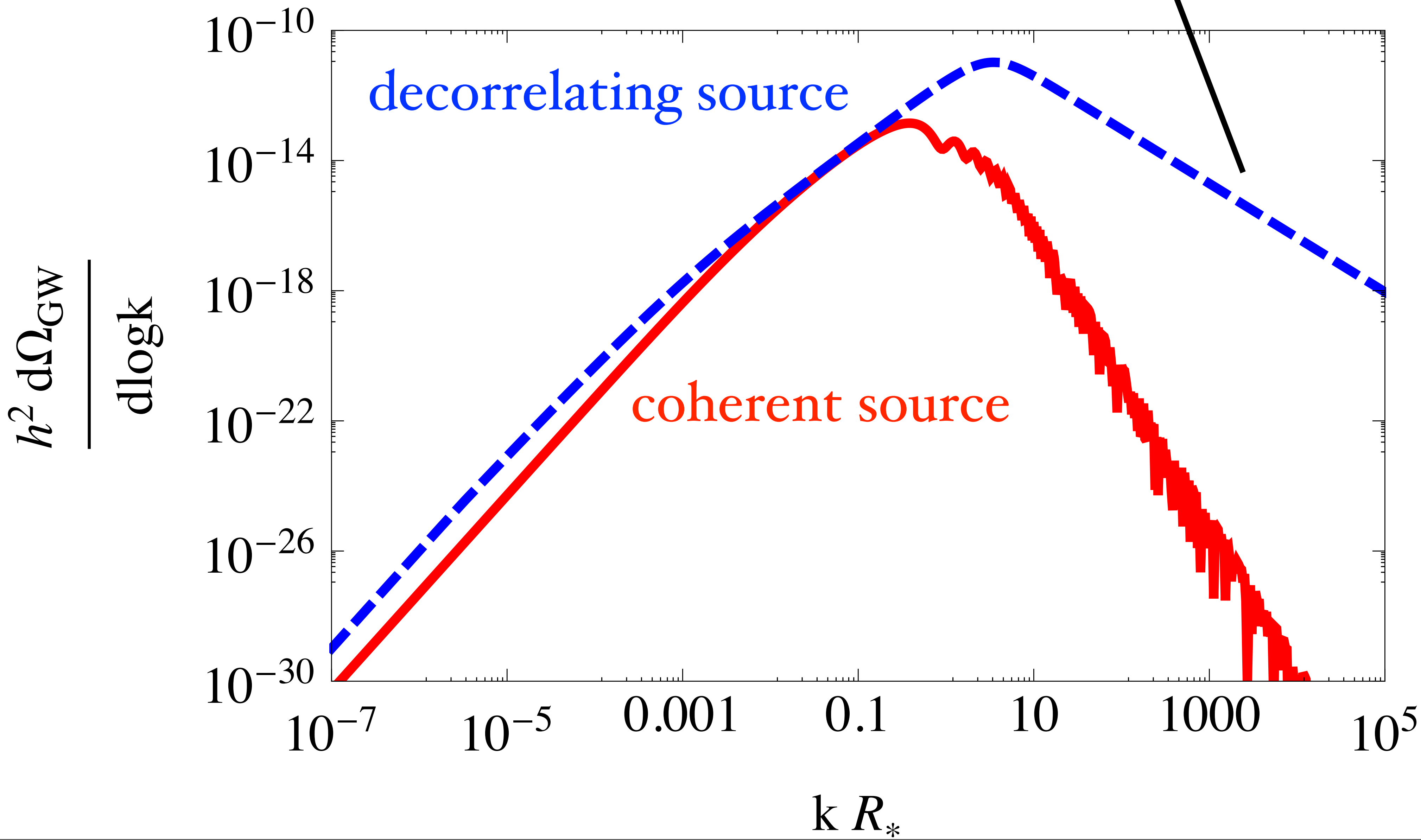
peak position:

$k_* \sim \beta \quad k_* \sim R_*^{-1}$

$R_* = v_b/\beta$

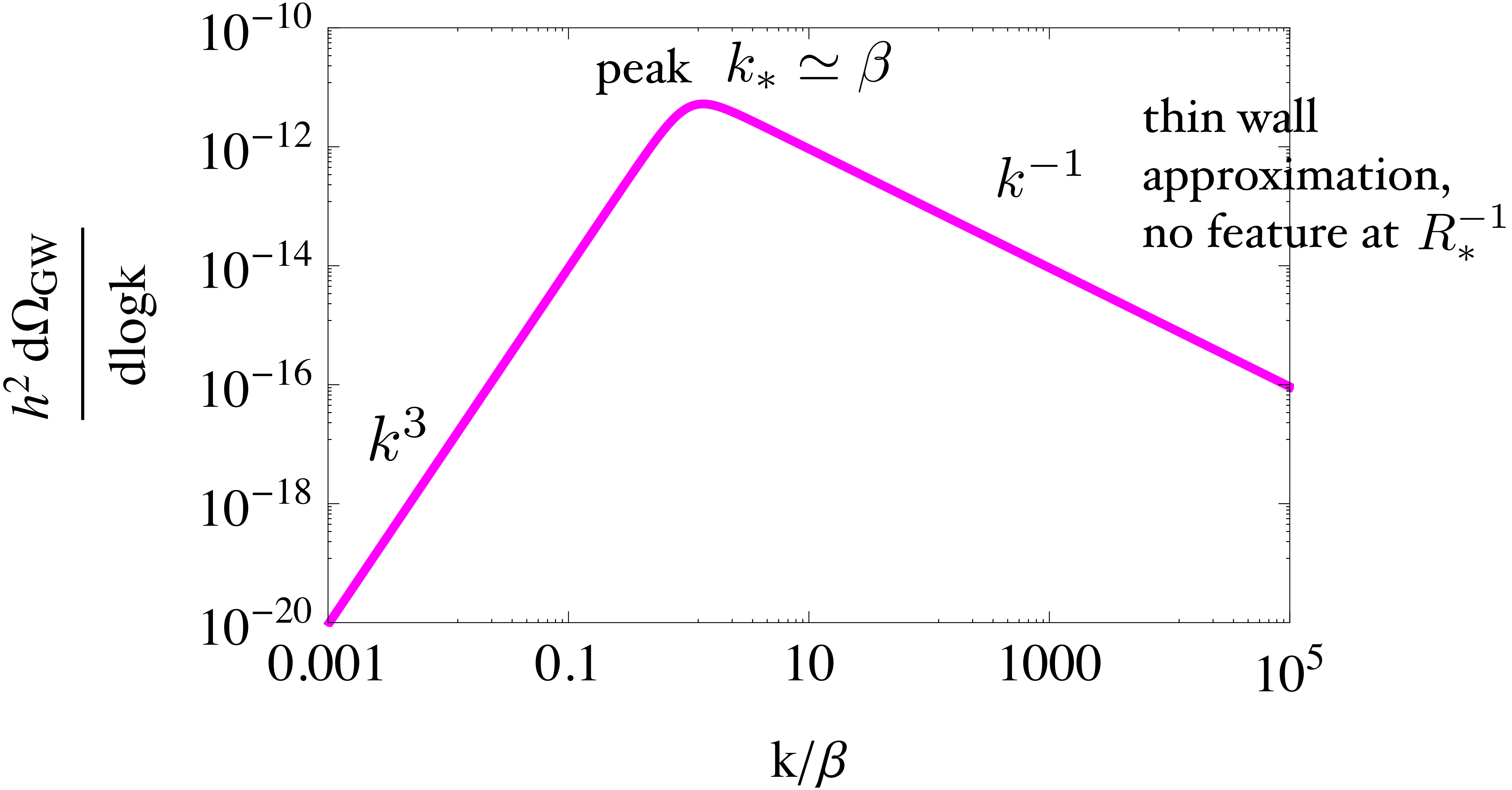
coherent source:
feature at
$k_* \sim R_*^{-1}$

high frequency slope:
depends on both power
spectrum and time
correlation of the source



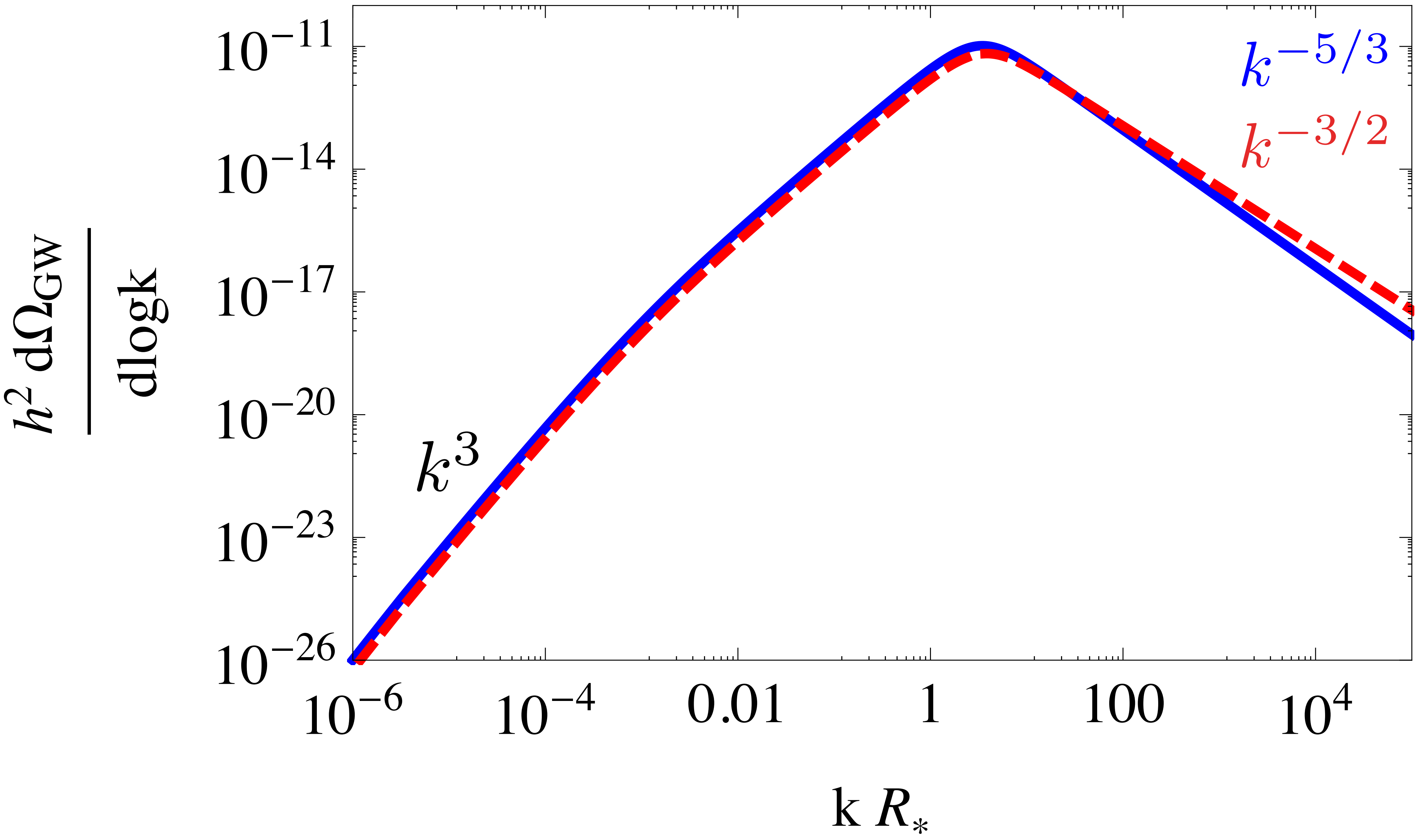
low frequency
slope: causality of
the source
k^3

GW spectrum from bubble collisions



GW spectrum from MHD turbulence

peak $k_* \sim R_*^{-1}$

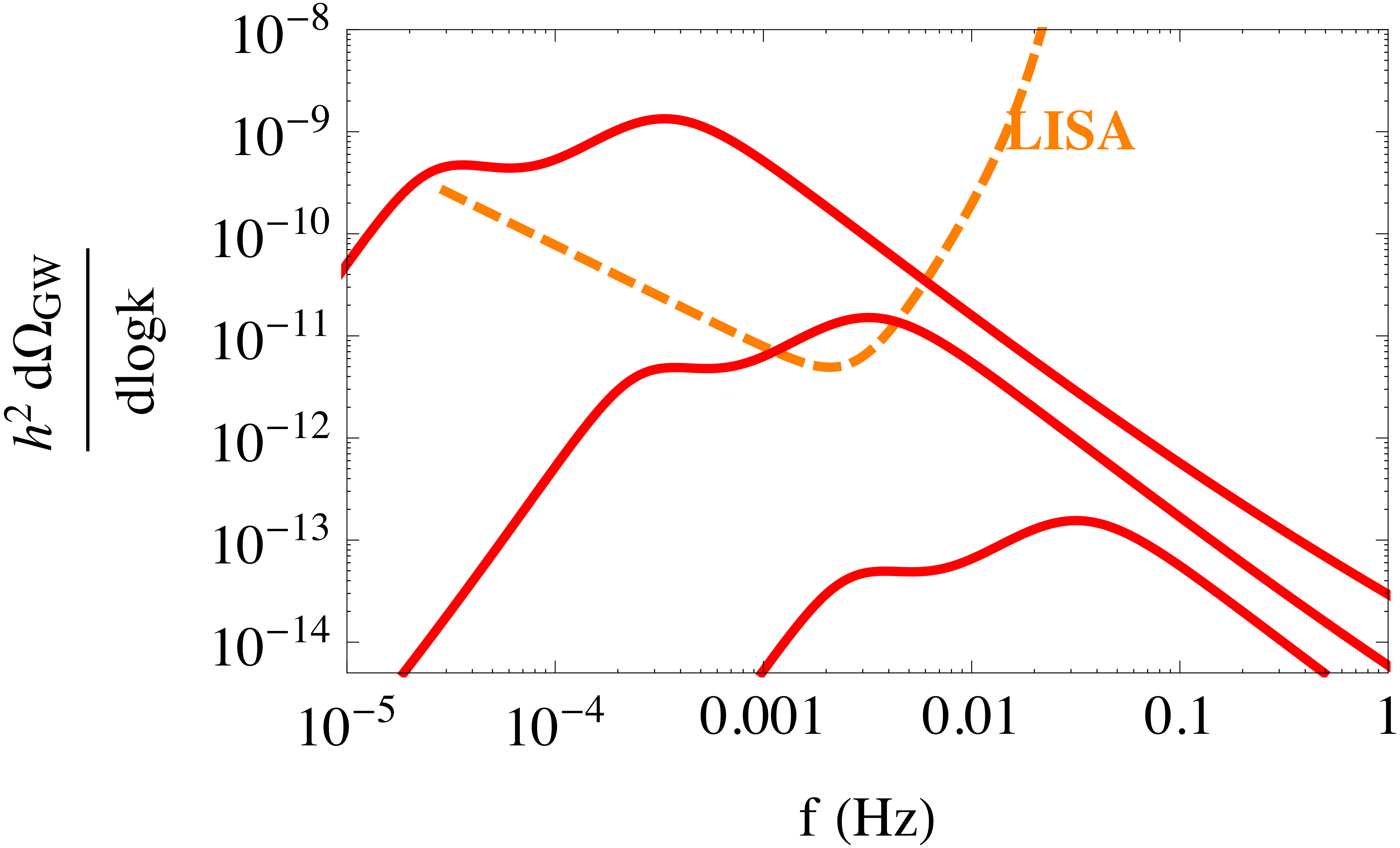


Kolmogorov
or Iroshnikov
Kraichnan
spectrum

total GW spectrum for the EWPT

$T_* = 100\,\mathrm{GeV}$ $\Omega_s^* = 0.2$ $v_b = 0.87$

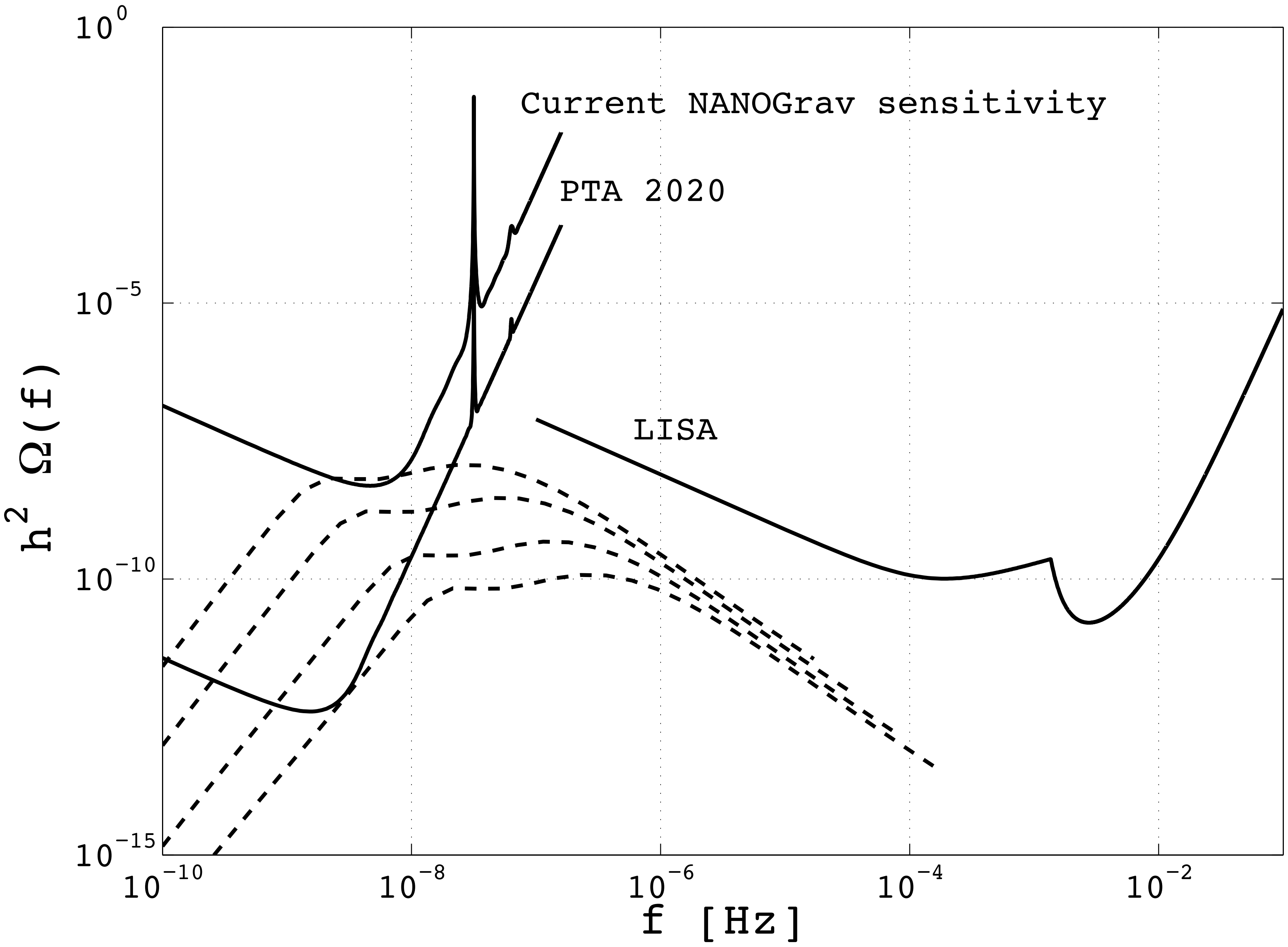
$$\frac{\beta}{\mathcal{H}_*} = 10\,,\ \ 100\,,\ \ 1000$$



total GW spectrum for the QCDPT

$$T_* = 100\,\mathrm{MeV} \qquad \Omega_{\mathrm{s}}^* = 0.1 \qquad v_b = 0.7$$

$$\frac{\beta}{\mathcal{H}_*} = 1\,,\ 2\,,\ 5\,,\ 10$$



Conclusions

- If EWPT is first order: GW generated are interesting for LISA if
  - energy in the bubble walls and turbulent motions is about 20% of radiation energy density
  - lasts for more than one hundredth of Hubble time

- If QCDPT is first order: GW generated are interesting for PTA2020 if
  - energy in the bubble walls and turbulent motions is about 10% of radiation energy density
  - lasts for more than one tenth of Hubble time

- Future improvements:
  - connection between phase transition parameters and kinetic energy in bubble walls and turbulence
  - for the bubble case: go beyond thin wall approximation?
  - for the turbulence case: confirm with simulations?