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GENERALIZED BIOMASS ESTIMATION EQUATIONS FOR

JACK PINE (*Pinus banksiana* Lamb.)

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## ABSTRACT

Two sets of biomass estimation equations were developed for several components of jack pine trees, one set using both diameter and height as independent variables and the other set using diameter alone. The data upon which the equations were based was collected by four different investigators working in Minnesota, Ontario, and Quebec. For most components, the standard error of the estimate was 10 to 15% of the mean of the dependent variable.

Detailed studies of biomass and/or nutrient dynamics of forest stands have usually included harvesting from and development of local or site-specific biomass estimation equations for the stands under investigation. Unfortunately, investigators who needed to estimate biomass over a wider geographic area, or who were not able to develop specific equations because of cost or time constraints, were forced to use the site-specific equations from the literature and hope that they were applicable. The equations presented here are a partial remedy for this situation.

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## INTRODUCTION

Biomass or weight of forest trees is becoming an important measure for forest managers and forest ecologists. As forest management moves towards whole tree harvesting, biomass may replace volume as a measure of yield (Young, 1973) and could be used as a measure of site productivity. Forest ecologists are currently using biomass to study nutrient cycling and energy flow in forest ecosystems (Crow, 1970; Foster and Morrison (1976).

Biomass is a measure of a forest product, just as is volume. As with volume variables such as board feet and cubic feet, it is useful to develop biomass estimation equations which are applicable over wide areas. In order to develop such biomass equations for jack pine, we contacted a number of investigators who graciously supplied us with the original data they used to develop site-specific estimation equations (Table 1). We combined these data and then derived a series of geographically generalized equations for jack pine biomass estimation. These equations should be applicable over the broad geographic range indicated in Table 1. The development of site-specific equations is the most accurate approach for localized studies; the equations given below should be a viable alternative when site-specific equations are unavailable and too costly to develop.

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In choosing a model for biomass estimation, we wanted to use independent variables that are commonly determined in forest mensurational and ecological research, keep the model simple, and explain most of the variation in mass of the trees measured. Two models commonly used for estimating biomass are:

$$Y = AD^B H^C \quad (1)$$

where Y is biomass, D is diameter at breast height (dbh), H is height and A, B, and C are model parameters, or, without height:

$$Y = AD^B \quad (2)$$

where the variables are defined as above (Crow, 1970; Husch, Miller, and Beers, 1972). The linearized form of these models:

$$\log Y = \log A + B \log D + C \log H \quad (3)$$

where variables are defined as above, is also commonly used for estimating biomass (Doucet et al., 1976; Hegyi, 1972; Husch, Miller, and Beers, 1972).

We chose the non-linear form of the models (equations 1 and 2) because their results are in the actual scale rather than log Y units. Also, corrections for bias in the estimates are necessary for the linearized form (3) because of the logarithmic transformations (Beauchamp and Olson, 1973), but this bias does not arise with the non-linear forms (1 and 2).

We developed equations both with and without height for the estimation of biomass of bark, wood, total stem, needles, live branches, total live mass and total mass using a FORTRAN program for nonlinear regression.<sup>1/</sup> Although both diameter and height are commonly determined in forest mensurational work, some investigators may wish to estimate biomass from diameter alone. These two sets of equations, with and without height, are analogous to standard and local volume tables (Husch et al., 1972).

#### RESULTS

Tables 2 and 3 show the coefficients for the resulting equations. Figure 1 shows the original data points for total live mass versus diameter and the corresponding regression line. The model

fits the data well ( $R^2 = 0.98$ ). Plotted residuals for the other regressions indicated that the models fit the data well in all cases.<sup>2/</sup>

Figure 2 shows seven of the eight regression lines. The line for dead branches is excluded because its values are too small to be adequately represented.

#### DISCUSSION

The equations including height are generally slightly better, in terms of  $R^2$  and standard error of the estimate, than those without height. Even so, the equations without height fit the data well and may be helpful for estimating biomass.

The sum of estimates of biomass for individual components differs slightly from the estimated total mass. Using the equations with diameter alone as an example, at the minimum diameter in our data, 2.8 cm (Table 1), the sum of the estimated mass of components is 1.66 kg and the estimated total mass is 1.22 kg. This difference decreases rapidly to a diameter of 16 cm where the sum equals the estimated total. The sum becomes less than the estimated total at diameters greater than 16 cm. At 32.3 cm, the maximum diameter in our data (Table 1), the estimated total mass, 413.5 kg, exceeds the sum by 15.6 kg or 3.8%. The best estimate for total mass is that from its predictive equation, not from the sum of components.

In using these equations, it is important to stay within the range of the data used in their development (Table 1). These equations should help fill the increasing need of foresters and ecologists for estimating the biomass of jack pine in the Great Lakes Region.

<sup>2/</sup> Readers may note that the heterogeneous variance evident in Figure 1 suggests weighted regression as a more efficient means of estimating model parameters. However, weighting was not employed since the following evidence suggested it would lead to little refinement of parameter estimates: a) most of the data lie at the lower end of the curve, b) the model is anchored at the lower end by the origin  $y = 0$ , independent variables = 0, and the plots of residuals.

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We would like to thank those cited in Table 1 who generously allowed us to use the data that they had so laboriously collected.

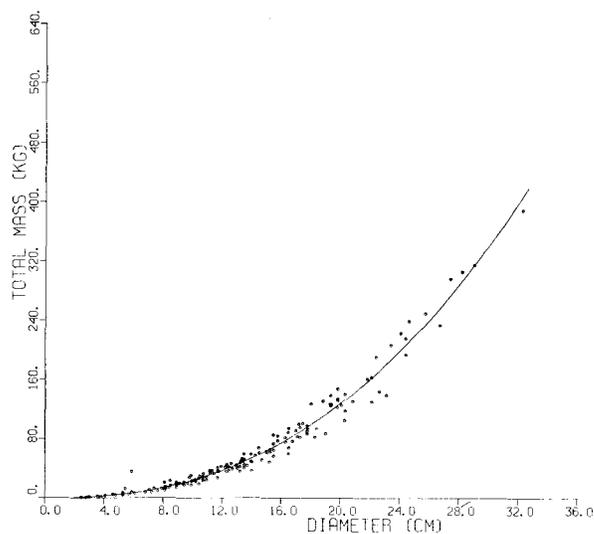


Figure 1. Total live oven-dry mass in kg versus DBH in cm for the data, and the corresponding regression line.  $Mass = .09114 (DBH)^{2.418}$ .

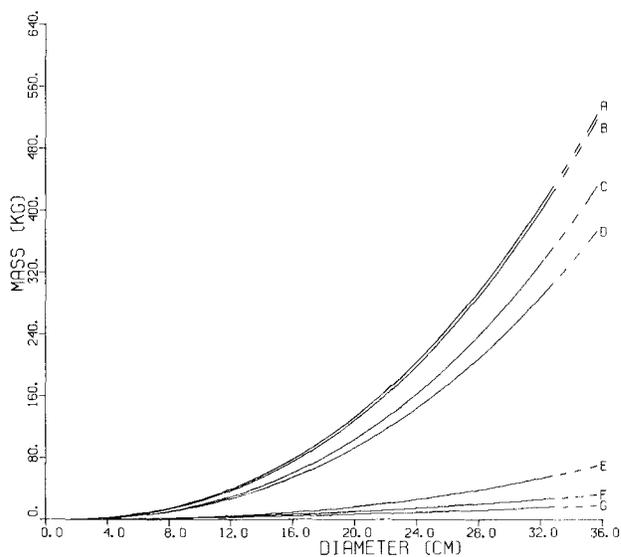


Figure 2. Regression lines (equation 2) for geographically generalized biomass estimation equations for jack pine. A) Total Mass, B) Total Live Mass, C) Total Stem, D) Stem Wood, E) Live Branches, F) Bark, G) Needles.

Table 1. Sources of data for generalized equations for jack pine biomass.

Source	Location	Number of Trees	Range of Data	
			Diameter (cm)	Height (cm)
Alban, 1977	Central Minnesota	10	10.2 - 20.1	13.4 - 20.1
Grow, 1970	Central Minnesota	40	5.8 - 17.8	6.7 - 14.1
Doucet et al., 1976	Southern Quebec	36	5.7 - 19.4	4.7 - 17.4
Hegyí, 1972	Northern Ontario	77	2.8 - 32.3	3.2 - 24.8
Total		163	2.8 - 32.3	3.2 - 24.8

Table 2. Coefficients for generalized biomass estimation equations for jack pine of the form  $Y = AD^B$ , where Y is oven-dry mass in kg and D is dbh in cm.

Component	A	B	R <sup>2</sup>	Sy.x (kg)	Sy.x/Y	df
Bark	.0326	1.925	.88	0.94	.213	86
Wood	.0677	2.410	.94	6.01	.181	86
Total Stem	.0642	2.466	.94	14.23	.264	163
Needles	.0471	1.664	.66	2.05	.517	163
Live Branches	.0094	2.493	.68	6.28	.713	163
Dead Branches	.0517	1.462	.32	2.66	.912	113
Total Live Mass	.0911	2.418	.98	10.16	.152	163
Total Mass	.1054	2.381	.98	11.45	.147	113

Table 3. Coefficients for generalized biomass estimation equations for jack pine of the form  $Y = AD^B H^C$ , where Y is oven-dry mass in kg, D is dbh in cm and H is height in m.

Component	A	B	C	R <sup>2</sup>	Sy.x (kg)	Sy.x/Y	df
Bark	.0319	1.919	0.014	.88	0.94	.212	86
Wood	.0328	1.828	0.859	.97	4.21	.126	86
Total Stem	.0230	1.722	1.136	.98	8.02	.149	163
Needles	.1159	2.778	-1.498	.84	1.38	.348	163
Live Branches	.0212	3.899	-1.884	.83	4.62	.542	163
Dead Branches	.0663	1.791	-.0427	.32	2.65	.908	113
Total Live Mass	.0601	2.090	0.490	.98	7.88	.118	163
Total Mass	.0726	2.091	0.435	.99	9.26	.119	113