

HOW IMPORTANT IS UNCERTAINTY IN ACCOUNTING
FOR DIFFERENCES IN INVESTMENT AND OUTPUT
ACROSS COUNTRIES?

by

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ABSTRACT

I offer differences in uncertainty of the industry-level investment environment as an explanation for the observed differences in the investment good price and output level across countries. I present a model economy where the tax rate on industry investment follows a stochastic process. In this environment, a higher level of uncertainty makes investment more inefficient (i.e. increase the expected cost of producing unit of capital), and this is reflected in higher investment good prices. An investigation of industry-level investment data across countries shows that the patterns of investment dynamics (i.e. features or statistics of investment sequence) in relation to output level are consistent with the implication of the model. I assign model parameter values to countries of different output levels so that the simulated investment sequences from the model mimick the actual investment sequences. I find that uncertainty can account for all of the observed differences in the investment good price, and a significant portion of the differences in the output level across countries. Differences in uncertainty between the US and Ethiopia can account for difference in the investment good price by factor of up to 4, and difference in the output level by factor of up to 16 between the two countries.

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1. INTRODUCTION

Parente and Prescott (1993) note that the output disparity between richer and poorer countries were large and persistent over the period from 1960 to 1985. An immediate explanation for this observed output disparity is that the poorer countries are poorer because they have less capital than richer countries. In the standard neo-classical framework with some inputs of production non-accumulable (typically labor), this implies the marginal product or the rental rate of capital in poorer countries is higher than that in richer countries. This implication is consistent with the observation that the price of investment good (or capital) relative to the price of consumption good is higher in poorer countries (see Summers and Heston (1993)): since the value of an investment good must be equal to the discounted sum of rents collected from renting that investment good, a higher rental rate implies a higher investment good price under a common interest rate,¹ and vice-versa. Then, an explanation for the differences in investment good prices may provide a (partial) explanation for the differences in rental rates and hence output across countries. Perhaps the conceptually simplest such explanation is a tax on investment since such a tax will increase the investment good price by the same amount of tax. This tax can be interpreted broadly as any policies that affect the cost of investment (e.g. direct taxes or subsidies, regulations affecting investment). If we assume that poorer countries have higher tax rates on investment than richer countries, this implies that the poorer countries have higher

¹ It is difficult to justify any significant differences in interest rates across countries. First, there is the international financial market: market forces equalize the interest rates across countries. Also, a typical implication of standard neo-classical models is that there is a unique interest rate determined from the fundamentals such as time preference, which prevails in balanced growth regardless of the output level. The persistent wealth disparity noted in Parente and Prescott (1993) suggests that when we consider wealth disparity across countries, it is plausible to think of both poorer and richer countries as in their balanced growth.

investment good prices and therefore lower output levels than richer countries.²

In this paper, I explore an alternative explanation for the higher relative price of investment good in poorer countries: uncertainty in the investment environment. I assume in a sense to be made precise that average tax rates on investment are the same across countries, but that these tax rates are more “uncertain” in some. This uncertainty may relate to the cost of producing investment good (e.g. uncertain taxes or subsidies on investment, frequent changes in regulations affecting investment) or to the price of the finished investment good through its effect on production of output that uses the finished investment good (e.g. uncertain taxes and subsidies on the output good, or changes in tariffs or quotas on the output good). Uncertainty in the price of investment good is qualitatively equivalent to that in the cost of producing the investment good in the sense that they both imply uncertain rate of return to investors.

In section II, I present a model economy with many industries. A key feature is that Investors choose among investment projects of different durations and projects of different durations have different before-tax rates of return (e.g. building a factory with a careful planning, adequate amount of ground of work, etc. will take longer time than building one without these, but will yield a higher rate of return.) There are industry-specific taxes on investment and these tax rates follow a stochastic process.³ In this environment, investors prefer to start projects in industries with lower tax rates. However, these lower tax rates are temporary and the expected tax rate that a project faces go up with its duration. Then, projects of shorter duration on average face lower tax rates over their duration than projects of longer duration. Then an investor may prefer a project with shorter duration to a project with longer duration even if the project with longer duration

² See Schmitz (1993) for discussion, and Parente and Prescott (1994) and Chari, Kehoe and McGrattan (1995) for models with this feature and adapted to data.

³ For simplicity I abstract away from uncertainty in the price of finished investment good. As discussed above, uncertainty of this kind is qualitatively equivalent to uncertainty in the cost of investment during the production of investment good in terms of its distortionary effect on investment. Therefore, essentially the same analysis applies to uncertainty of this kind.

has a higher before-tax rate of return. Also he may abandon an unfinished project if the project faces an unfavorable tax rate. If he cannot fully recover the cost of investment that he already made, this results in a waste of resources. Through these two distortionary effects, uncertainty can make investment inefficient. The more there is such inefficiency, the higher the cost of investment will be and this will be reflected in a higher investment good price.⁴ Given a set of before-tax rates of return for projects of different durations, different values of uncertainty parameters correspond to different levels of investment efficiency in the steady state of economy.

By assigning different values of the uncertainty parameters to countries of different output levels, we can (partially) explain the observed differences in investment good prices and output levels across countries. To accept this explanation as more convincing, however, we would want some more evidence as to why these values of parameters are plausible. Since these parameters are abstract, we cannot directly relate them to available data. However, uncertainty parameters determine not only investment efficiency, but also industry investment dynamics (i.e. aspects or statistics of industry investment sequences). In particular, I find that the model predicts three aspects of industry-level investment sequence to be related to the output level.

Section III is an investigation of industry-level investment data across countries to examine these relations. The data come from the industry-level surveys by the United Nations Industrial Development Organization (UNIDO) over the period 1967-1988 as well as Summers and Heston (1993). I find that the predictions of the model are consistent with the data. I also find that a significant portion of industry-level investment dynamics is idiosyncratic (i.e. industry-specific), and that this idiosyncratic component of investment dynamics is larger for poorer countries. This finding provides support to modeling

⁴ As mentioned earlier, I distinguish this effect of uncertainty on the investment good price and output level through affecting investment efficiency from that through affecting the expected tax rate on investment, and consider only the former in relation to different levels of uncertainty with a constant expected tax rate.

uncertainty at the industry level rather than at the aggregate level.

Secion IV is an attempt to determine to what extent uncertainty of the investment environment can account for differences in investment efficiency and output level across countries. For this, we need to assign uncertainty parameter values to countries of different output levels as well as other parameter values common to all countries. I find many sets of parameter values with different implications on the importance of uncertainty of investment environment, but overall the findings suggest that uncertainty of the investment environment is important in accounting for differences in investment good price and output level across countries. For instance, uncertainty can account for difference in the investment good price between the US and Ethiopia by a factor of 2 to 4. In general, uncertainty can account for all of differences in investment good prices across countries. Depending on the value of capital share in output production function, these differences in investment good price translates to large differences in output level across countries. Uncertainty can account for difference in output leve between the US and Ethiopia by a factor of up to 16.

In the remainder of this section, I will review some related literature. On the theoretical side, Rodrik (1989) considers policy reform that has the effect of increasing the after-tax rental rate of capital. However, it is uncertain whether the reform policy will be maintained. He finds that rather intuitively the more certain the policy is to be maintained, the more successful the reform will be in increasing investment. Hopenhayn and Muniagurria (1993) have a model where there are two tax/subsidy rates on investment, and these rates follow a stochastic process. This is a more general consideration of uncertainty than the one-shot policy reform considered in previous article. They find that if there is one sector in the economy, the persistence (low probability of switching) of tax rates can increase the long-run average investment and growth rate. However, if there are two sectors so that at any time one of the two sectors faces the lower tax rate and the other faces the higher tax rate, the persistence of tax rates can decrease the long-run average investment and growth rate. So there is some ambiguity as to the effects of uncer-

tainty. A source of this ambiguity is that a different level of uncertainty typically lead to a different long-run average share of investment carried out under the higher tax rate as opposed to the lower tax rate. This implies that uncertainty affects the long-run average tax rate on investment as well. Since different tax rates (even without uncertainty) affects investment, it is not clear how much of the reported effect of uncertainty can be attributed to changes in the average tax rate and how much to the 'pure' effect of uncertainty. In this paper, I distinguish the two and concentrate on the latter. Other main differences from the previous two articles is that I model uncertainty at industry-level as opposed to more aggregate levels, and that I consider investment to be carried out in different durations. Dixit and Pindyck (1994) consider various aspects of investment under uncertainty at the firm level, including making decisions on investment projects when the completion of project takes multi-periods and the cost of investment is uncertain. This is close to the investment environment that we consider in this paper. The main difference is that we consider investment in general equilibrium context to study the effects of uncertainty on aggregate variables. Also, we allow the investors to choose among investment projects of different durations, and to abandon projects if they want. These two features provide the sources of investment inefficiency, which is taken as a potential explanation for differences in levels of investment good price and output across countries.

There are also some related empirical studies that investigate the link between uncertainty and investment or output. Pindyck and Solimano (1993) investigate the relation between the variance of the marginal revenue product of aggregate capital (a proxy for uncertainty) and its maximal observed value (a proxy for the trigger value to initiate investment) for a sample of 14 developing countries and 16 OECD countries over the period from 1962 to 1989. Since the marginal product of capital is not directly observable, they estimate it using a rather stringent functional form. They find that these two measures are positively related, with the implication that uncertain investment environment discourages investment. In an investigation of 46 developing countries over the period from 1970

to 1985, Aizenman and Marion (1993) separate the unexpected component of aggregate policy (such as the government expenditure or inflation rate) change, again a proxy for uncertainty, assuming an autoregressive form of expectation formation. They find that the predictability of policy is positively related with the growth rate. Using stock return data for 600 US manufacturing firms over the period from 1981 to 1987, Leahy and Whited (1995) investigate the relation between the estimated expected variance of a firm's daily stock return, again a proxy for uncertainty, and its investment, and find it to be strongly negative. This approach has the merit of working with the uncertainty that an industry or a firm faces, unlike the previous two articles. However, since the stock market is an advanced financial market whose data are simply not available for many developing countries, we cannot extend this line of research for the purpose of explaining differences in levels of investment good price and output across countries. Finally, Ramey and Ramey (1994) find the variance of aggregate output to be negatively related to its growth rate for 92 countries over the period from 1960 to 1985.

2. THE MODEL ECONOMY

2.1 The Environment

There are measure 1 of industries indexed by $i \in [0, 1]$. The technology of industry i is represented by the production function

$$y(i) = k(i)^\alpha l(i)^{1-\alpha},$$

where $y(i)$ is output, $k(i)$ is capital, $l(i)$ is labor, and $0 < \alpha < 1$. Outputs of all industries are perfect substitutes. Aggregate output is

$$y = \int y(i) di.$$

The law of motion for capital in industry i is

$$k'(i) = k(i)(1 - \delta) + x(i),$$

where $k'(i)$ is capital next period, δ is depreciation rate, and $x(i)$ is new capital added this period. New capital is created by carrying out investment projects. Projects can be carried out in any scale $z \in (0, \infty)$. A project of type j and scale z requires z units of output to be invested each period for j periods, and yields $zh(j)$ units of new capital in the j th period. Investment is irreversible: capital (at any stage of the project) cannot be converted back to output. However, a project can be abandoned at any stage. If a project is abandoned, whatever remains of the project is lost.

There are measure 1 of identical people each of whom has 1 unit of time each period which can be converted to labor across industries. The representative person's preferences is given by

$$\sum_{t=0}^{\infty} \beta^t \log c_t,$$

where c_t is consumption.

There is an industry-specific tax/subsidy on investment. The tax rate in any particular industry takes on one of two values, ϕ_1 and ϕ_2 where $\phi_1 < \phi_2$. A negative value of either of these parameters implies subsidy. The tax rate of any industry follows a Markov chain. If this period's tax rate is ϕ_p , the probability that the next period's tax rate will be ϕ_p is π_p . I assume that $\pi_1 + \pi_2 > 1$. This implies that the probability of facing the lower (higher) tax rate in the next period is larger if this period's tax rate is the lower (higher) one. The parameters ϕ_1, ϕ_2, π_1 , and π_2 are common to all industries, but innovations are i.i.d. across industries.

Let T_t denote the aggregate tax revenue. If T_t is positive, the aggregate tax revenue is equally distributed to people, and if T_t is negative, it is financed by an equal lump sum tax on individuals.

2.2 Equilibrium

I will define equilibrium recursively. Before doing so I will introduce some notation and establish some useful results. First, note that labor is completely mobile across industries, so that in equilibrium in any period there will be a single wage rate. Because there is a constant-returns technology common to all industries, this implies that capital to labor ratios are equated across industries, which in turn implies that rental rates of capital are also equated across industries in any given period. Since the rental rate is equal across industries, the representative person would be indifferent concerning in which industries to start new projects, or to continue any projects, as long as they have the same current tax rates. For ease of notation, I will focus on symmetric outcomes, i.e.: I assume that if any two projects are identical except for the industries to which they belong, the representative person invests an equal amount in the two projects. The state variables for a given individual are total capital (summed over industries) owned by the individual, and the scales of ongoing investment projects, indexed by the type of the project, the age of the project,

and the current tax rate that the project faces. Let μ denote the set of individual state variables: $\mu = \{k\} \cup \{z_{jnp}\}$ where k is the individual's total capital and z_{jnp} is the scale of the projects with type j , age n , and the current tax rate ϕ_p . Note that the investment projects do not need to be distinguished by the industries to which they belong since any two industries with the same current tax rate share the same probability distribution of future tax rates.

The aggregate state variables are the sum of individual state variables. Let ν denote the set of aggregate state variables. Each period the aggregate state variables determine the rental rate, the wage, and the aggregate tax revenue: these variables are given by functions, $r(\nu)$, $w(\nu)$, and $T(\nu)$. Let Γ be the law of the motion of the aggregate state variables. The representative person takes as given Γ and therefore the current and future values of the rental rate, the wage, and the aggregate tax revenue. The decision variables are $\{z_{j0p}\}$ which is the sizes of projects that are started today, $\{g_{jnp}\}$ which represents the decisions of whether to continue projects (continue if $g = 1$, abandon if $g = 0$). Now we can formulate the representative person's decision problem as a dynamic programming problem. The Bellman's equation is

$$V(\mu, \nu) = \text{Max}\{\log c + \beta V(\mu', \nu')\}$$

s.t.

$$c + \sum_{j=1}^{\infty} \sum_{p=1}^2 (1 + \phi_p) z_{j0p} + \sum_{j=1}^{\infty} \sum_{n=1}^{j-1} \sum_{p=1}^2 g_{jnp} (1 + \phi_p) z_{jnp} \leq r(\nu)k + w(\nu) + T(\nu),$$

$$z'_{jn+1p} = \pi_p g_{jnp} z_{jnp} + (1 - \pi_q) g_{jnp} z_{jnp},$$

$$k' = k(1 - \delta) + x,$$

$$x = \sum_{j=1}^{\infty} \sum_{p=1}^2 z(j, j-1, p) h(j),$$

$$z_{j0p} \geq 0, \quad \text{and}$$

$$\nu' = \Gamma(\nu),$$

where $p = 1$ if $q = 2$ and $p = 2$ if $q = 1$.

Now we can define the equilibrium of this economy. An equilibrium is a value function $V(\mu, \nu)$, policy functions $c(\mu, \nu)$, $g_{jnp}(\mu, \nu)$, $z_{j0p}(\mu, \nu)$, $z'_{jnp}(\mu, \nu)$ for $n \geq 1$, and $k'(\mu, \nu)$, price functions $r(\nu)$ and $w(\nu)$, aggregate output function $y(\nu)$, aggregate investment function $z(\nu)$, aggregate tax function $T(\nu)$, and the law of motion of the aggregate state variables $\Gamma(\nu)$ such that

1) given $r(\nu)$, $w(\nu)$, $T(\nu)$, and $\Gamma(\nu)$, the policy functions are the solution to the representative person's problem;

2) the rental rate and the wage rate are competitively determined:

$$r(\nu) = \alpha k^{\alpha-1} \quad \text{and}$$

$$w(\nu) = (1 - \alpha)k^\alpha;$$

3) aggregate output is determined by the aggregate production function:

$$y(\nu) = k^\alpha;$$

4) aggregate investment is the sum of investment across projects:

$$z(\nu) = \sum_{j=1}^{\infty} \sum_{p=1}^2 z_{j0p}(\nu, \nu) + \sum_{j=1}^{\infty} \sum_{n=1}^{j-1} \sum_{p=1}^2 g_{jnp}(\nu, \nu) z_{jnp}(\nu, \nu);$$

5) aggregate tax is the sum of taxes across projects:

$$T(\nu) = \sum_{j=1}^{\infty} \sum_{p=1}^2 \phi_p z_{j0p}(\nu, \nu) + \sum_{j=1}^{\infty} \sum_{n=1}^{j-1} \sum_{p=1}^2 \phi_p g_{jnp}(\nu, \nu) z_{jnp}(\nu, \nu);$$

6) the sum of aggregate investment and aggregate consumption cannot be greater than aggregate output:

$$z(\nu) + c(\nu, \nu) \leq y(\nu); \quad \text{and}$$

7) the law of motion of aggregate state variables is determined by the values of the policy functions of the representative person when his state variables are equal to the aggregate state variables:

$$\Gamma(\nu) = \{k'(\nu, \nu)\} \cup \{z'_{jnp}(\nu, \nu)\}.$$

2.3 Investment in Steady State

A steady state is a state ν such that $\Gamma(\nu) = \nu$ in equilibrium. I will characterize the steady state investment (i.e. $\{z_{j0p}\} \cup \{g_{jn p}\}$). First note that since there are infinitely many industries, the representative person can fully eliminate the uncertainty of the returns to his overall investment by diversifying across industries. This implies that he will make investment decisions so as to simply maximize the expected rate of return to investment.

To think about this more precisely, first define $\pi(t; p)$ as the probability that an industry has tax rate ϕ_p t periods later, conditional on the current period's tax rate being ϕ_p . We then have $\pi(0; p) = 1$ and $\pi(t+1; p) = \pi_p \pi(t; p) + (1 - \pi_q)(1 - \pi(t; p))$ for $p = 1, 2$ and all $t \geq 0$, where $q = 1$ if $p = 2$ and $q = 2$ if $p = 1$. From these conditions, we can derive

$$\pi(t; 1) = a + (1 - a)b^t, \quad \text{and}$$

$$\pi(t; 2) = 1 - a + ab^t$$

where $a = (1 - \pi_2)/(2 - \pi_1 - \pi_2)$ and $b = \pi_1 + \pi_2 - 1$. As t increases, $\pi(t; p)$ converges to a , so a is the long-run probability of an industry having the lower tax rate. Alternatively, it is the fraction of industries with the lower tax rate in any period. I will call a the **frequency** parameter with higher a meaning greater frequency of the lower tax rate. The parameter b determines how quickly $\pi(t; p)$ converges to a . I will call b the **persistence** parameter with higher b meaning more persistence of tax rates. For instance, if $b = .9$, each period $\pi(t; p)$ approaches a by 10% of the gap between the two.

Let $\phi(t; p)$ be the expected (at period 0) tax rate of an industry t periods later, conditional on that the current period's tax rate is ϕ_p . We have

$$\begin{aligned} 1 + \phi(t; 1) &= (1 + \phi_1)\pi(t; 1) + (1 + \phi_2)(1 - \pi(t; 1)) \\ &= (1 + \phi_1)[a + d(1 - a) - (d - 1)(1 - a)b^t], \quad \text{and} \end{aligned}$$

$$\begin{aligned} 1 + \phi(t; 2) &= (1 + \phi_2)\pi(t; 2) + (1 + \phi_1)(1 - \pi(t; 2)) \\ &= (1 + \phi_1)[a + d(1 - a) + (d - 1)ab^t], \end{aligned}$$

where $d = (1 + \phi_2)/(1 + \phi_1)$. I will call d the **dispersion** parameter with higher d meaning

more dispersion of the two tax rates. We now have three uncertainty parameters, d , a , and b . Any set of values of these parameters and a lower tax rate ϕ_1 have a unique corresponding set of values of the primitive parameters ϕ_1, ϕ_2, π_1 , and π_2 .

Figure 1 illustrates $\phi(t; p)$. Note that the expected tax rate conditional on the current period's tax rate being the lower one is always less than the expected tax rate conditional on the current period's tax rate being the higher one: $\phi(t; 1) < \phi(t; 2)$ for all $t \geq 0$. Therefore, the representative person always starts all projects in industries with the lower tax rate; none are started in industries with the higher tax rate:

$$z_{j02} = 0 \quad \text{for all } j.$$

Now let's think about what type of projects the representative person will start, and whether he will continue or abandon his projects as the tax rates change in the following periods. First we can show that for any type of project j , if it is optimal to abandon the project when it faces the high tax rate and it is n periods old, it is better to abandon the project when it faces the high tax rate and it is less than n periods old. This is because when the project is younger, the return to investment is discounted by more periods, and also the expected cost is higher due to the longer remaining duration of investment. Similarly, we can show that for any type of project j , if it is optimal to continue the project when it faces the high tax rate and it is n periods old, it is better to continue the project when it faces the high tax rate and it is more than n periods old. This means that for any type of project j , there are at most two⁵ (consecutive) n 's such that abandoning the project is optimal if it is under the high tax rate and less than n periods old, and continuing it is optimal if it is under the high tax rate and at least n periods old.

I assume that there is a unique type of project J and a unique corresponding continuation/abandonment rule N ,⁶ that maximizes the rate-of-return to investment. Then we

⁵ there will be two if abandoning the project when it faces the higher tax rate and it is n periods old is as good as continuing it for some n .

⁶ Although we can construct examples where the set of J and N is not unique, numerical exercises show that it is unique for most reasonable sets of model parameter values.

have

$$z_{j01} = 0 \quad \text{for all } j \neq J,$$

$$g_{Jn2} = 0 \quad \text{for all } n < N, \text{ and}$$

$$g_{Jn2} = 1 \quad \text{for all } n \geq N,$$

for a unique set of J and N . For instance, if $J = 10$ and $N = 5$, the steady state investment projects will be of type 10 (i.e. of duration 10 periods); and during the first 5 periods, a project will be abandoned if it faces the higher tax rate, but once a project continues for the first 5 periods, it will be continued for the remaining 5 periods even if it faces the higher tax rate. The representative person maximizes the expected rate of return to investment, or equivalently, the ratio of the expected discounted stream of return to the expected discounted stream of costs. That is, J and N solve

$$Max \quad \frac{\pi_1^{n-1} \sum_{t=j}^{\infty} \beta^t (1-\delta)^{t-j} h(j) r}{\sum_{t=0}^{n-1} \beta^t \pi_1^t (1+\phi_1) + \pi_1^{n-1} \sum_{t=n}^{j-1} \beta^t (1+\phi(t-n+1; 1))}.$$

The numerator and denominator are the expected discounted streams of return and cost when a project of scale 1 is undertaken. Varying the scale of the project only changes the numerator and denominator by the same factor, and therefore will not change the ratio. Inspecting how the term $1 + \phi_1$ enters in the above maximization problem shows that we can write it as

$$Max \quad \frac{rR(j, n; d, a, b)}{1 + \phi_1},$$

where

$$R(j, n; d, a, b) = \frac{\sum_{t=j}^{\infty} \beta^t (1-\delta)^{t-j} h(j)}{\sum_{t=0}^{n-1} \beta^t (a+b-ab)^{t-n+1} + \sum_{t=n}^{j-1} \beta^t (a+d(1-a) - (d-1)(1-a)b^{t-n+1})}.$$

I here used the expression for $1 + \phi(t; 1)$ given earlier and the equality $\pi_1 = a + b - ab$, which can be easily derived from the expressions for a and b given earlier.

Now let's think about the scale of investment project (i.e z_{J01}) that the representative person undertakes in steady state. If the ratio of the expected return from investment to

the cost of investment is greater 1, he will invest an infinite amount; if the ratio is less than 1, he will invest none; and if the ratio is equal to 1, he will be indifferent about the scale of investment. In equilibrium the third case must hold:

$$\frac{rR(J, N; d, a, b)}{1 + \phi_1} = 1. \quad (1)$$

This equation determines the rental rate r in steady state. A value of r implies a value of capital k . Then the scale of a steady state investment project is such that the new capital produced each period, x , is equal to the depreciation of capital each period, δk .

2.4 Investment Efficiency, Investment Good Price, and Output

Let $\bar{\phi}$ denote the ratio of the expected discounted stream of tax payments to the expected discounted stream of investment for projects undertaken in steady state:

$$\bar{\phi} = \frac{\sum_{t=0}^{N-1} \beta^t \phi_1 \pi_1^t + \pi_1^{N-1} \sum_{t=N}^{J-1} \beta^t \phi(t-N+1; 1)}{\sum_{t=0}^{N-1} \beta^t \pi_1^t + \pi_1^{N-1} \sum_{t=N}^{J-1} \beta^t}.$$

Varying the size of project will not change $\bar{\phi}$. Using $\pi_1 = a + b - ab$ and the expression for $1 + \phi(t; 1)$ given earlier and doing some algebra, we can rewrite the above equality as

$$1 + \bar{\phi} = \frac{\sum_{t=0}^{N-1} \beta^t (1 + \phi_1) (a + b - ab)^{t-N+1}}{\sum_{t=0}^{N-1} \beta^t (a + b - ab)^{t-N+1} + \sum_{t=N}^{J-1} \beta^t} + \frac{\sum_{t=N}^{J-1} \beta^t (1 + \phi_1) (a + d(1-a) - (d-1)(1-a)b^{t-N+1})}{\sum_{t=0}^{N-1} \beta^t (a + b - ab)^{t-N+1} + \sum_{t=N}^{J-1} \beta^t}.$$

Substituting this in equation (1), we get

$$\frac{rE(J, N; d, a, b)}{1 + \bar{\phi}} = 1, \quad (2)$$

where

$$E(j, n; d, a, b) = \frac{\sum_{t=j}^{\infty} \beta^t (1 - \delta)^{t-j} h(j)}{\sum_{t=0}^{n-1} \beta^t (a + b - ab)^{t-n+1} + \sum_{t=n}^{j-1} \beta^t}.$$

E is the rate of return of investment if r is 1 and $\bar{\phi}$ is 0. We can interpret E as the measure of **investment efficiency** with higher E meaning more efficiency.

Consider two economies, s and u . Let ϕ_{1s} and ϕ_{1u} denote the lower tax rates of respective countries; $\{d_s, a_s, b_s\}$ and $\{d_u, a_u, b_u\}$ the uncertainty parameters of respective countries; and $\{J_s, N_s, \bar{\phi}_s, R_s, E_s\}$ and $\{J_u, N_u, \bar{\phi}_u, R_u, E_u\}$ the corresponding sets of the steady state values. From equation (2), we have

$$\frac{r_s}{r_u} = \frac{(1 + \bar{\phi}_s)E_u}{(1 + \bar{\phi}_u)E_s}.$$

This equation and equilibrium conditions (2) and (3) yield

$$\begin{aligned} \frac{y_s}{y_u} &= \left(\frac{r_u}{r_s} \right)^{\frac{\alpha}{1-\alpha}} \\ &= \left(\frac{(1 + \bar{\phi}_u)E_s}{(1 + \bar{\phi}_s)E_u} \right)^{\frac{\alpha}{1-\alpha}}. \end{aligned} \tag{3}$$

Hence, uncertainty affects the rental rate and output through two channels, first by affecting the expected tax rate on investment, and second by affecting investment efficiency. The first channel, differences in rental rate and output due to differences in (expected) tax rate on investment, have been studied by others (see Schmitz (1993), Parente and Prescott (1994), and Chari, Kehoe, and McGrattan (1995)). In this paper, we are interested in the second channel, differences in rental rate and output due to differences in investment efficiency. Since the investment good (i.e. capital) price is the discounted sum of rents, the ratio of the investment good prices between two countries is the same as the ratio of the rental rates between the two countries. As noted earlier, poorer countries tend to have higher investment good price relative to consumption good price. Equation (3) shows that differences in uncertainty can (partially) lead to the observed differences in the levels of investment good price as well as output across countries. Qualitatively, if poorer countries have “more”⁷ uncertainty in their investment environment, then the model will replicate the pattern found in data.

⁷ In next sub-section, the meaning of “more” uncertain investment environment will be discussed, and it will be shown that the more uncertain investment environment tends to be associated with less efficient investment.

2.5 Uncertainty and Investment Efficiency

To think about how uncertainty affects the rental rate and output by affecting investment efficiency, we narrow our attention to sets of values d , a , b , and ϕ_1 , that result in a common expected tax rate, and ask how large of differences in investment efficiency can result from different sets of values holding the expected tax rate constant. For this purpose, first we can derive from equations 1 and 2,

$$\frac{1 + \phi_1}{1 + \bar{\phi}} = \frac{R(J, N; d, a, b)}{E(J, N; d, a, b)}. \quad (4)$$

This shows that given any expected tax rate $\bar{\phi}$ and any uncertainty parameter values d , a , and b , we can find the lower tax rate ϕ_1 that results in the given expected tax rate $\bar{\phi}$ in steady state. This implies that for any given expected tax rate $\bar{\phi}$, the sets of d , a , b , and ϕ_1 that result in that expected tax rate $\bar{\phi}$ are described by any values of d , a , b , and ϕ_1 that satisfy equation (4).

Let's now examine how the investment efficiency E changes as the values of d , a , and b change. Investment will be most efficient when there is no uncertainty, that is when $d = 1$ (i.e. the two tax rates are the same), $a = 1$ (i.e. all industries face the lower tax rate), or $b = 1$ (i.e. the low tax rate that a project began with does not change). This can be easily verified by inspecting the expressions for $R(J, N; a, b, c)$ and $E(J, N; a, b, c)$: if $d = 1$, $a = 1$, or $b = 1$, R and E are the same so that maximizing R is equivalent to maximizing E . If $d \neq 1$, $a \neq 1$, and $b \neq 1$, that is if there is uncertainty, maximizing R is not equivalent to maximizing E , and therefore the investment may be inefficient. There are two reasons why investment may be inefficient. First, projects of types (i.e. durations) different from the most efficient type are undertaken and second, projects may be abandoned during their duration, resulting in a waste of resources.

Unfortunately, there are no analytical solutions for J and N . However, numerical exercises show the following. First, as d increases, J tends to decrease and N tends to increase, resulting in lower E . A higher d implies higher dispersion between the two tax

rates so that there is a higher penalty when the tax rate that a project faces changes from ϕ_1 to ϕ_2 . This increases the incentive to undertake projects of shorter duration to reduce the probability of facing ϕ_2 ; it also increases the incentive to abandon projects when they face ϕ_2 . Second, as a increases, J tends to increase and N tends to decrease, resulting in higher E . A higher a implies less probability for the tax rate that a project faces to change from ϕ_1 to ϕ_2 . This increases the incentive to undertake projects of longer duration. A higher a also implies more probability for the tax rate that a project faces to change from ϕ_2 to ϕ_1 . This decreases the incentive to abandon projects when the tax rate that they face changes from ϕ_1 to ϕ_2 . The relations between b and J , N , or E are more complicated. A lower b implies more probability for the tax rate that a project faces to change from ϕ_1 to ϕ_2 , increasing the incentive to undertake projects of shorter duration. However, a lower b makes the probability distribution of future tax rates for a currently low-taxed project more similar to that for a currently high-taxed project. This implies the reduction in the expected tax rate from undertaking projects of shorter duration is smaller, and so increases the incentive to undertake projects of longer duration. A lower b also implies more probability for the tax rate that a project faces to change from ϕ_2 to ϕ_1 , providing less incentive to abandon projects when they face ϕ_2 . The combined effect on E is ambiguous: it is not difficult to find examples where as b increases, E decreases and then increases.

2.6 Uncertainty and Investment Dynamics

Different uncertainty parameter values determine not only different levels of investment efficiency, but also different industry-level investment dynamics (i.e. features or statistics of industry-level investment sequence). More precisely, a set of d , a , and b determines J and N , and a set of J , N , a , and b in turn determines the dynamics of steady-state industry-level investment. Figures 2 to 5 present examples of industry-level investment sequences that illustrate how the dynamics of steady state industry-level investment may

change as these variables change. For each investment sequence, I assume that whenever the industry faces the lower tax rate, a new investment project of scale 1 is undertaken.⁸

Figure 2 and 3 illustrate the changes in investment dynamics as J increases with N , a , and b constant. Figure 2 shows that as J increases, the deviation of investment from the long-run average value (in fraction of the long-run average value)⁹ tends to decrease in absolute term. Figure 3 is the sequences of deviations of investment from the previous period's values for the sequences of investment in figure 2. This shows a related but distinct observation that as J increases, investment tends to be more persistent in the sense that the deviation of investment from the previous period's value (in fraction of the long-run average investment) is smaller in absolute term. Figure 4 and 5 illustrate the changes in investment dynamics as N increases with J , a , and b constant. They show that higher N tends to have the opposite effect as higher J . Figure 4 shows that as N increases, the deviation of investment from the long-run average value tends to increase in absolute term. Figure 5 is the sequences of deviations of investment from the previous period's value for the sequences of investment in figure 4. This shows that as N increases, investment tends to be less persistent in the sense that the deviation of investment from the previous period's value is greater in absolute term. Since higher J and/or lower N ¹⁰ imply higher levels of investment efficiency and output in the model, we may expect that industry-level investment data across countries show that industry-level investment in poorer countries tends to have greater deviation (from the long-run average value) and less persistence than that in richer countries.

Figure 6 illustrates the changes in investment dynamics as a increases with J , N ,

⁸ Varying the scale of investment (projects) only changes the scale of investment sequence, and does not change the investment dynamics. For ease of comparison among sequences, however, I drew the sequences so that the industry-wide investment (i.e. investment summed over projects of different ages) averaged over time is of the same magnitude.

⁹ Three aspects of investment sequence discussed here (i.e. amplitude, persistence, and frequency of high investment) will be formerly defined in section 3, and used to represent the industry-level investment dynamics across countries in the data.

¹⁰ In terms of model parameters, this can be thought of as lower d with a and b constant.

and b constant. Note that as a increases, as in the case of higher J or lower N , the deviation of investment from the long-run average value tends to decrease and investment tends to be more persistent. Since higher a tends to increase investment efficiency and the output level by increasing J and decreasing N , this finding reinforces the predictions of the last two illustrations: industry-level investment in poorer countries tends to have greater deviation and less persistence than that in richer countries. However, note also that as a increases, the fraction of elements of the sequence above the long-run average value tends to increase. In other words, (relatively) high investment is more frequent. Since higher a is, again, associated with higher output level, we may expect that the investment in poorer countries tends to have less frequency of high investment. Finally, Figure 7 illustrates the changes in investment dynamics as b increases with J , N , and a constant. Note that as b increases, there is no change in deviation of investment from the long-run average value. However, the investment tends to be more persistent.

In sum, the model predicts that the investment in poorer countries tends to have greater deviation (from the long-run average value), less persistence, and less frequency of high investment. The industry-level investment data analysis in section 3 shows that these patterns are actually found in the data.

3. DATA ANALYSIS

The example investment sequences considered in section 2 show that different uncertainty parameter values d , a , and b correspond to different industry-level investment dynamics. In particular, they suggest that the investment in poorer countries have greater deviation, less persistence, and less frequency of high investment. In this section, we explore industry-level investment data across countries to see if this prediction is consistent with the data. In section IV, we use the findings in this section to assign values of d , a , and b to countries of different output levels, and to assess the importance of uncertainty in accounting for differences in investment efficiency and output level across countries.

3.1 Data

The UNIDO has collected and published various industrial statistics for many countries on annual basis since the early 1960's. The statistics include gross investment of various industries, including 27 3-digit ISIC manufacturing industries and the manufacturing industry as a whole. Since the ISIC was revised in 1967, measures of investment before 1967 are not easily comparable to those thereafter. Therefore, I consider the period for my investigation to be from 1967 to 1988, which is the the last year of data available for most of the countries. For many developing countries, especially those in Sub-Saharan Africa, the data is incomplete, missing observations for many years and industries. This incompleteness of data limits the number of countries whose data we can use.

The 16 countries that I selected are listed in table 1. For the selection of the countries, I used the following general guidelines. First, I selected countries that have reasonably extensive data in terms of both years and industries. Table 1 list the years and the industries covered by the data used for each country. Second, I selected countries to

represent different levels of output and population and different geographical regions. Per-capita output and population averaged over the covered period are listed in the first and second columns of table 2. Finally, I tried to select countries whose per-capita outputs were not growing at rates much different from the worldwide average growth rate during the covered periods, which is about 2%, since the theoretical analysis in section 2 was carried out in the context of balanced growth. The trend growth rates¹¹ are listed in column 3 of table 2.¹²

The UNIDO survey data are measured in units of the current currencies of the respective countries. To compare the data across both time periods and countries, we need to convert them to be in units of a single currency for a single year. This is done using the Summers and Heston data set plus the US GDP deflator. We first convert each measure of investment into US dollars of the respective year using the Purchasing Power Parity of composite investment good¹³ of the respective country. The Summers and Heston data set provide this Purchasing Power Parity of investment good for each country for each year. Next, we convert the current US dollars of investment into US dollars of a particular year using the US GDP deflator. I chose the year 1985. The Summers and Heston data set also include for each country and each year the population, the per-capita output, the investment share of output, and the investment good price relative to consumption good price. Output is in units of 1985 US dollar so that we can directly compare these data with the investment data from the UNIDO survey. Table 2 list for each country the per-capita output, the population, the growth rate of per-capita output, the investment share of output, the manufacturing share of investment, and the investment good price relative to the consumption good price with this measure for the US normalized to be 1.

¹¹ The trend growth rate of output is derived the same way as the trend growth rate of investment is in section 3.2.

¹² Brazil and Korea grew much faster than 2%. But, they represent output level (both), population level (Brazil), and geographical area (Korea), for which the available countries in data are rare.

¹³ Of course, PPP of investment good varies across industries. However, PPP's of investment good for individual industries are not available.

3.2 Analysis and Findings

Investment Share, Manufacturing Share, and Output Level

Plot 1 plots the investment share of output and the manufacturing share of investment against per-capita income. The investment shares of richer countries are higher than the investment shares of the poorer countries. This is as noted in the analysis of Summers and Heston (1993) with a broader sample. The manufacturing share of investment varies across countries, averaging around 12 or 13 percentage, but does not have a clear correlation with the level of per-capita income. The coefficients and standard error of the regressions in all plots are reported in table 4.

Investment Good Price and Output Level

Plot 2 plots the investment good price relative to consumption good price against per-capita output. The investment good price tends to be higher for poorer countries, with those of some poorer countries more than twice as high as those of some richer countries. This is again as noted in Summers and Heston (1993).

Industry-Level Investment Dynamics and Output Level

The three aspects of investment sequence considered in the last section were deviation, persistence, and frequency of high investment. Here I define these formerly as statistics. First, I define a trend investment sequence so that we can consider investment as fraction of trend, independently of the scale. This is necessary for two reasons. First, the model does not feature long-run growth or decline of industry-level investment whereas we would expect to see this in data for many countries and industries. Second, we want to compare investment sequences for economies of different sizes (e.g. the US and Canada). Let

$$\begin{aligned} z_{sit} &= \bar{z}_{sit}(1 + d_{sit}) \\ &= \rho_{si}e^{\sigma_{si}}(1 + \lambda_{sit}), \end{aligned}$$

where z_{sit} is the actual investment of country s , manufacturing industry i , and year t ; \bar{z}_{sit}

is the trend investment; λ_{sit} is the deviation from the trend; and the intercept ρ_{si} and the growth rate σ_{si} are constants. The constants ρ_{si} and σ_{si} are determined by minimizing the squared deviations summed over t . The deviation is in percentage terms of trend investment, independent of the scale of investment.

The three statistics are defined as following. For country $s = 1, 2, \dots, 16$, $devi_s$ is the standard deviation of $\{\lambda_{sit}\}$ with regard to t , averaged over i ; $diff_s$ is the standard deviation of the first-order differences of $\{\lambda_{sit}\}$ with regard to t , averaged over i ; and $frac_s$ is the fraction of elements of z_{sit} that are less than their trend investment \bar{z}_{sit} , over i and t . Columns 1 to 3 of table 3 report these measures for each country. Plot 3 to plot 5 plot $\{devi_s\}$, $\{diff_s\}$, and $\{frac_s\}$ against output level. To relate these measures to the three aspects of investment sequence considered in last section, higher $devi$ means greater deviation; higher $diff$ less persistence; and higher $frac$ higher frequency of high investment. Then the model predicts that poorer countries have larger $devi$, larger $diff$, and smaller $frac$. Plot 3 to 5 in fact show these patterns, and therefore provide support for the model to think about the relations among uncertainty, investment, and output.

Comparison to Aggregate-Level Investment Dynamics

In the model economy, uncertainty is all at the industry-level; there is no uncertainty at the aggregate-level. This motivated us to examine the industry-level data as opposed to the aggregate data. Let's examine investment data at two levels of aggregation, at the level of manufacturing investment and at the level of aggregate investment, to see how investment dynamics differ from that of industry-level investment. Let λ_{smt} be the deviation of manufacturing investment of country s and year t , and λ_{svt} that of aggregate investment. Let $deviman_s$ and $deviinv_s$ be the standard deviations of λ_{smt} and λ_{svt} with respect to t . Columns 4 and 5 report these measures for each country. Plot 6 plots these measures as well as $\{devi_s\}$ against output level.

In general, $deviman$ is greater than $deviinv$, and in turn $devi$ is greater than $deviman$.

The magnitude of differences between these variables are large: for most countries, *devi* is more than twice as big as *deviinv*, with India's *devi* more than 5 times greater than its *deviinv*. This implies that large components of *devi* are idiosyncratic (i.e. industry-specific), which offset each other when aggregated. This is confirmed as we look at the correlations of an industry's investment with more aggregated measures of investment. Columns 6 and 7 of table 3 report for each country the average correlation coefficient of manufacturing industries' investment with manufacturing investment (*corman*) and aggregate investment (*corinv*). In general, *corman* is substantially lower than 1, and *corinv* is even lower.

Plot 6 also shows that *devi*, *deviman*, and *deviinv* are all negatively related to output level, but that *deviman* is more negatively related to output level than *deviinv* is, and that *devi* is more negatively related to output level than *deviman*. So the idiosyncratic component of *devi* is greater for poorer countries than richer countries. This is confirmed by comparing the correlations of an industry's investment with more aggregated measures of investment across countries of different output levels. Plot 7 plots *corman* and *corinv* against output level. In all three cases, the coefficients tend to be smaller for poorer countries.

The above findings, that there is a substantial idiosyncratic component in *devi* across countries, and that this idiosyncratic component of *devi* is greater for poorer countries, provide support to modeling uncertainty at the industry level rather than at more aggregate levels.

4. QUANTITATIVE ADAPTATION OF THE MODEL ECONOMY

Data analysis in section 3 showed that the qualitative predictions of the model economy are consistent with the data: poorer countries have larger *devi*, larger *diff*, and smaller *frac*. In this section, we explore the quantitative predictions of the model economy. In particular, we want to determine to what extent uncertainty can account for differences in investment efficiency and output level across countries. For this purpose, we assign parameter values to countries of different output levels, and derive the implied differences in investment efficiency and output level among them.

I adopt the following procedure to assign parameter values. Parameters β , α , δ and function h are assumed to be common to all countries. We consider the length of a period as a quarter and set $\beta = .99$. This implies an annual real interest rate of about 4%. We consider two values of α , $1/3$ and $2/3$. The first is the physical capital share typical in literature. The second is the share of broader capital including human capital (see Mankiw, Romer, and Weil (1992)) or technology capital (see Parente and Prescott (1994)). Assigning different values of δ does not affect the model's implications for investment efficiency and output, and therefore I do not assign any particular value to δ . Parameters d , a , and b are country-specific. Given h , different values of d , a , and b imply different industry-level investment dynamics. We can use the industry-level investment to determine values for d , a , and b for different output levels as well as function h .

Let a subscript s denote countries of different output levels. To find $\{(d_s, a_s, b_s)\}$ and h , we first represent the industry-level investment dynamics for countries of different output levels in the data by a line \bar{l} in the space of *devi*, *diff*, and *frac*. Next, we find $\{(J_s, N_s, a_s, b_s)\}$ whose simulated industry-level investment dynamics are "close" to \bar{l} . Finally, we find $\{(d_s)\}$ and h that are consistent with $\{(J_s, N_s, a_s, b_s)\}$. Each of these

steps will be explained in detail in next three sub-sections.

IV.1 Deriving \bar{l}

Plots 8 and 9 show that it is plausible to take as features of the data the positive and linear relationships between *devi* and *diff* and the negative and linear relationship between *devi* and *frac*, for the segment that is relevant to the data (i.e. $.2 \leq devi \leq .5$). Think of a three dimensional space where the axes are *devi*, *diff*, and *frac*. We can represent the values of these variables for each of 16 countries in the data as a point in the space. Let $\{p_s\} = \{(devi_s, diff_s, frac_s)\}$, $s = 1, 2, \dots, 16$, denote these points. Further, since the pairwise relationships among the three variables in the data can be taken as linear, we can draw a straight line in the space that represents these relationships. Let $l = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ denote a line in the space:

$$diff = \gamma_1 + \gamma_2 \times devi \quad \text{and}$$

$$frac = \gamma_3 + \gamma_4 \times devi,$$

where $\gamma_1, \gamma_2, \gamma_3$, and γ_4 are constants and $.2 \leq devi \leq .5$. Given l , the distance between two points $p = (devi, diff, frac)$ and $p' = (devi', diff', frac')$ in the space is defined as

$$\Delta(p, p'; l) = [(devi - devi')^2 + (\frac{diff - diff'}{\gamma_2})^2 + (\frac{frac - frac'}{\gamma_4})^2]^{\frac{1}{2}}.$$

The division of the second and the third terms by γ_2 and γ_4 are necessary since we do not want the units of measurement to matter. The distance between a point $p = (devi, diff, frac)$ and a line $l = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ is defined as

$$\Delta(p, l) = \min\{\Delta(p, p'; l) : p' \text{ is an element of the line } l\}.$$

The line that represents the relationships among the three variables in data is defined as

$$\bar{l} = \operatorname{argmin}\left\{\sum_{s=1}^{16} \Delta(p_s, l)^2\right\}.$$

Following this way of choosing constants, we can derive $\bar{l} = (-.2882, 2.0135, .6204, -.8677)$. Columns 5, 6, 7, and 8 of table 5 are the points on the line \bar{l} that are closest to the points $\{p_s\}$ and their respective distances.

IV.2 Finding $\{(J_s, N_s, a_s, b_s)\}$

Since we consider the length of a period to be a quarter, 88 consecutive elements of a simulated investment sequence will constitute 22 years of a yearly investment sequence, the sum of each four consecutive elements being a year's investment. Such a yearly investment sequence with its *devi*, *diff*, and *frac* will correspond to a point in the space. For the investment sequences simulated for a set of J , N , a , and b to be considered as resembling the investment sequences of, say, the US, we would want the range of points generated by d , a , b , and h to be in some sense close to the point for US on the line \bar{l} . One simple way to think about this closeness is by measuring the distance of the expected point over the distribution of points generated by J , N , a , and b from the point for the US on the line \bar{l} . For us to say that the expected point is close to the point on the line \bar{l} , the distance would have to be not very big in reference to the average distance between \bar{l} and country points $\{p_s\}$, which is .0346.

The detailed method that I used to find the sets of d , a , b and h is as following. First, I selected some candidate sets of values J , N , a and b , and simulated the investment sequences for each of them to see if its estimated expected point (*devi*, *diff*, *frac*) is close to some selected points on the line \bar{l} . I selected J and N to be 1 (quarter), 2, 3, ..., 27, or 28; a to be .1, .2, .3, ..., .8, or .9; and b to be 0, .1, .2, ..., .8, or .9. Selection of candidate values and simulation are inevitable since we do not have the analytical solution for the expected point. For simulation, I set the initial investment to be zero, and simulated 2876 periods of investment sequence for each set of J , N , a , and b . Then, I threw away the first 500 elements of the sequence. This leaves us with 2376 periods, or 594 years of yearly investment sequence after summing up each of four consecutive periods' investments.

Then, I divided the sequence into 27 of 22 years of sequences,¹⁴ which correspond to the numbers of industries and years in the data. Then, I derived *devi*, *diff*, and *frac* for each of 27 sequences. Averaging these values over 27 sequences gave me an estimate of the expected point in the space. Then, I derived the distance between this point and each of 7 points on line \bar{l} with *devi* = .20, .25, .30, .35, .40, .45, or .50. These 7 points represent the industry investment dynamics of countries of 7 different output levels. After deriving these distances for each of the candidate sets of J , N , a , and b , I selected approximately 20 sets¹⁵ of $\{(J_s, N_s, a_s, b_s)\}$ for each of $s = 1, 2, \dots, 7$, that have the smallest distances to the respective point on line \bar{l} .

IV.3 Finding $\{(d_s)\}$ and h

Given a set of (J_s, a_s, b_s) , a value of N_s corresponds to a set of values of d : this can be seen by inspecting the expression of R in equation 1. Further, we can show that a higher value of N_s corresponds to a higher value of d . So there is a range of values of d corresponding to a value of N_s . We can easily compute this range of d corresponding to a set of (J_s, N_s, a_s) , and b_s . For each set of (J_s, N_s, a_s, b_s) , I took the lowest value of this range of d as the corresponding d_s (with a margin of error equal to .1). Now we have sets of values $(J_s, N_s, d_s, a_s, b_s)$ for each of $s = 1, 2, \dots, 7$.

Next we need to derive the function h . This function h is common to all countries, so we are looking for function h corresponding to 7 sets of $(J_s, N_s, d_s, a_s, b_s)$, $s = 1, 2, \dots, 7$, each representing uncertainty of a country with a different output level. These 7 sets of values are derived by permutating the sets of values $\{(J_s, N_s, d_s, a_s, b_s)\}$, $s = 1, 2, \dots, 7$,

¹⁴ This implies *devi*, *diff* and *frac* of the 27 investment sequences are correlated. To avoid this, we would want 27 sequences generated independently of each other. Computational burden limited me from doing so. In this procedure, however, the estimates of *devi*, *diff* and *frac* are not biased.

¹⁵ It is easier to find these sets consistent with the data for countries either of high output level or of low output level, than for countries of middle output level. Therefore, I select somewhat higher numbers of sets for countries either of high output level or of low output level, than for countries of middle output level.

that I derived above. To derive the differences in investment efficiency among countries with these differences, we do not exactly need to derive h : we only need to derive the pairwise factor differences among $h(J_1), h(J_2), \dots, h(J_7)$ (see equation (3)). To derive these factor differences, first note that the following conditions must be satisfied:

$$R(J_s, N_s; d_s, a_s, b_s) \geq R(j, n; d_s, a_s, b_s)$$

for all j, n , and $s = 1, 2, \dots, 7$. Let $\tilde{R}(j, n; d, a, b) = R(j, n; d, a, b)/h(j)$. \tilde{R} is independent of h . Now the above conditions can be written as

$$\frac{h(J_s)}{h(j)} \geq \frac{\tilde{R}(j, n; d_s, a_s, b_s)}{\tilde{R}(J_s, N_s; d_s, a_s, b_s)}$$

for all j, n , and $s = 1, 2, \dots, 7$. In particular, for any $s, u = 1, 2, \dots, 7$, we have

$$\frac{\tilde{R}(J_u, N_u; d_s, a_s, b_s)}{\tilde{R}(J_s, N_s; d_s, a_s, b_s)} \leq \frac{h(J_s)}{h(J_u)} \leq \frac{\tilde{R}(J_u, N_u; d_u, a_u, b_u)}{\tilde{R}(J_s, N_s; d_u, a_u, b_u)}.$$

Solving this set of inequalities gives us the range of factor differences among $h(J_1), h(J_2), \dots, h(J_7)$. Of course, there is no guarantee that there will be a solution. In fact, for the majority of the cases, there are no h that solve the inequalities.

Once we have 7 sets of $(J_s, N_s, d_s, a_s, b_s)$, $s = 1, 2, \dots, 7$ and a range of factor differences in $h(J_1), h(J_2), \dots, h(J_7)$, the range of factor differences in investment efficiency (or price of investment good) and output level among countries of the 7 different output levels that are accountable for by differences in uncertainty are derived from equations (2) and (3).

IV.4 Findings and Interpretations

For sets of values $\{(J_s, N_s, d_s, a_s, b_s)\}$, $s = 1, 2, \dots, 7$, and h , that I considered, I found ranges of differences in investment good prices and output levels across countries that are accounted for by uncertainty of the investment environment. The first sets of $(J_s, N_s, d_s, a_s, b_s)$, $s = 1, 2, \dots, 7$, in table 6 illustrates the lower end of the range: they imply that if the level of investment efficiency, or equivalently the investment good price,

for a country with its *devi* equal to .2 (roughly equivalent to the US) is normalized to 1, that for a country with its *devi* equal to .3 (roughly equivalent to Spain) is .94; that for a country with its *devi* equal to .4 (roughly equivalent to Colombia) is .84; and that for a country with its *devi* equal to .5 (roughly equivalent to Ethiopia) is .58. The second sets of $(J_s, N_s, d_s, a_s, b_s)$, $s = 1, 2, \dots, 7$, in table 6 illustrates the upper end of the range: they imply the corresponding numbers to be .88, .55, and .24, respectively. Overall, these findings show that uncertainty of the investment environment is an important factor in accounting for differences in investment good price across countries. In fact, differences in uncertainty can account for all of the differences in investment good price among the 16 countries that we considered (see plot 2).

The magnitude of differences in output level accounted for by differences in uncertainty of the investment environment depends on the capital share α in the production function as well as the sets of $(J_s, N_s, d_s, a_s, b_s)$, $s = 1, 2, \dots, 7$ (see equation 3). For the lower end of these differences, we assume the physical capital share of 1/3 and the first sets of $(J_s, N_s, d_s, a_s, b_s)$, $s = 1, 2, \dots, 7$, in table 6. Then, when the output level for a country with its *devi* equal to .2 (roughly equivalent to the US) is again normalized to 1, the output level for a country with its *devi* equal to .3 (roughly equivalent to Spain) is .97; that for a country with its *devi* equal to .4 (roughly equivalent to Colombia) is .92; and that for a country with its *devi* equal to .5 (roughly equivalent to Ethiopia) is .76. For the high end of these differences, we assume a broader capital share of 2/3 and the second sets of $(J_s, N_s, d_s, a_s, b_s)$, $s = 1, 2, \dots, 7$, in table 6. Then, the corresponding numbers are .77, .30, and .06, respectively.¹⁶ Although these differences are not as big as the observed differences in output levels across countries, they are still quite large, and show that uncertainty of

¹⁶ Note that in the model, output is measured in units of the consumption good: we convert the value of investment good to the value of consumption good using the investment good price of respective countries. Since the investment good price relative to consumption good is higher for poorer countries, we overvalue the investment good of poorer countries in comparison to richer countries. Summers and Heston (1993) measure output using a weighted (across countries) investment good price common to all countries. In view of this practice, the output differences reported here are underestimates.

the investment environment may be an important factor in accounting for differences in output levels across countries.

5.CONCLUSION

In this paper, I offered differences in uncertainty of the industry-level investment environment as an explanation for differences in the investment good price and output level across countries. I presented a model economy where more uncertain investment environment leads to lower level of investment efficiency through two channels. First, under uncertainty investors undertake shorter-term investment projects despite the fact that the longer-term projects yield higher before-tax rates of return. Second, investors may abandon unfinished investment projects under unfavorable shocks, resulting in a waste of resources. Uncertainty makes the investment good price higher to make up for the inefficiency, and the higher investment good price leads to the lower output level. If we assume that there is more uncertainty in poorer countries than richer countries, then we can qualitatively account for the observed differences in the investment good price and output level across countries.

In the model, differences in uncertainty also lead to differences in the industry-level investment dynamics. Then the model implies a certain relationship between the investment good price or output level for a country *and* the industry-level investment dynamics for that country. An investigation of industry-level investment data for 16 countries over the period from 1967 to 1988 shows that the relationship between the investment good price or output level *and* the industry-level investment dynamics among these countries are consistent with the implication of the model. Simulated investment sequences from the model can also quantitatively mimic actual investment sequences for countries of different output levels. I then used this criterion of mimicking actual investment sequences to assign uncertainty parameter values to countries of different output levels to determine to what extent the differences in uncertainty of the investment environment can account for

differences in the investment good price and output level across countries. I found that the differences in uncertainty can account for all of the observed differences in the investment good price, and a significant portion of differences in the output level across countries. For instance, differences in uncertainty between the US and Ethiopia can explain the difference in the investment good price between the two countries by factor of up to 4. If we assume the capital share in the output production function to be $2/3$, this translates to difference in the output level between the two countries by factor of up to 16. I emphasize that these differences in the investment good price and output level are attributable solely to differences in uncertainty across countries, with a common expected tax rate on investment. One may hypothesize that the poorer countries tend to also have higher expected tax rate on investment than richer countries. These differences in the expected tax rate could account for (some of) the remaining differences in the output level across countries that are not accounted for by differences in uncertainty of the investment environment.

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Figure 1: illustration of expected tax rate $\phi(t)$

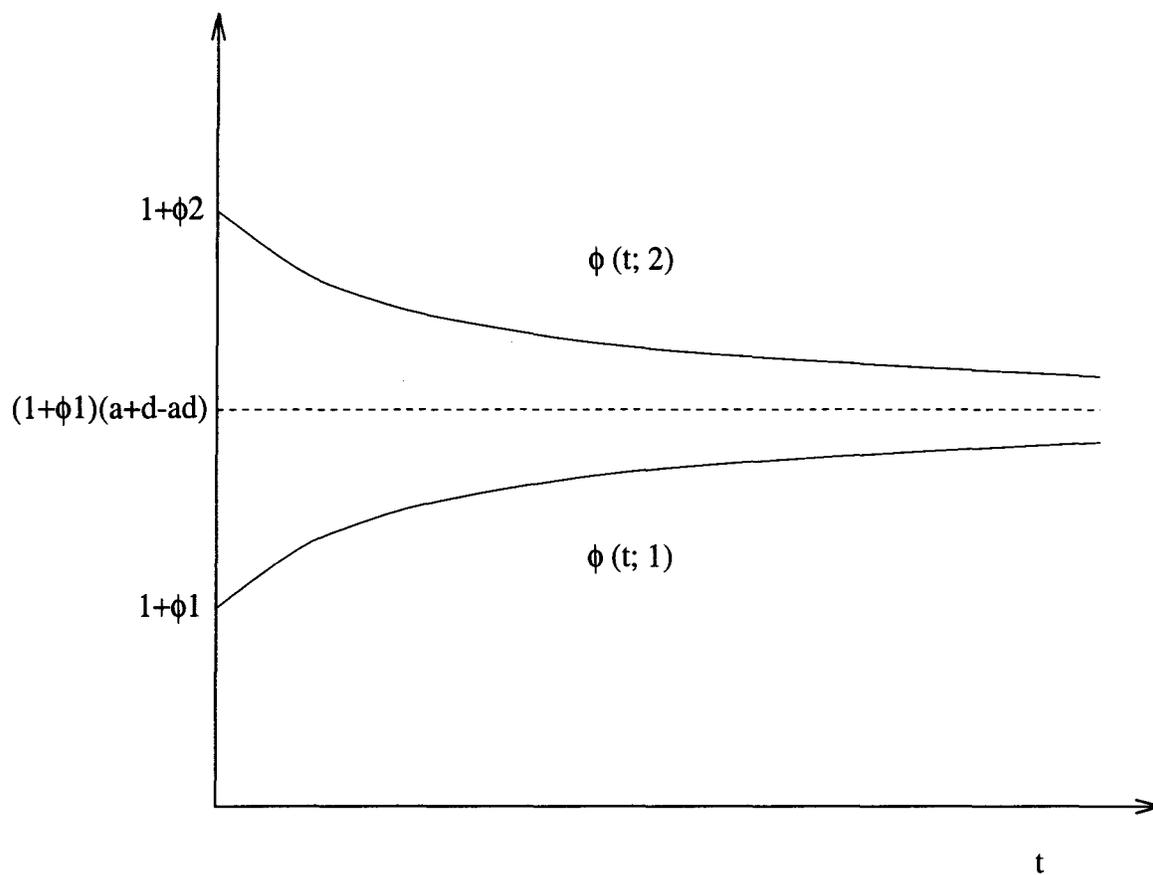


Figure 2: illustration of investment sequence 1

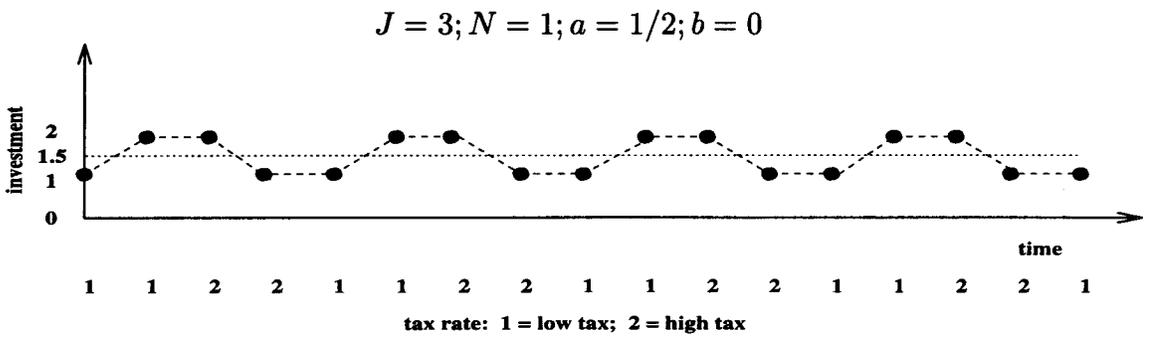
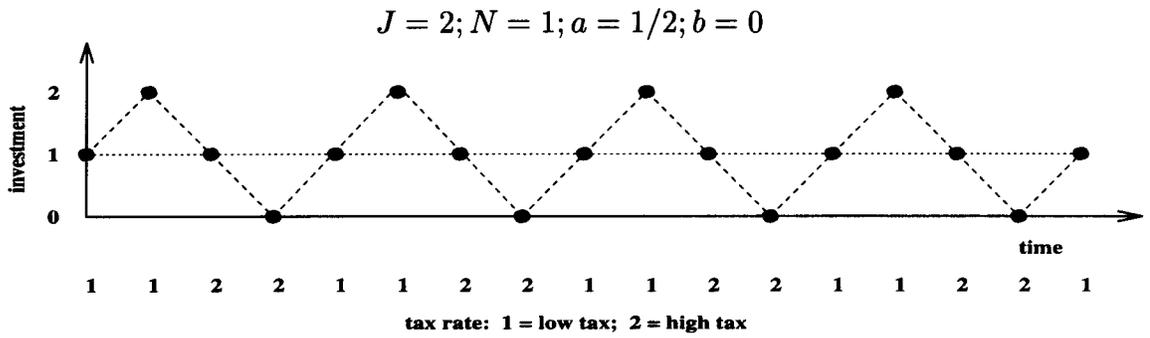
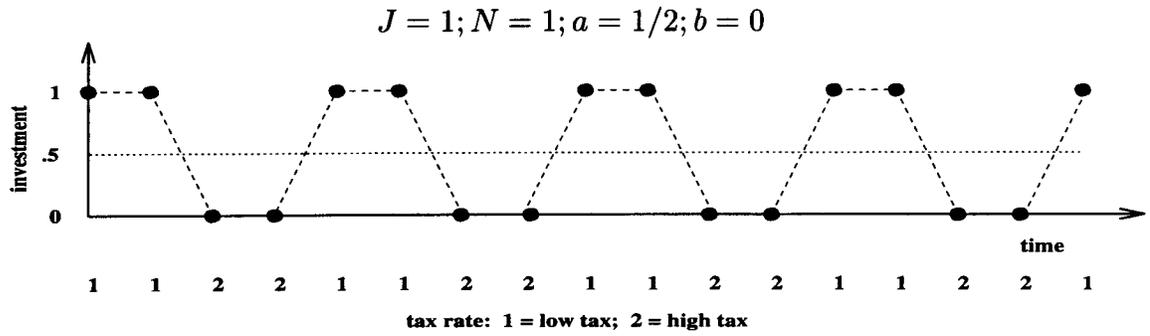


Figure 3: illustration of investment sequence 2

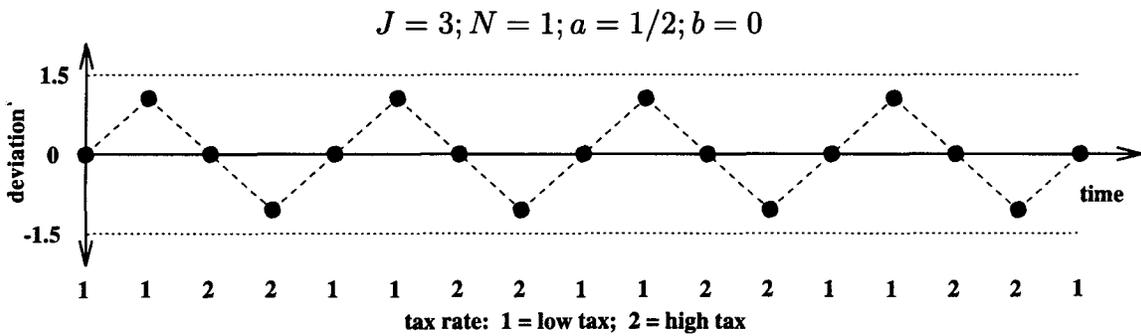
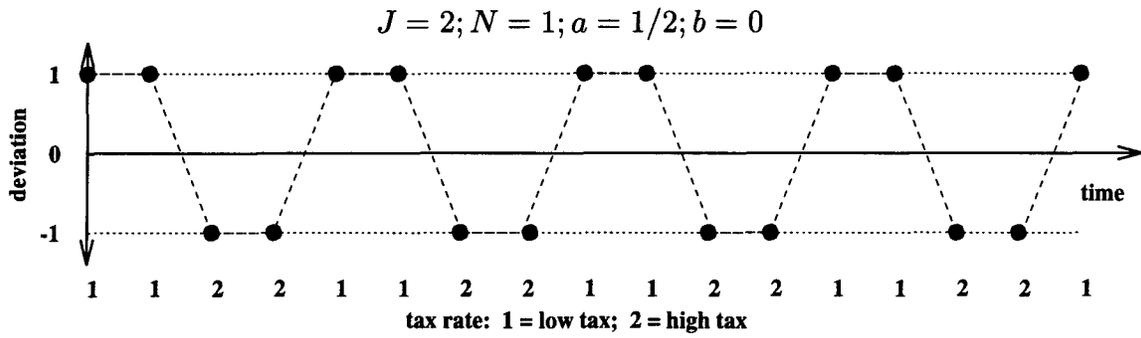
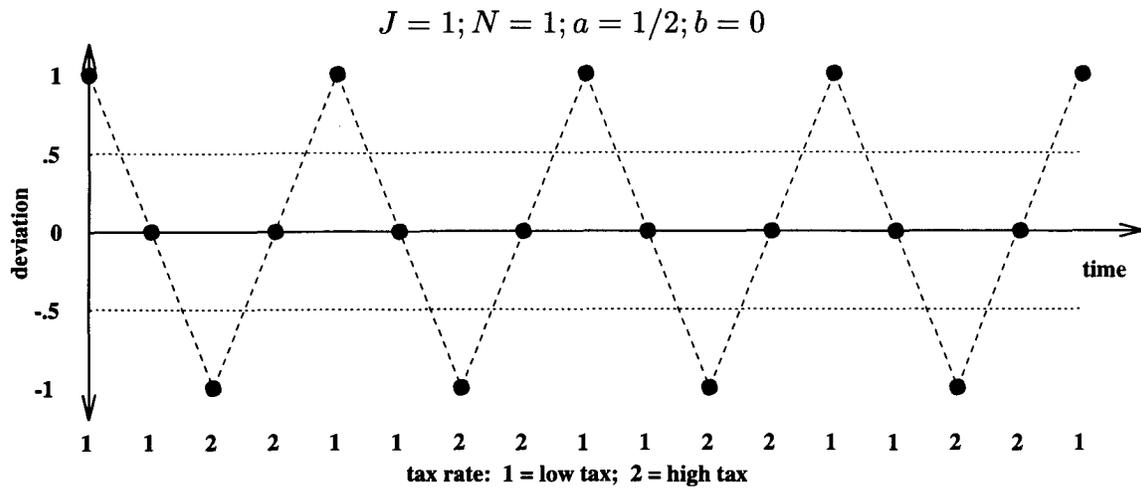


Figure 4: illustration of investment sequence 3

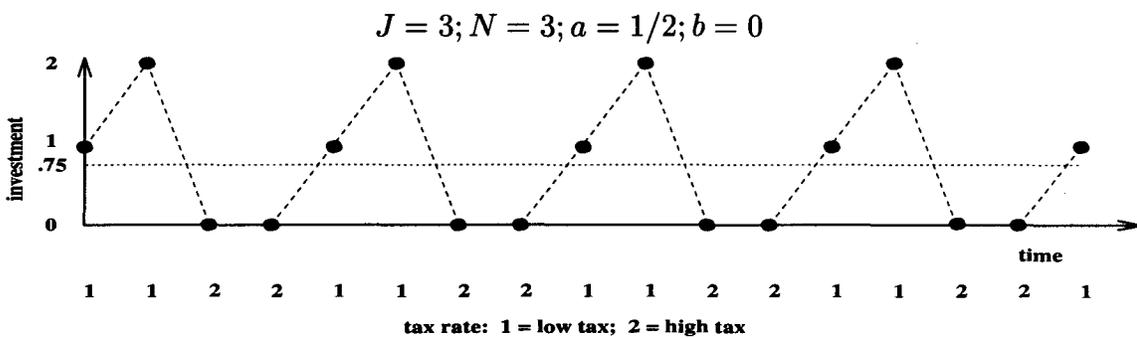
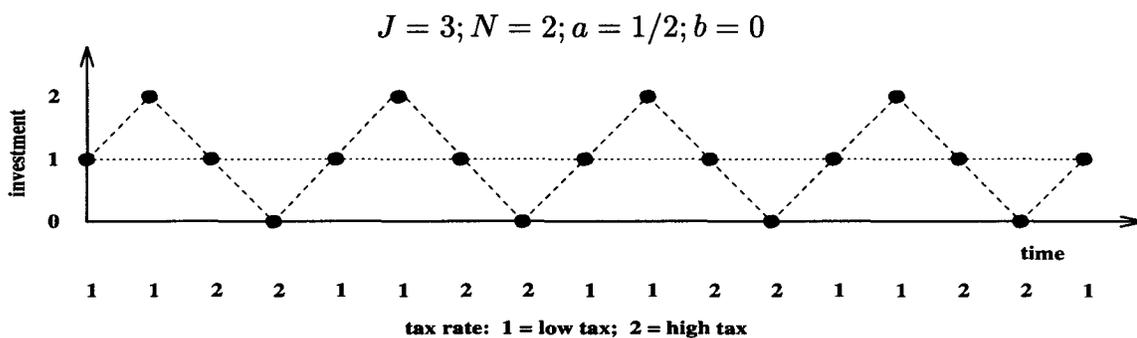
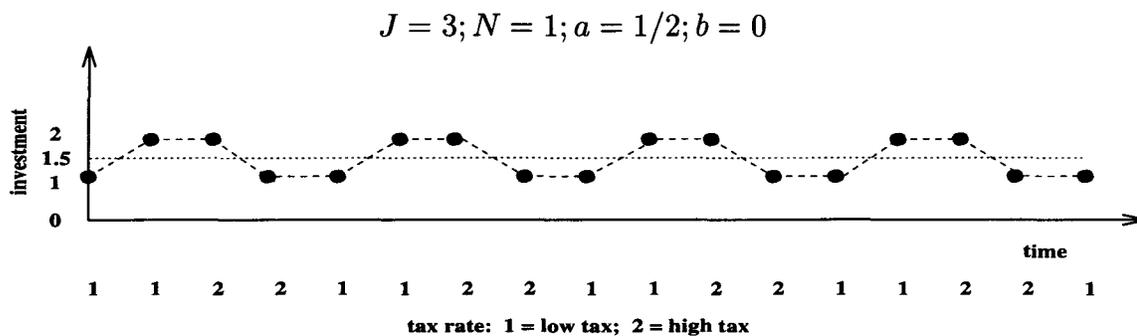


Figure 5: illustration of investment sequence 4

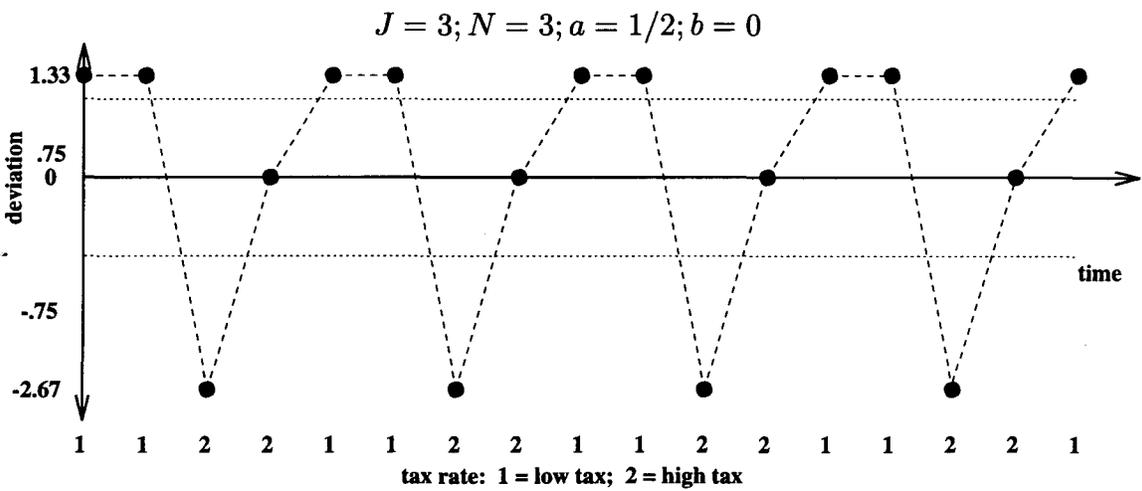
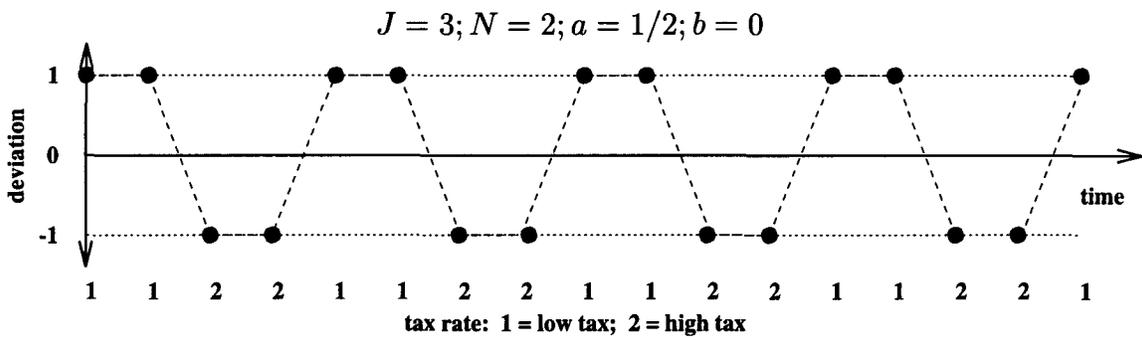
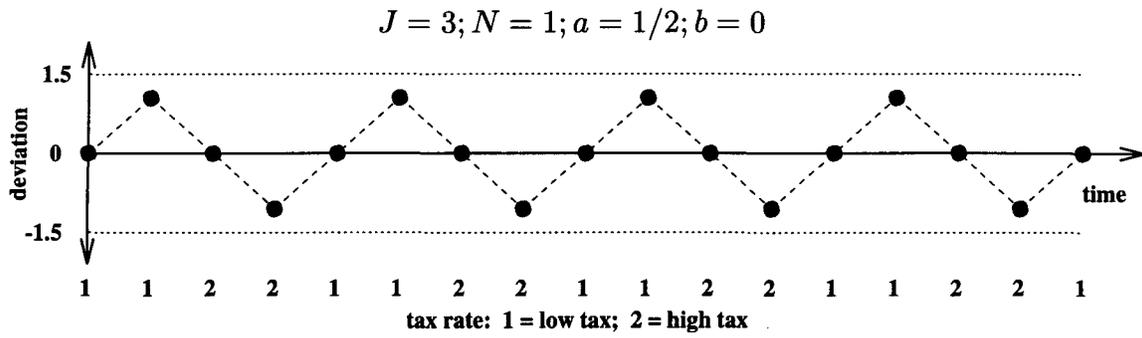


Figure 6: illustration of investment sequence 5

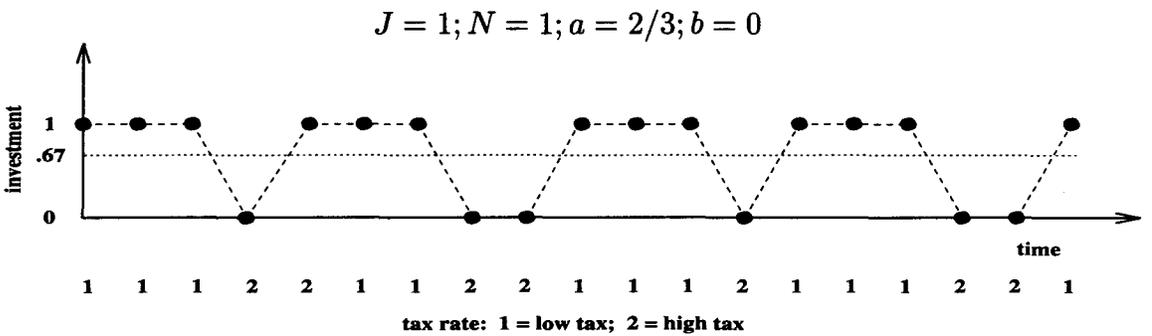
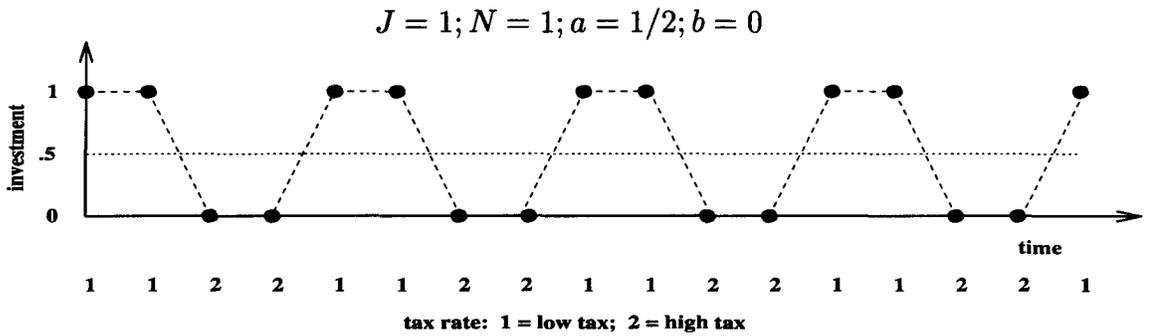
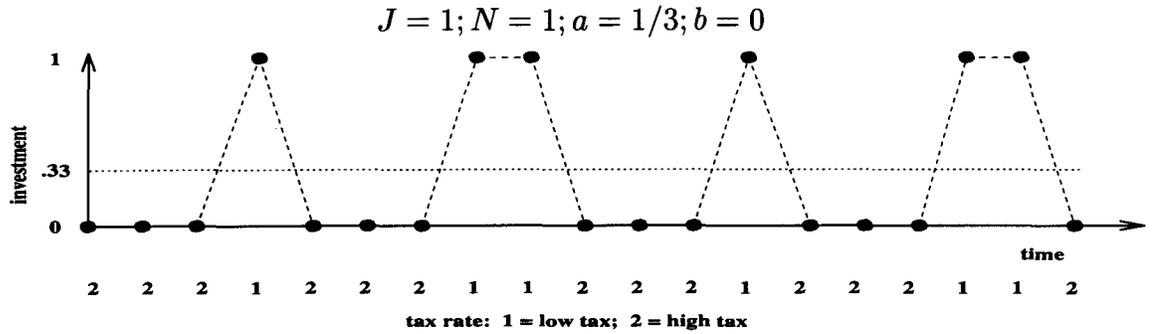
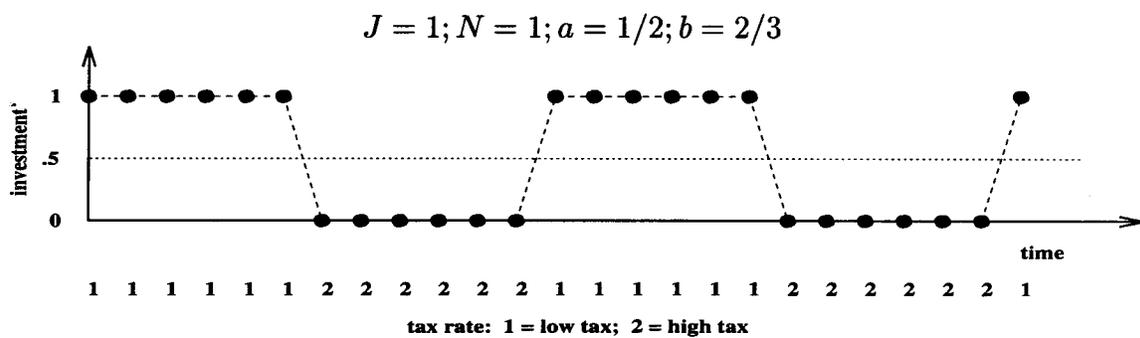
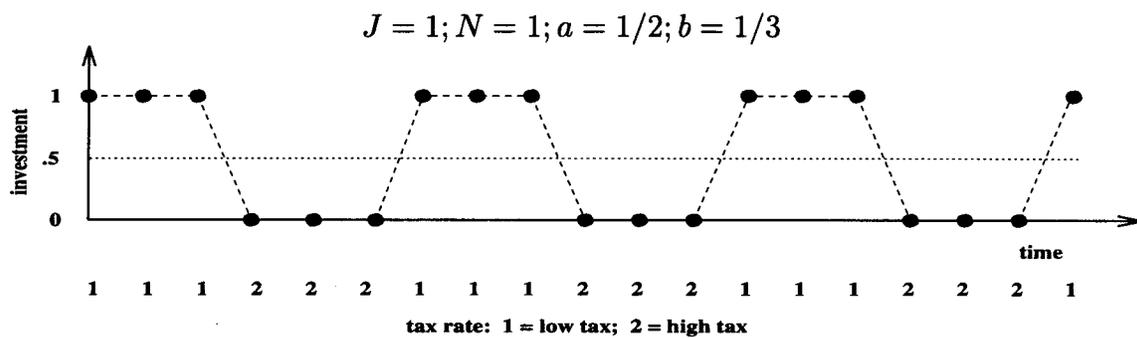
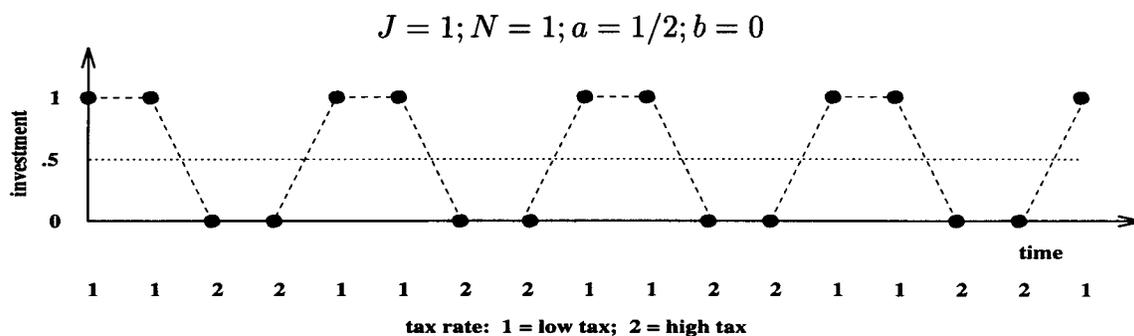
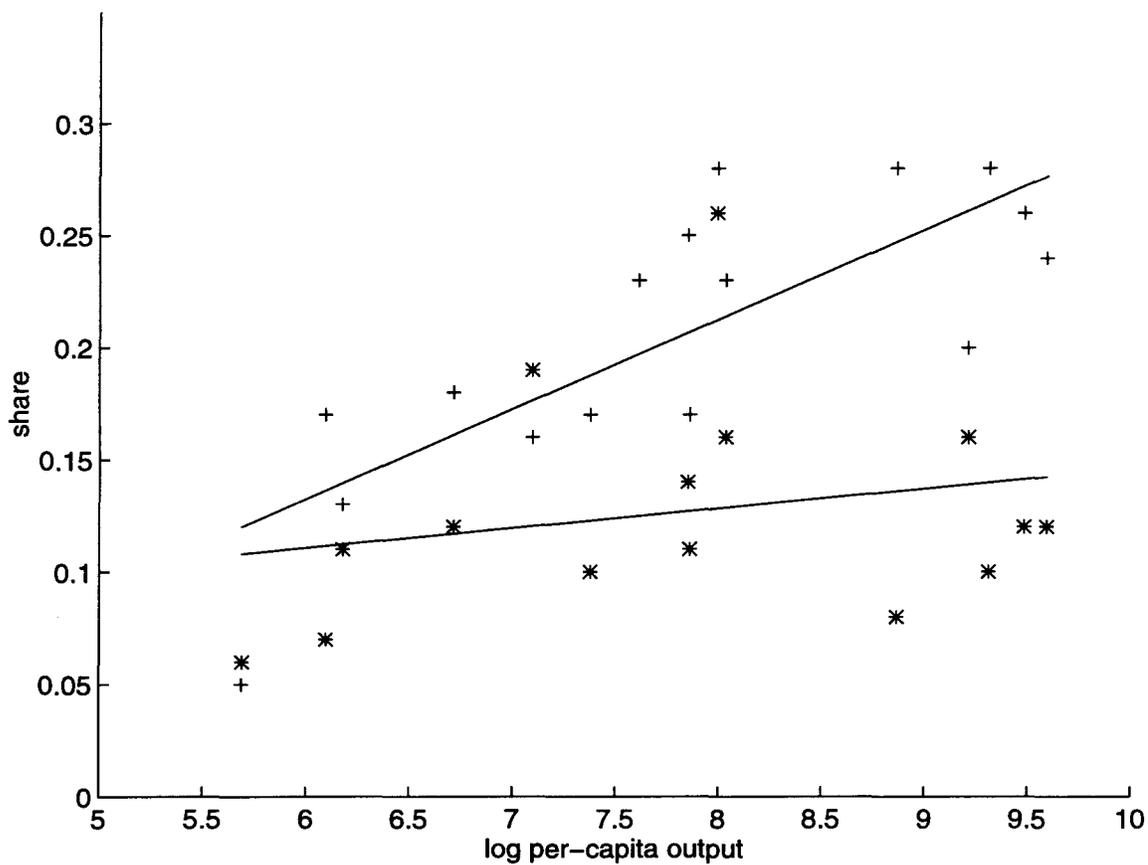


Figure 7: illustration of investment sequence 6

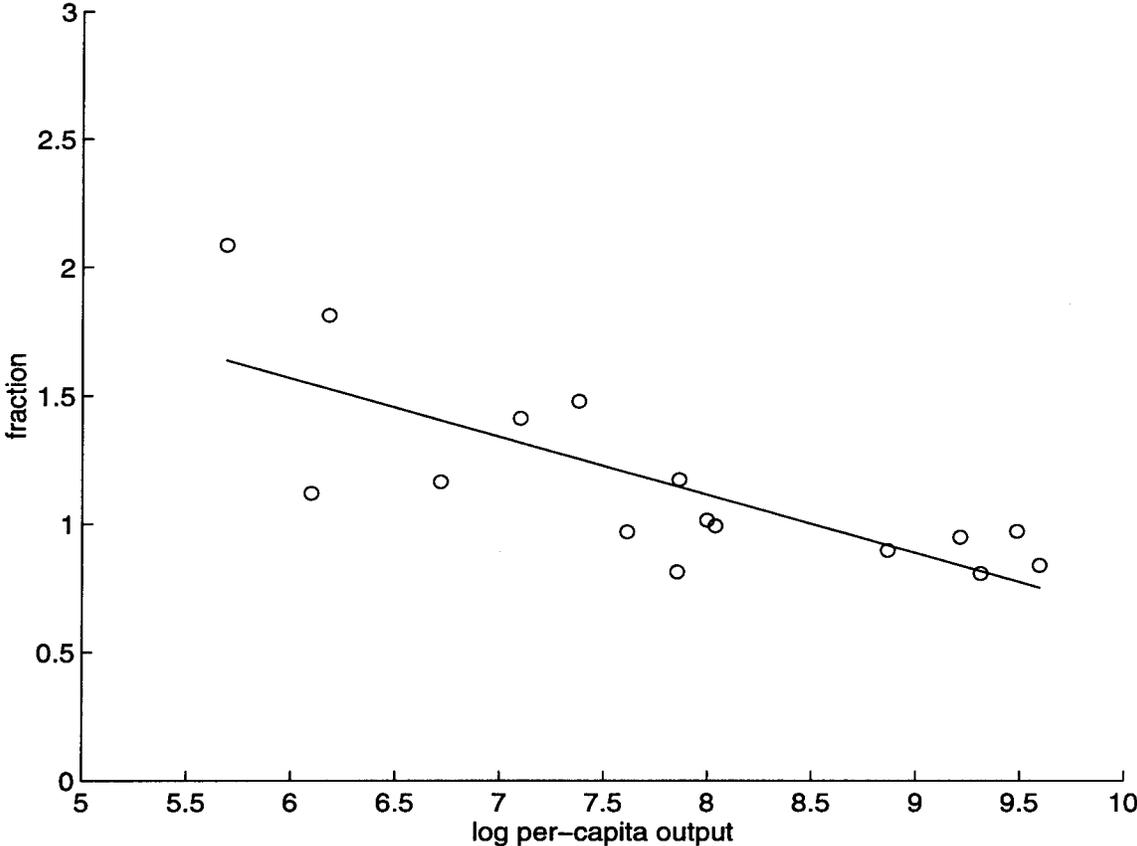


Plot 1: investment share of output and manufacturing share of investment

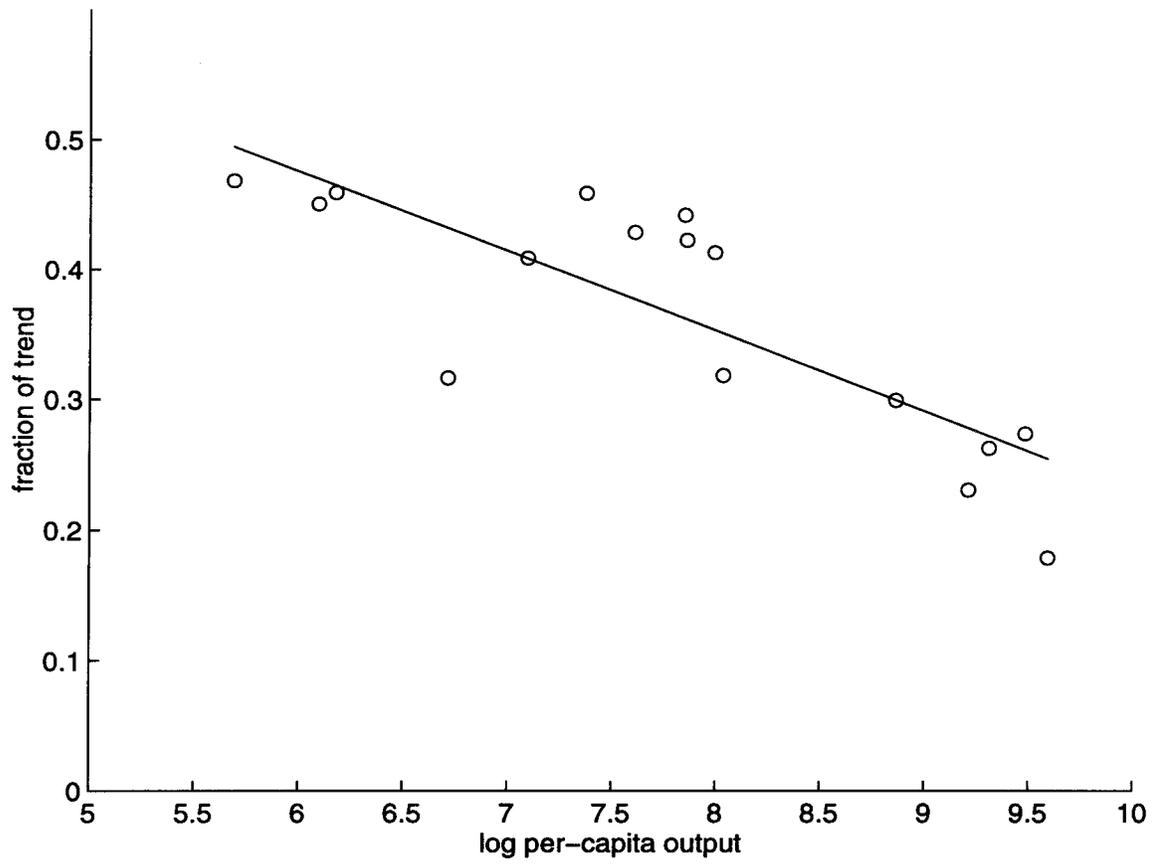


+ : investment share
* : manufacturing share

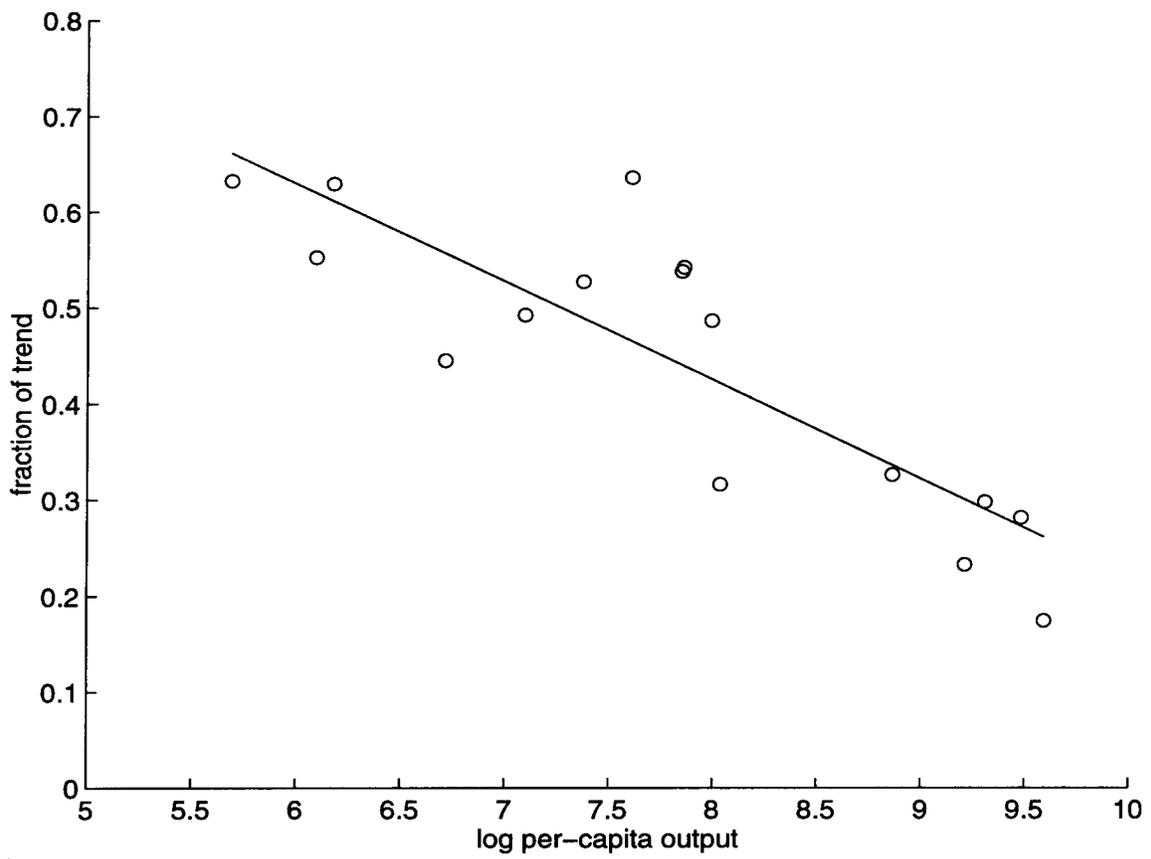
Plot 2: investment good price relative to consumption good price



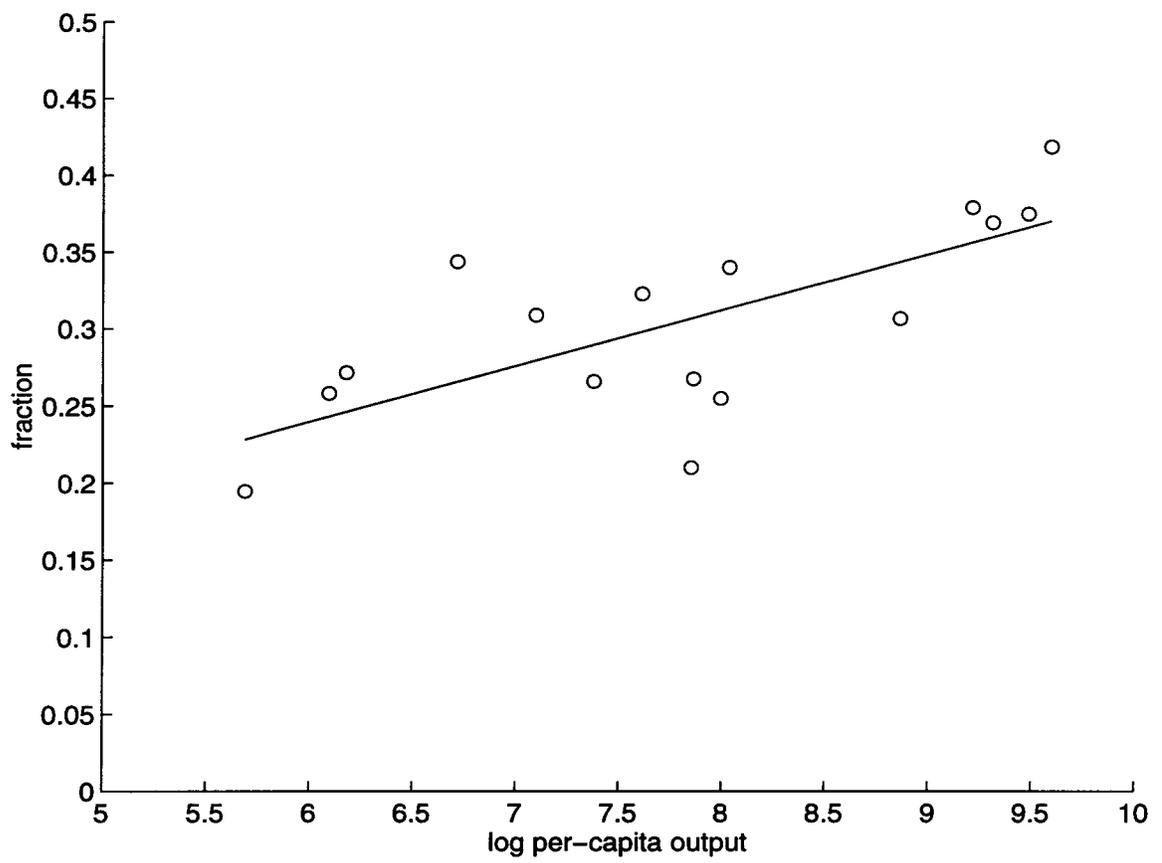
Plot 3: standard deviation (devi) 1



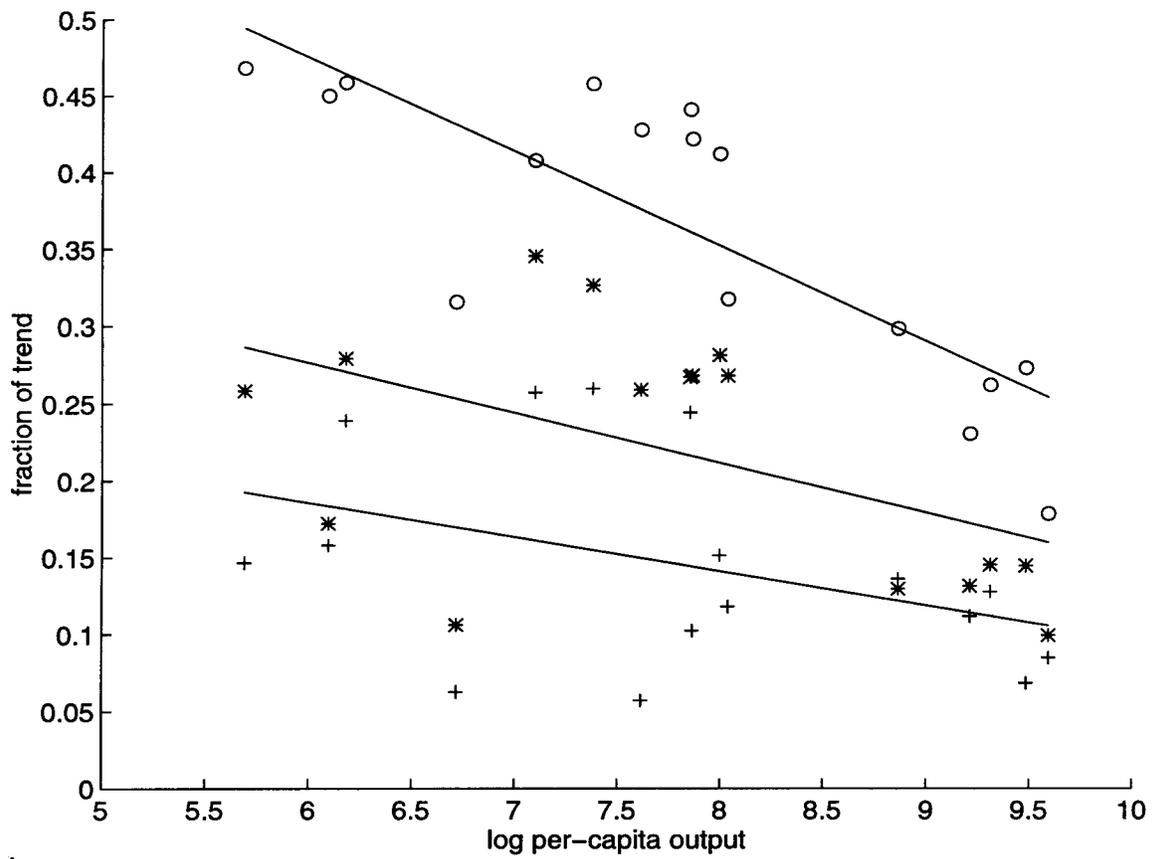
Plot 4: first-order difference (diff) 1



Plot 5: fraction above trend (frac) 1

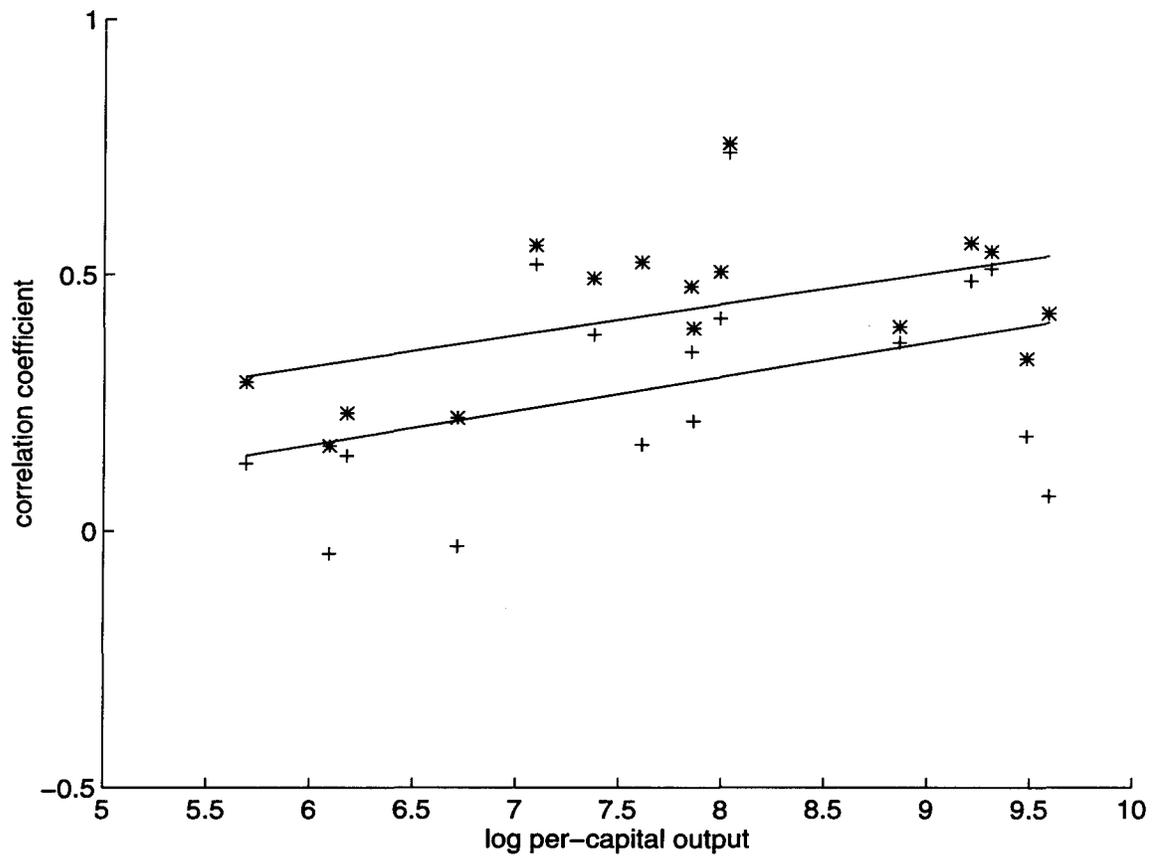


Plot 6: standard deviation (devi, deviman, deviinv) 1



o : manufacturing industries' investment (devi)
* : manufacturing investment (deviman)
+ : aggregate investment (deviinv)

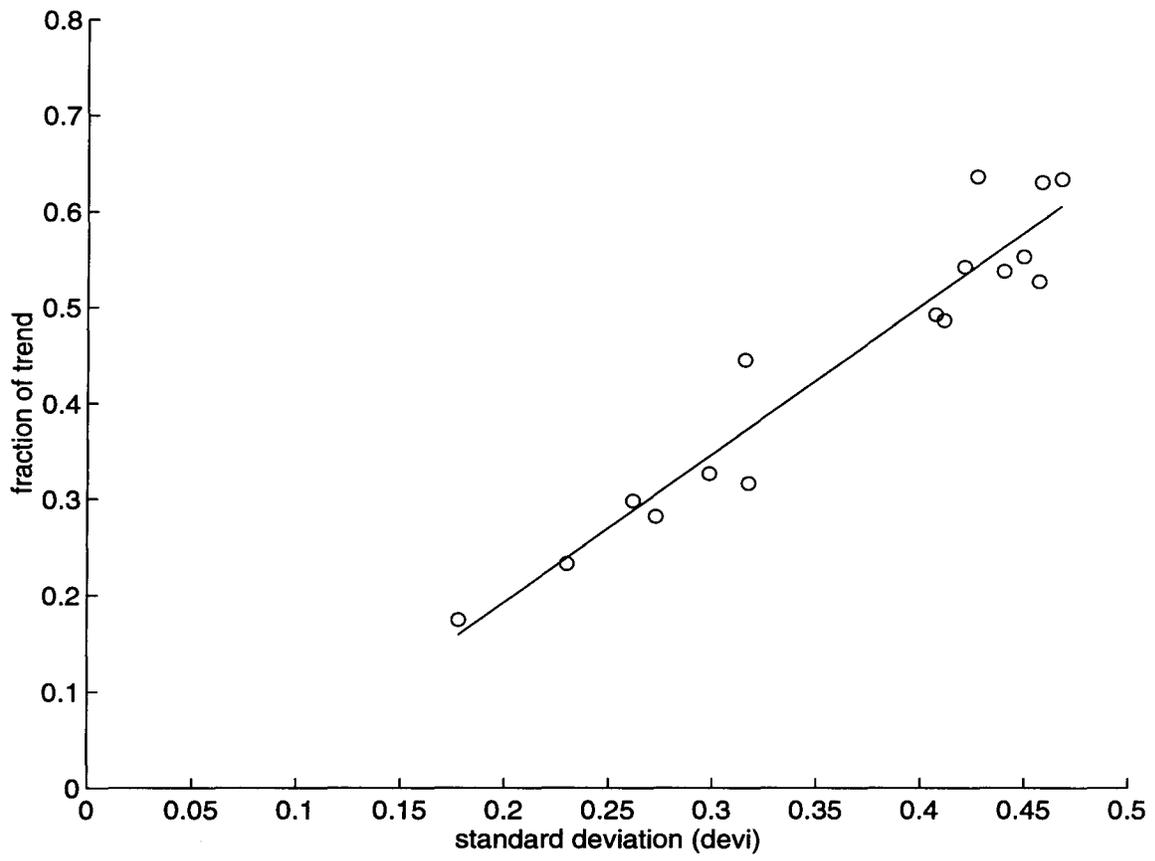
Plot 7: correlation (corman, corinv) 1



* : with manufacturing investment (corman)

+ : with aggregate investment (corinv)

Plot 8: first-order difference (diff) 2



Plot 9: fraction above trend (frac) 2

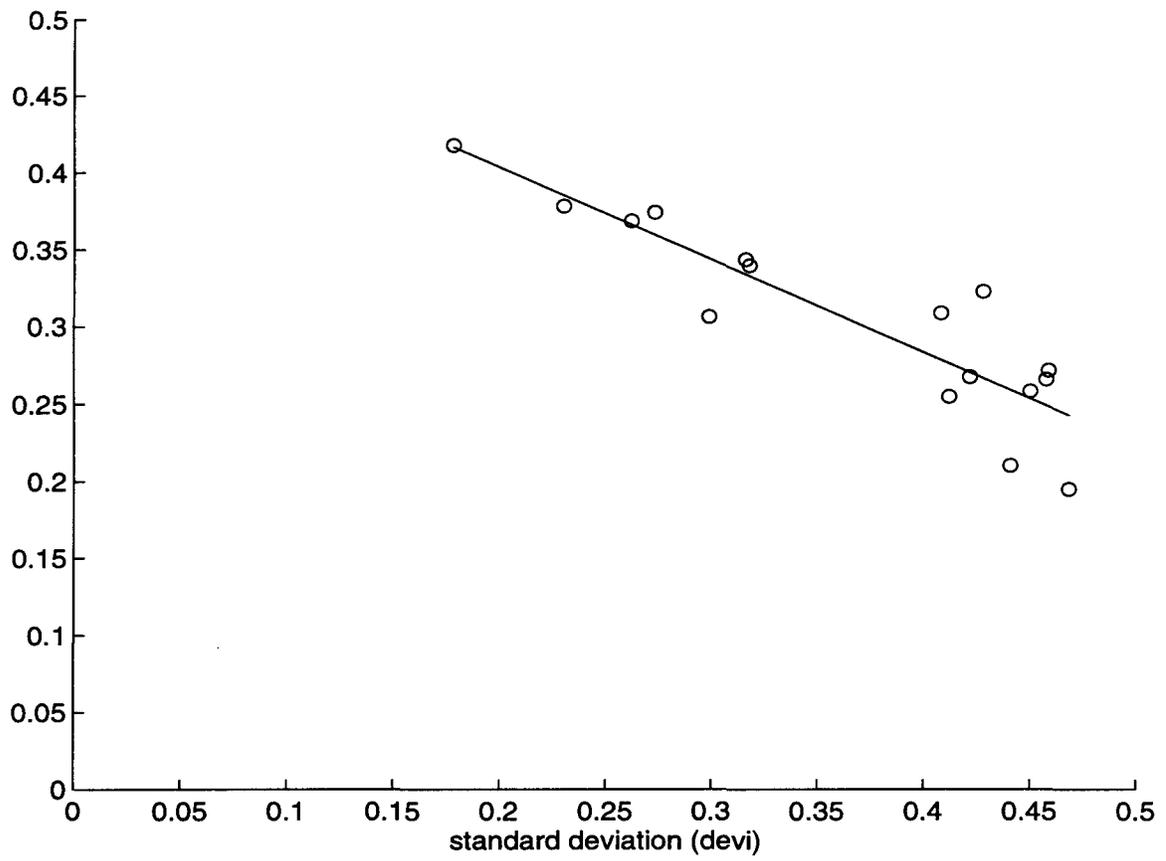


Table 1: years and industries of available data

country	years	industries
Brazil	67-69, 71-74, 76-78, 80	1-6, 8-11, 16-17, 24-26
Canada	67-88	1-17, 19-27
Colombia	67-70, 72-88	1-12, 14, 16, 18-20, 22-27
Denmark	67-88	1-6, 8-13, 17, 20, 23-27
Ecuador	67-88	1-6, 8-14, 16, 20, 23, 25-26
India	77-87	1-17, 19-26
Korea	67-88	1-17, 19-26
Malawi	69-75, 80-86	1-4, 8-10, 12, 20, 23
Philippines	68-75, 77, 79-88	1, 3-6, 8-12, 14, 16-26
Spain	67-76, 79-88	1-2, 4-7, 10-13, 16-20, 22
Sri Lanka	79, 81-86, 88	1-5, 7-10, 12-13, 16-18, 20, 23-26
Tanzania	67-74, 81-85	1-13, 16, 23-24, 26
UK	70-88	1-27
US	67-88	1-27
Zimbabwe	73-88	1-5, 7-13, 16-17, 25-26

index for industries

- | | | |
|-------------------------|------------------------------|--------------------------|
| 1: food products | 10: paper and products | 19: glass and products |
| 2: beverages | 11: printing, publishing | 20: non-metal products |
| 3: tobacco | 12: industrial chemicals | 21: iron and steel |
| 4: textiles | 13: other chemical products | 22: non-ferrous metals |
| 5: wearing apparel | 14: petroleum refineries | 23: metal products |
| 6: leather and products | 15: petroleum, coal products | 24: machinery |
| 7: footwear | 16: rubber products | 25: electrical machinery |
| 8: wood products | 17: plastic products | 26: transport equipment |
| 9: furniture, fixtures | 18: pottery, china etc. | 27: professional goods |

Table 2: country statistics 1

country	pout	pop	gpout	invs	mans	pinv
Brazil	3093	103773	6.3	23	16	1.18
Canada	13164	23291	2.8	26	12	1.16
Colombia	2600	25448	2.4	17	11	1.40
Denmark	11073	5049	1.8	28	10	.96
Ecuador	2573	7619	2.8	25	14	.97
Ethiopia	296	35579	.4	5	6	2.49
India	827	719013	2.6	18	12	1.39
Korea	2968	36454	6.7	28	26	1.21
Malawi	483	5772	.4	13	11	2.16
Philippines	1604	46686	1.2	17	10	1.77
Spain	7085	36225	1.7	28	8	1.07
Sri Lanka	2025	15526	3.1	23	N/A	1.16
Tanzania	444	15816	.3	17	7	1.37
UK	10029	56334	2.1	20	16	1.13
US	14681	222078	1.6	24	12	1.00
Zimbabwe	1211	7297	-.2	16	19	1.69

- pout: per-capita output in thousand 1985 US dollars averaged over the period
- pop: population in millions averaged over the period
- gpout: growth rate of per-capita output in percentage over the period from regression
- invs: investment share of output in percentage averaged over the period
- mans: manufacturing share of investment in percentage averaged over the period
- pinv: investment good price relative to consumption good price averaged over the period with the value for US normalized to 1

Table 3: country statistics 2

country	devi	diff	frac	deviman	deviinv	corman	corinv
Brazil	.3175	.3160	.3394	.2681	.1181	.7539	.7361
Canada	.2730	.2814	.3741	.1445	.0683	.3351	.1848
Colombia	.4214	.5415	.2671	.2681	.1024	.3937	.2138
Denmark	.2620	.2978	.3684	.1450	.1277	.5439	.5101
Ecuador	.4405	.5368	.2096	.2671	.2442	.4752	.3481
Ethiopia	.4680	.6323	.1944	.2583	.1466	.2903	.1316
India	.3158	.4444	.3434	.1061	.0627	.2201	-.0297
Korea	.4117	.4858	.2545	.2813	.1513	.5029	.4122
Malawi	.4585	.6291	.2714	.2792	.2389	.2296	.0464
Philippines	.4574	.5262	.2656	.3268	.2598	.4913	.3811
Spain	.2986	.3260	.3063	.1296	.1361	.3971	.3659
Sri Lanka	.4274	.6349	.3224	.2590	.0570	.5231	.1680
Tanzania	.4498	.5520	.2579	.1722	.1579	.1660	-.0440
UK	.2301	.2325	.3782	.1315	.1117	.5606	.4867
US	.1782	.1744	.4175	.0991	.0846	.4230	.0675
Zimbabwe	.4076	.4919	.3086	.3456	.2572	.5559	.5183

- devi: average standard deviation of manufacturing industries' investment in fraction of trend
- diff: average standard deviation of first-order difference of manufacturing industries' investment in fraction of trend
- frac: average fraction of manufacturing industries' investment above trend
- deviman: standard deviation of manufacturing investment in fraction of trend
- deviinv: standard deviation of aggregate investment in fraction of trend
- corman: average correlation coefficient between manufacturing industries' investment and manufacturing investment
- corinv: average correlation coefficient between manufacturing industries' investment and aggregate investment

Table 4: regression results

dependent variable	independent variable	coefficient of independent variable (standard error)	corresponding plot
invs	pout	.0401 (.0083)	plot 1
mans	pout	.0088 (.0106)	plot 1
pinv	pout	-.2271 (.0503)	plot 2
devi	pout	-.0615 (.0116)	plot 3
diff	pout	-.1024 (.0171)	plot 4
frac	pout	.0362 (.0095)	plot 5
deviman	pout	-.0325 (.0153)	plot 6
deviinv	pout	-.0223 (.0136)	plot 6
corman	pout	.0600 (.0287)	plot 7
corinv	pout	.0662 (.0431)	plot 7
diff	devi	1.5381 (.1215)	plot 8
frac	devi	-.6013 (.0799)	plot 9

- invs: investment share of output in percentage averaged over the period
- mans: manufacturing share of investment in percentage averaged over the period
- pinv: investment good price relative to consumption good price averaged over the period with the value for US normalized to 1
- devi: average standard deviation of manufacturing industries' investment in fraction of trend
- diff: average standard deviation of first-order difference of manufacturing industries' investment in fraction of trend
- frac: average fraction of manufacturing industries' investment above trend
- deviman: standard deviation of manufacturing investment in fraction of trend
- deviinv: standard deviation of aggregate investment in fraction of trend
- corman: average correlation coefficient between manufacturing industries' investment and manufacturing investment
- corinv: average correlation coefficient between manufacturing industries' investment and aggregate investment
- pout: per-capita output in thousand 1985 US dollars averaged over the period

Table 5: investment dynamics observed in data

country	devi	diff	frac	devi'	diff'	frac'	dist
Brazil	.3175	.3160	.3394	.3138	.3437	.3481	.0174
Canada	.2730	.2814	.3741	.2799	.2754	.3775	.0085
Colombia	.4214	.5415	.2671	.4135	.5445	.2616	.0102
Denmark	.2620	.2978	.3684	.2812	.2779	.3765	.0235
Ecuador	.4405	.5368	.2096	.4412	.6002	.2376	.0451
Ethiopia	.4680	.6323	.1944	.4720	.6623	.2108	.0244
India	.3158	.4444	.3434	.3330	.3823	.3315	.0379
Korea	.4117	.4858	.2545	.4059	.5292	.2682	.0273
Malawi	.4585	.6291	.2714	.4388	.5953	.2397	.0448
Philippines	.4574	.5262	.2656	.4236	.5647	.2529	.0415
Spain	.2986	.3260	.3063	.3219	.3599	.2649	.0494
Sri Lanka	.4274	.6349	.3224	.4098	.5369	.3411	.0841
Tanzania	.4498	.5520	.2579	.4283	.5742	.2488	.0264
UK	.2301	.2325	.3782	.2559	.2272	.3983	.0348
US	.1782	.1744	.4175	.2139	.1426	.4348	.0439
Zimbabwe	.4076	.4919	.3086	.3848	.4866	.2865	.0343

- (devi, diff, frac): investment dynamics observed in data
- (devi', diff', frac'): correspondidng point on the line \bar{l}
- dist: distance between (devi, diff, frac) and (devi', diff', frac')

Table 6: illustrations of differences across countries

illustration 1

devi	d	a	b	J	N	dist	E	y	y'
.20	1.9	.8	.3	20	2	.0084	1.00	1.00	1.00
.25	2.3	.7	.1	14	2	.0087	.97	.99	.94
.30	1.9	.7	.3	10	2	.0212	.94	.97	.89
.35	2.7	.6	.1	7	2	.0421	.89	.94	.80
.40	2.7	.6	.0	5	2	.0412	.84	.92	.70
.45	2.5	.5	.2	3	2	.0376	.71	.85	.51
.50	3.0	.3	.3	2	2	.0045	.58	.76	.34

illustration 2

devi	d	a	b	J	N	dist	E	y	y'
.20	1.1	.7	.6	20	1	.0093	1.00	1.00	1.00
.25	2.3	.7	.1	14	2	.0087	.91	.95	.83
.30	1.9	.7	.3	10	2	.0212	.88	.94	.77
.35	2.2	.6	.2	9	2	.0428	.86	.93	.74
.40	4.6	.7	.0	7	3	.0429	.55	.74	.30
.45	5.6	.6	.0	6	3	.0361	.50	.71	.25
.50	8.1	.4	.1	3	3	.0060	.24	.49	.06

- devi: standard deviation of investment on line \bar{l} representing different output levels
- d: dispersion parameter
- a: frequency parameter
- b: persistence parameter
- J: type (i.e. duration) of investment project
- N: continuation/abandonment decision
- dist: distance between the point on the line \bar{l} (with devi) and the expected point for d, a, and b from simulation
- E: investment efficiency
- y: output if α , capital share, is 1/3, with the value for US normalized to 1
- y': output if α , capital share, is 2/3, with the value for US normalized to 1