

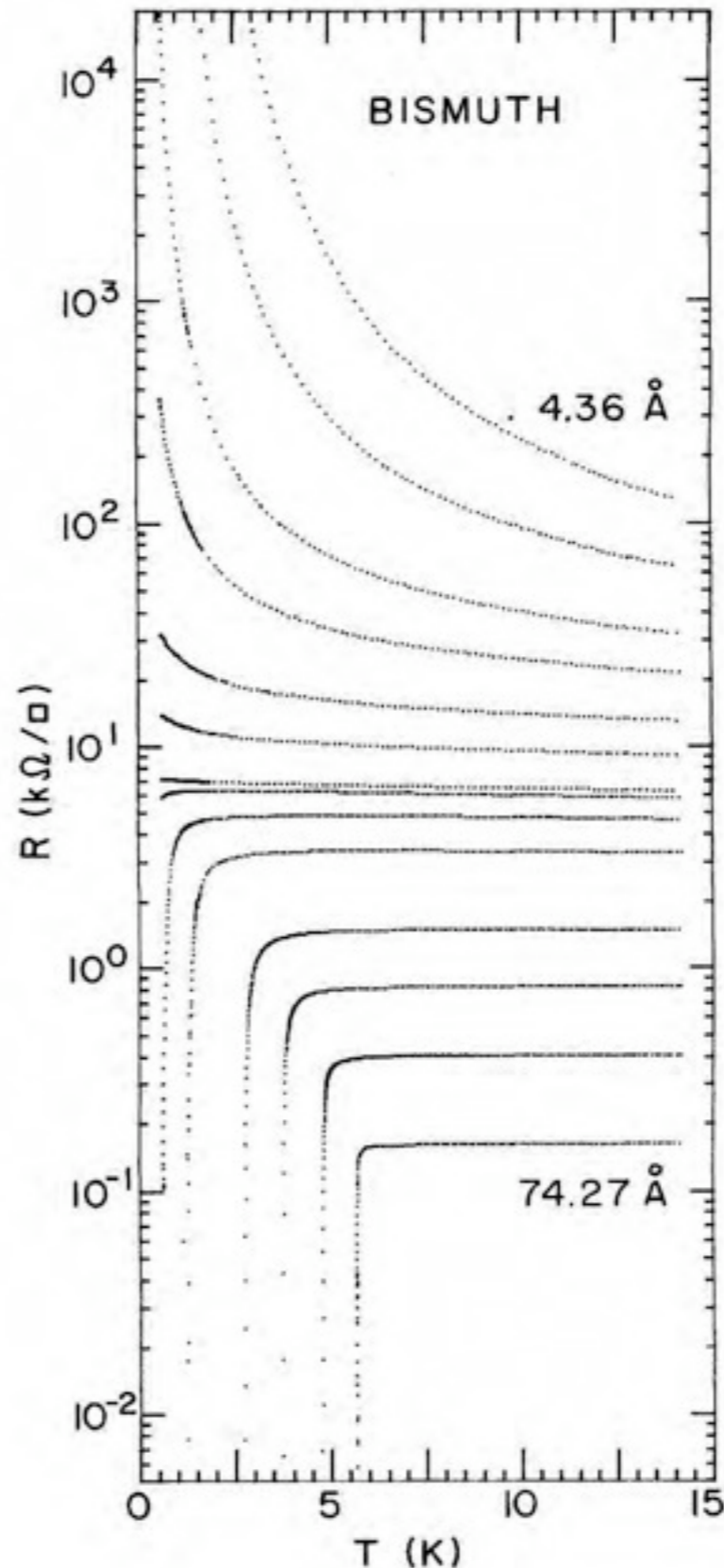
# Quantum-Criticality in the dissipative XY and Ashkin-Teller Model: Application to the Cuprates and SIT.

Jaeger, Orr, Goldman,  
Kuper (1986)

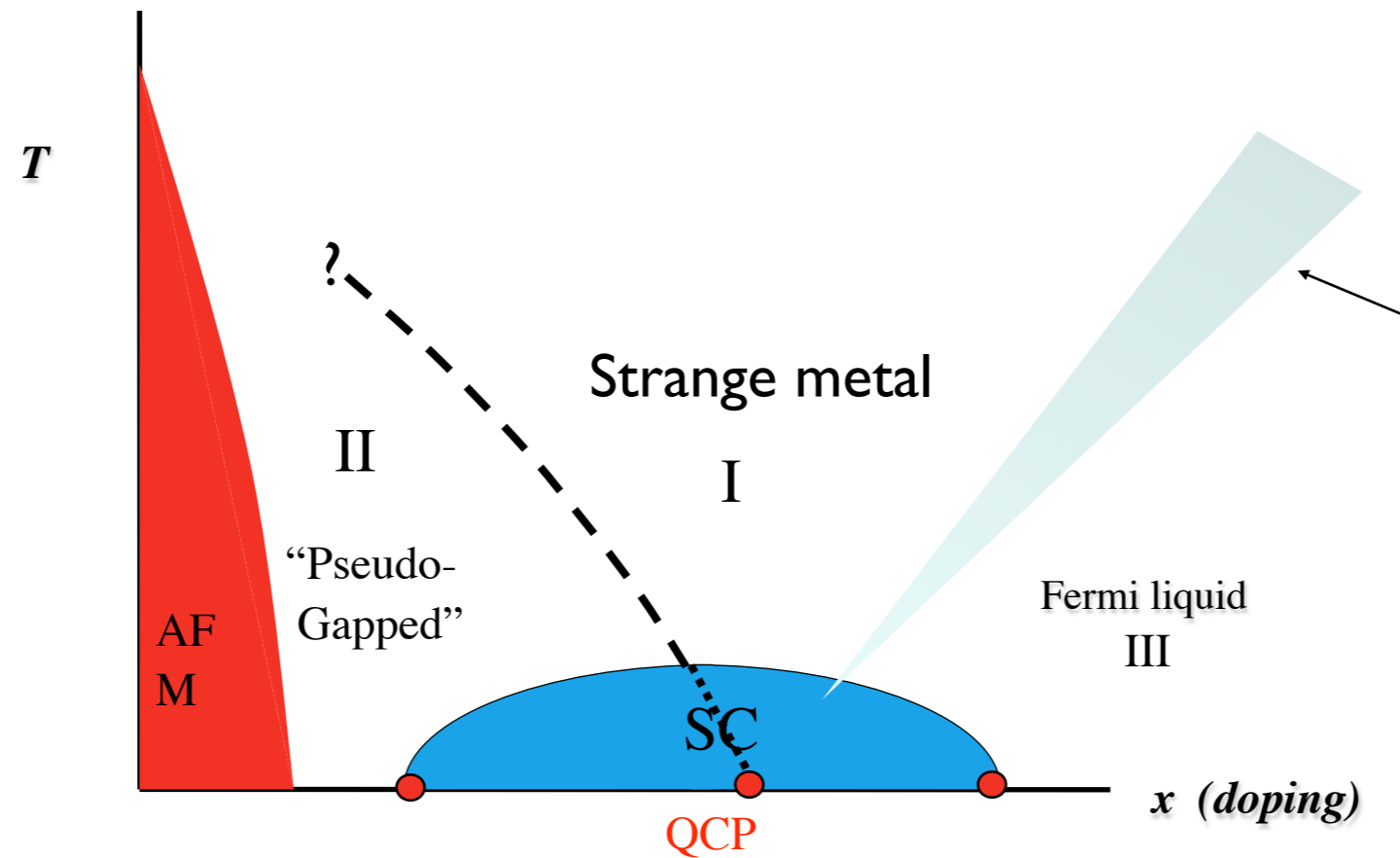
Dissipation driven  
QCP's

Haviland, Liu, and  
Goldman  
Phys. Rev. Lett. 62,  
2180 (1989)

Also, Hebard, Fiory,  
Paalanen (1985).



## Schematic Universal phase diagram of high- $T_c$ superconductors



(Based on Properties which occur in all Cuprates)

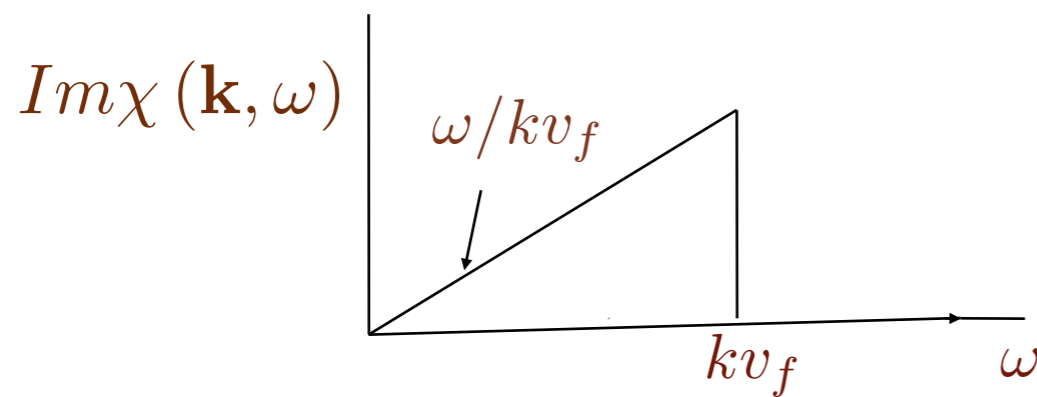
General Acceptance in the last few years of this phase Diagram. In particular that there is a competing Phase in region II and that it ends within the superconducting dome, i.e. there is a Quantum Critical Point.

# Anomalies in the strange metallic phase understood only if there exists a scale-invariant Fl.-spectra

## Excitation Spectra

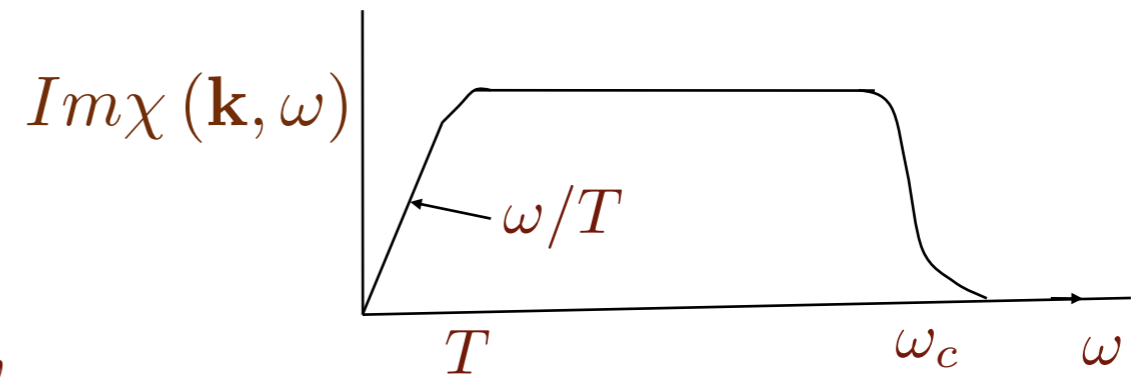
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Normal Metal



Scattering rate  $\propto T^2$

Marginal Fermi Liquid



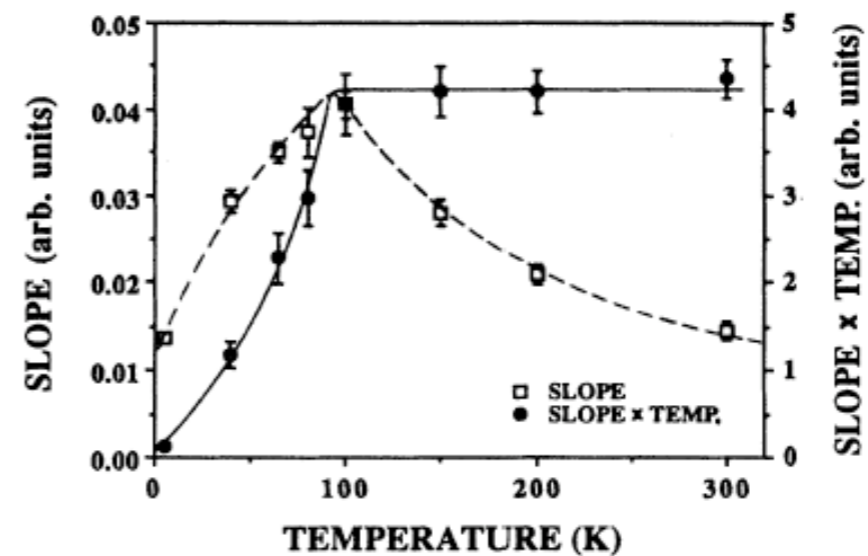
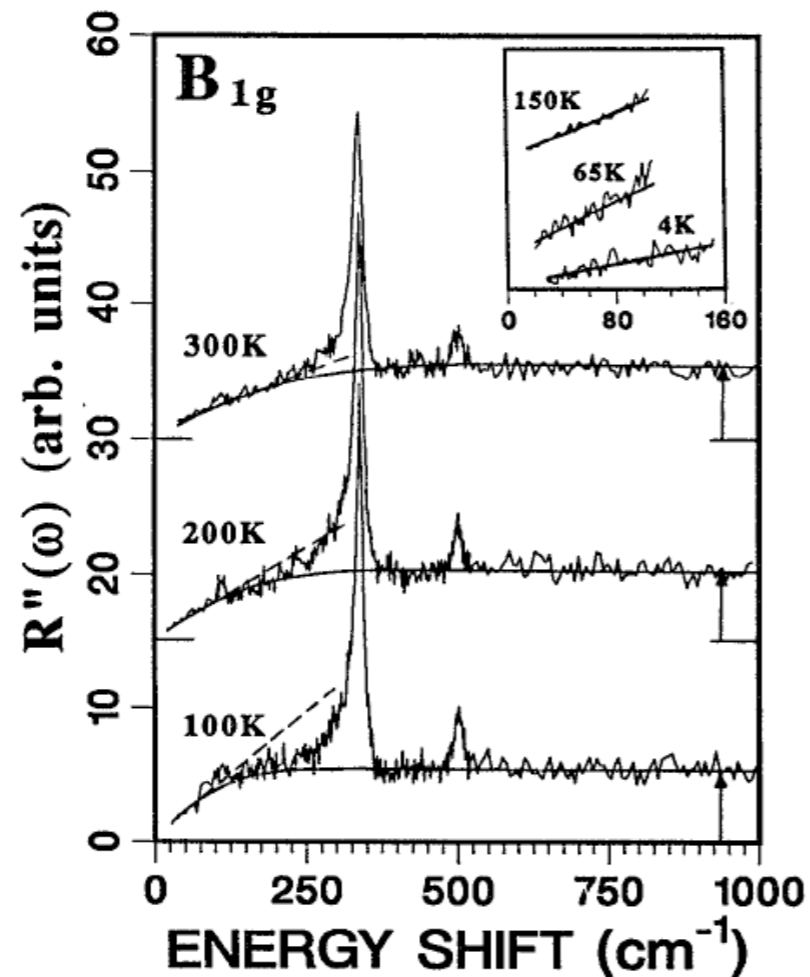
Scattering rate  $\propto T$

Quantum Critical Spectra: Very unusual, independent of momentum:  
**Local in real space, Singular in time**

# Experimental Evidence of the Derived Spectra

## Raman Spectra in Optimally doped Cuprates

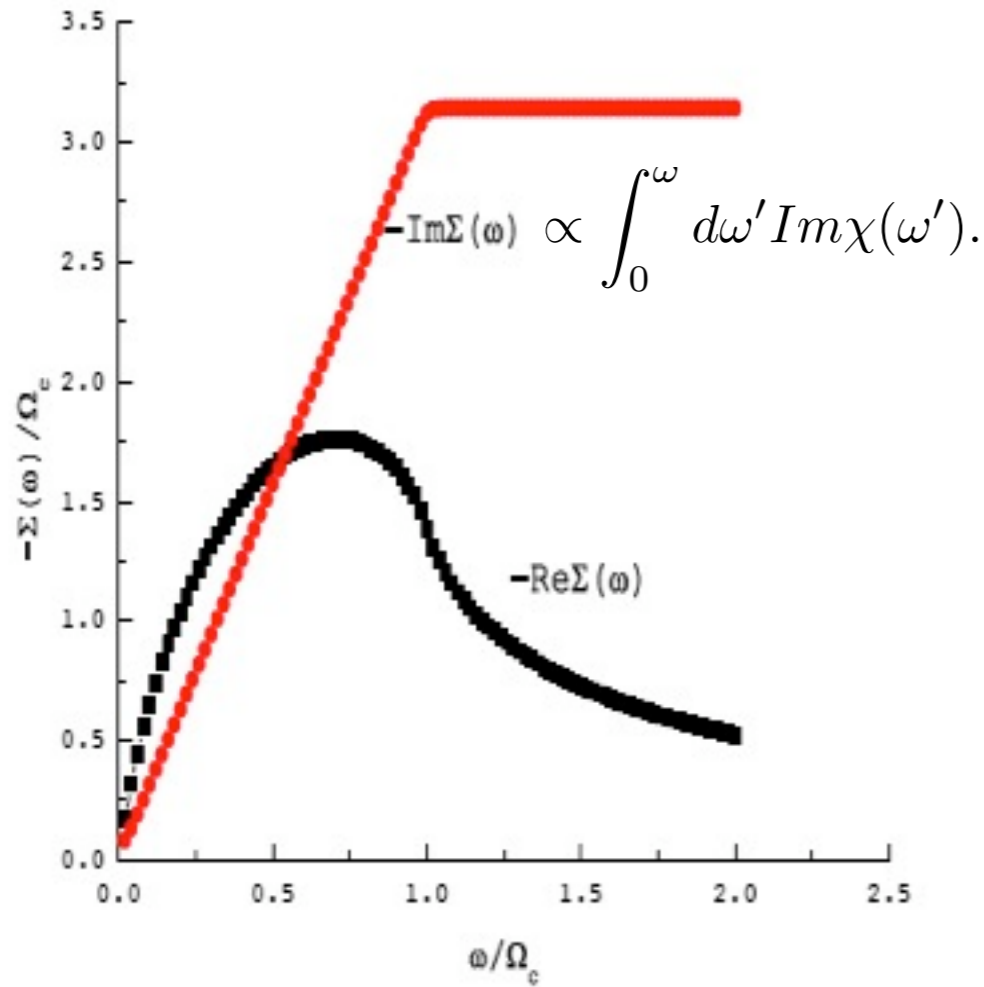
Direct observation of a Quantum critical fluctuations spectra of the derived form:  
for  $q \rightarrow 0$  in  $B_{1g}$  Symmetry



*F. Slakey et al. PRB 43, 3764 (1991)*

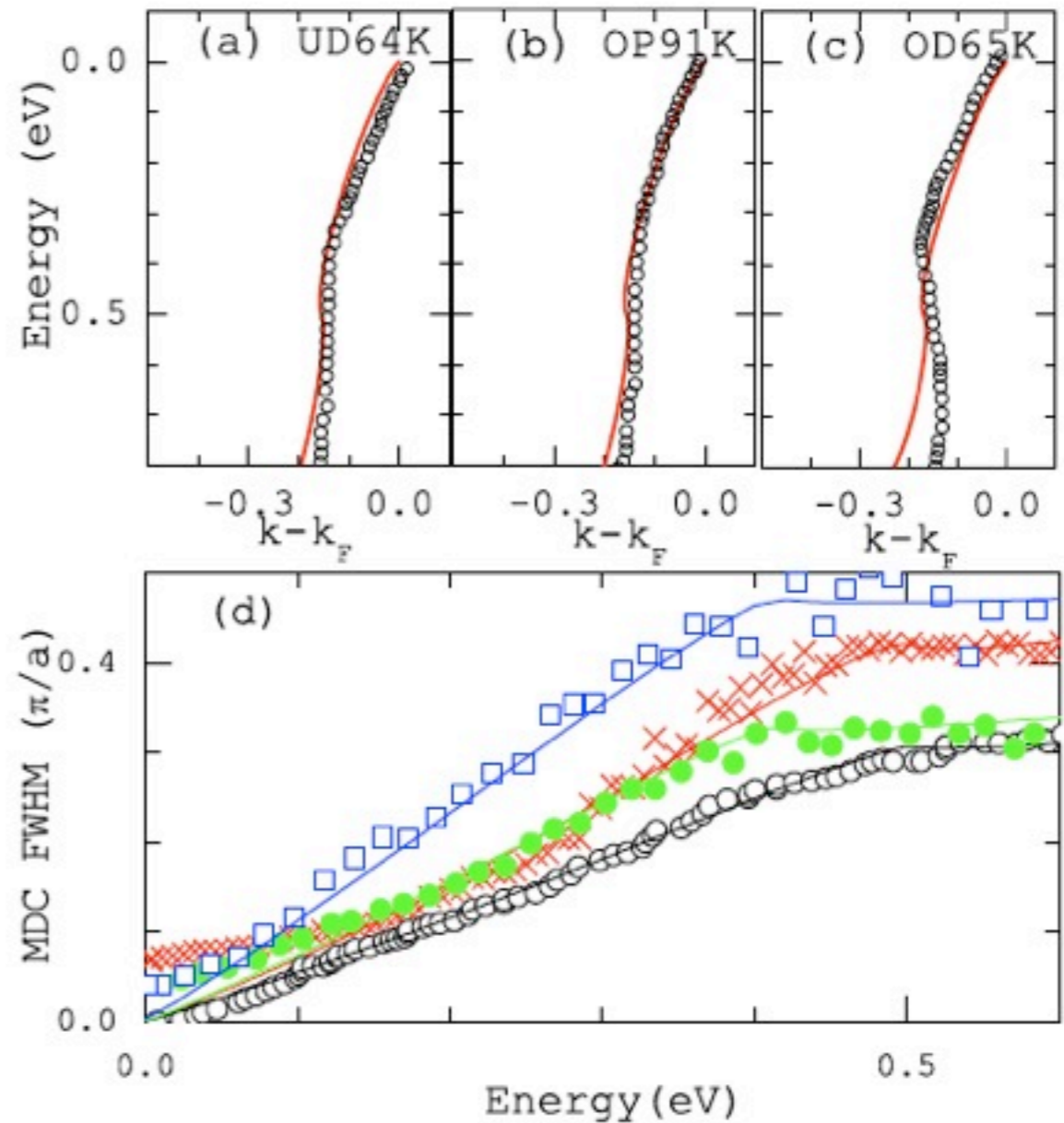
## Single-particle Spectra measured in ARPES

Linewidth proportional to  $\omega$  for  $\omega \lesssim \omega_c$  and constant beyond  
 Inelastic Scattering rate independent of  $k$ .



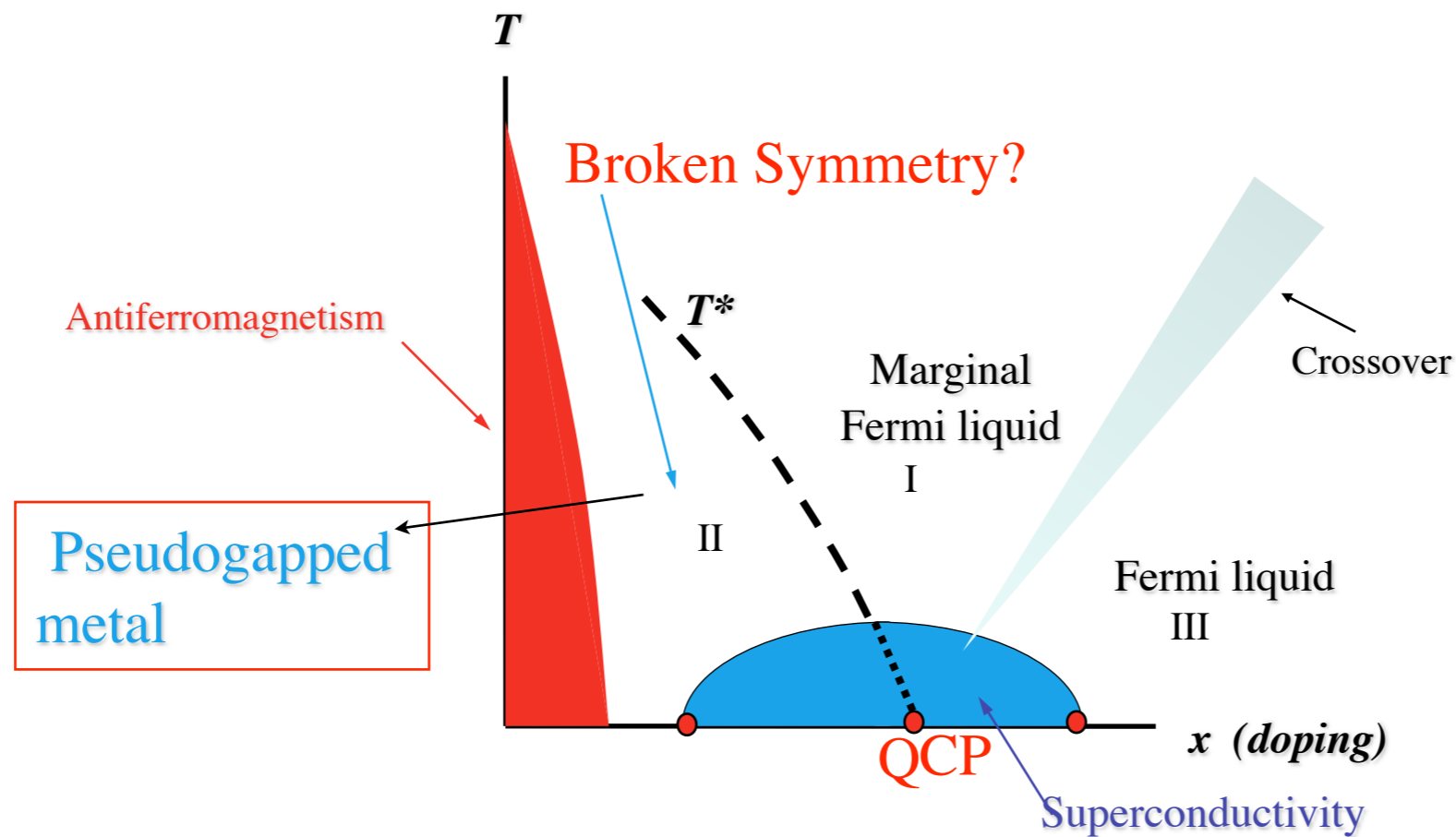
- OP Bi2201 Nodal, Meevasana et al.
- × OP-Bi2212 Nodal, Lanzara et al.
- LSCO OP, Nodal, Chang et al.
- LSCO Nodal underdoped, Chang et al.

Dispersion: Lanzara et al., Nodal cuts in Bi2212



Spectra in Region I is that for fluctuations due to a Quantum Critical Pt.  
THE QCP SUGGESTS A SYMMETRY BREAKING IN REGION II.

Mysterious Hidden Order Parameter in Region II ?  
Need large amplitude order parameter to be Thermodynamically significant.  
Must answer why No Specific heat Singularity at  $T^*(x)$ .



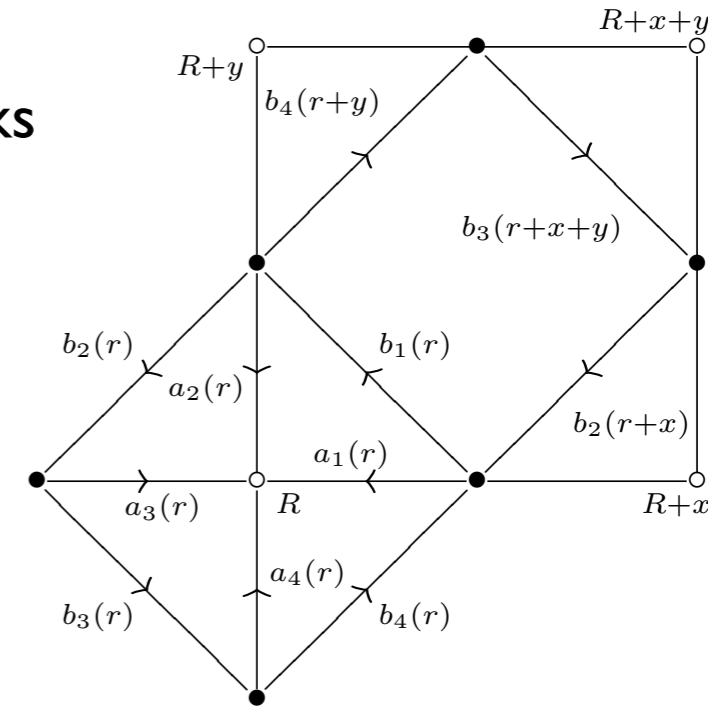
Deriving a Broken Symmetry requires a Microscopic Model.

# Theory of the Loop Order

Use Operator identity:  $Vn_in_j = -V/2(|J_ij|^2 + n_i + n_j)$ .

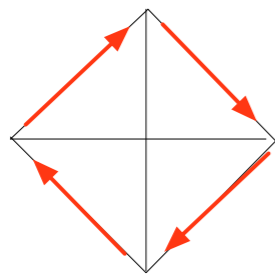
Mean-Field theory with collective variables on the 8 links per unit-cell constructed from the  $|J_ij|^2$ .

Organize links into irreducible representations of the lattice:

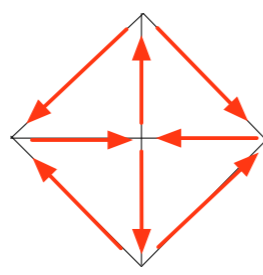


## Closed Loop Variables

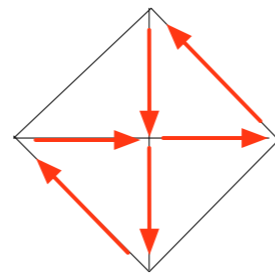
$A_{2g}$   
 $B_{2g}$   
 $E_u$   
 $E_u$



$L_s$



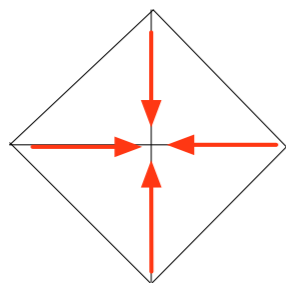
$L_d$



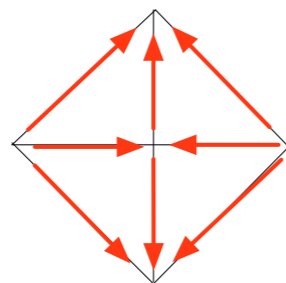
$\mathbf{L}=(L_x, L_y)$

## “Open Loop” Variables

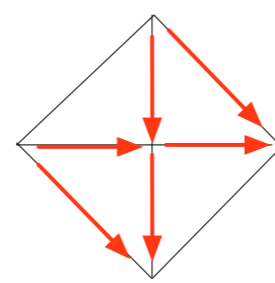
$A_{2g}$   
 $B_{2g}$   
 $E_u$   
 $E_u$



$A_s$

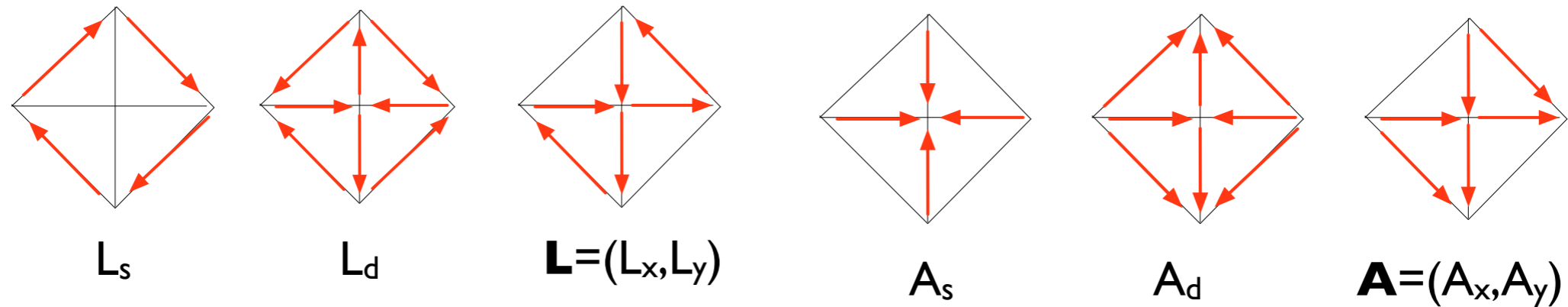


$A_d$



$\mathbf{A}=(A_x, A_y)$

(8 phase variables) - (3 sites) = (5 physical fields)



Physical Fields

$L_s$   $L_d$   $\mathbf{L}=(L_x, L_y)$ . Choose  $A_s = A_d = \text{div} \mathbf{A} = 0$ .

Only Fields  $\mathbf{L}$  and  $\mathbf{A}$  are relevant for cuprates

$L$ 's describe Gauge invariant objects and a finite expectation value for them can exist. Mean-field calculations to construct a Free-energy show possibility of the “anapole-order” state violating time-reversal and parity but preserving their product. This state is discovered in Expts.



## Experiments to look for time-reversal breaking in the pseudogap phase

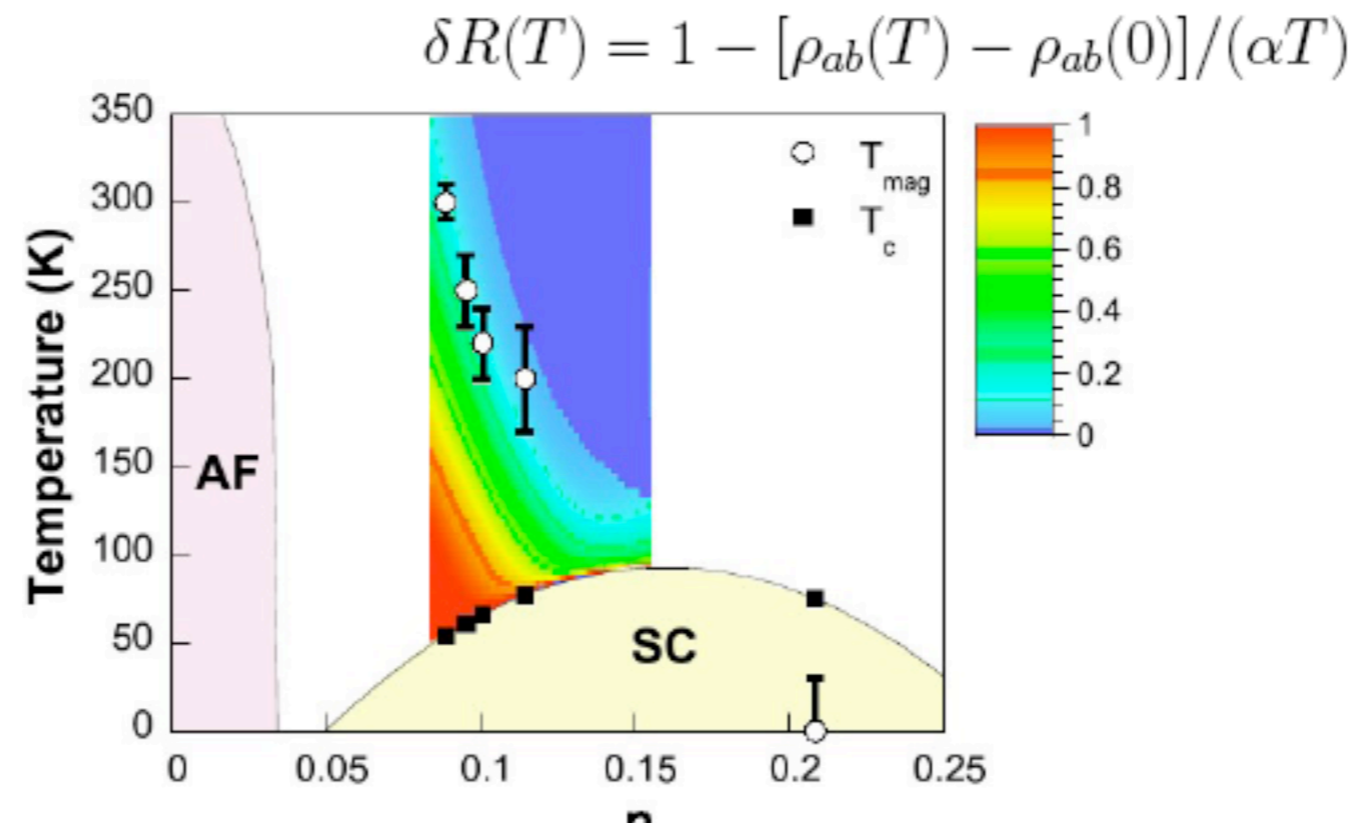
- I. Direct Observation by Polarized neutron Diffraction in underdoped YBaCuO of proposed order:(Bourges et al. 2005)  
Repeated by Mook et al. (2008)  
Exactly same order discovered in Underdoped single layer Hg-Cuprates (Greven et al., 2008)  
Same Order observed in LSCO (Bourges, private comm. 2009)
- II. Dichroism in Angle-Resolved Photoemission: Experiment by Kaminski et al. (2002) in underdoped BISCCO compounds
- III. Observation of Small ferromagnetic moment detected through Kerr effect in underdoped YBa<sub>2</sub>Cu<sub>3</sub>O<sub>x</sub> :(Kapitulnik)
- IV. Thermodynamics Evidence for a Phase Transition at T\*(x): Subtle Non-analytic Effect in magnetization even though no singularity in the specific heat. (Leridon, Monod and Colson).

## Phase diagram: magnetic order at $T^*$

B. Fauqué et al, Phys. Rev. Lett. 96, 197001 (2006)

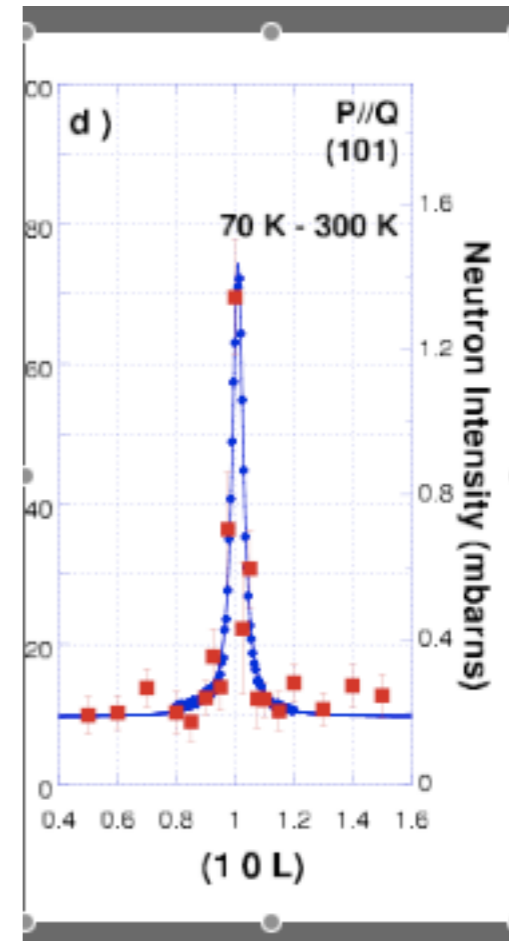
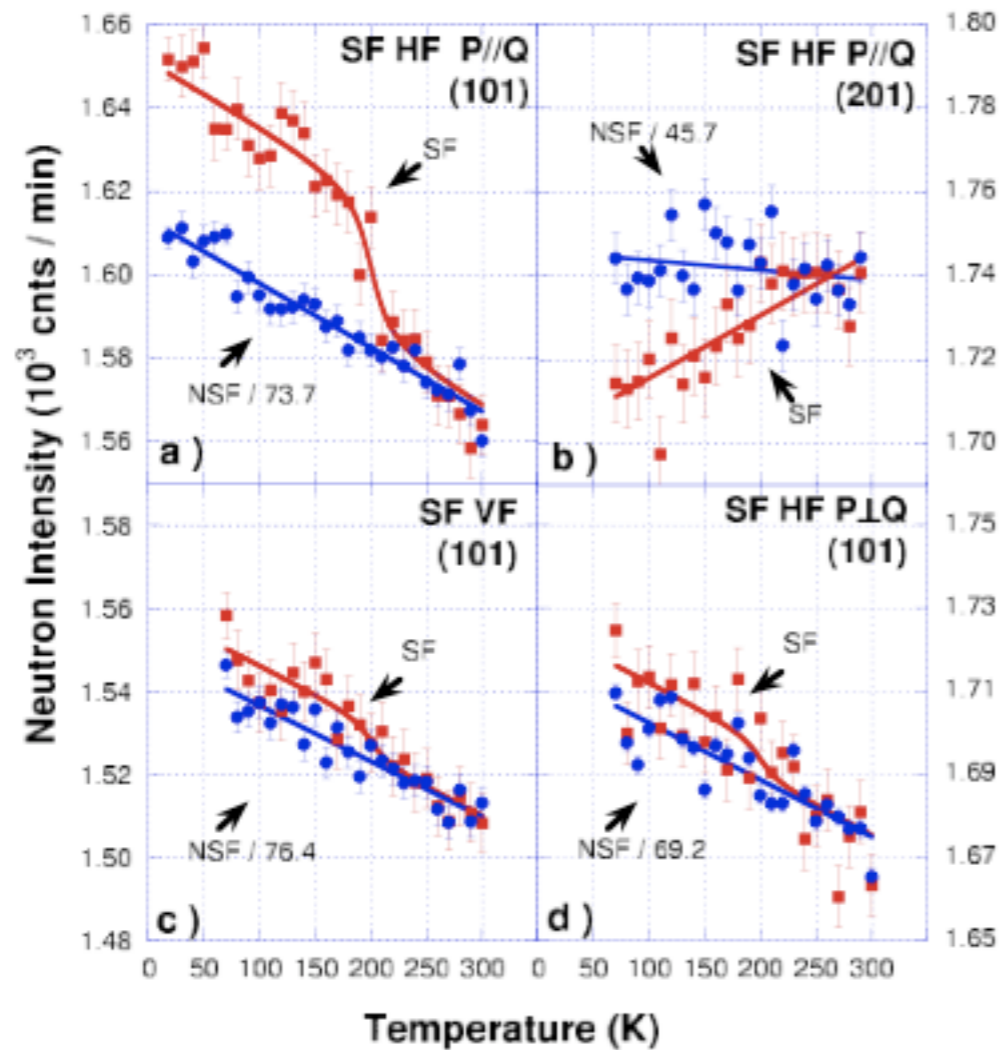
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⇒ Hidden order parameter for the pseudogap phase



1. Large magnitude Order,  $\approx 0.2\mu_B/\text{unit-cell}$  at the lowest doping.
2. **Universality.**
3. Difference from the prediction of the simplest model:  
Direction of the orbital moments.

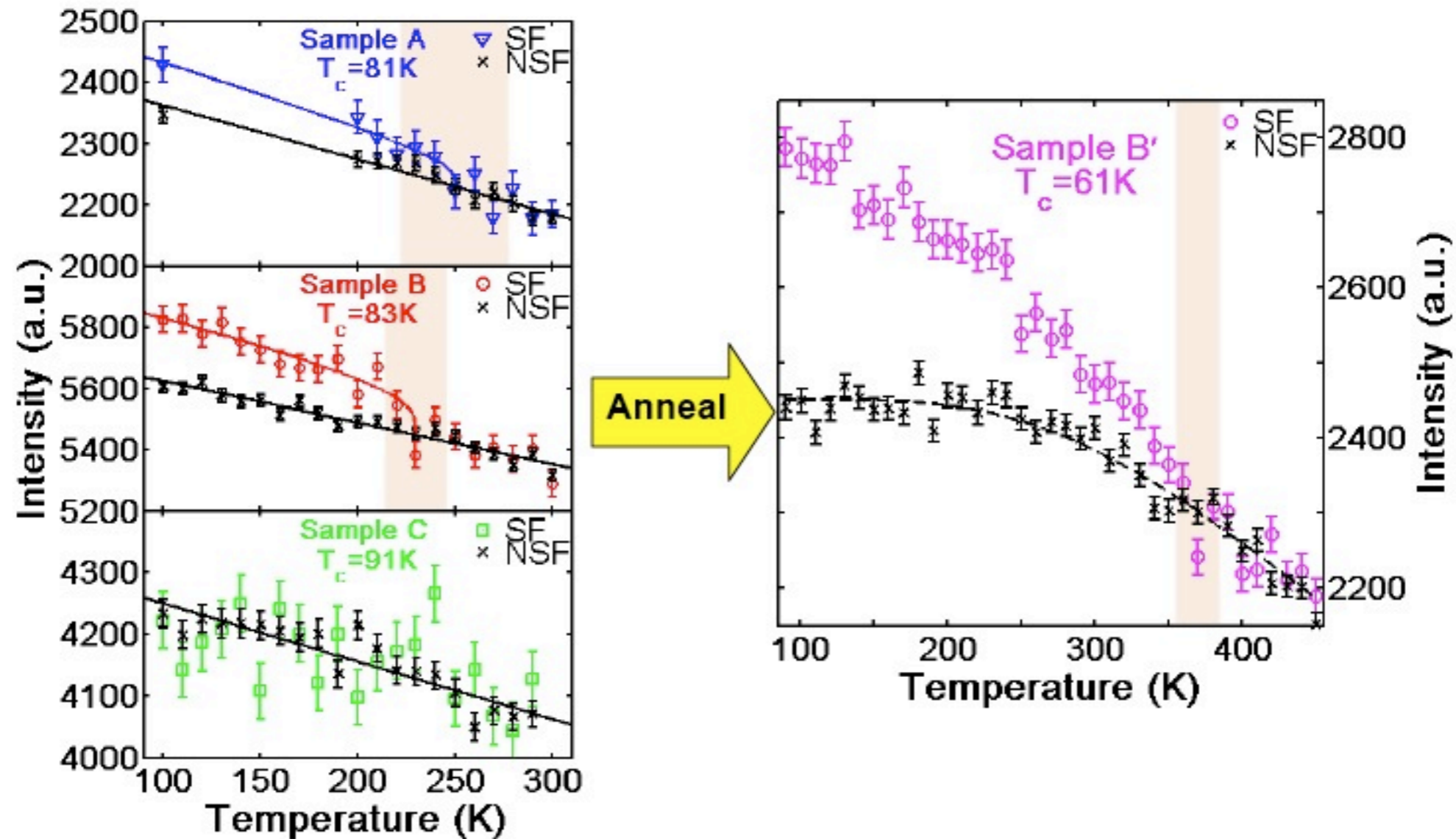
# Polarized Neutron scattering: YBCO



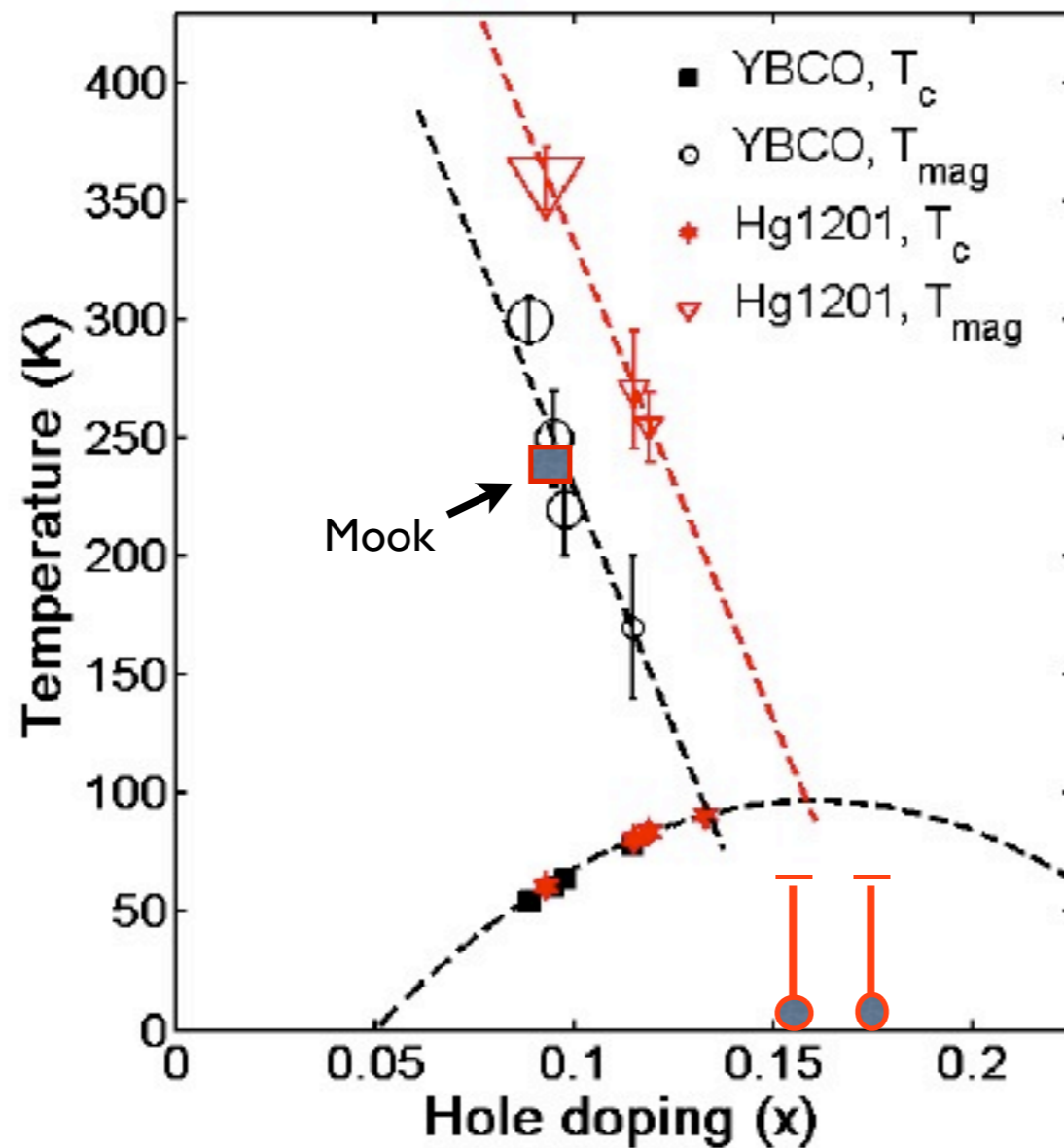
H.A. Mook, Y.Sidis, B. Fauque, V. Baledent and P. Bourges, PRB 78, 020506 (R) (2008 )

Polarized neutron Scattering in single layer Hg-Cuprates:  
Greven et al. (2008).

# Bragg Diffraction: (1 0 1), P//Q



# Phase diagram & Conclusion

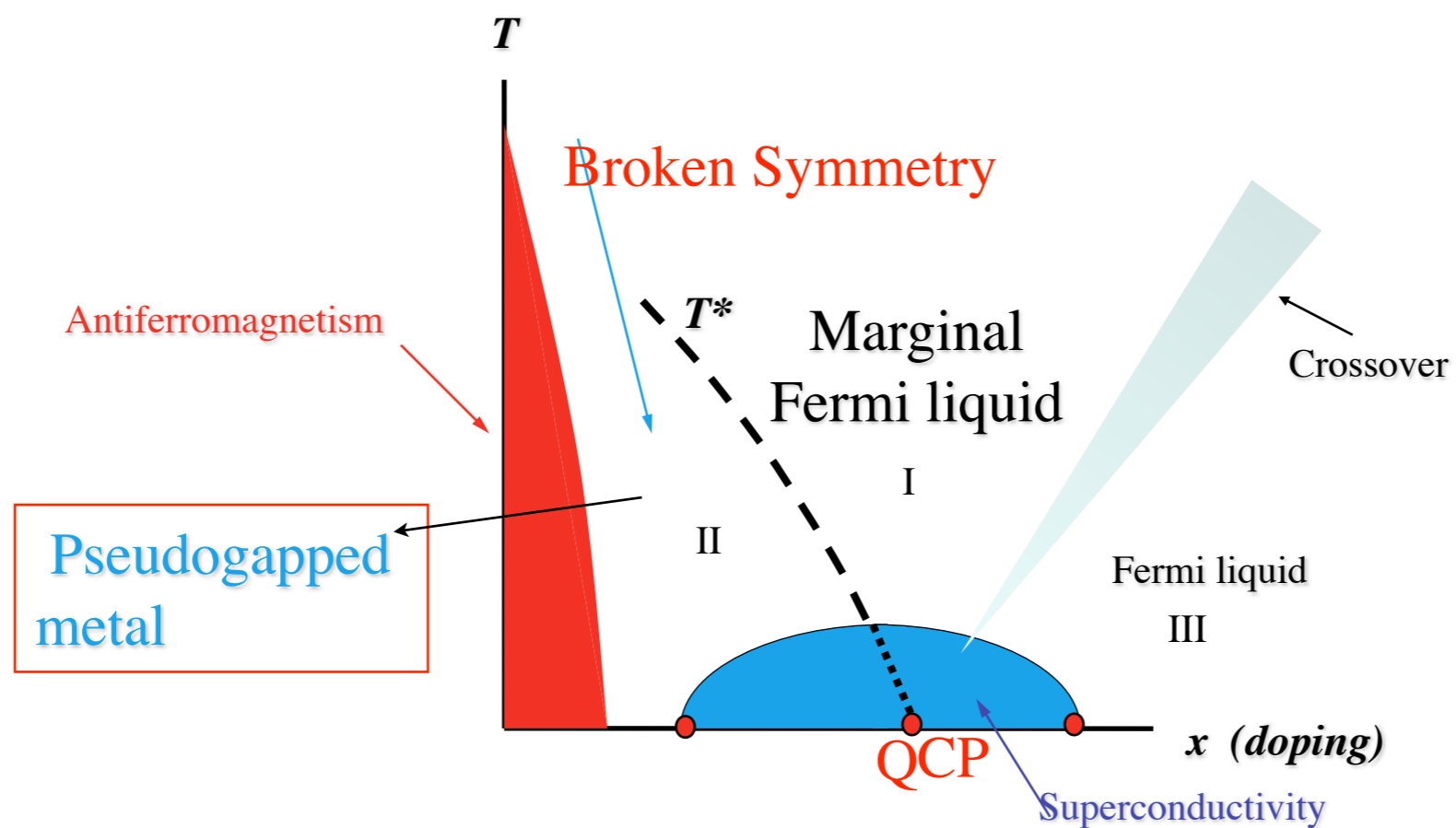


1. Magnetic order identified in underdoped Hg1201 by polarized-neutron diffraction
2. Temperature-dependence of magnetic intensity follows trend of resistivity deviation from linearity
3. Consistent with earlier result for YBCO, but with systematically higher  $T_{mag}$

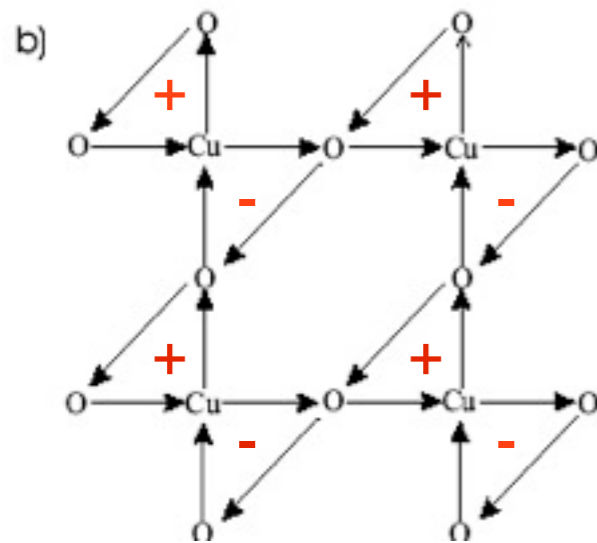
$T_c$ -doping correspondence is determined using the formula  
 $T_c = T_{c,max} * [1 - 82.6 * (x - 0.16)^2]$   
 (J. Tallon, *et al.*)



Next Task: To Derive the proposed Phenomenological spectra in Region I as the quantum Critical fluctuation spectra of the symmetry breaking in Region II.



## An effective Hamiltonian for the Loop-Current Order with four states per unit-cell. (ASHKIN-TELLER MODEL)



$$(\sigma, \tau) : \quad (1,1) \quad (-1,1) \quad (-1,-1) \quad (1,-1)$$

$$H_{eff} = - \sum_{\langle i,j \rangle} J_2(\sigma_i \sigma_j + \tau_i \tau_j) + J_4(\sigma_i \tau_i \sigma_j \tau_j)$$

+ Constrained kinetic energy of fermions

+ Interactions of fermion currents linearly with the  $\sigma_i$  and  $\tau_i$  operators.

For relevant range of parameters, the Ashkin-Teller Model has a smooth specific heat at the Transition even though there is an order parameter singularity (Baxter, Sudbo).

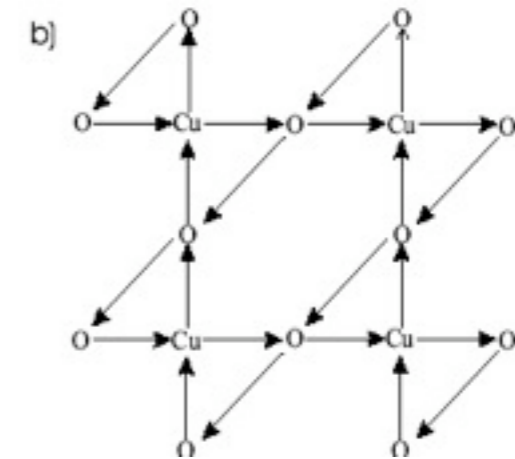
## Quantum Critical Fluctuations

(Vivek Aji, cmv : PRL07, PRB 09)

Class. AT model: 
$$H_{eff} = - \sum_{\langle i,j \rangle} J_2(\sigma_i \sigma_j + \tau_i \tau_j) + J_4(\sigma_i \tau_i \sigma_j \tau_j)$$

is equivalent for critical properties to a generalized xy model or Interacting Rotors Model with anisotropy:

$$H_{eff} = 2J_2 \cos(\theta_i - \theta_j) + J_4 \cos 2(\theta_i - \theta_j) + h_4 \cos 4\theta_i.$$



## Quantum Generalization of the Model: Dissipative xy or Quantum-Rotor Model

$\exp(i\theta_i) \rightarrow \mathcal{L}_i^+$  :  $\mathcal{L}$  is the angular momentum operator for the Rotors.

$$H = \mathcal{L}_i^z{}^2 / 2I + J(\mathcal{L}_i^+ \mathcal{L}_j^- + h.c.) + \text{Dissipative terms } (\alpha).$$

This model has been known to have a Quantum Critical Point at  $\alpha = \alpha_c$ .

(Chakravarty et al., M. Fisher, ... in connection with the superconductor-insulator transition.)

Several very interesting ideas proposed.

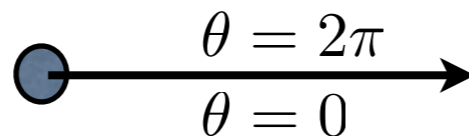
We have succeeded in verifying these as well as calculating the quantum-critical spectra.



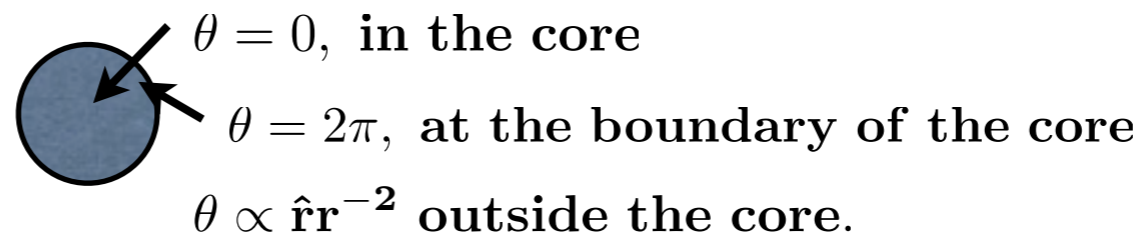
## Dissipative Quantum XY Model

$$S = \int d\tau \sum_{\langle ij \rangle} J \cos(\theta_i - \theta_j) + \int d\tau \sum_i \frac{\dot{\theta}_i^2}{C} + \int d\mathbf{k} d\omega \alpha |\omega| \mathbf{k}^2 |\theta_{\mathbf{k},\omega}|^2$$

Classical XY model: Physics dominated by vortex excitations  $\rho_v$  which count singularities in curl of velocity field  $\mathbf{v} = \text{grad } \theta$  as well as in singularities in  $\theta$ .



Dissipation introduces sources and sinks of the velocity field so that  $\text{div } \mathbf{v}$  field gets into the story. The handling of this requires another topological excitations characterized by discrete charges  $\rho_w$ , which we termed “warps”. They are a particular form of instantons.



# Theory of Criticality (Aji and cmv)

2+1 D XY model with dissipation

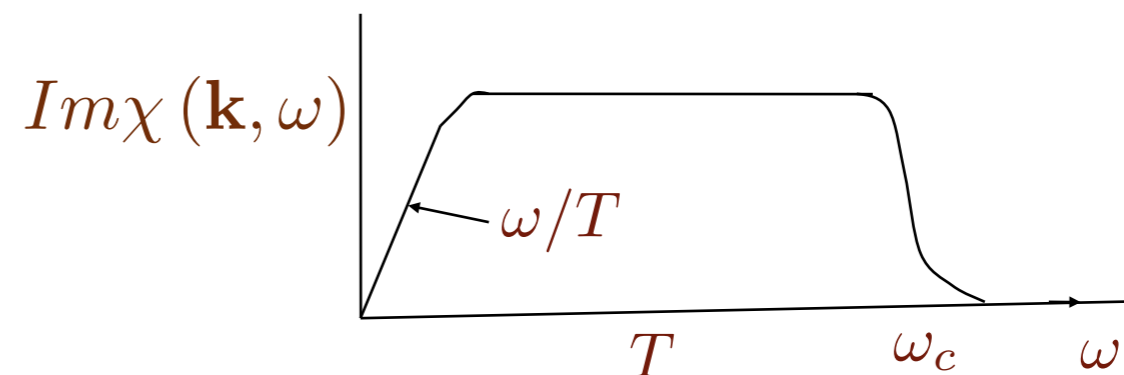
$$S = \int d\tau \sum_{\langle ij \rangle} J \cos(\theta_i - \theta_j) + \int d\tau \sum_i \frac{\dot{\theta}_i^2}{C} + \int d\mathbf{k} d\omega \alpha |\omega| \mathbf{k}^2 |\theta_{\mathbf{k}, \omega}|^2$$

Exact Transformation in QC-Region in terms of two sets of orthogonal variables:

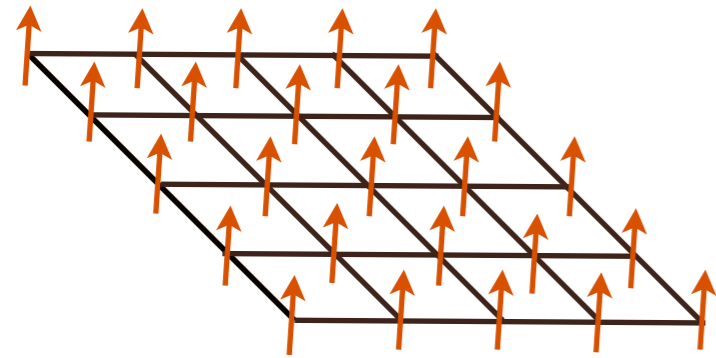
$$S = \int d\tau d\mathbf{r} d\mathbf{r}' \mathbf{J} \rho_{\mathbf{v}}(\mathbf{r}, \tau) \rho_{\mathbf{v}}(\mathbf{r}', \tau) \ln |\mathbf{r} - \mathbf{r}'| + \int d\mathbf{r} d\tau d\tau' \alpha \rho_{\mathbf{w}}(\mathbf{r}, \tau) \rho_{\mathbf{w}}(\mathbf{r}, \tau') \ln |\tau - \tau'|$$

Singularities decouple in space and time:  
Problem easily soluble in terms of these variables.

Fluctuation spectrum

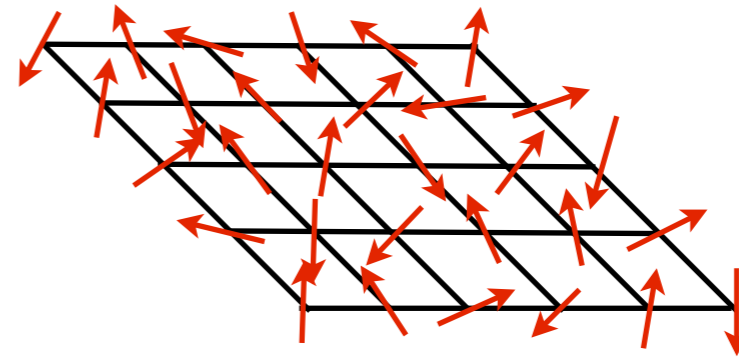


# Nature of fluctuations

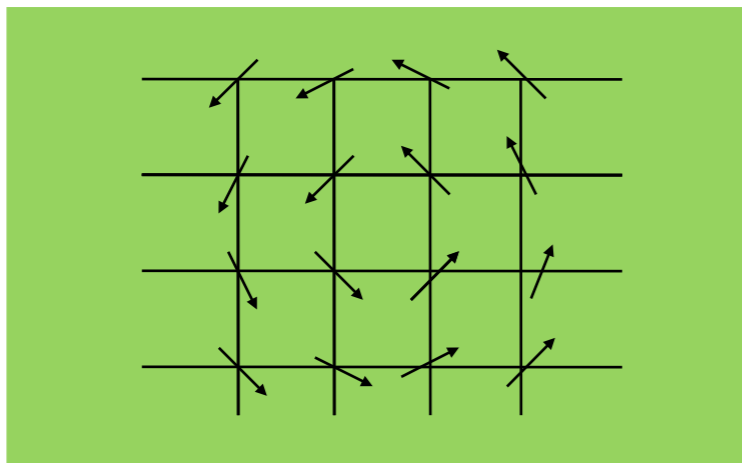
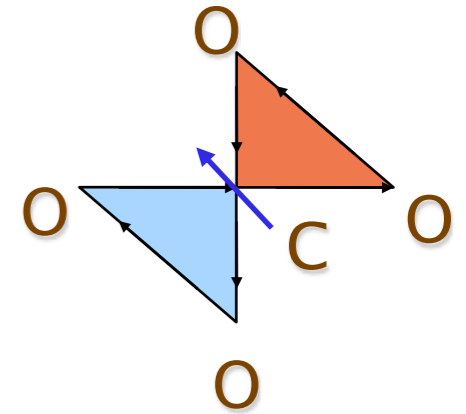


Ordered State

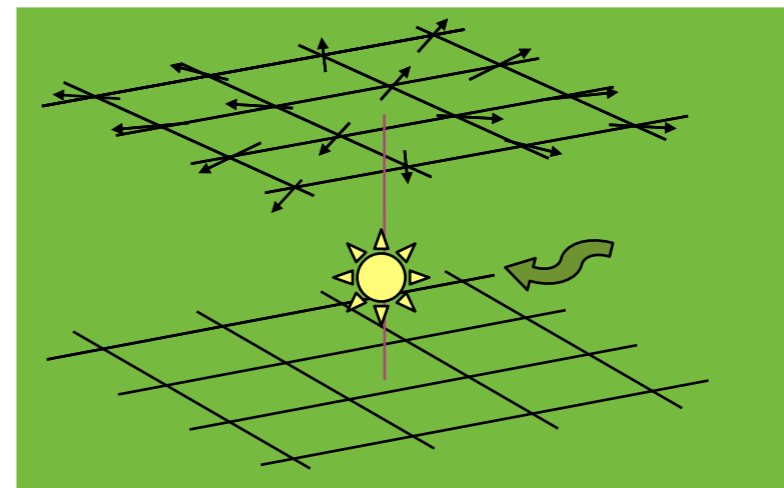
Fluctuations



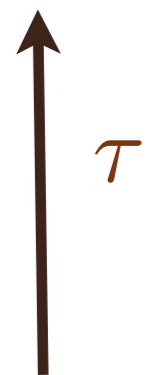
Disordered State



Vortex



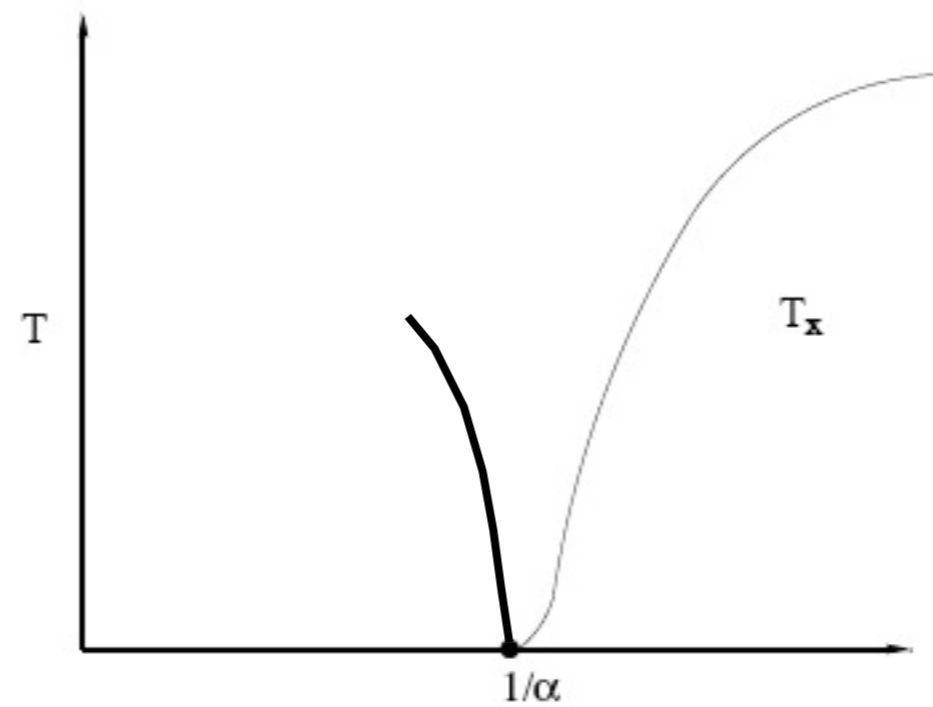
Warps



Vivek Aji and CMV, PRL 99, 067003 (2007)

Phys. Rev. B (appearing in May 09)

## Cross-over at finite T



Possible applications to SIT 's :

Test of Dissipation driven QCP: Look for scale-invariant fluctuations spectrum of the derived  $F(\omega/T)$  form.

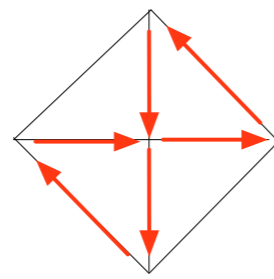
The resistivity and conductivity corrections close to the critical point driven by density of "Warps"  $\propto \ln(T/\omega_c)$ .

## Fluctuations in the Loop-ordered Phase in Cuprates

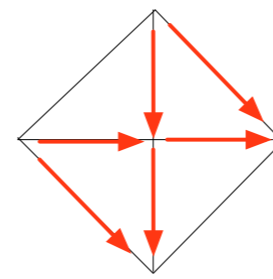
Fluctuations of  $L$  variables are of a discrete model in the ordered Phase and acquire gap.

But we also have the 'open-Loop' variables  $A$ :

### Physical Fields



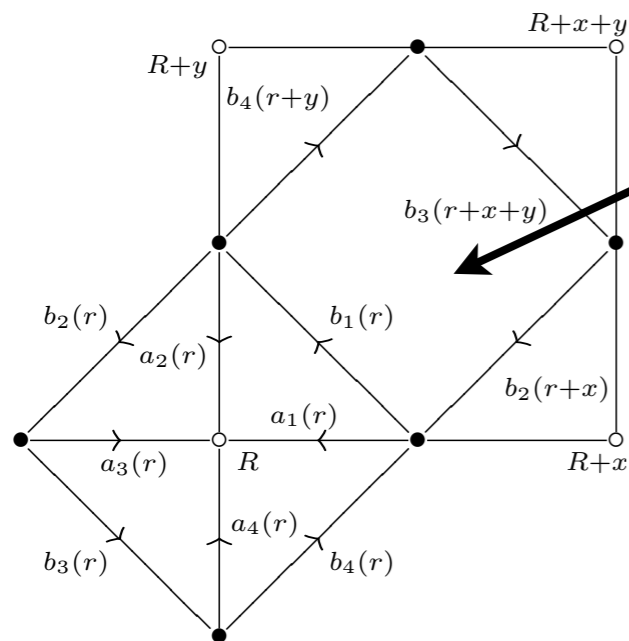
$$\mathbf{L} = (L_x, L_y)$$



$$\mathbf{A} = (A_x, A_y)$$

*Choose  $\text{div} \mathbf{A} = 0$ . So we are left with only  $\text{curl} \mathbf{A}$*

*$\text{curl} \mathbf{A} \propto \text{Flux in Regions not covered by } O - Cu - O \text{ Triangles.}$*



$\mathbf{A}$  is 0 in the ground state. It has all the properties of an emergent vector potential. One can derive the effective quantum Hamiltonian for it. And show that a photon with  $v$  of  $O(v_f)$  and coupling to fermions of  $O(V/t)$  should exist in the loop-ordered state. This has consequences.

## Summary

A Good and well posed problem reappears in various different Physical contexts. Such is the case for the dissipative XY model introduced by Allen Goldman and his students in the context of the SIT.

There appears to be a wide range of interesting physical contexts in which “local’ quantum-criticality is apparent. The only way to achieve this is if the physical model is expressible in terms of operators which are spatially local but with power law correlations in time. Such is the case for the dissipative XY model.

If SIT is due to a dissipation driven Quantum-Criticality, the noise spectrum should have a particular scaling form, which is testable.

With some luck and patient effort,  
and a few more careful experiments,  
the Cuprate problem may in fact be soluble.