# Examining the Implementation of an Innovative Mathematics Curriculum

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## DEDICATION

This doctoral dissertation is dedicated to my parents, Gary T. and Marsha L. Hansen. Without their constant encouragement and love, I would not be the person I am today. They taught me to appreciate a good education and to strive to reach my goals. They have stood by me on this crazy path that we call life, even when they didn't necessarily understand my choices. Thank you, Mom and Dad. I love you!

**ABSTRACT**

Reform in mathematics instruction at the college level has been slow to arrive (Dossey, Halvorson, & McCrone, 2008), and many institutions of higher learning still follow the calculus model, while fewer and fewer students need calculus for their chosen areas of study (Ganter & Barker, 2003). Instead, mathematics that is applicable and transferable to other disciplines is more useful to many of today's college students. The Introduction to the Mathematical Sciences course that was the subject of this research study is a standards-based laboratory class that integrates algebra, statistics, and computer science. It was designed for students at both the high school and college levels who have struggled in mathematics. The intent of the course is to provide students with mathematics that they will find useful in their future careers, or future classes. The course is intended to reflect the ideals of reform mathematics at the college level. The purpose of the study was to examine the implementation of this curriculum, and its impact on student thinking and learning of algebra.

In exploring the research questions, the researcher found that the Introduction to the Mathematical Sciences course provided a reform-instruction setting where students were able to demonstrate their understanding of algebra, statistics and computer science. The students showed skill at moving between a number of representations of algebra concepts, indicating they were developing deeper understanding of those concepts. One of the key components of this course that reflected reform ideals was the extensive discussion that took place in the course. This instance of the implementation of the Introduction to the Mathematical Sciences course provides an example of how reform instruction in line with the recommendations of NCTM, MAA and AMATYC (Baxter

Hastings, et al., 2006) can be successful in helping students at the introductory college level gain understanding of mathematics. This research study describes a course that successfully plays out using reform instructional methods that are in sharp contrast to other college courses taught using traditional lecture style methods. High DWF rates among students who take college algebra (Lutzer, et al., 2005) indicate that the current model of instruction at the college level is not working. For students who lack confidence in their mathematical abilities and have seen little success in mathematics, this type of course may be a tool that can provide students the mathematical skills necessary to move forward in their studies and their careers.

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# CHAPTER I: INTRODUCTION

## Rationale

Mathematics at the high school and college levels has traditionally been taught with a focus on preparing students for calculus at the university level with little input from other disciplines as to the actual needs of their students. However, studies have shown that many students do not need much calculus for their future career choices and may be better served by study of other mathematical topics (Ganter & Barker, 2003; Lutzer, Rodi, Kirkman, & Maxwell, 2005). Reform efforts at both the high school and college levels are now focusing on teaching students mathematics that is applicable and transferable to many disciplines (National Council of Teachers of Mathematics, 2000; Committee on the Undergraduate Program in Mathematics of the MAA, 2004; American Mathematical Association of Two-year Colleges, 2006). While change in mathematics has been advanced through the National Science Foundation (NSF) supported standards-based curricula at the high school level (Senk & Thompson, 2003), it has been slower to arrive at the college level and has been only partially implemented (Dossey, Halvorson, & McCrone, 2008). Calculus texts that support the ideas of the National Council of Teachers of Mathematics (NCTM) and Mathematical Association of America (MAA) standards have appeared (Hughes-Hallett & Gleason, 1998; Ostebee & Zorn, 1997), as have a number of college algebra texts with similar focus on teaching to the standards (Herriott, 2005; Harshbarger & Yocco, 2004; Small, 2004). Additional college texts have been produced that contain reform ideas that include discussion, active learning and technology alongside traditional teaching methods (Baxter Hastings, Gordon, Gordon, & Narayan, 2006). The *Statistical Abstract of Undergraduate Programs in the*

*Mathematical Sciences* from 2005 even suggested (although hotly debated by some) that it is becoming more difficult to separate texts as “reform” or “traditional” college texts, since the reform ideas being are embraced by many authors (Lutzer, et al., 2007). Despite these advances, many college courses are still taught using more traditional lecture methods.

For students who have consistently struggled in mathematics, it is especially important to design new reform-based curricula that engage and have meaning to students while still providing rigorous instruction. Such curricula need to be creative while still being grounded in teaching techniques that have been shown to work and follow the standards of the NCTM, AMATYC and MAA. These standards have been developed over the years as guidelines for teaching mathematics in a manner that provides students rich opportunities to gain knowledge and emphasize more active learning and discussion by the student rather than the more traditional instructor-imparted lecture.

The standards of the NCTM, AMATYC and MAA focus on exposing students to a variety of representations of mathematical concepts in the classroom, from word problems and real world settings, to graphs, tables and numerical expressions, often using technology. One measure of how well a student understands a concept can be through an examination of how well the student is able to move between different representations of the same mathematical concept (Lesh, Post, & Behr, 1987). For example, a student who can create a table, graph and equation for the same algebraic concept holds a deeper understanding than a student who focuses on a single representation.

In investigating the implementation of new curricula, it is vital to use effective research methods to carefully examine whether those curricula are meeting the goals of the course effectively (Clements, 2007; Schoenfeld, 2006). While the future may bring studies comparing the effectiveness of innovative curricula to more traditional curricula, the purpose of this study is to look closely at the college-level implementation of an innovative standards-based curriculum designed to meet the needs of students who have not been successful in mathematics in the past in order to shed light on how such a curriculum operates within its parameters.

## The Course

The course that is the subject of this research study is a newly-designed mathematics class entitled “Introduction to the Mathematics Sciences,” that integrates algebra, statistics and computer science in a computer lab setting. The course was originally created to help meet the needs of students who have struggled in mathematics in the past, while still providing rich, applicable problems intended to show the students the value of mathematics for their lives. This innovative standards-based course has been implemented at both the high school (for juniors and seniors) and college levels. The current study will examine the implementation of the course at the two-year college level, at a small technical school in particular. The students taking the examined course are generally older than traditional college students and primarily have not had a mathematics class in the recent past. In addition, the majority of students have little experience with the three areas of algebra, statistics and computer science. This research study explores the implementation of the course, how well the course relates to reform mathematics standards, and the course’s impact on student learning, particularly in the area of algebra and working with multiple representations of algebraic ideas.

## Statement of the Problem

Many students who have been unsuccessful in mathematics during their high school years also fail to succeed in college courses (Baxter Hastings, et al., 2006). The needs of these students differ from those of students who have the goal of taking calculus in order to prepare for majors in mathematics or the hard sciences (Committee on the Undergraduate Program in Mathematics of the MAA, 2004; American Mathematical Association of Two-year Colleges, 2006). In 2004, instructors at Bemidji State University in Bemidji, MN designed a curriculum to meet the needs of both high school students and college students who had to that point been less than successful in their mathematics courses. This standards-based course, Introduction to the Mathematical Science, integrates algebra, statistics and computer science in a computer lab setting where students use spreadsheets to explore and make sense of data and mathematics. Preliminary test scores gathered by the course designers during pilot and subsequent implementation years appear to indicate that these previously unsuccessful students can show gains in their mathematical abilities in this setting. The same data showed gains for all implementation sites in the areas of computer science and statistics, yet some sites did not show consistent significant increases in algebra skills as compared to control groups (Webb, Richgels, Wolf, Frauenholtz, & Hougen, 2009). Because of this, the current study has a focus on the impact of alternative curricula on students' algebra learning, in order to better understand how students' algebraic thinking is changing through the course.

Schoenfeld (2006) suggests that a close examination of the implementation of a curriculum at a small number of implementation sites that focuses on important variables should precede broader research comparing curricula. This study is designed to look at how algebra and its representations are implemented in this integrated curriculum in order to determine if the stated goals and objectives of the course regarding algebra are being met by college aged students the introductory college level course.

## Research Questions

The research questions to be examined in this study include the following:

1) What is the impact of a standards-based lab course integrating algebra, statistics, and computer science on two-year college students' mathematical thinking and learning specifically regarding algebraic learning and representations?
2) In what ways do students in this integrated standards-based laboratory class at the two-year college level demonstrate understanding through an ability to move between representations of algebra problems, specifically relating to the ideas of slope and linear equations?
3) How and to what extent does the course reflect fidelity of implementation of the course designers' vision of college reform in mathematics education for the two-year college?
4) How and to what extent does the teaching of this course reflect the NCTM, MAA, and AMATYC standards for college algebra reform in a two-year college setting in the areas of both content and pedagogy?

## Potential Significance of the Study

This study will have value for mathematics educators at the secondary and postsecondary levels. Educators at both levels have tried to design courses that can help students who have previously been unsuccessful in mathematics. This research will provide a snapshot of a course that is designed to do just that, in the context of a class that is intended to be taught in a manner consistent with the standards of the NCTM (2000), the American Mathematical Association of Two-year Colleges (AMATYC) (2006), and the MAA (2004). This picture of such a course should provide insight into new ways of approaching integrated mathematics at the undergraduate and secondary level.

In addition to providing the broader mathematics community with more detailed information about a course that may benefit students, this research study has the potential to assist the course designers in identifying strengths and weaknesses in the new curriculum. The researcher hopes that this work will also be valuable in helping the designers evaluate the implementation of their curriculum as they move forward with their research on developing curricula for struggling mathematics students.

## Overview of the Following Chapters

The following chapters will provide further information on the research study. Chapter II will discuss the relevant literature on reform in mathematics, important documents pertaining to mathematics standards at the high school and college level, and the use of multiple representations in mathematics instruction. Chapter III will cover the design of the research project including an overview and background of the course that will be studied, and the setting, instructor and students involved in the research. In addition, Chapter III will discuss planned data collection and analysis, validity and reliability of the study. Chapter IV will include a description of collection and analysis of the data while Chapter V will summarize conclusions, possible study limitations and implications for further research.

# CHAPTER II: LITERATURE REVIEW

The questions addressed in this research study involve examining the impact of the implementation of a standards-based mathematics course on student learning, particularly how well the students using a reform curriculum are able to move between representations of algebraic ideas; an examination of how well the implementation of the course meets the intended goals of the designers; and a discussion of how well the course meets the standards for algebra instruction created by the NCTM, MAA, and AMATYC.

The literature review will address research in the following areas: (a) mathematics standards at the high school and college levels, (b) reform mathematics instruction pedagogy and content, and (c) the use of multiple representations in mathematics teaching. Chapter III will go into greater depth on the history and background of the design of the course.

## Mathematics Standards

In order to better understand the ideas behind the course that is the topic of this research, and the research question of its potential impact on student learning, it is important to provide background on the teaching of mathematics in general, and how it has changed over the years. The need for reform in teaching mathematics has paved the way for the design of courses such as the Introduction to Mathematical Sciences. The next sections will discuss this change in mathematics education.

### *How Math Has Been Taught*

Mathematics is a field that not only stands alone as an important program of study, but also serves as a supporting field to many other subject areas. At the elementary school level, students take mathematics first in order to learn to work with basic ideas of number sense, counting, adding and subtracting, multiplying and dividing, all of which adults will use often in their daily lives. Elementary school children also begin study of ideas of quantity, shape and size, and other geometry topics. In middle school and high school years, students are met with new concepts of generalized mathematics in algebra, as well as basic statistics, probability and more complex geometric ideas. Many of these concepts are necessary for students to make progress in other disciplines. Biology, chemistry and physics all require mathematics skills. Business and computer courses necessitate that students have the ability to work with formulas and data, as do many industrial technology and consumer sciences courses. Mathematics, therefore, does not exist in a vacuum. It is indispensable for success in the real world in a wide variety of disciplines (Ganter & Barker, 2003).

Unfortunately, mathematics is frequently treated as if it does exist in a vacuum (Schoenfeld, 2004). Mathematics courses are often isolated and disconnected. The traditional algebra, geometry, precalculus, calculus sequence in high school is an example of this separation of mathematical ideas also often severed from their real world applications. Many textbooks include applications, but as examples of the skills learned in a particular section of the text. A section on linear equations may have 10 word problems showing the use of linear equations in real world contexts. These problems in and of themselves are not bad things, but students quickly learn to pick out the key idea from the section and plug the numbers into the problem with little thought to the meaning of the problem itself. This isolation of mathematics topics does little to promote understanding of how to examine a real world problem and apply mathematics appropriately to the context (NCTM, 1989).

*Changing School Mathematics*

Teaching of rote procedural skills in mathematics versus conceptual understanding of mathematical ideas is a topic that has concerned mathematics educators for many years (Brownell, 1947; Hiebert et al., 1997; Fennema & Romberg, 1999). The need for change in how mathematics has been taught in the United States has become apparent with students' declining mathematics skills (Schoenfeld, 2004). Early efforts to change mathematics through the "new math" of the 1960s that addressed the rising need for mathematics in technology were less than successful. Ideas from topics such as Modern Algebra were too foreign to educators and parents, leading to a push for the back-to-basics movement. This led to more rote drill and practice in the classroom, and in many cases further declines in student conceptual understanding of mathematics. In the 1980s, the National Council of Teachers of Mathematics began to press for reform in mathematics education with the publication of its *Curriculum and Evaluation Standards for School Mathematics* (1989). Other standards volumes followed, through the most recent *Principles and Standards for School Mathematics (PSSM)* (NCTM, 2000). The ideas of psychology and cognitive scientists that helped clarify how students learn through making connections to prior knowledge were acknowledged by the K-12 mathematics community, and teaching for understanding, rather than speed and memorization has became more accepted (Schoenfeld, 2004). New curricula grounded in the NCTM standards and funded by the National Science Foundation have stressed teaching methods that encourage group work, communication, technology use and representing mathematical ideas in a variety of ways in order to promote student understanding (Senk & Thompson, 2003). Content focuses have also changed. Instead of teaching many mathematical concepts with just a cursory overview, the *PSSM* recommend teaching fewer topics in more depth. While these new style curricula, or hybrids of them, are becoming more common in school mathematics, there is still a long way to go in moving all teaching away from the disconnected and lecture style mathematics.

*Changing college mathematics*

Collegiate mathematics has been even slower to accept curricular reform and the attendant changes in teaching methods. Lecture continues to be the primary method of teaching students mathematics in many college settings (Lutzer, et al., 2005; Dossey, Halvorson, & McCrone, 2008). However, inroads have been made in the last two decades. The Tulane Conference in 1986 (Douglas, 1986) spurred reform in the teaching of calculus, when attendees agreed that "the calculus syllabus should be leaner, contain fewer topics, but it should have more conceptual depth, numerically and geometrically" (p. v).

Reform in calculus brought about new methods of teaching calculus, such as the Calculus, Concepts, Computers and Cooperative Learning ($C^4L$) program founded by Ed Dubinsky, Schwingendorf and Mathews of Southwest Michigan College (Schwingendorf, McCabe and Kuhn, 2000), which focused on a teaching mathematics from a constructivist learning perspective. Calculus books appeared such as those produced by the Harvard Consortium (Hughes-Hallett, Gleason, & McCallum, 1998), and others (Ostebee & Zorn, 1997), which focused on using multiple representations of mathematics ideas and real world applications to promote understanding. This was in contrast to the many traditional calculus texts that often present mathematics topics followed by 80 plus practice problems designed to promote speedy calculations of symbols. In these reform curricula, students often are required to explain their understanding, use technology, and interpret their answers in context.

Collegiate mathematicians have long considered calculus to be the introductory course for college students (Ganter & Barker, 2003). However, more and more students are coming to college needing or wanting lower-level courses. According to the *Statistical Abstract of Undergraduate Programs* there are nearly 1,000,000 students in the United States taking courses below the calculus level: 201,000 students taking courses considered to be pre-college level and 708,000 at the college level, but below calculus (Lutzer, et al., 2005). This is an impressive number of students who are often underserved at the college level, with college algebra students having up to a 50% DWF (D, withdraw or fail) rate (Baxter Hastings, et al., 2006). Some students choose not to follow the traditional calculus sequence at all, knowing that they are not pursuing a career requiring calculus while others have had difficulty with a particular discipline of mathematics, and become discouraged with their lack of understanding, or see little value in the mathematics they have taken in high school. Because of the low success rates of these students, the importance of addressing the needs of all students entering the college setting has become increasingly imperative.

Reform work in the area of precalculus college mathematics has followed slowly in the wake of calculus reform. In the late 1990s, mathematics instructors from across the country began meeting at the joint meetings for the American Mathematical Society (AMS) and Mathematical Association of America (MAA) to discuss the need for changes in the lower-level college mathematics courses, and reform to precalculus level courses began, with emphases on a number of areas that reflected similar philosophy to the NCTM standards. A variety of documents have been published regarding reforming college mathematics standards since then and have helped codify these ideas. These important publications are outlined in the following section.

*Documents in Support of Undergraduate Mathematics Reform*

The Mathematical Association of America (MAA) and the American Mathematical Association of Two-Year Colleges (AMATYC) have authored documents in support of undergraduate mathematic reform. These documents have influenced mathematics education at both the two- and four-year colleges.

*MAA publications.* The MAA's Committee on the Undergraduate Program in Mathematics (CUPM) is a body designed to make recommendations for mathematics departments at the collegiate level in order to help departments design their curricula (Committee on the Undergraduate Program in Mathematics of the MAA, 2004). Between 1999 and 2001, the CUPM began the study *Curriculum Across the First Two-years* (CRAFTY), a series of workshops with instructors from the partner disciplines (such as physical sciences, the life sciences, computer science, engineering, economics, business, education, and some social sciences) that inquired into what other departments might need students to know and understand in terms of mathematics. In these workshops, the other disciplines had their say while mathematicians were present mainly to clarify any questions and collect data. The eleven workshops gathered information from a wide variety of disciplines, and in 2003 the CUPM authored a report entitled *A Collective Vision: Voices of the Partner Disciplines* outlining the most important areas of mathematics as seen from outside of the mathematics departments (Ganter & Barker, 2003). The information gathered for the report was significant in that it helped the sometimes narrowly focused mathematics departments broaden their vision for undergraduate mathematics.

The recommendations stemming from the suggestions by other disciplines for mathematics departments teaching early college mathematics courses that were published in *A Collective Vision* were extensive, including emphasizing conceptual understanding, problem solving skills, modeling, communicating mathematically, and balance between mathematical perspectives. Content such as descriptive statistics, a greater emphasis on dimension and scale, two and three dimensional topics and real world applications of mathematics were suggested to be more important than complex algebraic symbolic manipulation. In terms of pedagogical techniques, the report recommended: "Use a variety of teaching methods since different students have different learning styles. In particular, encourage the use of active learning, including in-class problem solving opportunities, class and group discussions, collaborative group work, and out-of-class projects." (p. 6). Use of technology was also considered important, particularly the use of spreadsheets, as these are utilized in many areas across the disciplines. The report also stressed assessing student understanding using methods appropriate to what they have learned (Ganter & Barker, 2003).

In 2004, the MAA published the *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004* in which they made a series of further recommendations for mathematics programs. The MAA's board of governors accepted these recommendations, which were geared toward improving the mathematics courses designed for all students at the college level, not just those students with majors in mathematics. The recommendations included several main suggestions for mathematics departments:

1. Understand the student population and evaluate courses and programs.
2. Help students develop mathematical thinking and communication skills.
3. Communicate the breadth and interconnections of the mathematical sciences.
4. Promote interdisciplinary cooperation.
5. Use computer technology to support problem solving and promote understanding.
6. Provide faculty support for curricular and instructional improvement (CUPM, 2004).

The recommendations of the *CUPM Curriculum Guide 2004* detailed specific methods such as promoting use of classroom activities, approaching problems from multiple perspectives and using discussion to help students gain true understanding of mathematics rather than focusing on rote skill building. Use of technology was also recommended in the CUPM *Curriculum Guide 2004,* reflecting the suggestions of the partner disciplines.

Additional recommendations of the committee outlined specific approaches for students depending on whether they were taking courses that meet minimum requirements, majoring in partner disciplines (such as physical sciences, the life sciences, computer science, engineering, economics, business, education, and sometimes social sciences), or majoring in mathematics, as these groups of students vary widely in their mathematical needs. These recommendations showed that the MAA's CUPM was moving away from the focus on calculus as the priority of all students entering college.

For departments serving students intending to take the minimum mathematics requirements for their major, the CUPM (2004) suggested the following: (a) offer suitable course that engage students, increase quantitative reasoning skills, strengthen mathematical abilities applicable to other disciplines, improve student communication of quantitative ideas, and encourage students to continue to take mathematics; (b) examine the effectiveness of college algebra to clarify whether the course meets the needs of their students; and (c) ensure the effectiveness of introductory courses in preparing students for future classes and career areas by examining whether students succeed in future coursework (CUPM, 2004).

*AMATYC standards.* In 2006, the American Mathematical Association for Two-year Colleges (AMATYC) also produced a document entitled *Beyond Crossroads: Implementing Mathematics Standards in the First Two-years of College*, a follow-up document to the 1995 AMATYC publication *Crossroads in Mathematics: Standards for Introductory College Mathematics*. These works outline standards similar to those of the MAA, but with a greater focus on providing for the needs of students who may only attend college for two-years. The document states that it integrates recommendations from the NCTM's PSSM as well the CUPM guide. It outlines a variety of topics, including focusing on assessment, broadening students' career options through mathematics, improving equity and access, instruction using innovation in mathematics, utilizing inquiry in the classroom, improving quantitative literacy, showing students the relevance of mathematics, using research to back practices, and integrating technology (AMATYC, 2006).

*Agreement in the documents*

In general, these documents agree on the following recommendations for change in lower-level courses including the following, summarized by Baxter Hastings et al. (2006):

- lessen the traditional amount of time performing algebraic manipulations;
- decrease time spent executing algorithms simply for the sake of calculation;
- restrict the topics covered to the most essential;
- decrease the amount of time spent lecturing;
- deemphasize rote skills and memorization of formulas.

Recommendations for changes in pedagogy included the following:

- embed the mathematics in real life situations that are drawn from the other disciplines that mathematics departments serve;
- explore fewer topics in greater depth;
- emphasize communication of mathematics through discussion and writing assignments;
- utilize group assignments and projects to enhance communication in the language of mathematics;
- use technology to enhance conceptual understanding of the mathematics;
- give greater priority to data analysis than in traditional precalculus courses of the past;
- emphasize verbal, symbolic, graphical, and written representations of mathematical concepts and objects;
- focus much more attention on the process of constructing mathematical models before finding solutions to these models.

The statements of the NCTM standards, the MAA and AMATYC are all in agreement as to the need to move away from teacher-centered teaching to student-centered instruction. All suggest the need for active learning, less rote skill work, more discussion, use of multiple representations and use of technology in mathematics classes at any level.

The abovementioned ideas, central to the NCTM, MAA, and AMATYC standards, were all integral to the design of the Introduction to the Mathematical Sciences curriculum examined in this study. In order to clarify discussion in the rest of the paper, a definition of reform or standards-based instruction is given in the following section.

## Reform or Standards-Based Instruction

The term reform has been used in many situations, so it is necessary to discuss more specifically what reform means in mathematics instruction for the purposes of this research study. Documents have been cited above that suggest change is necessary in teaching mathematics, but further ideas associated with the broad concept of reform mathematics instruction, specifically regarding reform of instruction and content in the mathematics classroom, are outlined in the following sections.

### *Reform and teaching methods*

The instruction of school mathematics of the past (and sometimes present) has been characterized as teaching by telling (Smith, 1996). It has also sometimes been likened to the filling of an empty vessel. In this conception of teaching, instructors carefully plan presentations of material in which they demonstrate how to solve mathematics problems. Learning for students in this setting is based on students absorbing and understanding the materials that are presented. Little attention is paid to what the students bring to the classroom, in terms of either conceptions or misconceptions. The students' role in this setting is perceived as passive, while the teacher plays a more active role.

The tenets of mathematics reform as defined by the NCTM in the *Principles and Standards for School Mathematics* (2000) call instead for active learning by students. These ideas are based on a constructivist understanding of learning, in which students actively build upon prior knowledge, making connections between what they already understand and any newly encountered material. For the purposes of this paper, *reform or standards-based mathematics* will be defined as a approach to teaching in accordance with the process and content standards of the NCTM *PSSM* 2000 as well as being consistent with the recommendations of the MAA and AMATYC standards, that is, emphasizing teaching students how to understand and utilize mathematics in a meaningful way through the processes of problem solving, reasoning and proof, communication, making connections, and representations of mathematical ideas. Content covered in reform courses varies dependent on the high school or college grade level, but the processes are essential to providing students with the ability to work with mathematics throughout their lives. Because the course of interest in this research is geared for the high school as well as the college level, the notions of instructional reform that have followed from high school to college level must be considered. Although many of the abovementioned processes overlap, a brief outline of each process area as originally defined in the NCTM's PSSM (2000) follows.

*Problem solving.* Regardless of the level of mathematics being taught, problem solving has long been a focus of emphasis, albeit with some disagreement on what it means to problem solve (Schoenfeld, 1992). Problem solving has sometimes been approached using heuristics such as those developed by Polya (1957), which outline specific methods for solving problems. Some suggest, however, that only teaching problem solving strategies is too simplistic (Begle, 1979, as cited in Lesh, Zawojewski, 2006). More recent research has moved away from the heuristic point of view and more to a mathematical modeling perspective (Lesh & Zawojewski, 2006). From a practical point of view, problem solving is essential to mathematics. The real world uses of mathematics do not usually lend themselves to the type of cut and dried problem represented in a textbook. Practical applications of mathematics require that mathematicians be able examine data, see patterns and create mathematical models that represent those real world situations (Lesh & Doerr, 2003).

The NCTM PSSM (2000) suggests that students should be able to solve mathematical problems that arise in and outside of the mathematics classroom by choosing and using a variety of strategies to solve those problems. The application of prior knowledge to problem solving, use of inquiry or discovery, and reflecting on methods and models used to solve problems helps students build a set of tools that they can apply to new settings in the classroom and in the real world. Reform teaching includes a focus on applications of problem solving from the real world.

*Reasoning and proof.* The use of reasoning is vital in mathematics. From ancient times on, proofs have been used in mathematics to ascertain mathematical truths and to convince others of those truths. The uses of proof are not simply to convince others, however. Proof in mathematics is used to verify, explain, discover, systematize, provide intellectual challenge, and to communicate (de Villiers, 1999). How that is done involves a personal or societal proof scheme. Development in students from a more primitive example-based proof scheme to a more deductive type of proof scheme such as is common to those of contemporary mathematicians is a goal of instruction (Harel & Sowder, 2006).

During school years, students should learn to make conjectures and test those conjectures in the context of math problems and real world settings. Students of all ages should be able to formulate arguments based on mathematical rules and to be able to identify when arguments are weak or flawed (NCTM, 2000). Ability to argue a mathematical point shows a stronger conceptual understanding of mathematics.

*Communication.* The ability to communicate mathematics is also an important part of reform mathematics at both the high school and college levels (AMATYC, 2006; CUPM, 2004; NCTM, 2000). Quantitative literacy is essential in today's workplace and world, and part of quantitative literacy is being able to express mathematical concepts to others in a coherent manner (Madison & Steen, 2003). Just like any other field of study, mathematics vocabulary must be learned, and used appropriately. The symbolic language of algebra must also be learned. Without vocabulary and symbolic language, quantitative literacy cannot be expressed. In addition, for individuals, creating communication about mathematics does not only present ideas to others, but also helps solidify the mathematical understanding in the person communicating the mathematics.

In the classroom, students need to be able to talk and write about mathematics with teachers and peers. This communication of mathematics can help students clarify their own understanding while helping others gain a greater understanding through hearing a different perspective. The use of alternative explanations of solutions is supported in reform classrooms. Small group as well as large group discussions in the classroom can encourage students to express their understanding. Communication between students and between students and teacher, as opposed to simple “telling” by the teacher is one of the necessary ideas of reform instruction (Smith, 1996). Use of accurate mathematical language and vocabulary are essential to understanding and communicating mathematics (NCTM, 2000).

*Connections.* Too often mathematics has been taught as separate areas of study: geometry separated from algebra, trigonometry separated from calculus. However, the fundamental ideas of mathematics are not truly separate. Algebra is utilized in geometry, algebra and geometry in trigonometry and algebra, geometry and trigonometry in calculus. Emphasizing the relationships and connections between the branches of mathematics and to fields outside of mathematics is important in mathematics instruction because it can help students see mathematics as a coherent body rather than distinctly separate ideas (NCTM, 2000).

In the classroom, reform-minded teachers must use rich examples that show the connections between geometry and algebra. Utilizing a variety of representations for concepts that draw from the various branches of mathematics is essential for creating a coherent picture of what mathematics is for students. For example, an instructor might illustrate the geometric Pythagorean Theorem $a^2 + b^2 = c^2$, using algebra and geometry, proving the theorem using both geometric formulas from pictures and also using algebra. In calculus, the basic geometric formulas for area come into play in the idea of finding the value of an integral numerically. Making these connections helps strengthen the students’ understanding of the concepts across the mathematics curriculum.

*Representations.* Representing mathematics in multiple ways is a central part of reform mathematics. In higher education, reform calculus texts often refer to the Rule of Four. The Rule of Four emphasizes that mathematical ideas should be presented numerically, graphically, symbolically, and verbally (Hughes-Hallett, Gleason, 1998, Ostebee Zorn, 1997).

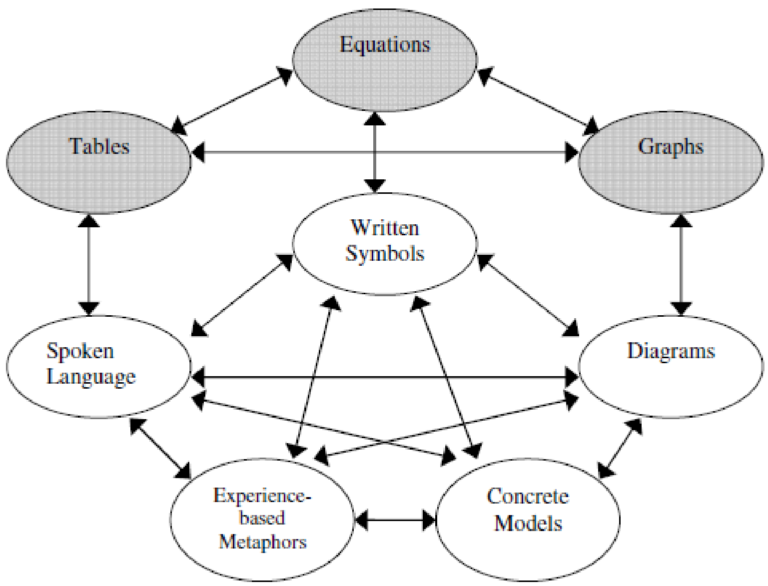


*Figure 1*. Lesh Translation Model

The translation model attributed to Lesh (1979) differs slightly from the rule of four, emphasizing five representations: real world settings, static pictures, spoken language, written symbols and manipulative models. Lesh's model includes tables, equations and graphs as parts of the static pictures and written symbols. If a student has the ability to move between mathematical representations, it is an indication that a student truly understands the mathematics behind the representations (Lesh, Post, & Behr, 1987). A student who can explain the salient features of a graph and write an equation or make a table of values for that graph more clearly understands the problem better than a student who can only represent a problem in one way.

The course that will be examined in this research study has a strong focus on teaching students using multiple representations (Webb, et al., 2009). Therefore, some of the research questions concentrate on looking at this very important aspect of the course, specifically in the context of algebra instruction.

*Reform and Content*

While the reform movement in mathematics instruction has involved change in how mathematics is taught, it has also influenced the content that is taught. The NCTM standards (2000), as well as the reform textbooks in calculus and college algebra have changed their focus from teaching mainly symbolic manipulation and a large number of topics to teaching fewer topics with a stronger emphasis on conceptual understanding and applications of mathematics. The concentration on fewer topics is intended to help students learn the mathematics in greater depth. The United States has been criticized in the past, particularly after the Third International Mathematics and Science Study of 1995, for teaching too many mathematics topics in too little depth, resulting in students knowing little about any of those topics. As a result, designers of reform curricula often reduce the number of topics taught in the course, in addition to changing the manner in which the topics are taught.

The emphasis in the 2004 NCTM *PSSM* content standards of number and operation, algebra, geometry, measurement, and data analysis and probability across all grade levels is on not only learning algorithms, but also on gaining deeper understanding about the mathematics. This, in many cases means concentration on fewer topics.

Similarly, at the college level, the calculus and algebra texts that are reform driven also strive to expose students to multiple ways of viewing topics (Hughes-Hallett & Gleason, 1998) in an effort to strengthen the understanding of students, rather than encouraging rote memorization of many symbolic methods of solving equations.

Research on Reform and Standards-based Curricula

Although there continue to be arguments as to whether traditional or reform curricula are more effective (Schoenfeld, 2004), evidence is mounting that students are provided with adequate mathematics skills and have increased mathematical reasoning power after having standards-based instruction with reform curricula. Senk and Thompson (2003) provided a useful overview of studies that have taken place regarding the K-12 curricula funded by the National Science Foundation. The studies covered the curricula for elementary and middle school as well as high school curricula. While some researchers in the text were criticized for their research designs, in her chapter on high school curriculum research, Jane Swafford concluded, “Taken as a group, these studies offer overwhelming evidence that the reform curricula can have a positive effect on high school mathematics achievement. It is not that students in these curriculum [sic] learn traditional content better but that they develop other skills and understandings while not falling behind on traditional content” (p. 468). Harwell, Post, Maeda, Davis, Cutler, Anderson, and Kahan (2007) also concluded that experience with reform or standards-based curricula does not hurt students’ achievement levels on standardized tests when variables such as socioeconomic status and other background variables were taken into account. However, it is important to realize that the teacher plays a role as well.

McCaffrey, et al. (2001) noted that students have greater learning gains when use of standards-based curricula is accompanied by appropriate changes in instruction methods.

A study done by Post, Medhanie, Harwell, Norman, Dupuis, Muchlinski, Anderson, and Monson (2010) also showed that although some of the lower-level ACT students who had studied reform curricula in high school did not necessarily enroll in the higher level courses initially, once they were enrolled in a college mathematics class, there were no significant differences in student grades in those classes between students who had taken more traditional high school courses as compared to students in the standards-based courses. In addition, they found no difference in persistence or course taking patterns between the two groups. So it seems that students who have taken NSF funded standards-based curricula in high school can perform as well in traditional college classes as students taught in traditional curricula, and it appears that in some cases they are gaining a deeper understanding of other mathematical ideas. At the college level, studies of reform based curricula (Hofacker, 2006; Goerdt, 2007) have shown that students enrolled in reform classrooms perform better than their peers in moving between representations of mathematical ideas. Further discussion of the importance of translating between representations follows.

## Importance of Multiple Representations in Mathematics

The important idea of the use of multiple representations in teaching mathematics is rooted in cognitive psychology. Bruner (1966) advocated a learning theory that stated that students should interact with mathematical ideas in three ways, enactive, or working with concrete materials; iconic, or working with visual representations or pictures; and symbolic, or working with symbols such as algebraic equations. Students must be able to make the logical connection between working with the physical objects or pictures to working with the symbolic representations. Pape and Tchoshanov (2001) stated that children represent mathematical ideas internally in order to make sense of concepts as they create schemata through experience. Numerals, algebraic expressions, graphs, tables, diagrams and charts are external representations of those ideas as well. Understanding of a mathematical concept involves an interaction, stated Pape and Tchoshanov (2001), between these internal and external representations.

According to Lesh, Post and Behr (1987), ease in moving between representations of mathematical ideas is an indication of greater understanding of the mathematics. Through their work with elementary students and middle school students in numerous grant projects, they found that not only is exposure to the different representations important, but ability to move, or translate between the representations is even more important. They determined that many students use multiple representations of a mathematical idea in order to solve a given problem. In fact, they noted that ability to translate between representations is part of being good at mathematics. “Good problem solvers tend to be sufficiently flexible in their use of a variety of relevant representational systems that they instinctively switch to the most convenient representation to emphasize at any given point in the solution process” (p. 39).

Sandoval, Bell, Coleman, Enyedy, and Suthers (2000) (as cited in Zbiek, Heid, & Blume, 2006) defined the term *representational fluency* as follows: “…being able to interpret and construct various disciplinary representations, and to be able to move between those representations appropriately. This includes knowing what particular representations are able to illustrate or explain, and to be able to use representations as justification for other claims. This also includes an ability to link multiple representations in meaningful ways."

Other studies have shown that use of multiple representations can be helpful in learning mathematics. Tchoshanov (1997) (as cited in Pape & Tchoshanov, 2001) conducted an experiment using high school students divided into three groups, a group that used strictly algebraic approaches to trigonometric problem solving, one that used strictly manipulatives and pictures to teach the same material and one that used a combination of the two, translating between the representational modes. The third group scored significantly higher on an assessment than both of the other groups, with the greatest difference between the purely analytic group and the translation group. He concluded that use of only one type of representation does not improve student conceptual understanding. He also found that the interaction between students was greater when they used a variety of representations, as they wanted to share their ideas with others. Pape and Tchoshanov (2001) used this and other research to support their three part theory: students must be given opportunity to practice with multiple representations; representation is a social activity; and that classroom instruction should use a variety of techniques to support use of a numerous representations.

In studying the use of representations in high school algebra, Friedlander and Tabach (2001) also noted that use of multiple representations must be supported in the classroom. They found that while high school algebra students were able to move between representations, they tended to use numerical representations over other types of representations. However, they stated, "We cannot expect the ability to work with a variety of representations to develop spontaneously. Therefore, when students are learning algebra in either a technologically based or a conventional environment, their awareness of and ability to use various representations must be promoted actively and systematically" (p. 175). This confirms that the style of instruction matters, and that the teacher plays a crucial role in helping students learn to move between representations.

At the college level, dissertation studies on reformed calculus have shown that students enrolled in standards-based reform courses are better able to move between representations of the derivative than those in traditional courses (Goerdt, 2007). In addition, Hofacker (2006) found that students taking reformed college algebra were better able to move between representations of linear and exponential functions than students in more traditional courses. Interestingly, Herman (2007) found that although students in a 10-week college algebra class determined that use of multiple representations deepened their understanding of the concepts, many felt that use of symbols was the "correct" way to solve the problem. Herman felt that this was based on their perceptions of what the instructor believed was appropriate. This indicates the important role of the beliefs of the instructor, as well as the teaching method.

*Technology and multiple representations*

The use of technology to aid in teaching with multiple representations has been studied by a number of researchers. Graphing calculators, geometry and algebra software, and computer programming are some of the tools that have been examined (Kaput, 1992; Zbiek, Heid & Blume, 2006). Technology can aid greatly in the use of multiple representations because of the possibility of creating dynamic and interactive representations. Recent work includes Balyta (2007) who examined the role of calculators (TI-84 Silver), the calculator based ranger (CBR), and Math Worlds computer programs in assisting students in translating between representations when studying algebraic functions. Using his results, he was able to make recommendations on how best to teach functions in algebra using technology.

Less work has been done on the topic of spreadsheet use and its relation to multiple representations in mathematics courses. Sutherland and Rojano (1993) looked at prealgebra student spreadsheet use in Britain and Mexico over five months. They found that students were able to learn to move from data-based spreadsheets to algebraic notation. Mitchell (1997) found that use of spreadsheets in statistics instruction seemed to result in greater student learning and greater student motivation. Alagic and Palenz (2006) studied middle school mathematics teachers in a professional development setting and their use of spreadsheets as a cognitive tool, and found that the teachers also gained a deeper understanding of linear and exponential functions through the use of the tool, more flexibility in moving between representations, and felt more prepared to deal with student questions following their experiences. Because the Introduction to Mathematical Sciences course that is the subject of this study takes place in a computer lab-based environment, it is important to be aware of the impact of technology in teaching reform courses.

## Summary

This review of the literature has discussed documents relevant to teaching according to the NCTM, MAA, and AMATYC standards. It has also examined the introduction of reform teaching in mathematics at the high school and college levels. In addition, the discussed research pertained to the use of and fluency with multiple representations, and use of technology use that supports that fluency.

The literature shows that use of multiple representations in teaching mathematics is helpful in learning mathematics, and technology can be an effective tool in teaching the use of those representations. Research indicates that standards-based reform curricula provide an environment that promotes the use of multiple representations, within their constructivist learning setting. The course to be studied in this research project has an intended goal of standard-based teaching using multiple representations in a computer lab locale. Determining how and whether the course is implemented in accordance with its goals is one of the objects of this research. The next chapter will outline research design and methods for the study.

# CHAPTER III: RESEARCH DESIGN AND METHODS

## Introduction

To better understand the methods selected to address the study's research questions, it is important to discuss the type of research design chosen for this study. The study involves the examination of an innovative mathematics curriculum and its impact on student learning. The type of research questions that the researcher is asking should and does drive the design of the study. The research questions for this study ask what the impact of this course is on student thinking and learning, particularly on learning the algebraic ideas of slope and linear equations, and the ability of students to move between representations of those ideas as well as how and to what extent the course meets its intended goals and reflects the standards of the NCTM, MAA and AMATYC. Research that can provide answers to these questions requires a specific type of design. The reasons for the selection of this type of design will be discussed below.

Research in mathematics education has often been experimental or quasi-experimental in design. In such experimental types of research, randomized or nonrandomized groups of students in control groups are compared to students who are included in some sort of treatment groups, such as classes of students being offered a particular instructional method. During this treatment, variables are manipulated in order to examine the impact of those manipulations (Campbell & Stanley, 1966). Students are usually given some sort of quantitative test (or pre- and posttest) and the test results are then statistically analyzed in order to determine if the students in the treatment group differ from than those in the control group. Randomized experimental research has been touted by those at the highest levels of government as being the recommended type of research for determining the effectiveness of curricular treatments (Coalition for Evidence Based Policy, 2003; National Mathematics Advisory Panel, 2008).

While the governmental entities have suggested that randomized experiments are the strongest types of research, many in the mathematics community have argued that research designs must fit the questions asked, the setting of the research, and the level at which the research object is being studied (NCTM Research Committee, 2009), particularly in the area of curriculum research (Clements, 2007; Schoenfeld, 2006).

Schoenfeld (2006) points out that there must be a relationship between the theory behind the research and the research method. Different questions and settings may be more appropriate to qualitative research than to quantitative. He argues that "the serious question to be considered is not, 'is this research of one type or another' but 'what assumptions are being made and how strong is the warrant for the claims being made?'" (p. 80). He states that what theory and conceptual model lie behind the research drive the questions asked, the data gathered, and the resulting conclusions.

## Research Framework

Schoenfeld (2006) has created a model in which he states that research on curriculum development follows these general processes: conceptualization, creation of a framework, use of a representational system, interpretation, and attribution to the original situation. He also specifies issues of trustworthiness, generality and importance as criteria for judging the quality of research. With these concepts, Schoenfeld claims, any research, qualitative or quantitative can be judged fairly by those who prefer either paradigm. He outlines phases of research that he deems necessary to complete effective research on curriculum development in mathematics education and that are loosely based on research in the medical fields. His first phase involves testing curricula in small samples, in order to identify which variables are important to examine in particular contexts. This is an early design type phase. His second phase focuses on trying the curricula in various settings to further examine those variables in more of the trial contexts. His third phase follows these first two, and it is only at that stage where curricula can be compared in randomized trials and it can be determined which curriculum to select for which setting, with which students and with what training and implementation depth for teachers.

The research presented in this paper fits partially into Schoenfeld's phase two. The curricular intervention being studied has been tested in several settings and after some revision is now being implemented in a wider manner. This study examines specific variables in an educational context that will be important to study more deeply in future randomized trials, at which time the curriculum could more appropriately be compared to other curricula. However, this research involves examining a single site, rather than multiple sites.

Clements (2007) has also put forward a framework on how to design research-based curricula, which may be better aligned with the research process of developing and evaluating this curriculum than Schoenfeld's framework. Clements notes that the lack of connection between research and curriculum development is one reason curriculum in mathematics education in the United States does not improve at the pace that it could. Curriculum should be based on research, and research that is scientific and transparent. However, scientific research can take many forms, and Clements notes that different levels of research on curriculum development require different approaches. His ten phase framework moves through three categories, from a priori foundations, through the learning model, and to evaluation. In the a priori category (phases 1-3), Clements states that research must be done first on what we know about the areas of content, pedagogy and general issues surrounding education in the field. The learning model category (phase 4) stresses the idea that curriculum should be consistent with models of student thinking and learning. The evaluation category (phases 5-10) includes phases of research that gather evidence to evaluate the curriculum, from individual and small group studies to large randomize field tests.

The research for this study fits to some extent into Clements phase 7 in evaluation category of his framework, that of formative research in a single classroom. This phase involves evaluating student learning, in order to make sense of how classroom activities are experienced by students. In addition, the whole class is observed to determine how effective the curriculum is as well as how usable it is (Clements, 2007). In this phase, the class is examined for effectiveness, not only in student learning, but also with implementation of the curriculum. This phase consists of qualitative research, rather than quantitative, as the goal is to determine what exactly is happening in the classroom. Clements recommends that the teacher of the course work closely with the course designers at this phase. While the current research study is not research undertaken by either the course designers or the teachers, close communication between the course designers and the instructor took place throughout the semester, with the researcher communicating with the instructor. The research undertaken by the course designers thus far has moved through a number of Clements' phases. The a priori category of research is complete. The designers have examined the subject matter, pedagogy and issues in mathematics education that would impact this curriculum. They pilot tested the curriculum, and have looked at student work in small groups and individuals. This study fits well with their current overview of the implementation of the course at a number of sites. It has a focus on one class, in one setting.

The research frameworks of both Schoenfeld and Clements influenced the design of this research, although the study does not fit perfectly into either framework. A reform mathematics course must be designed carefully with an eye to the needs of students, and also must be implemented as intended. Before larger trials comparing curricula can take place, the actual enactment of the curriculum in the classroom with real instructors and real students must be examined. The emphasis today on STEM education as exhibited by the Discovery Research K-12 program of the NSF also supports research that is grounded in innovative experiences in the classroom. The research in this study was designed to examine the implementation of one case of the Introduction to the Mathematical Sciences course in one classroom, as it occurred. The examination by an outside observer rather than the instructor or the course designers can help provide an unbiased view of what is happening in the course. If a course is not implemented as intended, research on the success of the course in terms of student learning outcomes compared to that of other courses is a moot point. If the course is determined to be implemented as intended, the effects of the course compared to other courses can then be studied. The researcher hopes that the results of this study are beneficial to the course designers and the instructor as they continue to refine the Introduction to the Mathematical Sciences course.

## Research Design: Case Study

This research study design is a single-case case study. The case study design is a form of qualitative or interpretive research (Cohen, Manion & Morrison, 2000). Using a case study allows the researcher to examine a phenomenon set in a bounded system in a real life context (Yin, 2009). The present research is undertaken as a case study design examining the case of the implementation of an innovative integrated mathematics course entitled "Introduction to the Mathematical Sciences." Embedded units of analysis include the teachers and students involved in the implementation.

Case studies focus on research questions that ask "how" or "why" rather than "who," or "what," in settings that are contemporary and not controlled by the researcher (Yin, p. 8). The aim of such interpretive research is just that: to interpret what is happening in the particular setting using the research questions as guides and based on a theoretical framework. According to Stake (1995), the researcher is a vital part of the research because it is the researcher who must "observe the workings of the case, one who records objectively what is happening, but simultaneously examines its meaning and redirects observation to refine or substantiate those meanings" (p. 9). Because a case study researcher must attempt to be objective in recording the events that unfold, it is important to recognize and set aside personal biases toward the research and research setting (Yin, 2009).

In a case study, there are many variables, yet one result must be reached. In order to bring meaning and coalescence to these multiple variables, the researcher must gather multiple sources of evidence, and triangulate the gathered data to identify themes reflected by that data. In this manner, the researcher can obtain answers to the research questions embedded in the researcher's design (Yin, 2009). In this case study, the research questions include:

1) What is the impact of a standards-based lab course integrating algebra, statistics, and computer science on two-year college students' mathematical thinking and learning specifically regarding algebraic learning and representations?
2) In what ways do students in this integrated standards-based laboratory class at the two-year college level demonstrate understanding through an ability to move between representations of algebra problems, specifically relating to the ideas of slope and linear equations?
3) How and to what extent does the course reflect fidelity of implementation of the course designers' vision of college reform in mathematics education for the two-year college?
4) How and to what extent does the teaching of this course reflect the NCTM, MAA, and AMATYC standards for college algebra reform in a two-year college setting in the areas of both content and pedagogy?

In an attempt to answer the above questions, this case study involved gathering data from a variety of sources. Data gathered included observations (using a modified Oregon Teacher Observation Protocol or OTOP), task interviews (planned for six students, although only four interviews were completed), formal and informal interviews with the instructor, and classroom documents and artifacts that included tests, homework, or classroom projects. Some classroom activity was video- or audio-recorded to ensure accurate recall by the observing researcher. Details of the data collection will be further outlined in subsequent sections. First, however, an overview of the course and participants will be provided.

## Overview and Background of the Course

This section will include background information on Introduction to the Mathematical Sciences course as well as the types of students who take the course. The course is a standards-based mathematics course that integrates algebra, statistics, and computer science in a computer lab setting. It is intended to be taught to high school students and students at the college level who traditionally have been less than successful in their mathematics courses. Further description follows.

### *Origins of the Course*

The course under study in this research project originated with instructors from Bemidji State University in Minnesota. To provide background for the stimulus behind the creation of the course, a brief overview of the basis for and development of the course follows.

The university that produced the course, Bemidji State University (BSU), is a medium sized liberal arts college in northern Minnesota, serving a population of approximately 5000 students. Originally a teaching college, the college is now fairly typical of a Midwestern liberal arts college.

Between 2001 and 2006, Dr. Glen Richgels, one of the professors at Bemidji State University in the department of Mathematics and Computer Science, examined the programs at BSU in order to determine the needs of students enrolled at the university in terms of mathematics courses. He found that 78% of the graduates needed one or more statistics courses to graduate in their chosen program of study while just 12% needed one or more calculus courses. He, along with several of his colleagues, determined that the "calculus model," which is the model of most high school mathematics curricula, was not preparing the average college student for needed coursework at the university level. The calculus model (as defined by Dr. Richgels and colleagues at BSU) at the high school level follows the traditional algebra, geometry, precalculus, calculus sequence. Often students who are not interested in pursuing particular hard sciences or who find mathematics difficult choose not to follow this sequence fully, instead taking only required courses in high school. Some of these students take no mathematics at all in their junior or senior years. This choice leaves many students underprepared for college and increases the number of developmental courses required at the college level. In addition, according to Dr. Richgels (personal communication, June 12, 2009), past studies at BSU have shown that students enrolled in the beginning college algebra courses who were tested with pre- and post-tests showed no significant gains in knowledge after a semester-long course as measured using a commonly utilized placement test. After his 2001-06 study, Dr. Richgels and his colleagues began looking at how they could create a course that could better serve the students both at the high school level and at the college levels.

In 2007, Dr. Glen Richgels, Dr. Derek Webb, Dr. Todd Frauenholtz, Ms. Ann Hougen and Dr. Marty Wolf, all faculty members at Bemidji State University, began designing an experimental course to better serve these students. They interviewed 24 faculty members from a wide variety of departments and programs ranging from theatre and physics to psychology and business administration in order to determine the mathematics skills that those faculty members felt their students needed in order to succeed in their majors. Through discussion following the interviews, the course designers distilled the skills into five main needs:

1. Students need to be able to read graphs and express meaning.

2. Students need to be able to understand and decipher word or contextual problems.
3. Students are already able to perform elementary manipulations, but should be able to manipulate equations with multiple variables so that the conceptual meaning of the variables and equation is maintained.
4. Students need to be able to communicate how they solve problems.
5. Students need to understand the differences in contexts of problems (G. Richgels, personal communication, June 12, 2009).

As solving equations and using variables was one of the identified needs, and because many college students struggle in algebra, algebra needed to be a focus of the new course. Many students are placed in developmental and pre-algebra courses upon college entrance. The need for statistics material was clear from the course designers' interviews with faculty in other departments. In addition, there is a need for students to be comfortable with technology and understand basic computer science topics. These are important skills for all contemporary students and can be utilized in many of the five areas identified above. The course designers determined that their best plan of attack would be to design a course that integrated algebra, statistics, and computer science into a single class.

*Content*

After extensive discussion of the needs of students, the designers outlined the various algebra, statistics and computer science topics that were to be included in the course. A listing of these topics appears in Table 1 (Webb, et al., 2009).

| Algebra Topics | Statistics Topics | Computer Science Topics |
|---|---|---|
| Functions<br>• Represented by formula, | Collecting and displaying data<br>• Types of data | Syntax and Semantics<br>Understanding Processes |

| | | |
|---|---|---|
| table, graph, words<br>Graphical and Tabular Analysis<br>• Tables and trends<br>• Graphs<br>• Solving linear equations<br>• Solving nonlinear equations<br>• Optimization<br>Linear Functions<br>• The geometry of lines<br>    • Linear Functions<br>    • Modeling data with linear functions<br>    • Linear regression<br>    • System of equations<br>Rates of Change<br>    • Velocity<br>    • Rates of change of other functions | • Creating data files in spread sheets<br>• Displaying data in tabular format<br>• Bar charts, histograms, pie charts, box plots, scatter plots<br>Populations and samples<br>Measures of central tendency<br>• Sample mean, median, and mode<br>Measures of dispersion<br>• Sample range, standard deviation, and inter quartile range<br>Shapes of distributions<br>• Skewness, symmetry, and modality<br>Correlation and association<br>Introduction to linear regression | • Describing processes used to solve specific problem<br>• Generalizing processes to solve general problem<br>• Converting processes into computer solutions<br>The notion of a "variable" in computing<br>• Variable names, references, and values<br>Formulas and expressions<br>• Operations, evaluation order, results, and errors<br>Making decisions<br>• Logical and rational operators and their values<br>• Conditional syntax<br>• Conditional semantics<br>Using functions<br>• Function syntax and semantics |

*Table 1*. Content of Introduction to the Mathematical Sciences

According to the course designers (Webb, et al., 2009), the course title of "Introduction to the Mathematical Sciences" was chosen by the researchers to both reflect the level and content of the course. The course was specifically designed for learners that are typically not as successful in traditional mathematics classrooms, have negative attitudes about mathematics, do not perform well within traditional mathematics pedagogies, or have weak/negative histories in mathematics classes. The class also provides statistics and computer science subject matter that students have likely not seen in their past. The course creators wanted to reach those students who were not being served by the calculus model and traditional mathematics pedagogy, at both the high school and college level. Therefore, the course was designed to be taught to seniors in high school as well as college students who needed to improve their background in mathematics prior to taking further coursework in their chosen program of study. Bemidji State University faculty members originally petitioned to have the course included on an experimental basis for one year as fulfilling a liberal arts requirement when they piloted the course at BSU in 2007 in order to encourage enrollment in the course while it was being piloted. The course is now an annual offering in the BSU department of Mathematics and Computer Science.

*Support for the Course and Research on the Course*

The course has also been piloted voluntarily at additional colleges and high schools in Northern Minnesota over the past two-years through grant support from the Blandin Foundation and the Minnesota State Colleges and Universities (MnSCU) Office of the Chancellor. Substantial grants from the Blandin Foundation were awarded in September 2009 as part of the Northern Minnesota College Readiness Partnership grant to the MnSCU system. These funds are awarded for college and high school programs that can address underserved populations and sustain long term success. Additional grants have been awarded through MnSCU to implement this course in schools throughout northern Minnesota (Webb, et al., 2009). This monetary support indicates that this course has been considered a worthwhile project and deserving of additional research.

Preliminary research reflecting improved student achievement through pre- and post test analysis, as well as results of student and teacher evaluations, have been presented at the Minnesota Council of Teachers of Mathematics Spring Conference (Duluth, MN, 2009) and a paper outlining this research was presented in Dresden, Germany, at the Tenth International Conference on Models in Developing Mathematics Education in Fall 2009 (Webb, et al., 2009). Early favorable results indicate that the course is successful at helping students who have struggled in the past to make significant gains, and further research continues.

*Pedagogy*

According to the course designers (Webb, et al., 2009), the course is designed ideally to be taught in a computer lab, using spreadsheet software (most often Microsoft Excel®) as a medium for many of the applications and calculations in the course. When taught at a college, the course is taught as a "lab" course and has twice the contact hours of a traditional 3 credit mathematics course. Because students taking this class at a high school typically have more seat time additional material has been added to the high school version of the course. Typically, this material is institution (and even class) specific and focuses in the weaknesses that that class may have in mathematics. Each day, students are ideally taught an integrated mixture of algebra, statistics and computer science.  Students see the relevance of each of the three mathematical science topics and how they complement each other.  Topics are not taught in isolation without any connections to other topics or practical applications.

The pedagogy of the course departs radically from traditional mathematics instruction and, often, experienced instructors take many weeks to become comfortable teaching this new course. The pedagogy of the course is strongly rooted in the ideals and recommendations of the *Principles and Standards for School Mathematics* (2000) of the National Council of Teachers of Mathematics. Group work, projects, multiple solution paths, student presentation and student discussion are integral to the course. The integrated nature of the course subject matter with its focus on algebra, statistics and computer science is designed to engage the students in real world problems that require multiple representations of the problems. Solving the problems using a variety of representations (verbal, concrete or pictorial, graphical, algebraic, and tabular forms) is an important part of the course. Ability to move between these multiple representations is stressed in the teaching of the intended curriculum along with explaining or communicating solutions in written and verbal format. Students are encouraged to share novel ways of solving problems, rather than having the instructor show one “right” way of solving problems with students mimicking the teacher’s actions.

The course philosophy reflects a constructivist philosophy of learning through which students build upon their prior knowledge through inquiry style learning. If students have weaker skills in a particular area, the instructors of the course are to help students identify what they already know and through exploring the topics of the course, enhance that knowledge base.

The course designers outlined several pedagogical focuses to complement the content focuses in a paper presented at a conference in Dresden, Germany, during the fall of 2009 (excerpt from Webb et al., 2009):

- Algebra, statistics and computer science topics are all presented in the context of real-world problems taken from many disciplines. This is especially critical in the teaching of algebra topics. In this class, algebra is not taught as a set of rules and symbol manipulation skills, which is what students typically see in traditional algebra classes. Students readily see the applicability of the algebra topics they are studying.

- Algebra, statistics, and computer science topics are interwoven and not taught in isolation. The course content is not three topics taught separately. Rather, it is three topics taught in concert making use of natural relationships. Students understand how the “mathematical sciences” is a cohesive discipline, not silos of information.
- Most algebra, statistics, and computer science topics are taught using spreadsheets. Students are much more engaged in the learning of algebra topics using spreadsheets and they also have a much better understanding of, and need for, proper order of operation and algebraic syntax.
- Students spend at least half their class time in a computer laboratory environment. This pedagogical aspect of the course depends on the available facilities the school. If possible, it is preferred that the course be taught entirely in a computer laboratory environment. If not, students should spend at least 50% of classroom time in a computer laboratory.
- The classroom time commitment for this class is approximately double that of a typical three credit college algebra course. This is very important because it allows enough classroom time for students to work together on their own, in student groups, and with the instructor to complete the majority of their “homework.” That way, they know they are being successful and do not struggle in isolation at home. This ensures that the majority of work is completed and students remain engaged in learning.

*Example Lesson*

An example lesson from early in the Introduction to the Mathematical Sciences course is described below to provide the reader with an opportunity to see how the classroom learning progresses. Students are presented with the following problem:

> You are part of a construction company that is supposed to build houses. An architect has left plans to build houses on islands. Bridges connect the islands. Island A and Island B have a total of 15 houses. Island A and Island C have a total of 17 houses. Island B and Island C have a total of 12 houses. A total of 22 houses are to be built on the three islands. Determine how many houses are to be built on each island.

The students are allowed to work on this problem for a time (amount depends on how the students progress). Once the students have shown some progress, they are asked to discuss, display or report on their solution methods. Emphasis is placed on solving the word problem in four ways: concretely using some type of manipulative, using vertex edge graphs, using algebra, or using tables in spreadsheets. If students as a class have not discovered all of these methods, the instructor may show students how they can use the spreadsheet, or graphs, for example. Use of multiple representations is discussed in class and students are shown the model of multiple representations. Students are then given additional island/house problems with different numbers to work on. This lesson may take two to three days; and student work, questioning and discussion eventually elicits, from even the weakest algebra students, a symbolic method that can be used to solve this three island problem. The emphasis in the lesson on the different representations is to help students more deeply understand the mathematics that leads to the solution. On the other hand, more traditional teaching may have emphasized the use of the formulas alone, with less conceptual understanding and no student discussion and presentation. As a follow up, students are later quizzed on a similar problem and their ability to solve this in multiple ways, including on the spreadsheet. The entire lesson with follow up may take three or four days to complete. However, by the end of this time, students will ideally have gained a facility with solving systems of equations.

## Setting

This research project examined the implementation of the Introduction to the Mathematical Sciences course in the setting of a two-year technical college. The course was taught as a for-credit college course, intended to serve students who have traditionally struggled in mathematics. The following sections describe the setting, students and instructor.

Small Technical College (STC) (pseudonym) is a small technical college in the Midwest. It is typical of technical colleges of its size in the Midwest in that it serves both traditional and nontraditional age students in a variety of programs ranging from nursing to the mechanical trades. The college offers associate of arts degrees, diplomas and certificates in a wide variety of areas to 1000+ students. Many of the courses at STC are included in the transfer curriculum and can be transferred to four year colleges. The students, according to faculty at the college, are usually taking mathematics courses as requirements for their fields of study. The range of student ability varies, as does the motivation of the students to learn the material. Nontraditional age students appear to be more motivated and willing to ask questions than traditional age students (Mr. Smith, personal communication, June 22, 2009). The Introduction to the Mathematical Sciences class was first offered at STC in the spring semester of 2009. There were two sections of the class offered, one with approximately 15 students and the other with approximately 40 students enrolled.

The Introduction to the Mathematical Sciences course observed at STC in the fall of 2009 was taught as a 3 credit course, instructed entirely in the lab, from 3:55PM to 6:55PM two days a week. The course work was designed to be completed in the lab, with little or no homework (by design), so attendance was vital to the course. In the fall 2009 course, homework, projects, and attendance were 20% of the grade, as the instructor noted that discussion is an essential part of the class and those students who do not attend cannot discuss or learn from their peers (personal communication, June 20, 2009). Other details of the grading scheme include 15% for quizzes, and 40% for exams. Exams included pencil and paper tests, and project type assessments, often performed on the computer.

## The Instructor

The instructor, who will be referred to as "Mr. Smith" for the purposes of identification in this report, who taught the course at STC has over 35 years of junior and senior high school teaching experience. He taught at a variety of schools, instructing courses ranging from 7th grade math to Calculus II. After retiring from high school teaching, he began teaching college courses at the technical school. He possesses a masters' degree in mathematics education and has worked with the BSU instructors in the past. For all of his classes, including the Introduction to the Mathematical Sciences course, he focuses his instructional approach on teaching for understanding using nontraditional reform methods of teaching. He follows the tenets of the NCTM standards, and that he places a strong emphasis on helping students learn to express their understanding of mathematics and to explain their reasoning.

Mr. Smith's stated goal for the Introduction to the Mathematical Sciences course was to help all students move from their current knowledge base to a deeper understanding of mathematics over the course of the semester. He stated clearly that he did not expect all students to advance at the same pace, nor to reach the same level of mathematical understanding, but to show that they had made gains and built on their prior understanding. In general, his philosophy of teaching represents a constructivist bent toward how students learn and he looks for gains in all of his students regardless of background or ability. Past teaching experience, he noted in an interview (personal communication, June 22, 2009), has shown him that teaching for understanding works with high school and college students and that in both college and high school settings his students made gains equal or greater to those in traditionally taught courses. He has used technology extensively in his past mathematics courses and also taught the Introduction to Mathematical Sciences course in the spring of 2009. He planned to build on what he learned from teaching in the spring as he taught the Fall 09 course (Mr. Smith, personal communication, June 22, 2009).

*Training the Instructor*

Two course designers, Dr. Webb and Dr. Richgels, worked closely with Mr. Smith during the spring semester of 2009 when he first taught the course. Mr. Smith was mentored during the teaching of class and outside of class on pedagogy and content weekly through site visits by Dr. Webb and Dr. Richgels. Dr. Webb and Dr. Richgels would also discuss with Mr. Smith the goals of various lessons and what level the students could be expected to progress from and to. Key ideas were emphasized in these meetings, but Mr. Smith was allowed to use his teaching experience with use of non-traditional methods in order to determine how to best instruct the course. He reported in an interview (personal communication, June 22, 2009) that rarely did he lecture during his teaching of this course in the spring of 2009, using instead inquiry-based methods; asking students to attempt problems and then discuss how they reached their conclusions. Often, he stated, he found that often the nontraditional age students were more prone to discussion than the traditional age students. Mr. Smith speculated that this may be the result of the traditional age students' more recent high school mathematics backgrounds where they expected to be told what to do and how to do it rather than reasoning about how or why things are done in mathematics. He noted that his meetings with Dr. Webb and Dr. Richgels helped him anticipate student misunderstandings and determine some best methods for drawing out understanding of particular concepts such as slope units, correlation, measures of variability, or differences between the concepts of ratios and rates. Through this process, he reported that he determined areas he would change for his teaching of the course in the fall 2009 (personal communication, June 22, 2009).

## The Students

At Small Technical College the number of students participating in the study was smaller than expected, with an enrollment of only 16. The students were primarily male, with only four female students in the class. The ages ranged from 18 to 57, with only four of the students in the traditional age group of 18-19. Students were studying a variety of fields ranging from nursing to manufacturing and engineering technology to accounting.

Some of the students were taking part in displaced worker programs. These programs pay for reeducation of workers after industries where they were employed have failed.

## Data Collection

Data collection began in mid-September following Internal Review Board approval from the University of Minnesota and permission from the college. A variety of instruments were used during the data gathering process. These instruments and their purposes are outlined in Table 2, and explained in more detail in the following sections.

| **Instrument** | **Purpose** |
|---|---|
| Consent Form | To obtain informed consent from participants |
| Revised Oregon Teacher Observation Protocol (OTOP) | To identify ways in which reform teaching was implemented |
| Student Interviews | To determine how students were able to utilize and move between multiple representations of algebraic ideas, particularly slope and the use of linear equations. To determine how students were approaching the problems in order to identify their thinking processes. |
| Instructor Interviews (informal and formal) | To determine if the instructor felt the goals of the course are being met and if course implementation was proceeding as desired |
| Classroom Artifacts | To examine on what goals/objectives students were being assessed and to discover which algebraic ideas were being covered throughout the course as compared to the intended topics and goals of the course |
| Recordings of Group Discussion | To determine how well students were able to translate between representations, and how well they were meeting objectives of the lessons, as well as determine whether the course was being taught in a manner consistent with the reform ideals and standards |

*Table 2*. Data Collection Instruments

*Consent Form*

The researcher provided consent forms on the first day possible, about two weeks into the class, and explained the research to the participants, answering any questions that arose in the course of the explanation. Students who agreed to participate and who were of the age of majority signed, dated, and returned the form. There were no minors and all forms were returned. Students were allowed to return the form to the course instructor or the researcher on the researcher's second visit in order to provide students opportunity to consider their participation and not rush them in decision making. However, all students quickly signed and returned the forms. The instructor was also asked to sign a form. The researcher also signed and dated the forms. The forms were stored electronically as soon as feasible, following which the paper copies were destroyed. The researcher provided blank copies of the form for the participants' future reference. See Appendix A for the consent form.

*Oregon Teacher Observation Protocol*

A number of instruments have been created in mathematics education research in order to record observations of reform teaching. One observation protocol used to measure how well reform teaching is implemented is the Oregon Teacher Observation Protocol (OTOP). This protocol was designed specifically for a National Science Foundation (NSF) project to examine reform teaching practices in college science and mathematics courses (Wainwright, Flick, Morrell, & Schepige, 2004). The instrument has been tested for validity and reliability (Wainwright, Flick, & Morrell, 2003). This protocol examines the pedagogy used in the classroom in order to determine if the ideals of reform teaching are being implemented. Using this instrument, the observer records the events that take place in the classroom, as well as noting whether various facets of reform instruction are occurring, from small group interaction to technology use or support of alternative solutions and so on. A revised OTOP (See Appendix B) was used for observations in this study in order to answer the question of whether the course designer's vision for reform instruction was being implemented in the classroom. It was also used to determine if the class was aligned with the standards of the NCTM, MAA, and AMATYC, that is, to answer research questions:

> 3) How and to what extent does the course reflect fidelity of implementation of the course designers' vision of college reform in mathematics education for the two-year college?
>
> 4) How and to what extent does the teaching of this course reflect the NCTM, MAA, and AMATYC standards for college algebra reform in a two-year college setting in the areas of both content and pedagogy?

Modifications made to the OTOP for this study were minor, including elimination of some columns of the grid, some numerical coding, and instructions on how to use the OTOP, as the researcher was the only one utilizing the tool.

Note that the OTOP focuses on the teacher as well as students, and on how the course is implemented. This was intended to help answer the research question regarding how/whether the goals of the course are being met. The OTOP was used to record how well reform teaching was implemented in the classroom. Since this course was designed to be taught from a reform or standards-based perspective, this type of teaching was one of the goals of the course. Although the OTOP includes assigning numerical scores to the teaching, this is not a quantitative study. These data were only used as a guideline to help determine whether the goals of instructing in a manner consistent with reform ideals were being met. The emphasis in a case study is on providing rich descriptions of what is occurring, not to quantify.

*Classroom Artifacts*

It was important to examine the types of materials students utilized in this class in order to help determine the answers to the research questions regarding student ability to move between representations, as well as to discern which topics are being covered in the class. Therefore, example work or classroom artifacts were gathered from the class. These artifacts ranged from instructional handouts to test and project work. The focus was on gathering items pertaining to algebra that reflected the use of multiple representations, but all handouts were collected by the researcher, as well as select copies of work completed by students. Test examples were also collected, in order to determine if the classroom assessments were assessing what the students were purported to be learning, i.e., to help determine whether the topics were implemented as intended. Select samples of students' projects were collected. These data were used to help answer all four of the research questions:

1) What is the impact of a standards-based lab course integrating algebra, statistics, and computer science on two-year college students' mathematical thinking and learning specifically regarding algebraic learning and representations?

2) In what ways do students in this integrated standards-based laboratory class at the two-year college level demonstrate understanding through an ability to move

between representations of algebra problems, specifically relating to the ideas of slope and linear equations?

3) How and to what extent does the course reflect fidelity of implementation of the course designers' vision of college reform in mathematics education for the two-year college?

4) How and to what extent does the teaching of this course reflect the NCTM, MAA, and AMATYC standards for college algebra reform in a two-year college setting in the areas of both content and pedagogy?

*Recordings of Discussion*

Discussion is an important part of the intended curriculum. Recording how that discussion proceeds is of value to the researcher in determining whether the goals of the course are being met. In addition, recordings of discussion in the classroom can help identify what types of algebraic representations the students are using, and to what extent they are using those representations, in order to help answer the two research questions regarding fidelity of implementation and alignment with standards as well as research questions one and two.

*Interviews*

In this study, both the students and the instructor were part of the case being examined. Therefore, it was vital to look at both perspectives in order to determine if the goals of the course are being met. This was delved into through the use of interviewing.

*Interviews with the instructor.* The instructor was interviewed briefly and informally either before or after each visit by the observer. This was done to gauge how the instructor perceived that the course was progressing and to determine the topics to be addressed in each class. The researcher investigated how successful the implementation seemed to the instructor through informal discussion. A brief summary was written up after the observation and included as part of the OTOP form. A more formal interview took place later in the course after the researcher had opportunity to observe the course instruction multiple times. Questions for the formal interview were developed over the course of the semester and administered after the course ended. The questions centered on how the specific algebra topics were developed and implemented in the course and areas where multiple representations of algebraic ideas were used, as well as how students responded to those multiple representations. Additional questions delved into how the course progressed overall and the instructional techniques the instructor was using. Interviews about how the instructor was implementing the course were intended to help identify whether the course goals were being met and whether the course was in line with standards for instruction. See Appendix C for Instructor Interview Protocol.

*Task interviews with students*. During the course of the study, specific task interviews were developed to address the algebra topics covered in the course. While six students were selected for the task interviews, only four students completed the interviews, due to absences and other scheduling problems. Students were interviewed whose skill levels were high (above 18), medium (13-17) and low (below 13), as based on scores on the 25 question introductory algebra test given in the class as part of the course designers separate research project. This was not a test designed for this research, and was simply used as a guideline for selecting students for interviews. Of those students who completed the interviews, two had low scores, one medium and one high. The majority of the students in the course scored in the low and middle range on this test.

These students were interviewed outside of class at a time convenient for them (before or after class). The task interviews were approximately 20-30 minutes long, and focused on questions that assessed how well the students were able to move between representations of algebraic ideas.

The specific questions for the student task interviews were developed throughout the course by the researcher. The first three questions of the interview focused on how the students perceived that this class was different from the classes they had taken in the past, in terms of algebra content, differences in material in general and differences in teaching style. These questions were intended to help determine if the course was being implemented as intended.

The other questions in the task interviews focused on how students were able to solve algebra problems and move between representations. Question Four examined how well students were able to interpret a graph of distance versus time and discuss the idea of slope in the graph, and its meaning in that context. Question Five involved solving a single variable equation with the variable on both sides of the equation. Question Six was a real-world context word problem that required writing and solving a system of equations. The focus was on determining if the students could recall that type of problem and interpret it correctly, as well as to see what types of solution methods they chose to use after being exposed to a number of possible methods. Question Seven focused on how well students were able to move from a table to a graph, with follow-up questions to determine if the student could explain and interpret slope in a real-world context. In addition, the students, if they could interpret the question, were asked to write an equation that represented the situation, or a similar situation. Question Eight, the final question, asked students to interpret a simple proportional word problem and explain how they would find the solution.

All of these questions were intended to discover how students were thinking about and recalling the algebra material, and how well they were able to translate between representations in the different scenarios with a focus on slope and linear equations. Students were allowed to use pencil, paper, calculators or spreadsheets while working out their solutions.

Important to note is that the interview tasks were piloted prior to use in the Introduction to the Mathematical Sciences class with three students in an unrelated beginning algebra class to test the types of answers the students provided. The first three questions did not collect applicable data, and the students' answers to the other questions resulted in minor revisions in the wording of the questions. See Appendix D for Student Task Interview Protocol.

## Data Analysis

Data analysis began following the conclusion of the fall 2009 course at STC. Classroom observation OTOP forms, discussion recordings and field notes, interview transcripts and classroom artifacts were used to analyze the case. The data were collected in a database in electronic form as much as possible, primarily in pdf form. In this way, the data were accessible to the researcher, stored securely, and stored in one place.

According to Yin (2009), using multiple sources of evidence can help the case study researcher identify converging lines of inquiry that may show support for themes in the data. In order to determine if the goals of the course were being met, it was necessary to triangulate the data. Triangulating data means examining multiple sources of data in order to see if the data are in agreement on any themes. If data disagree, it is important to examine why this may be the case as well. If the data all converge on specific facts, the researcher can be assured that the facts are more likely to be accurate.

Within the case, the researcher used open coding to identify concepts or themes that occurred in the data sources. Miles and Huberman (1994) suggest classifying and organizing data into manageable pieces to avoid data overload. Therefore, each source of data was first systematically analyzed and coded, then cross-source analysis was used. For the observation data, the OTOP was analyzed by category to determine which aspects of reform teaching were most prevalent in the course. In addition to OTOP data, the researchers' observation notes were coded for themes of reform teaching or other themes that arose in the course of the class periods. Similarly, field notes of data from class discussions were coded for themes in the discussions, with a focus on the NCTM process standards of problem solving, reasoning and proof, communication, connections, and representation as well as content themes. A greater emphasis was placed on looking for instances of the use of multiple representations in class than on the other process standards in order to answer the research questions.

Following observation analysis, each interview transcript was read through multiple times in a search for larger meanings and coded in detail for units of meaning relevant to the research questions. Hycner (1985) suggested that looking for both the large themes and those unique to the interview helps the researcher condense what the participant said while still maintaining the participant's perspective. Each student's response to individual questions was recorded in a table, with researcher comments.

Classroom artifacts were examined for two purposes: to identify student thinking and learning through work on particular problems, and to determine which topics were being covered in the class. In this manner, the researcher could discover whether the goals of the course for content were being met, and could identify how students were thinking about the problems and working with different representations of algebraic ideas.

After individual sources were analyzed for instances of reform teaching and learning and use of multiple representations, all data sources were examined at from a broad perspective (cross-source analysis) to see which themes are prevalent.

In terms of subject matter, the list of algebra subject matter outlined by the course designers was used as a guideline in looking at how algebra was implemented in the course. Each occurrence of the algebra subject matter was recorded along with the source and date of that occurrence. Sources may include student worksheets, discussion notes, class projects, and interview data. Once data from each source was coded for the algebra topics, again an overview of all the data was used to determine if some topics in the algebra subject area were being neglected in the implementation of the course.

Finally, all data were reviewed in order to look for common themes within the various data pieces to determine if any themes stood out after all of the more detailed analysis had already taken place. Movement from a search for individual themes to common themes in the data is consistent with Hycner's views of analyzing interview data (1985), and Yin's (2009) suggestions for data analysis.

## Validity and Reliability

Two important factors that must be considered in any research project are the factors of validity and reliability. In regards to validity, the researcher must be concerned with construct validity, internal validity, and external validity. Yin (2009) suggests that these concerns can be addressed using various tactics during a case study. To address construct validity in this study, methods were used that included utilizing multiple sources of evidence, keeping a record of the chain of evidence via a database of the collected data, and having key participants review portions of the report prior to publication. To address internal validity, the researcher used pattern matching or cross source analysis between data sources to examine rival explanations that might impact the implementation. To address external validity, examining the implementation of the course as it is occurring is a major interest in the study. Theoretically, the course is a standards-based or reform course, and the course should have the required elements of reform teaching. Similar implementations of the course in similar settings should result in similar themes. However, determining how well that implementation is occurring is the one of the objects of this study.

Reliability, or the likelihood that the operations of the study can be repeated with the same results, is addressed through the use of the case study outline or protocol for data collection. The site has a specific numbers of interviews, with students chosen for a specific reason, and specified types of observations and artifact collection. Interviews involved asking the same questions (with the exception of follow-up questions), and observations all used the OTOP. The data were stored in a database that could be examined by others. Yin (2009) suggests that these utilizing these methods will help strengthen the reliability of the study.

The following chapter reports on the results of the analysis of the data, and Chapter V includes a discussion of the conclusions, limitations and suggestions for future research.

# CHAPTER IV: DATA ANALYSIS

This chapter outlines the results of the qualitative analysis of data performed during the study. The purpose of the study is to examine the impact of the Introduction to the Mathematical Sciences course on student thinking and learning regarding algebra, particularly the ideas of slope and linear equations, to determine to what extent students are able to move between representations of mathematical ideas, and to determine if the course is being implemented according to the course designers' vision for reform and the standards of the NCTM, MAA and AMATYC. By gathering data in a variety of forms including student task interviews, instructor interviews, the Oregon Teacher Observation Protocol and classroom observations and artifacts, and triangulating that data, the researcher was able to make inferences that help answer the research questions.

The research questions and data used to help answer these questions are as follows:

1) *What is the impact of a standards-based lab course integrating algebra, statistics, and computer science on two-year college students' mathematical thinking and learning specifically regarding algebraic learning and representations?*

   Data used to answer this question: Student interviews, classroom artifacts, observations.

2) *In what ways do students in this integrated standards-based laboratory class at the two-year college level demonstrate understanding through an ability to move between representations of algebra problems, specifically relating to the ideas of slope and linear equations?*

Data used to answer this question: Student interviews, classroom artifacts and observations.

3) *How and to what extent does the course reflect fidelity of implementation of the course designers' vision of college reform in mathematics education for the two-year college?*

   Data used to answer this question: Student interviews, instructor interviews, classroom artifacts, observations and OTOP data.

4) *How and to what extent does the teaching of this course reflect the NCTM, MAA, and AMATYC standards for college algebra reform in a two-year college setting in the areas of both content and pedagogy?*

   Data used to answer this question: Student interviews, instructor interviews, classroom artifacts, observations and OTOP data.

Each question is addressed in this chapter, along with accompanying data analysis that provides support to help answer the questions.

Because the first two research questions are concerned with the impact of this course on student thinking and learning, the portions of the data analysis that address these two questions are presented together. Because the next two research questions deal with how the course is implemented and how it relates to reform teaching and national standards, the pertinent data analyses that address those questions are also presented together.

Research Questions 1 and 2

1) *What is the impact of a standards-based laboratory course integrating algebra, statistics, and computer science on two-year college students' mathematical*

*thinking and learning specifically regarding algebraic learning and representations?*

2) *In what ways do students in this integrated standards-based laboratory class at the two-year college level demonstrate understanding through an ability to move between representations of algebra problems, specifically relating to the ideas of slope and linear equations?*

Data used to answer these questions included student interviews, classroom artifacts, and observations. Each of these sources helped provide insight into how students were impacted by taking this innovative mathematics course, Introduction to the Mathematical Sciences, and how they were thinking and learning about algebra. The focus is on how the students learned to work with and move between a variety of representations of the algebraic ideas of linear equations and slope. The next section outlines how each of these data sources assisted in answering these questions.

*Analysis of Task Interviews for Research Questions 1 and 2*

Because the majority of students taking this class had struggled with mathematics, it was important to explore what impact the course had on students and student thinking regarding algebra and representations of algebra. The task interviews provided the researcher an opportunity to get an in-depth examination of student views on the class and student ability to work problems related to the material presented in the course. Students were first asked three questions, Task Interview Questions 1-3, which probed how students perceived the class. TIQ 1 inquired about what algebra topics the students had learned. TIQ 2 asked the students how the material they were learning in this class differed from material in previous mathematics classes they had taken. TIQ 3 posed the question of how the teaching style of the class differed from courses the students had taken in the past. These questions were helpful in determining if reform based methods were used in the class, in order to answer the questions about fidelity of implementation and alignment with reform ideals. The next five questions (TIQ 4-8) were mathematics problems the students were asked to solve and discuss in the presence of the researcher. These questions were used to determine how students were able to interpret and solve algebra problems, and how they were able to move between representations of problems relating specifically to the concepts of slope and linear equations. Task Interview Questions 4-8 were the questions that were helpful in describing student thinking about algebra and its representations, and are presented in this section.

In order to analyze the data from the student interviews, the researcher read each task interview question in an individual interview in search of points that stood out for that question. The researcher first made notes on each interview transcript of the outstanding themes, and then returned to each interview to determine more specific codes. Following the coding of individual interviews, the codes were compared across the interviews to look for common themes. A description of the students' answers and discussion of how the results of the analysis pertain to Research Questions 1 and 2 are presented.

Task Interview Questions Four through Eight examined student thinking about algebra problems. In particular, questions four through eight attempted to elicit student thinking regarding movement between representations of algebra topics involving the ideas of slope and solving linear equations and systems of equations. The Lesh Translation Model (Lesh, 1979) was used to determine a framework for the types of questions asked regarding translations between representations. The Lesh Model (see Figure 1 on page 22) shows possible translations between and within modes of representation that are possible for a given mathematical concept. The researcher looked primarily at the representations of graphs, tables, equations and real world scenarios, as well as verbal descriptions. Not all translations appeared during the student task interviews, but those that did were examined. Due to the nature of the task interview, verbal descriptions were required for nearly every question. This placed a greater focus on student explanation and discussion than may have been seen during a traditional test.

Different questions addressed different translations. Task Interview Question Four examined how well students were able to interpret a graph of distance versus time and discuss the idea of slope in the graph, and its meaning in that context. This question addressed moving between graphs and real world scenarios. Task Interview Question Five involved solving a single variable equation with the variable on both sides of the equation. This question did not address translations, but examined how students were able to work with the symbolic representation. Task Interview Question Six was a real-world context word problem that required writing and solving a system of equations. The focus was on determining if the students could recall this type of problem and interpret it correctly, as well as to see what types of solution methods they chose to use after being exposed to a number of possible methods. The researcher wanted to see which translations students would choose to make from the real world setting. For example, the researcher was interested in whether they would move to equations, or attempt a concrete, graphical, or pictorial solution. Task Interview Question Seven focused on how well students were able to move from a table to a graph, with follow-up questions to determine if the student could explain and interpret slope in a real-world context. In addition, the students, if they could interpret the question, were asked to write an equation that represented the situation or a similar situation and thus move to the abstract representation. Task Interview Question Eight, the final question, asked students to interpret a simple proportional word problem and explain how they would find the solution. This translation involved moving from a real-world scenario to an equation. As students were expected to explain their answers as they solved the problems, translation to a verbal description was expected for all problems as well.

The analysis of the data follows with a statement of each task interview question followed by a table displaying student responses and a discussion of important themes that arose during the interviews. First, however, a brief description of each student interviewed is provided in order to give the reader background on the students.

*Description of Interviewed Students*

Students were selected for interview based on scores on a 25 problem algebra pre-test utilized by the course designers for a different research project on the course. The researcher selected students who scored in the high (over 18), medium (13-17) and low (below 13) ranges in order to determine the impact of the course on students of varying ability. Six students were chosen to be interviewed, but scheduling problems and absences resulted in only four students being interviewed. None of the students interviewed were of traditional college student age. Only four of the students in the class were between 18 and 22. Since the majority of students were older-than-average, students who were selected for interview were also from this age group. Students were interviewed in the first and second week of November.

*Student 1.* Student 1 was a female in her mid fifties. She was enrolled in the nursing program at STC. She exhibited a fairly high level of math anxiety in class, and appeared unsure of her abilities with not only the math, but also the computer skills necessary for the class, as she had little background in computer use. She frequently asked the instructor for help. She did, however, demonstrate a positive attitude and was very willing to learn the material. Her score on the course designer's initial algebra pre- was in the low range.

*Student 2.* Student 2 was a female in her upper 20s. She was enrolled in the nursing program like Student 1, but this class was her final class in the program. She stated that she did not like mathematics in the past and had not had an algebra class in 16 years. She frequently stated that she was "no good at math," but her score on the initial algebra test for the class placed her in the medium range of scores, and she was quick to learn a new skill or concept. She worked closely throughout the class with a student who was in the high range of scores.

*Student 3.* Student 3 was a male in his upper 20s. He was a manufacturing and engineering technology student at STC. He exhibited no fear of mathematics and clearly had taken mathematics classes in the past that were beyond the scope of the algebra in this class. His score on the initial algebra test placed him in the high range of scores, one of the highest in the class. He frequently asked questions that delved deeply into the material in class, and needed little help from the instructor on his work.

*Student 4.* Student 4 was a male in his lower 50s. He was a student in the manufacturing and engineering technology program like Student 3. He was attending the college as part of a displaced worker program after his employer company failed. He stated that he had not had algebra in the past at all, and that almost all of the material was new to him including the statistics and computer science portions of the class. He also had a very positive attitude toward the class, noting that he felt it would be useful in the future. He frequently second-guessed his abilities, however. This student had a low score on the course designer's algebra pre-test.

The next sections outline the results of each of the task interview questions four through eight, and their analysis pertaining to Research Questions 1 and 2, the research questions that focus on student thinking and learning.

*Task Interview Question 4 (Translating from Graph to Real-world)*

Question 4:

a) I am going to show you a graph of distance (in feet) versus time (in seconds). I'd like you to tell me what you think is going on in the situation depicted by this graph (Figure 2).
b) When is the person moving the fastest? How do you know?
c) Which direction is he/she going at that time? How do you know?
d) What kinds of calculations could make using this graph?

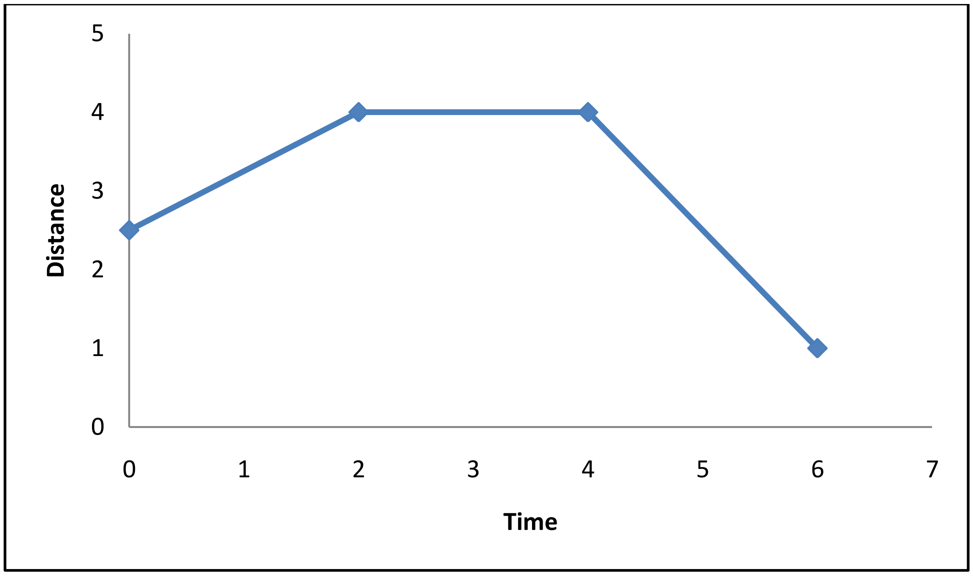


*Figure 2*. Graph for Task Interview Question 4

Student comments are shown in Table 3 below. The interviewer's comments or prompts, where applicable, are shown in brackets.

| Student | Comments |
|---|---|
| Student 1 | a. "We are at zero seconds and at 2 1/2 meters. And we then are going away from the timeline. And we go for approximately two seconds and we are at four meters. Then we stay at 4 meters so distance does not increase or decrease, it stops, and time continues on all the way up until 6, I mean four seconds. And then at four seconds we are going towards the line for an additional 2 ½ seconds. [What do you mean towards the line, in the context of the problem?] Coming forward. Well, um, that's the kind of thing that I have a problem with. I have trouble visualizing. The only thing I can visualize is this timeline. To me it's like a monitor sits down here. (points at timeline) [A monitor?]. A monitor. Here I'm going away from the monitor, (indicates first segment) here (indicates second segment) I'm at the same distance and here (indicates third segment) I'm coming closer to the monitor. And it takes six seconds to get here."<br>b. "I would be moving the fastest when I am coming towards the monitor, here (points at last segment). This point here. Because we're starting off at 4, and over a period of 2 seconds we have gone about 3, 3 ½ meters, and as opposed to this one |

| | |
|---|---|
| | (points at the first segment) we've only gone 2 meters."<br>c. "I was going forwards, towards the line. [And how do you know that?]. How do I know that? Well, I visualize this for example as a starting line, or a starting gate. And at this gate, when you are going away from it you are going backwards away from the line, and when you are going towards it, you have going forwards, towards the line."<br>d. "Um. you could do set up a calculation where you put in your times, put in all your times and then put in all your distances, and take change in time and change in distance, and then your rate would be the change in distance divided by the change in time." |
| Student 2 | a. "Ok. So I would say starting…they are starting at about zero seconds. Your points are at (0, 2 ½). And they are going, in about two seconds have gone four feet total, so from two and a half to four. Then they are stopped. I mean time is still moving, but in about four seconds, four feet, then they start to go, I guess, negative or go back, and they from four feet down to one foot and it's at six seconds, so in about two seconds they go from four to one."<br>b. "They are moving the fastest on the last section when they are coming back or whatever…that's the steepest slope and they are going from four to one, and that's the biggest …intervals as far as when the slope is changing, is changing and that's the biggest change in distance in seconds."<br>c. "Back, towards where they started."<br>d. "You could calculate the slopes of each, which the middle one where this distance is zero, which the slope would be zero, but you could figure out from 0 to two seconds, what the slope is, and then from the four to six seconds you could figure out how big the slope is, on the way back. [And what would that tell you in a practical sense?] Um, like how fast they are going." |
| Student 3 | a. "Ok. So you want to know like, where? [sure] So he started it looks like about 2.5 meters away, in about 2 seconds he went up to about 4 meters and stood still for looks 2 seconds, then he went closer to the…closer down to about 1 meter in two seconds, went down towards it."<br>b. "The last part, from the four meters down to the one meter in 3 seconds. [How do you know?] Steeper graph. …steeper graph, or steeper decline than increase."<br>c. "Closer to the thing checking it, or going back down towards zero, closer to the one, or closer to the zero. It's going down, or a negative slope." |

| | |
|---|---|
| | d. "As in…How do you mean? Oh, like checking the slope or unit rate, rate of decline, or slope? [And what does that tell you in the context of this problem?] Well the line is going to have, each section, each point is going to have its own slope, from here to here, here to here and here to here is going to have its own slope. (points at segments in graph) [And what kind of units is that slope going to have?] What kind of units? This one is 4 and 4 this is…you can make a unit rate out of it (calculates rates)…whatcha looking for? [In a practical sense, what does that slope tell you?] The rate of increase or decrease. If you're going away or going towards….Meters over seconds, distance over time." |
| Student 4 | a. "It looks like we start at our set point about two and a half meters. In two seconds we move one and half meters. We stay still for two seconds and we move closer for two more seconds and end up about one meter away."<br>b. "Moving away in the last segment. That to me is the sharpest angle. [OK, you said sharpest angle?] Yeah the downhill is two seconds compared to the first angle we went 1 ½ meters at two seconds. In the third segment he moved 3 meters at two seconds."<br>c. "On the third segment, towards you. Towards the monitor. [And you know that because?] It's a negative slope."<br>d. "I guess maybe a motion study. I guess you could do some kind of motion study with a car moving, I guess. [You mentioned the word slope; could you calculate that slope?] I'm not sure if I could do that now or not. It's rise over run…I would have to…I probably could not do that right now. I'd need to review that. I have the notes, probably, to solve that." |

*Table 3*. Task Interview Question 4

*Discussion of Question 4.* This question was intended to determine whether students were able to move between the representation of a rate in a distance-time graph to a real world context, and to examine student understanding of the concept of slope in a graph, and also as a rate. The students were all able to explain what was happening in the distance time scenario, as far as direction of movement. The students had had quite a bit of practice with a motion detector and problems like this in class, so it was not a surprise to the researcher that three out of four had good facility with it. Interestingly, three out of four students discussed the distance in meters, rather than feet, although the question stated that distance was in feet. This may have been a consequence of having worked consistently with meters in class.

Student 1 was able to interpret the segments of the graph as movement away, no movement, and then movement toward the “monitor” as she termed it. She was able to identify when the movement was the fastest based on the distance traveled over time, but did not mention the term “slope” here. Student 2 was also able to interpret the graph as movement away, no movement, and movement towards the starting position. She was able to identify the fastest motion by recognizing the slope would be steepest there. She did not attempt to find the rates, although when asked what calculations could be made, she noted that she could find the slope. Student 3 was able to interpret the graph in terms of direction and rate based on the segments. He recognized the relationship between rate, “steepness” and slope of the graph. Although he needed some prompting as to what the researcher was asking, he was able to explain the meaning of the slope. When asked about calculations, he noted, “You can find the unit rate.” He voluntarily computed the rates from the graph.

Student 4 also was able to interpret the graph to some extent, stating that, “the person moved away, stood still, and then came back at a faster rate.” He contradicted himself about direction when talking about when the person was moving the fastest, saying that in the last segment he was moving away, although he’d stated the opposite earlier. When asked about what types of calculations could be made from the graph, he agreed that you could find slope and although he stated that it was rise over run, said he would “refer to his notes to solve that”. He did not appear to have a strong grasp of the idea of slope at this point.

When asked what type of calculations Student 1 could do, she stated she could enter the data, and was referring to doing the calculation in a table using the spreadsheet program. The students had in class computed change in distance and change in time, then slope, using the spreadsheet program. While three out of four students discussed slope, one voluntarily found all the slopes of the segments in the graph, although was not required as part of the question. All students interviewed were able to recognize the relationship between rate and the “steepness” of the graph, as well as direction of the motion in relation to increasing or decreasing slope in the graph. These four students displayed an ability to translate from graph to real-world scenario in this context regarding the slope of a graph as a rate. Familiarity with the scenario may have played a role, as all of the interviewed students seemed to relate the problem to their experiences with the motion detector in the classroom.

*Task Interview Question 5 (Equation Solving, No Translations)*

Question 5: Please try to solve this equation for x: 3x + 5 = 7x -15

| Student | Comments |
|---|---|
| Student 1 | “Ok. So what we can do, do you want me to talk it through? [Yes, that would be great.]. Well what we can always do … our key here is to try to eliminate certain parts of this fraction (sic) to make it easier, to break it down. So what I do to one side I have to do to the other. So the easiest thing to do here is to subtract the five. And so I’ll subtract five from here. So that would be 3x equals 7x minus 10. Now, um. So how can I describe this? I like to do it this way; again, I’m going to turn it around so that I can see what maybe I can take out of the equation. As long as we keep these sides balanced, we’re OK. So now I have 7x minus ten is 3x. So now I can take out this 10, so I can subtract. No, I can’t. It |

| | |
|---|---|
| | doesn't work. I wonder if I… I'm a little confused at this point. [You can start over if you'd like] Yeah. Ok. There's supposed to be a way I can get rid of those. The x's maybe. Here you would have 2x equals…Could I get rid of, go down to 6x minus four? But that's not going to help me. I'm not sure on this problem. Maybe we should go on to something else. [Ok] But I'd like to come back, I'd like to learn how to do that one… [I know you said you like to switch sides sometimes, Would that help you? Perhaps?] I'll try that. 7x minus 10 equal 3x plus five. That's not going to help me…Oh maybe it will. [I think it was 7x minus 15]. 7x minus 10 equal 3x. Oh, that's not going to help either. That's the tough thing. (Picks up calculator) You do it on the computer and you lose track of how you do your formulas. Let me think for a while. Hmm. I suppose I could do that…I don't think that's right. Maybe we should go on… [That was a true statement but it doesn't find you the x. Ok, let's go on to something else, and perhaps we'll come back to that.] Ok." |
| Student 2 | "…My goal is to figure out what x is, or to simplify as much as I can. So I need to get rid of the seven x on this side, so then it would be minus twenty and then I subtract the seven…But don't we want to get the x's on one side? So then you'd subtract the 7x from both sides, and here, three, so you'd have negative four x equals negative 20, then you can divide by your four. But I don't know if it's negative, now I'm (types on calculator) so I'm getting x equals five. So let's see. (pauses and works on calculator) [So what were you doing now?] Well I was just putting my 5 in I could see if …I think it's right… (unsure)… Well I think so, I mean I got my twenty, and as far as I got my fifteen, so I think it's right. I think so. I over-think things when I'm on the spot and I get myself confused." |
| Student 3 | (Student quickly writes). "5. [Ok. And what did you first when you solved that?] Got x to both sides. First I made sure that I got x to both sides, then I put the real number on its own side, then I came up with 20 equals 4x, then divide by the four. [And that's a familiar process to you?] Yes." |
| Student 4 | [student works for several seconds] "Can I grab my calculator? I don't know if I'm doing that right or not. [Sure.] Oh, I don't have it. Can I talk my way through how I would solve it? [Yes.]. I'd start out with 3x + 5 = 7x – 15. I want to solve for x, so I'd want to get the x to the left side of the equation, so I'd subtract 5 from that side. That would give me 3x equals 7x minus twenty. I'd have to take the 3x and I would divide by 3 to eliminate the 3x, so I'd have x equals seven x divided by three minus 20. And then I'd have to use my calculator to solve that 7x over three x minus twenty. The x would equal, ah, two point something. Oh, I don't have the x isolated…so…possibly I don't have the x isolated (pauses). Oh, I have to get the seven x over to the other side. [And how |

| | |
|---|---|
| | would you do that?] I would subtract the seven x from the other side. I'm going to start over. So 3x…(mumbles) …subtract five…equals twenty. Minus seven x equals negative four x. Twenty divide four is, would be…x equals...I'm saying negative five... (looks at original equation). It does not work. So maybe I'll reverse that to positive five. It's positive five. X equals positive five. It was that inserting it back into the equation. It was proven either right or wrong. I proved it wrong the first time. Reversing the negative to positive solved it. It was right." |

*Table 4*. Task Interview Question 5

*Discussion of Question 5*. This question was designed to determine if students were able to solve a traditional context-free multistep single variable equation. The question did not involve using translations, but the question was helpful in determining whether students were able to work with the symbolic representation. Three out of four students struggled somewhat with this process, and one was unable to complete the problem despite multiple attempts.

Student 1 was unable to complete the problem, although she had some correct processes. Her work is shown in Figure 3.

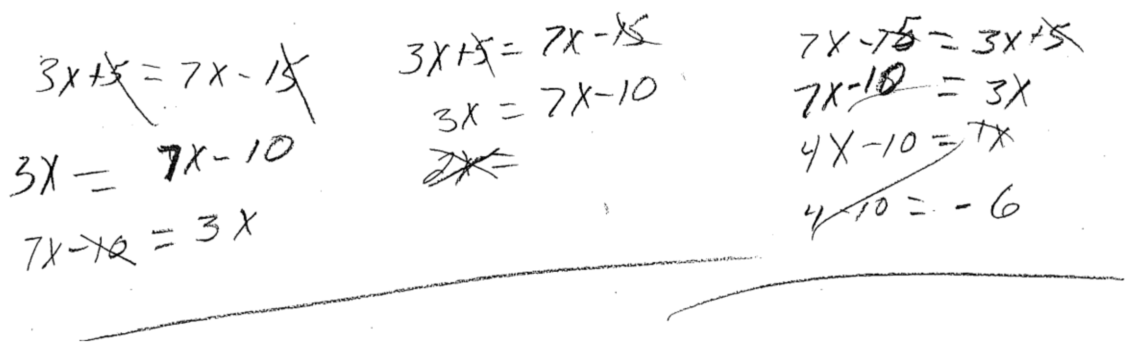


*Figure 3*. Student 1 Work for TIQ 5

Note that student 1 made three attempts to solve the problem. She realized she needed to subtract five from both sides, but incorrectly subtracted five from negative fifteen to get negative ten in all of her attempts. She did not appear to know what to do with the 3x on the right hand side of the equation. In the second attempt, she tried to get rid of an x by subtracting 1x, but this did not help her so she abandoned this path. In the third attempt, she reversed the equation to try to make it easier to solve, but began by trying to subtract the five again, when subtracting the 3x first would have been more fruitful. She appeared to be fixated on getting all of her constants on one side first.

Student 2 appeared intimidated by the problem at first and unsure of her process. She made two attempts at the problem. In the first attempt, she subtracted the five correctly, and then tried to divide by three in two terms of the problem, but not the third. In her second attempt, she correctly subtracted seven x, and then divided by negative four. She talked her way through the solution, stating each of her steps, but struggled briefly with dividing by a negative. She found a solution and then checked it to make certain it was correct. Her work is shown in Figure 4.

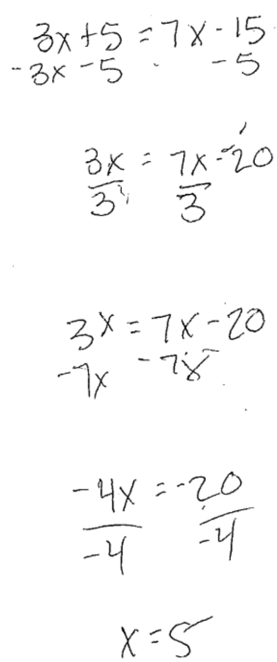


*Figure 4.* Student 2 Work for TIQ 5

Student 3 was able to solve the equation very quickly using traditional algebra methods, likely due to his stated strong background in mathematics. His work is shown in Figure 5.

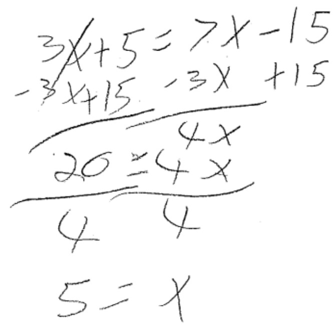


*Figure 5.* Student 3 Work for TIQ 5.

Student 4 questioned most of his steps, and wanted to use a calculator, which he had forgotten. He had a false start in that he did not isolate the x, and also tried to divide two terms by three, but not the third. When he began again, he was able to complete the majority of steps, although he had difficulty deciding on the result of division by a negative. He checked his answer, and was able to determine it was incorrect. Instead of reworking the problem, he changed the sign of his answer and tested it, finding the correct answer. This strategy was interesting, as it seemed somewhat arbitrary. His work is shown in Figure 6.

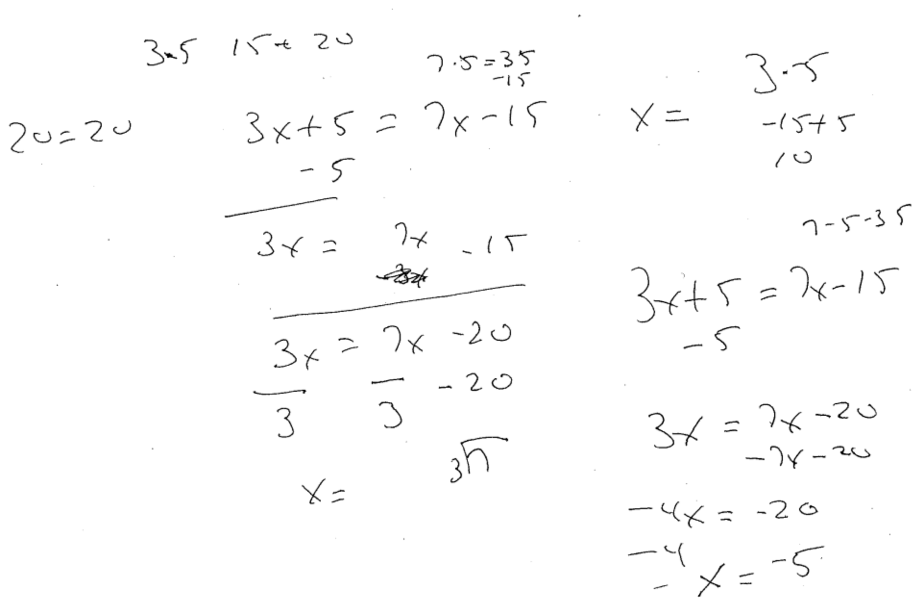


*Figure 6.* Student 4 Work for TIQ 5.

Like Student 2, Student 4 attempted to divide two terms of the equation by 3, but not the third. He also did not isolate the x in his first attempt. In the second attempt, he found the correct result, although he appeared to neglect the negative in front of the x, and did not state that he needed to divide by negative one. Above his work, he made notes to check his answer, and as his answer was at first x = -5, found that the answer did not work. He changed the sign, somewhat arbitrarily, and found x = 5. When he checked his work this time, his checking found 20 = 20, which confirmed his solution.

The students did not seem to have a strong facility with working with the symbolic form of an equation. The fact that this particular equation had a larger coefficient of x on the right seemed to play into the students' difficulty in solving this equation. The interviewed students, for the most part, immediately subtracted the five from both sides, but then appeared unsure of how to deal with the 3x and 7x. Two students eventually did subtract the 7x, resulting in a negative on each side. One student reversed the equation in order to avoid the negatives. Both of the students who were working with negatives expressed uncertainty about dividing by a negative, but were able to determine the correct answer. All three of the students who completed the problem checked the answer to see if they had found it correctly. Student 4 did not display clear understanding on why he should reverse the negative on his solution, and why doing that produced the correct answer, however.

At this point in the course, the students had not been presented with solving simultaneous equations using graphs, at least not linked to graphing two sides of an equation, and did not present any alternative solution methods. Work with the symbolic representation presented difficulty for three out of four of the students who were interviewed in the task interviews.

*Task Interview Question 6 (Real-world Scenario to Graph, Table, Pictorial, Concrete or Equation)*

Question 6: You are part of a construction company that is supposed to build houses. An architect has left plans to build houses on three islands. The islands are connected by bridges. Island A and Island B have a total of 20 houses. Island A and Island C have a total of 13 houses. Island B and Island C have a total of 15 houses. A total of 24 houses is to be built on the three islands. Determine how many houses are to be built on each island.

| Student | Comments |
|---|---|
| Student 1 | "Ok. We know that A is the same in each equation. Ok, so if we take 24 minus 20, we would have four. And um so we know that, ah… 24 minus 13 equals 11. So A is going to be one or the other of these. Um. So we took twenty minus…so 13…hang on just a second. So what I'm going to |

| | |
|---|---|
| | start with groups of five until I get to twenty-four. (makes hash marks on her paper) Ok, so we have 24 here. If A and C equal 13, we'll count off 13. Ten, 15,… So here we have four and let's see fourteen. That's not working. So that's not working…I want 13…I'm getting a little flustered here. So we know this group here is A and C. (circles a group of 13 hash marks) So then that leaves us… 11, which must be B. So if we take 24 minus 11, that would be 13. Which I had before but that wasn't right. So thirteen from 24. So 11. I don't think that will work. Yes it will. Gosh it's been a while since I've done this. Ok. So if B would equal 11, then, no I said… Ok, B equal 11. So A would be…nine. So say nine…so 15 minus 11 is four, so C should be four. So nine and 11 equal 20, 11 and 4 equal 13, no it doesn't, it equals 15. Oh, 'cause A and C….So 9 and 11 equal 20. A and C would be 9 and 4 are 13…which is correct there. And B which is 11 and C which is four equals 15. So we end up, after that whole mess. A equals 9, B equals 11, C equals four. Because if you take A and B, A is 9, B is 11, then 9 and 11 are 20. (writes, checking answers) Then you take A and C, A which is 9, C is 4 and 9 and 4 are 13. Then B and C, B is 11 and C is four, so A and C is 15." |
| Student 2 | "Ok (writes for a while). So let's see. I have to think about how we did this. [What are you thinking about now] I was thinking about how to set it up. I mean I could do the line way, It's like the first thing we did in here, so I can't remember. [Could you show me the line way, that might be interesting] Oh, yeah, so you make 24 lines. Then it says A plus B is 20, so you circle 20, I think I'm doing this right. Then A + C is thirteen so that's ten…thirteen, and B + C is 15, so then you're going to have all those. (circles groups) And then you have 24 total…now I can't remember… [What did your first circle represent?] A and B have twenty of them. So there are four left, so…those are C. If there's four left over. We can try it, and see…so that would be mean that A, if I put it into A plus four equal 13, then we're going to subtract the four and get A equals nine. If you have A plus B is 20, then subtract your nine, you'll have 11, and C equals four. So that equals the 24. [So you used kind of a combination of a drawing and a little algebra]. Trying to remember how I did it. Which that was how I was trying to figure out at first I just couldn't remember how we did it. I was trying to figure out how we did it because there was a way with just three, um, like scenarios, just to do it algebraic, by putting in a problem, which I'm thinking it was…which I could have just did, because A and B is… 20 plus C equals 24 and then subtract it and get your C and then plug your C into the rest of your equations and get your answers, that's what I was trying to do." |
| Student 3 | "Easy enough. You're not caring which way we do this. [No, whichever way is easiest for you.] Been a while since we've done these… (writes out a series of equations)…Ok. A is 9 B is 11, C is 4. [and I noticed you used a pretty traditional method to solve this] Yes, just algebraically, yes. [So |

| | |
|---|---|
| | what did you do in that first step?] First step I basically added the A + B plus A + C and their totals [to get 33?] Well to get the 33, and the equation becomes 2A plus B Plus C equals 33, then what you can do is you have B plus C right here, and take B plus C and put the 15, becomes 2A plus 15 equals 33 that's an algebraic equation and you can solve for A and then input back in. [So you used 3 equations and you didn't use the fact that there are a total of 24 houses on the 3 islands.] Never even needed to." |
| Student 4 | "First I'll try to lay it out. A plus B equals 20. A plus C equals 13. B plus C equals 15. Total of 24. (mumbles as he writes). Oh, 2A plus B equals 33. (pause) I'm not sure if I'm on the right path to solving this. A plus B plus C is 24…Boy, I don't remember how to solve this out. We know B and C is 15, that leaves me …A equaling nine….I can take off A and B. I want to subtract 20 off of 24. That leaves me C with four. I want to solve for B. C plus A is 13, getting 10. I think I did something wrong. [What did your numbers add up to?] 24. Oh, it is right. Just a matter of recalling that. Again, I'd refer to my notes. But thinking about it for a second, it came back." |

*Table 5.* Task Interview Question 6

*Discussion of Question 6.* Question 6 was designed with several purposes in mind. First, this type of question was utilized early in the course to illustrate that there can be multiple solution paths and multiple representations of a problem. During the classroom activities, students originally attempted the problem, and then displayed their solution methods in class. The instructor then pointed out that this type of real-world problem could be solved using graphs, algebra, concrete models and tables. Students were initially given total number of houses on all three islands, and used pictorial representations and later were give only the equations in two variables. They eventually also had to use the spreadsheet to determine large numbers of houses, which would not have been feasible using some of the pictorial methods. By asking this type of question, the researcher was interested in determining how the students were able to translate from a real-world type problem to any of the other representations. In addition, the researcher was interested in seeing which representation the students chose, and how they thought about the problem. Finally, solving systems of linear equations was one of the goals of the course, and the researcher was interested in whether the students were able to recall and complete this process.

The work of student 1 is shown in Figure 7. She created a pictorial set of hash marks and circled the marks that represented the sum of houses on two islands, leaving uncircled marks to represent the number of houses on the third island. The hash mark process is displayed, along with her system of equations and scratch work. Although she displayed her addition incorrectly on the right, she stated verbally during the interview that A= 9, B = 11, and C = 4.

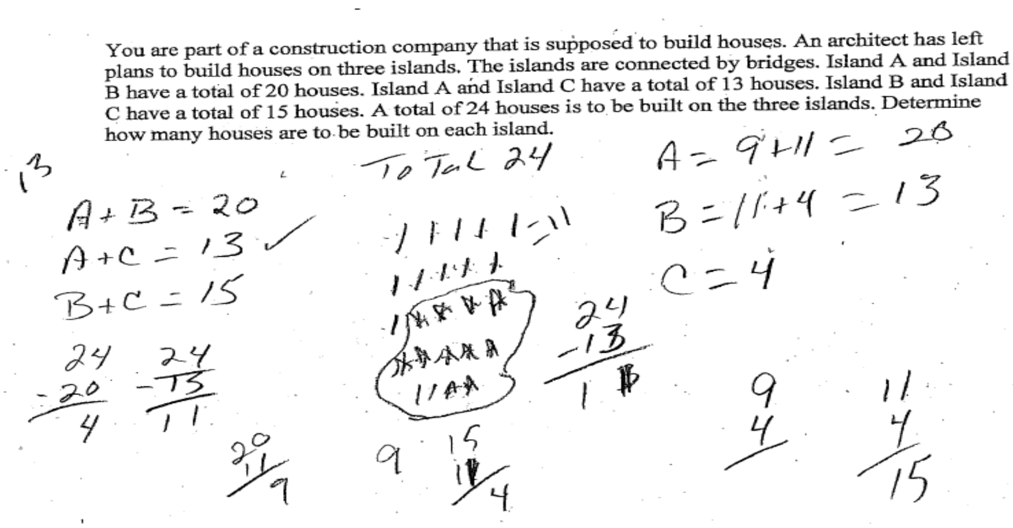


*Figure 7*. Student 1 Work for TIQ 6

Student 1 began the problem by setting up three equations, and initially tried to solve in an algebraic manner, but then reverted to a more pictorial model using hash marks. She drew 24 hash marks (the total houses), and circled 13 of them to represent the houses on Islands A and C. This left her 11 houses on Island B. Once she had found the solution for one of the variables, she went back to the equations to find the other variables. This student used the total of 24 houses in her process when using the pictorial model.

Student 2 also began her solution with three equations, and then went back to a pictorial model using 24 hash marks. She circled 20 marks to show that that represented A + B, then determined the four remaining marks must indicate C. Like Student 1, once she found one of the variables, she returned to the equations. She also did not set up the equation A + B + C = 24, but noted that the total was 24, and used the 24 marks in her pictorial model. After solving using this combination of methods, she explained the steps she could have used to solve it algebraically without using the picture. She noted that she could add A + B and A + C to get 33, then since B plus C was 15, that left 18 for the 2A, or A = 9, and continued from there.

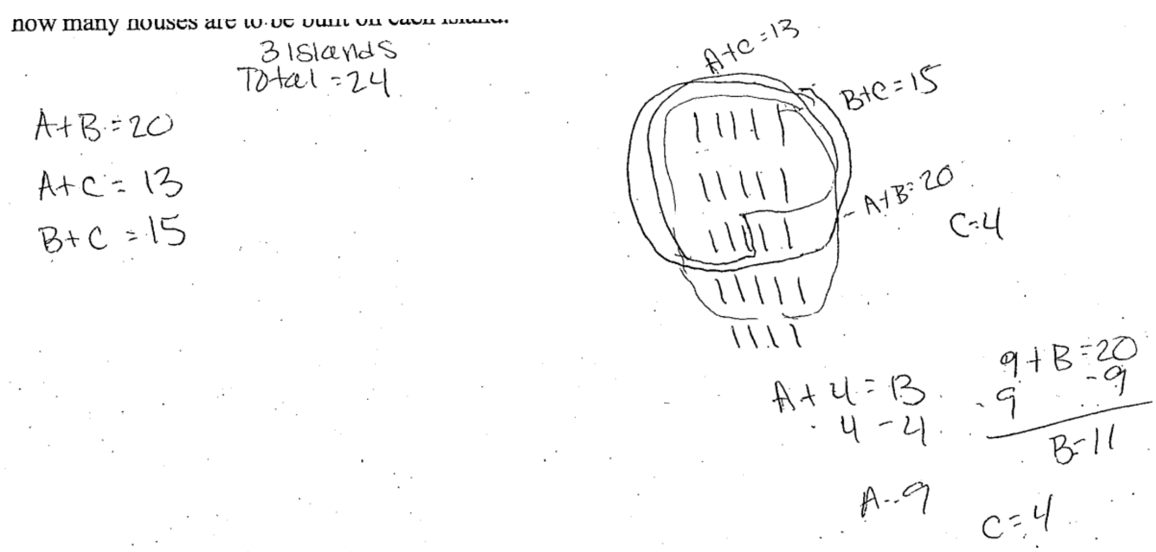


Figure 8. Student 2 Work for TIQ 6.

Student 3 immediately set up a system of three equations in three variables and solved it algebraically. He did not use the fourth equation involving the sum of 24 at all. When asked why, he stated, “Never even needed to.” He solved the equations and above his equations it is clear he checked his work following finding his solution. The work of Student 3 is shown in Figure 9.

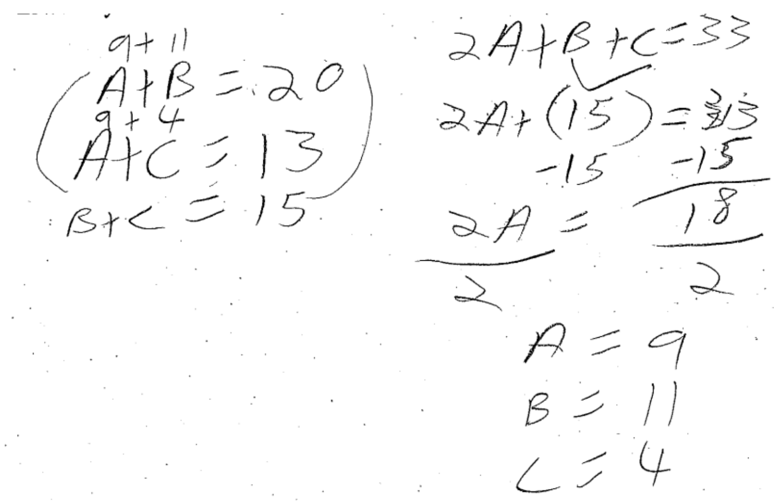


*Figure 9.* Student 3 Work for TIQ 6

The final student, Student 4, set up a system of equations in two variables to solve. He was unsure of his work at several points in the process of working with the equations and returned to try a different combination of equations, at one point stating, “Boy, I can’t remember how to do this.” His running commentary on what he was doing was somewhat disjointed as he stated his steps. He thought he had done something in error, but when the researcher asked what his numbers added up to, determined that he had done his work correctly, stating, “Oh, it is right! Just a matter of recalling that. Again, I’d refer to my notes. But thinking about it for a second…it came back.”

All of the students were able to solve the problem, that is, to translate between a real world problem and either a pictorial (hash mark) representation or an algebraic system of equations. Two students used a mixture of pictorial models and algebra, while two strictly used algebra. Lesh, Post and Behr (1987) noted that students who exhibit the greatest understanding of a problem are able to move instinctively between representations that help them the most. This appeared to be true for these two students, as they moved easily between the representations. Three out of the four students utilized the fourth equation, rather than simply solving three equations in three variables. All of the students noted that it had “been a while” since they had done this type of problem and that they needed a minute to recall the process. Student 1, while writing out equations in some places, also showed scratch work in simplified subtraction problems. Student 4, one of the low scoring students on the course designer’s algebra pre-test, expressed the most hesitation of the four students interviewed, and took the longest to finish the problem. Interestingly, like the students observed in research by Herman (2007), the students were willing to use different representations but tended to use the symbolic means of solving as the “best” way to solve the problem. Although the students had used the spreadsheet program to solve this type of system of equations in class, none of the four students interviewed turned to the computer as a tool for solving this problem during the task interview.

*Task Interview Question 7 (Table to Graph; Graph or Table to Real-world Scenario; Real world Scenario, Graph or Table to Equation)*

Question 7: I’m going to have you look at a table of data (Table A) and I want you to:

Table A.

| X | Y |
|---|---|
| 0 | 30 |
| 5 | 40 |
| 10 | 50 |

| | |
|---|---|
| 15 | 60 |
| 20 | 70 |

a. Draw a rough graph of the data on the graph paper I provide.

b. Can you describe for me what kind of relationship there seems to be between x and y?

c. If I tell you that the data in the x column stands for years, and the data in the y column stands for population of a city in thousands, tell me as much as you can about the situation.

d. Can you tell me when the city will have 100 thousand people?

e. If the data for a different city looks instead like the table below (Table B), please describe what is happening in this city and how it differs from the previous city. You may make a graph if you wish.

Table B

| X | Y |
|---|---|
| 0 | 33 |
| 5 | 31 |
| 10 | 29 |
| 15 | 27 |
| 20 | 25 |

f. If you can, please write an equation that represents the relationship between time and population for the Table A using x and y as the variables.

g. Do you think the cities will ever have the same population? If so, when? (You may estimate).

| Student | Comments |
|---|---|
| Student 1 | a. "Gosh this is a tough test. (Laughs). Ok so y is vertical, x is horizontal… (fills out scale on graph) So we are looking for…we are at zero and 30, start here. Can you see that? [Yes]. So then we have five and forty, then we're at 10 and 50, 15 and |

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|---|---|
| | sixty…Then 20 and 70. And it's like that." (shows graph)<br>b. "They are… the distance time are occurring at the same unit rates. Increasing by the same amount. A straight graph."<br>c. (Labels units on graph wrong, pop on x and time on y) "Um, this town started out at approximately smaller town of 30000 people then every five years it grew 10%. [10% or… by how many people? Time is on the x.] So by 5000 people. No 5%...because it's in increments of five. So every five years it has increased by 5000, no 10000 every five years. So then, if you were looking for a percent… (uses calculator) You're looking at it increasing by a half a percent...no? [Tell me how you calculated that] so you take 5 divided by 10000? It doesn't matter how you put the fraction, where you put your units here. You want to get it down to a one year rate. So if you are looking for people per year. If you have 10000 per five years that's 2000 people per year. I'm going to see if this works. You had 5000 to 10000 then it's doubling. From 5 to 10 you doubled. [What do you mean double?] If you were at 5000 people here, or 50000, then you went to 100000, [what's on the x axis again?] So that's the years and this is the population. So if we are at 5000 people. (having trouble reading the graph again...confusing x and y) So that's… we're not completed with this class. We're increasing by 2000 people per year. So if we took 2000 and times it by every five years. Oh, that does come out. So we we're every 5 years increasing by 10000."<br>d. "Ok, I can. So if it increases 2000 every year. Now is our population starting at zero? So here we have 5000, I mean 50000. Are we starting at 30,000 people? [you told me that] We want to know when it will equal 100000. Forty, fifty, sixty, seventy, ninety, 100000….I'm just going to double check here. 35 years. Sorry…so we have 35 years. [It's OK] You didn't expect my problems to take so long."<br>e. "This city is losing people. [Ok, and how do you know that?] As we can tell, over the years, (referring to the table) we were at 33000, five years later it was at 31000, 10 years it was at 29, and so in 20 years it has lost 8 thousand people, so that would be about 400 people a year. They are leaving, they are exiting. The Pied Piper has come."<br>f. [Interviewer inadvertently missed this question]<br>g. "No. because we're seeing a positive growth in this and a negative growth in that. So they are just growing away from each other [From the very beginning?] Well, from the beginning. From the very beginning. Yes, if that held true. But we don't have enough data really to know that. We assume that. |

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| | Project that out." |
| Student 2 | a. "Gosh now I've got to remember x and y. X goes up and down, no Y goes up and down. Y up, there you go. Ok, now…I don't even know. I'm not very good at this. At using my imagination and making my graphs work. Oh my gosh, now I wrote them on the wrong side. [you can just cross those off] So we are starting with thirty… (writes scale on graph). So as long I have everything scribbled out on there. (Shows graph)That's going to be as far as where they are meeting, (points at origin) and that's x and y."<br>b. "Um… [What kind of a graph would you call that, or anything in particular you notice about those points] I mean they steadily go up, I mean the slope goes up by, on the x side it goes up in increments of five on the x and going in increments of 10 on the y side. It's steadily going up each time it goes up. In, say, whatever it is, time and distance. It's going up at a equal amount; the slope is all the same. It's a line. Just a line."<br>c. "Um, so every five years, the population of the city is going up by ten thousand. I mean consistently every five years it goes up by ten thousand, as far as the population."<br>d. "If it keeps going in the same rate, you're going to have, so… (thinking)…35 years, according to this if the trend stays the same as far as the population and the years."<br>e. "It's going up by five years, and population is going down…It's going down 2000 every five years."<br>f. "I'm sure I could. So, I'm thinking of our spreadsheets, So this is my years, and this is my population. (writes on paper) So if it's five, we're going to take, like on the spreadsheet…33 minus five…not wait a second. (Turns to computer. Types in data, including years, and population values). So then 33… [What are you calculating there?] I need to figure out what to put in the equation. You're going to take the population and then you're going to need to know that you are losing 2000 people per year. Well according to this you can see it's going down by 2000 every year, I mean every five years. [How can you write that in an equation?] I wouldn't! I wouldn't make it any harder than it is…I don't know, I guess I could figure out how much it's going down a year. (types on spreadsheet) Well if it's …see I do things the hard way. So it's going down 400 a year. [How did you calculate that?] See my brain doesn't work the same as other…I figured out it goes down 2000 every five years so I divided by 5. That's 400 a year. [So if it says 2 instead of 2000 here, what does that mean?] Oh, it's gosh (types in calculator), .4. Yeah, 0.4. So goes down by .4. I can do the... (goes back to |

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| | spreadsheet) equals =33. You see I since I already have it figured out in my head, I can't figure out the equation, since I already have the answers figured out in weird ways in my head. Every year you're going to have to subtract, minus every year, you're going to lose 400 people or .4. [So your equation would be…?] 33 minus…I have no idea. [Ok, we'll go on then.] Well, I already know the answer in my head, and it's just confusing as to what I'm trying to figure out the equation. I think the first time we did the islands thing, we just had to figure it out, I just kind of played with numbers until they worked. [sort of a guess and check method] Yeah, but it only took me like 5 minutes. Sometimes it's just easier, I can look at it and, like you can think of it in your head, and can't necessarily put it down on paper you can come up with the answer and figure it out and know what numbers to punch into a calculator, but if you have to put it into a formula, it doesn't exist. Like you just can't I don't know how to explain it."<br>g. "Um. Probably at some point. You'd have to figure out both, and then graph them to see where they cross… but I couldn't tell you when. (goes to graph) Um, the second one…Oh I see at 20 years it's… thirty three, so…we're only going 2 years. I should have spaced out my graph. Oh it's going five years, but two. So then we have 31, I mean as this is going up and that is going down, they are going to cross, maybe I don't even know, about…well it's not going to be…it's going to be like about one or two years, just guessing according to my un-…very not great graph here. [that's ok you can estimate here] I don't know one or two years they are going to cross. Between when the first one is increasing in population and going from thirty thousand to forty thousand in the first five years, and the next one is going down from 33 to 31 in the first five years, I don't know say, at, I don't know, 32 years. I mean at 32000, they might have the same population and that's going to be guessing maybe two years into it the first five when this one is going down and the other is going up. [So where the two lines cross is where the populations are the same?] Yes." |
| Student 3 | a. "Sweet. You just want plotting? OK. (writes scale on graph). I suppose I could have made it bigger for you… but that's ok…10, 15. OK." (Draws line).<br>b. "The relationship? You mean the rate of increase? [that would be helpful] The relationship…whenever the x goes up 5, when y goes up 10. So you're looking at...ah… trying to think what you're looking for. It's a linear line, going positive."<br>c. "Every five years the population goes up 10000. Um…that rate. |

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| | Every five years, the population goes up 10000, so it's 5 to 10 thousand. The population started at 30000." (Writes linear equation y = 10x +30)<br>d. "100 thousand? (thinks for moment, adds dots to graph) 35 years. [And did you write an equation there?] Yeah, I figured you were going to ask that sooner or later. [And what kind of equation?] That's just a y intercept equation. [And it says…?] y = 10x +30."<br>e. "This one is a decreasing value. Same, years and population? Is it? [Yes still a city and population in thousands] This one started off at 33000 people. Every 5 years looks like its decreasing in 2000 people."<br>f. (student writes equation y = 2x + 33 quickly) [How did you know what kind of relationship the two variables had?] "Because there's a decrease, the y is decreasing so there actually would be a decreasing line. Because there would be a negative slope. What else do you need? [Why did you choose the numbers you did for the equation in part e?] Because y intercept starts at 33 so that would be the starting spot on the y axis of a graph. That's the y intercept there. (points to 33) Then 2 is the rate of slope, oh, should be negative two….(adds a negative)…yes, it should have been a negative 2."<br>g. "Could they of? Um…thirty… (pauses and looks at graph) I was going to say you could just graph the other line… (does so) That's going to be tough, because 33…add 5 there… and 31…Yeah... my graph is so damn small. Yes, they will have the same population. [When you estimated when the cities would have the same population, how did you decide this? Tell me what was going on in your head.] Because the lines intersect. [About when?] Um. (looking at graphs)…in about 2 or 3 years…" |
| Student 4 | a. (student plots points without comment)<br>b. "Positive slope, very high slope. So whatever the data is… [What else can you tell me about the relationship between x and y then?] Um the data is constant. It's on a one to one type ratio. For every five years it increases on the y, it increases ten…I mean every five it increases on the x, it increases ten on the y, so it s one to two ratio."<br>c. "Ok. The population is increasing dramatically as it ages. Every ten years… it's growing by 20 thousand every ten years. The population is going up every ten years. The population started at 30,000."<br>d. "Need more paper here…let's see…in fifty years….yeah, extend the trend line farther…you can predict it if the data stays |

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| | constant."<br>e. "This one's a declining slope, so the population is going down. The population started at about 33,000 and in 20 years it's gone down to 25,000. The city will be empty in about 60 years. If he data stays the same I predict there will be nobody left in that town."<br>f. "I don't know if I could. I'd go to the computer program. You can do that if you'd like]. (student opens excel, creates table, highlights it, creates scatter plot and finds trend line) Put in our data, go to the scatter plot function, choose graph, add trend line…data is…that appears to be in line with what I got. Slope is 2x + 30. [So the line is y = 2x + 30. What is the slope in that? ] The slope is y. You know I'm not exactly even sure. [But you got y = 2x + 30?] Yes."<br>g. (looks at graphs) "It's in about two years. At about 32,000, 32,500 people." |

*Table 6*. Task Interview Question 7

*Discussion of Question 7*. This question had many parts. It was designed to discover several aspects of the students' thinking and ability to move between representations. One aspect was to determine whether the students were able to translate from table to graph, table to equation or graph to equation. A second question was whether the students could interpret a graph or table given a real world context. A third question was whether the students could utilize the graph(s) to extrapolate values and recognize that the intersection of the graphs was where two equations had the same x and y values.

Student 1 was able to graph the data from the table easily. She was able to identify a linear relationship between x and y, and constant rate of change although she tried to put the graph into a context before given one. She noted that the "distance time are occurring at the same unit rates. Increasing by the same amount." Given the scenario, she was able to interpret that the population was growing at 2000 people per year, for

Table A, and losing 400 people a year for Table B. She had a tendency to try to find a percent, referring to growth by 10% and 5%, but changed her answer as she thought it through, to a people per year rate, noting, "You want to get it down to a one year rate. So if you are looking for people per year. If you have 10000 per five years that's 2000 people per year." Once she had determined the initial population and rate of growth in the first table, she quickly was able to find the values in the second table. She was also able to determine that the city in Table A would reach 100,000 people in 35 years. She was able to extrapolate using the graph she had created. By error, the interviewer omitted the question asking if the student could write the equation of a line. This student could find the starting value and the rate, but because of this omission, it is not clear whether she could write the slope-intercept equation of the line at this point. This student did not believe the populations were ever going to be the same, as one increased, and one decreased. She did not take into account that the decreasing population started at a higher value than the growing one.

Student 2 at first questioned the locations of the x and y axis. She was able to graph the data, however. She was able to describe the relationship between x and y as "It's going up at an equal amount. Slope is all the same. A line, just a line." She talked about the rate in five-year increments and did not find the unit rate, or slope, stating, "Every five years, it goes up ten thousand." She was able to extrapolate values using a graph, but was not able to write the equation of the line. She was able to find starting value and a rate, but did not make the link between them and the equation. This student demonstrated she was able to recognize increasing and decreasing slopes as population growth or loss, using the table and graph, but could not make the link to the abstract expression. She initially stated that she thought she could find the equation, and tried to use the spreadsheet program to do it, but became confused as to the process for finding an equation of the line using the program. She could estimate when the populations were the same using the graphs and knew that the point of intersection was where the populations were the same. "… One or two years they are going to cross. Between when the first one is increasing in population and going from thirty thousand to forty thousand in the first five years, and the next one is going down from 33 to 31 in the first five years, I don't know say, at, I don't know, 32 years. I mean at 32000, they might have the same population and that's going to be guessing maybe two years into the first five when this one is going down and the other is going up."

Student 3 was able to graph the data and describe the relationship as linear, positive slope. When writing the equation of the line, he did not find a unit rate, but instead used the five year rate of change as one increment, writing $y = 10x + 30$, instead of $y = 2x + 30$ or even $y = 10/5\ x + 30$ although he stated, "When x goes up 5, y goes up 10." He did not note that the population grew by 2000 people per year. He made a similar error in the equation of the decay graph. While he caught himself missing a negative when he wrote the decay equation, he did not notice the error in slope. His decay equation ended up to be $y = -2x + 33$, indicating that the population would have declined by 2000 people per year, rather than the correct decline of 400 people per year, which would have produced the equation $y = -.4x + 33$. He was able to extrapolate from the graph and predict when the city in Table A would have a population of 100,000, and use the intersection of the two graphs to correctly find the approximate time the two cities would have the same population, but did not use his equations to find this. An approximation was all that was asked for in the protocol, although the researcher suspected that this student could have found that exact value using the correct equations had he been asked, based on his demonstrated ability to work with equations in the other interview problems. His error appeared to be in finding the unit rate, not in solving equations. This student showed an ability to follow processes easily, but may have been following those procedures without thinking as deeply about the scenario as needed.

Student 4 translated from table to graph easily. He recognized that the growth rate was constant but did not find the unit rate right away. He stated that it was “one to two ratio,” and also said that the population went up 20 thousand every 10 years. He was able to interpret the graph into the scenario as growth, and find initial population. This student did not predict the time at which the city in table A would have 100,000 people correctly, stating instead that it would be 50 years. His constructed graph went off of the paper at this point, and he appeared to misread the anticipated location of the point. He was able to graph both tables and approximate when the cities would have the same population, at 32,000 people. However, he incorrectly stated that the second city would be empty in 60 years. He was not able to write the equation of the line by hand. In order to find the equation of the line, he turned to the spreadsheet program to calculate it for him after he had entered the data from table. He found the line correctly with the help of the computer. However, when asked a follow-up question of which number in the equation was the slope, he was not able to identify it, indicating he did not quite understand the meaning of the parameters in the equation.

The students had no problem translating from table to graph, as all were able to take the data from the table and plot the points. Student 2 initially questioned which axis was which, but was able to resolve this as she talked it through. All of the students drew their graphs for Table A using increments of five on the x-axis and increments of 10 on the y-axis. This then caused some of them difficulty when it came to graphing Table B, which changed by increments of 2 on the y-axis. The students were also able to interpret or translate from the table or graph to the context of a real world scenario, and determine when the cities represented by the tables were growing or losing population. All students knew that the slope represented a rate in units of people per year, although three out of four stated the rates in people per every five or ten years, rather than a unit rate. They understood that positive slopes indicated increasing populations and negative slope indicated declining population. Three students were also able to extrapolate from the graph by extending their lines. One had trouble because his graph went off the edge of his paper, so he did not determine the correct value, guessing instead where the approximate value would be. Three out of four were able to determine that the intersection of the graphs occurred where the two populations were equal at the same time.

None of the students were able to write an equation of the line for Table A correctly by hand, or to translate from graph/table/real world scenario to an equation. Student 4 used the computer spreadsheet program to find the regression line using Table A. It is important to note, however, that the first two students were interviewed only a day or two after the slope-intercept equation had been introduced, so the material was still very new to them. Student 3 used the five year rate rather than the unit rate, but otherwise had good facility with working with equations, quickly writing the equations of the lines and correcting himself when he realized he had missed a negative in the equation representing population decline. He did not catch his error in the slope, however. He had clearly used the slope-intercept form of the equation in the past. However, while he could perform speedy calculations throughout the process of the interview, his focus on change in y in his equation rather than the quotient representing slope was an indicator that he was not displaying a deep understanding the relationship between unit rate and the slope parameter in the equation. Throughout the interview, he consistently focused on the use of the abstract representation of the equation, although he was able to create graphs and use tables as well. This was in contrast to some of the other students in the class who seemed to avoid working with the formulas.

Student 4, on the other hand, clearly did not understand the locations of slope and y-intercept in the equation, although he was able to find the equation using the linear regression program on the computer.

Overall the researcher found that the students were able, in this context, to translate from table to graph fluently. The students were able to interpret the graph or table to the real world population scenario to verbal description in terms of describing what was happening to the populations represented by the two tables. They were not all equally able to extrapolate from the data or to identify when the two tables or graphs would be equal. Most were not able to move to the most abstract representation, or equation, at the time of interview, with only one student correctly finding the equation of the line using the computer program. Other data gathered during the course provided additional insight into the student thinking regarding development of student ability to translate from graph or table to equation.

*Task Interview Question 8 (Real World Scenario to Equation or Verbal Description)*

Question 8: If you need to tile your living room, and know that each tile covers 2.5 $ft^2$, please show me how you will calculate how many tiles you need to buy if you know how many square feet you have in your living room.

| Student | Comments |
| --- | --- |
| Student 1 | Student 1 was easily able to state that you divide the square footage by 2.5 to find the number of tiles. She said, "First of all what we want to do, is…we need to find out how big the living room is. Let's say for example it's 250 feet. Which isn't very big. Simply take …each tile is 2.5 inches? [square feet] We have the same unit there. So all we have to do is divide 250 by 2.5, then you need 100 tiles to um…and to do that, you just took 250 divided by 2.5…I know that the room is 250 square feet. Took the 250 square feet and divided by the size of the tiles, 2.5 square feet and got my answer. That told me I needed 100 tiles to cover the floor." She noted that they have the same unit, so it is a simple division. Her answer appeared to reflect personal experience with this type of scenario. She did not need a tool. She explained her answer, but did not abstract to a general equation, although she stated that she had to divide the room size by tile size. |
| Student 2 | Student 2 turned to the spread sheet to solve this problem, as the class had done with similar problems. She tried to create an equation on the spread sheet and was unsuccessful. She later turned to her calculator without success. Finally she talked it out using a specific example. … "Um. You would take your (goes to spreadsheet, deletes previous work, goes to look at programs used in the past). You're going to take, let's see, you would take the area of room in square feet, depending on how much square feet you have, say you have 275 and if each tile covers 2.5 square feet, and you're going to take divide it by your 2.5 square feet to get… (gets calculator) you'd need 110 tiles, if your floor was 275 square feet." With prompting [If your area was A, how could you write an equation to show that?], she finally stated her equation in terms of variables. She said, "A divided by 2.5 is B, and B is the number of tiles. So you take your square feet, A, and divide it by 2.5, to get B." She still exhibited a fear of having variables in her equations, stating, "I can think it and it makes sense and I can say it, but writing it is a completely different story." |

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| Student 3 | Student 3 turned to the spreadsheet, and a similar problem they had used in class. He stated that, “You just multiply the footage by 2.5,” but corrected himself when that answer apparently did not make sense to him. He then turned to a specific numerical example to check if it made sense, and determined you take total square footage divided by the square footage of the tile to get the number of tiles. “So let’s say 2.5 square feet and 500 square foot room, each tile covers 2 ½ square feet then you go….this divided by this (on spreadsheet)]. That gives you the total number of tiles needed. [Interviewer: so you wrote a little equation there?] Yes, it would be square footage divided by the unit of each tile. And that gives you 200. Tiles.” |
| Student 4 | Student 4 turned to the spreadsheet tool. He tried to create a table of example values, but in error multiplied his total footage again by number of tiles. He typed 4*2.5 on the computer, but then in error multiplied by the number of tiles again. He realized that 4 tiles could not cover 40 square feet, and when unable of find the answer using the spreadsheet, then apparently reverted to his number sense and personal experience using a specific example. “Let’s see, four tiles would cover 10 square feet. If I knew square feet, I’d divide it by the 2.5 to get total tiles. I can do that in my head. But I could do it on a calculator or on paper, I guess.” |

*Table 7.* Task Interview Question 8

*Discussion of Question 8.* This question was designed to discover if students could create an indirect proportion equation given a real world scenario or translate from real world scenario to equation. The students had done this in class by entering sample data in the spreadsheet program and then abstracting it and creating a general equation in the spreadsheet program that would allow them to enter any square footage and tile size to determine the number of tiles necessary in any case. The researcher also wanted to determine how well the students were able to work in this situation without being given particular numbers other than tile size, and determine whether they could use general variables without being given the variable letters themselves.

Three out of four of the students turned immediately to the spreadsheet tool to help them determine this equation, perhaps because this was a problem similar to those done in class on the spreadsheet. After working with the spreadsheet two of the students were unsuccessful using that method, but were able to use their number sense and personal experience in similar situations to find the equation and state it verbally. Both of these students stated that it was actually easier to find the equation just by thinking about the practical aspects of the problem than by using the tool. All of the students were eventually able to find an equation in terms of square footage and tiles, but they also all stated it verbally, rather than writing the equation on paper using variables. Therefore, the students were able to translate from a real world scenario to the verbal description of the equation, but not necessarily the written symbols. Two were able to create an algebraic equation in the computer, referencing cell locations, but did not write it with variables on paper. Prompting the students more explicitly to write down their equation may have resolved this issue.

*Summary of Task Interviews*

The task interviews provided information on how the students were approaching problems and thinking about their work. This information was helpful in answering the research questions surrounding student thinking and learning, and student ability to translate between representations of algebraic concepts.

During the task interviews, the students were asked to perform a number of translations between representations. As Lesh, Post and Behr (1987) suggest, ability to translate between representations reflects a deeper understanding of mathematical concepts. The following table summarizes the translations and the students' ability to perform those translations. A discussion reflecting themes regarding student thinking that stood out in this portion of the analysis follows the table.

| Translation | Student Performance |
|---|---|
| Graph to Real World Scenario (TIQ 4) | All students were able to perform this translation and interpret it verbally as well in the distance-time graph context. |
| Real-world Scenario to Graph, Table, Pictorial, Concrete or Equation (TIQ 6, TIQ 8) | In TIQ 6, the students moved from real world scenario to equations, and some utilized pictorial representations. They also stated verbally what was occurring in the situation. In TIQ 8, the students had difficulty with actually writing an equation using variables when not given actually variable names, but were able to state the equation or process verbally. |
| Table to Graph (TIQ 7) | All four students interviewed were able to make this translation. |
| Graph or Table to Real-World  Scenario (TIQ 7) | The students showed their ability to move from graph to real world scenario in TIQ 4, and had the opportunity to move from either the table or the graph to the real world scenario in TIQ 7. The students appeared to be using a combination of the two in determining what was happening to the cities in problem TIQ 7. Some looked at the graphs they had created to determine the starting value of the population, while others used the table. |
| Real world Scenario, Graph or Table to Equation (TIQ 7) | None of the students were able to perform this translation to a slope-intercept equation by hand. One student found the equation of the line using technology, although his understanding of the slope-intercept for of the line was questionable. |
| Translation to Verbal from Other Representations | During the interviews, students were asked to explain their thinking during all of the above processes. Where students were able to make the translations, they were also able to explain to the researcher their process. |

*Table 8*. Translations Performed in Task Interviews

*Student thinking*. One of the primary purposes of this research study was to determine how students enrolled in the Introduction to the Mathematical Sciences course thought about different representations of mathematics and were able to translate between representations. The task interviews provided information to the researcher on this important aspect of the study. The following bulleted points express some of the larger themes that stood out to the researcher during this portion of the analysis.

- Students were able to represent real-world scenarios and to use mathematics to solve contextual problems.
- Students showed an ability to move between a number of different representations of mathematics (table to graph, graph to real-world scenario, real-world scenario to equations—in some cases, and real-world scenario to pictures). They were able to read graphs and tables and make sense of the data in context.
- Students showed the least facility at the time of the interviews with moving to and working with the most abstract representation, the equation.
- Students were willing to utilize technology in the form of a spreadsheet program to solve mathematics problems and to produce the different representations such as tables, graphs and equations. However, some of the interviewed students relied on the technology without exhibiting deep understanding of the meaning behind their processes. Some also had difficulty recalling processes on the computer.
- Students who were interviewed readily discussed what they were thinking about their work with the different representations as they worked the problems.

*Classroom Artifacts*

During the semester, the researcher collected several types of classroom artifacts in order to determine what was being taught in the class and the manner in which it was being taught, and thus to help answer the research questions. These artifacts included: blank student worksheets from every class that the researcher observed (to determine topics covered), student work on two exams, student work on various worksheets and homework and copies of student created computer projects (to determine student thinking about the mathematics problems from the class).

It is important to note some of the artifacts were gathered from the instructor after grading. The researcher asked the instructor for copies of students' work. While the researcher would have liked a more systematic selection of the completed student work, with work from students at high, low and middle level (as determined by the course designers' algebra pre-test), she was not able to select the data herself. Nor was she able to obtain all of the artifact data for the students interviewed, although some was collected. The instructor stated that there was "no rhyme or reason" to those items he selected for the researcher, so the items were not picked for any particular reason, but rather were those he had at hand. Many of the worksheets were discussed in class, but not handed in for credit, so these also were not available to the researcher. The researcher did note that, based on student identification, student work artifacts came from students at a variety of skill levels, rather than all from one category or level. This may actually be beneficial in that it provides information from a wider selection of students. A table of the algebra related artifacts gathered by the researcher is shown in Table 9.

| Artifact | Research question |
|---|---|
| Exam II (6 copies) | R1 & R2 |

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|---|---|
| Exam III (6 copies) | R1 & R2 |
| Systems of Equation worksheet (1 copy) | R1 & R2 |
| Systems of equations 2 worksheets (8 copies) | R1 & R2 |
| Solve for y worksheet 1 (1 copy) | R1 & R2 |
| Solve for y worksheet 2 (13 copies) | R1 & R2 |
| Computer printouts | Intended for R1 & R2, but deemed not useful |
| Blank worksheets from all classes observed (1 copy of each) | R3 & R4 |

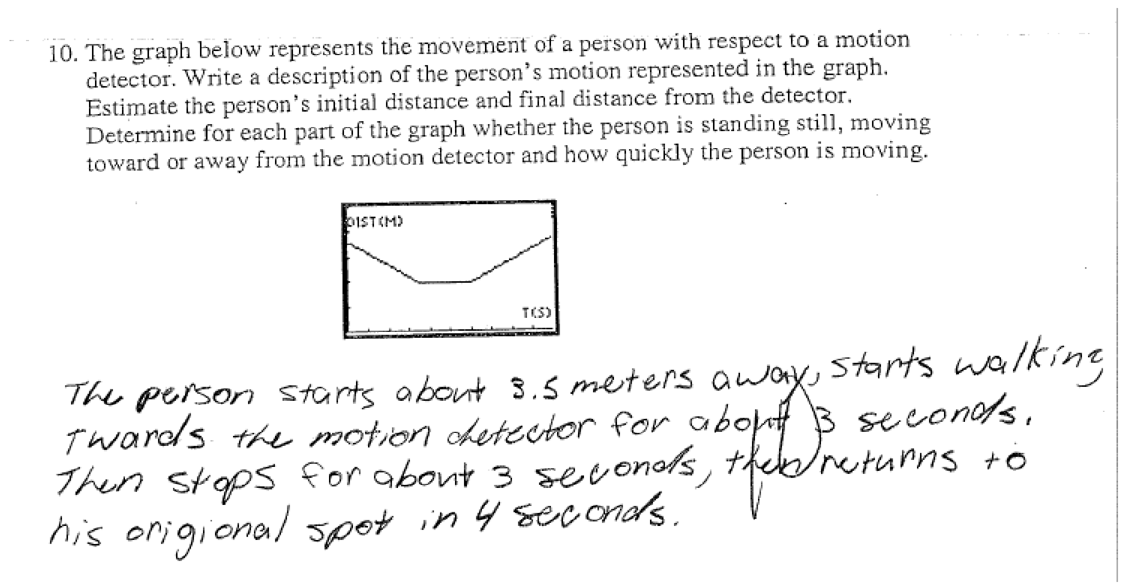
Table 9. Artifacts and Research Questions

The gathering of these data was intended primarily to inform the researcher on Research Questions 1 and 2, which attempted to pinpoint the impact of the class on student thinking and learning regarding algebra and its representations, particularly concerning slope and linear equations. The following analysis of classroom artifacts added to the data that could help answer these questions.

*Description of Analysis of Artifacts*

Because the focus of the first two research questions was on student thinking and understanding of algebraic concepts, particularly slope and linear equations, the researcher concentrated on problems in the artifacts that were related to these topics. Some of the artifacts were not helpful, as they contained material that was not as pertinent to the topics. For example, one of the exams (Exam II) concentrated more on proportions, ratios, and some statistics topics, because it was taken earlier in the semester. Many of the computer printouts were less than useful as well, since they did not provide the researcher with information on the students' equations that were entered into the spreadsheets, and thus did not help with answering questions about student thinking. A number of artifacts did provide information on the important concepts of slope and linear equations. These are outlined in the following sections.

*Interpreting rates in a graph.* Exam II did contain a problem that asked students to interpret motion in a graph showing distance versus time. Because this problem was of the type that led students to the concept of rate, then to slope, it is of interest to examine how students were thinking about this problem. The students had not yet learned the term "slope" at this point. Examples of student work are shown below. Different students are assigned different letters. Some student work on the different problems may be by the same students, however. If so, they are assigned the same letter. Work by Students A-D is shown in Figures 10-13.


10. The graph below represents the movement of a person with respect to a motion detector. Write a description of the person's motion represented in the graph. Estimate the person's initial distance and final distance from the detector. Determine for each part of the graph whether the person is standing still, moving toward or away from the motion detector and how quickly the person is moving.



The person starts about 3.5 meters away, starts walking twards the motion detector for about 3 seconds. Then stops for about 3 seconds, then returns to his origional spot in 4 seconds.

*Figure 10.* Student A Work on Motion Detector Problem

Student A's work is shown in Figure 10. Note that this student was able to identify distances and times, as well as the direction of motion. Although the question asked how quickly the person was moving, the student did not calculate the rate.

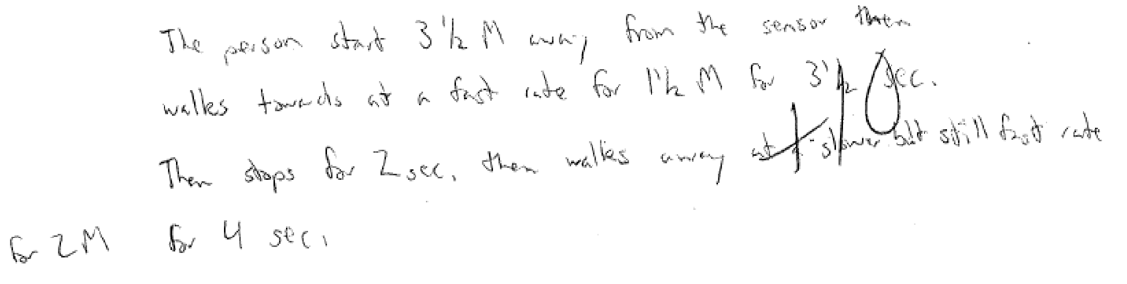



*Figure 11*. Student B Work on Motion Detector Problem

Student B was able to identify locations, direction and a rate, but not a unit rate. He stated that the person moved 2M in 4 seconds, but did not change to a rate of ½ m/sec, which would indicate how quickly the person was moving. He did notice that the rate in the final segment was close to, but slightly slower than the rate in the first segment.

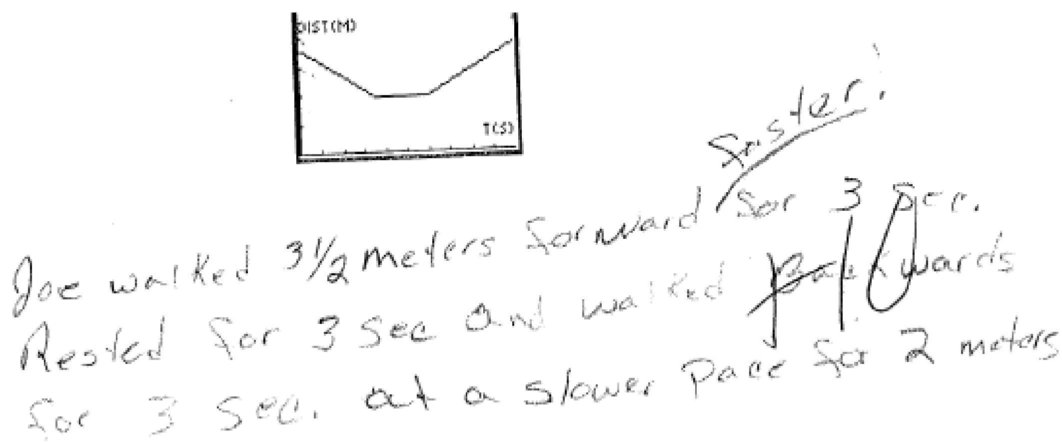



*Figure 12*. Student C Work on Motion Detector Problem

Student C did not correctly state the distance the person moved in the first 3 seconds. She instead mistakenly referred to the fact that the person started at 3 ½ meters away, rather than calculating that the person moved 1 ½ meters forward in 3 seconds, or at ½ a meter per second. She did state correctly that the pace was different for the first and last segments and that the first rate was faster than the last. She did not state initial and final positions, however. Note that this student is also Student 1 from the interviews.

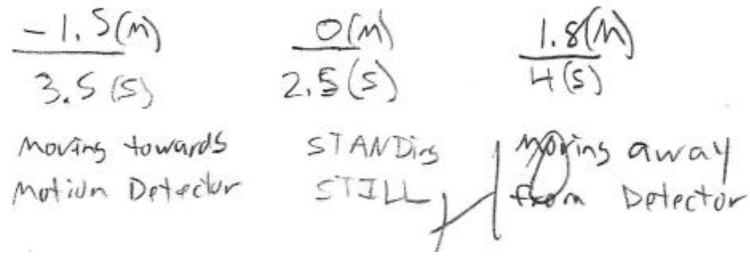



*Figure 13.* Student D Work on Motion Detector Problem

Student D created rates for each segment, using distance over time, and clearly noted direction of the motion. He did not convert to a unit rate in order to state how quickly the person was moving in meters per one second, however. It is unclear whether he was able to state this, from his work. He recognized that the first rate needed to be negative as the change in distance was negative. The student shows a variety of rates, including the units, but did not simplify his fractions. Nor did he note the initial distance or final distance asked for in the question and did not write his answer in sentences.

From these examples, students were at that point in the course (about a month and a half into the semester) to some extent able to interpret a graph in terms of direction and rate, although not necessarily a unit rate and had not yet been introduced to the term “slope”. The other two examples collected from this exam also showed ability to find the rate and discuss it thus showing that the students had some ability to translate from graph to written word.

*Solving for a variable*. Other artifacts showed that the students were able to solve a linear equation for y but these were not particularly interesting in terms of accessing the student thinking, as they were solved (as requested) using traditional algebraic methods. These worksheets were worksheets that asked students to solve for y in an equation. Some examples of student work follow. The researcher was able to obtain worksheets for three of the interviewed students in this case. The following three examples are from students who participated in interviews. They are titled by their interview numbers here, Students 2-4. Two examples from each worksheet are shown below. Other problems on the worksheet were set in the same format. The students were required to distribute and rearrange the equations to solve for y.

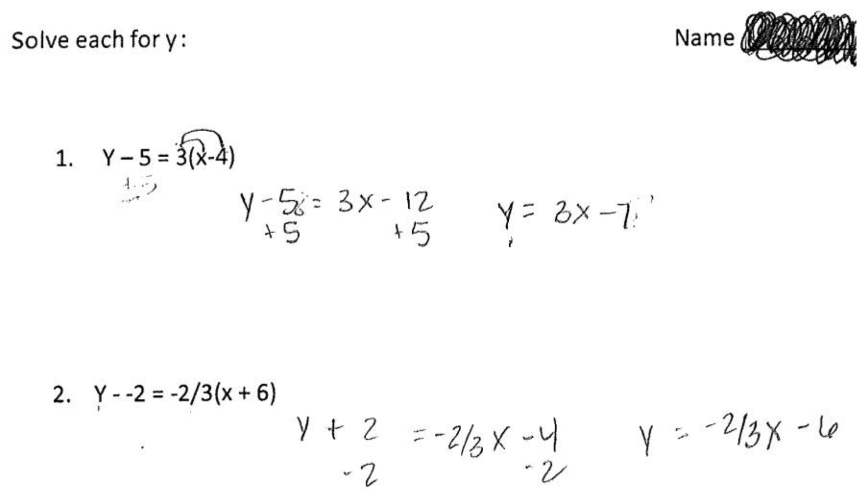



*Figure 14.* Student 2. Solve for y Worksheet 2.

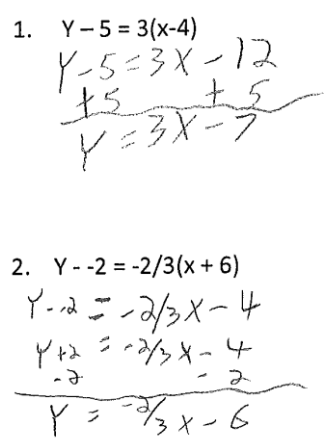



*Figure 15.* Student 3. Solve for y Worksheet 2.

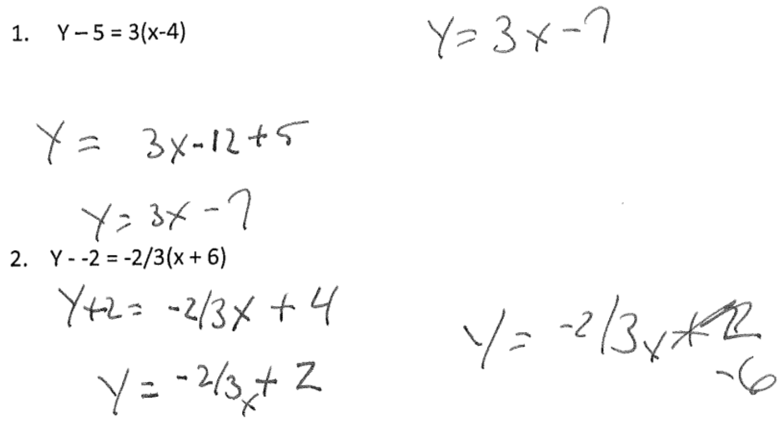


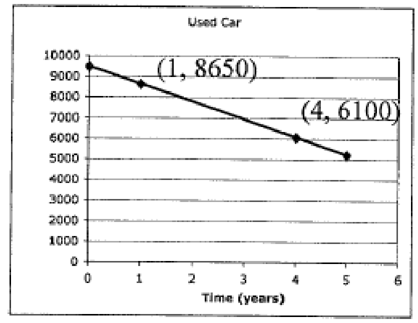

*Figure 16.* Student 4. Solve for y Worksheet 2

All of the students' work shown here indicated that the students were able to do the symbolic manipulation necessary, and only one of the students out of the 13 examples collected got more than one problem wrong on the six problem worksheet. While this assignment does not help in examining student thinking, it does show process and how the students were able to work with this representation. The data provide evidence that students were increasingly able to work with some symbolic representations of equations, although these examples show ability to manipulate variables, not understanding of the parameters in the equations. Exam III and a worksheet on systems of equations provided more insight into how students were thinking about the ideas of slope and linear equations. Further examples of student work pertaining to these ideas as evidenced in the artifacts are found below.

*Computing slope from a graph.* Questions on Exam III that asked students to compute slope from a graph and interpret that graph were also enlightening. Work by Students E and F are shown.

2. The graph below shows the approximate decline in the value of a used car over a 5-year period.
    a. Use the two points graphed to find the slope of the line segment.

-850




6

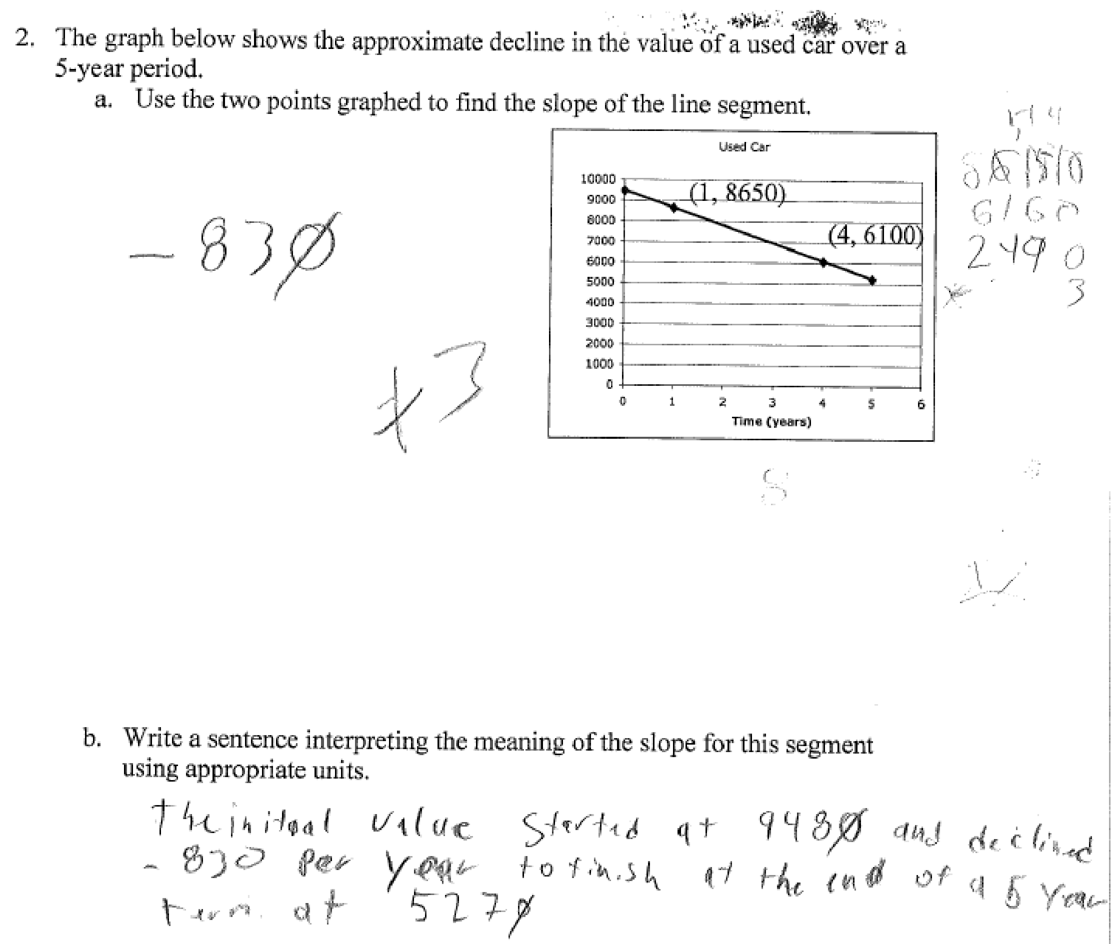
    b. Write a sentence interpreting the meaning of the slope for this segment using appropriate units.

The car decreased in value/at a rate of $850 per year

*Figure 17.* Student E Slope Problem

Student E was clearly able to calculate the value of the slope and interpret that slope as a rate of decrease in value, including the appropriate units. During classes, the student had learned how to enter two points into the spreadsheet and to calculate the slope by finding change in y, change in x and the quotient in the table portion of the spreadsheet. Part a. of the question here involved whether the students could do this, and part b. involved whether they could interpret that slope in the context of the problem. Since the students were always allowed access to the computers during exams, it is likely the students utilized the spreadsheet program while working this problem. Student E did not show work on his paper, so this use of the computer is presumed. The number 6 is a score written by the instructor and not part of the problem.

Student F's work shows a different skill level and thought process.


2. The graph below shows the approximate decline in the value of a used car over a 5-year period.
   a. Use the two points graphed to find the slope of the line segment.

−830



8650
6100
2490
3

   b. Write a sentence interpreting the meaning of the slope for this segment using appropriate units.

The initial value started at 9480 and declined −830 per year to finish at the end of a 5 year term at 5270

*Figure 18.* Student F Slope Problem

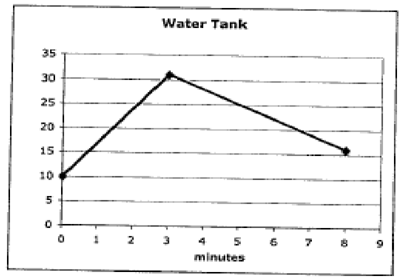
In part a., Student F subtracted incorrectly while finding the slope, and did not show a very clear method for finding the slope. It appears that the student did the subtraction necessary and indicated on the far right that a 3 was used to divide. As noted above, some students did their work on spreadsheets, but it appears this student was working the problem by hand, based on the error. In part b., the student estimated the initial value as 9480 using the graph, and final value as 5270 using the graph, but did not use his slope value of -830 to do the calculations. If he had, he would have needed to subtract 830*5 from 9480, resulting in 5330, which did not agree with his result. He did not use the fact that the rate of decrease was -$830/year at all in his calculations, which may indicate problems with his understanding of the constant rate of decline.

The five out of six examples of student work examined on this particular problem indicated that students were able to calculate slope from a graph using two points and to interpret that slope in the context of the problem. Some of the students attempted to estimate values of other points on the graph, however. Students were not asked to write the equation of the line given the two points, although one student did this, using the calculated slope and a guess at the y-intercept, rather than using a point and the slope. Students were not exposed to the use of the point-slope form of equation in this class, and could not be expected to utilize this method.

*Using graphs and tables to find and interpret slope*. Exam III also contained a problem that was based on flow rates into and out of a tank. Students were expected to find the slopes and interpret their answers in the context of the problem. Two examples of the six collected artifacts of student work on this problem follow. The first example, from Student G, is student work from one of the stronger students in the course based on exam scores, although his algebra pre-test score placed him in the middle range. The second student, Student H, was one of the weaker students in the class on the algebra pre-test. She was very detail oriented, but based on researcher observations, required a great deal of help from the instructor for many of the new concepts and had frequent absences from the class.

3. A water tank has an inlet pipe with a flow rate of 7 gallons per minute and an outlet pipe with a flow rate of 3 gallons per minute. A pipe can be either completely closed or completely open. The graph shows the number of gallons of water in the tank after x minutes have elapsed.

| Time(min) | Water(gal) |
|---|---|
| 0 | 10 |
| 3 | 31 |
| 8 | 16 |




a. There are two line segments in this graph. Give the slope of the segment that has a negative slope.

-3 ~~4~~ 3

b. Write a sentence that interprets the meaning of the slope you gave in part a using appropriate units, and state which pipes are open and closed.

inlet Pipe is closed

Outlet Pipe is loosing 3 gallons a minute

c. Interpret what (0, 10) means for this graph.

how many gallons were in tank

apun start up y-intercept

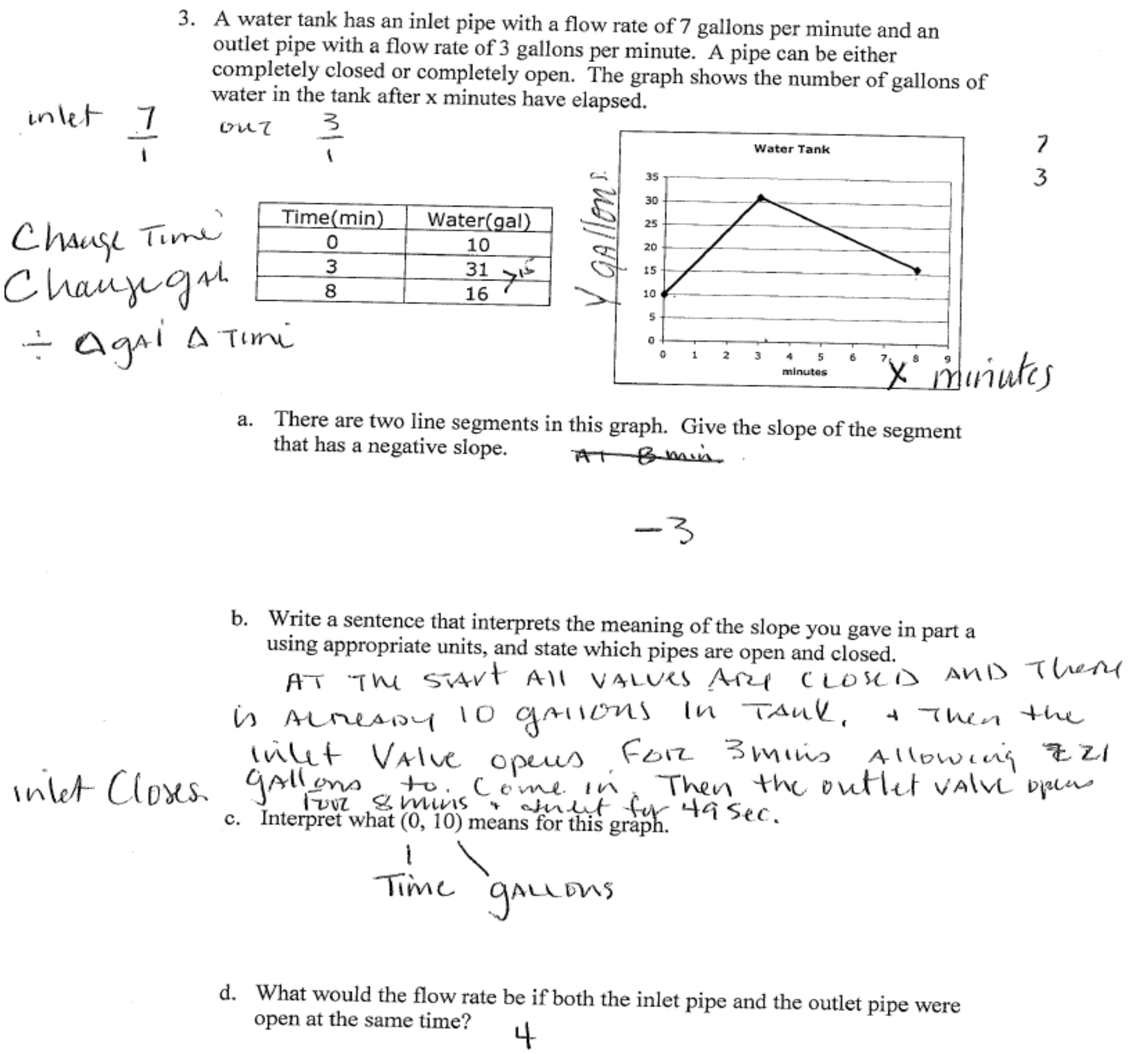
d. What would the flow rate be if both the inlet pipe and the outlet pipe were open at the same time?

7

+4 gallons a minute 3 1/2

*Figure 19.* Student G Tank Problem

Student G, who was one of the stronger students in the class, appeared to understand the problem clearly, based on his answers. He found the correct slope, and interpreted the slope and intercept on the graph correctly. As noted earlier, students were allowed to use the spreadsheet program for their work, which may explain the lack of work shown on the paper. Students who worked the problems by hand generally showed their work. The next example illustrates another student's solution.


3. A water tank has an inlet pipe with a flow rate of 7 gallons per minute and an outlet pipe with a flow rate of 3 gallons per minute. A pipe can be either completely closed or completely open. The graph shows the number of gallons of water in the tank after x minutes have elapsed.

inlet 7/1 out 3/1

Change Time
Change gal
÷ Δgal Δtime

| Time(min) | Water(gal) |
|---|---|
| 0 | 10 |
| 3 | 31 |
| 8 | 16 |



a. There are two line segments in this graph. Give the slope of the segment that has a negative slope. ~~AT 8 min~~

−3

b. Write a sentence that interprets the meaning of the slope you gave in part a using appropriate units, and state which pipes are open and closed.

AT THE START ALL VALVES ARE CLOSED AND THERE IS ALREADY 10 GALLONS IN TANK, + THEN THE inlet VALVE OPENS FOR 3 mins Allowing 21 gallons to come in. Then the outlet valve opens for 8 mins + shut for 49 sec. inlet Closes.

c. Interpret what (0, 10) means for this graph.

Time gallons

d. What would the flow rate be if both the inlet pipe and the outlet pipe were open at the same time? 4

*Figure 20.* Student H Tank Problem

Student H commonly wrote helpful notes to herself on her papers, which are useful in understanding how she was thinking about the problem. In this case, when giving and the rate in part d., she did not note the units in the problem. In part b, she explained what was happening in the tank, but again did not note the units, despite being asked for this in the problem, although based on her work, she was clearly calculating change in gallons over change in time. It was unclear from her work whether she was unsure of her units at this point. All other student examples indicated appropriate units in part d.

Five out of six examples of student work on this problem collected by the researcher reflected that students were able to find the slope, and interpret that slope in the context of the problem, indicating they were able to move between table and graph and the real world scenario in this context including finding slope and interpreting units.

*Writing equations and interpreting slope and intercept in a real-world context.* An additional problem on Exam III looked at how students were able to take a real world linear growth problem, write an equation and interpret the slope and intercept in the context of the problem. Examples by Students I and Student F follow.

4. Carpenter-4-U charges a consultation fee of $50. Once he starts a job, he charges $45 an hour.
    a. Give a linear equation to represent the cost y of hiring Carpenter-4-U for x hours.

       $y = 45x + 50$

    b. What is the slope of this equation? Write a sentence interpreting the meaning of the slope using appropriate units.

       45
       The charge is 45 dollars/hour.

    c. What is the y-intercept of this equation? Write a sentence interpreting the meaning of the y-intercept.

       50
       The initial charge is 50 dollars.

    d. Determine the cost of having Carpenter-4-U remodel a basement if it takes him 20 hours.

       $950

    e. What would the linear equation be if the consultation fee is $60 instead of $50?

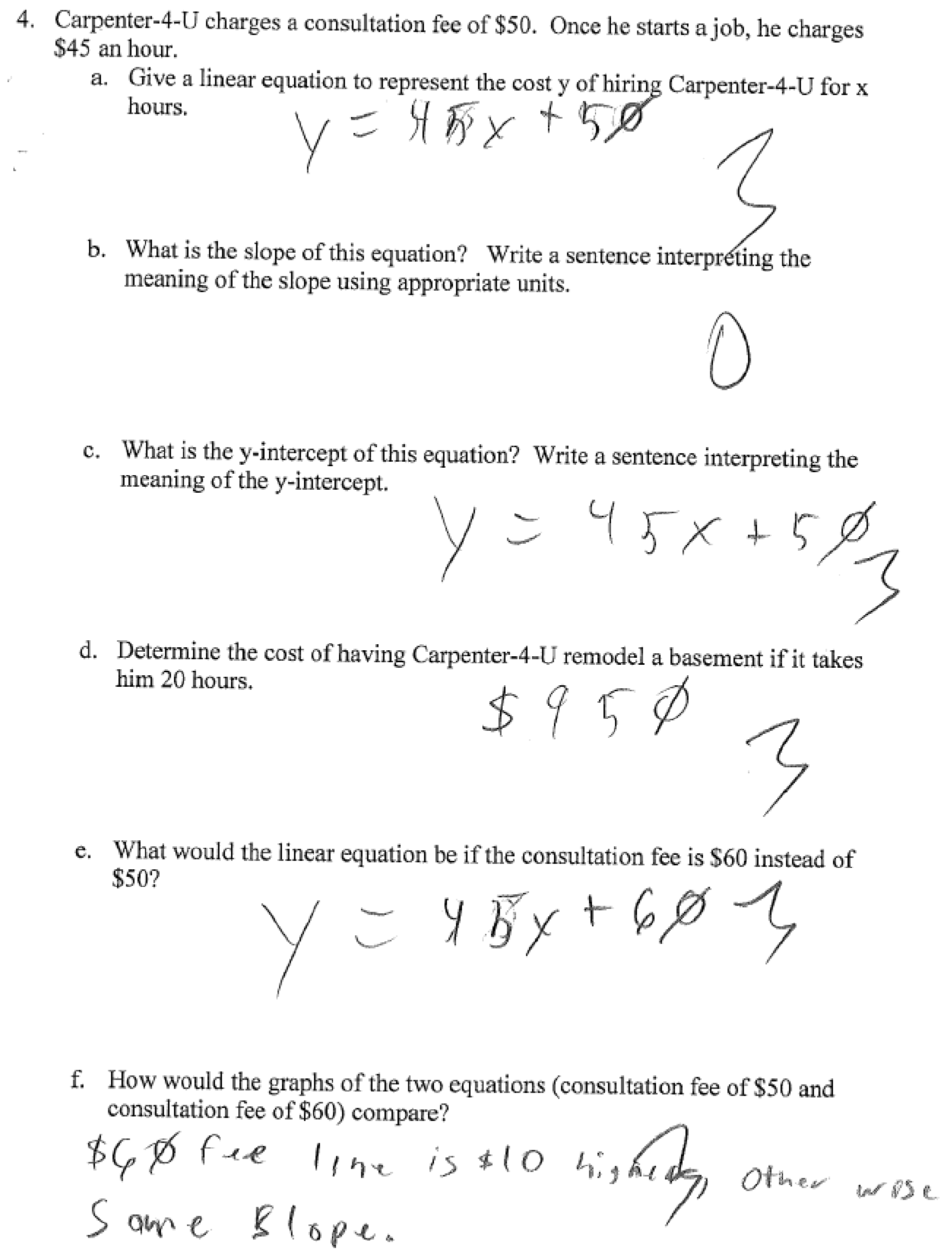
       $y = 45x + 60$

    f. How would the graphs of the two equations (consultation fee of $50 and consultation fee of $60) compare?

       they would be 2 parallel lines

*Figure 21*. Student I Linear Equation Problem

Student I had a clear understanding of slope, intercept, and linear equations in the context of the problem. He was able to write the equation from the context and explain the slope as well as what happened to the line when the y-intercept was changed. Student I was also a strong student in the class, who frequently asked and answered questions.



4. Carpenter-4-U charges a consultation fee of $50. Once he starts a job, he charges $45 an hour.
    a. Give a linear equation to represent the cost y of hiring Carpenter-4-U for x hours.
    $y = 45x + 50$ 3
    b. What is the slope of this equation? Write a sentence interpreting the meaning of the slope using appropriate units.
    0
    c. What is the y-intercept of this equation? Write a sentence interpreting the meaning of the y-intercept.
    $y = 45x + 50$ 3
    d. Determine the cost of having Carpenter-4-U remodel a basement if it takes him 20 hours.
    $950 3
    e. What would the linear equation be if the consultation fee is $60 instead of $50?
    $y = 45x + 60$ 3
    f. How would the graphs of the two equations (consultation fee of $50 and consultation fee of $60) compare?
    $60 fee line is $10 higher, otherwise same slope. 3

*Figure 22.* Student F Linear Equation Problem

Student F was able to write the equation of the line, but was not able to find slope and intercept in the problem, or interpret them. He was able to calculate total cost and find the equation of a line that had the same slope but a different intercept, and indicated that the cost was higher, but did not specifically mention that the lines would be parallel, although he indicated that the two lines would have the same slope. Understanding that parallel lines have the same slope is an important concept in algebra.

Of the six examples of student work collected on this problem, this student was the only one who did not complete all the problems on this particular page correctly, so his answers were not typical of those collected, but rather serve to illustrate difficulties other students in the class may have had with this concept. In general, however, students appeared to have no difficulty at that later point in the semester in moving from the real-world scenario to the linear equation in slope-intercept form. The students also had little difficulty in part f. in moving between the equations graphs and verbal description.

*Interpreting simultaneous linear equations and finding intersection graphically*. The final algebra topic in the class was finding and interpreting the intersection of two lines graphically.  Students were introduced to an online calculator and instructed in how to use it to graph linear equations, as opposed to generating the equation using the spreadsheet program as they had done previously in the class. This tool was introduced because the spreadsheet did not calculate intersection points of lines. Students practiced using the calculator in class and learned how to find the intersection of two lines using the program. They were then given a scenario of linear growth involving tuition and credits taken and asked to find when the total cost of education equaled a particular value. In other words, they needed to write and find the intersection of the equations of a horizontal (constant education cost) and non-horizontal line (the line indicating cost dependent on number of credits taken) using the calculator, and then interpret that result. They were able to do this relatively easily. Following this assignment, they were given a second assignment that involved cost and revenue for which they had to write two equations and were asked to find the break-even point. An example of student work by Student K on that assignment is show below.

1. The BSU Track Team is planning to sell t-shirts at the NSIC Conference Track Meet. Thunderbird Graphics charges $25 to set up a screen print with the NSIC Logo ($25 is called a fixed cost. No ma how many t-shirts they purchase they still need to pay $25.) and $5.75 a t-shirt (this cost depends on many t-shirts they sell, so it is a variable cost.). They plan to sell the t-shirts for $10 a shirt.

   a. Consider the linear equation that gives the amount spent in terms of the number of t-shirts purchased.

   What should the independent variable represent?

   5.75

   What should the dependent variable represent?

   25

   Write the linear equation that gives the amount spent in terms of the number of t-shirts purchase

   $y = 5.75x + 25$

   What is the slope? Write a sentence interpreting the meaning of the slope.

   5.75 per shirt

   What is the y-intercept? Write a sentence interpreting the meaning of the y-intercept.

   25 starting cost

   b. Consider the linear equation that gives the amount of money you take in for the sale of the t-shirt This equation is called your revenue equation.

   What should the independent variable represent?

   rate

   What should the dependent variable represent?

   y intercept starting cost

   Write the linear equation that gives the amount of money you take in for the sale of the t-shirts.

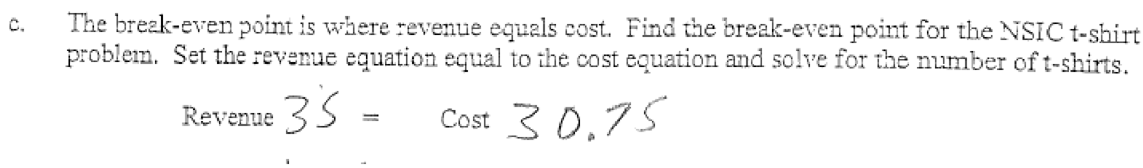
   $y = 10x + 25$

   What is the slope? Write a sentence interpreting the meaning of the slope.

   -10 Per shirt

   What is the y-intercept? Write a sentence interpreting the meaning of the y-intercept.

   25 starting cost



c. The break-even point is where revenue equals cost. Find the break-even point for the NSIC t-shirt problem. Set the revenue equation equal to the cost equation and solve for the number of t-shirts.

Revenue 35 = Cost 30.75

*Figure 23*. Student K Intersection of Lines Problem

Student K understood the linear equation for cost, but had several errors in his work. He exhibited a typical error seen in many of the students' assignments in thinking that the dependent variable, y, was the same as the y-intercept, 25. He was clearly confused about the meaning of dependent and independent variables. He also made a second mistake in including the 25 as the y-intercept in the revenue equation, which should simply have been $y = 10x$, not $y = 10x + 25$. He was not able to find the correct intersection point of the two equations at this point, because the two lines would be parallel. Later in his work, he was corrected by his work partner and was able to find that the break-even point was approximately at $x = 6$ shirts and interpret this correctly. However, this error in understanding the dependent variable (differentiating y-intercept and y as the dependent variable) occurred in several of the student assignments, as did including the 25 in the revenue equation. This was the last day of new material for the students, so it was unclear whether the misconceptions regarding the dependent variable were completely addressed in class before the semester ended. In addition, this assignment was collected on the review day, and all but one of students did not complete it fully. Because of this, only one example of student work was shown here to illustrate these errors. However out of the eight examples, only one student completed the entire worksheet correctly. The other students all had errors in their work regarding dependent and independent variables.

Based on the activity the researcher observed in the class, the students were able to graph lines on the computer and find the intersection point graphically, but interpreting this scenario on paper with two equations gave students some difficulty. They could interpret each line in a context, but were not necessarily able to interpret the cost-revenue scenario and break-even point, and had difficulty with the concepts of dependent and independent variables.

*Summary of Artifact Analysis*

The artifact analysis was used to examine student thinking in the Introduction to the Mathematical Sciences class and student ability to move between representations of slope and linear equations. Classroom artifacts showed that students were able to move between a number of representations of slope and linear equations as exhibited in their solutions to various problems on in-class assignments and exams. The following table summarizes those translations.

| Translation | Student performance |
|---|---|
| Translation from Graph to Real World Scenario I (Figures 10-14) | Most student artifacts indicated that students were able to describe a real world scenario from a graph. Some students did make errors in interpreting the rate units while doing this, however. This occurred more often in the earlier artifacts than those collected near the end of the semester. |
| Translation from Graph to Real World Scenario II (Figures 17-18) | Students were to use points on the graph to find slope and interpret this slope. The students were able to interpret the slope in the real world context, although some made subtraction errors during their calculations by hand. |
| Translation from Table and Graph to Real World Scenario to Verbal Description (Figures 19-20) | Students had to use a graph and table to find slope and interpret that slope. They also had to explain their solution. Five out of 6 examples of student work showed that the students were able to make these translations. |

| | |
|---|---|
| Real World Scenario to Equation I and vice versa (Figures 19-20) | All six student examples showed that students were able to write an equation in slope-intercept form from the real world scenario at this point late in the course, and also were able to discuss the meaning of slope and y-intercept in this context. |
| Equation to Graph to Written Language (Figures 21-22) | Students were asked to use their equations to describe graphs and compare those graphs. Students were able, on most examples, to perform these translations, although some did not state that the lines being discussed would be "parallel," using instead terms like "higher than" or "above." |
| Real World Scenario to Equation II (Figure 23) | Students were able write an equation given a scenario which provided a rate and initial start value, but several became confused when given only a rate, but no start value, electing to use a prior unrelated start value that was previously given. Note that this artifact was given on the last day of new material for the class, presented new ideas of cost and revenue, and may not have been indicative of student ability and may have reflected difficulty with the concepts of cost and revenue. |

*Table 10.* Translations Observed in Student Artifacts

Students showed a facility in interpreting rates from a graph, finding slope from points on a graph in a contextual problem, using graphs and tables to find and interpret slope, and had some facility with writing equations and interpreting slope from a real world context. While several students were not able to write equations of the line in slope-intercept form during the interviews, by the time they completed these assignments and tests, the class demonstrated greater ability to write the equations when given a rate and a starting value in a real world context. This indicates that students were progressing in their skills at writing linear equations as the semester moved along. As in any class, there were varying abilities and varying facility with working with the different representations. One type of artifact of student work showed that students appeared to be confused about writing an equation of the line when not given a starting value. The instructor addressed this in class, but no later artifacts provided data to indicate whether this clarified the matter for the students who were confused. The students were also able to find intersection points graphically using an online graphing calculator program, but based on student work, many were not successful in identifying dependent and independent variables in equations on a worksheet at the end of the semester.

These artifacts help inform the first and second research questions by providing insight into student thinking about algebra topics and student ability to move between different representations of algebraic topics, particularly slope and linear equations.

*Observation Data Addressing Research Questions 1 and 2*

A third source of data was used to provide insight into student thinking and understanding of algebra topics and ability to move between representations. This source was from field notes and recordings of classroom observations. The researcher observed the classroom 14 times over the course of the Fall 2009 semester. In a case study, field notes can help provide a rich description of the case that is being studied (Yin, 2009). Field notes can be used to present a picture of what is occurring in the setting being examined and through triangulation with other data, can add to the validity of qualitative inquiry (Cresswell & Miller, 2000) In this case, the setting is the classroom of the Introduction to the Mathematical Sciences course. By providing such a description of the events in the classroom, the researcher will add to evidence to help answer the questions regarding student thinking and ability to move between representations of algebra topics, with a focus on the ideas of slope and linear equations. Table 11 outlines the topics discussed in class during each observation.

| Date of Observation | Topics Discussed |
|---|---|
| 9-24 | Combining like terms, solving single variable equations, ratios rates and proportions. |
| 9-29 | M & M project, averaging, data validity, fraction review, standards for measurement, percent increase and decrease. Quiz on ratios, rates & proportions |
| 10-1 | Fractions, percents, analyzing histograms, simple spreadsheet programs using algebraic expressions, order of operations. |
| 10-6 | Histograms using the spreadsheet program, identifying qualitative graphs, rate and graphs. |
| 10-8 | Rate-time graphs (motion detector), rate and time-distance and velocity. Introduction to Discrimination problem (normal data), test review for Exam II. (mean, median, mode, simple programs on spreadsheets, rates, ratios, and proportions) |
| 10-20 | Mean and standard deviation, simulation, randomization. Rates, distance-time graphs, calculating rates using spread sheets. |
| 10-22 | Normal distribution and the empirical rule, quiz on normal distributions, more distance-time graphs, introduction of slope as a rate |
| 10-27 | Calculating slope using two points, finding an equation of the trend line in slope-intercept form on the computer, if-then statements in the spreadsheet program. |
| 10-29 | Slope and vertical and horizontal lines. Interpreting slope of a line from a graph (units), more if-then statements. |
| 11-3 | Decision making in the spreadsheet program, slope, rate of change and linear equations in slope-intercept form. |
| 11-10 | Review of trend lines (slope-intercept) and review for Exam III |
| 12-1 | Correlation, finding line from scatter plots and interpreting equation, solving for a variable. |
| 12-8 | More solving for a variable, using the online calculator to find intersection points of two lines, cost-revenue and the break-even point. |
| 12-15 | Review for final exam (simultaneous equations, normal curves, mean, median mode, solving equations, proportions, simple proportional programs on spreadsheet, rates and slope, If-then and Or statements) |

*Table 11.* Topics by Observation Date

The following section will describe the classroom as observed by the researcher in the fall of 2009 at STC. The description is provided through a compilation of the field notes taken throughout the semester of the class in order to help elucidate the type of activities and atmosphere in the class. This description can be used to provide evidence that can help answer the research questions regarding how students may be have been thinking about algebra topics and algebraic representations. Field notes were recorded on the OTOP protocol and many classes were also videotaped. Those notes will be used to describe the teaching and learning that occurred in the classroom. This section will also be referenced when addressing Research Questions 3 and 4.

*Describing the Case*

Although the students, setting and instructor were described earlier in this paper, a more detailed description is provided here in order to help the reader better understand the atmosphere of the class.

*Students and setting.* The Introduction to the Mathematical Sciences class was taught during the fall semester of 2009 at a small technical college in the Midwest. The students who participated in the course were mostly older than the average college student, and the students ranged from 18-57 years in age. The students also ranged in ability and area of study. A number of the students were changing to new careers as a consequence of having lost their jobs due to the recent economic downturn. Other students were just beginning their programs of study. Several of the students were in the nursing program, but many of the male students were in the manufacturing and engineering technology program. The students in the manufacturing and engineering technology program had most of their classes together, and were together most of the day on the class days. This commonality seemed to create a bond among many of the students.

The class took place in a computer lab at the technical college. The room was arranged in hexagonal computer pods, with six computers per pod. Each student had a computer in front of them, as well as an additional screen for every two computers, mounted above the computers, on which the students could observe the instructor’s computer work. This way, all students could see the examples that the instructor was showing on the computer. The sixteen students tended to sit close to each other, so some of the pods had no students at them, while others were full. How well the students could see the instructor and the whiteboard also determined where the students sat. The classroom had one whiteboard, on the west side of the room, but it was set in a position that was difficult for all of the students to see. Students had to move away from their computers in order to see the whiteboard, and no one sat at the pods on the side of the room farthest from the whiteboard.

The instructor’s desk was centrally located on the south side of the room. Although he did not spend much time at the desk, at times it was necessary for him to illustrate problems on the computer, which was located at his desk, since the students used spreadsheets for many of the problems. Most students could see the instructor from where they sat, but regardless, they could always see the computer screens.

*The class.* On a typical day, several students would be present in the classroom when Mr. Smith arrived from his office. Sometimes students asked him questions about math when he arrived, but most often, Mr. Smith engaged the students in conversation about topics he knew interested them. The students filtered in as class time neared, and they sat down and turned their computers on. Mr. Smith asked students to bring laptops if they had them, so that they could keep their data and could get used to using their own computers with the spreadsheet program, even though the class was conducted in a computer lab. Some students did use the classroom computers, but many had their own laptop computers.

For the most part, students arrived at class in time, and were ready for class to start. Mr. Smith usually handed out a problem for the students to work on immediately. Sometimes it was a problem related to problems they had done the class period before, but many times it was not. It was Mr. Smith's habit to give the students small bits of work at a time, so that they did not feel overwhelmed by the work. For example, he handed out a worksheet saying, "I want you to try number 1. Just try number one. It's OK if you don't know how to do it. Just try number 1. What are you going to try?" Students responded, "Number 1!" This repetition of instructions was typical of Mr. Smith's teaching style. While the students tried the problem, Mr. Smith walked around the room to each computer pod to see if the students were progressing. As he approached students, he asked the students probing questions such as, "What did you do? Why did you decide to do that?"

After students had completed the first problem, Mr. Smith usually asked a student to explain the problem and how to reach the solution. After the student explained the solution, Mr. Smith almost always asked, "Did anyone do it differently?" and often there were several possible ways the students found for reaching the solution. For example, students were asked to find out how much fertilizer they needed to cover a 2000 square foot area, given that 5 lbs of fertilizer were required for every 100 square feet. Some students calculated the solution by hand, by dividing by 100 and multiplying by 5, while others used the spreadsheet or made a table by hand to determine how many sets of 5 lbs they needed to cover the area, by following the pattern. After students had worked on this problem and discussed their methods, Mr. Smith showed them how they could quickly make a program in the spreadsheet that allowed them to enter any size of lawn and application rate, using algebraic formulas referencing cells.

The discussion of varying methods continued throughout the class. After several weeks of the class, students often volunteered, “I did it a different way,” before Mr. Smith even asked the question. Even the quieter students became acclimated to this explaining of answers as the class progressed.

During class time, Mr. Smith engaged the students in many ways. The class had a focus on three areas of mathematics: algebra, statistics and computer science. At the beginning of the semester, the students spent a class period gathering data from each other such as height, shoe size, foot length, forearm length and so on. During this process, Mr. Smith continually guided the students with his questions, asking, for example, about what units they thought they should use, and questioning how they decided exactly what the forearm was.

As another example of engagement, on a day when he was developing the idea of slope in the class, the students were asked to look at graphs of distance versus time as person moved to and from a motion detector. The students were to identify the rates of each of the segments of the graphs. Mr. Smith asked students when the rates were positive or negative. Some students responded, “It goes up or down.” Mr. Smith asked what they meant; forcing them to actually state that the line rises or falls from left to right. He also asked when the person whose motion was detected was moving fastest, and how the students knew. After the students had found some of these, Mr. Smith related the rate of the floor and the walls to zero and undefined slopes, without actually using the term slope at that point. Sometimes he “played dumb” in order to have students provide clearer answers. On the day the discussion above had occurred, the following ensued:

Mr. Smith: “If you have the roof line of a house, what’s that called?”

Student: “Pitch.”

Mr. Smith: “Pitch, what’s pitch? Who knows what pitch is?”

Second student: “Rise over run.”

Mr. Smith: “What do you mean? I don’t know what you mean by rise.”

Second student: “Rise is going up. Like the distance it’s going up.”

Mr. Smith: “So if you went from the edge of the house. Or better yet, if you went from right below the peak of the house, and you went up and then over. [Draws on board]. What’s that called, what’s the common name for it?”

Third student: “Slope.”

Mr. Smith: “Slope! What is slope?”

Fourth student: “An incline.”

Mr. Smith: “What do you mean, incline? I don’t understand.”

Other students: “An angle.” “A rate.”

Mr. Smith continued in this vein until the students realized that slope was a type of rate comparing rise to run, or change in vertical to change in horizontal. Through his questioning, he forced the students to clarify their understanding of slope. During this process, he related the mathematical ideas to the everyday world in which the students lived. Throughout the class, he also tapped the knowledge of the students who were more advanced in order to have them explain the ideas to those who were not as far along. The student in the previous discussion clearly knew that slope was rise over run, but needed to articulate more clearly what the idea meant for him and for the other students.

Mr. Smith often led the students in this type of a discussion, and then had them return to the problems, stating, “Ok, now try number two. Just try the next one. We’re only interested in the next one right now.” The students at this point were much more comfortable with what they were doing after having the discussion. The class was characterized by short assignments with discussion and explanation throughout the class period. Students completed the assignments in the classroom, and rarely had homework. Occasionally, Mr. Smith would give them a problem to take home and say, “I want to you look at this problem and think about it. Don’t do it, just think about it. We’re going to do it next class. I don’t expect you to know how to do it right now.” This provided students with a new concept to consider, without forcing them to complete the problem on their own.

Students usually worked in pairs, sometimes groups of three. Mr. Smith frequently stated, “Ok, now work together. We don’t live in a vacuum. Don’t work alone. I want you to work together.” In some cases, he allowed them to consult with each other on quizzes, and always asked them to compare their answers on homework with others, and then talk about why they might be different if they didn’t agree.

The pace of the class was geared so that students were provided enough work time to complete the problems. Mr. Smith frequently commented. “It’s not a race. Don’t worry; we’re not in a hurry here.” At the same time, when students had completed the work and the class had discussed answers, Mr. Smith would quickly pass out a new problem or worksheet that was sometimes completely different from the topic they had just studied. If the students had been doing an algebra problem, they might have been next presented with a new statistics problem. Although algebra and computer science were used in the statistics problems, changing topics frequently kept the students engaged. Mr. Smith noted in an interview that this was necessary for these college students, and to keep the pace of the class going. Because the assignments were, for the most part, limited to 8-10 problems and sometimes fewer, this change of topics did not seem to intimidate the students.

During the class, students were frequently asked to work problems using the spreadsheet program. Many of the algebra problems for computing slopes were done on the computer. Students entered points, then computed change in x, change in y, and then the quotient, all on the computer. Mr. Smith also introduced linear equations on the computer, having the spreadsheet program find the linear regression equation. Students did not compute these by hand initially. Some of the students had technical difficulties with the computers. In an interview, Mr. Smith informed the researcher that some of the students had not used computers at all prior to the class, so they were learning the mechanics of the computer as well as the mathematics. He stated that some of the students from his previous classes now were self-proclaimed computer nerds. During the classes observed by the researcher, some of the students required Mr. Smith’s attention to help with this aspect of the class as well. This was the only time during the class when the other students appeared restless or bored. During such times, some students played computer games or checked their email if they had finished their work. This only occurred on two out of fourteen observations of the class, however, so it was not a frequent occurrence. The students were usually engaged in the class. If students did not seem to be paying close attention, Mr. Smith would sometime poll the class on their answers in order to get them reengaged. Mr. Smith also kept the students' attention by telling them interesting facts. For example, when discussing the rate gallons of water usage per day, Mr. Smith used real numbers from his hometown. He had the students guess how much water was used with a shower, a bath, a whirlpool, or for flushing a toilet. This helped give students an idea of the magnitude of water usage, again in relation to their real lives.

*Assessment.* Formal assessment in the class took the form of short quizzes and several exams. The quizzes were frequently graded in class so that the students had immediate feedback on how they had done. These quizzes seemed to be much less formal than the exams. Prior to exams, Mr. Smith provided the class with a review for the exam, which contained problems similar to those that were on the exam. The exam itself contained multiple choice questions, manipulation problems the students had to work out, interpretation problems, and problems that needed to be completed on the computer. The students seemed to like the reviews, although during the review for the final exam, some seemed stressed out and complained about a few of the problems.

Informal assessment was occurring throughout each class period through Mr. Smith's questioning of the students. Because they were continually responding to him, he was able to hear each day where the students were in their understanding of the material. He frequently would spend time at the beginning of class reinforcing a concept from the previous class by giving the students a problem like they had done the day before, then advancing the problem to stretch their understanding. For example, Mr. Smith asked students to do a problem relating to the motion detector, in which they had to calculate the various slopes of the segments, and then offered a graph that had a vertical slope, to challenge their thinking. This way, the students were able to review, but also were introduced to a new concept.

*Summary of Field Note Analysis Pertaining to Research Questions 1 and 2*

*Research question 1.* The first research question focused on the impact of this integrated standards-based course on student thinking and learning, particularly regarding algebra and its representations. The term impact can be considered in several ways. Student behavior can be observed. What the students demonstrate they can do in a class can also show the impact of the class on those students. It is clear from the field notes that students were engaged in this class, and participated actively in the class. They readily asked questions about mathematics and responded to questions asked by the instructor. The students did not hesitate to discuss mathematical concepts and were willing to work on algebra problems set in context and also problems that were not contextual, although the majority of problems were set in practical applications in the class. When discussing rates and slope, examples included flow rates from pipes in gallons per minute, rates of lawn mowed in percent of lawn per hour, and depreciation rates in dollars per year.

Throughout the course, students were presented with a variety of representations when studying the algebra in the class. Some problems were presented in tables, some in word problems and others in graphs. Students frequently used the computer to move between representations, often being given a scenario, entering data into a table and creating a scatter plot and trend line, a pictorial representation. Yet others were presented strictly as algebraic symbols in the form of equations to solve. Problems were often posed verbally in the class, or as written word problems and students were asked to interpret those problems.

Students were allowed to work problems independently and to present their solutions to the class, often using multiple solution paths. The instructor probed the students' thinking and the students responded by clarifying their statements about their work. In the examples shown above, students were willing to re-explain their understanding of slope as a rate of change, and as rise over run. This helped the instructor determine the student level of understanding as well as helping other students understand the concept.

*Research question 2.* This research question is centered on how students in this class demonstrated understanding through an ability to move between representations of algebra problems, specifically relating to slope and linear equations. During two class periods, the students utilized the motion detector generated distance/time graphs to "walk the graph" in class. The students were able, through this activity, to discuss the impact of increased rates of speed on the slope of the graph, and effects of direction of the motion on the positive and negative slopes. This aided their ability to recognize direction and rate of increase or decrease in a graph immensely. In addition, they were able to verbalize what was occurring in a particular graph after this activity. This is one example of how students demonstrated in class an ability to interpret a graph, or translate from graph to real world scenario to verbal description. In addition, students spend quite a bit of class time explaining in class what a particular table on the spreadsheet meant, or creating an equation from a written description.

In developing the concept of slope, the instructor initially began with the idea of rates. Early in the semester, the students studied rates and unit rates that they were familiar with, and such as miles per hour, feet per second, and dollars of pay per day. The class was then introduced to the motion detector, and distance-time graphs representing the motion. The students were asked to "walk the graph" in class, or replicate the graph by their movements. At first students walked the graph while looking at the graph, and then some students where blindfolded and the other students were told to direct the student. Discussion in the class elicited from students the idea that a steeper graph indicated a faster rate, and direction forward and back were determined by the decreasing or increasing graphs, respectively. The class was asked to actually determine the rates by stating how far a person moved in what amount of time and then convert to a unit rate. Students sometimes argued about numerical values as they read the graphs, and Mr. Smith allowed them to interpret some of the graphs slightly differently, but elicited from them consensus that direction of motion based on the increasing or decreasing graph must agree.

Once the students were able to understand and discuss these concepts, the class was presented with pictorial representations of the graphs in assignments and asked to interpret the rates and direction of motion. The students were next provided graphs with two points and learned to utilize the spreadsheets to calculate the unit rates by finding change in y, change in x, and the quotient. It was only after the students had discussed rates for some time that the term "slope" as a rate of change was introduced. Because this process was slow and took place over the course of the semester, the students did not appear intimidated by learning about slope. After working with numerous examples that required the students to calculate slope from a graph, including the introduction of vertical and horizontal lines, the students learned to utilize the spreadsheet program given two points to generate a scatter plot and then a linear trend line. After finding a variety of lines using the spreadsheet, the students were asked to calculate slopes using the spreadsheet and then generate lines, recording the slope and the linear equation. The lines on the spreadsheet program were always given in slope-intercept form. As the students continued to work with the slopes and lines, and through Mr. Smith's questioning, they realized that slope is the "m" parameter of the equation and that the "b" parameter in the $y = mx + b$ formula was the y-intercept on the graph. This was an "aha!" moment for many of the students. During the discussion on that particular day, students compared answers and corrected each other if they had signs or values incorrect, making statements such as, "No, it goes down, so the slope must be negative!" or, "Look at the graph, that can't be right."

Because slope was initially presented as a rate, it was not a jump for the students to be provided with real-world scenarios that depicted linear growth, such as a plumber charging a rate of $25/hour. Since the students now understood (based on observations of class discussions) slope as a rate, they were able to make the link to writing equations of the line in slope-intercept form when given initial values, such as cost for the plumber to come to the house, and rates. These scenarios were usually applications of linear equations that the students could relate to in their real lives. Students were also asked to explain in class what the slope or intercept mean in a particular equation that represented total cost of labor for a carpenter, for example.

Because the students were presented with slope a number of representations, they were able to make connections between those representations. Observations indicated that the students were able to discuss graphs, tables and equations when working with linear equations and slope. The students moved from tables to graphs to equations using the computer multiple times during the observations. They wrote equations given real-world scenarios using rates and initial values, and identified and interpreted slope and y intercept in contexts.

As with the case of the classroom artifacts, not all students showed the same facility in working with every type of representation, and ability to work a problem in class when discussing it with others and the instructor is no guarantee that students can replicate the problem outside of class. However, the observations showed that students were able to move between the above-discussed representations in the classroom.

*Summary of Data Addressing Research Questions 1 and 2*

Evidence has been provided in the previous sections that indicates that students who were taking the Introduction to the Mathematical Sciences course have been impacted by taking that course. Based on data gathered in the form of student task interviews, classroom artifacts, and observations of the class, students were able to perform a number of translations between representations of algebraic ideas and insights into student thinking about algebra has been provided. The specific focus of this study has been on how students thought about and were able to translate between different representations of linear equations and slope. A summary table of the data outlining the data sources used to indicate understanding through ability of students to move between different representations of linear equations is shown below.

| | |
|---|---|
| Graph to Real World Scenario (TIQ 4) | **Task Interviews:** All students were able to perform this translation and interpret it verbally as well in the distance-time graph context. **Supported by observations.** |
| Graph to Real World Scenario I (Figures 10-13) | **Classroom Artifacts:** Most student artifacts indicated that students were able to describe a real world scenario from a graph. Some students did make errors in interpreting the rate units while doing this, however. This occurred more often in the earlier artifacts than those collected near the end of the semester. |
| Graph to Real World Scenario II (Figures 17-18) | **Task Interview:** Students were to use points on the graph to find slope and interpret this slope. The students were able to interpret the slope in the real world context, although some made subtraction errors during their calculations by hand. **This translation supported by observations.** |
| Graph or Table to Real-World Scenario (TIQ 7) | **Task Interviews:** The students showed their ability to move from graph to real world scenario in TIQ 4, and had the opportunity to move from either the table or the graph to the real world scenario in TIQ 7. The students appeared to be using a combination of the two in determining what was happening to the cities in problem TIQ 7. Some looked at the graphs they had created to determine the starting value of the population, while others used the table. |
| Real world Scenario, Graph or Table to Equation (TIQ 7) | **Task Interviews:** None of the students were able to perform this translation to a slope-intercept equation by hand. One student found the equation of the line using technology, although his understanding of the slope-intercept form of the line was questionable. |
| Real World Scenario to Equation I and vice versa (Figures 21-22) | **Classroom Artifacts:** All six student examples showed that students were able to write an equation in slope-intercept form from the real world scenario at this point late in the course, and also were able to discuss the meaning of slope and y-intercept in this context. |
| Real World Scenario to Equation II (Figure 23) | **Classroom Artifacts:** Students were able write an equation given a scenario that provided a rate and initial start value, but several became confused when given only a rate, but no start value, electing to use a prior start value that was given. Note that this artifact was given on the |

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| | last day of new material for the class, presented new ideas of cost and revenue, and may not have been indicative of student ability as it might have reflected difficulty with the concepts of cost and revenue. |
| Table and Graph to Real World Scenario to Verbal Description (Figures 19-20) | **Classroom Artifacts:** Students had to use a graph and table to find slope and interpret that slope. They also had to explain their solution. Five out of 6 examples of student work showed that the students were able to make these translations.<br>**Supported by observations.** |
| Equation to Verbal Description | **Observations:** Students discussed the meaning of the slope and intercept given an equation and a context. |
| Table to Equation | **Observations:** Students were able to create a line using two points through the spreadsheet program on the computer. |
| Equation to Graph to Language (Figures 21-22) | **Classroom Artifacts:** Students were asked to use their equations to describe graphs and compare those graphs. Students were able, on most examples, to perform these translations, although some did not state that the lines being discussed would be "parallel," using instead terms like "higher than" or "above." |

*Table 12*. Translations Performed

Research Questions 3 and 4

3) *How and to what extent does the course reflect fidelity of implementation of the course designers' vision of college reform in mathematics education for the two-year college?*
4) *How and to what extent does the teaching of this course reflect the NCTM, MAA, and AMATYC standards for college algebra reform in a two-year college setting in the areas of both content and pedagogy?*

The two above research questions are concerned with examining whether the course is being implemented in a manner consistent with the course designers' vision for reform mathematics, and also whether the course reflects national standards for college algebra instruction. In order to provide insight into the answers to these questions, the researcher analyzed all five forms of data collected for the study: Student interviews, instructor interviews, classroom artifacts, observations and OTOP data. Analysis of each of these sources as they apply to the above research questions is outlined in the following sections.

*Analysis of Task Interviews for Research Questions 3 and 4*

The task interviews posed a number of questions (TIQ 4-8) that examined linear equations and slope, in order to determine how students were thinking about those algebra topics. In addition to these questions, there were three questions that attempted to add to evidence of the manner in which the course was being implemented, Task Interview Questions 1-3. The results of the analyses of these three questions are given below. For each task interview questions, there is a statement of each question followed by a tabular analysis of the answers by each student and discussion of important themes that arose during the interviews.

*Task Interview Question 1*

Question 1: Tell me about some of the ideas about algebra you feel you have learned in this class.

| Student | Comments |
| --- | --- |
| Student 1 | "Um. You know I feel like I've learned some Algebra but I didn't realize I was learning it. Which is a really a good thing. Because too many times we walk into a situation like this, like I was just deathly afraid of algebra and didn't think that I was capable of doing it. And the way that Mr. Smith has explained it and walked us through it hasn't even seemed like a problem at all. Yeah and there's more people that feel the same way that I do. |
| Student 2 | "Actually, kind of a lot. I don't mind the algebra stuff now. I guess, just |

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|---|---|
| | kind of figuring out how do to the problems, um. Logically looking about them without just being overwhelmed by them. With being oh my gosh, it’s algebra. [Do you mean the written problems then?] Yeah, even the island problems, and just the different, not that they know what I’m talking about, but the stuff we were doing today. Like just knowing how to set it up to get your answer, versus just oh my gosh it’s algebra, I don’t want to do it. So I think it’s helped me so far. [So the island type problem, and the solving equations you’re saying?] Yeah. [And then what we did today which involved slope?]. Yeah but, understanding, besides understanding the slope, like the slope is…I don’t even think about that. Because if you didn’t know how to set it up, because then you’re just…going oh my gosh, it’s like an overwhelming thing. If you don’t even know how to set it up, like where you use the algebraic part to set it up to get the slope and the rest of your answers is I guess a way easier concept than before. I’d have been, like, I’m not doing this before. Yeah I said today algebra is great compared to those ratios and the stuff that we ... (inaudible)...and I was like dreading it, because this is the last, the very last class I have to take. I did not want to take a math class. [So you feel a little less intimidated than before?] Yes.” |
| Student 3 | “Algebra on its own, just algebra? [As a part of the class.] None. [Why do you say that?] Because I think I’m beyond the algebra of the class. Just in the actual algebra form. [So you learned it before?] Now the spreadsheet…and statistics, and all that stuff, that’s the stuff I’ve learned more.” |
| Student 4 | “First just solving algebraic equations. I was not exposed to algebra in high school which was over 25 years ago for me. [You weren’t exposed to it?] No. I’m not sure if it wasn’t offered or it was just an earlier period in history. I would guess it was offered but I didn’t take it. I just took basic math in high school.” |

*Table 13.* Task Interview Question 1

*Discussion of Question 1.* This question was intended to help determine which algebraic ideas stood out to the students, and to a lesser extent, which topics were being taught in the class, in order to help answer the research question regarding fidelity of implementation of the course. Each student had something to say about the algebra they were learning.

Student 1 discussed how she felt she was learning algebra without realizing it. She referenced her past fear of algebra and belief she was incapable of doing algebra, and stated that the feeling had lessened throughout the class.

Student 2 mentioned setting up and solving equations and learning about slope in the class. She, like Student 1, stated that she "didn't mind the algebra stuff now," whereas she stated that she had dreaded taking this math class.

Student 3 felt he had not learned any new algebra topics in this class, as he had studied those covered in the class in other classes in the past. He noted that the other two topics of the class, statistics and computer science had provided new material, however.

Algebraic equations were important to Student 4. He stated that he had not had algebra in the past and so this was all new material to him.

The interviewed students consistently mentioned equations as a topic of study, but were more inclined to talk about how they felt about the algebra, rather than the actual topics covered. Some went into discussion about way the material was taught, the topic of the next question, even before asked. Three of the four students interviewed clearly stated that they felt algebra was easier for them now, or less intimidating.

*Task Interview Question 2*

Question 2: How do you think the material in this class is different from math classes you have taken in the past?

| Student | Comments |
|---|---|
| Student 1 | "I think that mostly due to the professor, he takes an individual approach. And he makes sure that the class understands the concept before he goes on to a new concept. And he understands that there are a lot of different ways to learn math, and he tries to find that nugget in |

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| | each person so that they are able to get up to par with the rest of the class. [Ok, what are some specific things you think we have learned in this class?] Oh, um angles, and some equations, lots of equations. This is a class you know where it has, is a combination of three things. You know we've covered the statistics, the spreadsheets or the computer science and then the algebra. It's a nice compilation of all three. They bring it all together. And it's kind of nice, a lot of the equations you would normally do on the calculator, you can accomplish on the computer." |
| Student 2 | "Because you work, I guess just in the concept of you work through it, and you hear everyone else's ideas and you, they don't just give you a problem and explain one way of doing it and this is how you do it and expect everyone to understand. You, he, looks at everybody's ways and concepts and how you do it and so you can see the different ways but then you can find a way you understand or that works with how you can do it, versus this is it and this is how we're doing it. [And the type of material we are doing?] Well…it's been a long time since I had a math class. Maybe 9th grade? Oh my gosh, 16 years? I didn't like the ratios part…but…um, I guess it has more I guess, you deal with figuring out things in everyday life versus just an algebra problem or just a something you have out of a textbook, with just x and y and they don't mean anything. [So you're saying this seems more practical?] More practical and deals with things you would look at or see in your daily life versus just an equation that someone thought up and put in a book. So, yeah." |
| Student 3 | "Not so cut and dried. He teaches that you can have more than one way to find one answer. That's the biggest thing….as long as you get the solution or the answer, there's many logical ways of doing it." |
| Student 4 | "It seems to be, I'm not sure what the good word is, it's um, interesting. The teacher walks around the room and either uses the computer screen or the whiteboard to walk you through the problems. We move along at the pace of the slowest student, which could possibly be me sometimes. He makes sure we've got it before we move along. I appreciate that. [So the pace is different?] The pace is different. [What about the material?] The material, for me is material I've not been exposed to at all. Statistical stuff. Almost every day is something brand new. Things I wasn't aware of, and I find very interesting because it applies to a lot of everyday things. Glad I'm taking the class, I guess. It's going to help. |

*Table 14*. Task Interview Question 2

*Discussion of Question 2.* This question was intended to help answer the research questions three and four regarding fidelity of implementation of the course and whether the course was being taught as a reform course. Reform courses are intended to be taught in a manner that provides material to students which is more applicable, shows greater connections between topics, and covers fewer topics in greater depth (Baxter Hastings, et al., 2006). Student commentary on how the material differs from classes they had taken in past was helpful in determining whether the course is being taught according to the course designers vision and according to national standards.

Student 1 noted that this class is a compilation of the three topics (algebra, statistics, and computer science). In terms of topics taught in the class, she mentioned solving equations, and angles (relating to slope). She noted that she appreciated how the three topics, algebra, computer science and statistics were tied together. She also noted that it was good to be able to do some of the work by using the computer that may have normally been done on a calculator. In addition, she mentioned that the class moved along at a different pace than the other classes she had had. They did not move on to a new concept until the previous concept had been learned.

Student 2 stated that the material she had learned in this class was much more practical than other classes she had studied in the past. She noted "you deal with figuring out things in everyday life versus just an algebra problem or just something you have out of a textbook, with just x and y and they don't mean anything." She also noted that different ways of solving the problems were accepted in this class, and that the class differed in that you got to hear other students explain their answers in class.

Student 3 felt that this material was not so “cut and dried,” as other classes he had had and he appreciated that the instructor accepted many logical solutions to a particular type of problem.

In terms of material covered in this class, Student 4 noted that the majority of it was new to him, but he found the class interesting because it applied to “a lot of everyday things,” and believed that the material was going to help him in the future.

The major theme in this interview question was that all of the students who were interviewed perceived the material to be useful to their lives. The students saw value in the algebra, statistics and computer science aspects of the course. A second theme that is related to Task Interview Question 3, but was mentioned by several students during this question was that the material was covered thoroughly, so that all students had some grasp on the material before a new concept was presented. A third theme that emerged showed that students liked the fact that multiple solution paths were allowed in this class. Finally, a fourth theme that appeared was that students were able to integrate the material. The computer was used to learn algebra and statistics as well as computer science. The students were able to see the connections between the three topics.

*Task Interview Question 3*

Question 3: How do you think the teaching style of this class is different from math classes you have taken in the past?

| Student | Comments |
|---|---|
| Student 1 | “Um like I said before, he takes an individual approach. And he does make sure that you learn it. You know he wants us to walk out of here, you can tell, he wants us to walk out of here knowing something, not just necessarily concerned about just getting them through, getting a |

| | |
|---|---|
| | passing grade. He really wants you to understand what you're doing." |
| Student 2 | "Yeah, you do everything as a group, and everything is explained versus expecting you to know how…I mean, I guess that's the way I see it… I know we do everything in class versus…he looks at every aspect instead of giving you just one way to do it." |
| Student 3 | "It's more of an everyone-included class rather than the teacher up front, preaching to the class. It works really well." "A lot more hands on, feedback-ish, I guess. That's the best way I can think of. It's not just here's the book, here's the assignments, this is how you do it, do it. It's more of a discussion, walk through. [Walk through by him, or…] Well by everybody included. It's more of everyone-included class rather than the teacher up front, preaching to the class. [And how do you feel about that?] I like it. It works really well I think." |
| Student 4 | "To me, the math classes I've had before where the teacher stands up front of the class and lectures, you see the back of his or her head the whole day, it's kind of blah, blah, blah…and Mr. Smith, I don't know if it's unique to him or his style, but he's entertaining and animated. You can apply the problem to an everyday function you'd see so it helps, makes it clearer, makes sense. [You had mentioned before that he walks around the room and kind of interacts with the students more?] Yes, very good one on one. [And the pace was applied to making sure that every student gets it?] Yes he's always open to 'Everybody got this? Anybody not? Raise your hand if you don't.' Moves along at the pace of the slowest student like I said. To me that helps a lot." |

*Table 15*. Task Interview Question 3

*Discussion of Question 3.* This question was intended to help provide evidence to answer research questions three and four, as were the previous two questions. Reform mathematics courses require a different teaching style from traditional mathematics classes. This question examined how the students perceived the course and was used as an aid to determine if the course was being taught according to the vision of the course designers and the reform standards.

Student1 focused on several aspects of the class. She remarked on the slower pace of the class in a previous answer, and mentioned individual attention from the instructor, and her perception that the instructor was concerned about their learning concepts, not just getting them through the class.

Student 2 noted that the class had a discussion focus, and talked about the emphasis on working in class, rather than having a lot of homework. She also again remarked on the acceptability of multiple solution paths in this class.

Student 3, like Student 2, commented on the instructor's emphasis on allowing multiple way of solving a problem. He noted the importance of discussion as well, and mentioned the fact that everyone was included in the discussion, not just the instructor telling the students what to do.

Student 4 discussed the difference in having applicable problems presented that emphasized the usefulness of the math. In his answer to a previous question, he noted that that the pace was different. "We move along at the pace of the slowest student, which sometimes could be me. That helps a lot." In addition, he commented on the focus of the instructor on the involvement of the students in the class, rather than the standard teacher lectures he had seen in the past.

In the examination of the student answers to this question regarding the difference in teaching style from what they had seen in the past, several themes emerged. First, all of the students commented on the group discussion, or student-centered aspects of the class. Second, they all were impressed by the individual attention and pace geared to the slowest students. Third, three out of four commented on the fact that the instructor made sure that everyone understood the concepts before moving on. Additionally, two of the students commented on the hands-on aspects of the course, in that they were allowed to work on problems in class rather than have a lot of homework. Finally, the students noted in answering this question as well that multiple solution paths were acceptable in this class, and all explanations were accepted.

*Summary of Task Interview Question Analysis Pertaining to Research Questions 3 and 4*

The comments by the students in the class provide evidence that will help answer Research Questions 3 & 4 by in several ways. Outstanding themes are summarized in the following sections.

*Research question 3.* This research question inquired as to whether the course was being implemented according to the course designers' vision for reform mathematics. Their vision must be inferred from the statements of the course designers on how they intended the class to proceed. These expectations were outlined in Chapter III. The class, according to the designers, has a content focus on integrating algebra, computer science and statistics in a class taught in a computer lab using spreadsheets for much of the work. The content is meant to be presented through applications that have meaning to the students. In addition, the subject matter presentation of the class is intended to introduce fewer topics than those presented in more traditional courses, but in greater depth. The majority of the actual work for the class is supposed to be completed in the classroom, rather than as homework, and the class is designed so that students are to spend twice as much time in the classroom as a regular college course would require.

In terms of pedagogy, the instruction of the class is designed to utilize group work, projects, multiple solutions paths, student presentation and student/student and student/instructor discussion. Utilizing multiple representations to solve problems is also an important part of the class, and the designers stated that students should be encouraged to share novel ways of solving problems rather than having the instructor show just one way to solve the problems.

Through the task interviews, several themes emerged, as discussed in previous sections. Task Interview Question 1 did not provide extensive information about the course from the student's perspective, although the students mentioned solving equations as a focus of the class, and the ideas of ratio, rate, and slope. The students commented on the use of computers as a plus in the classroom.

Task Interview Question 2 provided more information about the course. The themes that arose during this question included the first theme, a perception of the practical nature of the problems used in the class. A second theme, which is related to Task Interview Question 3, but was mentioned by several students during TIQ 2, was that the material was covered thoroughly, so that all students grasped the material before a new concept was presented. A third theme that emerged showed that students liked the fact that multiple solution paths were allowed in this class. Finally, a fourth theme that appeared was that students were able to integrate the material. The computer was used to learn algebra and statistics as well as computer science. The students were able to see the connections between the three topics.

Task Interview Question 3 provided even more information about the course. This question delved into how the students perceived the instruction of the course was different from math courses they had encountered in their pasts. The themes that emerged from the student's comments were the group discussion, or student-centered aspects of the class, the individual attention and pace geared to the slowest students, the fact that the instructor made sure that everyone understood the concepts before moving on, the hands-on aspects of the course, where they were allowed to work on problems in class rather than have a lot of homework, and finally, the acceptance multiple solution paths and alternative explanations.

These topics are in line with the expectations of the course designers intended curriculum. Therefore, these interview questions provide evidence that the students, at least, perceive these events to be occurring. Additional data will examine whether the evidence supports these themes overall.

*Research Question 4.* This research question asks to what extent the Introduction to the Mathematical Sciences course reflects the NCTM, MAA and AMATYC standards for college algebra reform in the areas of content and pedagogy. As noted in Chapter III, Baxter Hastings et al. (2006) outlined a number of areas where these standards agree on changes that need to be made for college reform in lower-level mathematics courses. Changes in subject matter included less time on rote manipulation and execution of algorithms, restricting topics to the most essential, exploring topics in greater depth, and presenting more application problems. Changes in pedagogy included less time lecturing, emphasizing communication, utilizing group work, using technology, giving greater priority to data analysis, emphasizing multiple representations, and focusing on models.

As can be seen from the discussion of Research Question 3 in the previous section, there is a great deal of overlap between the course designers' vision for their course and the recommendations for reform presented by the NCTM, MAA and AMATYC, in terms of content and pedagogical focuses. The same themes that arose in the previous commentary apply to this question as well. In addition, the NCTM process standards of problem solving, reasoning and proof, communication, making connections, and representations are important considerations. The TIQ 1-3 emphasized the student perception of the importance of communication between students and teacher and between students in this class. The students also noted the connections between the three topics they were studying, and how the problems were contextual and application based. They also commented on the multiple solution paths allowed in the classes and way they were allowed to solve the problems in any manner they choose, and then could explain their solutions. These comments provide evidence that there is some alignment of the course with the process standards. Additional data will be examined to determine to what extent the alignment continues.

*Instructor Interviews*

The instructor plays a key role in implementing any new curriculum. Studies have shown that if an instructor does not hold beliefs about teaching mathematics that are consistent with those that are the basis for the curriculum, the teachers do not use the curriculum as intended (Collopy, 2003). Therefore, in order to determine if a curriculum is being implemented as intended, it is vital to look at how the instructor views the class. The researcher attempted to find out more about the instructor's method and philosophy of teaching the Introduction to the Mathematical Sciences through interviewing the instructor. The researcher spoke with the instructor both before and after each classes, also interviewed the instructor in a more formal semi-structured interview. The questions for this semi-structured interview were developed throughout the course of the semester. They were used to inform the researcher on both a broader perspective of the class, and how the instructor perceived his teaching and the relationship of that teaching to the course designer's view of reform mathematics. The questions centered on how the specific algebra topics were developed and implemented in the course and areas where multiple representations of algebraic ideas were deliberately used, as well as how students responded to those multiple representations. Additional questions delved into how the course progressed overall and the instructional techniques the instructor was using. See Appendix C.

Questions about how the instructor was implementing the course were intended to help identify whether the course goals were being met and whether the course was in line with standards for instruction, that is, to help answer Research Questions 3 and 4. In order to analyze the data, the researcher used a whole-part-whole method to look for themes in the data. To do this, the researcher listened to the recording of the interview and read through the transcripts and notes for an overall view of the interviews, then looked again at individual questions asked in the interviews for coding for themes, then returned to reread the entire transcript again to return to the broader perspective. The results of the analysis of the interviews are summarized below.

*Themes in Instructor Interviews*

The interviews with the instructor overall reflected several themes: (a) the instructor had the goal of changing student thinking about mathematics; (b) the instructor adapted the class to fit the students; (c) the instructor developed the mathematical ideas in the class from concrete to abstract. Each of these themes will be discussed in the following sections.

*Changing student thinking*. Mr. Smith, the instructor, determined the focus of the class to be, in his own words, "Trying to get students who are not mathematically oriented to look at math as something they can accomplish and do, and to look at it in a way, in ways, that are exciting for them… to engage them and to get them thinking about things and talking about it" (personal communication, February 18, 2010). Mr. Smith also noted during interview times that he wanted students to see that algebra is something they can actually learn to use in their lives. He stated that use of the spreadsheet helped students realize this, as they were forced to write simple algebraic formulas to use in the spreadsheet. During the interviews, he repeated more than once that the integration of the spreadsheet as technology in the class was helpful. When asked about whether he thought students would use the technology and algebra in their futures, he was very clear that in his experience they would.

> …I have a group of students who took this class last spring, a year ago now that are in the manufacturing and engineering technology program and they're doing spreadsheets all the time for their classes. And they're seeing when they are job shadowing that the people they are shadowing are using Excel® all the time… (Personal communication, February 18, 2010).

Mr. Smith also discussed how some of the nursing students would be using various spreadsheet programs in their careers as well. He stated often that the real-world problems were used to show students that mathematics, including algebra, could be used to illustrate to the students how useful mathematics could be to their lives.

*Adapting the class to the students.* During the semi-structured interview, Mr. Smith discussed the need to adapt the class to the students. He mentioned that while the class the observed had typical student age distribution, with mostly older than average students, the class makeup fluctuated somewhat in terms of student majors. In general, Mr. Smith viewed the course as an evolving, changing class. At the time of interview, he was teaching his third semester of the course. He noted that his instructional style, which he described as "question oriented," was necessary for the type of students he had. He stated that it was essential to the class design to have the students explain what they were doing, and why they were doing it that way, and for them to “do more of the presenting than I do…” (Personal communication, February 18, 2010). When asked about whether the course designers’ philosophy of mathematics instruction, which he also held, was necessary to the course, he commented that the application-based idea behind the course was vital.

> … I think it’s not only necessary for the teaching of the class, but for the type of students I have. These are students that are technical students. They are going to go on to whatever jobs, there’s nursing, manufacturing engineering and technology, there are some electrician students in here, and those are all going to be doing hands-on type things, jobs, so they need hands-on type things to do. Applications based. Much more than theoretical based. (Personal communication, February 18, 2010)

While Mr. Smith generalized in this manner, he also stated clearly that he was adapting this class as he gained experience in teaching the course. During the first semester that he taught the course, he had met with the course designers every two weeks, during the second semester about once a month, and during the current semester had contacted them only via email. He now adapted the class to make the course fit his students. For example, while he noted that he tried to make his examples of using slope as broad as possible, so it was applicable to everyone, he also considered the makeup of the course. If the course had more nursing students, he might do more examples involving IVs and rates of change using milligrams/hour because they might be more applicable to that group. He was careful to say, however, that he wanted the students to see that there were many uses for the concept of slope or rate of change, not just the ones they were going to use in their jobs.

Mr. Smith also adapted his class to the pace of his students. When asked how he knew when the students had "gotten it" or could move on, he stated the following (personal communication, February 18, 2010):

> What I try to do is…figure out who the slowest learner is, and I try not to go on until the slowest learner has an idea of what [the concept] is. Have they got it totally? No. But they've got a good idea of what it is before we move on to something else. In the high school, you kind of shoot for the middle students. And try to, hope, that you help the lower ones keep up and keep the other students from getting bored. But at [this school] especially with the tech students, you want to make sure that the lowest student, slowest student, the one who's furthest behind, has the most trouble, is onboard before you go on.

Mr. Smith stated that in the past semesters of the class, he felt that at times he had gone too fast in the class, but now was more aware of the pace of his students. He felt that he had gotten to the material, but could even slow it down further if necessary for his students. The use of the real-world problems kept the students engaged, and they were truly learning the material. He mentioned a day when the students were working on an in-class project with some data the students had collected: "Two hours later, two HOURS later, one looked up and said, 'Geez, I gotta go home!' They were so engaged … interpreting what the data said…they just ate that up." Mr. Smith noted that this engagement is what he looks for. When asked about switching between the major topics in the class, (between algebra, statistics and computer science), he stated that the integration was necessary to the pace of the class and to engaging the students as well.

*Moving from concrete to abstract.* Mr. Smith explained in his interviews that his philosophy of teaching involved exploration and moving from informal to formal, or concrete to abstract. He liked to give the students a problem to work on, to try out, and then see if they could explain in words how they did the problem. He wanted students to come up with a variety of ways to solve the problem, using different representations, from concrete or pictorial first, then on to more abstract ways of representing ideas. He stated that he wanted students to think about how they were doing the problem, explain it, then write down what they actually did, in words, then later put those words into algebra. He noted that after writing down their words, they could then often try to generalize it in a spreadsheet. In this manner the students actually had to consider closely what they understood about the concept.

Mr. Smith also discussed the use of the motion detector in the classroom for developing the idea of a rate displayed on a graph, noting that the students could see the movement and the motion as the distance was getting greater or smaller. Students, he said, could then see why faster rates resulted in steeper slopes. He also referenced using the walls and floor to represent lines with no slope and zero slope and that those particular examples seemed to help students understand those concepts. The examples Mr. Smith discussed all were to be introduced prior to working with the actual formula for a line ($y = mx + b$). This abstract form using slope and intercept was to be a final discovery that the students made after working with graphs and data generated from real world examples. When asked about what he saw as the impact of developing fluency in moving between representations of mathematical ideas, Mr. Smith stated that he wanted students to see the connection that using algebra was a way of formalizing how the students were thinking about the problems, in whatever representation.

*Summary of the Instructor Interviews*

During the interview with Mr. Smith, three themes emerged, which may be helpful in answering the research questions. Those three themes were that the instructor had the goal of changing student thinking about mathematics, adapted the class to fit the students and developed the mathematical ideas in the class by moving from concrete to abstract. His energy and excitement about helping students, his students, learn was infectious in class and in the interviews. The students also noted this in their interviews. They appreciated that Mr. Smith really did want students to succeed in his class. Mr. Smith's interview comments also showed that he was continually adapting the class to his students. He followed the course designers' suggestions for content and pedagogy, but also included his own adaptations to the class, based on his experience with reform teaching as both a high school and college instructor. His philosophy of teaching according to reform ideals was clearly reflected in both conversations with the researcher and classroom behavior.

While teacher beliefs and attitudes are not the subject of this research, it is important note that the philosophy of the instructor should be in line with the philosophy of the course designers in order for fidelity of implementation to occur (Thompson, 1984). Mr. Smith had a strong commitment to reform mathematics instruction and clearly stated a commitment to the course designers' vision. This similarity in philosophy lends itself to enactment of the intended curriculum, and provides additional evidence that the instructor, as well as the students perceived the course as being taught in a manner consistent with the reform ideals. This helped add to the data used to answer Research Questions 3 and 4. However, further evidence beyond individual's perceptions was necessary in order to answer the research questions.

*Classroom Artifacts*

A number of artifacts were collected during the research on the Introduction to the Mathematical Sciences course. Some artifacts, which lend themselves to examining student thinking and learning regarding algebraic representations, have already been discussed. Additional artifacts helped answer whether the subject matter of the course is implemented as intended.

*Artifacts and fidelity of implementation of subject matter.* In order to further help answer question three regarding fidelity of implementation of the course designers view of reform mathematics, the researcher examined the handouts from the class collected during her observations for topics that were intended to be covered in the class. Table 11 provided a list of all mathematics topics observed during the researchers observation days, by date. Since algebra the topic of study for this paper, the algebra topics that were included in the course designers list of intended topics are provided in Table 16 along with the date the researcher observed those topics. It is important to note that the researcher was not present during every class, so some topics may have been presented again on days the researcher was not present. This list is meant only to add to the data gathered, and help identify topics that were covered more frequently than others.

| Algebra Topics | Date observed |
| --- | --- |

| | |
|---|---|
| Functions<br>• Represented by formula, table, graph, words | • 10/1, 10/8, 10/22, 10/27, 10/29,11/10, 12/1, 12/3 |
| Graphical and Tabular Analysis<br>• Tables and trends<br>• Graphs<br>• Solving linear equations<br>• Solving nonlinear equations<br>• Optimization | • 10/8, 10/29<br>• 10/1, 10/8, 10/22, 10/27, 10/29<br>• 9/24, 9/29, 12/1, 12/3<br>• 9/24, 9/29<br>• Not taught |
| Linear Functions<br>• The geometry of lines<br>  • Linear Functions<br>  • Modeling data with linear functions<br>  • Linear regression<br>  • System of equations | • 10/27, 11/10, 12/1, 12/3<br>• 10/27, 10/29, 11/10, 12/1, 12/3<br>• 10/27, 10/29, 11/10, 12/1, 12/3<br>• 12/3 |
| Rates of Change<br>  • Velocity<br>  • Rates of change of other functions | • 10/1, 10/8, 10/22, 10/29<br>• 9/24, 9/29, 10/1, 10/22, 10/27, 10/29, 11/10, 12/1, 12/3 |

*Table 16*. Algebra Topics Observed in the Course

The topic of optimization, which was one of the topics listed by the course designers, was not presented during the semester. The other topics were all observed by the researcher. On the days on which the topics were presented, they were not covered superficially, although the amount of time spent on each subject is not recorded here. The major focus of the course, in terms of algebra, was on slope as a rate of change. As an additional note: Although the students were working with functions throughout the course, function notation was seldom utilized and covered only briefly, not used throughout the course. Also, little work was done with functions that were not linear.

This collection of artifacts in the form of student worksheets therefore adds to the data to be used to answer the research question of to what extent fidelity of implementation of the course designers' vision of reform in terms of content for the course was occurring.

Use of contextual problems, technology, and multiple representations of mathematics topics are all common to the standards for reform teaching of mathematics, particularly algebra, put forth by the NCTM, AMATYC and MAA. The artifacts of student work collected during the course of the study helped contribute to an understanding of to what extent the course reflects the standards for mathematics reform as identified by these groups. The content shows that fewer topics were discussed in greater depth, and the students were provided opportunity to work with a variety of representations such as graphs, tables, and equations.

*Observations/Field Notes and Research Questions 3 and 4*

A description of the classroom activity as obtained through observations and field notes collected during the study was given in a previous section of this chapter. This section discusses how the observations and field notes provided insight into these two research questions.

*Research Question 3.* This question involves how and to what extent the course reflects the course designers' vision of college reform in mathematics education for the two-year college. In their description of the course, the course designers noted a number of factors important to the implementation of the course. Group work, projects, multiple solution paths, student presentation and student discussion are integral to the course. The integrated nature of the course subject matter with its focus on algebra, statistics and computer science is designed to engage the students in real world problems that require multiple representations of the problems. Solving the problems using a variety of representations (verbal, concrete or pictorial, graphical, algebraic, and tabular forms) is an important part of the course. Ability to move between these multiple representations is stressed in the intended curriculum along with explaining or communicating solutions in written and verbal format. Students are encouraged to share novel ways of solving problems, rather than having the instructor show one "right" way of solving problems with students mimicking the teacher's actions.

Evidence from observations suggests that many of these elements of the intended curriculum were implemented. The sample classroom discussion emphasized the interaction that was taking place in the classroom. During classroom observations, discussion integral to the class and occurred not only between students and teacher, but also between students in small and large group conversations.

The students utilized the spreadsheets extensively in class, creating tables, graphs and equations using the tool. Technology and the use of the spreadsheet program were important factors in this intended curriculum, particularly in moving between representations, and were observed to be present in the classroom. Students exhibited ability to move between representations, quite literally, in their "walking the graph" activity. The use of real world examples of slope, as in the pitch of a roof, or rate of change in velocity, reflected the instruction using applications of mathematics that were relevant to the students' lives. Other examples of applications of mathematics were shown in the examples of rates of fertilizer or paint application, and in classroom discussions of rates of water consumption.

In terms of content, only one topic, optimization, was missing from the subject matter suggested by the designers. Otherwise, the instructor followed the designers' suggestion of teaching problems in context and covering a reduced number of topics, as compared to a traditional curriculum. The course was intended to have the material interwoven in the algebra, statistics and computer portions, and the researcher observed the interwoven material. The course was to be taught in a computer lab, and students were to use spreadsheets for solving many of the problems. The spreadsheets were used to solve problems both in the computer portion of the class, and in the algebra and statistics portions of the class. The students often calculated values of slope or found algebraic equations using the spreadsheet program. They were required to write algebraic equations to enter into the spreadsheet in order to solve many of the problems as well.

While the instructor followed the pedagogical and content recommendations for the course in many aspects, the course was designed to have approximately twice as much time spent in class as a normal college course in order to allow students to complete their work in class. However, while the class was scheduled from 3:55PM until 6:55PM, the students were rarely there beyond 6PM. This could be seen as a deviation from the course designer's intent, although the students were able to complete the material in class most days and rarely had any homework despite the shortened class.

*Research question 4.* This question examines how and to what extent the teaching of this course reflects the NCTM, MAA, and AMATYC standards for college algebra reform in a two-year college setting in the areas of content and pedagogy.

Baxter Hastings, et al. (2006) outlined areas of agreement in the standards of the abovementioned entities regarding change necessary for reform in lower-level college courses. These changes included ideas such as lessening time spend on rote manipulations and execution of algorithms just for the sake of calculations, restricting topics to the most essential, and decreasing the amount of time spent lecturing. In terms of pedagogy, they suggested embedding the mathematics in context, exploring fewer topics in greater depth, emphasizing communication, utilizing group work, using technology to enhance conceptual understanding, giving greater priority to data analysis, emphasizing multiple representations, and focusing on construction of mathematical models before finding solutions to the models.

During the field observations, the researcher observed all of these emphases as present in this class. Much less time was spent on rote manipulation in the class than in a traditional algebra course, although the students did have some worksheets to complete that required "solving for y" or "evaluating x" in the problems. For the most part, the problems presented in the class were contextual, and as described earlier, were investigated either individually or in groups and discussed extensively in class. The students performed data analysis using data either gathered in class, or collected from the internet in projects that allowed them to select data they were interested in. The students utilized computers extensively and translated between multiple representations such as tables, graphs, verbal descriptions and real world scenarios. The students were asked to construct their own models for solving problems and presented those models in class. The models were fairly simple, however, for the most part, and rarely were multiple days spent on creating a model, due to the structure of the course.

*OTOP Data and Research Questions 3 and 4*

Examination of the OTOP data was designed to help determine the answers to the research questions: 3) How and to what extent does the course reflect fidelity of implementation of the course designers' vision of college reform in mathematics education for the two-year college? and 4) How and to what extent does the teaching of this course reflect the NCTM, MAA, and AMATYC standards for college algebra reform in a two-year college setting in the areas of both content and pedagogy?

This section will discuss the results of the Oregon Teacher Observation Protocol (OTOP) analysis and will include an examination of frequency with which reform ideals as identified in the OTOP occurred in this classroom. Since the course designers were interested in implementing a course that reflected mathematics reform, this is an important aspect to examine. The OTOP is in also in line with the NCTM standards for reform instruction as published in the Principles and Standards for School Mathematics (2000), and thus the ideals of reform in the MAA and AMATYC standards. Table 17 shows which NCTM process standard(s) align with the various OTOP items.

| OTOP Item | NCTM Process Standard(s) |
| --- | --- |
| **1. This lesson encouraged students to seek and value various modes of investigation or problem solving.**<br>Teacher/Instructor:<br>Presented open-ended questions<br>Encouraged discussion of alternative explanations<br>Presented inquiry opportunities for students<br>Provided alternative learning strategies<br>Students:<br>Discussed problem-solving strategies<br>Posed questions and relevant means for investigating<br>Shared ideas about investigations | Problem Solving<br>Communication |
| **2. Teacher encouraged students to be reflective about their learning.**<br>Teacher/Instructor: | Communication |

| | |
|---|---|
| Encouraged students to explain their understanding of concepts<br>Encouraged students to explain in own words both what *and* how they learned<br>Routinely asked for student input and questions<br>Students:<br>Discussed what they understood from the class *and* how they learned it<br>Identified anything unclear to them<br>Reflected on and evaluated their own progress toward understanding | |
| **3. Interactions reflected collaborative working relationships and productive discourse among students and between teacher/instructor and students.**<br>Teacher/Instructor:<br>Organized students for group work<br>Interacted with small groups<br>Provided clear outcomes for group<br>Students:<br>Worked collaboratively or cooperatively to accomplish work relevant to task<br>Exchanged ideas related to lesson with peers and teacher | Communication |
| **4. Intellectual rigor, constructive criticism, and the challenging of ideas were valued.**<br>Teacher/Instructor:<br>Encouraged input and challenged students' ideas<br>Was non-judgmental of student opinions<br>Solicited alternative explanations<br>Students:<br>Provided evidence-based arguments<br>Listened critically to others' explanations<br>Discussed/Challenged others' explanations | Communication<br>Reasoning & Proof |
| **5. The instructional strategies and activities probed students' existing knowledge and preconceptions.**<br>Teacher/Instructor:<br>Pre-assessed students for their thinking and knowledge<br>Helped students confront and/or build on their ideas<br>Refocused lesson based on student ideas to meet needs<br>Students:<br>Expressed ideas even when incorrect or different from the ideas of other students<br>Responded to the ideas of other students | Communication |
| **6. The lesson promoted strongly coherent conceptual understanding in the context of clear learning goals.**<br>Teacher/Instructor: | Problem Solving<br>Reasoning & Proof<br>Communication |

| | |
|---|---|
| Asked higher level questions<br>Encouraged students to extend concepts and skills<br>Related integral ideas to broader concepts<br>Students:<br>Asked and answered higher level questions<br>Related subordinate ideas to broader concept | |
| **7. Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.**<br>Teacher/Instructor:<br>Accepted multiple responses to problem-solving situations<br>Provided example evidence for student interpretation<br>Encouraged students to challenge the text as well as each other<br>Students:<br>Generated conjectures and alternate interpretations<br>Critiqued alternate solution strategies of teacher and peers | Problem Solving<br>Reasoning & Proof<br>Communication |
| **8. Appropriate connections were made between content and other curricular areas.**<br>Teacher/Instructor:<br>Integrated content with other curricular areas<br>Applied content to real-world situations<br>Students:<br>Made connections with other content areas<br>Made connections between content and personal life | Connections |
| **9. The teacher/instructor had a solid grasp of the subject matter content and how to teach it.**<br>Teacher/Instructor:<br>Presented information that was accurate and appropriate to student cognitive level<br>Selected strategies that made content understandable to students<br>Was able to field student questions in a way that encouraged more questions<br>Recognized students' ideas even when vaguely articulated<br>Students responded to instruction with ideas relevant to target content<br>Appeared to be engaged with lesson content | Communication |
| **10. The teacher/instructor used a variety of means to represent concepts.**<br>Teacher/Instructor:<br>Used multiple methods, strategies and teaching styles to explain a concept<br>Used various materials to foster student understanding (models, drawings, graphs, concrete materials, manipulatives, etc.) | Multiple Representations of Mathematical Ideas |

*Table 17.* Relationship of OTOP Items to NCTM Process Standards

By examining the activity of the class in relation to the OTOP items, the OTOP helps provide additional evidence on whether or not reform teaching is being implemented. The OTOP also includes room for comments on the classroom activity and descriptive field notes. Field notes taken on the OTOP have been discussed in a previous section.

*OTOP Scores*

The OTOP scoring section is made up of 10 questions. Each of these can be scored from 0 to 4, or NA, according to the following scale (Table 18).

**Score descriptions:**

| | |
|---|---|
| 0 | Practice could have been included in the lesson, but wasn't |
| 1 | Some attempt is made at the item |
| 2 | Elements of the item are clearly present, but not fully carried out |
| 3 | Areas in the item of good quality, but there is room for improvement |
| 4 | Highest level of quality evident in this item |
| N/A | Practice was not included and was not appropriate for the lesson |

*Table 18*. OTOP scoring scale

In order to provide the reader with an idea of how the scoring worked, or how the researcher selected the scores, examples of scoring choices using Item 1 from the OTOP are briefly outlined in the next paragraphs.

A score zero on the OTOP Item 1 (see below) indicated that there was no problem solving or investigation involved in that class period. This zero score on Item 1 did not occur in any of the classes, as the students were presented with problems to attempt every day, even on review days. A score of 1 on Item 1 meant that there was some investigation taking place in the class, but not as much as other days. On a day like this, the instructor may have had students solving equations for y, or learning how to use the spreadsheet program. The students were still trying new things and experimenting on the computer, with the instructor asking questions, but there was not much discussion and opportunity for exploring different solutions paths. A score of 2 meant that there was more investigation and discussion of problem solving than on days with scores of 1, but students might have been working on new topics that required more instructor presentation and instructor directed investigation, but less opportunity to investigate or problem solve on their own. A score of 3 meant that students were working in partners to investigate a new problem, and had discussion in class about the problem, but were not provided much opportunity to present their different solutions to the problems. A score of 4 meant that students investigated a new problem, discussed it in the large group with the instructor and their peers, and presented their solutions to the class. NA occurred on a day when the students were only taking a test for a different research project and not having regular class. Scoring for the other OTOP items can be interpreted in a similar manner.

Although this research is not focused on quantifying these scores, it is helpful to make note of the frequency with which the researcher observed the various scores in the course. This will give the reader an idea of how well the reform ideals were implemented in the class. Each of the questions in the OTOP will be listed, followed by the frequency 0s, 1s, 2s, 3s, and 4s observed during the researcher's 14 observations of the class over the semester. Each observation involved scoring on each of the OTOP items. This data is displayed in the following table (Table 19).

| OTOP Item | 0 | 1 | 2 | 3 | 4 | NA |
|---|---|---|---|---|---|---|
| 1. This lesson encouraged students to seek and value various modes of investigation or problem solving. | | 1 | 1 | 3 | 8 | 1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 2. Teacher encouraged students to communicate their learning. | | 1 | | 1 | 11 | 1 |
| 3. Interactions reflected collaborative working relationships and productive discourse among students and between teacher and students. | | | 1 | 1 | 11 | 1 |
| 4. Intellectual rigor, constructive criticism and challenging of ideas were valued. | | 1 | 1 | 6 | 5 | 1 |
| 5. The instructional strategies and activities helped students make connections. | | 1 | 1 | 7 | 4 | 1 |
| 6. The lesson promoted strongly coherent conceptual understanding in the context of clear learning goals. | | 1 | 1 | 5 | 6 | 1 |
| 7. Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence. | | 1 | 4 | 2 | 6 | 1 |
| 8. Appropriate connections were made between content in the real world and/or other curricular areas. | | | | 1 | 12 | 1 |
| 9. The teacher had a solid grasp of the subject matter content and how to teach it. | | | | 4 | 9 | 1 |
| 10. The teacher used a variety of means to represent concepts. | | | 1 | 2 | 10 | 1 |

*Table 18*. Frequency of OTOP scores

*Discussion of OTOP Data*

From the table above, it is clear that the majority of items on the OTOP reflected the presence of reform teaching in the Introduction to Mathematical Sciences class. The item receiving the lowest scores was number 7. This item involved determining whether students were encouraged to interpret evidence in different ways, and involved critiquing each others' explanations and the text. The students did not always present alternative solution strategies to the class, although alternative paths were readily accepted by the instructor. This may have been a time issue on some days, as the class moved along at a fairly fast pace if students appeared to understand the material. In many classes, however, students argued their points of view in class and justified their opinions in class. The other items on the OTOP all had a high frequency of scores of three or four. The NA occurred on a day on which the students spent the day taking tests for a different research project. Note that the OTOP items reflect the NCTM process standards of problem solving, reasoning and proof, communication, making connections, and representations of mathematical ideas. Therefore, the OTOP provides additional that the Introduction to the Mathematical Sciences course is indeed being implemented in a manner consistent with the course designers' view of reform teaching of mathematics at the two-year college level, and the NCTM standards. Since these goals are consistent with those of the MAA and AMATYC, the OTOP data also provides support for the implementation of these reform standards.

*Summary of Data Addressing Research Questions 3 and 4*

Five sources of data were used to address the third and fourth research questions, which examined whether the course was being implemented as intended by the course designers, and whether the course was being implemented in accordance with the standards of the NCTM, MAA, and AMATYC for the teaching of algebra. These sources were interviews with students, interviews with the instructor, classroom artifacts, observations, and the OTOP data.

Interviews with students brought out several themes. The students commented on the practical nature of the mathematics they were learning, the hands-on aspects of the course, and the thorough coverage of material presented before moving on. They noted the use of multiple solutions paths, the connections formed between the three topics of study (algebra, statistics and computer science). The students liked the student-centered aspect of the course with the emphasis on group discussion and the pace of class being geared toward the students.

The interview with the instructor provided insight into the instructor's philosophy of the course. This philosophy is highly aligned to the philosophy of the course designers and reform philosophy. This fact lends credence to instructor's statements about how he teaches the class, as instructors who hold a similar philosophy to the designers of the curriculum are more likely to teach the intended curriculum (Thompson, 1984).

Classroom artifacts indicated that the content covered in the class is in accordance with the course designers' vision. The subject matter indicates fewer topics were taught in more depth, and that multiple representations of algebra topics were utilized in the class, as well as technology.

Observations of the classroom activity also indicated that the course is being taught pedagogically according to the course designers' vision for this course, with the exception of the length of time spent in the classroom. The course designers envisioned a greater amount of time spent in class. Because the course designers' pedagogical focus reflects the tenets of the NCTM standards, this puts the course in line with the NCTM standards as well. The reform recommendations common to the NCTM, MAA &

AMAYTC outlined by Baxter Hastings, et al. (2006) for changes in lower-level college courses are exhibited in this class as well, in terms of pedagogy.

OTOP scoring data supported the presence the aspects of reform teaching as indicated by alignment with the process standards of the NCTM. While not all process standards were in place every day the course was taught, the OTOP provides additional support for the other data that indicates that reform methods were utilized in the instruction of the course.

In terms of content, during the fall of 2009, the topic of optimization, a topic recommended by the course designers was not presented in the class at all. Discussion of function notation was minimal and was not used throughout the course. These topics were part of the intended subject matter of the course, so these aspects are not in line with the course designer's vision for the class. In terms of algebra content, the standards of the MAA cover far more algebra topics than were presented in this class, so it is important to note that this class cannot be considered a substitute for a complete College Algebra class.

This summarizes the results of the data analysis for Research Questions 3 and 4. The next chapter will synthesize the results of the research and provide conclusions, implications and suggestions for further research.

# CHAPTER V: CONCLUSION, IMPLICATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

This chapter consists of five sections. Those include a discussion of the findings from the data analysis in relation to the research questions, limitations of the study, recommendations for the course designers, suggestions for further research, and finally, the concluding remarks.

## Discussion of Findings

The purpose of this study was to examine the implementation of an innovative mathematics curriculum that was designed for students who had traditionally struggled in mathematics. Because more and more students today are coming to college needing or wanting lower-level mathematics courses (Lutzer, et al., 2005), and large numbers of students are failing college algebra courses (Baxter Hastings, et al., 2006), there is a demand for courses that can assist students in preparing for the entry-level mathematics skills that will be needed in many career paths. The Introduction to the Mathematical Sciences course has been implemented at both the high school and college levels, and is designed to provide students with an introduction to algebra, statistics and computer science that is needed in a broad range of fields (Webb, et al., 2009). This course is intended to be taught from a reform mathematics perspective, and the current study examined this new curriculum and its implementation, focusing on the area of algebra and its representations in relation to student thinking and learning. Schoenfeld (2006) and Clements (2007) both recommend that a close examination of a curriculum at a small number of implementation sites should precede broader research comparing curricula.

This single case study examined the implementation of the course at the two-year college level in the setting of a small technical college in the Midwest.

The next sections outline each of the research questions of the study and discuss the findings of the researcher as supported by the data analysis. The data gathered from observations, the OTOP, classroom artifacts and instructor and student interviews will be used to provide insight into answering the questions in the context of the study.

*Research Question 1*

*What is the impact of a standards-based lab course integrating algebra, statistics, and computer science on two-year college students' mathematical thinking and learning, specifically regarding algebraic learning and representations?*

In order to answer this question, it is important to consider all of the data gathered throughout the study. No one source of data is sufficient to respond to the research question completely. Yin (2009) states that triangulation of data using multiple sources of evidence can help the case study researcher identify converging lines of inquiry that may show support for themes in the data. The three sources of data were used to answer this question included student task interviews, classroom artifacts and observations of the class. The various sources of data indicated that participating in this class had two major impacts on student thinking and learning about algebra and representations. First, the students were able to find meaning in the mathematics while utilizing a variety of representations of the data. Second, the students were able to use technology in this class to create representations and work with algebra in ways they might not have had the opportunity to in other mathematics classes.

Radford (2004) stated that students find meaning in algebra through a variety of sources, including from the algebraic structure itself, the representations of the algebra, the problem context, and meanings constructed by the learner. Kieran (2006) noted that many studies have shown that different types of representations "offer different meaning-building experiences" (p. 712). The Introduction to the Mathematical Sciences course provided opportunity for students to find significance in the algebra through the use of various representations of algebra and in contextual problems. Most of the problems in the Introduction to the Mathematical Sciences course were presented in real-world contexts, rather than meaningless abstract forms. Many traditional mathematics courses do not provide students with the opportunity to reflect on the meaning of a graph or table in the context of a practical setting, while this class offered students algebra problems in a wide variety of forms. Hofacker (2006) determined that students in a contemporary college algebra class were better able to discuss mathematical situations when they had used a variety of representations of algebraic ideas. This study, while not specifically a college algebra course, similarly presented algebra topics using a variety of representations. From the evidence, it appears that, like Hofacker's students, students in this course were able to make sense of their answers as they worked problems and could explain their answers in context. The evidence presented through the task interviews, classroom artifacts and observations combined indicates that the students in the Introduction to the Mathematical Sciences course demonstrated that they were able to utilize different representations of algebraic ideas and interpret the mathematics in the problems.

Throughout the class, the students were required to explain their answers in words as they found solutions to problems. During the task interviews, the students were asked to interpret tables in the context of a population that was growing or losing population. They also were able to discuss the meaning of a graph representing motion toward and away from a motion detector. On tests and homework, students demonstrated an ability to decipher the mathematics in word problems, graphs and data sets and display understanding of the mathematics in the problems through their explanations. They demonstrated that they were able to extrapolate using graphs and tables to predict values and explain in words why a particular graph had a positive or negative slope. The students were also, by the end of the semester, able to interpret and write equations given linear growth scenarios, explain the meaning of the intercept and slope, and relate it back to the problem. Further discussion of the specific topics of linear equations and slope will follow when addressing Research Question 2. The researcher concludes that this ability to apply and interpret the various representations is one of the impacts of taking this course.

Students were able to work with many of the different representations of algebra topics, but there were flaws in their abilities as well. Students were not able to use all of the representations of a given algebra topic with the same fluency. Some of these deficiencies appeared during the task interviews. Students showed the least facility at the time of the interviews with working with the most abstract representation, the equation. During the task interviews, students sometimes abandoned the attempt to create equations on the computer and instead, like the students in algebra classes studied by Stacey and MacGregor (1999), reverted to using easier arithmetic methods to find their answers. As

Pape and Tchoshanov (2001) noted, students must be given opportunity to work with the different representations in order to gain skills. According to their theory, with practice and classroom support for using a variety of representations, student ability to work with the different representation increases. In agreement with their theory, as the semester progressed, it was clear from the classroom artifacts that students in the Introduction to the Mathematical Sciences course had gained greater facility with working with the symbolic form of equations.

Working with multiple representations must be supported in the classroom. Friedlander and Tabach (2001) stated that ability to work with multiple representations does not develop spontaneously and use of multiple representations must be supported actively and systematically. One way that use of multiple representations can be supported is through the use of technology. In the Introduction to the Mathematical Sciences course, students had an opportunity to utilize computers in the classroom daily, and this aided the students in creating representations of mathematics topics. Students were able to employ technology in the form of a spreadsheet program to solve mathematics problems and to produce the different representations such as tables, graphs and equations. Students used the spreadsheet program to perform calculations in many of the problems, and because of the contextual nature of the problems mentioned earlier, were able to determine if they had input an equation incorrectly. For example, in the task interviews, Student 4 realized he had used the spreadsheet technology incorrectly when he found that four 2.5 square foot tile supposedly covered 40 square feet. Students not only used the computer while working with the algebra problems, but in many cases were utilizing the technology in statistics problems, which required that they create algebraic formulas in the spreadsheet program to complete tasks. The integrated nature of the course provided opportunity for students to learn additional algebra skills even while studying the other topics of statistics and computer science. This daily use of computer technology makes the Introduction to Mathematical Sciences course different from traditional mathematics courses and provides unique opportunity for students to create and make meaning of representations of algebra.

However, the presence of technology does not guarantee the understanding of the mathematics. During the task interviews, some of the interviewed students relied on the technology without exhibiting deep understanding of the meaning behind their processes. Some also had difficulty recalling the detailed processes on the computer. In order for the technology to be effective, students must be able to understand each step of the process. A recommendation for future instruction of the course would be to have students explicitly discuss the “why” behind the steps they are taking on the computer, to prevent students from blindly mimicking the instructor. Some of the students in the class had not used this type of spreadsheet technology prior to the course, and greater support for students having difficulty understanding the computer processes could be beneficial.

*Summary of Research Question 1*

The two major findings regarding the impact of this course of student thinking and understanding regarding algebra and its representations were that students were able to make meaning of the mathematics from the various representations, and students were able to generate a variety of representations using the computer technology to assist in their understanding of the algebra.

*Research Question 2*

*In what ways do students in this integrated standards-based laboratory class at the two-year college level demonstrate understanding through an ability to move between representations of algebra problems, specifically relating to the ideas of slope and linear equations?*

This question contains some overlap from the previous question, as the students' ability to describe and make meaning of the mathematics was related to their understanding of the various representations of mathematics. Lesh, Post and Behr (1987) stated that students who show an ability to move between representations of a mathematical concept have a deeper understanding of the concept than those who cannot. The students in the Introduction to the Mathematical Sciences course demonstrated an ability to move between representations in many cases. Data supporting this ability came from classroom artifacts, student interviews, and classroom observations. To what extent the students in this class displayed ability to move between various representations will be discussed below.

*Table-Graph.* Students who were interviewed displayed a good facility with creating a graph based on a table when asked to create a graph by hand. They were able to plot points, although at least one student confused her x and y coordinates at first. In the task interviews, students were able to look at data given in a table and determine that the graph was linear. During classroom observations, the students showed that they were able to take a set of data, enter it in a spreadsheet and construct a graph using the computer as well. They were able to plot points on the computer and find trend lines. The students were not tested on plotting points by hand during exams.

*Table-Real-world scenario.* During the student task interviews, students were given a table of data that represented the population of a city in different years. The students were asked to explain what was happening to the city. The students were able to explain how the city was growing, for example at 10000 people every five years. Not all stated the annual growth rate of the population, however, or made the connection between rate of growth and unit rate when given x-increments other than one. Students were also able, based on the tables of data, to describe a city that was losing population as well as the starting populations of both cities.

In-class activities also required students to describe what was happening in a real-world scenario based on a table of data. One example of this involved finding which inlet and outlet pipes were open for a particular tank of water, given a set of data and the capacities of the inlet and outlet pipes. Students needed to find the rates at which water was entering or exiting the tank from the data in the table, and then determine which pipes were open. While students were able to identify flow rates from the table, some students in the class had difficulty finding the appropriate numerical combinations of inlet and outlet values that led to the correct overall flow rate, a problem with combining sets of numbers, not interpreting the graphs. In the problems presented to the students, the students' understanding and interpretation of what the table depicted in context was generally supported by the data, although some problems with determining unit rates persisted.

*Table-Equation.* This movement was the most difficult for students to do by hand. During classroom observations, all of the students displayed ability in class to enter data into the spreadsheet program and find the regression line using the computer. This was how students were first introduced to trend lines. None of the four students who were interviewed in the task interviews were able to correctly find the linear equation by hand from the table. During task interviews, the students were able to find unit rate from a table, and also initial value, but were not able to make the leap to writing the equation. However, it is important to note that the interviews were done when the students were just beginning the slope-intercept equation topic. Later in the course, when students were given rates and initial values on classroom artifacts, they showed that they were able to write equations. It appeared that during the course, the students were not necessarily expected to write equations by hand using data points. The instructor did not place an emphasis on creating the equation from a table using any type of point-slope work. The point-slope form of the equation was not presented and students were not expected to learn how to find equations just using two points. The students frequently calculated slope using two points, but did not have a method for finding the y-intercept when it was not readily apparent. A recommendation by the researcher to the course designers is that greater emphasis be placed on having students find equations of the line from data without relying on the computer technology.

*Graph-Real-world scenario.* The classroom observations, classroom artifacts and student task interviews provided data on this translation between representations. During the task interviews and in the classroom, students were asked to view graphs and interpret those graphs, particularly the slopes, in real-world situations. Many of the graphs the students were given were distance-time graphs, but others depicted depreciation, payments vs. mortgage values, water flow rates, or tax rates. Students were asked to find the slopes and then discuss what the slopes meant in the real-world situation.

The students were able to do this in most cases, including identifying the units required. Tax rates and mortgage values vs. payment, both of which were rates in dollars per dollar were trickier for the students to explain. The students had no problems with interpreting the distance-time graphs. This may have been due to the fact that most of the students were familiar with ideas such as miles/hour, or that they used the motion detector problems throughout the class.

*Graph-Equation.* During the student task interviews, students were asked if they could write an equation based on a table of data. They also constructed a graph based on this data. None of the students were able to write the equation of the line based on the graph at that time, despite the fact that most could identify the growth rate and the starting value. However, as stated above, the interviews took place just as the students were learning the slope-intercept form of the line. The emphasis on writing equations based on graphs and data was lacking, as noted in the section on Table-Equation. Again, the researcher recommends that this topic be concentrated on more heavily.

*Real-world scenario-Equation.* Near the end of the course, Mr. Smith provided the students with numerous scenarios in class and on exams depicting linear growth and asked them to find the equation of the line. For example, a problem stated that a plumber charges $30 to come to your house and then $15 per hour for his work. The students were then asked to find an equation for total cost. When the initial value was clear, as well as the rate, the students had little trouble doing this. As noted when discussing the classroom artifacts, the students did exhibit some difficulty when scenarios had an initial value of zero when discussing cost and revenue scenarios. Mr. Smith addressed this specifically in an attempt to clarify the matter, but no later data were available to determine if the students were understood it in this context. After students had learned the slope-intercept equations of the line, being given the initial value and rate made it easy for the students to find the equation of the line. As noted in the previous discussions on translating to equations, whether the students could write the equation from two points without the help of computer is questionable. If students had been given the cost of labor after one hour and after three hours, (two points), the researcher questions whether the student would have been able to find an equation.

*Graphs, tables, real-world scenarios or equations - Written or verbal descriptions.* As Madison and Steen (2003) noted, part of quantitative literacy is being able to express mathematical concepts to others in a coherent manner. This ability to describe mathematical thinking was a strong emphasis in the course. Throughout the course, the instructor asked students to explain and discuss in class what was going on in a problem. Task interviews and classroom artifacts also required that students to explain verbally or in written words the meaning of some aspect of a problem or describe a scenario given a graph or table. In example artifacts, students were not always able to write complete sentences explaining their processes, but this at times appeared to be more of a lack of English skills or an interest in saving time than a lack of understanding of the problem. Because the course had a strong emphasis on discussion, this type of translation was present during almost all class periods. More about this will be said about the discussion aspect of the class in response to Research Question 3.

*Summary of Research Question 2*

Students were able to demonstrate an ability to move between representations of linear equations and slope in many cases, but initially struggled the most with writing equations of lines from other representations. By the end of the semester most students had mastered writing equations in slope-intercept form from real-world scenarios, and could find equations of lines from sets of data using the computer. As noted in the comments above, the researcher believes that the course could have a stronger emphasis on writing the equations of the line using graphs or sets of data without using the computer as a tool.

The students showed facility with movement from table to graph and interpreting graphs and tables into real-world scenarios. The researcher believed that some of this had to do with the fact that the majority of problems in the class were grounded in contexts that the students could understand. Very few problems in the class were given without any context. This allowed the students to reason logically about the problems and recognize when their answers were not correct.

As Lesh, Post, and Behr (1987) noted, students who demonstrate an ability to move between representations have a deeper understanding of the mathematical concept behind the representations. The students in the Introduction to the Mathematics Sciences course were primarily students who had not had much success in mathematics in the past. The ability to move between representations that the students exhibited in the class provides an indication that the students were gaining knowledge and increasing their understanding about the mathematics taught in the class. The researcher observed that during the task interviews, some of the students were able to switch between representations, particularly on the Islands Problem, using the representation that they found most useful for solving the problem, and then switching back to a previous representation to confirm the solution was correct. This demonstration of representational fluency indicated that the students were mastering the mathematical concepts. As Sandoval, Bell, Coleman, Enyedy, and Suthers (2000) (as cited in Zbiek, Heid, & Blume, 2006) noted, students should be able to know what the different representations are best able to illustrate, and be able to link multiple representations in meaningful ways.

As is the case in any classroom, it is important to be mindful of what students show that they are able to do, and what material the instructor presents to the students. Students showed that they were able to work with motion detector problems after having been confronted with that type of problem. They showed that they were less able to work with the abstract form of the equation, which may be a reflection of the lesser emphasis on this representation. Interview tasks were designed to examine specific types of translations that reflected material that the researcher anticipated the students had studied, when in reality, some of the material was presented more in depth slightly later on in the course. However, while the students in this class were not able to perform all translations perfectly, their increased ability to move between a variety of representations was an indicator of at least their growing understanding of the mathematics. This was an important step for many of the students.

*Research Question 3*

*How and to what extent does the course reflect fidelity of implementation of the course designers' vision of college reform in mathematics education for the two-year college?*

Mathematics education at the college level today is faced with increasing numbers of students who are entering college needing lower-level or developmental courses, and students who are not being well served by current courses (Lutzer, et al, 2005). The course under study in this research was designed to provide an alternative course for such students in hopes that it better serves the needs of those students. Clements (2007) recommends a number of phases of research in his framework on how to design research-based curricula. His phase 7 in the research process is a part of his evaluation curricula, and a type of formative research. In this phase, the class is examined for effectiveness, not only in student learning, but also in implementation of the curriculum. It is important for designers of the curriculum to determine exactly what is occurring in the classroom. While this research does not follow Clements framework to the letter, use of an outside observer to determine the events of the classroom can be helpful to the course designers. Therefore, this research question attempts to delve into the how well the intended curriculum is enacted in a classroom for single case of the Introduction to the Mathematical Sciences course. It is hoped that this snapshot of the classroom will aid the course designers in further development of the course.

To answer Research Question 3, data were used from classroom artifacts, observations, the OTOP, and interviews. Using cross-source analysis the researcher looked at the broad picture of the implementation of the curriculum in relation to the course designer's vision as described in the Overview and Background of the Course in Chapter 3. The researcher examined the course in terms of subject matter as well as pedagogy. Results are described below.

*Content.* The course designers provided a list of topics that they viewed as essential for the course (see Table 1, p. 40). The course was designed to integrate algebra, statistics and computer science in a class taught in a computer lab. The list of topics was shorter than those provided by most traditional curricula, which often provide an overview of many topics with little depth.

The researcher found that the data from classroom observations and classroom artifacts supported the implementation of the suggested algebra topics, with the exception of optimization in the algebra strand. This topic was not taught during the course in the Fall 2009 semester. The researcher also determined that although the course presented relations that were functions, function notation was seldom used in the classroom beyond the day that the topic was presented. During the study, the researcher focused on the algebra strand primarily, but classroom artifacts and observations also showed that the suggested topics from computer science and statistics were also presented in the class. Therefore, the researcher concluded that the course designers' vision for reform mathematics in terms of the subject matter of the course was, for the most part, implemented as desired.

*Pedagogy.* As noted in Chapter 3, in the vision of the course designers, the pedagogy of the course was strongly rooted in the ideals and recommendations of the *Principles and Standards for School Mathematics* (2000) of the National Council of Teachers of Mathematics. Group work, projects, multiple solution paths, student presentation and student discussion were deemed integral to the course. The integrated nature of the course subject matter with its focus on algebra, statistics and computer science was designed to engage the students in real world problems that utilize multiple representations of the problems. Solving the problems using a variety of representations (including verbal, concrete or pictorial, graphical, algebraic, and tabular forms) was an important part of the course. Ability to move between these multiple representations was to be stressed in the teaching of the intended curriculum along with explaining or communicating solutions in written and verbal format. Students were encouraged to share novel ways of solving problems, rather than having the instructor show one “right” way of solving problems with students mimicking the teacher’s actions.

According to the course designers (Webb, et al., 2009), the course philosophy reflected a constructivist philosophy of learning where students build upon their prior knowledge through inquiry style learning. If students had weaker skills in a particular area, the instructors of the course were to help students identify what they already know and through exploring the topics of the course, enhance that knowledge base.

The researcher determined through observations, classroom artifacts, the OTOP data and interviews that all of these desired aspects of the course were present. Perhaps the most outstanding feature of the class was the extensive discussion and questioning that took place in the classroom. Observation data indicated that group work and discussion were evident during nearly every class period. The discussion sessions were very productive at identifying levels student understanding. During classroom discussions, Mr. Smith probed student comments as the students provided answers to questions in order to force them to clarify their answers in their own minds, and then explain their thinking to the class. If a statement was unclear, and the student became confused, other students often jumped in to explain how they were thinking about the problems. By the end of the semester, observations showed that almost all of the students were volunteering answers, and explaining their thinking about the problems. This reaction by students was viewed to be in sharp contrast to many lecture based mathematics classes that are taught by “telling” (Smith, 1996). The students exhibited a high level of engagement and appeared to appreciate the class and the instructional methods. Comments by students during the task interviews indicated that they found the discussion and presentation of student answers in class to be extremely helpful.

In addition to the above reform characteristics, the course designers identified several other more unique pedagogical features that were to be implemented in the course. These included the following: (a) presenting problems in real-world contexts, (b) integrating algebra, computer science and statistics in a coherent blend, (c) teaching the course topics using spreadsheets to enhance the understanding of syntax and algebraic formulas, (d) spending at least half of the class time in a computer lab, and (e) spending approximately double the standard class time of a 3 credit college mathematics course in the class in order to allow students to complete projects and class work.

The researcher verified through observation and artifacts that the class was indeed presented with the majority of the problems in real-world contexts. As Radford (2004) noted, context is one way students make meaning from algebra problems. Students were given problems that related to their lives, thereby engaging them in the learning process better then strictly abstract problems might have. Responses to task interview questions indicated that the students found that the material had use in their lives. One student commented on this, "…you deal with figuring out things in everyday life versus just an algebra problem or just a something you have out of a textbook, with just x and y and they don't mean anything. [It's] more practical and deals with things you would look at or see in your daily life versus just an equation that someone thought up and put in a book." Observations and artifacts also provided evidence that the students were engaged in the class when discussing the contextual problems.

The course did integrate the three topics of algebra, statistics and computer science suggested by the course designers, as evidenced in the observations and artifacts. Students also seemed to appreciate this aspect of the course. One student noted, “This is a class you know where it has, is, a combination of three things. You know we’ve covered the statistics, the spreadsheets or the computer science and then the algebra. It’s a nice compilation of all three. They bring it all together.” Generally the material was integrated, as students used the computer to solve algebra problems or used algebra when doing calculations for the statistics portion of the class. Occasionally transitions seemed abrupt, but this continuous variation was, according to Mr. Smith, partly to keep the class moving and to keep the class engaged by changing topics frequently.

The students used the spreadsheet program daily and spent all of their class time in the lab. This, the researcher believes, is a feature that is unique to this class, and one of the strengths of the class. The students always had access to the computer, which not only allowed them to generate different representations of problems, but also provided them with skills they might be able to use in their future careers, unlike the graphing calculators more commonly seen in a college mathematics class, which may not be used beyond the scope of the class.

One major difference the researcher observed from that of the intended curriculum and the implementation of these specific aspects of the class was in the amount of time spent in the classroom. The class was scheduled from 3:55-6:55 PM. On most evenings, the students were done with class before 6:00 PM, so they were not spending the suggested amount of time in the classroom. This is not to say that the intended material was not discussed, but that the time frame of the course designers was not always followed. The students did complete their work in class, with little homework, which was also a desired feature of the course, so the time may not have been deemed necessary by the instructor. This may be something for the course designers to consider, if this same issue is true of the implementation of the Introduction to the Mathematical Sciences in other classrooms.

*Summary of Research Question 3*

Overall, the researcher concluded that, based on the data, the course was in almost all ways implemented in accordance with the reform-based ideals envisioned by the course designers. Simple addition of optimization as a topic and increased use of function notation would correct the discrepancy in intended subject matter, and increasing the in class time, adding activities or slowing the pace of the class would correct the issue regarding intended pedagogy.

*Research Question 4*

*How and to what extent does the teaching of this course reflect the NCTM, MAA, and AMATYC standards for college algebra reform in a two-year college setting in the areas of both content and pedagogy?*

The NCTM, MAA, and AMATYC standards agree on many aspects of change that are necessary to improve instruction in lower-level college mathematics courses (Baxter Hastings, et al., 2006). Since the Introduction to the Mathematical Sciences course is designed for students who have been less than successful in mathematics, these standards should apply to the course. This question examines how and to what extent those standards for college algebra are reflected in the course. The researcher will address the areas of pedagogy and content in relation to the data gathered during the study in order to help answer this research question.

*Pedagogy.* As noted in chapter 2, the statements of the NCTM, the MAA and the AMATYC are all in agreement as to the need to move away from teacher-centered teaching to student-centered instruction. All suggest the need for active learning, less rote skill work, more discussion, use of multiple representations and use of technology in mathematics classes at any level.

The researcher found that the Introduction to the Mathematical Sciences course did implement all of these suggested changes. These topics are in accordance with the vision of the course designers, and have been discussed in answering the previous research question. In addition, the course addressed the NCTM process standards outlined in the *Principles and Standards for School Mathematics* (2000), which are utilized in this study as guidelines for reform instruction. The MAA and AMAYTC standards are also aligned with these process standards. Specifically, the course puts into practice the standards of problem solving, reasoning and proof, communication, making connections, and representations of mathematical ideas. Classroom observation, OTOP data, artifacts and interviews all provide evidence to support the presence of these standards in the Introduction to the Mathematical Sciences course.

Nearly all problems presented in the class were set in real-world contexts, and students were generally expected to attempt to solve a problem on their own prior to receiving instruction, thus gaining problem solving skills and stronger skills at independently investigating application problems.

The process standard of reasoning and proof was not implemented in a traditional sense, that is, the students did not write formal proofs. However, they were asked to justify and explain their reasoning on almost all problems, if not always in writing, at least verbally in the classroom. Many problems on exams and worksheets asked the students to explain in words their thinking processes, or explain their answers. For a course of this level, designed to reach students whose mathematical skill levels are not well developed, this type of more informal justification seemed reasonable to the researcher.

Communication between students and between students and instructor was an overarching theme that was observed daily in the classroom. The instructor continually asked the students questions and asked them to present material in the class. They were asked to work in pairs and in groups throughout the class, and had to write up reports on their findings for some projects. They were asked to explain their answers to students near them, and to the class. Students also had to come to agreement as a class on various standards for measurement during data collection for the statistics portion of the course. Their success at these activities clearly emphasized communication as central to the Introduction to the Mathematical Sciences course.

The process standard for making connections was apparent in the integrated nature of the course. Observations showed that students were able to utilize algebraic formulas in the spreadsheets used in the computer portion of the class, and in the statistics strand as well. In addition, the students were also able to see connections between their daily lives and the mathematics that they were learning in the classroom because of the contextual problems they were asked to solve in the class.

Multiple representations of mathematical ideas were prevalent in the class. During class assignments, and on exams, students were asked to work with and translate between graphs, tables, real-world problems (sometimes through verbal descriptions) and equations. Students who were interviewed in the task interviews for the research study were also presented with multiple representations of the ideas of slope and linear equations. Because the students were presented with these different representations throughout the course, they did not shy away from interpreting or working with the various representations. The students were successful in moving between many of the representations. The students exhibited the most difficulty with writing linear equations from the other representations, and wrote linear equations utilizing only slope-intercept form. Further emphasis on exploring the abstract representations of algebraic ideas is a recommendation of the researcher.

*Content.* The Introduction to the Mathematical Sciences course is an introductory course designed to present algebra, statistics and computer science to students who have traditionally struggled with mathematics. The course reflected the recommendations on which the NCTM, MAA, and AMATYC show agreement, summarized by Baxter Hastings et al. (2006): (a) lessen the traditional amount of time performing algebraic manipulations; (b) decrease time spent executing algorithms simply for the sake of calculation; (c) restrict the topics covered to the most essential; (d) decrease the amount of time spent lecturing; and (e) deemphasize rote skills and memorization of formulas.

Students rarely did calculations with algebra outside of context, as has been mentioned in previous discussions. As evidenced by classroom observations and artifacts, they also did little work in memorizing formulas and repetitively practicing skills. The students were presented with a few worksheets to practice solving equations over the course of the semester, but overall, this work was a small portion of the class. These worksheets generally had 10 or fewer problems on them.

As mentioned in the discussion of Research Question 3, the class was primarily student-centered and discussion-based, rather than being a lecture-based course. The instructor guided the students' discussions, by asking questions, but elicited from the students their understanding of the topics. Occasionally direct instruction was used in presenting how to utilize the computer program, but the instructor still employed questioning techniques during this time.

The course was intended follow the content standards for reform mathematics in the sense that it was designed to present fewer topics in greater depth than the traditional mathematics course. Based on the observation and artifact data gathered throughout the study, the researcher concludes that the course achieved this goal for a lower-level mathematics course, particularly regarding the concept of slope and linear equations. This course presented fewer topics that a traditional algebra course. Students studied the concept of rate, which led to unit rate, and to the concept of slope throughout the semester. The concept was repeated throughout many class periods in order to solidify the idea of slope prior to presenting the linear equation. When linear equations were first presented, it was through the computer in terms of finding a trend line, and later the slope and intercept were tied to their locations in the abstract form of the equation, $y = mx + b$. This focus on developing ideas throughout the semester and revisiting them, building each class on the previous class, which was also evident for other topics in the class, reflect the reform ideal of presenting fewer topics in greater depth. Therefore the researcher concludes that the course is in line with the standards for reform instruction in terms of content in lower-level courses. However, the aspect of *algebra* instruction specifically must also be considered.

The Introduction to the Mathematical Sciences is not intended to be a complete course in algebra, and the course reflected this. The focus of the algebra portion of the course was on linear equations, and the students did not explore other types of functions in any depth. The class did not study exponential, quadratic, or logarithmic functions. These subjects are recommended in the standards of the MAA and AMATYC as part of the content in a college algebra course. Therefore, the researcher concludes that the course should not be considered a replacement for a college algebra course in terms of content studied, if students will need this content for their future careers.

As a point to consider for the course designers, this course provided students with an alternative to the traditional mathematics courses that focus on the abstract representations of mathematical ideas. Hofacker (2007) found that students who take part in a contemporary college algebra course that utilizes multiple representations are better able to move between different representations of algebraic ideas and exhibit a deeper understanding of the ideas behind the representations. A question to consider is how students are able to perform when those students move from a course such as the Introduction to the Mathematical Sciences to a more traditional college algebra course that primarily emphasizes working with the abstract form. While the researcher does not believe that this course contains sufficient content to be considered a replacement for college algebra, a second course that extends the material, but is taught according to reform ideals and continues the integration of statistics, computer science and algebra could provide students with the material they need for their future careers and also satisfy the requirements of the standards. This extension of the course could provide students the bridge that they need to become more fluent in the use of the abstract formulas and yet continue the process of building conceptual understanding through practical applications and skills the students find valuable in their lives.

*Summary of Research Results*

Reform in mathematics instruction at the college level has been slow to arrive (Dossey, Halvorson, & McCrone, 2008), and many institutions of higher learning still follow the calculus model, while fewer and fewer students need calculus for their chosen areas of study (Ganter & Barker, 2003). Instead, mathematics that is applicable and transferable to other disciplines is more useful to many of today's college students. The Introduction to the Mathematical Sciences course that was the subject of this research study is a standards-based laboratory class that integrates algebra, statistics, and computer science. It was designed for students at both the high school and college levels who have struggled in mathematics. The intent of the course is to provide students with mathematics that they will find useful in their future careers, or future classes. The course is intended to reflect the ideals of reform mathematics at the college level. The purpose of the study was to examine the implementation of this curriculum, and its impact on student thinking and learning of algebra.

In exploring the research questions, the researcher found that the Introduction to the Mathematical Sciences course provided a reform-instruction setting where students were able to demonstrate their understanding of algebra, statistics and computer science. The students showed skill at moving between a number of representations of algebra concepts, indicating they were developing deeper understanding of those concepts. One of the key components of this course that reflected reform ideals was the extensive discussion that took place in the course. This instance of the implementation of the Introduction to the Mathematical Sciences course provides an example of how reform instruction in line with the recommendations of NCTM, MAA and AMATYC (Baxter Hastings, et al., 2006) can be successful in helping students at the introductory college level gain understanding of mathematics. This research study describes a course that successfully plays out using instructional methods that are in sharp contrast to other college courses taught using traditional lecture style methods. High DWF rates among students who take college algebra (Lutzer, et al., 2005) indicate that the current model of instruction at the college level is not working. For students who lack confidence in their mathematical abilities and have seen little success in mathematics, this type of course may be a tool that can provide students the mathematical skills necessary to move forward in their studies and their careers. The next section discusses the limitations of the study.

## Study Limitations

There are several limitations to this study that must be pointed out. One limitation of this research project is that it was a case study, which by nature is a study unique to the case. Therefore, it is unlikely that the findings are generalizable to other courses, even those in which reform methods of teaching are utilized. However, the purpose of the study was to obtain insight into student thinking and learning, and discover how well the specific course itself was being implemented, in particular the algebra portion of the course. The results of the study may be helpful to the designers of the course and to those who choose to implement this course in their schools, and may identify key components essential to the successful implementation of the course. It is not intended to be generalizable to other courses.

A second possible limitation of this study is the instructor had worked closely with and even taken courses from some of the designers of the course. The implication is that implementation of the course by those who have not studied under or worked as closely with these professors may not have results similar to those obtained in this study. In addition, the instructor had extensive experience teaching reform and standards-based courses in the past. His energy and commitment to reform was extraordinary. Future course instructors may not have similar backgrounds.

Finally, it is important to state that the researcher is a former student of the course designers and holds a similar philosophy regarding mathematics education. All attempts were made to remove personal bias from the research, but it is still important to make note of this fact.

## Suggestions for Further Research

Research on affect in mathematics education has shown that student beliefs and attitudes towards mathematics can have a significant effect on student learning (McLeod, 1992). One theme discovered in the analysis of the data was that the students in this course stated that their fear of mathematics and algebra in particular had been reduced through participating in this class. During the interviews with students, three of the four students stated that they did not find algebra as intimidating as they had in the past. One woman noted the following:

> I don't mind the algebra stuff now. I guess, it's just kind of figuring out how do to the problems…logically looking at them without just being overwhelmed by them. Because if you didn't know how to set it up, because then you're just…going oh my gosh, it's like an overwhelming thing… Yeah, I said today algebra is great…

A second student who had exhibited a fear of mathematics had a similar comment on the class:

> I feel like I've learned some algebra but I didn't realize I was learning it. Which is a really a good thing. Because too many times we walk into a situation like this, like I was just deathly afraid of algebra, and didn't think that I was capable of doing it. And the way that Mr. Smith has explained it and walked us through it hasn't even seemed like a problem at all…and there's more people that feel the same way that I do.

This may have been unique to this particular class and the instructor, but a study of the impact of this course on student affect and belief about mathematics would be of interest, in order to determine if this style of teaching truly does change the views about mathematics held by students who have traditionally struggled in their math classes. A study on mathematics anxiety in students taking the Introduction to the Mathematical Sciences course designed to determine if fear of math is truly reduced would be valuable as well.

This study looked at student understanding of and ability to move between representations of slope and linear equations in this course using the Lesh (1979) translation model as a guide to types of representations. An examination of why the students had the most difficulty in translating to the abstract equation writing in this particular setting, and how to correct that problem may be worthwhile. Studies that follow these students into later algebra courses to determine how these students fare in algebra compared to students who have not had the course would also be of interest, and would also be in line with recommendations from the MAA's Committee on the Undergraduate Program in Mathematics Guide (2004) for follow-up studies.

## Recommendations for the Course Designers

While the course followed the vision of the course designers in most ways, there were some aspects of the course that did not. In terms of subject matter, the suggested topic of optimization was not discussed during this semester. Whether that was an oversight by the instructor or a flaw in the course materials should be examined and corrected, or the topic omitted. As a second aspect of subject matter, function notation was only covered briefly in the Fall 2009 semester. The researcher suggests that if this notation is introduced, it should be used throughout the semester to solidify student understanding.

Regarding the topic of linear equations, the researcher had several concerns. First, students showed the weakest facility in working with the abstract representation of the line, the equation. Students struggled during task interviews to write equations of the line and solve equations. While the students did develop greater skill over the semester, the researcher recommends further emphasis on working with equations and linking the equation to the other representations. In particular, students did not display ability to write an equation of the line from a table or graph without assistance of the computer, unless given initial value, and not always with that given value. Additional work with this topic, perhaps introducing the point-slope form for the equation of the line may be helpful to students in writing the equation. Further emphasis on different forms of the equation and their relationships to graphs or tables may be beneficial. Additional discussion of extrapolating back to the initial value using tables on the spreadsheet might help as well.

The aspect of practicing with working with the abstract form of the equation was not highly focused on in this class. The students were given worksheets with 8-10 problems to work through at various times during the course. This lack of emphasis on rote equation manipulation was by design, while there was a greater focus on using a variety of representations. This provides students a broader picture of the concept, but does not necessarily prepare the students for further work with equations. As mentioned earlier, a second course extending the material in this course, but maintaining the integration of algebra, statistics and computer science could be of value for students who need to complete a course that fulfils the requirements of college algebra. Moving directly into a traditional college algebra course might otherwise prove a difficult transition for students who have taken part in the Introduction to the Mathematical Sciences. As mentioned in the suggestions for further research, an examination of how these students fare in later mathematics courses would be of interest in determining whether students can easily make this transition or not. While many college algebra courses include some measure of reform teaching, not all do, and some students who have seen success in this course may revert to less successful behaviors when confronted with a traditional mathematics class.

Pedagogically, the instance of the implementation of the Introduction of the Mathematical Sciences course that the researcher observed followed all the course designers recommendations with the exception of the amount of time spent in the class. This could easily be corrected by addition of material or changing the pace of the course.

In this particular class, the instructor showed dedication to teaching the course as intended and the researcher believes that other instructors of the class could benefit from his level of energy and enthusiasm. A recommendation of the researcher to the course designers is to have future instructors of the course observe the instruction of teachers such as Mr. Smith in order to better understand the goals of the course designers and of reform teaching.

## Concluding Remarks

The examination of this innovative mathematics course, Introduction to the Mathematical Sciences, involved the study of student thinking and learning about algebraic representations in the context of a course that integrated algebra, statistics and computer science into a single class. This reform mathematics course was designed to meet the needs of students who were not following the traditional course sequence of the calculus model and/or had struggled with mathematics in the past.

This particular study adds to the literature on student thinking and learning by presenting a case in which students who have traditionally been underserved in the college system are exhibiting successful behaviors in learning mathematical concepts. The course enacts the use of mathematics reform ideals from the high school and college level (Baxter-Hastings, et al., 2006) in creating opportunity for students to achieve greater goals in mathematics.

In addition to providing the broader mathematics education community with a course supporting reform mathematics instruction, it is hoped that this research study also may provide the course designers with a snapshot of a particular instance of the implementation of their course, assisting them in the continuing evolution of their new curriculum.

## References

American Mathematical Association of Two-Year Colleges, Cohen, D. (Ed). (1995). *Crossroads in mathematics: Standards for introductory college mathematics before calculus*. Memphis, TN: AMATYC.

American Mathematics Association of Two-Year Colleges. (2006). *Beyond crossroads: Implementing mathematics standards in the first two-years of college*. Memphis, TN: AMATYC.

Balyta, P. (2007). Using multiple representation systems to deepen understanding of functional relationships in mathematics. *Dissertation Abstracts International, 69*(5), 1708, 0419-4209.

Baxter Hastings, N., Gordon, F., Gordon, S., & Narayan, J. (Eds.). (2006). *A fresh start for collegiate mathematics: Rethinking the courses below calculus*, MAA Notes 69. Washington: DC. Mathematical Association of America.

Brownell, W. A. (1947). The place of meaning in the teaching of arithmetic. *Elementary School Journal*, 47, 5, 256-265.

Campbell, D. T., & Stanley, J. C. (1966). *Experimental and quasi-experimental designs for research*. Chicago: Rand McNally.

Clements, D. H. (2007). Curriculum research: toward a framework for 'research-based curricula.' *Journal for Research in Mathematics Education, 38*(1), 35-70.

Coalition for Evidence Based Policy. (2003). *Identifying and implementing educational practices supported by rigorous evidence: A user friendly guide*. Washington, D. C.: U. S. Department of Education, Institute of Education Studies, National Center for Education Evaluation, and Regional Assistance.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education*. (5th ed.), New York: RoutledgeFalmer.

Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America. (2004*). Undergraduate Programs and Courses in the Mathematical Sciences: A CUPM curriculum guide*, MAA Reports, Washington, DC: Mathematical Association of America. Retrieved June 30, 2009 from http://www.maa.org/cupm/curr_guide.html.

Cresswell, J. W., & Miller, D. L. (2000). Determining validity in qualitative inquiry. *Theory into Practice*, *39*(3), 124-130.

de Villiers, M. D. (1999). *Rethinking proof with the Geometer's Sketchpad*. Emeryville, CA, Key Curriculum Press.

Douglas. R. (1986). *Toward a lean and lively calculus*. Washington, DC: MAA.

Dossey, J., Halvorson, K., & McCrone, S. (2008). *Mathematics education in the United States: A capsule summary fact book*. Reston, VA: NCTM.

Fennema, E., & Romberg, T. A. (1999). *Mathematics classrooms that promote understanding*. New York: Lawrence Erlbaum.

Friedlander, A., & Tabach, M. (2001). Promoting multiple representations in algebra. In A. A. Cuoco (Ed.), *2001 Yearbook of the National Council of the Teachers of Mathematics: The Roles of Representation in School Mathematics* (pp.173-185). Reston, Virginia: The Council.

Ganter, S.L. and Barker, W. (Eds.) (2003). *A collective vision: Voices of the partner disciplines*. MAA Reports, Washington, DC: Mathematical Association of America.

Goerdt, S. (2007). The Effect of emphasizing multiple representations on calculus students' understanding of the derivative concept. *Digital Dissertations*. UMI 3277946.

Harshbarger, R., & Yocco, L. (2004). *College algebra in context*. New York: Addison Wesley.

Harel, G., & Sowder, L. (2006). Toward comprehensive perspectives on the learning and teaching of proof. In F. K. Lester, Jr., (Ed.) *Second handbook of research on mathematics teaching and learning.* (pp. 805-842). Charlotte, NC: Information Age Publishing.

Harwell, M., Post, T., Cutler, A., Maeda, Y., Anderson, E., Norman, K. W., & Medhanie, A. (2009). The Preparation of students from National Science Foundation–funded and commercially developed high school mathematics curricula for their first university mathematics course. *American Educational Research Journal*, 46(1), 203-231.

Herman, M. (2007). What students choose to do and have to say about use of multiple representations in college algebra. *Journal of Computer in Mathematics and Science Teaching, 26*(1), 27-54.

Herriott, S. (2005). *College algebra through functions and models*. Pacific Grove, CA: Brooks-Cole.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K.C., Wearne, D., Murray, H., Oliver, A., & Human, P. (1997). *Making sense: teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Hughes-Hallett, D., & Gleason, A. M. (1998). *Calculus: single and multivariable*. Sommerset, NJ: Wiley.

Hofacker, E. B. (2006). The effect of different college algebra courses on students' understanding of linear and exponential function concept. *Digital Dissertations*. UMI 3227552.

Hycner, R. H. (1985). Some guidelines for the phenomenological analysis of interview data. *Human Studies*, 8, 279-303.

Kaput, J. (1992). Technology and mathematics education. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 515-556). New York: MacMillan.

Kieran, C. (2006). Learning and teaching algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning.* (pp. 707-762). Charlotte, NC: Information Age Publishing.

Lesh, R. (1979). Mathematical learning disabilities: Considerations for identification, diagnosis, and remediation. In R. Lesh, D. Mierkiewicz, & M. G. Kantowski (Eds.), *Applied mathematical problem solving* (pp. 111-180). Columbus, OH: ERIC/SMEAR.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier, (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier, (Ed.), *Problems of Representations in the Teaching and Learning of Mathematics* (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

Lesh, R., & Zawojewski, J. (2006). Problem solving and modeling. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763-804. Charlotte, NC: Information Age Publishing.

Lutzer, D. J., Rodi, S. B., Kirkman, E. E., & Maxwell, J. W. (2007). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States: Fall 2005 CBMS report*. Providence, RI: American Mathematical Society. Retrieved July 12, 2009 from http://www.ams.org/cbms/full-report.pdf.

Madison, B. L., & Steen, L. A. (Eds.) (2003). *Quantitative literacy: why numeracy matters for schools and colleges*. Princeton, NJ: National Council on Education and the Disciplines.

McCaffrey, D., Hamilton, L., Stecher, B., Klein, S., Bugliari, D., & Robyn, A. (2001). Interactions among instructional practices, curriculum, and student achievement: The case of standards-based high school mathematics. *Journal for Research in Mathematics Education*, *32*(5), 493-517.

McLeod, D. B. (1992). Research on affect in mathematics education: a reconceptualization. In D. A. Grouws (Ed.) *Handbook of research on mathematics teaching and learning* (pp. 575-596). New York: MacMillan.

Miles, M., & Huberman, M. A. (1994). *Qualitative data analysis* (2nd ed.). Beverly Hills: Sage Publications.

Mitchell, M. (1997). The use of spreadsheets for constructing statistical understanding. *Journal of Computers in Mathematics and Science Teaching*, 16(2-3), 201-222.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics Research Council. (2009). Research in mathematics education: Multiple methods for multiple uses. *Journal for Research in Mathematics Education, 40*(3), 216-240.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, D. C.: U. S. Department of Education.

Ostebee, A., & Zorn, P. (1997). *Calculus from graphical, numerical, and symbolic points of view*. Boston: Houghton Mifflin.

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory into Practice*, *40*(2), 118-127.

Pólya, G. (1957). *How to solve it*. Garden City, NY: Doubleday.

Post, T. R., Medhanie, A., Harwell, M., Norman, K., Anderson, E., Muchlinski, T., & Dupuis, D. (2010). The impact of prior mathematics achievement on the relationship between high school mathematics curricula and post-secondary mathematics performance, course-taking, and persistence. *Journal for Research on Mathematics Education, 41(3),* 274-308.

Schoenfeld, A. H. (1992). Learning to think mathematically: problem solving, metacognition and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: MacMillan.

Schoenfeld, A. H. (2004). The math wars. *Educational Policy, 18*(1), 253-286.

Schoenfeld, A. H. (2006). Method. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 69-110). Greenwich, CT: Information Age Publishing.

Schwingendorf, K., McCabe, G., & Kuhn, J. (2000). A longitudinal study of the $C^4L$ calculus reform program: comparisons of the $C^4L$ and traditional students. *CBMS Issues in Mathematics Education, 8,* 63-76.

Senk, S. L., & Thompson, D. R. (Eds.). (2003) *Standards-based school mathematics curricula: What are they? What do students learn?* Mahwah, NJ: Lawrence Erlbaum.

Small, D. (2004). *Contemporary college algebra*. (5th ed.). New York: McGraw-Hill.

Smith, John P., III. (1996). Efficacy and teaching mathematics by telling: a challenge for reform. *Journal for Research in Mathematics Education, 27*, 387-402.

Stake, R. (1995). *The art of case study research*. Thousand Oaks, CA: Sage Publications.

Sutherland, R., & Rojano, T. (1993). A spreadsheet approach to solving algebra problems. *Journal of Mathematical Behaviour*, 12(4). 351-383.

Wainwright, C. L., Flick, L. B., & Morrell, P. D. (2003, Fall). Development of instruments for assessment of instructional practices in standards-based teaching. *Journal of Mathematics and Science: Collaborative Explorations, 6*(1), 21-46.

Wainwright, C. L., Flick, L. B., Morrell, P. D., & Schepige, A. (2004). Observation of reform teaching in undergraduate level mathematics and science courses. *School Science and Mathematics*, 104(7), 322-335.

Webb, D., Richgels, G., Wolf, M. J., Frauenholtz, T., & Hougen, A. (2009, September). *Improving student interest, mathematical skills, and future success through implementation of a novel mathematics bridge course for high school seniors and post-secondary students.* A paper accepted for publication in the proceedings of the Tenth International Conference on Models in Developing Mathematics Education in Dresden, Germany.

Yin, R. (2009). *Case study research: Design and methods*, 2nd ed. Thousand Oaks, CA: Sage.

Zbiek, R. M., Heid, M. K., Blume, G. W. (2006) Research on technology in mathematics education: a perspective of constructs. In F. K. Lester, Jr., (Ed.) *Second handbook of research on mathematics teaching and learning* (pp. 1169-1207). Charlotte, NC: Information Age Publishing.

# APPENDICES

## Appendix A

### CONSENT FORM

A Study of the Introduction to the Mathematical Sciences Course.

You are invited to be in a research study of the implementation of an innovative mathematics course designed to improve student learning. You were selected as a possible participant because you are taking this course. We ask that you read this form and ask any questions you may have before agreeing to be in the study.

This study is being conducted by: Heidi Hansen, graduate student at the University of Minnesota, Department of Curriculum and Instruction, under the supervision of Drs. Tamara Moore and Kathy Cramer.

**Background Information**

The purpose of this study is: to examine how the Introduction to Mathematical Sciences course is being implemented, and how the course goals regarding algebra instruction are being met. This study is a case study and we will be looking at two sites where this specific course is being taught. Your classroom is one of them.

**Procedures:**

If you agree to be in this study, we would ask you to do the following things:
Students in the class will be observed 12-15 times during their normal class times over the course of the class. Student discussions will sometimes be audio or videotaped in order for the researcher to recall events in the classroom. Samples of student work will be collected. Six students in each class will be asked to voluntarily be interviewed while completing math tasks. Instructors will also be interviewed.

**Risks and Benefits of being in the Study**

The study has minimal risk: You may feel uncomfortable being recorded; if you are interviewed, you may feel uncomfortable if you have difficulty completing the tasks. If you choose, you may decide not to participate in this study at any time.

The benefits to participation are: you can provide valuable input for future instructors and students in this class.

**Compensation:**

You will not receive any compensation.

**Confidentiality:**

The records of this study will be kept private. In any sort of report we might publish, we will not include any information that will make it possible to identify a subject. Research records will be stored securely and only researchers will have access to the records. Audio and videotapes will be used only by the researchers in order to transcribe events. Any tapes will be stored securely and will be destroyed five years after the conclusion of the study.

**Voluntary Nature of the Study:**

Participation in this study is voluntary. Your decision whether or not to participate will not affect your current or future relations with the University of Minnesota, or the technical college. If you decide to participate, you are free to not answer any question or withdraw at any time without affecting those relationships.

**Contacts and Questions:**

The researchers conducting this study are: Heidi Hansen under the supervision of Drs. Tamara Moore and Kathy Cramer. You may ask any questions you have now. If you have questions later, **you are encouraged** to contact Ms. Hansen at home, phone: 218-370-0676, or via email at hanse758@umn.edu . You may also contact Dr. Moore at tamara@umn.edu, or by phone at 612-624-1516 or Dr. Cramer at crame013@umn.edu or by phone at 612-624-7312.

If you have any questions or concerns regarding this study and would like to talk to someone other than the researcher(s), **you are encouraged** to contact the Research Subjects' Advocate Line, D528 Mayo, 420 Delaware St. Southeast, Minneapolis, Minnesota 55455; (612) 625-1650.

***You will be given a copy of this information to keep for your records.***

**Statement of Consent:**

I have read the above information. I have asked questions and have received answers. I consent to participate in the study.

Signature:_____________________________________________ Date: __________________

Signature of parent or guardian:__________________________ Date: __________________
*(If minors are involved)*

Signature of Investigator:_______________________________ Date: __________________

Appendix B

# Observation Protocol

**I. Background Information**

| | | | | | | |
|---|---|---|---|---|---|---|
| **Teacher Name** | | | | **School** | | |
| **Subject observed** | | | | **Grade level** | | |
| **Observation is (circle or bold one)** | | In-field | | Out- of-field | | |
| **Date** | | **Start time** | | **End time** | | |
| **Traditional/Block** | | | **Meet 5 days or 2 evenings** | | | |
| **Observer** | | | **Observation#** | | | |
| **Number of students in class:** | | | | | | |
| **Brief description of students in class:** | | | | | | |

**II. Contextual Background and Activities**

**A. Objective for lesson (ask teacher before observing):**

| |
|---|
| |

**B. How does the lesson fit in the current context of instruction (e.g. connection to previous or other lessons)?**

| |
|---|
| |

**C. Classroom setting: (space, seating arrangements, etc. Include a diagram, if possible).**

| |
|---|
| |

**D. Any relevant details about the time, day, students, or teacher that you think are important?** (i.e.: teacher bad day, day before spring break, pep rally previous hour, etc.)

| |
|---|
| |

**III. Detail log/transcript of the classroom observation (indicate time when the activity changes)**

| **Elapse Time**<br>Start time: | **Observation Notes**<br>(Note: *=Critical Incident) |
|---|---|
| 0-5 | |
| 5-10 | |
| 10-15 | |
| 15-20 | |
| 20-25 | |

| | |
|---|---|
| 25-30 | |
| 30-35 | |
| 35-40 | |
| 40-45 | |
| 45-50 | |
| 50-55 | |
| 55-60 | |
| 60-65 | |
| 65-70 | |
| 70-75 | |
| 75-80 | |
| 80-85 | |
| 85-90 | |
| 90-95 | |
| 95-100 | |
| 100-105 | |
| 105-110 | |
| 110-115 | |
| 115-120 | |
| 120-125 | |
| 125-130 | |
| 130-135 | |
| 135-140 | |
| 140-145 | |
| 145-150 | |
| 150-155 | |
| 155-160 | |
| 160-165 | |
| 165-170 | |
| 170-175 | |
| 175-180 | |
| End time: | |

**IV. OTOP**

This instrument is to be completed following the observation of classroom instruction. Prior to instruction, the observer will review planning for the lesson with the teacher. During the lesson, the observer will write an anecdotal narrative describing the lesson and then complete the instrument. Each of the ten items should be rated "globally"; the descriptors are possible descriptors **not** required as a check-off list.

**Score descriptions:**

| | |
|---|---|
| 0 | Practice could have been included in the lesson, but wasn't |
| 1 | Some attempt is made at the item |
| 2 | Elements of the item are clearly present, but not fully carried out |
| 3 | Areas in the item of good quality, but there is room for improvement |

| | |
|---|---|
| 4 | Highest level of quality evident in this item |
| N/A | Practice was not included and was not appropriate for the lesson |

| | |
|---|---|
| **1.** This lesson encouraged students to seek and value various modes of investigation or problem solving. (Focus: Habits of Mind) | 0 1 2 3 4 NA |
| **Teacher:**<br>Presented open-ended questions<br>Encouraged discussion of alternative explanations<br>Presented inquiry opportunities for students<br>Provided alternative learning strategies | **Student:**<br>Discussed problem-solving strategies<br>Posed questions and relevant means for investigating<br>Shared ideas about investigations |
| **Comments:** | |
| **2.** Teacher encouraged students to be reflective about their learning. (Focus: Metacognition)<br>*Note*: Consider both the teacher and student for the score, focus on what is being said. | 0 1 2 3 4 NA |
| **Teacher:**<br>Encouraged students to explain in their own words both what and how they learned | **Student:**<br>Discussed what they understood from class and how they learned it<br>Identified anything unclear to them<br>Reflected on and evaluated their own progress toward understanding. |
| **Comments:** | |
| **3**. Interactions reflected collaborative working relationships and productive discourse among students and between teacher and students. (Focus: Student discourse and collaboration)<br>*Note:* focus of what students are involved in engagement. | 0 1 2 3 4 NA |
| **Teacher:**<br>Organized students for group work<br>Interacted with the small groups<br>Provided clear outcomes for groups | **Student:**<br>Worked collaboratively or cooperatively to accomplish work relevant to the task<br>Exchanged ideas related to the lesson with peers and teacher |
| **Comments:** | |
| **4.** Intellectual rigor, constructive criticism, and the challenging of ideas were valued. (Focus: Rigorously challenged ideas)<br>*Note:* Focus students and teachers rigorously challenged ideas. | 0 1 2 3 4 NA |
| **Teacher:**<br>Encouraged input and challenged students' ideas<br>Was non-judgmental of student opinions | **Student:**<br>Provided evidence-based arguments<br>Listened critically to others' explanations |

| | |
|---|---|
| Solicited alternative explanations | Discussed/challenged others' explanations |
| **Comments:** | |
| **5.** The instructional strategies and activities probed students' existing knowledge and preconceptions. (Focus: Student pre- and mis-conceptions)<br>*Note:* Must see this from the teacher and then see the students explicitly stating their conceptions. | 0 1 2 3 4 NA |
| **Teacher:**<br>Pre-assessed students for their thinking and knowledge<br>Helped students confront and/or build on their ideas<br>Refocused lesson based on students ideas to meet needs | Student:<br>Expressed ideas even when incorrect or different from the ideas of other students<br>Responded to the ideas of other students |
| **Comments:** | |
| **6.** The lesson promoted strongly coherent conceptual understanding in the context of clear learning goals. (Focus: Conceptual thinking ) | 0 1 2 3 4 NA |
| **Teacher:**<br>Asked higher level questions<br>Encouraged students to extend concepts and skills<br>Related integral ideas to broader concepts | **Student:**<br>Asked and answered higher level questions<br>Related subordinate ideas to broader concepts |
| **Comments:** | |
| **7.** Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence. (Focus: divergent thinking)<br>*Note:* Primary focus of the teacher. | 0 1 2 3 4 NA |
| **Teacher:**<br>Accepted multiple responses to problem-solving situations<br>Provided example evidence for student interpretation<br>Encouraged students to challenge the text as well as each other | **Student:**<br>Generated conjectures and alternative interpretations<br>Critiqued alternative solution strategies of teacher and peers. |
| **Comments:** | |
| **8**. Appropriate connections were made between content in the real world and/or other curricular areas. (Focus: Interdisciplinary connections) | 0 1 2 3 4 NA |
| **Teacher:**<br>Integrated content with other curricular areas<br>Applied content to real-world situations | **Student:**<br>Made connections with other content areas<br>Made connections between content and personal live |
| **Comments:** | |

<table>
<tr><td colspan="2"></td></tr>
<tr><td>9. The teacher had a solid grasp of the subject matter content and how to teach it. (Focus: PCK)<br>Note: can’t double ding against #6.</td><td>0 1 2 3 4 NA</td></tr>
<tr><td>Teacher:<br>Presented information that was accurate and appropriate to students’ cognitive level<br>Selected strategies that made content understandable to students<br>Was able to field students’ questions in a way that encouraged more questions<br>Recognized students’ ideas even when vaguely articulated.</td><td>Student:<br>Responded to instruction with ideas relevant to target content<br>Appeared to be engaged with lesson content</td></tr>
<tr><td colspan="2">Comments:</td></tr>
<tr><td>10. The teacher used a variety of means to represent concepts. (Focus: Multiple representations)</td><td>0 1 2 3 4 NA</td></tr>
<tr><td>Teacher:<br>Used multiple methods, strategies and teaching styles to explain a concept<br>Used various materials to foster student understanding (i.e. models, drawings, graphs, concrete materials, manipulatives)</td><td>Student:</td></tr>
<tr><td colspan="2">Comments:</td></tr>
</table>

**VI. Post Observation Comments**

| |
|---|
| |

# Appendix C

**Interview Protocol for Interviewing the Instructor**

I: Thank you for allowing me to interview you today. As you know, I have been observing your class as part of my dissertation research as I pursue a doctorate in Curriculum and Instruction, Mathematics Education from the University of Minnesota. I appreciate this opportunity.

You have signed an agreement to participate in my research and today I will be interviewing you as part of my study. I will be recording and later transcribing this interview, but your identity will be kept confidential in my dissertation.

Today I'll be asking you a series of questions about your Introduction to Mathematical Sciences class. I'll ask questions generally about the class, and specifically about the algebra portion of the class.

You may discontinue the interview at any time if you feel uncomfortable. We will begin when you are ready.

Q1: What do you see as the major focus of this class?

Q2: Where do you see the students moving from and to in this class? In other groups who have taken the Intro to Math Sciences class?

Q3: How much do you rely on the instruction of the course designers for planning your classes and how much on your teaching background?

Q4: When teaching algebra ideas, do you follow a particular philosophy?

Q5: [Followup] If so, do you feel your philosophy is in line with that of the course designers?

Q6: [Followup] Do you think that philosophy is necessary for the teaching of this class?

Q7: Do you strive to teach to any particular set of national standards? (NCTM, MAA, AMATYC?)

Q8: What do you see as the impact on students of developing fluency in moving between representations of mathematical, specifically algebraic, ideas?

Q9: [Followup] Do you try to focus on this in your teaching of this class?

Q10: [Followup] Is there a particular representation or representations you focus on?

Q11: Please give a brief outline of how you develop the solving of linear equations.

Q12: [Followup] Do you focus on any particular representation or representations of linear equations in this class?

Q13: Please give a brief outline of how you develop the idea of slope.

Q14: [Followup] Do you feel it is important to focus on any particular representation or representations of slope in this class?

Q15: How do you know when students have "gotten it", i.e. How do you know when to move on?

That was the last question. Thank you for taking the time to meet with me. I appreciate your input and enjoyed working with you.

Appendix D

**Student Task Interview Protocol** **Date**_______________ **Student Code**____________

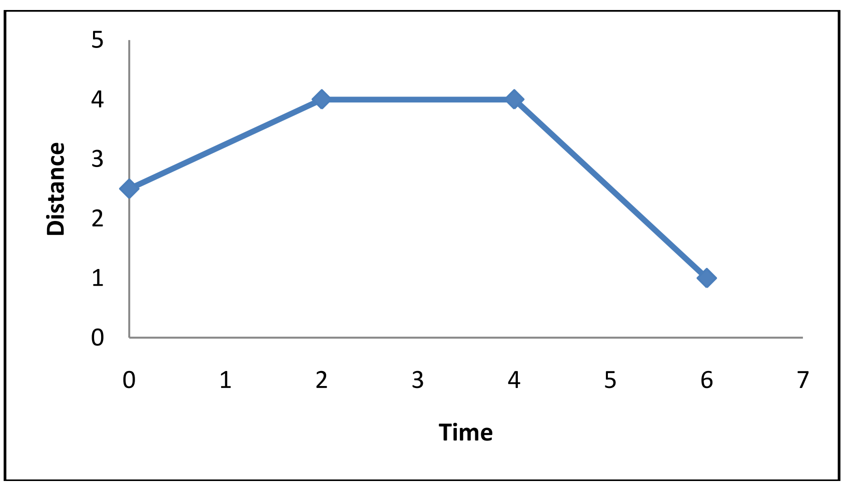
*Introductory remarks*:
My name is Heidi Hansen, and I'm doing a research study on the Introduction to Mathematical Sciences course in order to fulfill the requirements to complete my doctoral degree. You have already signed a consent form to participate in my research, and today I will be interviewing you. This interview will take about 20-30 minutes and will be audio or video recorded. During that time I will ask you some questions about the class you are taking and also ask you to work some problems. I might ask questions about your solutions and how you thought about or solved the problem. During this process, there is a chance that you may feel uncomfortable if you are unsure about how to do the problem, otherwise there is little risk involved in the process. If you so choose, you may discontinue the interview at any time. If you don't know how to do the problem, it's OK. Just tell me and we will go on to the next one. You can use the spreadsheet program, paper and pencil, or your calculator to help you solve the problems. Do you have any questions before we begin?

Q1: "Tell me about some of the ideas about algebra you feel you have learned in this class."

Q2: "How do you think the material in this class is different from math classes you have taken in the past?"

Q3: "How do you think the teaching style of this class is different from math classes you have taken in the past?"

Q4: I am going to show you a graph of distance (in feet) versus time (in seconds). I'd like you to tell me what you think is going on in the situation depicted by this graph.




b. When is the person moving fastest? How do you know?

c. Which direction is he/she going at that time? How do you know?

d. What kinds of calculations you could make using this graph?

Q5: Please try to solve this equation for x:

$3x + 5 = 7x - 15$

Q6: You are part of a construction company that is supposed to build houses. An architect has left plans to build houses on three islands. The islands are connected by bridges. Island A and Island B have a total of 20 houses. Island A and Island C have a total of 13 houses. Island B and Island C have a total of 15 houses. A total of 24 houses is to be built on the three islands. Determine how many houses are to be built on each island.

Q7: a. I'm going to have you look at a table of data and I want you to draw me a rough graph of the data on the graph paper I provide.

Table A

| X | Y |
|---|---|
| 0 | 30 |
| 5 | 40 |
| 10 | 50 |
| 15 | 60 |
| 20 | 70 |

b. Can you describe for me what kind of relationship there seems to be between x and y?

c. If I tell you that the data in the x column stands for years, and the data in the y column stands for population of a city in thousands, tell me as much as you can about the situation.

d. Can you tell me when the city will have 100 thousand people?

e. If the data for a different city looks instead like the table below, please describe what is happening in this city and how it differs from the previous city. You may make a graph if you wish.

Table B

| X | Y |
|---|---|
| 0 | 33 |
| 5 | 31 |
| 10 | 29 |
| 15 | 27 |
| 20 | 25 |

f. If you can, please write an equation that represents the relationship between time and population for Table A using x and y as the variables.

*If f was completed:

i. How did you know what kind of relationship the two variables had?

ii. Why did you choose the numbers you did for the equation in part e?

g. Do you think the cities will ever have the same population? If so, when? (You can estimate.)

*For part g, when you estimated when the cities would have the same population, how did you decide this? Tell me what was going on in your head.

Q8: If you need to tile your living room, and know that each tile covers 2.5 $ft^2$, please show me how you will calculate how many tiles you need to buy if you know how many square feet you have in your living room.

*Concluding remarks*:

That was the last question. I will be transcribing what you said in this interview, but your name will be kept confidential so no one but us will know your answers. Do you have any questions now? Thank you for taking the time to talk with me. If I would like to ask some further questions near the end of the semester, would you be willing to answer more questions? ________ (Y/N). I appreciate your help with my research.