Review and Analysis of Rainfall and Runoff Data For Selected Watersheds in Minnesota

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FOREWORD

This Bulletin is published in furtherance of the purposes of the Water Resources Research Act of 1964. The purpose of the Act is to stimulate, sponsor, provide for, and supplement present programs for the conduct of research, investigations, experiments, and the training of scientists in the field of water and resources which affect water. The Act is promoting a more adequate national program of water resources research by furnishing financial assistance to non-federal research.

The Act provides for establishment of Water Resources Research Institutes or Centers at Universities throughout the Nation. On September 1, 1964, a Water Resources Research Center was established in the Graduate School as an interdisciplinary component of the University of Minnesota. The Center has the responsibility for unifying and stimulating University water resources research through the administration of funds covered in the Act and made available by other sources; coordinating University research with water resources programs of local, State and Federal agencies and private organizations throughout the State; and assisting in training additional scientists for work in the field of water resources through research.

This report is the eighth in a series of publications designed to present information bearing on water resources research in Minnesota and the results of some of the research sponsored by the Center. In the present investigation, a thorough study of available rainfall and runoff data for selected watersheds in Minnesota was made to: 1) provide information on loss rates, hydrograph characteristics and related factors; and 2) assemble basic data on precipitation and runoff for use on this research project and other future projects. The results of the study should assist in the development of peak rates of runoff design criteria and flood routing procedures.

Review and Analysis of Rainfall and Runoff Data for Selected Watersheds in Minnesota[a]

by C. E. Bowers and A. F. Pabst

ABSTRACT

The objective of this study was the analysis of available rainfall and runoff data for selected watersheds in the state of Minnesota to assist in the evaluation of peak rates of runoff for design purposes.

Six watersheds were selected for study. Rainfall and runoff data were analyzed for 51 flood events in five of the six watersheds. An optimization program prepared by the U. S. Army Corps of Engineers Hydrologic Engineering Center was used to optimize nine variables associated with the watersheds, develop characteristic unit hydrographs, and evaluate loss rates for the watershed in terms of the mathematical model represented by the optimization program.

Data on annual maximum floods as well as maximum summer floods were plotted on log-probability paper and theoretical flood-frequency curves determined by the log-Pearson Type III distribution.

Approximately 200 figures relating to the rainfall-runoff data and analysis thereof have been included in Appendices for future use. This might involve further work with the same program or possibly other programs. A limited number of copies of the Appendices has been prepared for reference purposes.

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[a] *Limited number of copies of this report were printed as Project Report No. 97, St. Anthony Falls Hydraulic Laboratory, University of Minnesota, June 1968.*

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INTRODUCTION

The design of many structures associated with the control and use of water is based on criteria involving the rate and volume of runoff. The selection of these criteria in turn depends on the size, life, and consequence of failure of the structure. In some instances the design can be based on generalized estimates of runoff as determined from a frequency analysis of available data for the area of interest. This may involve measured runoff data, precipitation data, or both. The design criteria for large structures will frequently use the probable maximum precipitation as determined from hydro-meteorological studies, while smaller structures may involve a frequency analysis of precipitation data. The methods used to arrive at a design runoff value from basic data may also involve empirical equations (for small structures), unit hydrograph theory, flood routing, or some form of mathematical model.

If precipitation data are used to arrive at a design flood, the relationship between precipitation and runoff must be determined. U. S. Weather Bureau Technical Paper No. 40 and other publications present excellent generalized data on precipitation with various durations ranging up to 24 hours and recurrence intervals ranging up to 100 years. These provide an excellent basis for the supply rate to the watershed system; it is then necessary to determine the amount and time distribution of runoff from the given or design rainfall. Such relationships between rainfall and runoff can be based on generalized data for the area, analysis of available data on precipitation, runoff and other factors for selected watersheds, or the determination of representative infiltration indices for the area.

A thorough analysis of rainfall runoff data for a selected area would involve the intensity, duration, total precipitation, direction of travel, time distribution, and areal distribution of precipitation in selected storms, and antecedent moisture conditions in the watershed. For comparison with other watersheds the physical characteristics of the watershed, including such items as the average slope, stream slope, soil types, and cover of the particular watershed are also necessary. Some information is available on infiltration rates for some soils, based on the measured runoff rate from small test plots supplied by a sprinkler system. The difference between rainfall and runoff rates would then be the infiltration rate. While this is of considerable interest, where it is available for the particular soils and cover of the watershed under study, it does not include other losses such as depression storage and interception which might occur for the overall watershed. Thus, the analysis of rainfall and runoff on a basin- or watershed-wide basis is of interest relative to the use of precipitation for design runoff from areas of this type.

WATERSHEDS

A selection of watersheds or basins for this study was based on geographical location and climatological characteristics within the state of Minnesota. The initial list is as follows:

Root River watershed – southeast Minnesota
Le Sueur River watershed – south central Minnesota
Chippewa River watershed – western Minnesota
The Middle River watershed – northwest Minnesota
Baptism River watershed – northeast Minnesota

Subsequently, the Embarrass River watershed in north central Minnesota was added to the study and the Chippewa River watershed was dropped after initial studies of some of the hydrographs associated with this watershed. The Root River watershed has had at various times six gaging stations of which three were subjected to substantial analysis during the present study. The other three were also studied in part, but the data were not adequate for a thorough analysis.

To assist in the analysis of data for the various watersheds a number of basin characteristics were computed. For some watersheds these included length of watershed, length to center of area, drainage density, form factor, compactness coefficient, and average basin slope. A portion of these data are included in Table 1. Two methods were used to determine average basin slope. One of these involved the measurement of the length of all contours of a given contour interval and computation of the mean slope by the following equation:

$$S = (\Sigma L) \times \frac{D}{A}$$

where Σ L is equal to length of contours, D is equal to contour interval, and A is equal to area of basin. A second method involved the use of a grid system laid over the watershed. This involved the determination of average slope at each point on the grid and the computation of the mean value of the slopes. This was considerably less time-consuming than the contour method. The drainage density was determined by measuring the length of all streams within each basin and relating it to the corresponding area, where

$$\text{Drainage density} = \frac{\text{Total length of all streams}}{\text{Basin area}}$$

For the Root River area the drainage density was relatively constant but the average slope varied by a factor of at least two.

Table 2 lists some hydrologic data for the watersheds including items such as mean annual precipitation, mean discharge, and maximum flood of record.

The following information includes a brief description of additional physical and hydrologic characteristics of each of the primary watersheds used in the study.

Table 1
WATERSHED CHARACTERISTICS

Watershed	Area (sq. mi.)	L (mi.)	L_c (mi.)	Elevation (ft.) Gaging Station	Elevation (ft.) Headwater	Average Stream Slope (ft./mi.)	Average Stream Slope %	Average Basin Slope %	Drainage Density (mi./mi?)	Form Factor
Root River near Houston	1,270	61	29	672	1425	5.5	.00104	5.26	0.472	0.348
Root River near Lanesboro	615	42	22	792	1425	6.4	.00125	3.92	0.505	0.346
South Fork of Root River	275	25	13	680	1025	14.2	.00269	8.45	0.486	0.382
Root River below South Fork	1,560	64	27	660	1425	5.5	.00104	5.85		0.381
Rush Creek	129	14	7	735	1025	17.5	.00332	6.44	0.537	0.684
Le Sueur River	1,100	39	18	776	1170	7.4	.00140	0.65		72
Middle River	265	48	26	829	1150	3.5	.00066	0.22		115
Embarrass River	93.8	12	4	1415	1700	2.8	.00053	1.21		.65
Baptism River	140	20	9.5	610	2000	42.3	.00801	4.04		.43

L = Overall length of watershed
L_c = Length to center of area
Drainage density = Summation of channel length divided by area of watershed
Form factor = Area divided by square of the stream length

Table 2
HYDROLOGIC CHARACTERISTICS OF WATERSHEDS

Watershed	Area sq. mi.	Avg. Annual Precip. in.	Avg. Annual Runoff in.	Mean Discharge cfs	Peak Flood of Record cfs	Q_p/A csm	$Q_p/\bar{Q}$
Root River near Houston	1270	31.8	6.86	644	37,000	29.2	57.5
Root River near Lanesboro	615	31.4	6.80	319	22,100	70.5	69.4
Root River below South Fork	1560	31.8	7.35	845	10,500	-	–
South Fork of Root River	275	32.0	6.01	122	8,420	30.6	68.8
Rush Creek	127	31.8	5.70	54.2	11,600	90.0	214.0
North Branch Root River – Stewartville	0.73	31.8	–	–	328	450.0	–
Le Sueur River, Rapidan	1100	28.5	4.58	372	24,700	22.5	66.3
Middle River near Argyle	265	20.0	2.04	39.8	2,590	9.8	65.0
Embarrass River near Embarrass	93.8	25.4	10.8	74.8	1,740	18.6	23.2
Baptism River	140	26.0	15.4	159	9,350	66.7	58.7

Root River Watershed

Photographs 1 through 4 are aerial views of portions of the Root River watershed. Figure 1 is a map of the area showing the primary streams and the geographic location within the state of Minnesota, and Figure 2 shows the stream profiles. The watershed has a length of about 61 miles above the downstream gaging station used in this study. As noted in the hydrologic atlas of Minnesota [1]:

> The headwaters of the Root River are in an area of large spring-fed sloughs. Streams in the headwater region of the west flow in wide, shallow valleys cutting the thin mantles of clayey glacial drift which overlie the bedrock. Several miles west of Fillmore the valleys are incised into the bedrock to depths of 100 to 300 ft. below the upland surface. East of Fillmore the valleys are deep, sinuous gorges that gradually deepen and widen toward the east. The gorges at Lanesboro, Rushford, and Hokah are 400 to 550 ft. deep and a quarter of a mile to one mile wide. Tributary streams flow in steep-walled coulees cut into the main stream valleys.
>
> Bedrock exposed in the valley walls is composed of sedimentary beds of limestone, dolomite, sandstone, and shale. Beneath the uplands groundwater has dissolved some of the limestone and created cavities in the formation. Where the cavities have extended to the surface of the limestone the overlying drift has collapsed and sink hole topography has developed.
>
> The western part of the Root River basin is drained by the north, middle, and south branches of the Root River and many small tributary streams. The approximate fall of all branches from their sources to Lanesboro is about 550 ft. The eastern part of the Root River basin below Lanesboro is drained chiefly by the main stem and the South Fork of the Root River and by Rush and Money Creeks. The fall of the main stem from Lanesboro to the Mississippi River, a distance of about 40 miles, is about 150 ft.
>
> The well-defined drainage pattern throughout the watershed has eliminated most of the undrained depressions and consequently runoff is rapid on all of the watershed streams. In the eastern part of the watershed the deeply incised streams and steep valley slopes accelerate runoff and cause high, short duration peak flows.
>
> The Root River valley has a long history of floods which have occurred nearly every year at some point in the watershed unit. Spring floods, generally caused by snow melt and augmented by small amounts of rainfall, occur during March and April. Summer floods, following periods of heavy rainfall, occur frequently.
>
> During recent years major floods have occured in 1933, 1934, 1938, 1942, 1945, 1950, 1952, 1960, and 1965.

As the watershed is essentially devoid of lakes, low flows of streams are regulated and sustained at relatively high rates by the slow release of ground water from storage in the sandstone, limestone, and dolomite strata. As noted in reference [2], the lower portion of the watershed is a sloping to steep area covered by Fayette silt loam and Dubuque silt loam. Both are light-colored soils

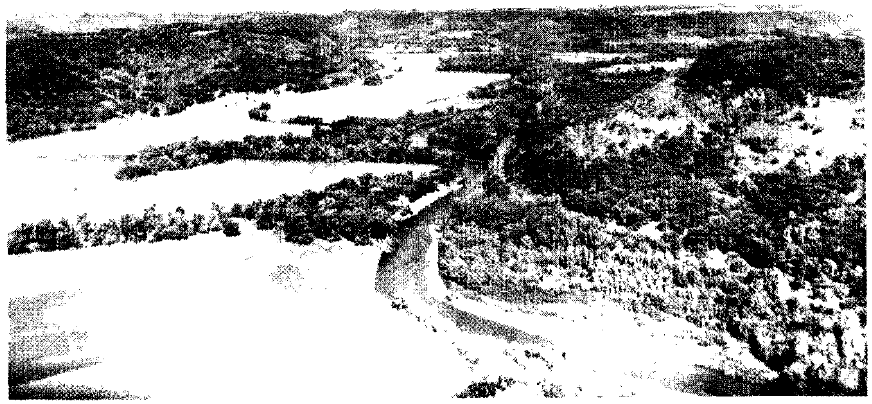


PHOTO 1 (Serial No. 157-1-15) View of Root River upstream of Houston, showing the river gorge 400 to 500 feet deep. The bottom of this gorge is flat and has a width ranging from one-fourth to three-fourths mile.

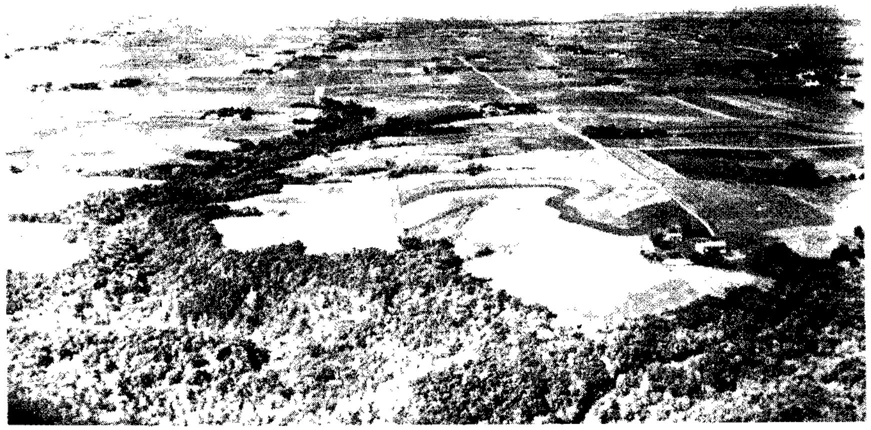


PHOTO 2 (Serial No. 157-2-1) View of typical upland area and the upstream end of a valley or gorge cut through the bedrock. The bedrock generally consists of sedimentary beds of limestone, dolomite, sandstone and shale, overlain with a thin layer of clayey glacial drift.

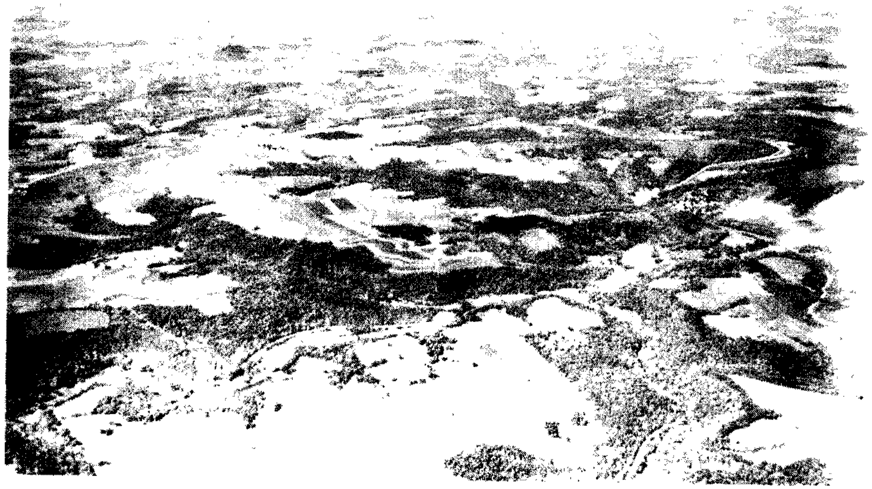


PHOTO 3 (Serial No. 157-2-9) View of the Root River Watershed near its center, about 25 miles upstream of the gaging station at Houston. Lanesboro is near the upper left center of the photograph. The gorge of the main stream at right center of the photograph is about 400 feet deep.

PHOTO 4 (Serial No. 157-15) View of the upland portion of the Root River Watershed near Stewartville. The terrain is quite flat.

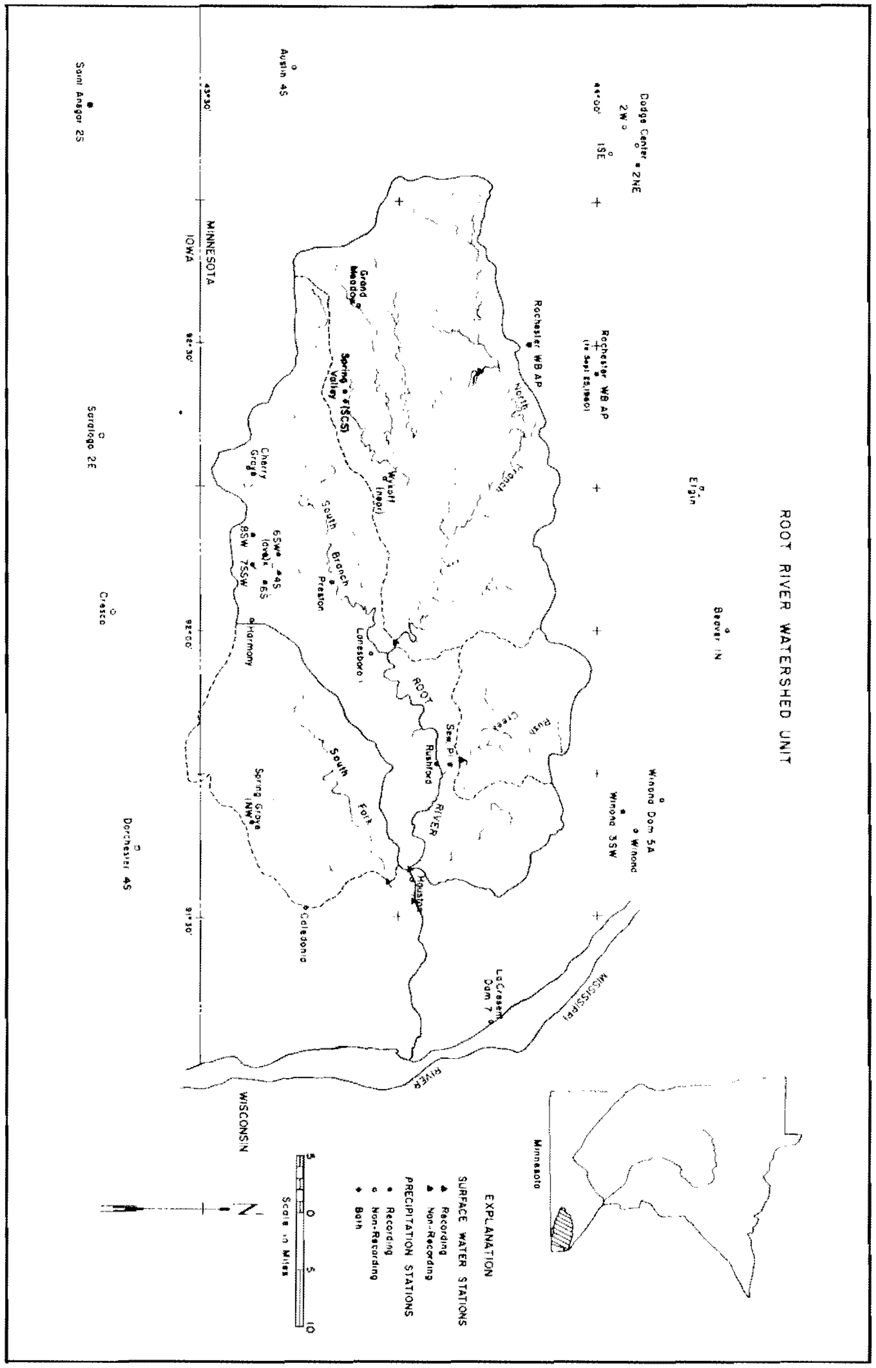


Figure 1. Map of the Root River watershed

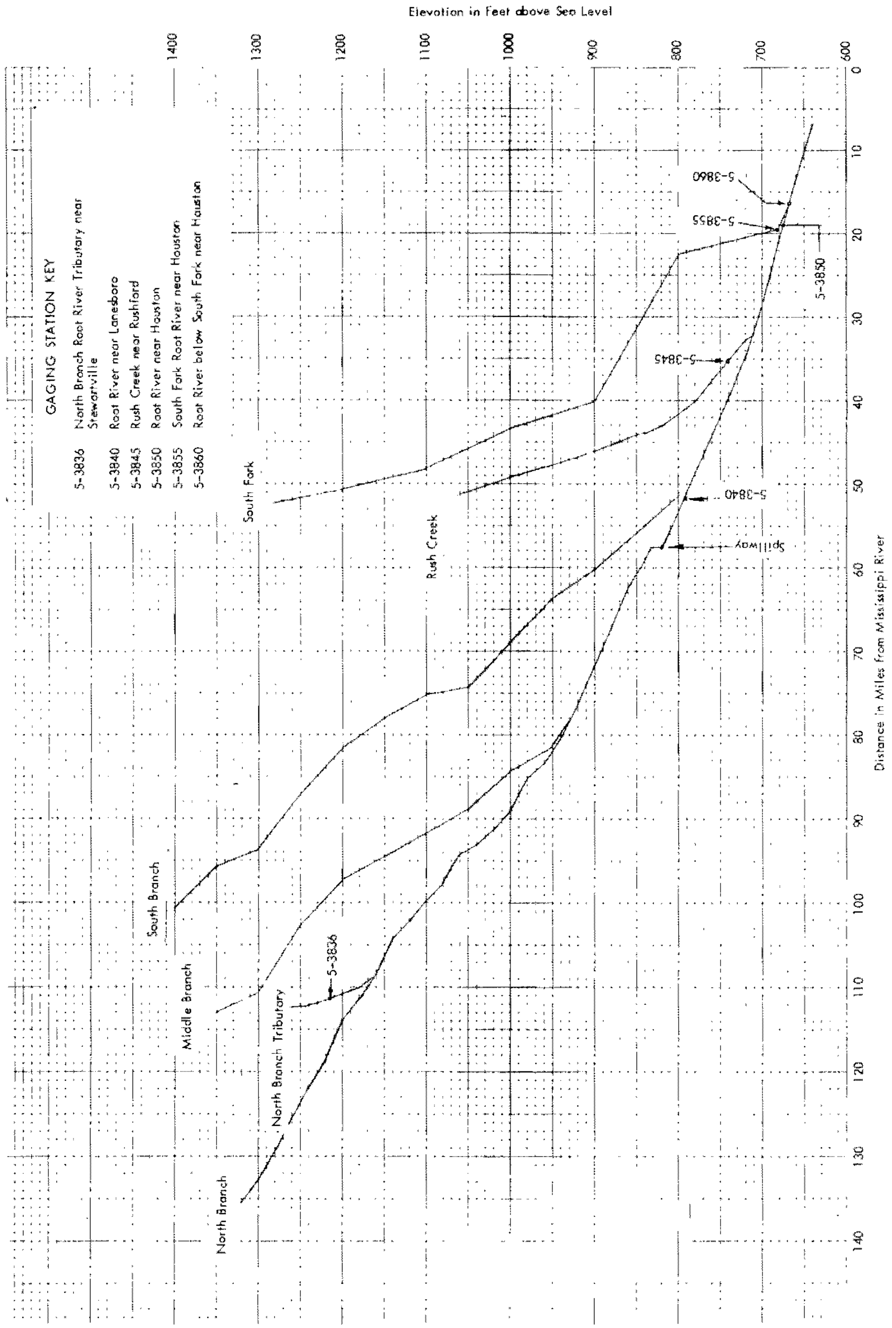


Figure 2. Stream slopes for the Root River watershed

having good internal drainage and rapid surface drainage dependent on the slope. Erosion control is a major problem. Rock outcrops are common on many of the slopes. The upper part of the watershed is covered by Ostrander-Kenyon-Floyd soils over a nearly level to sloping area. These dark-colored soils are well to poorly drained and formed in a thin loess mantle overlying firm medium-textured glacial till. These can generally be described as silty forest and prairie soils of southeastern Minnesota and fall within the Soil Conservation Service soil group "B".

Le Sueur Watershed

The Le Sueur River above the gaging station at Rapidan is roughly circular in shape and approximately 40 miles in diameter with an area of 1100 sq. miles, as shown in Figure 3. It is drained by the Le Sueur River and two main tributaries, the Cobb River and the Maple River. The average slope of the area is 0.65 per cent as compared to 5.26 per cent for the Root River above Houston. Figure 4 illustrates the stream profiles. The average fall of the main stem of the Le Sueur River is about 7.4 feet per mile.

The Le Sueur watershed is a gently undulating glacial till plain with surface deposits of glacial drift ranging up to several hundred feet thick. The major streams have eroded channels 40 to 75 ft. deep in the headwater regions and on the order of 150 ft. deep near the Minnesota River valley. Photos 5 and 6 illustrate the general character of the Le Sueur topography.

High flow in the streams usually occurs during the spring as a result of snow melt and spring rainfall. There are numerous lakes in the watershed, one of 2900 acres, one of 2200 acres, and one of 1200 acres. Soils are medium- to fine-textured prairie border soils (Hayden, Kilkenny, Lester, Le Sueur, Glencoe), loams generally SCS type "B".

Middle River Watershed

The Middle River watershed is within the basin of glacial Lake Agassiz in the northwestern corner of Minnesota. It has a length of about 48 miles above the gaging station at Argyle, as shown in Figure 6. The main stream has a total length of about 67 miles above its junction with the Snake River, which in turn is a short distance above the junction of the Snake with the Red River of the north. The nature of the glacial lake soils makes them particularly suitable for agriculture if they have proper drainage. Consequently, drainage enterprises are extensive and agriculture, with resultant markets and distributing centers, is a dominant industry. The glacial deposits of the western 2/3 of the area are underlain by Cretaceous sediments consisting of soft bluish-gray shale and basil beds of fine light sand and thin streams of lignite. Reference [2] indicates that the soils in this area are course- to fine-textured prairie and organic soils of glacial lake plains, primarily of the Fargo and Grimstead soils. These are in the SCS soil groups "C" and "D". Wind erosion on the better drained areas and drainage on the poorly drained areas are major problems. As may be noted in Figure 6, numerous drainage ditches have been constructed throughout the watershed. According to the 1950 U. S. Census of Agriculture more than half of the total watershed area is in drainage enterprises.

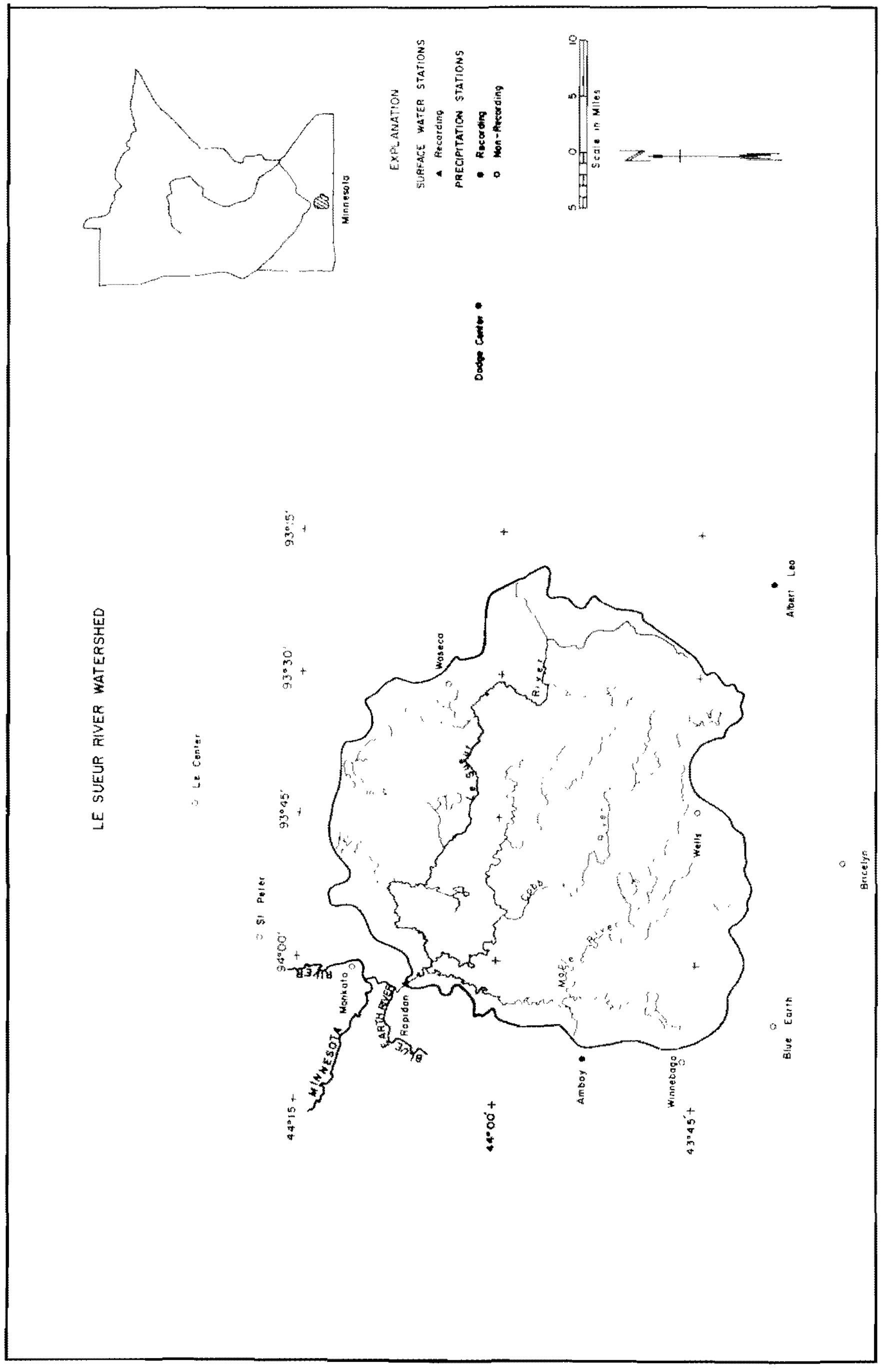


Figure 3. Map of the Le Sueur watershed

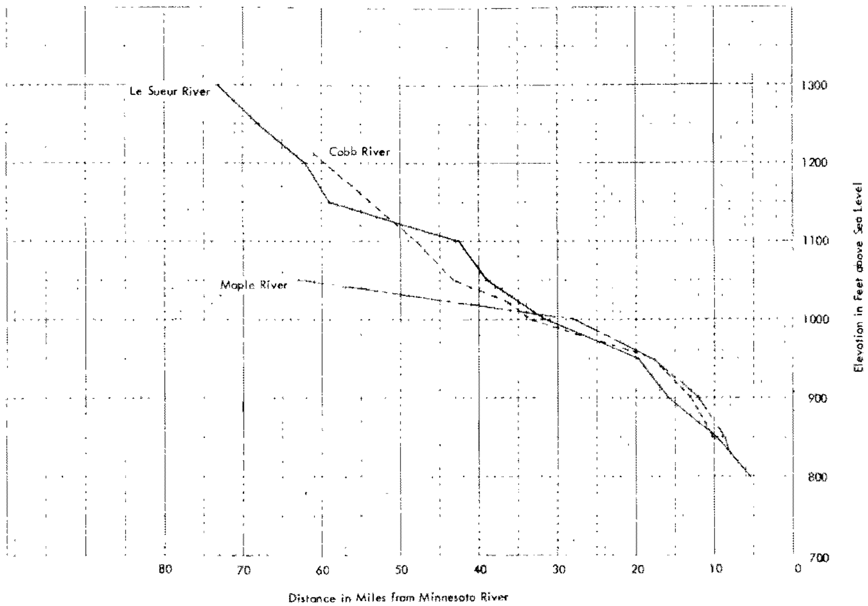


Figure 4. Stream slopes for the Le Sueur watershed

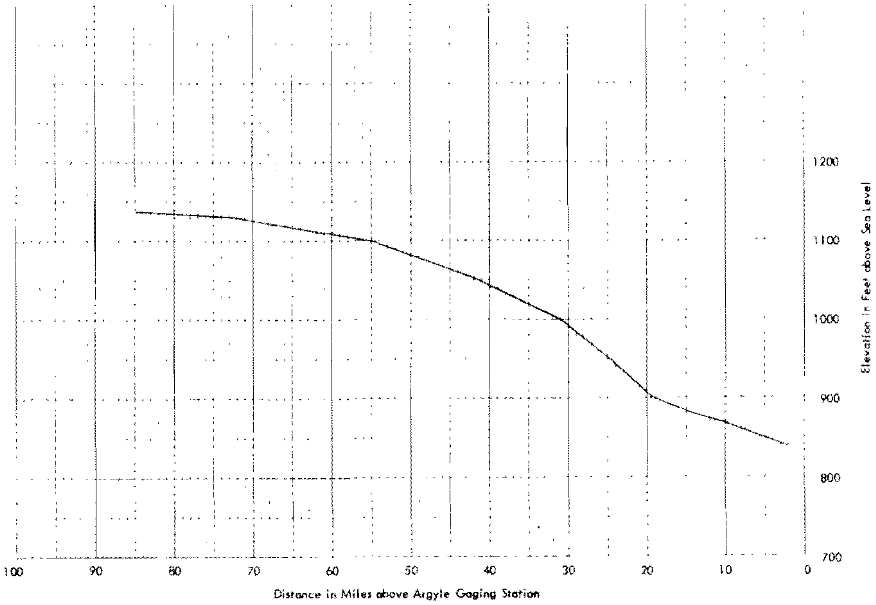


Figure 5. Stream slopes for the Middle River watershed

PHOTO 5 (Serial No. 157-13) Le Sueur River near New Richland in upper one-third of watershed. Main watercourse is outlined by a row of trees across upper section of photo. Flow is from upper right to left.

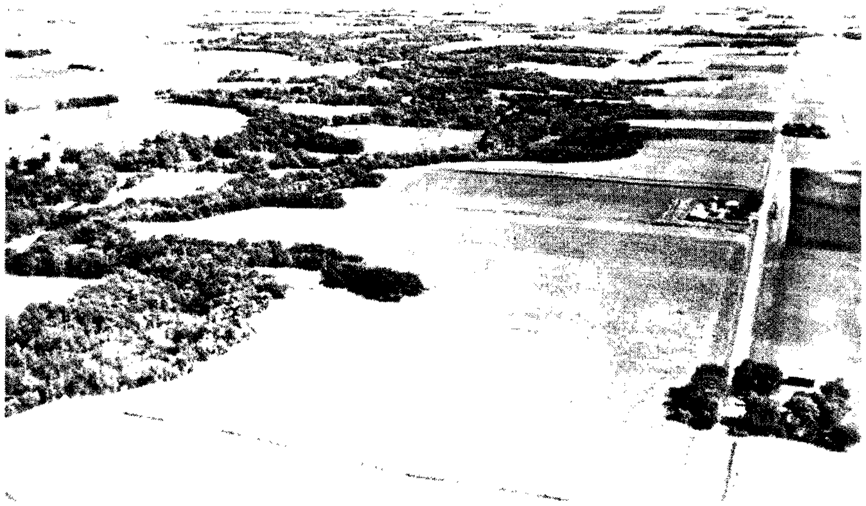


PHOTO 6 (Serial No. 157-4-2) View of Le Sueur River near junction with Blue Earth River. The main stream is outlined by trees. Terrain is gently rolling, to flat.

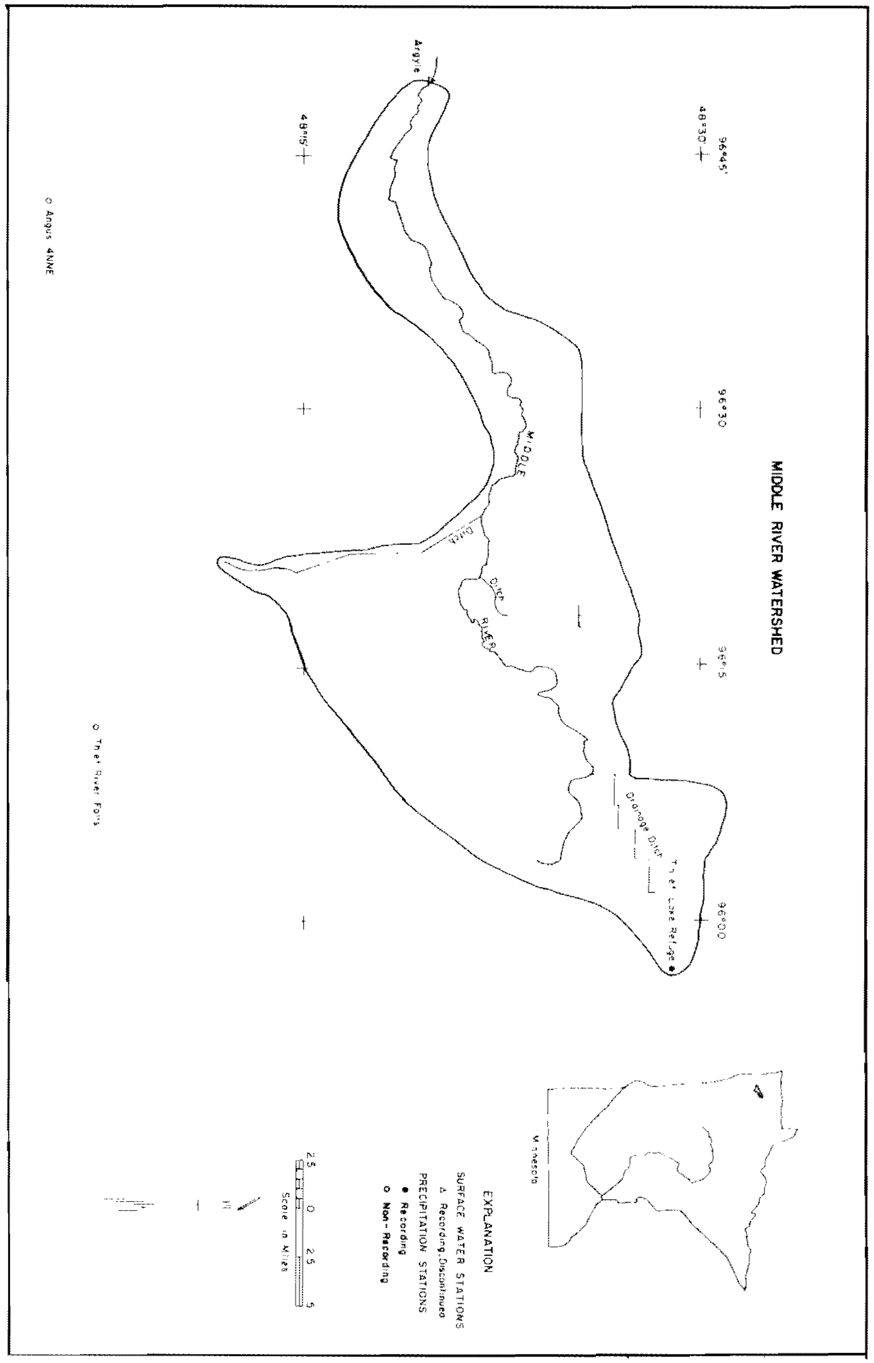


Figure 6. Map of the Middle River watershed

Peak flows in the watershed occur as a result of snow melt in the spring and thunder showers in the summer. The glacial till and lake clays which cover the watershed have low permeabilities and therefore tend to increase direct runoff. Runoff is further increased during the spring when the glacial deposits are frozen, making them even more impermeable. Floods generally occur in March, April, and May. Damage to agricultural lands is ordinarily slight because of the frozen condition of the ground. The maximum discharge of record of the Middle River at Argyle occurred on April 12, 1965, with a discharge of 2590 cfs. The average basin slope of the Middle River is 0.22 per cent, about 1/3 of that for the Le Sueur River and about 1/24 that of the Root River above Houston.

Embarrass River above Embarrass, Minnesota

The Embarrass River above Embarrass, Minnesota drains an area of 93.8 sq. miles as part of the St. Louis River watershed unit. It is 14 miles long and about 12 miles wide. Maximum fall in the watershed is from an elevation of about 1750 ft. above sea level near the southeastern edge of the watershed to the elevation of 1410 at the gaging station, a total drop of about 340 ft. Figure 7 illustrates the watershed shape and Figure 8 the stream profiles. The main stream has a fall of only about 1.2 ft. per mile over most of its length and flows through a rather marshy area over a considerable portion of the watershed. This is the bed of glacial Lake Norwood. The headwater area and the tributaries have steeper slopes. There are numerous lakes in the northern part of the watershed, a region of relatively low relief as compared to the ridges and knobs along the southeastern boundary. Approximately three per cent of the total area is in lakes. The average slope of the watershed is 1.21 per cent or about six times that of the Middle River and about one-fourth that of the Root River above Houston. The bedrock, which consists mainly of granite, gabbro, basalt, and slate, is exposed at the surface in parts of the basin, and in some areas formations of the Messabi Iron Range exist.

Reference [2] describes the soils as Ahmeek rock outcrops. The Ahmeek soils are dark-colored soils formed from a reddish-brown sandy, stony glacial till. Rock outcrops of basic igneous rocks are common. There is no agriculture to speak of in this area, most of which is in the Superior National Forest. The area supports a good growth of aspen and white spruce.

Baptism River

The Baptism River (Figure 10) drains an area of 140 sq. miles along the shore of Lake Superior near Beaver Bay, Minnesota. The watershed has a length of about 20 miles and a width of 17. Altitudes in the watershed range from elevation 2000 in the northern corner to elevation 610 at the gaging station, for a total fall of about 1390 ft. The average stream slope of the main stream is about 42 ft. per mile, as shown in Figure 9, and the average slope of the watershed is about four per cent. Basalt, gabbro, and diabase are the principal types of bedrock in the area. During the ice age glaciers completely covered the area several times. The upland areas generally are covered by glacial drift, whereas the segment closest to Lake Superior has some areas of exposed bedrock.

The watershed is entirely contained within the Finland State Forest and the Superior National Forest and is heavily timbered. The upper half of the basin or upland area generally has a much milder slope than the lower half.

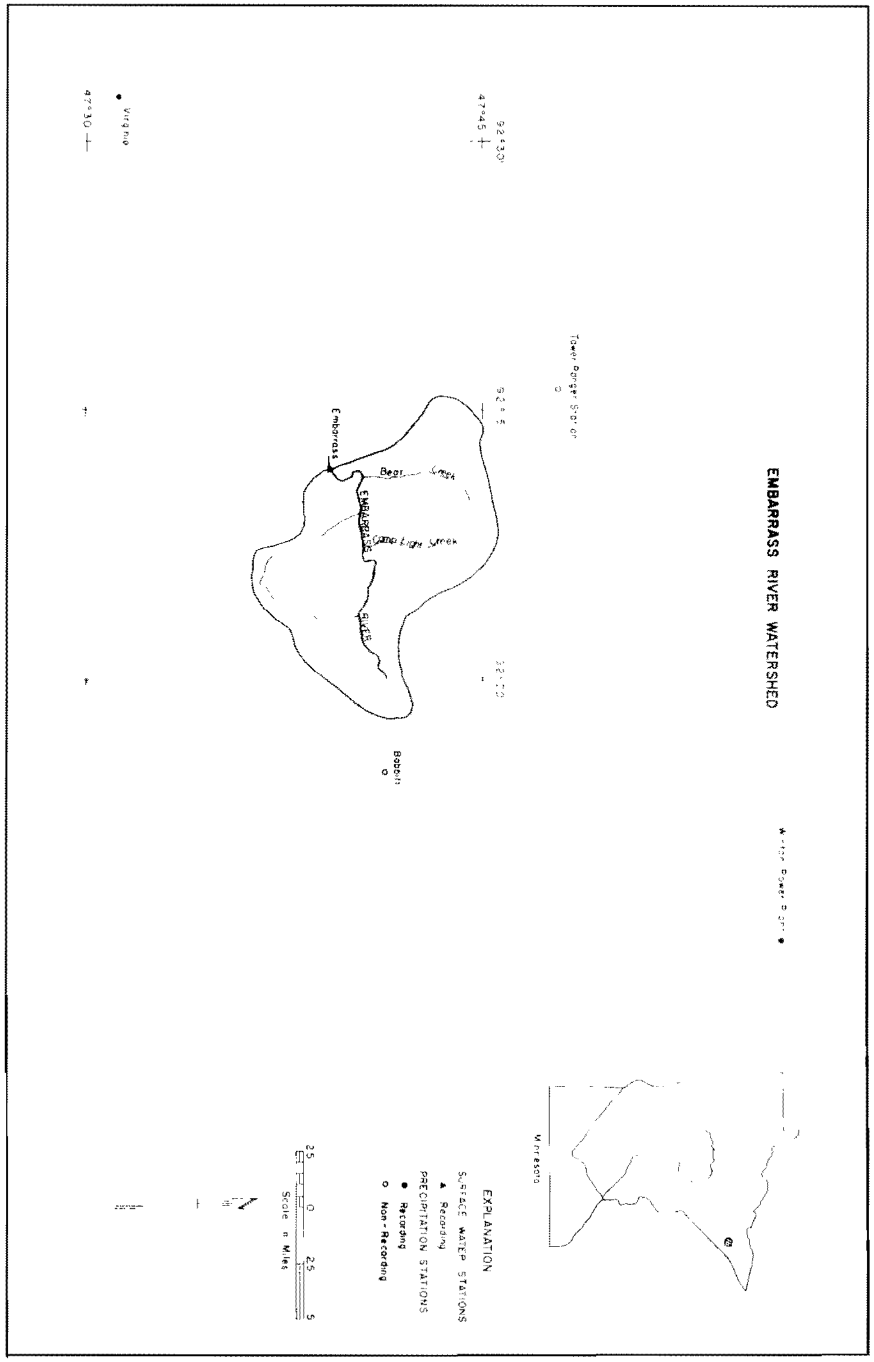


Figure 7. Map of the Embarrass River watershed

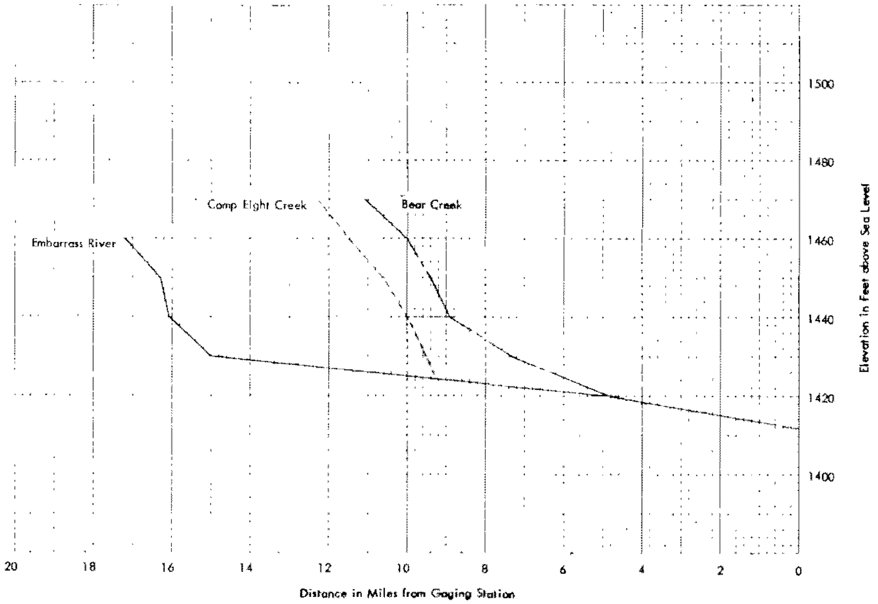


Figure 8. Stream slopes for the Embarrass River watershed

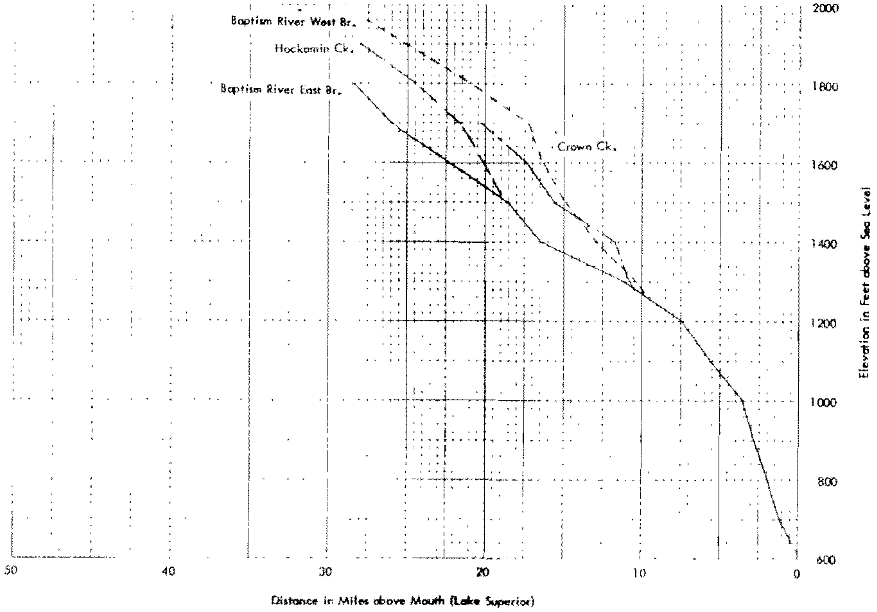


Figure 9. Stream slopes for the Baptism River watershed

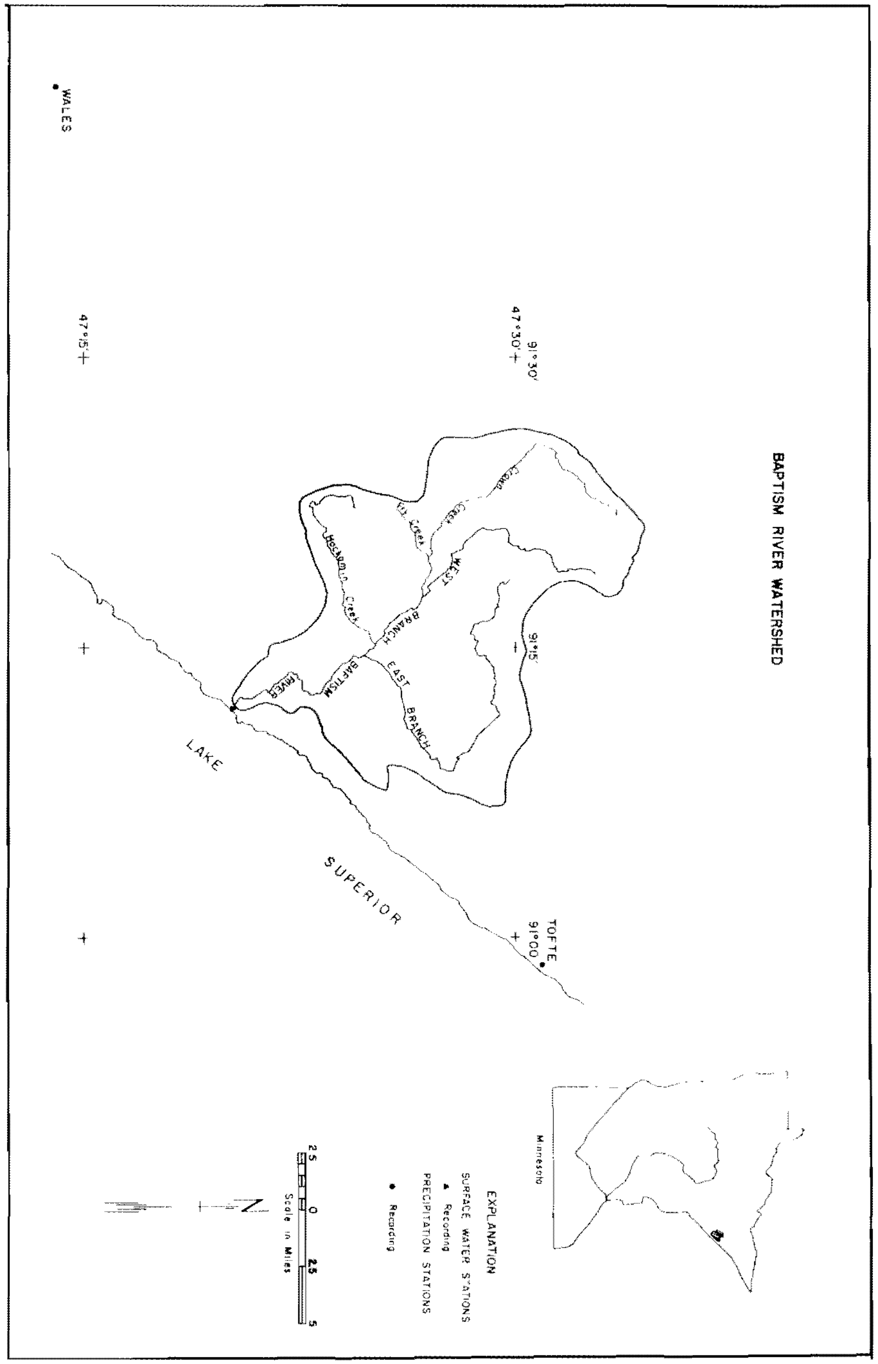


Figure 10. Map of the Baptism River watershed

PHOTO 7 (Serial No. 157-4) View of Baptism River at crossing of Highway US No. 1. Scene is about 3 miles upstream of gaging station. Several falls occur downstream of the site; the average stream slope is 42 feet per mile.

PHOTO 8 (Serial No. 157-2) View of Baptism River Watershed about 1.5 miles above gaging station. The terrain is generally quite steep. Most of the area is covered with a mixture of broadleaf and evergreen trees.

ANALYTICAL PROCEDURES

Basic Data

While the past record of floods in the watersheds was of interest, the primary interest centered on severe floods generated by rainstorms. The first step, then, involved preparation of graphs of the past flow records at each gaging station of interest in the watersheds. These are shown in the Appendices. Figure 11 is a typical illustration of a short portion of one of these records. From these records significant floods were selected outside the region of the normal spring snow melt flood. In most instances, then, the floods of interest were between the first of May and the end of October. While it would have been desirable to analyze a great many floods, limitations on time and funds did not permit this, and an attempt was made to select major floods caused by a fairly well-defined burst of rain rather than by a rainstorm lasting several days. This ultimately involved 11 floods for the Root River area, 8 floods for the Le Sueur River, 4 floods for the Middle River, 5 floods for the Embarrass River, and 6 floods for the Baptism River. Considering the multiple gaging stations in the Root River area 28 station-events were involved in that watershed. Relative to the analysis of precipitation, the daily precipitation records at a selected station were used to compute the antecedent precipitation index. A recession coefficient of 0.9 was used to compute the API. Hourly precipitation data from recording rain gages in or adjacent to the basin were used together with pro-rated precipitation data from non-recording gages in the area to compute average hourly precipitation for the watershed. The Thiessen polygon method was used to compute average precipitation over the watershed. The Root River watershed had the best coverage relative to precipitation gages of any of the watersheds studied. The number of precipitation stations usually used for analyses of precipitation was as follows:

Watershed	Recording Stations	Non-Recording Stations	Total
Root River	6	3	9
Le Sueur	3	6	9
Middle	2	2	4
Embarrass	2	2	4
Baptism	2	0	2

The discharge data for selected floods at the various gaging stations were obtained directly from gaging station charts and rating curves in the St. Paul District Office, U. S. Geological Survey. This was necessary in order to obtain instantaneous discharge values rather than the mean daily discharges available in the published water resources data. The compilation of instantaneous discharge data and the hourly precipitation data is very time-consuming; the data are available in the files associated with this project. Average hourly precipitation data over the watershed for each storm and the instantaneous discharge data are also available in computer printouts and graphs.

Figure 11. Typical graph showing average daily discharge data (Root River near Houston)

Frequency Analysis

To assist in analysis of hydrologic characteristics of the watersheds the annual floods at each gaging station were tabulated and empirical plotting positions determined by the following equations:

$$Tr = \frac{N + 1}{M} \text{ and } P_{o.c.} = \frac{1}{Tr}$$

where

Tr = recurrence interval in years,
$P_{o.c.}$ = probability of occurrence of a flood in one year,
N = number of years of record, and
M = order of a given flood event

In view of the fact that a major portion of the study was concerned only with summer floods or those in which snow melt was not an important factor, it was thought that a frequency analysis should also be made of the summer floods. Accordingly, the annual summer floods were determined from the gaging station records. Many of the values in this group were not necessarily the maximum instantaneous discharge, but rather the average discharge for the day in which the flood peaked. In some instances the major storm for that year may have been one in which instantaneous peak was available from the water resources data. A graphical relation between instantaneous peak and mean daily flow was used to estimate an instantaneous peak for the summer floods when the peak was not available.

In the initial stages of this study both the annual floods and the summer floods were plotted on log-probability paper and on Gumbel probability paper together with theoretical probability distributions based on the Gumbel theory of extreme variables, the log-normal probability theory, and the log-Pearson Type III, as used by the Corps of Engineers and described in Reference [3]. Subsequently the Water Resources Council published a report in December, 1967 [4] recommending the adoption of the log-Pearson Type III as a uniform technique for analysis of flood frequency data. This method involves the computation of the logarithms of the annual floods to the base ten and then the mean, the standard deviation, and the coefficient of skewness of the logs of the annual floods. The method is described in Reference [4].

Runoff Analysis

Initially the rainfall and runoff data were analyzed by conventional methods as described in References [5] and [6]. This included the separation of direct runoff from base flow in the hydrographs and determination of unit hydrographs for each of the initial floods analyzed. A portion of the records of the Root River area was analyzed by these means. However, soon after the start of the studies a series of computer programs was received from the Corps of Engineers Hydrologic Engineering Center, Sacramento, California. Among these was one entitled "Unit Graph and Loss Rate Optimization," Hydrologic Engineering Center, Computer Program 23-J2-L211, prepared by Leo R. Beard [7]. The following notes are from that report:

1. This program written in Fortran II will determine the best unit hydrograph and loss coefficients within the limits of the mathematical model to reproduce a number of flood hydrographs at a given location from

specified rainfall amounts. Best reproduction is defined as that which gives the least squares of differences between computed and observed flows, with greater weight given areas associated with higher flows.

2. The unit hydrograph is computed from the Clark coefficients, time of concentration, and routing coefficient and given time-area tabulation. An artificial time-area tabulation in the program can be used if desired. After the best TC and R are determined, Snyder's T_p and C_p for the unit graph are computed.

3. Losses are a complex function of rain intensity and accumulated loss (as an index of ground wetness). Five loss-rate variables represent average loss, initial loss, rate of decrease of loss with wetness, relation of loss to rain intensity, and rate of recovery of loss rate between storm periods. Any of these can be specified and held constant in order to simplify the analysis, but probably at the expense of adequate results. In addition, one variable (No. 4) represents the ratio of imperviousness, the proportion of basin rainfall that is considered to run off without any losses.

4. While it is advantageous to suggest approximate values for variables to begin each analysis, the program will initiate any or all of them, if not supplied. It is best to supply at least an initial estimate of time of concentration.

5. All variables to be changed for optimization purposes are designated VAR with a subscript. The first two are Clark's TC and R/TC. The given time-area curve expressed in any units of area at uniform time intervals is converted to a base length of TC and ordinates of cfs by linear interpolation. If an artificial time-area curve is to be used, VAR(3) represents the exponent of a parabolic time-area curve for each half of the area as follows:

$$A = kT^E \quad (0<T<.5) \tag{1}$$

$$1\text{-}A = k(1-T)^E \quad (.5<T<1) \tag{2}$$

where A is area as ratio to total drainage area and T is time as ratio to time of concentration. These are routed through basin storage by the following standard Clark equations, which is equivalent to the Muskingum routing with R = K and X = 0:

$$C1 = TRHR/(R + .5\ TRHR) \tag{3}$$

$$C2 = 1 - C1 \tag{4}$$

$$0_2 = C1(I_2) + C2(0_1) \tag{5}$$

$$QUNGR_2 = .5(0_1 + 0_2) \tag{6}$$

where TRHR is the tabulation interval in hours.

6. Losses for each period are computed by the following equation [see sketch on page 14 of this report]:

$$ALOSS = AK(RAIN)^E \tag{7}$$

The coefficient AK is a function of 4 variables (average value and initial loss increment, which differ from flood to flood, and recovery rate and exponential recession rate, which are uniform for all floods). If the first ordinate of the time-area curve (at zero time) is not zero, its value is considered to be reservoir area and contributing 100 per cent runoff.

7. No return flow is added to the computed flow except an exponential recession of flow that existed at the start of the storm. Thus, the unit hydrograph obtained includes subsurface flow as well as surface runoff. After flow recedes to a specified value, the recession flow each period is computed and used whenever flow computed from the unit hydrograph falls below that recession value. This exponential recession rate is the same as that used for recession from antecedent runoff.

8. After initializing all variables, the program will start optimizing with the loss rate variable E (VAR 7) unless directed to start elsewhere. Each approximation is accomplished by computing all flood hydrographs and overall standard error of reproduction with all variables fixed and then with the one variable decremented by 10 and 20 per cent, respectively. The standard error differences indicate the direction in which the variable should be changed to improve the reproduction. The amount of change is computed as follows:

$$X' = X(.95 + DSER1/DIF2) \tag{8}$$

where X stands for any variable, DSER1 is the difference in standard error in the second and first computation, and DIF2 is the increase in this difference for the third and second computations. If DIF2 is negative, divergence is indicated, and a maximum change in the direction of improvement is made. All changes are limited to a factor of 1.5 and are checked to assure that the standard error is being reduced by the change and that the new value is logical. If the standard error increases, the change is reduced 70 per cent. If divergence still exists, it is reduced 70 per percent of the remainder. If the standard error still increases, it is set to where it was and the next variable considered.

9. In order to improve the reproduction of peak flows, errors associated with high flows are weighted heavier than those associated with low flows. Each error squared is multiplied by $(Q + Q)/(2Q)$. Also, if a reproduction is not satisfactory, considerable improvement can be made in a second run by a routine that artificially changes 1 or 2 flows in each flood temporarily to force a better reproduction without impairing the validity of the results. For example, a portion of a reconstituted hydrograph that is too low can be fitted better by increasing a key flow (using

input items G6-9) by about double the discrepancy. Since the reconstituted hydrograph is derived from the known unit hydrograph and loss rate functions, the only test of validity is its comparison with the observed hydrograph.

10. After all variables have been optimized 3 times (after the 4th cycle), the program continues optimizing, selecting each time the variable that made the greatest change in its latest test. When the greatest change is less than 1 per cent of the remaining standard error, all variables are reviewed once more and the routine of selecting variables causing maximum improvement is repeated, after which optimization is declared and results printed out.

The program has about 600 input statements, and the author recommends that it be used only on high speed computers of 7090 class. In this study it was used with the CDC 1604 and the CDC 6600. Figure 12 illustrates a sample output from the program. Reading from left to right across the top the first item labeled DA is the drainage area, in this case 1270 sq. miles; the next is the rainfall interval TR in minutes, in this case 60 minutes; the next 7 items are 7 of the variables to be optimized; the last item on the top line, QRECSN, is the flow below which recession rates are miantained as a minimum, and is fixed by the operator of the program. Items in the next row, reading from left to right, are labeled FLAG 1 through FLAG 7 and FLGNH 1 and FLGNH 2; a positive value inserted under these items prevents a change of variable having the same subscript. The item RTIOR is the ratio of the recession flow to that 10 periods later, or in effect a recession coefficient. Relative to the variables 1 through 7 the following additional tabulations may be helpful:

VAR (1) = Clark's T_c or time of concentration. As defined by Clark [10], this is the time from the end of runoff-producing rainfall to the point on the hydrograph with most rapid relative decrease in runoff; the latter point would usually be the inflection point. As a first approximation the program takes the last rain increment exceeding 7/10 of the maximum rain increment as the end of runoff-producing rainfall and the peak of the hydrograph as the end of the time of concentration.

VAR (2) = Ratio of Clark's R to T_c, where
$R = Q/S$,
Q = discharge at point of inflection, and
S = slope at point of inflection = $\Delta Q/\Delta t$.

VAR (3) = Exponent of an artificial time-area curve.

VAR (4) = Ratio of impervious area to total drainage area.

VAR (5) = Ratio of K of straight line portion of loss rate curve to K at 10 in. more accumulated loss.

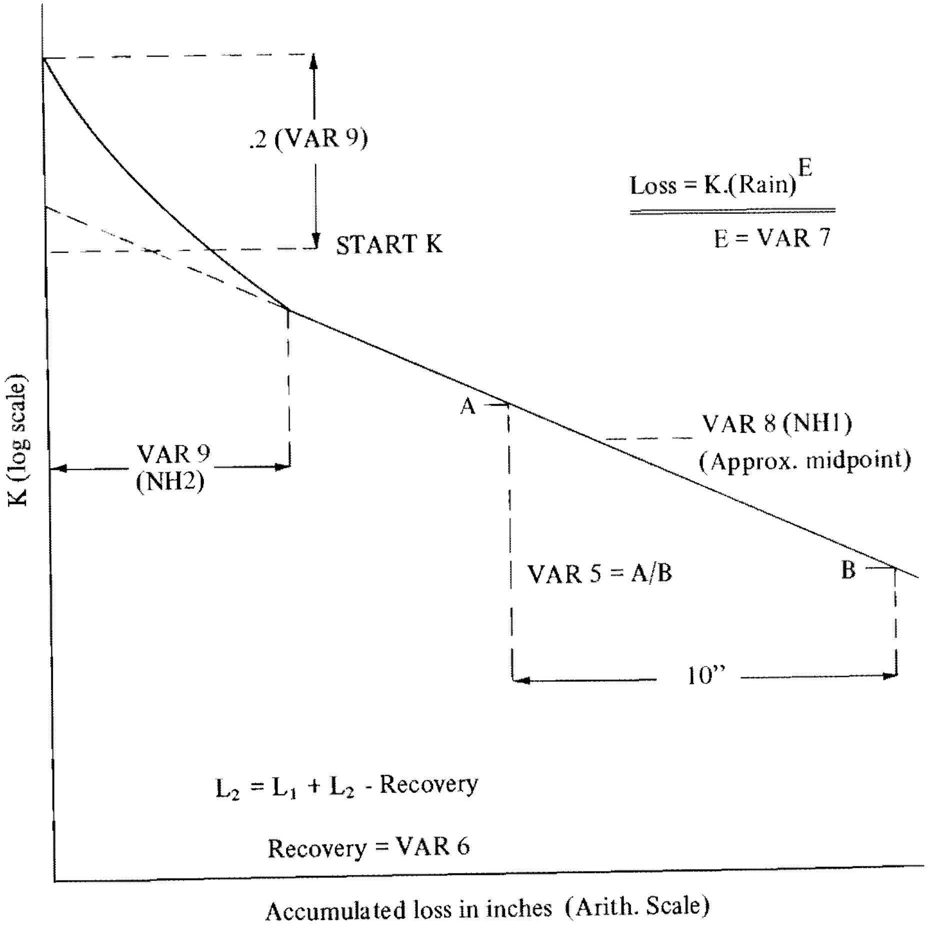
VAR (6) = Recovery loss index in inches subtracted from accumulated loss for each period.

VAR (7) = Exponent of rain in loss computation.

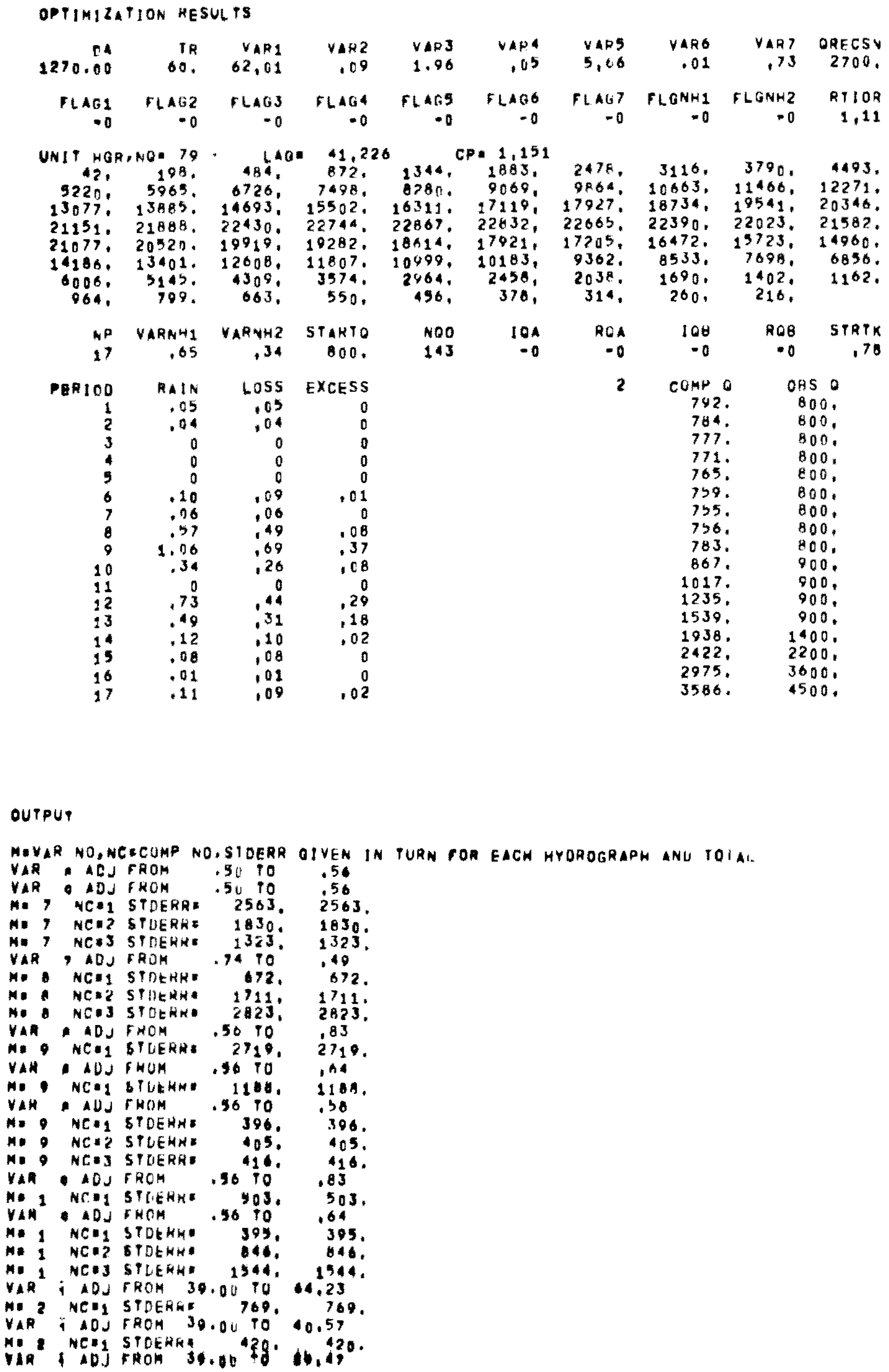
VAR (NH1) = VAR (8) = Value of K on straight line portion of loss rate curve when accumulated loss is 1/2 of storm loss.

VAR (NH2) = VAR (9) = Accumulated loss increment during initial loss period. It adds an increment of 0.2 [VAR (NH2)] to K when accumulated loss is equal to zero.

Some of the preceding variables have been defined on the following sketch:



C_p equals Snyder's C_p and is equal to $Q_{max} \times \frac{lag}{645 \times DA}$, where DA is equal to the drainage area. Other items that might be defined are NP, which equals the number of observed precipitation periods in the storm, and NQO, which equals the number of observed flows in the hydrograph. Referring again to Figure 12, the unit hydrograph output of the program is defined in the large block of numbers in the upper half of the page starting with the discharge of 42 cfs and going to a peak of 22,867 cfs, then receding to 216 cfs.



```
OPTIMIZATION RESULTS

     DA        TR      VAR1      VAR2      VAR3      VAR4      VAR5      VAR6      VAR7    QRECSN
1270.00       60.     62,01      ,09      1.96       ,05      5,66       .01       ,73     2700.

  FLAG1     FLAG2     FLAG3     FLAG4     FLAG5     FLAG6     FLAG7    FLGNH1    FLGNH2     RTIOR
     -0        -0        -0        -0        -0        -0        -0        -0        -0      1,11

UNIT HGR,NO= 79 .    LAG=  41,226       CP= 1,151
    42,      198.      484,      872.     1344,     1883,     2478,     3116,     3790,     4493.
  5220,     5965.     6726,     7498,     8280.     9069,     9864,    10663,    11466,    12271.
 13077,    13885.    14693,    15502,    16311.    17119,    17927,    18734,    19541,    20346.
 21151,    21888.    22430,    22744,    22867,    22632,    22665,    22390,    22023,    21582.
 21077,    20520.    19919,    19282,    18614,    17921,    17205,    16472,    15723,    14960.
 14186,    13401.    12608,    11807,    10999,    10183,     9362,     8533,     7698,     6856.
  6006,     5145.     4309,     3574,     2964,     2458,     2038,     1690,     1402,     1162.
   964,      799.      663,      550,      456,      378,      314,      260,      216,

     NP    VARNH1    VARNH2    STARTQ       NQO       IQA       RQA       IQB       RQB     STRTK
     17       ,65       ,34      800.       143        -0        -0        -0        -0       ,78

 PERIOD      RAIN      LOSS    EXCESS                                 2     COMP Q      OBS Q
      1       ,05       ,05         0                                         792.       800,
      2       ,04       ,04         0                                         784.       800,
      3         0         0         0                                         777.       800,
      4         0         0         0                                         771.       800,
      5         0         0         0                                         765.       800,
      6       .10       ,09       ,01                                         759.       800,
      7       ,06       ,06         0                                         755.       800,
      8       .57       ,49       ,08                                         756,       800,
      9      1.06       ,69       ,37                                         783.       800,
     10       .34       ,26       ,08                                         867.       900,
     11         0         0         0                                        1017.       900,
     12       ,73       ,44       ,29                                        1235,       900,
     13       .49       ,31       ,18                                        1539,       900,
     14       ,12       ,10       ,02                                        1938.      1400,
     15       .08       ,08         0                                        2422,      2200,
     16       .01       ,01         0                                        2975.      3600,
     17       .11       ,09       ,02                                        3586.      4500,
```

```
OUTPUT

M=VAR NO,NC=COMP NO,STDERR GIVEN IN TURN FOR EACH HYDROGRAPH AND TOTAL
VAR  8 ADJ FROM    .50 TO    .56
VAR  9 ADJ FROM    .50 TO    ,56
M= 7  NC=1 STDERR=   2563,   2563,
M= 7  NC=2 STDERR=   1830,   1830.
M= 7  NC=3 STDERR=   1323,   1323,
VAR  7 ADJ FROM    .74 TO    ,49
M= 8  NC=1 STDERR=    672,    672.
M= 8  NC=2 STDERR=   1711,   1711.
M= 8  NC=3 STDERR=   2823,   2823,
VAR  8 ADJ FROM    .56 TO    ,83
M= 9  NC=1 STDERR=   2719,   2719.
VAR  8 ADJ FROM    .56 TO    ,64
M= 9  NC=1 STDERR=   1188,   1188,
VAR  8 ADJ FROM    .56 TO    .58
M= 9  NC=1 STDERR=    396,    396.
M= 9  NC=2 STDERR=    405,    405.
M= 9  NC=3 STDERR=    416,    416.
VAR  9 ADJ FROM    .56 TO    ,83
M= 1  NC=1 STDERR=    503,    503.
VAR  9 ADJ FROM    .56 TO    ,64
M= 1  NC=1 STDERR=    395,    395.
M= 1  NC=2 STDERR=    846,    846.
M= 1  NC=3 STDERR=   1544,   1544.
VAR  4 ADJ FROM  39.00 TO  44,23
M= 2  NC=1 STDERR=    769,    769.
VAR  4 ADJ FROM  39.00 TO  40.57
M= 2  NC=1 STDERR=    420,    420.
VAR  4 ADJ FROM  39.00 TO  40,49
```

Figure 12. Example of computer print out of optimization data (Root River near Houston, event of June 28, 1942)

In the middle left-hand section of Figure 12 are the numbers of the input period for both rain and observed discharge; to the right of these are the hourly average rainfall over the watershed, the computer evaluation of the hourly loss over the watershed, and the computer evaluation of the rainfall excess.

In the middle right-hand section of Figure 12 are initial values of both the computed discharge and the actual measured or observed discharge; the latter two columns continue for a total of 143 periods, but in this case they have been cut off in the process of reproducing this figure. Figure 12 also contains a portion of the output during the optimization process. As may be noted thereon, the variables are adjusted one at a time and the standard error (STDERR) computed for each change in a variable. At each time interval for the complete hydrograph (one-hour intervals in this record) the difference is determined between the computed and observed hydrograph and the square root of the weighted summation of the values determined as the standard error. Figure 13 illustrates the form of plotting both the hourly precipitation data and the hourly excess, the dark portion of the isohyetal diagram in the upper left-hand corner, and both the observed and computed discharge curves. The measured or observed discharge is the solid line; the dashed or broken line is the computer curve. This shows very good agreement in this particular case between the two values.

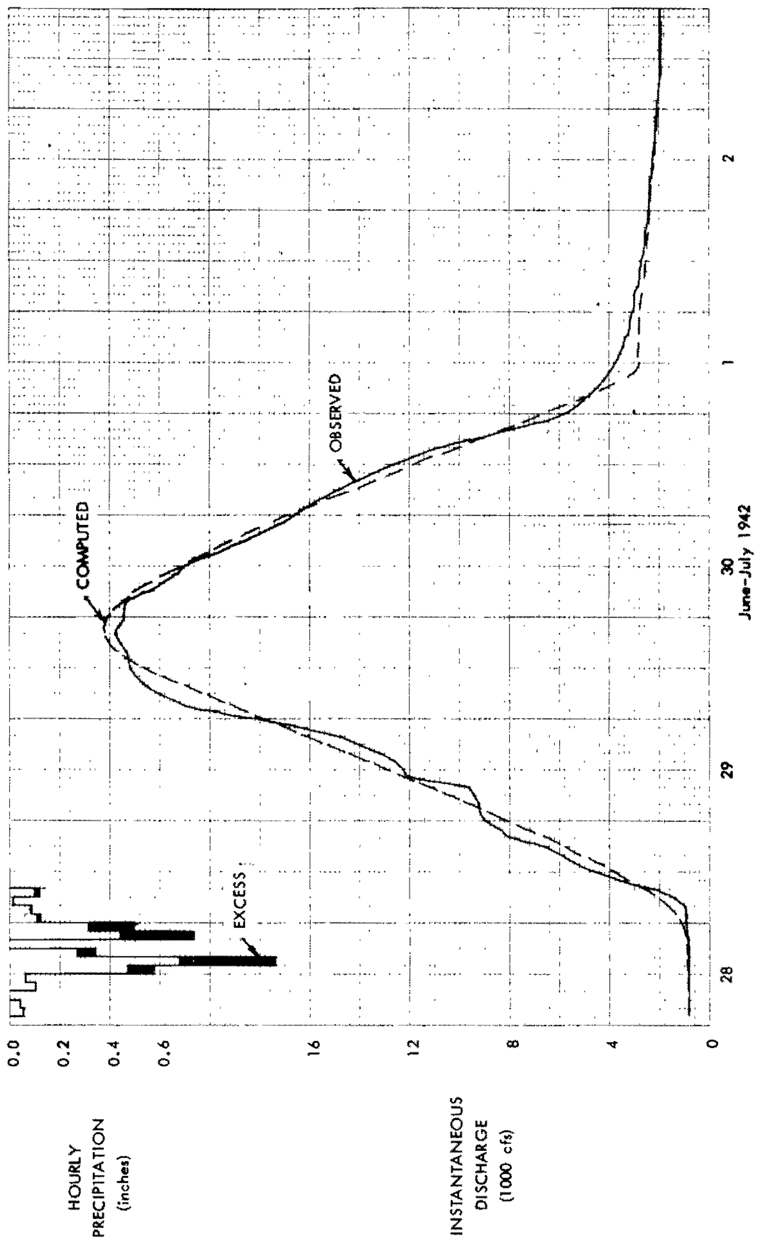


Figure 13. Comparison of computed and observed discharges of Root River near Houston, event of June 28, 1942

COMPARATIVE RESULTS

Frequency Analysis

Figures 14 through 31 are graphs on log probability paper of flood frequency data for each station with a reasonably adequate length of record. The upper graph on each page shows the annual maximum series for each station and the lower graph the annual maximum summer storm series for each station. The annual floods are shown and plotted in accordance with the plotting equation previously defined. Two theoretical curves are shown on the graphs, one for a skewness as computed for the actual record and the other for a skewness of zero.

During the course of the studies a computer program was developed which arranges the floods in descending order and computes the empirical recurrence interval and probability of occurrence, the log of the floods, and the mean, standard deviation, and skewness of the logs. It then computes the magnitude of floods having recurrence intervals of 100, 50, 25, 10, 5, 2, 1.25, 1.11, 1.05, and 1.01 years. The corresponding probabilities are 1, 2, 4, 10, 20, 50, 80, 90, 95, and 99 per cent chance of occurrence in any one year.

The program computes the desired discharges for the skewness determined for that particular set of data as well as other values of skewness that might be specified. When adequate data are available many hydrologists prefer to use a regional value of skewness coefficient due to the wide range of values which may occur in relatively short records. The number of stations covered in this study was not adequate for the determination of a regional skewness coefficient. Some hydrologists in the area have indicated a preference for a skewness approaching zero where the record is sufficiently long. A log-Pearson Type III distribution with a skewness of zero would correspond to the log-normal probability distribution.

The objective in presenting a separate set of data and graph for the summer floods was to permit a comparison with runoff computations based on a frequency analysis of precipitation data. Time did not permit such an analysis, but the data have been provided for reference purposes. A comparison of the annual maximum summer storm series results in a lesser discharge for a given recurrence interval than does the annual series. This results in part from the fact that the annual series may contain very large spring floods resulting either from snow melt or from a combination of snow melt and rain, producing some of the larger floods.

Table 3 is a summary of data on the flood frequency analysis including the number of years of record used in each analysis, the mean and standard deviation of the floods, and the mean, standard deviation, and skewness of the logs of the floods.

Runoff

A majority of the data, graphs, and computer printouts developed in the study are included in Appendices with a total of 201 graphs and computer printouts. One appendix has been assigned to each basic watershed or a total in this case of five appendices. The data have generally been arranged with initial

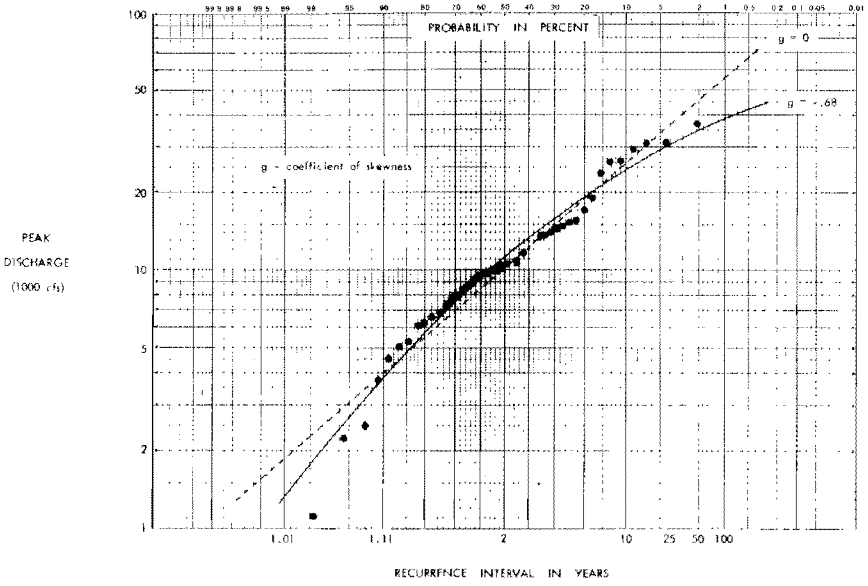


Figure 14. Flood frequency curves of annual maximum series for Root River near Houston

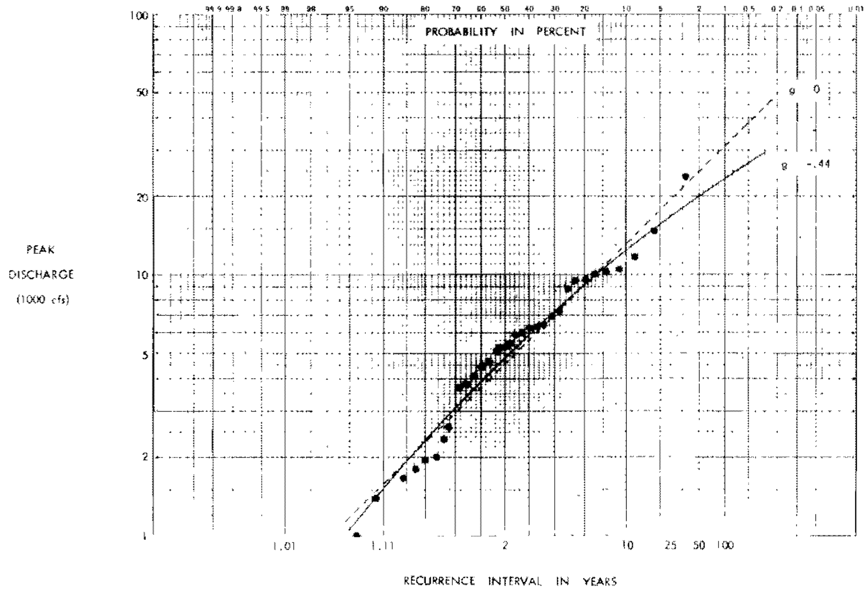


Figure 15. Flood frequency curves of annual maximum summer flood series for Root River near Houston

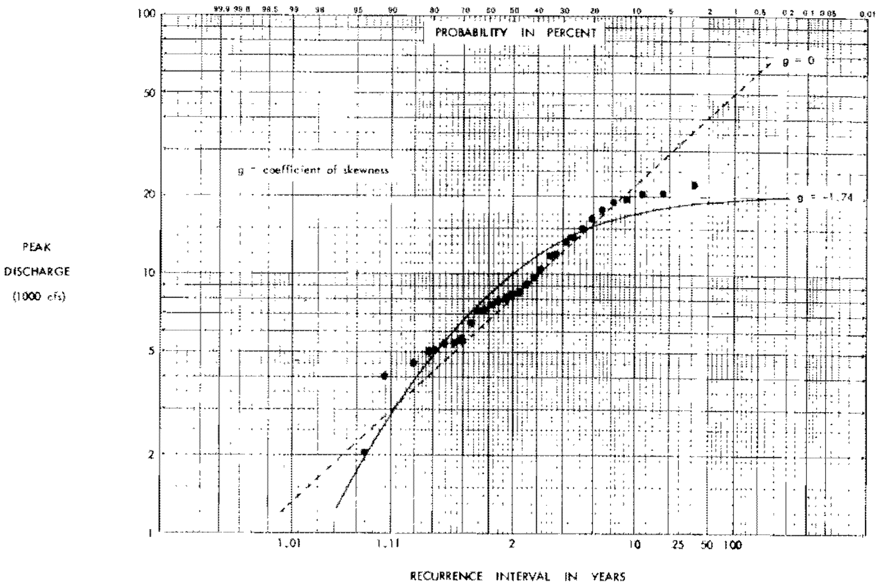


Figure 16. Flood frequency curves of annual maximum series for Root River near Lanesboro

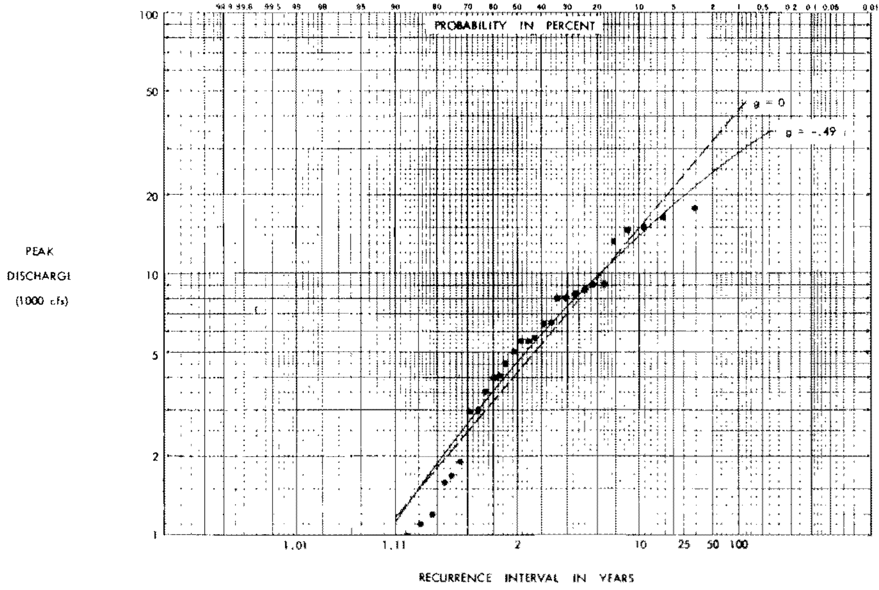


Figure 17. Flood frequency curves of annual maximum summer flood series for Root River near Lanesboro

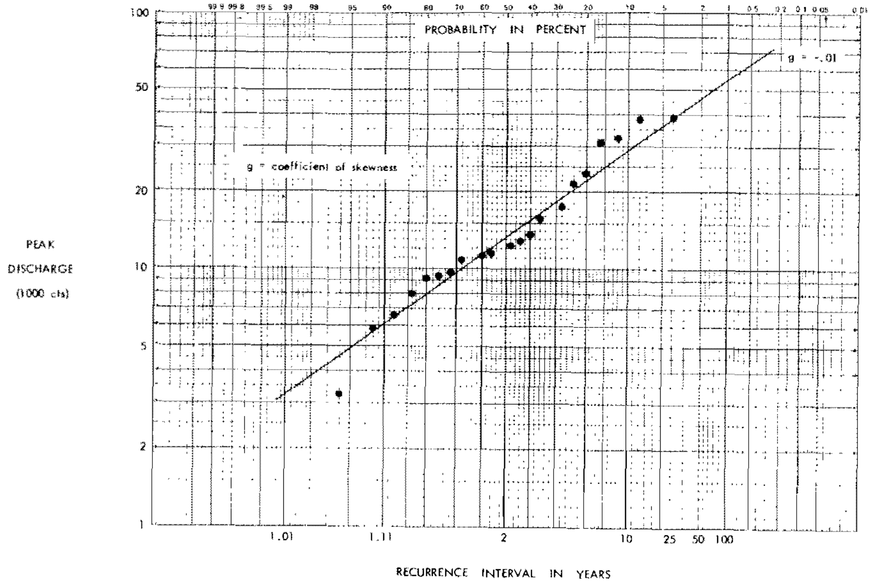


Figure 18. Flood frequency curves of annual maximum series for Root River below South Fork near Houston

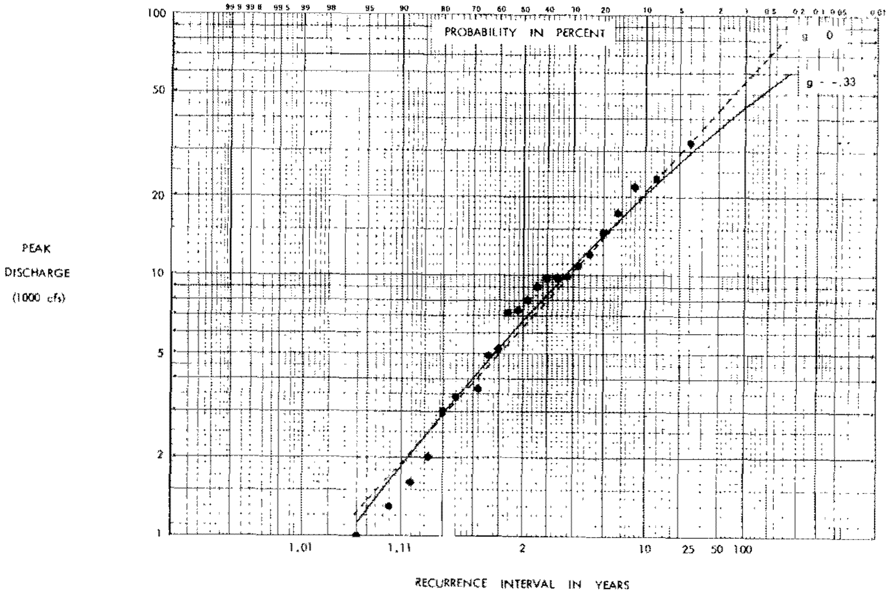


Figure 19. Flood frequency curves of annual maximum summer flood series for Root River below South Fork near Houston

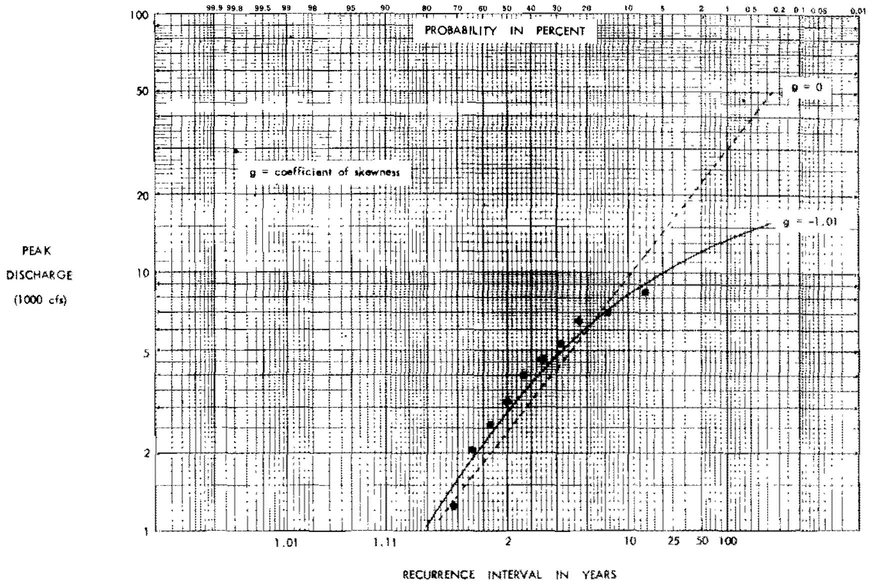


Figure 20. Flood frequency curves of annual maximum series for South Fork Root River near Houston

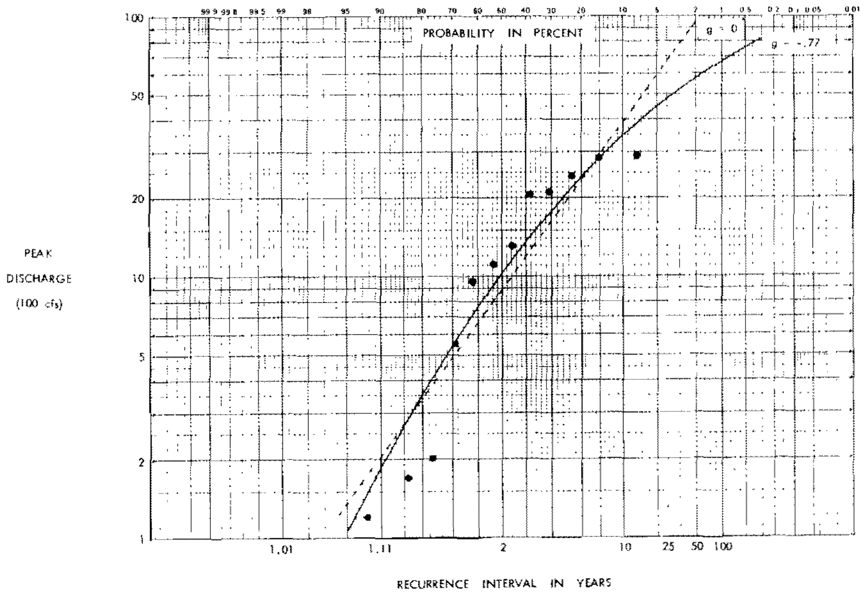


Figure 21. Flood frequency curves of annual maximum summer flood series for South Fork Root River near Houston

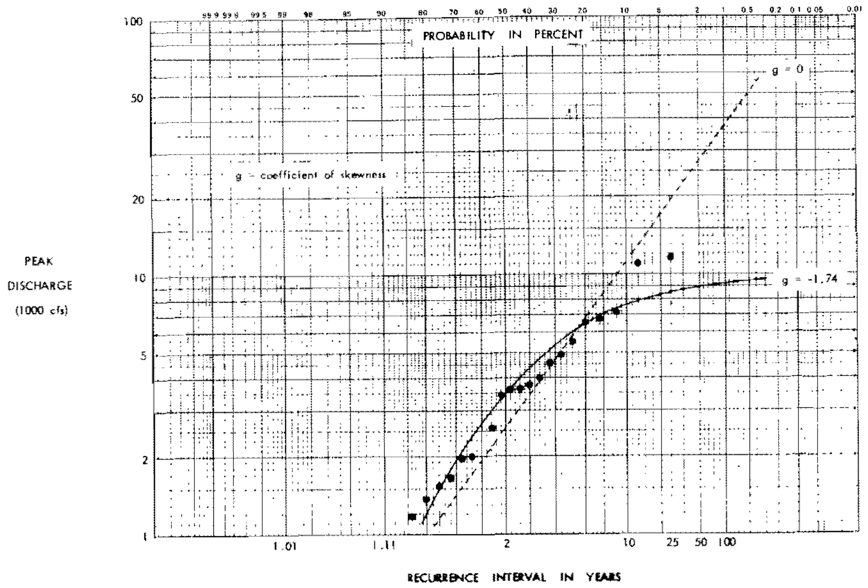


Figure 22. Flood frequency curves of annual maximum series for Rush Creek near Rushford

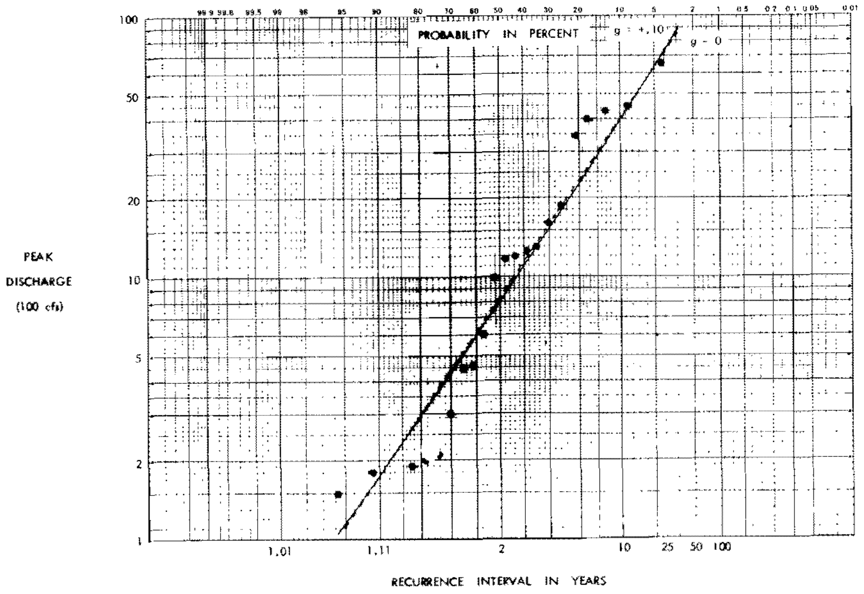


Figure 23. Flood frequency curves of annual maximum summer flood series for Rush Creek near Rushford

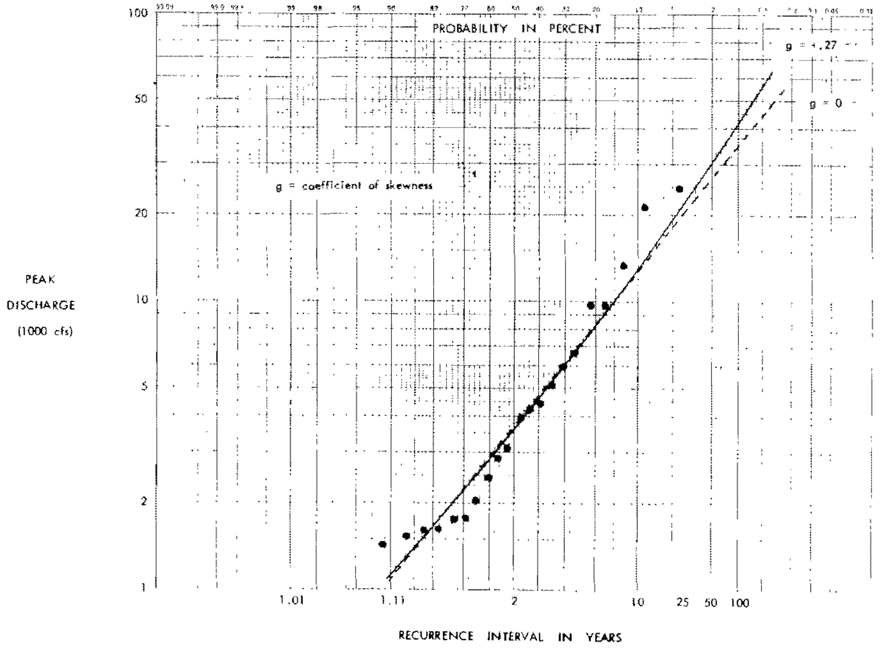


Figure 24. Flood frequency curves of annual maximum series for Le Sueur River near Rapidan

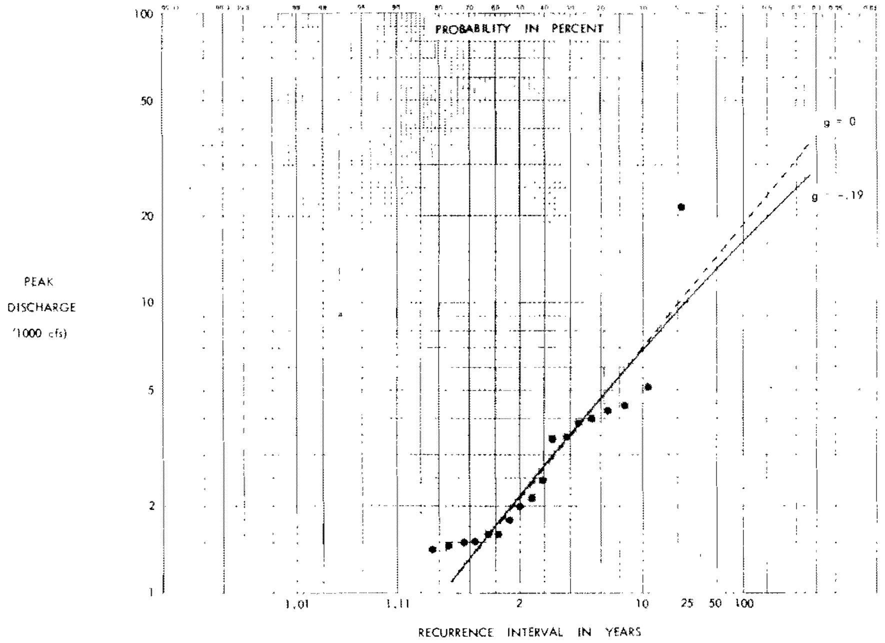


Figure 25. Flood frequency curves of annual maximum summer flood series for Le Sueur near Rapidan

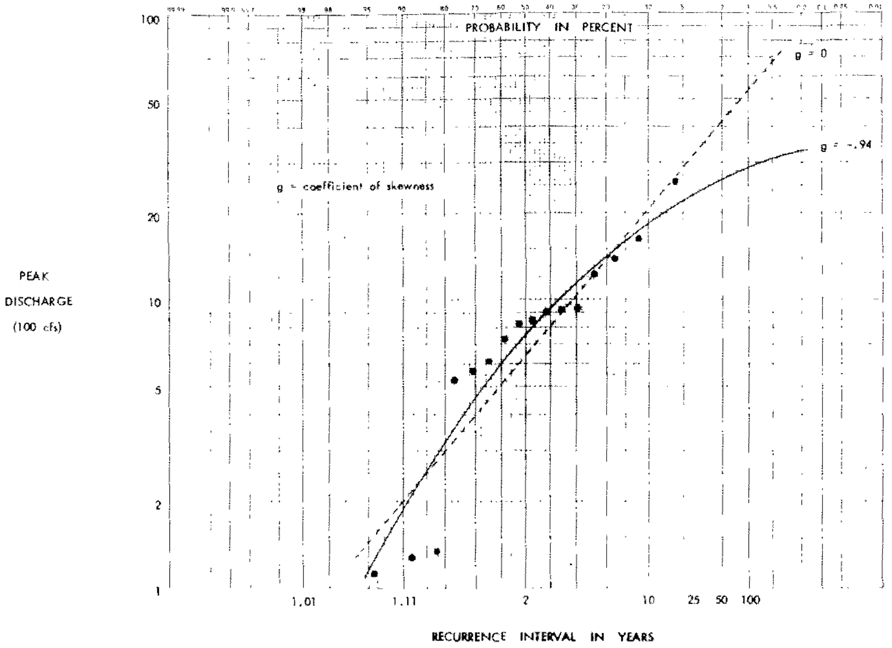


Figure 26. Flood frequency curves of annual maximum series for Middle River near Argyle

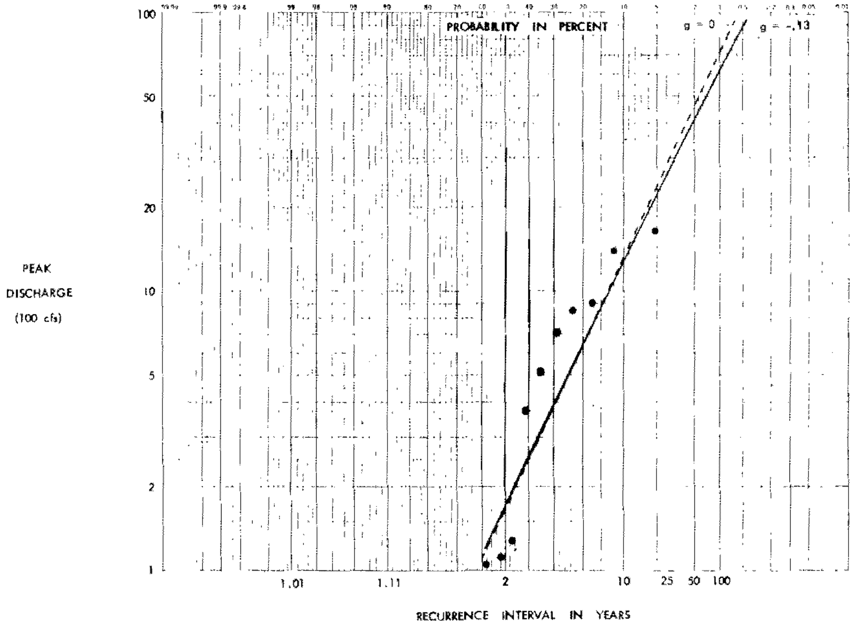


Figure 27. Flood frequency curves of annual maximum summer flood series for Middle River near Argyle

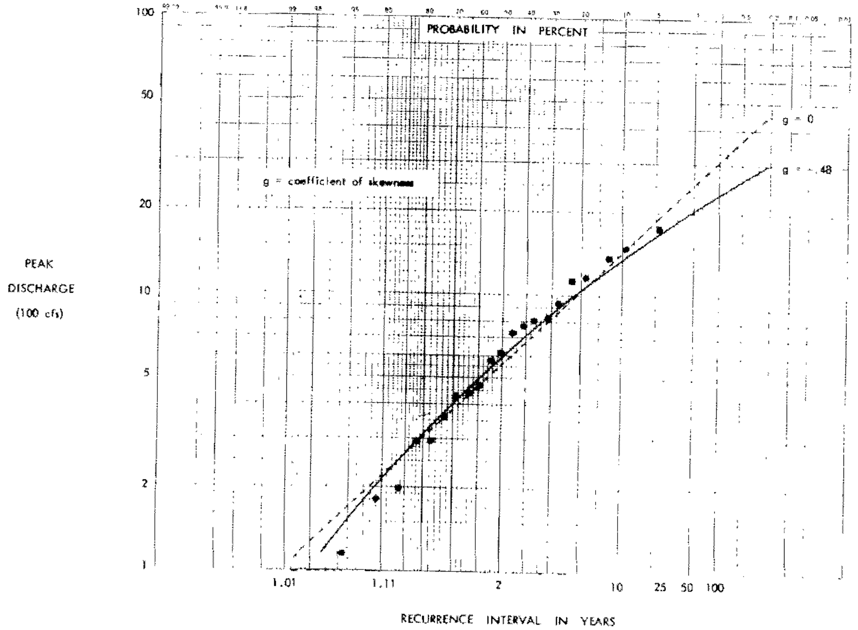


Figure 28. Flood frequency curves of annual maximum series for Embarrass River near Embarrass

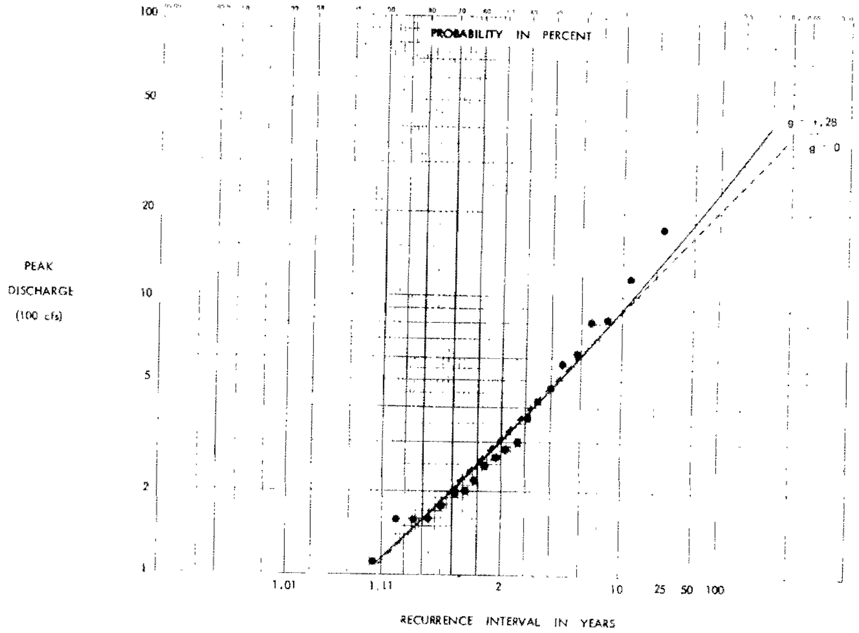


Figure 29. Flood frequency curves of annual maximum summer flood series for Embarrass River near Embarrass

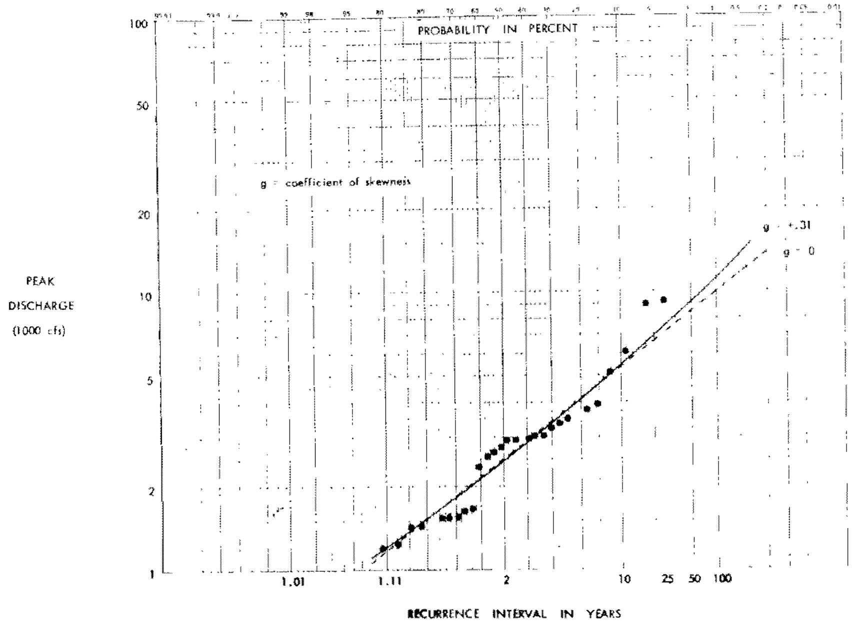


Figure 30. Flood frequency curves of annual maximum series for Baptism River near Beaver Bay

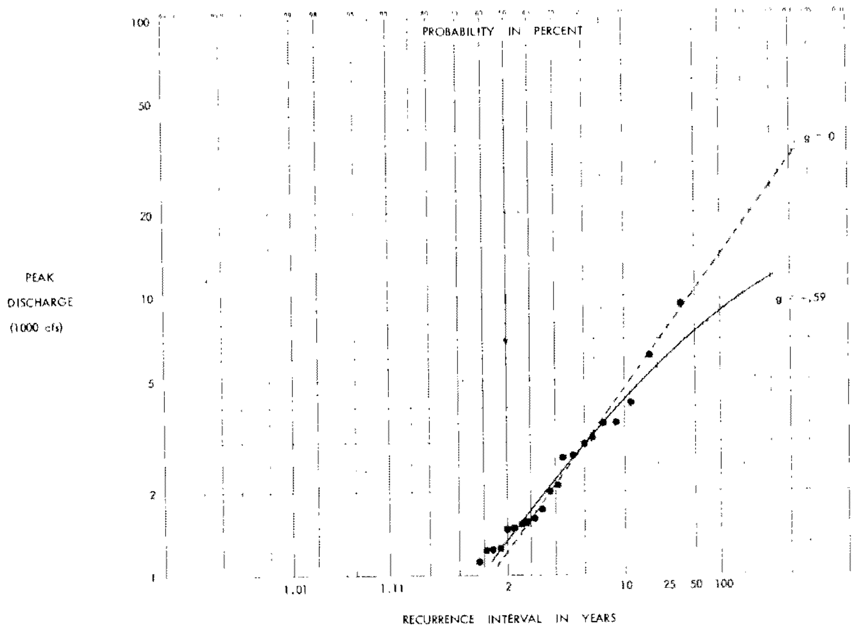


Figure 31. Flood frequency curves of annual maximum summer flood series for Baptism River near Beaver Bay

Table 3

STATISTICAL PARAMETERS RELATING TO FLOOD DATA

			Floods		Log of Floods		
Watershed	Series	Years	Mean (cfs)	St'd. Dev. (cfs)	Mean	St'd. Dev.	Skewness
Root River – Houston	A	44	12,960	8620	4.01	0.317	-0.68
	S	34	6,080	4670	3.66	0.358	-0.44
Root River – Lanesboro	A	33	10,120	5880	3.91	0.341	-1.74
	S	32	6,130	–	3.62	0.429	-0.49
Root River – below	A	24	15,850	9990	4.12	0.263	-0.01
South Fork	S	24	9,270	7880	3.81	0.404	-0.33
South Fork of	A	13	3,590	2640	3.38	0.469	-1.01
Root River	S	12	1,390	1040	2.95	0.500	-0.77
Rush Creek – Rushford	A	24	3,840	3040	3.41	0.498	-1.74
	S	22	1,600	1800	2.92	0.533	+0.10
Le Sueur – Rapidan	A	22	5,880	6410	3.57	0.418	+0.27
	S	21	3,280	4330	3.33	0.404	-0.19
Middle River – Argyle	A	16	880	630	2.81	0.396	-0.94
	S	16	440	520	2.22	0.704	-0.13
Embarrass River	A	32	3,000	2020	3.40	0.256	-0.48
near Embarrass	S	33	1,900	1880	3.08	0.456	+0.28
Baptism River	A	21	700	450	2.75	0.317	+0.31
Beaver Bay	S	22	430	400	2.50	0.350	-0.59

graphs illustrating the total flow record for a given gaging station and then a collection of graphs and printouts associated with each storm in that particular watershed.

This list usually includes the following:

1. A continuous hydrograph for the period of record of the gaging stations.
2. A six-month graph showing the discharge in the stream, significant precipitation, and an antecedent index over the period of interest.
3. A map of the basin with isohyetal lines of precipitation for that particular storm.
4. Graphs showing the hourly precipitation and a continuous record of computed and observed discharge for the period of the flood.
5. Computer printouts showing the finally optimized values of each of the nine variables, the unit hydrograph developed with that particular storm, the hourly rain, hourly loss, hourly excess, and a portion of the computed and observed runoff.

Figures 32 through 44 include a typical set of curves and data for one storm in the Root River area as an illustration of the type of data.

Figure 32 is an illustration of one of the graphs showing antecedent moisture conditions above Houston for July 20 and 21, 1951. Figure 33 is a map with isohyetal lines for that particular storm superimposed on a map of the area. Also shown on this map is an arrow indicating the general direction of travel of the

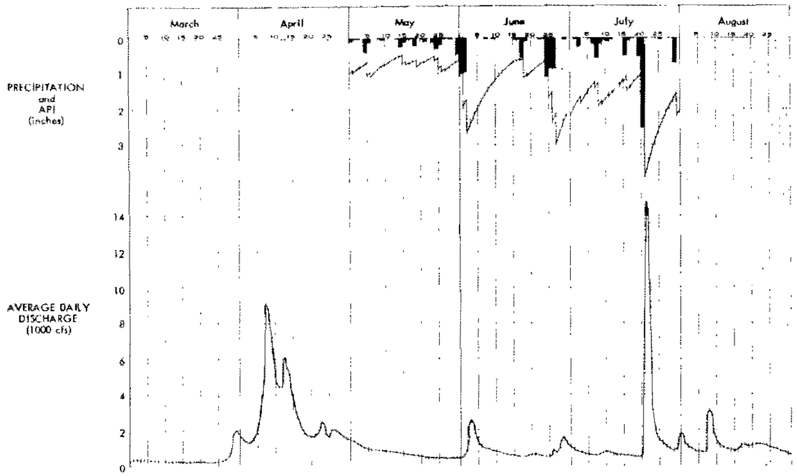


Figure 32. Antecedent moisture conditions above Houston, storm event of July 20, 1951

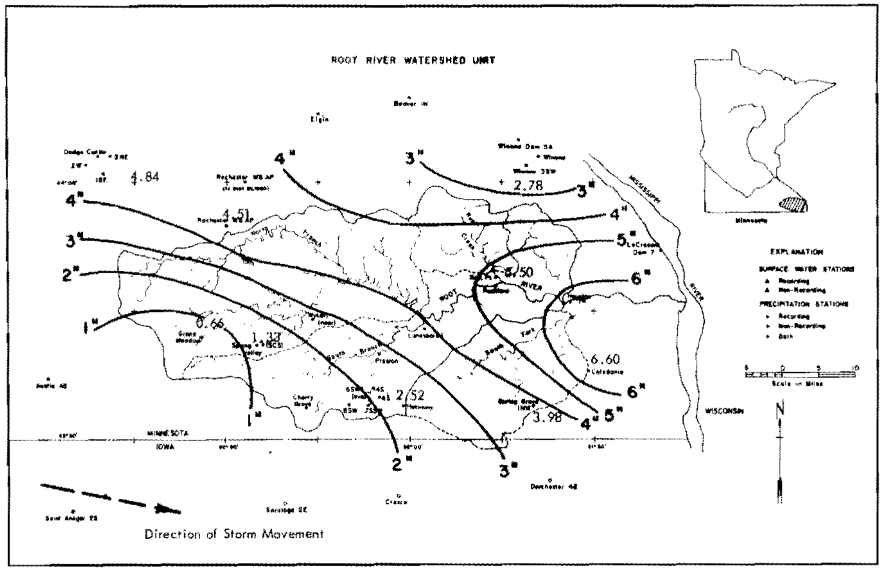


Figure 33. Isohyetal map for storm event of July 20, 1951

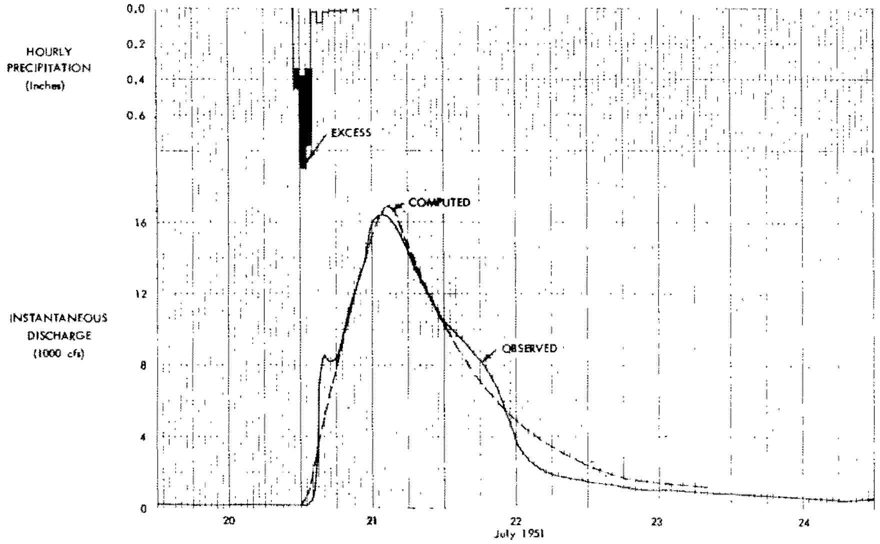


Figure 34. Computed and observed hydrographs for Root River near Lanesboro, storm event of July 20, 1951

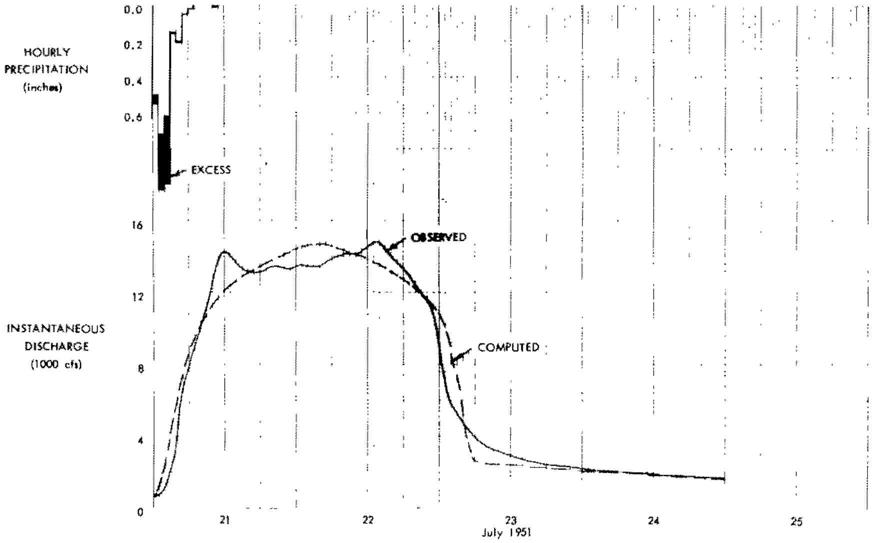


Figure 35. Computed and observed hydrographs for Root River near Houston, storm event of July 20, 1951

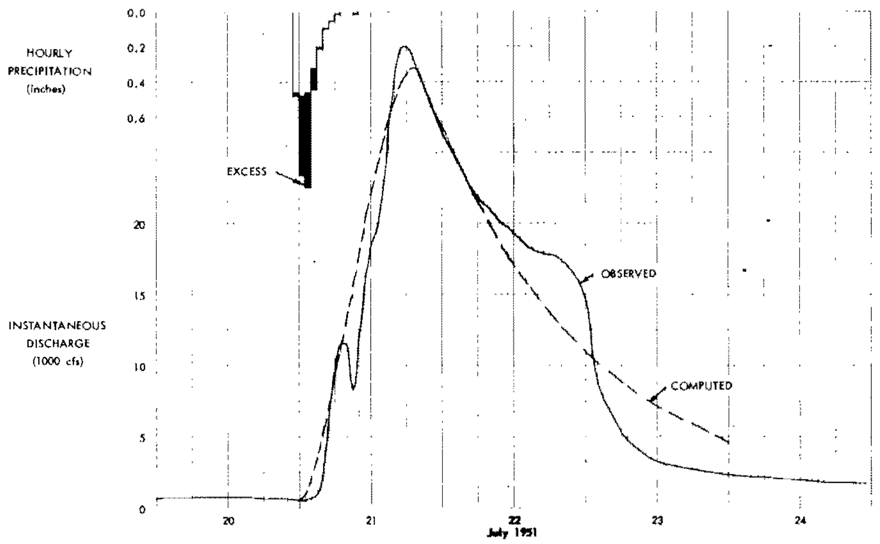


Figure 36. Computed and observed hydrographs for Root River below South Fork near Houston, event of July 20, 1951

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OPTIMIZATION RESULTS

      DA        TR      VAR1      VAR2      VAR3      VAR4      VAR5      VAR6      VAR7    QRECSN
  615.00       60.     14.52      1.12      1.12       .00      6.68       .05       .26     1600.

   FLAG1     FLAG2     FLAG3     FLAG4     FLAG5     FLAG6     FLAG7    FLGNH1    FLGNH2     RTIOR
      -0        -0        -0        -0        -0        -0        -0        -0        -0      1.24

UNIT HGR,NO= 91        LAG=  14.556        CP=  .578
    647.     1993.     3426.     4855.     6258.     7625.     8949.    10220.    11406.    12488.
  13463.    14323.    15073.    15671.    15759.    15127.    14228.    13383.    12587.    11839.
  11135.    10474.     9851.     9266.     8715.     8197.     7710.     7252.     6821.     6415.
   6034.     5675.     5338.     5021.     4722.     4442.     4178.     3929.     3696.     3476.
   3270.     3075.     2892.     2721.     2559.     2407.     2264.     2129.     2003.     1884.
   1772.     1666.     1567.     1474.     1387.     1304.     1227.     1154.     1085.     1021.
    960.      903.      849.      799.      751.      707.      665.      625.      588.      553.
    520.      489.      460.      433.      407.      383.      360.      339.      319.      300.
    282.      265.      249.      235.      221.      207.      195.      184.      173.      162.
    153.

      NP    VARNH1    VARNH2    STARTQ       NQO       IQA       RQA       IQB       RQB     STRTK
      11       .37       .02      200.        70         5       .25        15      1.02       .41

  PERIOD      RAIN      LOSS    EXCESS                                      COMP Q     OBS Q
       1       .45       .34       .11                                        269.      200.
       2       .90       .38       .52                                        755.      200.
       3       .77       .34       .43                                       1901.      650.
       4       .02       .02         0                                       3395.     5000.
       5       .08       .08         0                                       4920.     8500.
       6       .02       .02         0                                       6425.     8200.
       7       .01       .01         0                                       7895.     8300.
       8       .01       .01         0                                       9321.     9500.
       9       .01       .01         0                                      10692.    11000.
      10        -0         0         0                                      11983.    12100.
      11       .01       .01         0                                      13171.    13100.
```

Figure 37. Partial computer print out for Root River near Lanesboro, event of July 20, 1951

```
OPTIMIZATION RESULTS

      DA        TR      VAR1      VAR2      VAR3      VAR4      VAR5      VAR6      VAR7    QRECSN
 1270.00       60.     49.25       .05      1.19       .05      2.94       .00       .48     2300.

   FLAG1     FLAG2     FLAG3     FLAG4     FLAG5     FLAG6     FLAG7    FLGNH1    FLGNH2     RTIOR
      -0        -0        -0        1.        -0        -0        -0        -0        -0      1.13

UNIT HGR,NO= 61        LAG=  27.510        CP=  .654
   1437.     4253.     6778.     8826.    10465.    11779.    12841.    13709.    14429.    15035.
  15554.    16005.    16403.    16758.    17079.    17372.    17643.    17894.    18130.    18351.
  18561.    18763.    18950.    19131.    19302.    19424.    19468.    19447.    19379.    19277.
  19150.    19001.    18840.    18663.    18473.    18278.    18054.    17824.    17579.    17318.
  17036.    16731.    16402.    16037.    15630.    15167.    14624.    13956.    13038.    10779.
   7644.     5221.     3569.     2439.     1667.     1139.      778.      532.      363.      248.
    170.

      NP    VARNH1    VARNH2    STARTQ       NQO    STARTK         N
      11       .18       .91      600.        97       .76         1

  PERIOD      RAIN      LOSS    EXCESS     RAIN2     LOSS2   EXCES2     COMP Q     OBS Q
       1       .55       .52       .03                                    632.      600.
       2      1.03       .72       .31                                   1154.      600.
       3       .99       .62       .37                                   2632.      800.
       4       .16       .15       .01                                   4523.     1700.
       5       .21       .20       .01                                   6170.     3500.
       6       .05       .05         0                                   7522.     6000.
       7       .02       .02         0                                   8612.     8000.
       8        -0         0         0                                   9490.     9500.
       9        -0         0         0                                  10203.    10300.
      10        -0         0         0                                  10787.    11500.
      11       .01       .01         0                                  11274.    12300.
```

Figure 38. Partial computer print out for Root River near Houston, event of July 20, 1951

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OPTIMIZATION RESULTS

      DA        TR      VAR1      VAR2      VAR3      VAR4      VAR5      VAR6      VAR7    QRECSN
 1560.00       60.     18.50      1.43      1.25       .03      1.70       .02       .42     2600.

   FLAG1     FLAG2     FLAG3     FLAG4     FLAG5     FLAG6     FLAG7    FLGNH1    FLGNH2     RTIOR
      1.        -0        -0        -0        -0        -0        -0        -0        -0      1.13

UNIT HGR,NO=100        LAG=  13.532        CP=  .498
    580.     1935.     3560.     5331.     7171.     9062.    10983.    12922.    14868.    16794.
  18434.    20330.    21879.    23277.    24514.    25577.    26437.    27033.    26993.    26237.
  25266.    24330.    23428.    22561.    21725.    20920.    20145.    19399.    18680.    17988.
  17322.    16680.    16063.    15468.    14895.    14343.    13812.    13300.    12807.    12333.
  11876.    11436.    11013.    10605.    10212.     9834.     9469.     9118.     8781.     8455.
   8142.     7841.     7550.     7271.     7001.     6742.     6492.     6252.     6020.     5797.
   5582.     5376.     5176.     4985.     4800.     4622.     4451.     4286.     4127.     3974.
   3827.     3685.     3549.     3418.     3291.     3169.     3052.     2939.     2830.     2725.
   2624.     2527.     2433.     2343.     2256.     2173.     2092.     2015.     1940.     1868.
   1799.     1732.     1668.     1606.     1547.     1490.     1434.     1381.     1330.     1281.

      NP    VARNH1    VARNH2    STARTQ       NQO       IQA       RQA       IQB       RQB     STRTK
      11       .18       .77      650.        77        43       .50        47       .35       .50

  PERIOD      RAIN      LOSS    EXCESS                                      COMP Q     OBS Q
       1       .48       .46       .02                                        651.      650.
       2       .93       .48       .45                                        725.      650.
       3      1.00       .46       .54                                       1860.      790.
       4       .44       .32       .12                                       3409.     1150.
       5       .21       .20       .01                                       5262.     3320.
       6       .09       .09         0                                       7262.     7200.
       7       .05       .05         0                                       9347.     8860.
       8       .01       .01         0                                      11485.    11300.
       9        -0         0         0                                      13656.    11300.
      10        -0         0         0                                      15844.     8360.
      11       .01       .01         0                                      18029.    14750.
```

Figure 39. Partial computer print out for Root River below South Fork near Houston, event of July 20, 1951

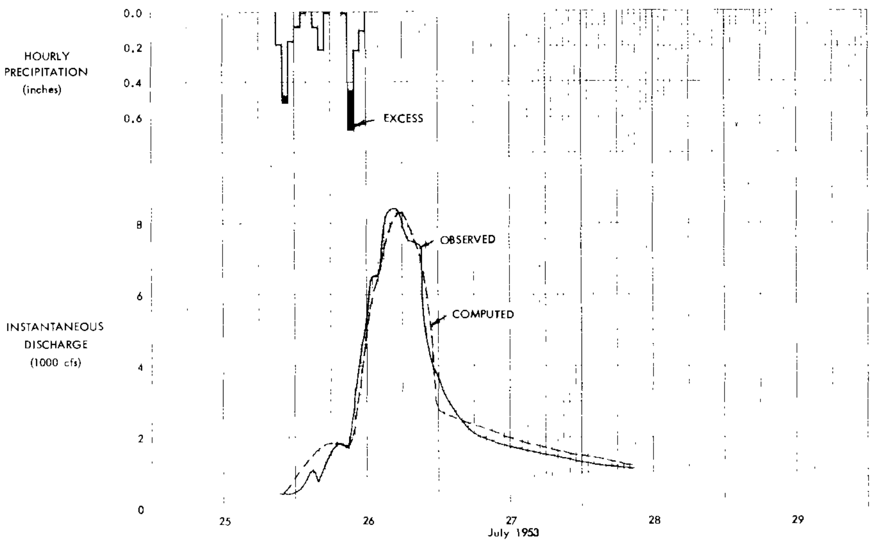

Figure 40. Computed and observed hydrographs for Root River near Lanesboro, event of July 25-26, 1953

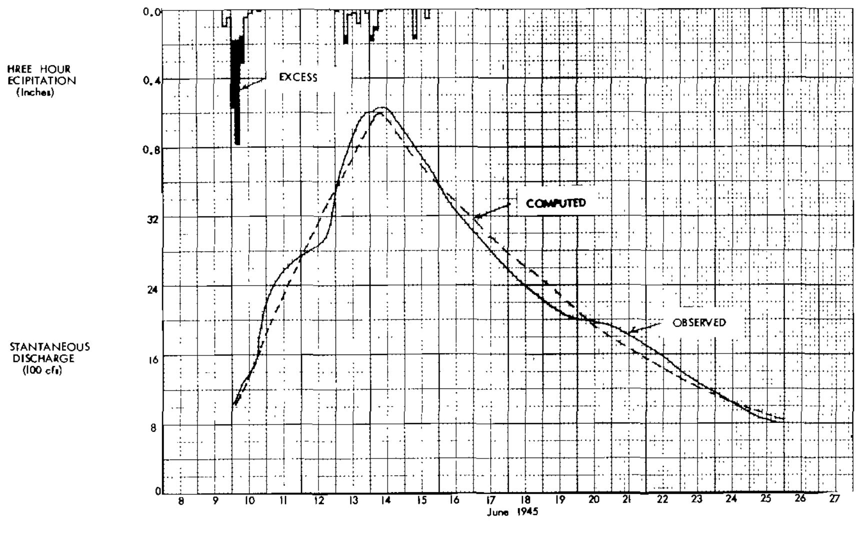

Figure 41. Computed and observed hydrographs for Le Sueur River near Rapidan, event of June 10, 1945

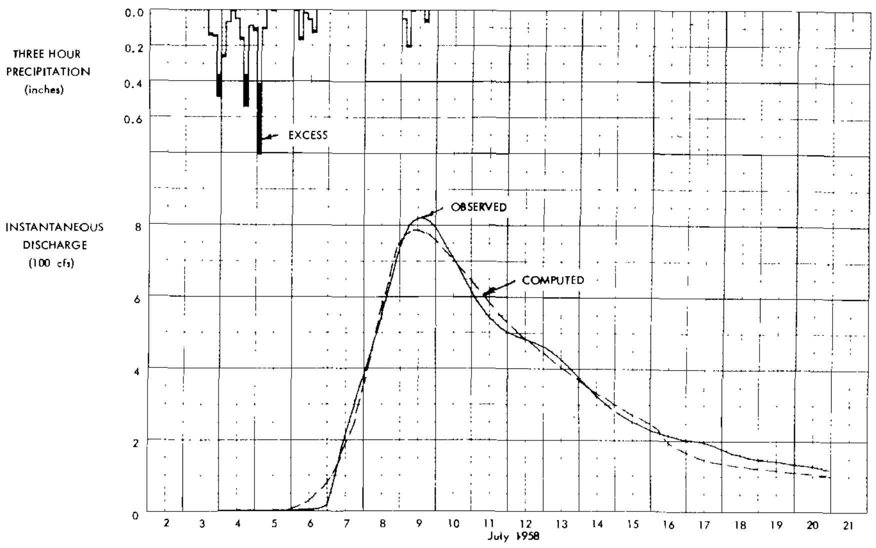

Figure 42. Computed and observed hydrographs for Middle River near Argyle, event of July 3-6, 1953

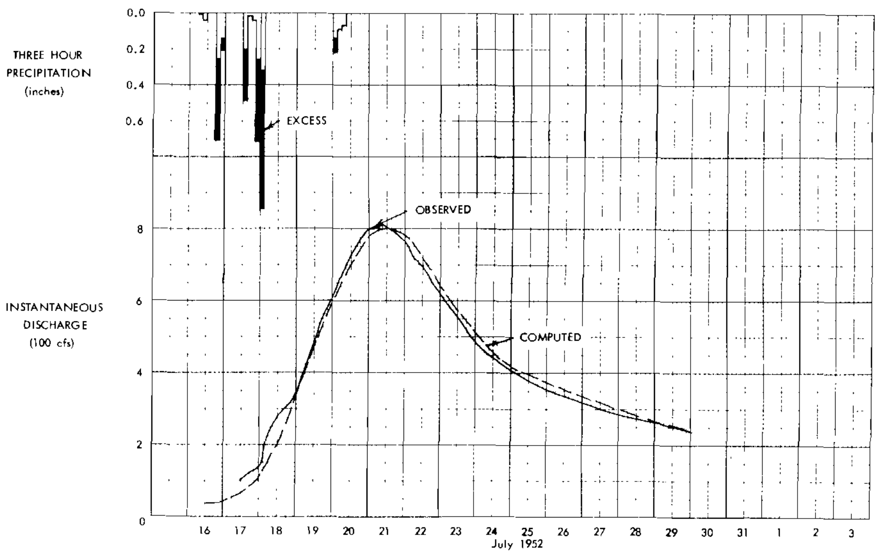

Figure 43. Computed and observed hydrographs for Embarrass River near Embarrass, event of July 16-18, 1952

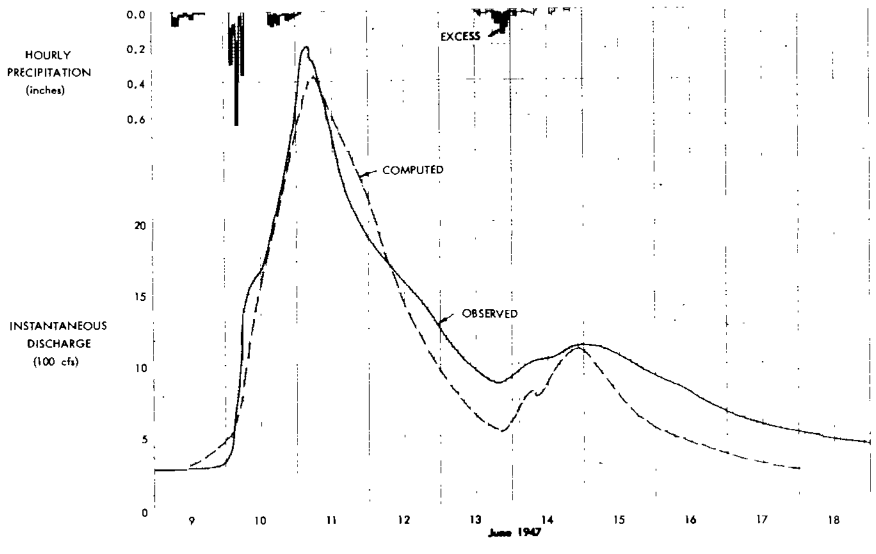

Figure 44. Computed and observed hydrographs for Baptism River near Beaver Bay, event of June 10, 1947

storm as determined from mass diagrams obtained from the recording rain gages in the area. In this particular storm the maximum rain was 6.6 in., which was a very substantial storm. It produced a peak flood of about 16,000 cfs in the Root River near Lanesboro, almost 15,000 cfs further downstream near Houston, and more than 32,000 cfs in the Root River below South Fork near Houston. Approximately 80 per cent of the rainfall fell in a 3-hour period.

Referring to Figure 34 concerning the Root River near Lanesboro, it may be noted that the computed curve is in rather good agreement with the observed curve for this particular flood. Also of interest is the fact that the average loss rate during the three-hour main portion of the storm was on the order of 0.36 in. per hour. This may also be noted in the computer printout of Figure 37.

Also referring to Figure 37, it may be noted that under variable 1 the time of concentration of the 615 sq. mile area was on the order of 14.5 hours. The unit hydrograph as determined by the mathematical model had a peak value of 15,750 cu. ft. per sec. The rainfall excess for the three hours during the heaviest portion of the storm as determined again by the mathematical model were 0.11, 0.52, and 0.43 in.

Proceeding downstream to the gaging station at Houston it may be noted that with the drainage area of 1270 sq. miles the peak rate of runoff was on the order of 15,000 cfs, or less than that produced by the 615-sq-mile area above Lanesboro. Referring to Figure 35 it may be noted that the hydrograph at Houston had a rather steep front, a flat top, and a rather steep recession. In view of the unusual shape of this hydrograph, the computer solution appears to be in very good agreement with the measured or observed runoff. Referring to Figure 38 it

	June 28 1942	July 20 1945	July 13 1950	July 20 1951	July 25 1953	June 4 1958	June 24 1959	July 2 1960	Aug. 28 1960	Aug. 29 1962	Average
Total Rain (in)	3.76	2.00	2.00	3.02	2.76	3.07	5.91	1.78	2.27	5.94	3.25
Total Loss (in)	2.71	1.74	1.81	2.29	2.50	2.72	5.40	1.51	2.10	5.61	2.84
Total Excess (in)	1.05	0.26	0.19	0.73	0.26	0.35	0.51	0.27	0.17	0.33	0.41
Avg. % Loss	72.1	87.0	90.5	75.8	90.6	88.6	91.4	84.8	92.5	94.0	86.7
Max. Loss	.69	.23	.54	.72	.46	.90	1.03	.40	.88	.64	.65
Max. Rain	1.06	.31	.65	1.13	.51	1.14	1.28	.46	.97	.59	.81
Vol. Obs. (sfd)	43,450.	13,808.	9,922.	29,325.	16,745.	16,820.	25,230.	14,521.	9,340.	17,761.	19,692.
Vol. Comp. (sfd)	43,047.	13,948.	8,976.	29,325.	16,121.	17,323.	24,400.	14,328.	8,662.	16,930.	19,306.
% ERR	-0.9	+1.0	-9.5	+0.	-3.7	+3.0	-3.3	-1.3	-2.9	-4.7	3.0
QRCSN	2,700.	2,000.	780.	2,300.	2,200.	2,150.	2,400.	2,000.	3,300.	2,200.	2,203.
RTIOR	1.11	1.15	1.09	1.13	1.08	1.13	1.07	1.11	1.25	1.11	1.123
Var. 1	62.01	26.81	18.31	49.25	40.78	72.31	64.01	39.0	16.10	85.00	47.34
Var. 2	0.09	0.22	0.36	0.05	0.09	0.06	0.15	0.25	0.63	0.10	0.200
R	5.581	5.898	6.60	2.463	3.670	4.339	9.602	9.750	12.898	8.00	6.880
Var. 3	1.96	1.55	1.43	1.19	2.89	2.78	2.59	1.62	2.74	3.89	2.26
Var. 4	0.05	0.05	0.06	0.05	0.08	0.07	0.05	0.05	0.05	0.05	0.056
Var. 5	5.66	7.74	2.78	2.94	4.23	4.78	7.30	9.91	1.45	7.10	5.389
Var. 6	0.01	–	–	–	0.01	0.01	0.01	0.01	0.01	0.01	0.007
Var. 7	0.73	0.63	0.33	0.48	0.41	0.64	0.62	0.35	0.96	0.60	0.575
Var. 8	0.65	0.54	0.51	0.68	0.67	0.92	1.01	0.53	0.95	0.80	0.726
Var. 9	0.34	0.37	0.55	0.91	0.51	0.59	0.85	0.38	0.72	0.78	0.600
START K	0.78	0.62	0.55	0.76	0.74	1.10	1.56	0.60	0.98	1.30	0.899
Lag (hrs)	41.2	18.1	42.4	27.5	23.3	51.1	40.1	27.5	11.9	49.2	33.2
CP	1.15	0.84	2.40	0.65	1.31	1.68	1.17	0.87	0.72	1.58	1.24
U.H.P.	22,867.	38,196.	46,374.	19,468.	46,120.	26,908.	23,841.	25,956.	49,442.	26,301.	32,547.
U.H.T.P.	35.	18.	14.	27.	23.	40.	39.	27.	12.	48.	28.3
QP (cfs)	23,800.	10,200.	8,780.	14,800.	10,400.	9,600.	10,100.	7,870.	8,627.	8,000.	11,218.

Table 4
SUMMARY OF OPTIMIZATION RESULTS
Root River Near Houston

may be noted that the optimization process resulted in loss rates of 0.52 in., 0.72 in., and 0.62 in. for the three hours during the major portion of the storm. This is somewhat higher than the values arrived at for the watershed above Lanesboro. Rainfall excess amounted to 0.03, 0.31, and 0.37 in.

Figure 36 illustrates the hydrograph at the next gaging station downstream below South Fork, with a drainage area of 1560 sq. miles. For the watershed above this site substantial rain occurred over a period of four hours with two hours during the center of the storm having almost double the intensity of the other two hours. The observed hydrograph at this site is very similar in shape to that upstream at Lanesboro. The program produced a computed hydrograph similar in shape to that at Lanesboro. Agreement is not quite as good, but still must be considered to be a fairly good approximation of the actual observed flood. Loss rates during the first three hours of the flood, as may be noted in Figure 39, were on the order of 0.46 in. per hour. Time of concentration, variable 1, and the basin lag – both on the order of 18.5 hours – do not seem to be in agreement with the time of concentration for the Root River near Houston. The reason for the discrepancy is not apparent. However, it may be associated with the storm pattern, which is not uniform over the watershed, or possibly with errors in the gaging station's records due to silting of the intakes.

In addition to the data on the Root River area presented in the body of the report, additional graphs and computer printouts are presented in Appendices available through St. Anthony Falls Hydraulic Laboratory. Data have been assembled for a total of ten flood events on the Root River at Houston, eleven events for the Root River near Lanesboro, and lesser numbers for three other gaging stations in the watershed. In some instances, a storm produced significant flooding at all gaging stations. In others the flood event was too small to be significant, or it may have had multiple peaks or other complexities interfering with an optimization of the results. Appendix A contains 82 figures relating to flood events in the Root River area.

The data in the Appendices were assembled in formal graphs with the thought that they might be of interest relative to analytical procedures involving mathematical models other than the one used to make the current study. The past runoff records, the storm pattern, antecedent moisture conditions, and the actual graphs of discharge as a function of time for the selected storms all may be applicable to other methods of analysis. For example, the graphs of computed and observed runoff or discharge show the observed or measured discharge as determined from the actual gaging station charts. These would require considerable effort to prepare or reproduce if they were not already available. Also shown on the same charts are the average hourly precipitation data over the watershed above the gaging station in question. These are based on Thiessen polygon analysis of the hourly precipitation data. Rather than prepare special tables to show the hourly average precipitation, these have been carefully plotted on the graphs showing the computed and observed discharge.

Due to the length of the Appendices only a limited number of copies has been prepared for reference purposes.

Table 4 is a summary of the results of the optimization program for the Root River near Houston. One column has been assigned to each of the storms analyzed. The data include:

1. The total value of average rainfall over the watershed.
2. The total average loss over the watershed.
3. The excess rain over the watershed expressed in inches.
4. The average loss over the watershed expressed in per cent of the total rainfall for the same storm.
5. The maximum one-hour loss in inches.
6. The rainfall for that same one-hour period indicated as max. rain.
7. The volume of observed runoff during the course of the flood. This includes both base flow and direct or storm runoff and is expressed in sec-ft-days. One sec-ft-day is about equal to two acre-feet.
8. The computed volume of runoff.
9. Per cent error between the computed and observed runoff. In effect this is a measure of the accuracy with which the computed hydrograph corresponds to the observed or measured hydrograph.
10. QRCSM and RTIOR (concerning low flow recession).
11. Variables 1 through 9, which have been defined.
12. STARTK, initial value of loss coefficient in the equation. $\text{Loss} = K(\text{Rain})^E$.
13. CP, Snyder's coefficient $= \frac{Q(\text{Lag})}{645A}$.
14. UHP, the peak value of the unit hydrograph developed by the optimization program for that particular storm.
15. UHTP, the time from the beginning of the unit hydrograph to the peak ordinate or value of the unit hydrograph.
16. QP, the observed peak discharge.

As may be noted in Table 4, the total rainfall in the storms analyzed in this study ranged from 1.78 in. to 5.94 in. The maximum one-hour rainfall in these storms ranged from 0.31 in. to 1.28 in. The corresponding maximum one-hour loss ranged from 0.23 in. to 1.03 in. In other words, the maximum one-hour loss over the watershed as determined by this mathematical model ranged up to 1.03 in. in one hour. Actually, the loss is a function of many factors, including antecedent conditions, intensity, duration, path and pattern of the storm. In the ten storms analyzed for the Root River above Houston the average loss was 86.7 per cent, which meant that the rainfall excess was equal to about 13.3 per cent of the total rainfall.

Referring to item 3 it may be noted that the total rainfall excess for the storm ranged from 0.17 in. for two storms to 1.05 in. In general, it is desirable to have large values of rainfall excess or direct runoff in arriving at a unit hydrograph. Values as small as 0.17 in. could introduce considerable error in developing a unit hydrograph.

Relative to item 9 it may be noted that the average error for all ten storms in terms of volume of runoff was only 3.0 per cent. This indicates good agreement between computed and observed volumes of runoff.

In using this program the operator usually selected values of QRCSM and RTIOR, then permitted the program to operate and optimize the nine variables. If a good fit was not obtained the values of QRCSM and RTIOR were adjusted in an attempt to improve the fit. Average value of QRCSM for the ten storms in the Root River above Houston was 1993 cfs, and the average value of RTIOR was 1.12.

Referring to variable 1, it may be noted that the time of concentration varied from 16.1 hours to a maximum of 85 hours with an average of about 47.3 hours. Apparently two of the most significant factors affecting this variable were the storm center or location and the direction of travel of the storm. Storms of July 13, 1950, and August 28, 1960, had the shortest times of concentration, the smallest amount of excess rainfall (0.17 and 0.19 in.), and the highest unit hydrograph peaks. As would be expected, a short time of concentration would produce a steep, high-peaked unit graph. Storms for these two floods were generally centered over the lower part of the watershed where the basin slope is quite steep. In the storm of June 13, 1950, there were 4.5 in. of rain near Rushford, very near the downstream end of the watershed, and considerably less than one inch over most of the upper third of the watershed. In the storm of August 28, 1960, the storm was centered at Lanesboro with the point precipitation of 3.5 in. A large portion of the North Branch of the Root River had less than 2 in. of rain.

The storm of August 29, 1962, resulted in a time of concentration, or variable 1, of 85 hours. This produced rainfall on the order of 6 in. from the headwaters to the gaging station at Houston on an east-west axis through the watershed. Thus, in this storm there were substantial contributions from the complete watershed resulting in a long time of concentration. This is probably much more realistic relative to a true time of concentration than the values on the order of 15 to 18 hours resulting from the storms referred to above.

The storm of June 28, 1942, had an excess rainfall of 1.05 in. and was generally centered in the upper part of the watershed; its direction of travel was downstream. Variable 1 as determined by the program was 62 hours, which seems somewhat large when compared with the data as plotted in Figure 13.

Item 23 in the table is Snyder's CP, which ranges from 0.65 to 2.40. Snyder originally found values of this variable ranging from 0.56 to 0.69. The value of 2.40 was obtained in the storm of June 13, 1950, corresponding to a rainfall excess of 0.19 in., a short time of concentration (18 hours), and a very high peak on the unit hydrograph. The value of 2.40 is probably unrealistically high and may form a basis for either further study or rejection of this particular flood in an overall study of the watershed. A hydrologist making a study of this particular watershed would probably reject some of the floods and storms included in this study. It would appear that the optimization program used here or a similar model would also provide data for the selection or rejection of the storms to be used in a given study. For example, unrealistic values of some of the variables should provide a basis for rejection of some of the data, or at least further study of that particular storm.

The peak values of the unit hydrograph are shown as item 25 in the table; in addition, the unit hydrographs obtained through the optimization program have been plotted in Figure 45. These show considerable variation in the shape and magnitude of the peak discharge. Further study of these hydrographs and of the optimization data would be necessary in order to select a representative unit hydrograph. Actually, the program will accept data for several storms and optimize the results. In the present study it was thought that the results would be of most interest if each storm were analyzed on an individual basis. This would show the range of variation in the variables associated with the program and with the watershed.

Tables 5 and 6 summarize the optimization results of other gaging stations or watersheds of the Root River watershed area.

Figure 41 shows computed and observed hydrographs of the Le Sueur River for a storm event of June, 1945, as an example of comparative results. Additional floods are included in Appendix B. Due to the excessive length of the flood on this particular watershed, as may be noted in Figure 41 as a flood length in excess of 16 days, it was necessary to use three-hour periods instead of one-hour periods in the optimization program. Table 7 is a summary of the optimization data for the Le Sueur watershed and Figure 46 summarizes the unit hydrographs obtained from this program.

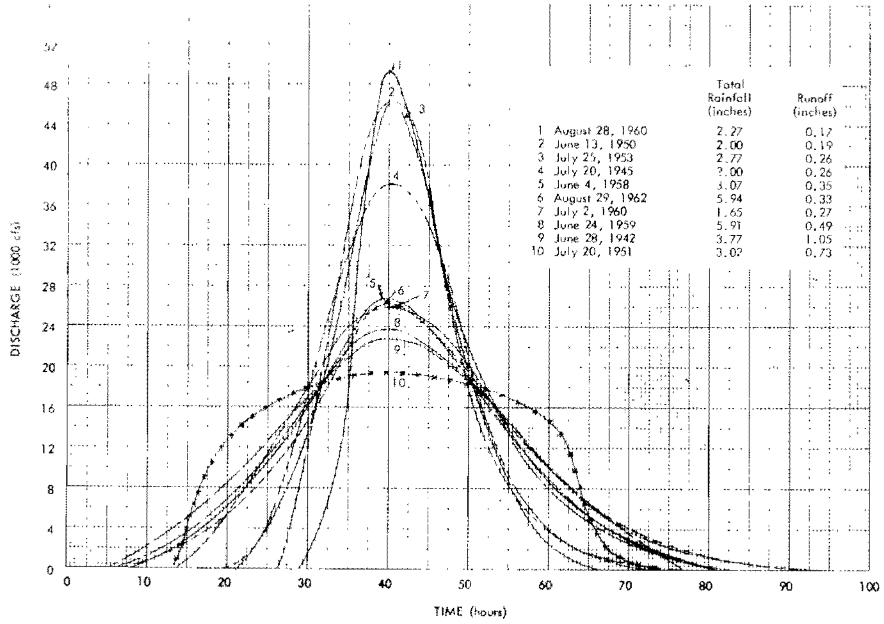

Figure 45. Unit hydrographs for Root River near Houston, as determined by optimization program

	June 28 1942	July 20 1945	June 13 1950	July 20 1951	July 25 1953	June 4 1958	June 24 1959	June 27 1960	July 2 1960	Aug. 28 1960	Aug. 29 1962	Average
Total Rain (in.)	3.97	1.63	.97	2.28	2.31	3.40	5.22	.54	1.29	2.54	5.41	2.69
Total Loss (in)	2.79	1.48	.79	1.22	2.05	2.55	4.28	.21	.76	2.46	4.89	2.14
Total Excess (in)	1.18	.15	.18	1.06	.26	.85	.94	.33	.53	.08	.52	.55
Avg. % Loss	70.3	90.8	81.4	53.5	88.7	75.0	82.0	38.9	58.9	96.9	90.4	75.2
Max. Loss	.78	.27	.22	.38	.48	.67	.60	.09	.19	.84	.35	.44
Max. Rain	1.43	.30	.30	.90	.52	.82	1.23	.41	.48	.87	.45	.70
Vol. Obs. (sfd)	22,204.	3,992.	3,308.	17,052.	6,710.	16,450.	14,390.	5,209.	11,335.	3,279.	11,397.	10,484.
Vol. Comp. (sfd)	20,282.	3,869.	3,564.	17,766.	6,929.	16,422.	14,898.	5,576.	10,997.	2,556.	10,818.	10,334.
% ERR	-3.8	-3.1	+7.7	+4.2	+3.3	-0.2	+3.5	+7.0	-3.0	-22.0	-5.1	-1.0
QRCSN	700.	650.	250.	1,600.	2,000.	2,000.	1,300.	1,200.	1,050.	500.	1,050.	1,118.
RTIOR	1.20	1.12	1.04	1.24	1.30	1.24	1.07	1.22	1.10	1.14	1.12	1.16
Var. 1	23.05	7.43	4.93	14.52	13.13	46.29	20.71	6.90	17.89	2.64	62.10	19.96
Var. 2	0.56	.53	1.83	1.12	.14	.03	1.00	2.03	.84	1.25	.11	.86
Var. 3	1.00	1.07	4.36	1.12	1.33	2.80	1.00	2.42	1.35	2.39	6.38	2.293
Var. 4	.05	.03	.01	0.	0.	.05	.05	0.	.08	.03	.05	.0318
Var. 5	4.56	1.80	6.44	6.68	5.92	1.60	1.95	20.25	2.00	4.00	4.43	4.93
Var. 6	0.	.02	.01	.05	0.	.02	.01	.08	.03	.01	.01	.02
Var. 7	.18	.12	.16	.26	.53	.01	.33	.08	.24	.01	.09	.18
Var. 8	.73	.31	.23	.37	.59	.68	.61	.09	.25	1.22	.38	.50
Var. 9	.61	.05	.14	.02	.13	.19	.52	.05	0.	.59	.28	.23
START K	.89	.32	.24	.41	.70	.72	.69	.09	.25	1.42	.53	.57
Lag (hrs)	23.5	6.3	3.9	14.6	-.8	25.3	21.6	6.1	17.0	2.1	35.0	13.9
CP	.84	.70	.35	.58	-.07	1.41	.65	.35	.67	.44	2.0	.72
U.H.P.	14,211.	44,085.	35,285.	15,759.	36,652.	22,139.	11,937.	22,636.	15,601.	81,213.	21,931.	29,222.
U.H.T.P.	23.	7.	4.	15.	9.	25.	21.	7.	17.	3.	35.	15.09
QP (cfs)	15,000.	6,450.	6,500.	16,400.	8,400.	17,900.	9,150.	7,940.	8,100.	5,300.	8,700.	9,985.

Table 5
SUMMARY OF OPTIMIZATION RESULTS
Root River Near Lanesboro

	Root River Below South Fork Near Houston					South Fort Root River Near Houston			Rush Creek Near Rushford
	June 13 1950	July 20 1951	July 25 1953	June 4 1958	Average	Aug. 28 1960	Aug. 29 1962	Average	July 20 1945
Total Rain (in)	1.93	3.22	2.53	2.60	2.57	1.06	6.88	3.97	3.90
Total Loss (in)	1.71	2.08	2.27	2.31	2.09	1.02	6.56	3.79	3.16
Total Excess (in)	0.22	1.14	0.26	0.29	.48	.04	.32	.18	.74
Avg. % Loss	88.7	64.6	90.0	89.0	83.0	96.4	95.5	96.0	81.0
Max. Loss	.52	.48	.39	.68	.52	.59	.73	.66	1.46
Max. Rain	.60	.93	.40	.97	.73	.63	.96	.79	1.50
Vol. Obs. (sfd)	10,984.	42,812.	15,616.	15,403.	21,200.	572.	2,930.	1,750.	2,330.
Vol. Comp. (sfd)	10,523.	44,827.	15,641.	16,511.	21,875.	576.	3,204.	1,890.	2,705.
% ERR	-4.4	4.7	0.2	7.2	4.1	0.6	9.4	5.0	16.2
QRCSN	2,000.	2,600.	2,600.	3,500.	2,675.	130.	500.	315.	170.
RTIOR	1.25	1.13	1.03	1.25	1.16	1.09	1.25	1.17	1.30
Var. 1	27.85	18.50	31.64	75.	38.25	21.6	14.	17.8	6.78
Var. 2	.07	1.43	.25	.04	.44	.35	.43	.39	1.28
Var. 3	3.44	1.25	1.36	2.61	2.16	3.66	1.00	2.33	1.00
Var. 4	.04	.03	.04	.01	.03	.02	.02	.02	.03
Var. 5	1.00	1.70	2.64	1.58	1.73	5.46	1.00	3.23	3.77
Var. 6	.08	.02	.02	.02	.04	.03	.01	.02	.01
Var. 7	.19	.42	.33	.08	.25	.08	.10	.09	1.00
Var. 8	.47	.48	.49	.68	.53	.54	.75	.64	.79
Var. 9	.60	.77	1.15	.63	.79	.35	.96	.65	1.02
START K	.47	.50	.54	.71	.56	.58	.75	.66	.98
Lag (hrs)	16.0	18.5	23.4	41.4	24.83	14.3	13.7	14.0	6.8
CP	1.56	.50	.83	1.31	1.05	1.00	.87	.93	.51
U.H.P.	98,390.	27,030.	35,670.	31,748.	48,210.	12,400.	11,307.	11,853.	6,314.
U.H.T.P.	16.	18.	23.	40.	24.	14.	14.	14.	7.
QP (cfs)	20,200.	32,500.	10,900.	9,700.	18,325.	1,460.	2,820.	2,140.	4,200

Table 6
SUMMARY OF OPTIMIZATION RESULTS

	June 9 1945	July 5 1945	June 1 1951	May 25 1953	June 17 1956	May 16 1960	Aug. 30 1962	Sept. 7 1964	Average
Total Rain (in)	2.75	1.65	2.69	1.96	4.24	6.61	5.20	4.61	3.71
Total Loss (in)	1.51	1.05	1.99	1.01	3.68	3.80	4.21	3.94	2.65
Total Excess (in)	1.24	0.61	0.70	0.95	0.56	2.81	0.99	0.67	1.07
Avg. % Loss	55.0	63.7	74.0	51.6	86.8	57.5	81.0	85.6	69.4
Max. Loss	.18	.42	.38	.65	1.35	.18	.87	1.50	0.69
Max. Rain	.57	.50	.72	1.56	1.79	.63	1.39	1.95	1.14
Vol. Obs. (sfd)	38,909.	19,381.	18,233.	29,546.	17,288.	98,052.	32,097.	17,474.	33,873.
Vol. Comp. (sfd)	39,102.	19,866.	18,708.	27,884.	15,686.	95,011.	31,388.	17,255.	33,238.
% FRR	+0.5	+2.5	+2.6	-5.6	-9.3	-3.1	-5.5	-1.3	3.8
QRCSN	2,000.	900.	890.	1,850.	1,760.	10,000.	1,020.	1,200.	2,452.
RTIOR	1.21	1.23	1.20	1.25	1.36	1.29	1.14	1.24	1.24
Var. 1	100.	82.54	87.84	46.45	23.00	75.69	62.	96.05	71.70
Var. 2	1.53	1.83	1.99	2.41	3.15	0.78	2.01	1.28	1.87
Var. 3	1.00	1.00	1.58	1.00	1.00	7.54	1.00	1.00	1.89
Var. 4	.05	.00	.05	.11	.05	.05	.12	.08	.06
Var. 5	1.00	4.52	1.00	5.58	1.52	2.19	1.00	1.95	2.35
Var. 6	.00	.05	.01	.08	.01	.02	.02	.01	.03
Var. 7	.14	.26	.34	.29	.30	.15	.02	.30	0.23
Var. 8	.08	.18	.22	.29	.53	.08	.34	.57	.29
Var. 9	.05	.18	.23	.29	.53	.15	.25	.57	.28
START K	.08	.19	.22	.31	.57	.09	.34	.64	.31
Lag (hrs)	105.0	85.8	86.7	48.8	25.3	49.7	64.9	99.6	70.7
CP	.50	.43	.39	.35	.29	.67	.40	.56	.45
U.H.P.	3,371.	3,568.	3,195.	5,086.	8,105.	9,529.	4,403.	3,962.	5,152.
U.H.T.P.	102.	84.	84.	48.	27.	51.	63.	96.	69.
QP (cfs)	4,450.	2,575.	2,230.	4,405.	3,990.	21,230.	3,405.	2,440.	5,590.

Table 7
SUMMARY OF OPTIMIZATION RESULTS
Le Sueur River Near Rapidan

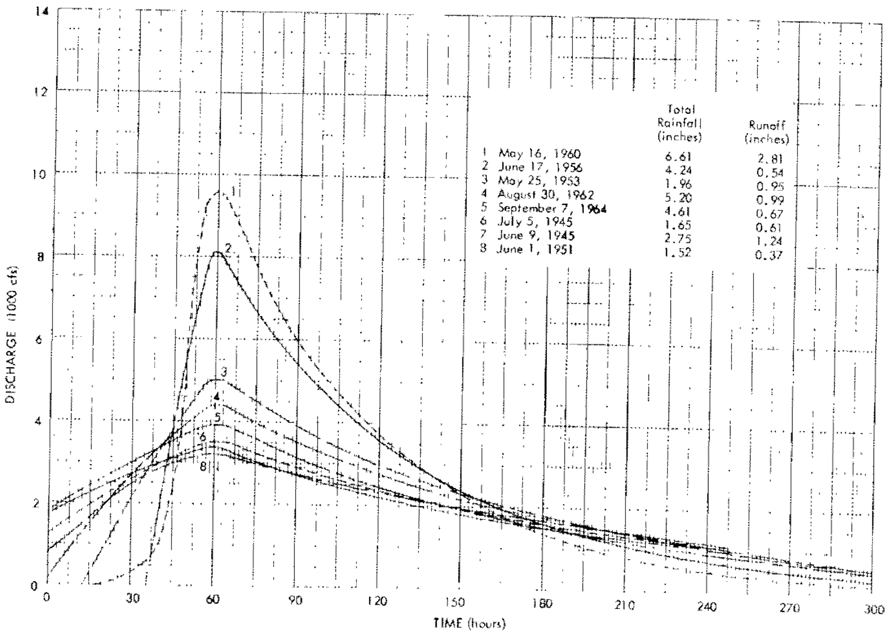

Figure 46. Unit hydrographs for Le Sueur River near Rapidan, as determined by optimization program

Referring to the individual storms presented in Appendix B it may be noted that in some instances double peaks have resulted. This is probably associated with the fact that the three main streams comprising the drainage system the Le Sueur, the Cobb, and the Maple Rivers have approximately equal areas and have a fan-shaped layout. The three streams join just above the gaging station. A variation in the time distribution of precipitation over the three main streams would produce multiple peaks.

The most severe storm studied in this series (May 1960) had an average precipitation over the watershed of 6.6 in. Official point rainfalls as high as 8 in. were noted, and some unofficial figures went as high as 9.63 in. This was distributed over a three-day period. It produced a flood of 21,230 cfs, the second largest flood of record. Also of interest is the fact that it resulted in the largest unit graph, even though the storm was quite long (Figure 46).

The second largest unit graph was produced by the storm of June 17, 1956. This had an average precipitation over the watershed of 4.24 in. and produced a peak flow of 3990 cfs. It also produced a double peak on the hydrograph and rather poor agreement between computed and observed runoff.

During the storm of September 7, 1964, most of the rain occurred in a very short period (approximating a single burst), which is of interest for comparison with the May 1960 storm. A rather low unit graph resulted.

Referring to Table 7, it may be noted that the time of concentration, variable 1, ranged from 23 hours to 100 hours, with an average of 71.7 hours. An inspection of the hydrographs in Appendix B indicates that the time to peak was generally on the order of four days.

Figure 42 illustrates a typical computed and observed hydrograph for the Middle River. Figure 47 summarizes the unit hydrographs obtained from the computer program, and Table VIII summarizes the computer output for the Middle River. Additional data are in Appendix C. While other storms were studied, only four were completely analyzed and satisfactory results obtained. The largest flood of this series produced a peak discharge of around 1390 cfs resulting from a rain of about 4.7 in. over the watershed in July 1956. This storm had precipitation ranging from 2.3 in. near the gaging station to about 7 inches in the headwaters of the Middle River. The flood was caused by rather severe rains on July 4 and 5 followed by another severe rain on July 7. This included 1.05 inches in 3 hours on July 4 and 0.91 inches in 3 hours on July 7. A single peak flood resulted. This storm also produced the largest unit hydrograph, with a peak value of 1486 cfs.

In September 1957 a storm produced 4.1 in. of precipitation over the watershed, apparently very evenly distributed. This produced a peak flood of 740 cfs and a unit hydrograph peak of 1430 cfs. Most of the rain occurred over a period of about 36 hours. Very good agreement was obtained between observed and computed runoff. In July 1958 a storm deposited precipitation with an average value of 3.8 in. over the watershed, ranging from 2 in. at the gaging station to 4.5 inches in the headwater area. This is the flood shown in Figure 42. Most of the rain occurred over a period of about 30 hours on July 3 and 4, 1958. A single peak flood resulted and very good agreement was obtained between computed and observed runoff. The unit hydrograph had a substantially lower peak than the preceding two floods due in part to a flatter recession curve on the flood hydrograph.

In June, 1955, 1.6 in. of rain fell over a three-day period. Due to the fact that most of these storms extended over a substantial period of time, it is somewhat difficult to define the time to peak or time of concentration. Variable 1 in the computer program ranged from 166 hours to 217 hours. These values appear long as compared to visual inspection of some of the graphs.

Figure 43 is a typical graph showing computed and observed discharges on the Embarrass River, Figure 48 shows the unit hydrographs obtained from the computer program, and Table 8 is a summary of the computer output data for the Embarrass River. A total of five floods were analyzed during the course of the study. These had an average precipitation over the 94-sq-mile area of the watershed ranging from 3.1 in. to 5.36 in. These produced floods with peaks ranging from 406 cfs to 875 cfs. As may be noted in Figure 43, good agreement was obtained between computed and observed discharges. This was generally true for all floods analyzed. The 1952 flood had a rather even distribution of precipitation over the watershed and produced about an average unit hydrograph. As may be noted in Table 8, the time of concentration ranged from 70 to

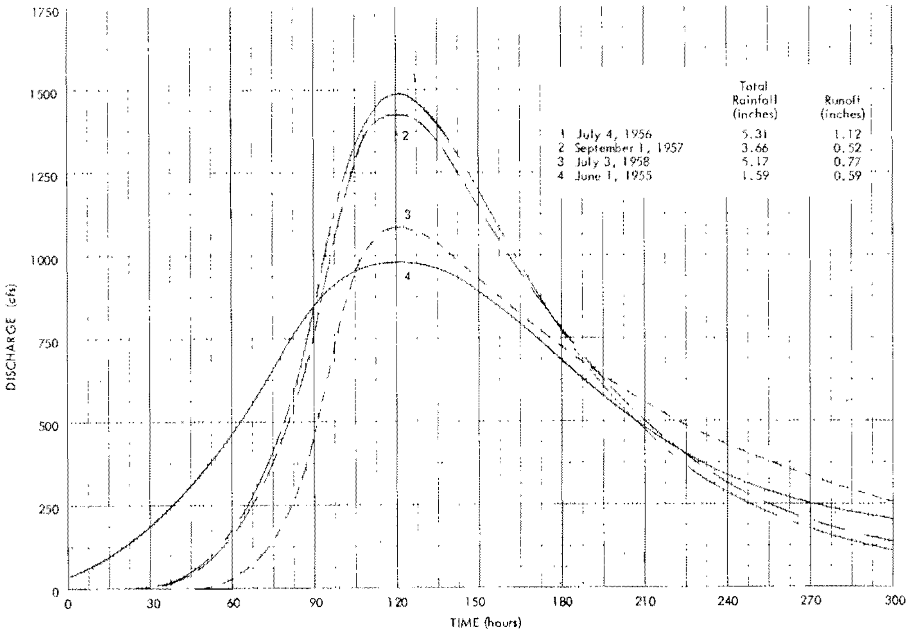

Figure 47. Unit hydrographs for Middle River near Argyle, as determined by optimization program

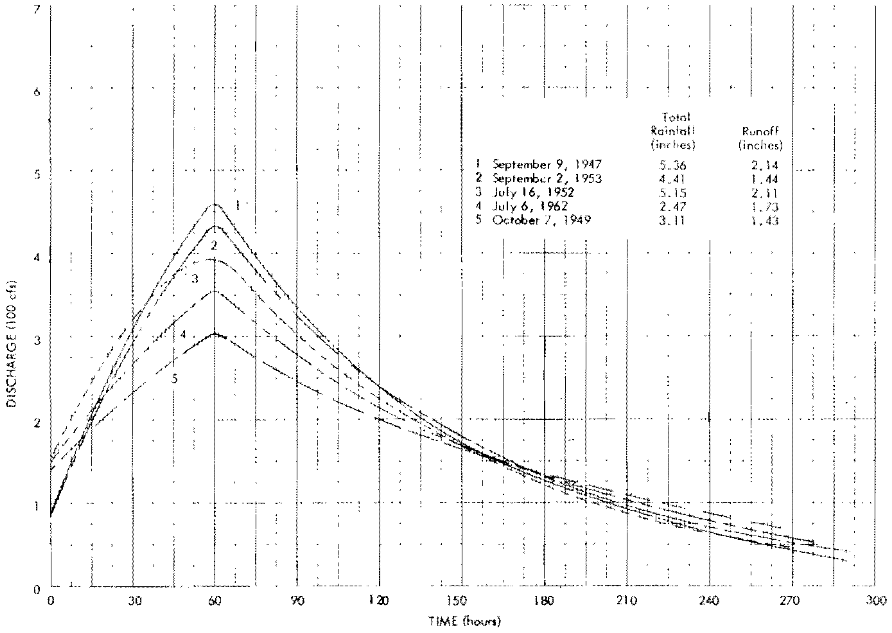

Figure 48. Unit hydrographs for Embarrass River near Embarrass, as determined by optimization program

	Middle River Near Argyle				Embarrass River Near Embarrass					
	June 1 1955	July 4 1956	Sept. 1 1957	July 3 1958	Average	Sept. 9 1947	Oct. 7 1949	July 16 1952	Sept. 2 1953	July 6 1962	Average
Total Rain (in)	1.59	4.72	4.10	3.82	3.56	5.36	3.11	3.72	4.41	2.47	3.81
Total Loss (in)	1.00	3.60	3.56	3.05	2.80	3.22	1.68	1.61	2.97	.74	2.04
Total Excess (in)	.59	1.12	0.54	0.77	.76	2.14	1.43	2.11	1.44	1.73	1.77
Avg. % Loss	62.9	76.3	86.8	79.8	76.4	6.01	54.0	43.3	67.3	29.9	50.9
Max. Loss	.25	.73	.75	.42	.54	.78	.28	.32	.92	.14	.49
Max. Rain	.26	1.05	1.02	.81	.78	1.58	.77	1.09	1.95	.43	1.16
Vol. Obs. (sfd)	4,397.	7,708.	4,108.	5,157.	5,342.	5,551.	3,120.	5,612.	4,097.	4,130.	4,500.
Vol. Comp. (sfd)	4,553.	8,296.	3,976.	5,074.	5,475.	5,269.	3,054.	5,630.	3,972.	4,134.	4,410.
% ERR	+3.5	+7.7	-3.2	-1.6	1.6	-5.0	-2.0	+0.3	-3.0	+0.1	2.1
QRCSN	200.	400.	200.	150.	238.	245.	130.	425.	205.	190.	239.
RTIOR	1.27	1.20	1.27	1.14	1.22	1.02	1.16	1.15	1.17	1.30	1.16
Var. 1	217.05	168.12	213.11	166.75	191.26	70.	96.78	100.	70.22	90.	85.40
Var. 2	.39	.37	.32	.69	.44	1.30	1.49	.97	1.40	1.31	1.29
Var. 3	2.42	3.54	4.92	5.42	4.08	1.00	1.00	1.38	1.00	1.00	1.08
Var. 4	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05
Var. 5	7.57	19.10	3.25	1.52	7.86	1.00	1.95	1.00	1.00	1.00	1.19
Var. 6	0.	.01	.04	.02	.02	.04	0.	0.	.03	0.	.01
Var. 7	.60	.27	.33	.33	.38	.47	.61	.52	.26	.83	.54
Var. 8	.17	.26	.32	.23	.24	.36	.22	.19	.36	.18	.26
Var. 9	.97	.55	.31	.23	.51	.02	.08	.04	.29	.59	.20
START K	.18	.41	.39	.24	.30	.36	.23	.19	.36	.18	.26
Lag (hrs)	155.2	113.7	136.0	114.8	129.9	72.8	101.2	96.7	73.7	93.4	87.5
CP	.89	.98	1.13	0.73	.93	0.55	0.50	.62	.53	.55	.55
U.H.P.	980.	1,486.	1,430.	1,087.	1,246.	459.	303.	392.	433.	355.	388.
U.H.T.P.	50.	37.	44.	37.	42.	24.	33.	31.	24.	31.	29.
QP (cfs)	613.	1,390.	745.	821.	892.	875.	406.	808.	618.	573.	656

Table 8
SUMMARY OF OPTIMIZATION RESULTS

100 hours, which seems large for the small area represented by this watershed. Likewise, an inspection of Figure 43 indicates a time to peak from the end of excess rainfall of about 3-1/2 days. This is probably associated with rather high depression storage in the glacial lake beds through which the main stream passes.

The storm of September 2, 1953, is of interest because it produced an average precipitation of 1.95 inches in one three-hour period, or a total of 2.98 inches in one six-hour period. TR periods of 180 minutes were used in the computer solution of this problem, similar to those for the Middle and Le Sueur Rivers.

The computer data on CP for the Embarrass River indicated values ranging from 0.50 to 0.62. These are in good agreement with Snyder's original values and show very little variation.

Figure 44 illustrates typical computed and observed hydrographs for the Baptism River, Figure 49 illustrates the unit hydrographs obtained from the computer program, and Table 9 is a summary of the computer output for the Baptism River. This watershed has an area of 140 sq. miles and a high average slope. The main stream and tributaries have slopes on the order of 40 to 50 ft. per mile, resulting in rather high velocities in the streams. The storms used in this study had average precipitations over the watershed ranging from 1.60 in. to 3.69 in. In general losses were quite small as compared to some of the other watersheds, with an average of only 30 per cent loss, giving 70 per cent runoff.

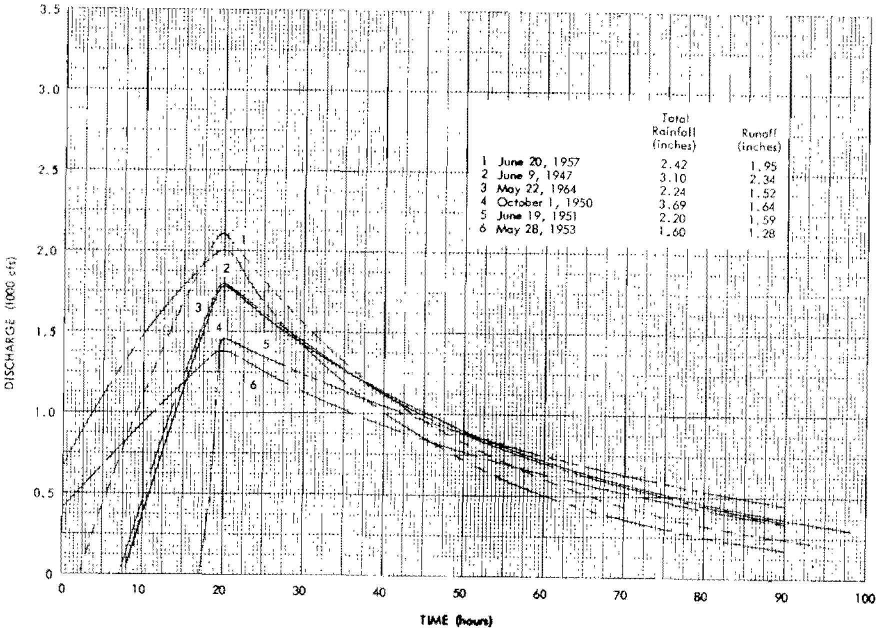

Figure 49. Unit hydrographs for Baptism River near Beaver Bay, as determined by optimization program

	June 9 1947	Oct. 1 1950	June 19 1951	May 28 1953	June 20 1957	May 22 1964	Average
Total Rain (in)	3.10	3.69	2.20	1.60	2.42	2.24	2.54
Total Loss (in)	.76	2.05	.61	.32	.47	.72	.82
Total Excess (in)	2.34	1.64	1.59	1.28	1.95	1.52	1.72
Avg. % Loss	24.4	55.6	27.8	20.0	19.4	32.2	29.9
Max. Loss	.17	.26	.07	.03	.06	.09	.11
Max. Rain	.64	.43	.32	.04	.32	.44	.36
Vol. Obs. (sfd)	10,285.	6,314.	5,330.	5,921.	8,542.	6,260.	7,110.
Vol. Comp. (sfd)	9,066.	6,200.	5,283.	5,503.	8,854.	6,065.	6,830.
% ERR	-11.8	-1.8	-0.8	-7.0	+3.6	-3.1	4.7
QRCSN	580.	500.	810.	460.	1,200.	500.	675.
RTIOR	1.18	1.15	1.14	1.05	1.15	1.13	1.13
Var. 1	26.00	12.07	4.00	25.84	18.05	12.88	16.47
Var. 2	1.13	3.60	15.03	1.05	1.78	3.30	4.47
Var. 3	1.00	1.00	6.63	1.00	1.00	1.00	1.94
Var. 4	.02	.06	.13	.11	.09	.02	.07
Var. 5	4.67	30.24	7.75	11.39	15.19	1.58	11.80
Var. 6	0.	.02	.01	0.	0.	0.	.01
Var. 7	.99	.18	.79	.29	.60	.45	.55
Var. 8	.25	.14	.21	.03	.05	.12	.13
Var. 9	.54	.73	.39	.28	.55	.14	.44
START K	.25	.18	.21	.03	.06	.12	.14
Lag (hrs)	26.7	12.6	3.8	27.0	18.8	14.1	17.2
CP	.59	.25	.06	0.41	0.44	0.28	.34
U.H.P.	2,018.	1,789.	1,461.	1,384.	2,116.	1,809.	1,763.
U.H.T.P.	26.	13.	4.	17.	19.	14.	15.
QP (cfs)	3,250.	2,840.	2,120.	1,635.	2,700.	2,160.	2,450.

Table 9
SUMMARY OF OPTIMIZATION RESULTS
Baptism River Near Beaver Bay

Large storms with a single burst of rainfall were difficult to find for this watershed, and as a result most of the storms analyzed were rather complex, with rain lasting from one to four days. The storm of June 1947 involved several periods of rain, but most of it occurred on June 10. A second storm on June 13 produced a second peak to the hydrograph. It was for this reason that this hydrograph was included in the body of the report, to illustrate a record with multiple peaks. The six storms analyzed resulted in peak discharges ranging from 1635 to 3250 cfs. The peak ordinates of the unit hydrographs ranged from 1384 to 2116 cfs.

The analysis of precipitation for this watershed is somewhat difficult due to the fact that there are no precipitation stations in the watershed. One station is located about 15 miles northeast of the center of the watershed; the second is located about 28 miles southwest of the center of the watershed. A lack of accurate precipitation data for the watershed would seriously influence the optimization process, the variables optimized, and the unit hydrograph developed by the program. Referring to Figure 49, it may be noted that there is considerable variation in the shape of the rise portions of the unit hydrographs. This is probably influenced in part by the time distribution of rainfall in the storm, but may also be affected by inadequate rainfall data. The unit hydrograph for the storm of June 19, 1951, has an extremely steep front or rise curve; this is characteristic of the actual hydrograph, in which the flow increased from about 100 cfs to 2100 cfs in 6 hours. The major portion of the rainfall in this storm occurred over a period of about 6 to 8 hours. Agreement between computed and observed hydrographs was very good. The storm of June 1957 produced a flood of 2700 cfs and was the largest unit hydrograph of the group. It was generated by a rather complex storm involving 3 periods of rain over a 4-day period.

Variable 1, the time of concentration, ranged from 4 hours for the 1951 storm to 26 hours for the storm of June 1947. The 4-hour value appears to be somewhat low. Likewise, the rise of 26 hours appears to be on the high side. There was considerable variation in the value of CP, ranging from 0.06 to about 0.59. The nominal value is on the order of 0.4.

Multiple Storm Analysis

As noted in an earlier section, the optimization program prepared by the Corps of Engineers can optimize up to six storms or hydrographs for the same station. In this study it was considered desirable to optimize only one storm at a time to observe the variation in some of the variables. Also, some difficulty was encountered with this feature on multiple storms, although it was probably due in part to lack of experience on the part of the operators. In Tables 4 through 9 one column has been devoted to an average of the variables for the storms that were studied. It would have been of interest to utilize average values of these variables in computing new runoff hydrographs. Time did not permit this study. It is probable that an optimization of the variables for up to six storms would result in different values of the variables than would an average of storms that were individually optimized.

Loss Rate Analysis

The optimization and loss rate program determines from the observed rainfall the amount of rainfall excess. The rainfall excess is the quantity that is left to run off the land surface after all losses have been satisfied. The losses may include, but are not limited to, initial wetting of plant and ground surfaces, depression storage, and infiltration.

These losses are lumped together and are determined by an equation of the form:

$$\text{Loss} = K(\text{Rain})^E$$

where Loss is the loss rate in inches per time interval, K is a loss coefficient determined by optimization which decreases as a function of the accumulated loss during the storm, Rain is the average rainfall over the watershed in inches per time interval, and E is an exponent also determined by optimization in the program.

Average values of K were determined for each basin which are plotted in Figure 50. A comparison of the initial values and slope of each line gives an indication of how the loss coefficient changes during a storm. Referring again to the loss equation, it may be seen that the actual loss depends not only on K but also on a function of the rainfall during the time period under consideration.

A comparison of the losses in different basins should reflect the influence of K and of the rainfall exponent E. Such a comparison was made for the condition when the rainfall would just equal the loss. This is plotted in Figure 51. For a given value of accumulated loss, if the rainfall rate were above a particular curve it would indicate that rainfall excess (runoff) would occur. If it were below a given curve it would indicate that there would be no runoff. It should be emphasized that the loss rate shown in the curve would not be the correct value if the rainfall were either greater or less than the value of the curve. Thus, if the rainfall were greater than the curve, the loss rate would also be greater than the value shown by the curve, but the rainfall would exceed the loss. The reverse is true for a point below the curve.

The primary purpose of Figures 50 and 51 is to provide a comparison between the various watershed tested. For example, in referring to Figure 50, the curve of K for the Baptism River initially starts out with a very low value of START K and continues to decrease at a rapid rate. The value of K for the Le Sueur River is less than that for the Root River at Houston and Lanesboro for all losses less than 8 in. Figure 51 brings in the value of E as well as K and provides a comparison of the watersheds studied. Again the Baptism River has the lowest loss rates of any of the watersheds studied. These curves appear to provide an interesting numerical basis for comparing the various watersheds studied with this program. In fact, Figure 51 is one of the more interesting figures in the report. It combines average values of the variables optimized for each watershed studied in a form that has more physical meaning than some of the variables themselves.

Figure 52 presents a summary of data from the frequency analysis of annual floods for all the watersheds studied. Fifty year floods were determined from Figures 14 through 31 for zero skewness as well as the skewness computed for the particular set of data. The results indicate that floods in the Embarrass and Middle Rivers were substantially less than those in the Root and Le Sueur Rivers

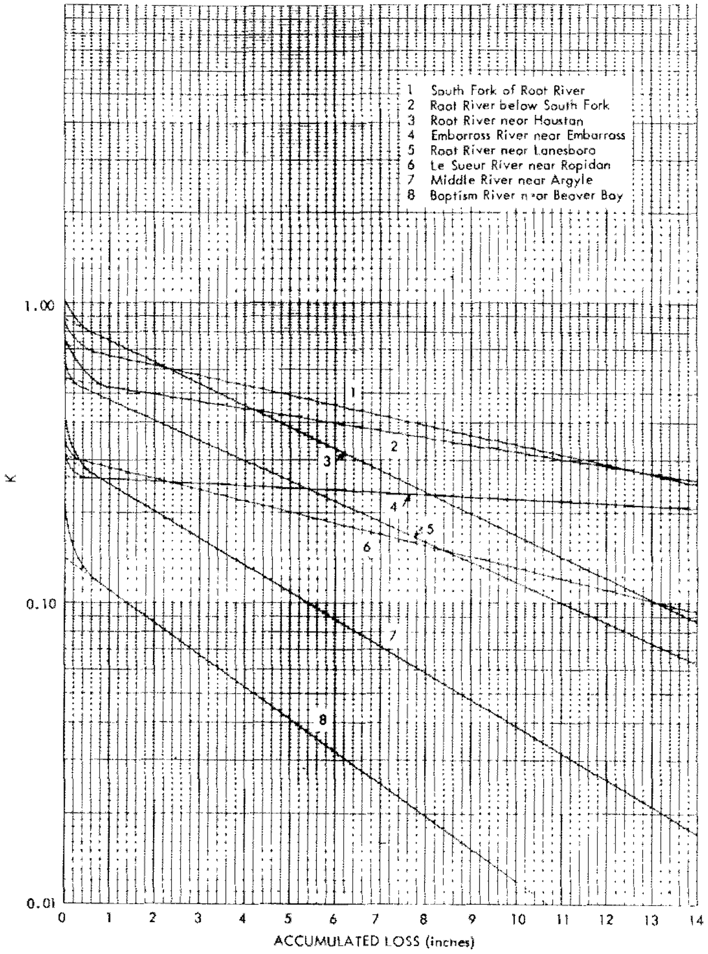

Figure 50.Average curves of loss rate K values.

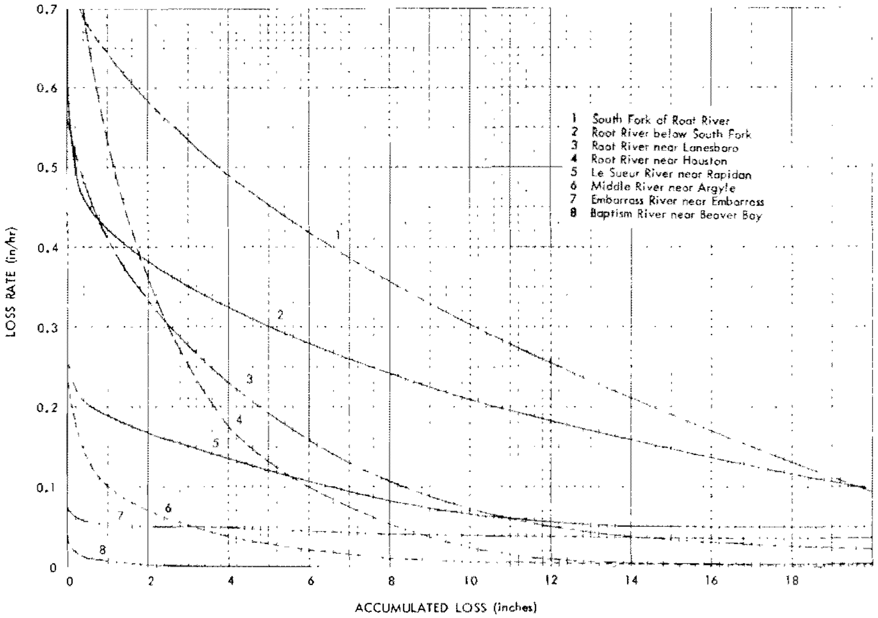

Figure 51. Loss rate in inches per hour as function of accumulated loss, for rain equal to loss

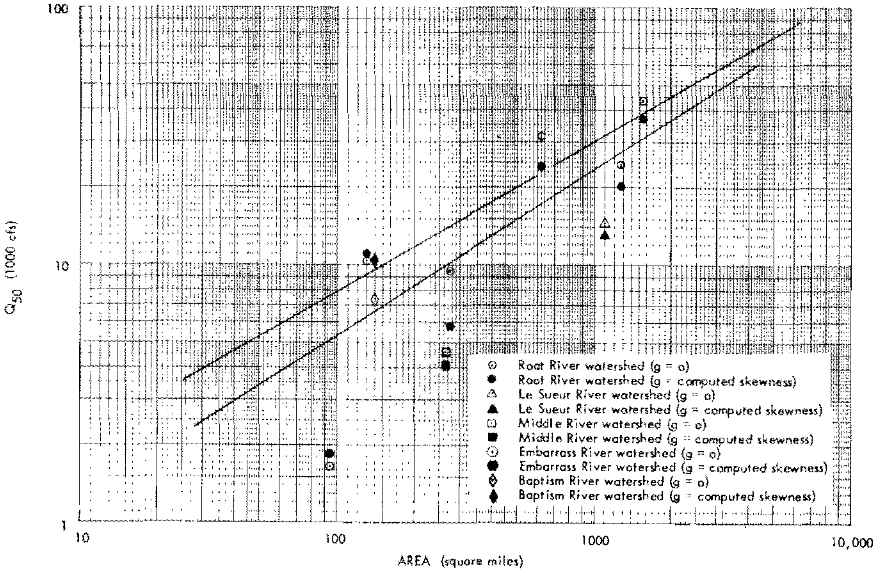

Figure 52. Fifty year floods as a function of basin area for both computed skewness of record and for zero skewness, annual maximum flood series

even though loss rates were small. Similar data have been plotted in Figure 53 for the summer floods, and the same trend is apparent.

Figures 54 and 55 illustrate rainfall loss data taken from the computer output. For example, data are plotted on Figure 54 for the various stations on the Root River. For each storm the top six rainfall increments were selected and the "computed loss" for each hour was plotted as a function of the corresponding rain for that hour. Thus if there were 100 per cent loss the data would form a 45-degree line. All values above 0.10 in. of rainfall were used. Two envelope lines are apparent, one approaching the 45-degree line and the other of a lesser slope. Of the storms tested in the Root River area the minimum loss rate for a one-inch-per-hour rainfall was on the order of 0.45 in. The maximum loss rate was on the order of 0.9 in. These data are of interest as an indication of the hourly loss rates indicated by the mathematical model used by this optimization program. Presumably other mathematical models would produce other results. Further study of these data would be desirable in order to arrive at conclusions as to their effectiveness on design procedures in any period.

The high loss rates noted above may prevail for relatively short storms of high intensity; they would not necessarily hold for lengthy storms of high intensity.

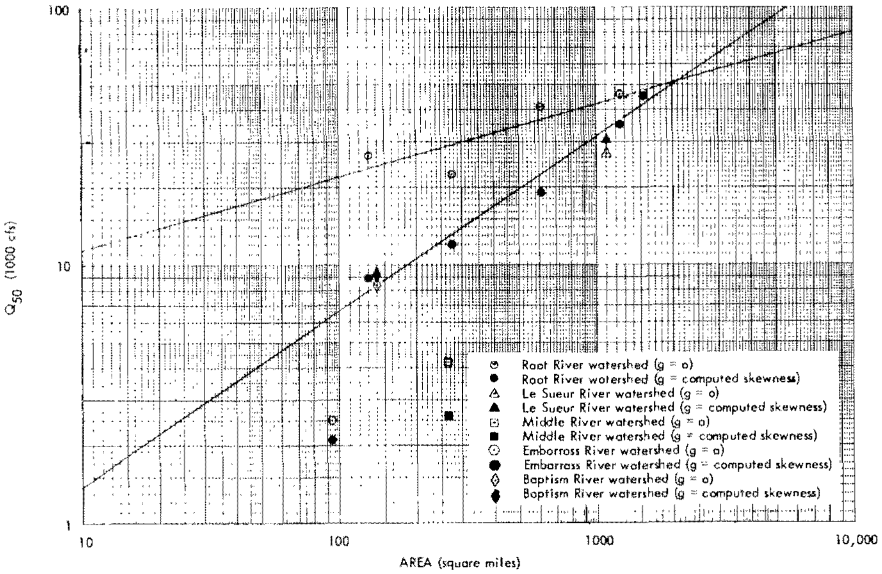

Figure 53. Fifty year floods as a function of basin area for both computed skewness of record and zero skewness, annual maximum summer flood series

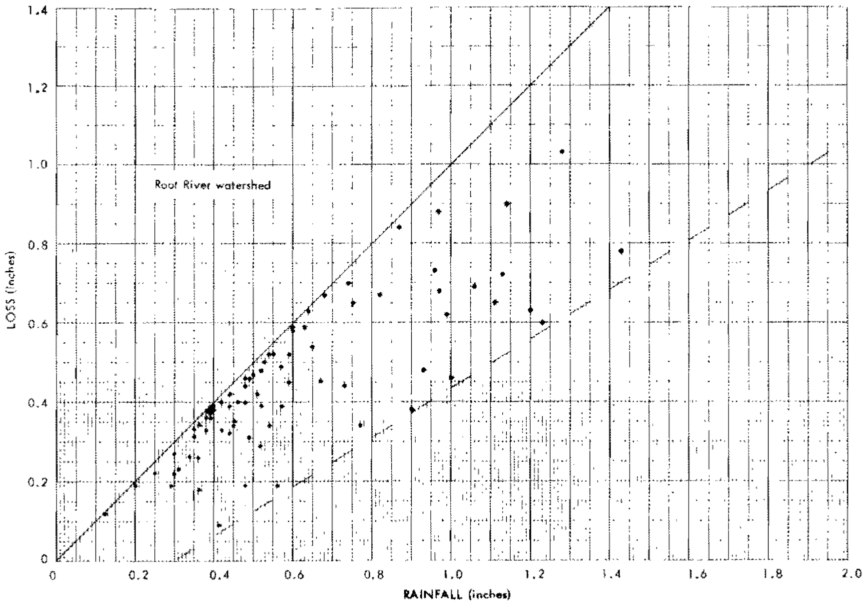

Figure 54. Hourly rainfall loss in inches as function of corresponding hourly rainfall, Root River area

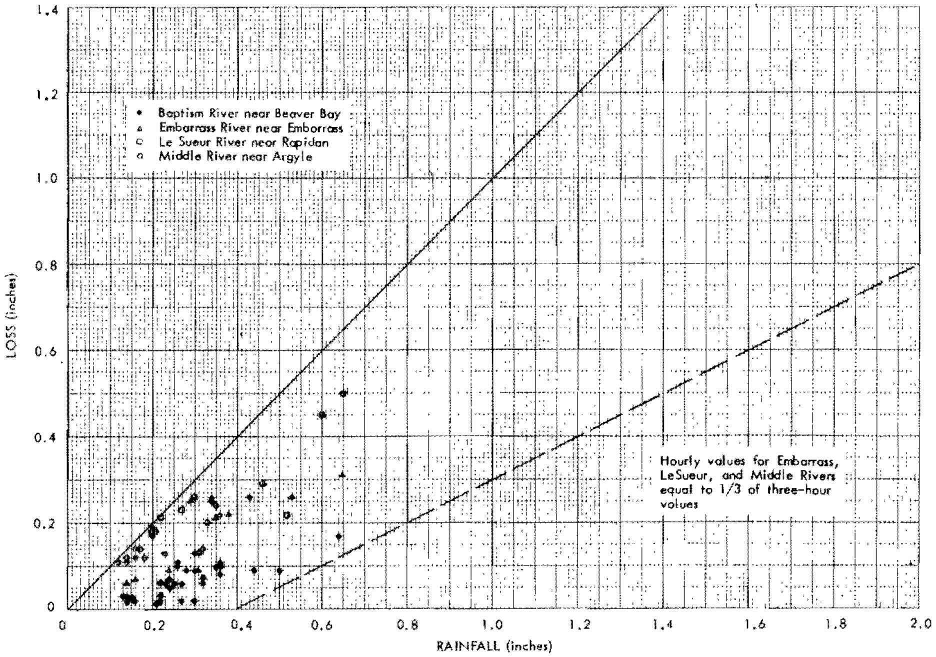

Figure 55. Hourly rainfall loss in inches as function of corresponding hourly rainfall, Le Sueur, Middle, Embarrass, and Baptism Rivers

CONCLUSIONS

The objective of this study was the analysis of available rainfall and runoff data for selected watersheds in the state of Minnesota to assist in the evaluation of peak rates of runoff for design purposes. Of special interest were the basin-wide or watershed-wide loss rates. This involves an analysis of rainfall and runoff as a function of time using average values for the watershed. The results of such an analysis depend to a considerable extent on the analytical procedures or models used. The "unit graph and loss rate optimization program" used in this study is one possible approach. It has the advantage of depending to a considerable extent on established procedures while taking advantage of computer technology to optimize some of the many variables involved in hydrologic processes.

During the course of the study a total of about 12 students were used to assist in the analysis of the data. While this had many advantages, it also resulted in a lack of continuity in some of the work as compared to the processing of data by established federal and state organizations. Further checking of some of the data is desirable.

Conclusions reached during the course of the study included the following:

1. Basin-wide or watershed-wide hourly loss rates were quite high for the storms and watersheds used in this study. These may be noted in Figures 54 and 55 and in the computer output sheets (in the appendices) for each of the storms analyzed.

2. Storm pattern, storm location, and to some extent storm direction apparently had a considerable effect on the shape of the unit hydrographs developed for the larger watersheds such as the Root and Le Sueur Rivers.

3. The use of the computer permits a much more complex loss rate analysis than that usually used in design studies.

4. The unit graph and loss rate optimization program was very helpful in the overall study and should be very useful in design procedures. It is a complex program requiring considerable work to utilize its full potential. Initial assignment of preliminary values of the variables considerably speeded up the optimization process and resulted in much better results. Experience was needed in selecting values of RTIOR and QRECSN. The artificial time area curve built into the program was used for most of the studies reported herein. Further work with this feature and the use of actual time area curves might have improved the results.

5. The use of optimized variables to determine an average loss rate as a function of accumulated loss, as shown in Figure 51, provides an interesting method for comparing various watersheds. While perhaps not intended for this purpose by the authors of the program, it does provide a useful numerical basis for comparing different watersheds.

6. Further study of the program and of the data is desirable in order to determine the effect of inaccuracies in precipitation data on the overall optimization and unit graph computation procedures.
7. It would have been desirable to compute floods for some of the watersheds based on average values of the variables and also to utilize the feature in which several storms are optimized at one time. Time limitations prevented further studies of these features under this project.

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