CREDIT IMPERFECTIONS, INEQUALITY

AND ECONOMIC GROWTH

by

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"Credit Imperfections, Inequality and Economic Growth"

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Abstract

This paper presents a dynamic general equilibrium model to investigate the interaction of financial markets with economic growth, change of industry structure and the evolution of wealth across households along the development process. I find that in the early stages of development the economy experiences "extensive growth," in which the growth rates are increasing and the fraction of entrepreneurs is positively correlated to the level of aggregate output. The engine of growth in this stage comes from the reallocation of resources from low to high productivity sectors. In the middle and mature stages of development the economy experiences "intensive growth," that is, the fraction of entrepreneurs is negatively correlated with the level of output and the source of growth in these stages is the higher average productivity achieved by the competition among entrepreneurs. As a result, the growth rate could be increasing in the middle stage and then displays a decreasing pattern during the mature stage.

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I .- INTRODUCTION

This paper studies how financial markets and accumulation of financial assets interact with economic growth, change of industry structure, productivity and distribution of wealth across households. To carry out this study, a dynamic general equilibrium model with imperfections in the credit market is developed.

In particular I want to focus on the following observations:

(i) In the early stages of development we observe low or moderate rates of growth. In the final stages, the economy's growth rate converges to a value that is usually higher than those rates experienced at the beginning, but lower than the rates observed in the middle phase of development.

(ii) The structural change in the economy. The population movement from the subsistence sector to the industrial sector.

Following Kuznets (1971), the developed countries have experienced a significant acceleration of growth when they entered the stage known as modern economic growth. The recent experience of Japan , is also a clear example of the nonlinear pattern displayed by the growth rate. During almost 20 years, from 1950 to 1970, the Japanese economy grew at rates above ten percent. Before and after this period the average growth rate was below five percent (see figure 1). The behavior of the growth rate in Japan has been studied by Parente and Prescott (1994), Minami (1973), among others.

The same generalization about the growth rate can be drawn from cross-countries comparisons. Table 1 , presents the average growth rate of group of countries ranked by income, according to the World Development Report 1983. The growth rate is higher for the middle-income countries and slows down for high-income countries. This behavior is the same for total and per capita income. Also, Cho (1992) and Chenery, et al. (1986) present evidence of the humped pattern of growth across countries.

TABLE 1

Growth of Total and Per Capita Income of All but Centrally Planned Countries by Income Group

Per Capita Income of Country Group	Annual Growth Rate of Per Capita Income (%) 1960-81	Annual Growth Rate of GDP (%) 1960-81
India	1.4	3.5
China	3.4	5.4
Other Low-Income	0.8	4.1
Lower Middle-Income	3.4	5.3
Upper Middle-Income	4.2	6.0
Industrial Market Economies	3.4	4.0

Source: Chenery, et. al. (1986)

In this paper, I want to stress the relationship between the pattern of growth and the structural change.

> " In the presence of significant differences in factor returns across sectors, structural change becomes an essential element in accounting for the rate and the pattern of growth. On the one hand, that change can retard growth if its pace is too slow or its direction inefficient. On the other hand, it can contribute to growth if it improves the allocation of resources" (Syrquin, 1986 page 252).

The observation that rapid economic growth is accompanied by a relative reduction in the employment in agriculture has been documented by Deane and Cole (1969) for the case of UK and by Maddison (1982) for the case of US. In more recent years, Kim (1978) studies the patterns of growth and the structural change in Korea, and Minami (1973) studies the economic take-off and the evolution of the employment in Japan. For the case of Japan, in figure 2 , we can observe that agricultural employment shrinks consistently from 1950 to 1970. This structural change in Japan coincides with the acceleration of growth over this period.

In order to explain the relationship between growth rates and structural change , a model base on the recent literature on distributional dynamics is developed. This literature emphasizes the role of the occupational choice and intergenerational transition of wealth in the development process. Banerjee and Newman (1993) show that the economy can either become prosperous or stagnate depending on the initial wealth distribution. Galor and Zeira (1993) point out that with credit constraints and indivisibility in human capital investment, the initial distribution of wealth affects the growth rate in the short and long-run. Lloy-Ellis and Bernhardt (1994) focus on the dynamic evolution of inequality in an economy. Their results are consistent with the Kuznets's hypothesis.

This paper not only tries to describe the income distribution dynamics but also attempts to explain the process of economic take-off. If we eliminate the credit-constraints, the result would be exactly that of the Solow model, where the economic equilibrium depends on aggregate level of wealth and not on the distribution of wealth across households. On the other hand, for a complete credit constraint economy, we would have a development trap (as in Banerjee and Newman, 1993). Thus, the economic growth is affected by the level of imperfection in the credit market and the mechanism that produces economic take-off is endogenous in the model.

The model is very simple. There exists a continuum of individuals who live one period. They receive utility from consumption and from leaving bequests to their children. Given the bequest the individuals received, they decide if they are willing to pay the entry cost to use a modern technology. If they do so, they will draw a productivity shock and then decide whether or not to use this technology. For any set of prices and individual wealth, there exists a threshold level of the productivity shock, such that, any agent with a shock greater (lower) than this value will (will not) become an entrepreneur. If the individual becomes an entrepreneur, he will borrow funds in order to hire labor and capital, receiving the output and repaying the debt at the end of the period. If the individual does not become an entrepreneur, they either use a subsistence technology or become a worker in the modern sector.

The financial market is incomplete. The reason is twofold. First, lenders cannot enforce borrowers to repay their debt unless the debt is secured. Second, the returns from investment are only partially collateralizable. This fact implies that the entry cost must be self-financed and loans will be an increasing function of the individual's wealth. Borrowing constraints arise endogenously in this case.

The development process is as follows. At the early stages the growth rates are low because only a small fraction of the population is able to pay the entry and demanding labor and capital, so wages and interest rates are at their lowest levels. This is the stage of "extensive growth." In this period the economy grows because there are more entrepreneurs and the wealth accumulation process is very slow because wages and interest rate are low.As the dynasties accumulate wealth, the fraction of entrepreneurs becomes bigger, and so does the demand for labor and capital. As the interest rate and wages start to increase, the threshold level of the productivity shock also increases. This has two opposite effects. On one hand, since prices are higher, fewer people will choose to be an entrepreneur. On the other hand, because the threshold level is higher, only the more productive entrepreneurs will remain in the market.. If the last effect dominates the first effect, we would expect increasing growth rates and productivity levels in this stage. This is the stage of "intensive growth," the fraction of entrepreneurs tends to decrease, but they become more productive. There is also and intertemporal effect, because the prices are increasing the wealth of non-entrepreneurs accumulates at faster rates. The potential number of entrepreneurs in the next stage becomes bigger, enhancing the competition among entrepreneurs.

Eventually, more and more entrepreneurs become wealthy enough to self-finance their projects and to invest in financial assets. At this point, the interest rate starts to decline. However, the wage would be increasing and converging from below to its steady state level. Note that this result depends on the fact that there is no population growth in the model and aggregate wealth is strictly increasing. The threshold level of the productivity shock will be decreasing and the fraction of new entrepreneurs would be declining too. Under these conditions, we observe decreasing sequences for the growth rate and productivity levels. In the long-run, the wealth distribution converges to an invariant distribution.

As we said above, if credit imperfections are too severe, the economy would never take-off. The reason for this is very simple: an important part of the population will persistently remain in the subsistence sector, so the interest rate and wages will remain at their lowest levels. This means that the mechanism for development is broken.

The rest of the paper is organized as follows. In Section 2, we will discuss the environment. Section 3 is devoted to explain the occupational choices in this economy. Section 4 studies the evolution of wealth within a dynasty. In Section 5 the equilibrium is presented. Section 6 discusses the development process in the economy. Section 7 focuses on the results from simulation. Finally, the conclusions are presented in Section 8.

II.- ENVIRONMENT

There is a continuum of one-period-lived agents of measure one. At the end of the period, the agents die and have a child who lives next period. Each agent is endowed with one unit of labor and some units of the consumption good that was inherited from his/her parent. The agents receive utility from consumption and from leaving bequests to their children. They have identical preferences represented by the utility function:

$$c^{\delta} b^{(1-\delta)} \tag{1}$$

Where c stands for consumption and b for bequest and $0< \delta < 1$. Note that the utility depends on the level of bequest and not on the child's utility. This structure is the same used in Andreoni (1989). There are three technologies available to the agents:

(i) Subsistence technology.- In this technology the agent puts one unit of labor and gets D units of the consumption good.

(ii) Storage technology.- For each unit of the good that the agent invests in this technology at the beginning of the period, he will receive z units at the end. In here we assume that $(1-\delta)z < 1$.

(iii) Advanced technology.- If the agents want to use this technology they have to pay a fixed entry cost equal to ϕ at the beginning. After this, they realize a productivity shock a with support on $[o,a^*]$, probability distribution function $f(a)$ and cumulative distribution function $F(a)$. Given this shock, they can hire labor (l_t) and capital (k_t) and get output at the end of the period, according to the following production function: $af(k_t,l_t)$. For simplicity we will assume that the capital depreciates 100 percent and the consumption good can be convertible to capital one to one without cost. We also assume that f(. , .) is a twice differentiable, decreasing returns to scale function, with $f_k > 0$, $f_l > 0$, $f_{kl} > 0$, $f_{kk} < 0$, $f_{ll} < 0$.

If the individual chooses to become an entrepreneur, then he/she has to self-finance the entry cost. Thus, any agent with initial wealth w_t less than ϕ, cannot operate the advanced technology. However, after the agent pays the entry cost, he/she can use external funds to finance the inputs.

The financial market in this economy is incomplete: there exists an enforcement problem. The lender cannot force the borrowers to repay the debt, but they can seize a fraction θ of the borrower's final income. Then the amount of goods that the agent has to repay cannot be greater than his enforceable income, i.e.:

$$r_t C_t \leq \theta I_t \tag{2}$$

Where r_t is the interest rate, C_t is the amount of credit and I_t is the final income.

If the agent cannot be an entrepreneur, he can use the subsistence technology or become a worker in the modern sector. This puts a lower bound for wages. The wages cannot be lower than D. By the same logic, the interest rate cannot be lower than z.

In this model the decisions are sequential. The agents live for one period. At the beginning of the period they have one unit of labor and some units of the consumption good inherited from their parents. Given their endowments and the equilibrium prices in the economy, they have to decide whether or not to pay the entry cost. If they do not pay it, they will either use the subsistence technology or become workers. This is the problem in the first stage. In the second stage, those who paid the entry cost will draw a productivity shock. For any set of prices and individual wealth, there exists a threshold level of the productivity shock, such that any agent with a shock greater (lower) than this value will (will not) become an entrepreneur. If they decide to use this technology, they can borrow funds in order to hire capital and labor. In the final stage, the individuals receive the output, realize the financial contracts (as a borrower or lender) and then decide between consumption and bequest. Figure 3 shows the timing of decisions. Note that at the beginning of the period the heterogeneity depends on the level of wealth, there are type (w) individuals. In the second and final stages, the individuals will be type (w,a), because the realization of the shock is a source of differentiation.

III.- OCCUPATIONAL CHOICE

There are three stages within the period. We can find optimal policies by solving the model backwards. The solution in the final stage is trivial. Given the final income, the agents will choose consumption and bequest policies to maximize their utility function.

For the second stage, assume that the agent has decided to become an entrepreneur (given that he has already paid the entry cost and realized the productivity shock). Then, he/she must choose the quantities of capital, labor and financial assets (borrowing or lending) in order to maximize profits. Formally, the agent type (w,a) chooses k( ), l( ) and C( ) such that:

$$\pi(w,a,r_t,v_t) = \max_{k,\,l,\,C} \ \{af(k_t,l_t) - r_tC_t\} \tag{3}$$

$$\text{s.t.} \quad v_t\, l_t + k_t \leq C_t + (w-\phi) \tag{4}$$

$$r_t\, C_t \leq \theta\, af(k_t,l_t) \tag{5}$$

Where w is the initial wealth of the agent, a the realized productivity level, and v_t the wage. Equation (4) is the budget constraint: how many inputs the agents can hire depend on the total funds (internal plus external funds). Given the characteristics of the production and utility functions this constraint will always hold with equality (C_t can be positive or negative). Equation (5) is the borrowing constraint, which says that the agent cannot borrow more than a fraction of the final income. Note that the price of the good has been normalized to one. Also the cost of the inputs includes the financial gains foregone for using the funds in the production process (see Christiano and Eichenbaum, 1992).

The explicit solution to this maximization problem is given in the appendix. Note that if the borrowing constraint is binding, the capital and labor demand functions, $k(w,a,r_t,v_t)$ and $l(w,a,r_t,v_t)$, will depend positively on the level of wealth w and will be less than the capital and labor demand functions when the borrowing constraint is not binding ($k^*(a,r_t,v_t)$ and $l^*(a,r_t,v_t)$, respectively).

Definition 1.- Let $\underline{w}(\,)$ be the minimum level of wealth needed to get sufficient credit to hire the unrestricted optimal quantities of capital and labor, i.e.:

$$\underline{w}(\,) = k^*(a,r_t,v_t) + v_t\, l^*(a,r_t,v_t) - (\theta/r_t)\, a\, f(k^*,l^*) \tag{6}$$

Notice that $\underline{w}$ depends on a, r_t and v_t. If an agent has wealth less than $\underline{w}$ we will say that the agent is credit constrained.

If after a is realized the agent decides not to be an entrepreneur, his final income will be:

$$r_t\,(w-\phi) + v_t \tag{7}$$

We can define the final income in stage two of an individual of type (w,a), as:

$$V_2(\,) = \max\{\ \pi(w,a,r_t,v_t),\ r_t(w-\phi) + v_t\} \tag{8}$$

Proposition 1.- There exists a critical level for the technological shock $\underline{a}(w,r_t,v_t)$, such that if the realized productivity shock is greater (lower) than this critical level, then the individual will (will not) become an entrepreneur.

Proof: (See appendix).

Definition 2.- Let us define the function $e(w,a,r_t,v_t)$ as the occupational choice such that:

$$e(\,) = \begin{cases} 1 & \text{if } a > \underline{a} \\ 0 & \text{if } a < \underline{a} \end{cases} \tag{9}$$

At the first stage, the decision is whether or not to pay the entry cost.

Definition 3.- The expected final income at stage one of an agent who pays ϕ is:

$$V_e(w,a,r_t,v_t) = E_a\, V_2(w,a,r_t,v_t) \tag{10}$$

Definition 4.- The final income of an agent who decides to be a worker or to be at the subsistence sector will be:

$$V_s(w,r_t,v_t) = r_t\, w\ +\ v_t \tag{11}$$

Definition 5.- Let us define the payment function $\phi(w,r_t,v_t)$ such that:

$$\phi(\,) = \begin{cases} 1 & \text{if } V_e > V_s \\ 0 & \text{if } V_e < V_s \end{cases} \tag{12}$$

Since, we want to assure that at the earliest stages of development, any agent with wealth greater than the entry cost will be willing to pay it, we are going to assume:

Assumption .- $V_e(\phi,z,D) > V_s(\phi,z,D)$

Now, it can be proved that if there exists an equilibrium, the prices that support that equilibrium belong to a bounded set. Let this set be denoted by B'

Proposition 2.- The equilibrium prices belong to a bounded set.
Proof: (See appendix).

As we did before, we can find a critical value for initial wealth $\omega(r_t,v_t)$, such that, any agent with wealth greater (lower) than this critical level, will choose to (not to) pay the entry cost. The following proposition formalizes this statement.

Proposition 3.- For any (r,v) that belongs to B', there exists a critical level of wealth $\omega(r_t,v_t)$, such that:

$$V_e(\ ) \geq V_s \text{ if } w \geq \omega(r_t,v_t), \text{ and } V_e(\ ) < V_s(\ ) \text{ if } w < \omega(r_t,v_t).$$

Proof: (See appendix).

The intuition behind this result is that, if we take a pair of prices (r',v') outside this set, all the individuals would be better off as workers (for v_t too high, given r_t) or as lenders (for r_t too high, given v_t). In any case, the excess supply functions in the respective market would be positive, so the prices will decrease. The same logic applies for $r_t < z$ and $v_t < D$.

IV.- INDIVIDUAL DYNAMICS

The intergenerational evolution of wealth will depend on the prices, the occupational decision, the idiosyncratic shock and, of course, the level of wealth. We have three cases:

(i) If an agent chooses to be a worker and invests in financial assets the off-spring will have wealth:

$$w_{t+1} = (1-\delta)(r_t w_t + v_t) \quad (13)$$

(ii) If the agent chooses to pay, but the productivity shock realized is too low, the bequest that he leaves would be:

$$w_{t+1} = (1-\delta)(r_t (w_t-\phi) + v_t) \quad (14)$$

(iii) If the agent becomes an entrepreneur, the child will inherited:

$$w_{t+1} = (1-\delta)(\pi(w,a,r_t,v_t) - r_t C_t) \quad (15)$$

The transition diagram in Figure 4 represents the dynamics of the dynasty's wealth, given prices and the productivity shock. If the level of wealth is less than $\omega(\,)$, the agent will be a worker and wealth will move along the segment AB. The intercept is given by $(1-\delta)v_t$ and the slope is equal to $(1-\delta)r_t$.

Now, if the agent decides to pay the entry cost and $a < \underline{a}$; the evolution of wealth will be described by the line CD. This line starts for wealth equal or greater to $\omega(\,)$ as the slope is the same as in the previous case. Point C would be at the level of point A if $\omega(\,) = \phi$. However, if the agent's productivity is greater than $\underline{a}$, the wealth dynamics will be described by EFG. Notice that in the interval $[\omega(\,), \underline{w}(\,)]$, the agent is credit-constrained, this means that, given the prices, the capital and labor demand functions are lower than they would be if there were no enforcement problem. If $w \geq \underline{w}(\,)$, credit restriction is not binding and additional increments in wealth will be allocated to financial assets and not to hire productive factors. The slope of the curve for this case will be the same as before $(1-\delta)r_t$.

In summary, given w_t at the beginning of the period, the individual wealth evolves according to the following stochastic process:

$$w_{t+1} = \Gamma(w_t; a, r_t, v_t) \quad (16)$$

Where Γ is a stochastically monotone operator, in the sense that given $w_1 < w_2$, $\Gamma(w_2; .)$ dominate $\Gamma(w_1; .)$ in the first order stochastic sense. The transition function for the entire wealth distribution H is given by:

$$H_{t+1}(w') = \int\int_{\{(w,a):\ \Gamma(w,a,r,v) < w'\}} d\,F(a)\, d\,H_t(w) \tag{17}$$

V.- EQUILIBRIUM

Definition 6.- Let $\xi_c(r_t,v_t)$ and $\xi_l(r_t,v_t)$ be the excess demand function for credit and labor respectively.

$$\xi_c(r_t,v_t) = \int_{\omega(r_t,v_t)}^{\infty} \int_{\underline{a}(w,r_t,v_t)}^{\alpha^*} [v_t\, l(\,) + k(\,) - (w - \phi)]\, d\,F(a)\, d\,H_t(w) - \int_0^{\omega(r_t,v_t)} w\, d\,H_t(w)$$

$$- \int_{\omega(r_t,v_t)}^{\infty} \int_0^{\underline{a}} (w - \phi)\, d\,F(a)\, d\,H_t(w) \tag{18}$$

The first term is the demand for credit by the agents who pay the entry cost and get a good productivity shock. The second term is the supply represented by those who paid the entry cost but got low productivity. The last term is the supply of credits of those who did not pay the entry cost. Note that for an agent with $w_t > k^*(\,) + v_t\, l^*(\,) + \phi$, the demand for credit will be negative. That is, they start to have positive investment in financial assets. Doing the same for the excess demand function for labor:

$$\xi_l(r_t,v_t) = \int_{\omega(r_t,v_t)}^{\infty} \int_0^{\underline{a}} l(\,)\, d\,F(a)\, d\,H_t(w) - \int_0^{\omega(r_t,v_t)} d\,H_t(w) - \int_{\omega(r_t,v_t)}^{\infty} \int_0^{\underline{a}} d\,F(a)\, d\,H_t(w) \tag{19}$$

The first term is the demand for labor given by the entrepreneurs, and the second and third term are the supply of labor. Now, the equilibrium can de defined.

Definition 7.- An equilibrium is the sequences of consumption and bequest decisions $\{c_t(w,a), b_t(w,a)\}$, the occupational choice and payment function $\{e_t(w,a), \phi_t(w)\}$, the demand for labor, capital and the credit decisions $\{l_t(w,a), k_t(w,a), C_t(w,a)\}$; the sequences for distribution of wealth $H_t(\ )$ and the prices r_t and v_t, such that:

- Given $e_t(\ )$, $\phi_t(\ )$, $k_t(\ )$, $l_t(\ )$, $C_t(\ )$, r_t, v_t and H_t; the individual type (w,a) chooses $c_t(\ )$ and $b_t(\ )$ to maximize the utility.

- Given $\phi_t(\ )$, $e_t(\ ) = 1$, r_t, v_t and H_t; the individual type (w,a) chooses $l_t(\ )$, $k_t(\ )$ and $C_t(\ )$ to maximize the final income.

- Given $\phi_t(\ )$, r_t, v_t, H_t; the individual type (w,a) chooses $e_t(\ )$ to maximize the final income.

- Given r_t,v_t, H_t; the individual type (w) chooses $\phi_t(w)$ to maximize expected final income.

- Given H_t; r_t and v_t clear the markets, i.e.:

$$\xi_c(r_t,v_t) \leq 0 \quad ; \quad \text{with equality if } r_t > z$$
$$\xi_l(r_t,v_t) \leq 0 \quad ; \quad \text{with equality if } v_t > D$$

- The transition rule for the wealth distribution:

$$H_{t+1}(w') \;=\; T_t(H_t)$$

where T is the transitional operator defined in equation (17).

VI.- AGGREGATE DYNAMICS : DEVELOPMENT PROCESS

1.- Underdevelopment Stage

This is the initial stage in the economic process. A characteristic of this stage is that some positive fraction of the population is using the subsistence and the storage technology. Some societies can stay in this phase of development (underdevelopment trap) and never take-off. The next proposition is formalizes this statement.

Proposition 4.- If the credit constrain is too severe (θ closed to zero), the economy will converge in the long-run to a subsistence economy.
Proof: (See appendix).

The key to understand this phenomenon is the persistence on wealth patterns ("if you are poor, so will be your child"). Given that θ is closed to zero, you are constrained by your own funds to finance not only the entry cost but also the inputs. The level of wealth that makes you indifferent between being an entrepreneur or not is greater than ϕ, even if the prices are at their lowest level. If the dynasty's initial wealth is less than the threshold level of wealth, and if the aggregate conditions in the economy do not generate increments in the level of prices, the dynasty's wealth is going to be a sequence that converges to a fixed point. If we take any family with a level of wealth greater than $\omega(\ )$. At some point in time, a member of this dynasty will draw a very low productivity shock, so he will choose to be at the subsistence sector, and from then on, every member of the dynasty will remain in the subsistence sector. This pattern can be observe in Figure 5.

The line from w_0 indicates the wealth dynamics for a dynasty with initial wealth lower than ω. This will converge to w^* in the long run. The line from w_1 indicates a possible wealth evolution for a dynasty with initial wealth higher than ω. As we can see, the dynasty's wealth also converges to w^*. In the long-run we have stagnation.

If the condition for growth is satisfied, meaning that the credit restrictions are not too severe, then we can observe the following dynamics:

<u>Proposition 5</u>.- During this stage, r_t=z and v_t=D, then:

(i) The fraction of entrepreneurs and workers are strictly increasing.

(ii) The demand function for credit is also strictly increasing.

(iii) The growth rate could be increasing

<u>Proof</u>: (See appendix).

The intuition behind these results is the following: In the early stages, only a small fraction of the population is able to pay the entry cost. The demand for labor and capital are very low, so wages and interest rates are at their lowest levels. This implies that the accumulation of wealth of the non-entrepreneurs is very slow in this period. Because of this, the potential number of entrepreneurs grows at a very low rate. The economy grows because a high fraction of the potential entrepreneurs become actual entrepreneurs. The main source of growth in this period is the structural change in the economy. Given the higher factor-productivity in the advanced technology compare to the labor productivity in the subsistence sector and the capital productivity in the storage technology, the reallocation of resources from these sectors to the modern sector produces a positive impact in aggregate output. Notice that the imperfection in the credit market is the reason of the inefficient allocation of resources in this economy.

A corollary from this proposition is that the number of people in the subsistence sector declines monotonically during this stage. Given that the population is constant at each period of time, we have that the size of the subsistence sector (S_t) would be given by:

$$S_t = 1 - E_t - L_t$$

Since E_t and L_t are increasing, S_t must be decreasing.

2.-The Economic Take-off

The next phase of development is when the economic break-through takes place. At some point in time S_t would be zero and the excess demand function of credit at interest rate equal to z would be positive. Then, prices will start to increase. As we said before, this could have two opposite effects. On one hand, this increment in prices would put some potential entrepreneurs out of the market. On the other hand, those who remain in the market are the most productive ones. If the "productivity effect" dominates the "population effect", the economy will experience increasing growth rates. We formalize this in the following proposition:

Proposition 6.- At this stage:

(i) The interest rate and wages are increasing.

(ii) The growth rate could be increasing.

(iii) The fraction of entrepreneurs is decreasing.

Proof: (See appendix).

The dynamics in this stage will depend on the change on the relative prices. First, suppose that labor is the first output in became scarce. To restore the equilibrium wages will increase over subsistence level. The credit-constrained entrepreneurs will have a better borrowing position this period, because their wealth is higher. This scale effect could offset the negative effect of higher wages this period, so that the dynasty's wealth next period will increase for the next period. For the unconstrained entrepreneurs the scale effect does not exist because he is operating at the optimal scale. This means that profits this period will be lower than the profits last period. However, this effect can be canceled out if the interest rates increase to generate sufficient financial profits to compensate the lower managerial earnings. In this situation, the wealth distribution grows in the first order stochastic sense. For simplicity, it is assumed that this is the case. If the interest rate is decreasing or does not increase enough, the wealth of the unconstrained dynasties will be decreasing and the wealth distribution will start to grow in the second order sense. This is the case for the mature stage of the development process.

We can have some intuition about this proposition just from looking Figure 4. The curve EFG has slope equal to $\alpha\pi'(\ )$ which is bigger than one when w is close to ω, and declines monotonically with w, converging to $(1-\delta)r_t$ when w goes to $\underline{w}(\ )$. If the fraction of credit constraint entrepreneurs grows over time (or not decline too much), and given that the average productivity level goes up, we will observe increasing growth rates in the economy. This situation is not sustainable in the long-run. The next section deals with convergence in the economy.

3.- Convergence

Eventually, more and more entrepreneurs become wealthy enough to self finance their projects and to invest in financial assets. At this point, the interest rate starts to decline, however, the wage could be increasing and converging from below to the steady state value. Note that this result depends on the fact that there is no population growth in this economy and aggregate wealth is strictly increasing. The threshold level of productivity shock will be decreasing and the increment in the fraction of new entrepreneurs would be declining (if positive at all). At this stage, the "population effect" and the "productivity effect" work in the same direction. Under these conditions we observe decreasing sequences for growth rate.

The following proposition formalizes this statement.

Proposition 7.- At this stage,

(i) The interest rate r_t is decreasing and converges to r^*.

(ii) The wage v_t is increasing and converges to v^*.

Proof: (See appendix).

In this stage. a big fraction of the entrepreneurs is unconstrained entrepreneurs. They are operating their firms at the optimal scale: $vl^*(\ ) + k^*(\ )$, at this level the modern technology exhibits decreasing returns to scale. When most of the agents have overcome the credit constraint, the economy behaves as could be predicted by the neoclassical growth model.

Now, in order to find the long run equilibrium in this economy, the wealth distribution must converge. The following theorem establishes this fact.

Theorem .- The wealth distribution will converge to a unique invariant distribution.
Proof: (See appendix).

The intuition behind this theorem is that, given that the prices converge and that the transition of wealth is (stochastically) monotone, if the wealth process is globally ergodic, meaning that one agent can move from one wealth interval to another in a finite time, then H_t converges to a unique invariant distribution. In the case where the credit constraint was too severe, we did not have communication between two states (for any given $w < w^*$, the dynasty's wealth will be always less than w^*). In this case the wealth distribution collapses into a one-point distribution.

VII.- A NUMERICAL EXAMPLE

This section presents some numerical exercises. Since I am interested in the qualitative features of the variables, there is not explicit intention to choose the parameters to match the real data of any particular development experience. The same claim applies to the time-period of the model economy. In this case each period is one generation, that can be equivalent to 30 to 40 years.

Keeping this mind, the parameters chosen to realize the simulations are in table 2. The functional form of the technology in the advanced sector is a Cobb-Douglas production function:

$$f(k,l) = k^{\alpha}\, l^{\beta}$$

TABLE 2

Parameters

$\delta = 0.5$	$\theta = 0.4$	$\alpha = 0.25$
$\beta = 0.5$	$\phi = 1$	$D = 0.5$
H_0 (w) unif. on [0,0.9]	f(*a*) unif. on [0,3]	$z = 1.2$

The results are shown in figures 6 and 7. The growth rate displays a non monotone behavior. The first three periods, the growth rate is increasing and then tends to decline. However, there is an increment in the growth rate from period five to six. The reason for this is that in the sixth period the productivity effect dominates the population effect. After this, the growth rate converges monotonically to zero.

Now, in figure 7 we can observe that from period one to five the population moves from the subsistence sector to the modern sector. Fewer people are using the subsistence technology and become workers or managers. Both fractions, managers and workers are increasing in this stage. This is the stage of "extensive growth": The economy grows because there are more entrepreneurs. From period six , the fraction of entrepreneurs is strictly decreasing because they are competing for the scarce resources. This is a period of "intensive growth": The economy grows because the entrepreneurs are richer and more productive.

VIII.- CONCLUSIONS

The paper presents a complete description of the development process, focusing on the behavior of the growth rate. I find that in the early stages of development the economy experiences a period of "extensive growth," in which the growth rates are increasing and the fraction of entrepreneurs is positively correlated with the level of aggregate output. The engine of growth in this stage comes from the reallocation of resources from low to high productivity sectors. In the middle and mature stages of development the economy experiences "intensive growth." At the middle stage the growth rate could be increasing if there is an increasing fraction of credit-constrained entrepreneurs using the modern technology when it exhibits increasing returns to scale. At the mature stage the growth rate will be decreasing because more and more people overcome the credit constraint, using the modern technology when there are decreasing returns to scale. During the middle and mature stages the fraction of entrepreneurs is negatively correlated with the level of output and the source of growth is the higher average productivity achieved by the competition among entrepreneurs.

There are implications for other variables in the economy. The real wage is strictly increasing during the process of development and the real interest rate follows the same nonlinear pattern as the growth rate. The explanation for this is that the labor force becomes scarce as the demand for labor increases and the population remains fixed. Also, credit becomes scarce at the early stages when the entrepreneurs are competing against each other for credit. Then, as they accumulate wealth, most of the entrepreneurs become net lenders. The capital-labor ratio will be weakly increasing during the entire process. The reason for this is that this ratio depends only on the real wage. The wealth distribution dynamic resembles the Kuznets's hypothesis: At the early stages of development, income inequality tends to expand with economic growth but then the inequality shrinks.

Finally, a comment about growth and development in an open economy. In a world with two countries, the country with less inequality (given the same aggregate wealth) or the country with higher aggregate wealth (given the same wealth distribution) will lead the world economy. Both countries will converge to the long-run steady state faster than in the case of a closed economy. However, the disparity, in terms of total output, between these two countries grows bigger at the beginning, but also shrinks faster with respect to the closed economy case.

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APPENDIX

The problem is:

$$\Pi\,(a,w,r_t,v_t) = \max_{k,l,C} \{a\, f(k,l) - v_t C_t\}$$

s.t.

$$v_t + k_t \le C_t + (w - \phi) \qquad \text{(A.1)}$$

$$r_t C_t \le \theta\, a\, f(k_t,l_t) \qquad \text{(A.2)}$$

The first order conditions are, (assuming that (A.1) is binding and $k > 0$, $l > 0$):

$$a\, f_k - r_t - \lambda\,[r_t - \theta\, a\, f_k] = 0 \qquad \text{(A.3)}$$

$$a\, f_l - vt\, r_t - \lambda\,[v_t\, r_t - \theta\, a\, f_l] = 0 \qquad \text{(A.4)}$$

$$\theta\, a\, f(k,l) - r_t\,[v_t\, l_t + k_t - (w - \phi)] \ge 0 \qquad = 0 \text{ if } \lambda \ge 0 \qquad \text{(A.5)}$$

It is easy to see that k/l depends only on the level of v_t, i.e.:

From (A.3) and (A.4) we obtain:

$$f_k/f_l = 1/v_t \qquad \text{for any } \lambda \ge 0$$

From (A.3) - (A.5) we can obtain implicitly the functions $k\,(a,w,r_t,v_t)$ and $l\,(a,w,r_t,v_t)$, (the constrained demand functions for capital and labor, respectively), when $\lambda > 0$; i.e. the restriction is binding.

Also when $\lambda = 0$, we obtain from (A.3) and (A.4) the unconstrained demand functions for capital and labor, $k^*\,(a,r_t,v_t)$ and $l^*\,(a,r_t,v_t)$, respectively.

It is easy to see that $k^*\,(a,r_t,v_t) \ge k\,(a,w,r_t,v_t)$.

From (A.5) we have that $af_k = r_t\,(1 + \lambda)/(1 + \lambda\theta)$, since $\theta < 1$ then $f_k > f_{k^*}$ (remember that when $k = k^*$, $\lambda = 0$). Since f_k is decreasing in k, this implies that $k^*(a,r_t,v_t) > k(a,w,r_t,v_t)$. By the same reason, $l^*\,(a,r_t,v_t) \ge l\,(a,w,r_t,v_t)$.

After some algebra is easy to show that:

$$0 > \partial\, k^*\,(a,r_t,v_t)/\partial\, r_t > \partial\, k\,(a,w,r_t,v_t)/\partial\, r_t$$

$$0 > \partial\, k^*\,(a,r_t,v_t)/\partial\, v_t > \partial\, k\,(a,w,r_t,v_t)/\partial\, v_t$$

$$\partial\, k\,(a,w,r_t,v_t)/\partial\, a > \partial\, k^*\,(a,r_t,v_t)/\partial\, a > 0$$

$$\partial\, k\,(a,w,r_t,v_t)/\partial\, w > \partial\, k^*\,(a,r_t,v_t)/\partial\, w = 0$$

The same is true for the demand for labor.

It is easy to see that $k(a,w,r_t,v_t)$, $l(a,w,r_t,v_t)$, $k^*(a,r_t,v_t)$ and $l^*(a,r_t,v_t)$ are continuous functions of all the arguments.

Characterization of Π:

Proposition (A1): If the firm is using credit, then Π is decreasing in r_t and v_t; and increasing in a and w. If the firm is not using credit, then Π is increasing in r_t.

<u>Proof</u>:

Note that when $w = \underline{w}(a,r_t,v_t)$, $k(a,w,r_t,v_t) = k^*(a,r_t,v_t)$ and $l(a,w,r_t,v_t) = l^*(a,r_t,v_t)$.

$\partial \Pi/\partial v_t = (a f_k - r_t)\, \partial k/\partial v_t + (a f_l - r_t v_t)\, \partial l/\partial v_t - r_t l(a,w,r_t,v_t) < 0$

if $w < \underline{w}(a,r_t,v_t)$

$\partial \Pi/\partial v_t = - r_t\, l(a,w,r_t,v_t) < 0$

if $w \geq \underline{w}(a,r_t,v_t)$

$\partial \Pi/\partial r_t = (a f_k - r_t)\, \partial k/\partial r_t + (a f_l - r_t v_t)\, \partial l/\partial r_t - (v_t\, l(a,w,r_t,v_t) + k - (w - \phi)) < 0$

if $w < \underline{w}(a,r_t,v_t)$

$\partial \Pi/\partial r_t = - (v_t\, l^*(a,r_t,v_t) + k^*(a,r_t,v_t) - (w - \phi)) < 0$

if $w \in [\underline{w}(a,w,r_t,v_t),\ v_t\, l^*(a,r_t,v_t) + k^*(a,r_t,v_t) + \phi]$

$\partial \Pi/\partial r_t = (w - \phi) - v_t\, l^*(a,r_t,v_t) - k^*(a,r_t,v_t) < 0$

if $w \geq v_t\, l^*(a,r_t,v_t) + k^*(a,r_t,v_t) + \phi$

$\partial \Pi/\partial a = (a f_k - r_t)\, \partial k/\partial a + (a f_l - r_t v_t)\, \partial l/\partial a + f(k,l) > 0$

if $w < \underline{w}(a,r_t,v_t)$

$\partial \Pi/\partial a = f(k(a,w,r_t,v_t),\ l(a,w,r_t,v_t)) > 0$

if $w \geq \underline{w}(a,r_t,v_t)$

$\partial \Pi/\partial w = (a f_k - r_t)\, \partial k/\partial w + (a f_l - r_t v_t)\, \partial l/\partial w + r_t$

if $w \leq \underline{w}(a,r_t,v_t)$

$\partial \Pi/\partial w = r_t$

if $w \geq \underline{w}(a,r_t,v_t)$

Moreover, Π is continuous in all its arguments since $k(a,w,r_t,v_t)$, $l(a,w,r_t,v_t)$, $k^*(a,r_t,v_t)$ and $l^*(a,r_t,v_t)$ are continuous.

Proof of proposition 1.-

First note that $\Pi(0, w,r_t,v_t) = r_t (w - \phi) < r_t (w - \phi) + v_t$

Since Π is strictly increasing and continuous in a, $\exists\ \hat{a} \in R_{++}$, s.t.

$$\Pi(\hat{a},w,r_t,v_t) = r_t (w - \phi) + v_t$$

Now, define $\underline{a}(w,r_t,v_t) = \min[\hat{a}(w,r_t,v_t), a^*]$

The proposition follows by the continuity of Π in a.

We can further characterize the function $\underline{a}(w,r_t,v_t)$.

Proposition (A.2): $\underline{a}(w,r_t,v_t)$ is weakly increasing in r_t and v_t and weakly decreasing in w.

Proof:

$\partial \hat{a}(w,r_t,v_t)/\partial r_t = [(w - \phi) - \partial \Pi/\partial r_t]/\partial \Pi/\partial a > 0$	always
$\partial \hat{a}(w,r_t,v_t)/\partial v_t = [1 - \partial \Pi/\partial v_t]/\partial \Pi/\partial a > 0$	always
$\partial \hat{a}(w,r_t,v_t)/\partial w = [r_t - \partial \Pi/\partial w]/\partial \Pi/\partial a < 0$	if $w < \underline{w}$
$\partial \hat{a}(w,r_t,v_t)/\partial w = [r_t - \partial \Pi/\partial w]/\partial \Pi/\partial a = 0$	if $w \geq \underline{w}$

Proof of Proposition 2: The set of prices is a bounded set.

First, I will construct the set and then prove that the equilibrium prices must belong to this set.

We already know that the set (r,v) is bounded from below by (z, D).

Define for each level of w, the set A(w):

$$A(w): \{(r,v) \in [z,\infty) * [D,\infty) : g(w,r,v) \geq 0\}$$

where $g(w,r,v) = V_e(w,r,v) - V_s(w,r,v)$

$$\text{since} \quad \partial V_e/\partial w = \int_{\underline{a}(w,r_t,v_t)}^{a^*} \partial \Pi/\partial w\, dF(a) + \int_0^{\underline{a}(w,r_t,v_t)} r_t\, dF(a) \geq r_t$$

Then g (w,r,v) is weakly increasing in w.

Then for $w_1 < w_2$, $A(w_1) \leq A(w_2)$

Now, define the set B:

$$B: \{(r',v') \in [z,\infty) * [D,\infty): g(\underline{w}(a^*,r',v'), r',v') = 0\}$$

Then we have that for any $w > \underline{w}(a^*,r',v')$, $g(w,r',v') = 0$

Since $\partial V_e/\partial w = r_t$ and for any pair $(r,v) \geq (r',v')$ (where $\geq$ means that at least one component is greater); then $g(w,r,v) < 0$, for any w.

Now, let's extend the set B and define B' as:

$$B' = B \cup [(r,v) \in [z,\infty), [D,\infty): (r,v) \leq (r',v')]$$

Then for any w, $A(w) \subseteq B'$.

Since A(w) is a bounded monotone sequence, A(w) converges to A'.

<u>Claim 1</u>.- A' = B'

Suppose not; then $\exists\ (r,v) \in B' - A'$:

Since $(r,v) \leq (r',v')$, then $g(\underline{w}(a^*,r,v), r,v) \geq 0$ and since $(r,v) \notin A(w)$, then $g(w, r,v) < 0$ for any w. A contradiction.

<u>Claim 2</u>.- In any equilibrium $(r,v) \in B'$.

Suppose not. Then (r',v') are the equilibrium prices and belong to B'^{c}.

Then the agents will not pay the entry cost. They will choose to be workers and invest in financial assets. The excess demand functions for labor and credit are negative, then prices will have to decrease. A contradiction.

<u>Proof of Proposition 3</u>:

This is obvious. Just define: $w_m(r,v) = \min_{w} \{w \geq \phi : (r,v) \in A(w)\}$ and the result follows from proposition 2. Remember that $g(w,r,v) = V_e(w,r,v) - V_s(w,r,v)$ is increasing in w if $w \in [\phi, w_m]$.

<u>Proof of Proposition 4</u>:

Take $\theta = 0$ and then define $w_0(z,D)$ implicitly by $V_e(w_0(z,D), z,D) = V_s(w_0(z,D), z,D)$. Where $w_0(z,D)$ is the critical level of wealth when $\theta = 0$.

Now, if $w_0(z,D) > w^*$, then at the beginning of the period only the fraction $[1 - H(w_0(z,D))]$ will pay the entry cost and only a fraction $\int_{\omega_0(z,D)}^{\infty} \int_{\underline{a}(w,r_t,v_t)}^{a^*} d\,F(a)\, d\,H_0(w)$ will become entrepreneurs at the first period. The rest of the people will be at the subsistence sector.

We want to show that $w < w_0\,(z,D)$ is an absorbing state.

First of all, note that for any level of wealth w, $\exists\, M \geq 1$:

$$\text{Prob}\,[w_{T+M} > w_0\,(z,D) \mid w] < 1 \quad \text{and}$$

$$\text{Prob}\,[w_{T+M-1} > w_0\,(z,D) \mid w] = 1$$

If $w \leq w_0\,(z,D)$, then M=1 and $\text{Prob}\,[w_{T+M-1} > w_0\,(z,D) \mid w] = 0$.

Now, we can find for $N \geq M$:

$$\text{Prob}\,[w_{T+N} > w_0\,(z,D) \mid w_T \in [w_0\,(z,D), \infty)] = \prod_{j=M}^{N} [1 - F(\underline{a}\,(w_j,z,D)]$$

Note that if $w_T < \infty$, so is M. Then fixing M and letting $N \to \infty$ we have:

$$\lim_{N\to\infty} \text{Prob}\,[w_{T+N} > w_0\,(z,D) \mid w_T] = 0$$

Since $F\,(\underline{a}\,(w_T,z,D)) > 0$

This means that:

$$\lim_{N\to\infty} \text{Prob}\,[w_{T+N} < w_0\,(z,D) \mid w_T] = 1$$

Then $w < w_0\,(z,D)$ is an absorbing state.

Lemma 1:

If $r_t = z$, $v_t = D$ and H_t dominates H_{t-1} in the first order sense, then H_{t+1} dominates H_t in the first order sense.

Proof.-

Let $\varphi\,(w,w') = \int_{a:\,\Gamma(w,a) \leq w'} d\,F(a)$

Note that $\varphi_w\,(w,w') \leq 0$ since $\partial\,\Gamma/\partial\, w$ and $\partial\,\Gamma/\partial\, a$ are positive.

Now,

$$H_{t+1}(w') = \int_0^{\infty} \varphi(w,w')\, dH_t(w)$$

$$H_{t+1}(w') - H_t(w') = \int_0^{\infty} \varphi(w,w')\, dH_t(w) - \int_0^{\infty} \varphi(w,w')\, dH_{t-1}(w)$$

Integrating by parts:

$$H_{t+1}(w') - H_t(w') = \int_0^{\infty} \varphi_w(w)\, [H_{t-1}(w) - H_t(w)]\, dw < 0$$

Since H_t dominates (first order) H_{t-1} and $\varphi_w(w) \leq 0$, then H_{t+1} dominates (first order) H_t.
In order to assure that H_1 dominates H_0, we need that h'_0 be uniformly bounded. This seems a strong restriction, but this is only to simplify the analysis. The results about growth rate, prices and occupational decisions require that H'_t dominates H'_{t-1}, where H'_t is the restriction of H_t over the interval $[\phi,\infty)$.

Proof of Proposition 5:

(i) The fraction of entrepreneurs and workers are strictly increasing.

Let E_t be the fraction of entrepreneurs at period t.

$$E_t = \int_{\phi}^{\infty} \int_{\underline{a}(w,z,D)}^{a^*} dF(a)\, dH_t(w)$$

Now, define $\Phi(w) = \int_{\underline{a}(w,z,D)}^{a^*} dF(a)$

Now, $\Phi_w(w) = -(\partial \underline{a}(\,)/\partial w)\, f(\underline{a}(w,r,v)) > 0$

Then, since we are at the first stage of development, $r_t = z$, $v_t = D$ and $w(r,z) = \phi$.

So $$E_t = \int_{\phi}^{\infty} \Phi(w)\, d\, H_t(w)$$ and

$$E_t - E_{t-1} = \int_{\phi}^{\infty} \Phi(w)\, d\, H_t(w) - \int_{\phi}^{\infty} \Phi(w)\, d\, H_{t-1}(w)$$

Integrating by parts:

$$E_t - E_{t-1} = \int_{\phi}^{\infty} (H_{t-1} - H_t)\, \Phi_w(w)\, d\, w + [H_{t-1}(\phi) - H_t(\phi)]\, [1 - F(\underline{a})]$$

Since $\Phi_w(w) > 0$ and H_t dominates (first order) H_{t-1}, then $E_t > E_{t-1}$

Let L_t be the total number of workers; then:

$$L_t = \int_{\phi}^{\infty} \int_{\underline{a}(w,z,D)}^{a^*} 1(a,w,z,D)\, d\, F(a)\, d\, H_t(w)$$

Let $$\psi(w) = \int_{\underline{a}(w,z,D)}^{a^*} 1(a,w,z,D)\, d\, F(a)$$

Then $$\psi_w(w) = -(\partial\, \underline{a}(w,z,D)/\partial\, w)\, [f(\underline{a}(w,z,D))\;\; 1(\underline{a}, w,z,D)] + \int_{\underline{a}(w,z,D)}^{a^*} (\partial\, 1/\partial\, w)\, d\, F(a)$$

Then:

$\psi_w(w) > 0$	if	$w < \underline{w}(\underline{a},z,D)$
$\psi_w(w) = 0$	if	$w \geq \underline{w}(a^*,z,D)$

Then $$L_t - L_{t-1} = \int_{\phi}^{\infty} \psi_w(w)\, (H_{t-1} - H_t)\, d\, w + [H_{t-1}(\phi) - H_t(\phi)]\;\; [\psi(\phi)]$$

This expression is positive.

(ii) Excess demand for credit (EC_t) is increasing.

Define $$EC_t = F_t - W_t \tag{A.6}$$

Where F_t is the total funds used to finance the advanced technology (including the fixed costs) and W_t is the aggregate wealth in the economy, i.e.:

$$F_t = \int_{\phi}^{\infty} \int_{\underline{a}(w,r_t,v_t)}^{a^*} [v_t\, l\,(a,w,r_t,v_t) + k\,(a,w,r_t,v_t)]\, d\,F(a)\, d\,H_t\,(w) + \int_{\phi}^{\infty} \phi\, d\,H_t\,(w)$$

$$W_t = \int_{0}^{\infty} w\, d\,H_t\,(w)$$

Now, define $\Delta\,(w,a) = v_t\, l\,(a,w,r_t,v_t) + k\,(a,w,r_t,v_t)$

We know that $\Delta_w(w,a) > 1$ if $w \leq \underline{w}\,(a,z,D)$

$\Delta_w\,(w,a) = 0$ if $w \geq \underline{w}\,(a,z,D)$

Now, define:

$$\Delta\,(w) = \int_{\underline{a}(w,r_t,v_t)}^{a^*} \Delta\,(w,a)\, d\,F(a)$$

Then $\Delta_w\,(w) > 1$ for any $w < \underline{w}\,(\underline{a},z,D)$

$\Delta_w\,(w) = 0$ for any $w > \underline{w}\,(a^*,z,D)$

Then $$F_t - F_{t-1} = \int_{\phi}^{\infty} (H_{t-1} - H_t)\, \Delta_w\,(w)\, d\,w + [H_{t-1}\,(\phi) - H_t\,(\phi)]\,[\Delta\,(\phi) + \phi] \tag{A.7}$$

and $$W_t - W_{t-1} = \left[\int_{0}^{\phi} w\, d\,H_t - \int_{0}^{\phi} w\, d\,H_{t-1}\right] + \left[\int_{\phi}^{\infty} w\, d\,H_t - \int_{\phi}^{\infty} w\, d\,H_{t-1}\right] \tag{A.8}$$

Solving the first term of the right hand side we get:

$$\left[\int_0^{\phi} w\,dH_t - \int_0^{\phi} w\,dH_{t-1}\right] = [H_t(\phi) - H_{t-1}(\phi)]\,\phi + \int_0^{\phi} (H_{t-1} - H_t)\,dw \qquad (A.9)$$

Note that the first term is negative and offsets the second term, so this part is negative. Now, solving the second term in the brackets we get:

$$\left[\int_{\phi}^{\infty} w\,dH_t - \int_{\phi}^{\infty} w\,dH_{t-1}\right] = \int_{\phi}^{\infty} (H_{t-1} - H_t)\,dw + (H_{t-1} - H_t)(\phi) \qquad (A.10)$$

Now we need to show that $EC_t - EC_{t-1} > 0$. Remember that EC is negative in this stage; this means that:

$$F_t - F_{t-1} > W_t - W_{t-1} \qquad (A.11)$$

Plugging (A.9) and (A.10) in (A.8),and (A.7) and (A.8) in (A.11), and after some algebra we have:

$$(F_t - F_{t-1}) - (W_t - W_{t-1}) = \int_{\phi}^{\infty} (H_{t-1} - H_t)(\Delta_w(w) - 1)\,dw + (H_{t-1} - H_t)(\Delta(w) + (\phi))$$

$$- \int_{\phi}^{\infty} (H_{t-1} - H_t)\,dw > 0$$

The second term offsets the third term and $\Delta_w(w) - 1 > 0$ for $w < \underline{w}(\underline{a}, z, D)$. Since a big fraction of the population in this stage has wealth less than $\underline{w}(\underline{a}, z, D)$, the first term is positive.

(iii) The growth rate in this period could be increasing.

Let Y be the total output in the economy. After some algebra we get:

$$Y_t = \int_{\omega(r_t,v_t)}^{\infty} \int_{\underline{a}(w,r_t,v_t)}^{a^*} [a\,f(k(a,w,r_t,v_t), l(a,w,r_t,v_t)) + v_t\,l(a,w,r_t,v_t)]\,dF(a)\,dH_t(w)$$

$$- r_t\,EC_t - v_t\,EL_t$$

Where EC_t and EL_t are the excess demand in the credit and labor markets respectively.
Note that EC and EL are negative and increasing, where $r_t = z$ and $v_t = D$.

Also note that because of the utility function $W_t = (1 - \delta)\, Y_{t-1}$

Using the credit market equilibrium we get:

$$W_t = \int_{\omega(r_t,v_t)}^{\infty} \int_{\underline{a}(w,r_t,v_t)}^{a^*} [v_t\, l\,(a,w,r_t,v_t) + k\,(a,w,r_t,v_t)]\, d\,F(a)\, d\,H_t\,(w) + \int_{\omega(r_t,v_t)}^{\infty} \phi\, d\,H_t\,(w) - EC_t$$

This means that:

$$Y_t/Y_{t-1} = (1 - \delta)Y_t/W_t$$

Let equation (A.12) be:

$$\chi_t\,(w,a) = [af(k(\,),\, l(\,)) + v_t l(\,)]/\{\phi(1\text{-}H_t\,(\phi))^{-1} - EC_t\,[(1\text{-}F(\underline{a}(\,)))(1\text{-}H_t\,(\phi))]^{-1} + v_t l(\,) + k(\,)$$

and $\chi_t\,(w) = \int_{\underline{a}(w,r_t,v_t)}^{a^*} \chi_t\,(w,a)\, d\,F(a)$

Then $Y_t/(1\text{-}\delta)Y_{t-1} = \int_{\omega(r_t,v_t)}^{\infty} \chi_t\,(w)\, d\,H_t\,(w) - r_t\,(EC_t\,/W_t) - v_t\,(EL_t/W_t)$ (A.13)

We need to prove that $Y_t/Y_{t-1} > Y_{t-1}/Y_{t-2}$

First note that EC_t and EL_t are negatives and increasing since W_t is increasing (this will be shown later)

$$-\,[r_t\,(EC_t\,/W_t) + v_t\,(EL_t/W_t)] < -\,[r_t\,(EC_{t-1}\,/Wt_{t-1}) - v_t\,(EL_{t-1}/W_{t-1})]$$

This term gives us the loss in aggregate output by the fall in the output of the subsistence sector and the storage technology.

The term $-EC_t$ in the denominator in equation (A.12) gives us the increment in output by the mobilization of financial resources from the storage technology to the advance technology through the credit market.

Because there exists fixed costs, the production function in the advanced sector displays increasing returns to scale at the beginning but then it will have decreasing returns to scale.

i.e. $\exists\, w^*(a)$:

$$\chi_w(w,a) > 0 \quad \text{if} \quad w < w^*(a)$$

$$\chi_w(w,a) < 0 \quad \text{if} \quad w > w^*(a)$$

Note that $w^*(a)$ is increasing in a because $\chi_{wa}(w,a) > 0$.

Then since $$\chi_w(w) = \int_{\underline{a}(w,r_t,v_t)}^{a^*} \chi_w(w,a)\, dF(a) - (\partial \underline{a}(w,r_t,v_t)/\partial w)\, \chi(w,a)\, f(a)$$

$\exists$ some w": $\chi_w(w) > 0$ if $w < w''$

$\chi_w(w) < 0$ if $w > w''$

Now ignoring the second and third term in equation (A.13), we have:

$$Y_t/Y_{t-1} - Y_{t-1}/Y_{t-2} = \int_{\phi}^{\infty} \chi_t(w)\, dH_t(w) - \int_{\phi}^{\infty} \chi_{t-1}(w)\, dH_{t-1}(w)$$

$$\int_{\phi}^{\infty} \chi_t(w)\, dH_t(w) - \int_{\phi}^{\infty} \chi_{t-1}(w)\, dH_{t-1}(w) > \int_{\phi}^{\infty} \chi_{t-1}(w)\, dH_t(w) - \int_{\phi}^{\infty} \chi_{t-1}(w)\, dH_{t-1}(w)$$

Since $-EC_t > -EC_{t-1}$ and $H_t(\phi) < H_{t-1}(\phi)$, note that $r_t = z$ and $v_t = D$

Integrating by parts:

$$Y_t/Y_{t-1} - Y_{t-1}/Y_{t-2} = [H_{t-1}(\phi) - H_t(\phi)]\, \chi_{t-1}(\phi) + \int_{\phi}^{w''} \chi_w(w)\, (H_{t-1} - H_t)\, dw$$

$$+ \int_{w''}^{\infty} \chi_w(w)\, (H_{t-1} - H_t)\, dw$$

Agents with wealth in [ϕ,w"] are using the advanced technology when it displays the increasing returns. If this effect dominates the "decreasing returns" effect and the decreasing output of the subsistence and the storage technologies, the growth rate would be increasing in this period.

__Lemma 2__.-

If $v_t > v_{t-1}$, $r_t > r_{t-1}$, $\Gamma(w,a,r_t,v_t) > w$ for any a, and H_t dominates (first order sense) H_{t-1}, then H_{t+1} dominates (first order sense) H_t.

__Proof__:

We know that we can express $H_{t+1}(w') - H_t(w')$ as:

$$H_{t+1}(w') - H_t(w') = \int_{\omega}^{\infty} \varphi_w(w,w')\,[H_{t-1}(w) - H_t(w)]\, d\,w + \int_{\omega}^{\infty} \Big[\int_{\Delta r} \varphi_r\, d\,r + \int_{\Delta v} \varphi_v\, d\,v\Big]\, d\,H_{t-1}(w)$$

Since $\Delta r > 0$, $\Delta v > 0$, and φ_r and φ_v are positive in some range of w, the second term in the right hand side could be positive. Since by Lemma 1 the first term is always negative, then the sign is ambiguous. Now, taking derivatives with respect to w', we have:

$$h_{t+1}(w') - h_t(w') = \int_{\omega}^{\infty} \varphi_{ww'}(H_{t-1} - H_t)\, d\,w + \int_{\omega}^{\infty} \Big[\int_{\Delta r} \varphi_{rw'}\, d\,r + \int_{\Delta v} \varphi_{vw'}\, d\,v\Big]\, d\,H_{t-1}(w)$$

Since $\varphi_{ww'}$, $\varphi_{rw'}$ and $\varphi_{vw'}$ are positives, then $h_{t+1}(w) > h_t(w)$ at all w'. Then H_{t+1} dominates H_t in the first order sense.

__Proof of Proposition 6__:

(i) Note that in this case there is no subsistence sector and no investments in the storage technology. If the same conditions as in proposition 5 holds and since H_{t+1} dominates H_t, then wages and interest rates must be increasing.

(ii) We know that:

$$Y_t/Y_{t-1} = [(1-\delta)] \int_{\omega(r_t,v_t)}^{\infty} \int_{\underline{a}(w,r_t,v_t)}^{a^*} \chi_t(w,a)\, d\,F(a)\, d\,H_t(w)$$

Where $\chi_t(w,a) = [a\, f(k(\,),l(\,)) + v_t\, l(\,)] / [v_t\, l(\,) + k(\,)] + \phi\,[1-H_t(\omega(\,))]^{-1}$

Notice that in this case, because r_t and v_t are changing, χ_t depends on t.

$$[Y_t/Y_{t-1}] - [Y_{t-1}/Y_{t-2}] = \int_{\omega_t}^{\infty} \int_{\underline{a}_t}^{a^*} \chi_t(w,a)\, dF(a)\, dH_t\,(w) - \int_{\omega_{t-1}}^{\infty} \int_{\underline{a}_{t-1}}^{a^*} \chi_{t-1}(w,a)\, dF(a)\, dH_{t-1}\,(w)$$

$$[Y_t/Y_{t-1}] - [Y_{t-1}/Y_{t-2}] = \int_{\omega_t}^{\infty} \int_{\underline{a}_t}^{a^*} \chi_t(w,a)\, dF(a)\, dH_t\,(w) - \int_{\omega_{t-1}}^{\infty} \int_{\underline{a}_{t-1}}^{a^*} \chi_{t-1}(w,a)\, dF(a)\, dH_{t-1}\,(w)$$

$$+ \int_{\omega_t}^{\infty} \int_{\underline{a}_t}^{a^*} \chi_{t-1}(w,a)\, dF(a)\, dH_{t-1}\,(w) - \int_{\omega_t}^{\infty} \int_{\underline{a}_t}^{a^*} \chi_{t-1}(w,a)\, dF(a)\, dH_{t-1}\,(w)$$

$$+ \int_{\omega_t}^{\infty} \int_{\underline{a}_t}^{a^*} \chi_t(w,a)\, dF(a)\, dH_{t-1}\,(w) - \int_{\omega_t}^{\infty} \int_{\underline{a}_t}^{a^*} \chi_t(w,a)\, dF(a)\, dH_{t-1}\,(w)$$

$$[Y_t/Y_{t-1}] - [Y_{t-1}/Y_{t-2}] = \int_{\omega_t}^{\infty} \int_{\underline{a}_t}^{a^*} \chi_t(w,a)\, [H_t - H_{t-1}\,]\, dw - \int_{\omega_{t-1}}^{\omega_t} \int_{\underline{a}_{t-1}}^{\underline{a}_t} \chi_{t-1}(w,a)\, dH_{t-1}\,(w)$$

$$+ \int_{\omega_t}^{\infty} \int_{\underline{a}_t}^{a^*} (\chi_t - \chi_{t-1})\, dH_{t-1}\,(w)$$

Now, the first term is positive if the same conditions as in proposition 5 holds. The second term is negative since $\underline{a}$ and ω are increasing in prices. The sign of the third is ambiguos. The third term is equal to:

$$\int_{\omega_t}^{\infty} \int_{\underline{a}_t}^{a^*} \Big[\int_{\Delta r} \chi_r\, d\,r + \int_{\Delta v} \chi_v\, d\,v + \int_{\Delta H(\omega(\,))} \chi_{H(\omega(\,))}\, d\,H(\omega(r_t,v_t))\Big]\, d\,F(a)\, d\,H_{t-1}\,(w)$$

The first and second term are negative for w < w" and positive for w > w". Where w" is a level of wealth define in the same way as in proposition 5. For the third term note that H_t dominates H_{t-1} and $\omega\,(r_t,v_t) \geq \omega\,(r_{t-1},v_{t-1})$. It is possible to show that the first effect is bigger than the second. This is obvious when $\omega\,(r_t,v_t) = \omega\,(r_{t-1},v_{t-1})$, i.e. the second effect does not exist. The third term is the effect in growth because the ratio of invetment in fixed cost over total investment is decreasing with respect to wealth.

If the positive effect dominates we could expect increasing growth rate.

Lemma 3:

If $r_t \leq r_{t-1}$, $v_t \geq v_{t-1}$ and H_t dominates H_{t-1} in the second order sense, then H_{t+1} dominates H_t in the second order sense.

Proof.-

$$H_{t+1}(w') - H_t(w') = \int_{\omega_t}^{\infty} \varphi_t(w,w')\, dH_t(w) - \int_{\omega_{t-1}}^{\infty} \varphi_{t-1}(w,w')\, dH_{t-1}(w)$$

Integrating and taking derivatives with respect to w' we get:

$$h_{t+1}(w') - h_t(w') = \int_{\omega_t}^{\infty} [H_{t-1} - H_t]\, \varphi_{ww'}\, dw + \int_{\omega_{t-1}}^{\infty} \Big[\int_{\Delta r} \varphi_{rw'}\, dr + \int_{\Delta v} \varphi_{vw'}\, dv\Big]\, dH_{t-1}(w)$$

If H_t dominates H_{t-1} in the second order sense and $\varphi_{www'} < 0$ and $\lim_{w \to \infty} \varphi_{www'} = 0$, then it can be shown that the first term is positive. Since most of the entrepreneurs are unconstrained the positive part in the second term offsets the negative part.

Since $h_{t+1}(w') > h_t(w')$ and for the w_{max}, $\Gamma(w_{max}, a^*, r, v) < w_{max}$ (where w_{max} is the maximun amount of wealth), then H_{t+1} dominates H_t in the second order sense.

Proof of Proposition 7:

(i) From the labor market clearing condition we get:

$$\int_{\omega(r_t,v_t)}^{\infty} [\psi_t(w) + 1]\, dH_t(w) = \int_{\omega(r_{t-1},v_{t-1})}^{\infty} [\psi_{t-1}(w) + 1]\, dH_{t-1}(w)$$

Let r and v be fixed. Since H_t dominates H_{t-1}, for any given r, v_t must increase in order to clear the market. When r_t is decreasing, the increment in wages must be higher.

(ii) From the credit market we have:

$$W_t = \int_{w(r_t,v_t)}^{w'_t(r_t,v_t)} [\Delta(w,r_t,v_t) + \phi]\, d\,H_t(w) + \int_{w'_t(r_t,v_t)}^{\infty} [\Delta(w,r_t,v_t) + \phi]\, d\,H_t(w)$$

$$W_{t-1} = \int_{w(r_{t-1},v_{t-1})}^{w'_{t-1}(r_t,v_t)} [\Delta(w,r_{t-1},v_{t-1}) + \phi]\, d\,H_{t-1}(w) + \int_{w'_{t-1}(r_t,v_t)}^{\infty} [\Delta(w,r_{t-1},v_{t-1}) + \phi]\, d\,H_{t-1}(w)$$

Where $$w'_t = \min_{w} \left\{ \int_{\underline{a}(w,r_t,v_t)}^{a^*} \Delta(w,r_t,v_t) = v_t\, l^*(a,r_t,v_t) + k^*(a,r_t,v_t) \right\}$$

A bigger fraction of entrepreneurs becomes unconstrained, so since H_t dominates H_{t-1} and v_t is increasing, the demand for credit is decreasing. Since $W_t > W_{t-1}$, then the interest rate must decrease in order to clear the credit market.

Now, since (r_t,v_t) are monotone sequences that belong to a bounded set (proposition 2), then r_t must converge to $r^* \geq z$ and v_t must converge to $v^* > D$.

Claim: $W_t \geq W_{t-1}$, $\forall$ t

Proof.-

Since $W_t = (1 - \delta)\, Y_{t-1}$, then we just need to prove that $Y_t \geq Y_{t-1}$ $\forall$ t. This result is established by induction, knowing that in the first stages $Y_t > Y_{t-1}$ and that H_t dominates stochastically H_{t-1} (in the second order sense) $\forall$ t.

Proof of the Theorem:

First of all, we need to find a compact support for H^*.

Define w_1 as: $w_1 = \omega(r^*,v^*) - \phi$, $w_1 \geq 0$ since $r^* \geq z$, $v^* > D$ and $\omega(r^*,v^*) > \omega(z,D) = \phi$

Define w_2 as: $w_2 = \Gamma(w_2, a^*, v^*, r^*)$

Then w_2 is the maximum level of wealth attainable when the prices are fixed. The minimum level of wealth is given by w_1.

Claim.- $\omega(r^*,v^*) \geq [(1-\delta)v^*]/[1-(1-\delta)r^*]$

Suppose not. Then $[0, \omega(r^*,v^*))$ is an absorbing state. Then for any $w_t > \omega(r^*,v^*)$ $\exists$ some $N \geq 1$: $\forall \varepsilon > 0$ Prob $[w_{t+N} > \omega(r^*,v^*)] < \varepsilon$ (see proposition 4).

Then at some point, everybody will have wealth less than $\omega(r^*,v^*)$. This means that there will exist negative excess demand for labor and credit, then the prices have to decrease. A contradiction.

Now, $[w_1,w_2]$ is the compact support for H^*.

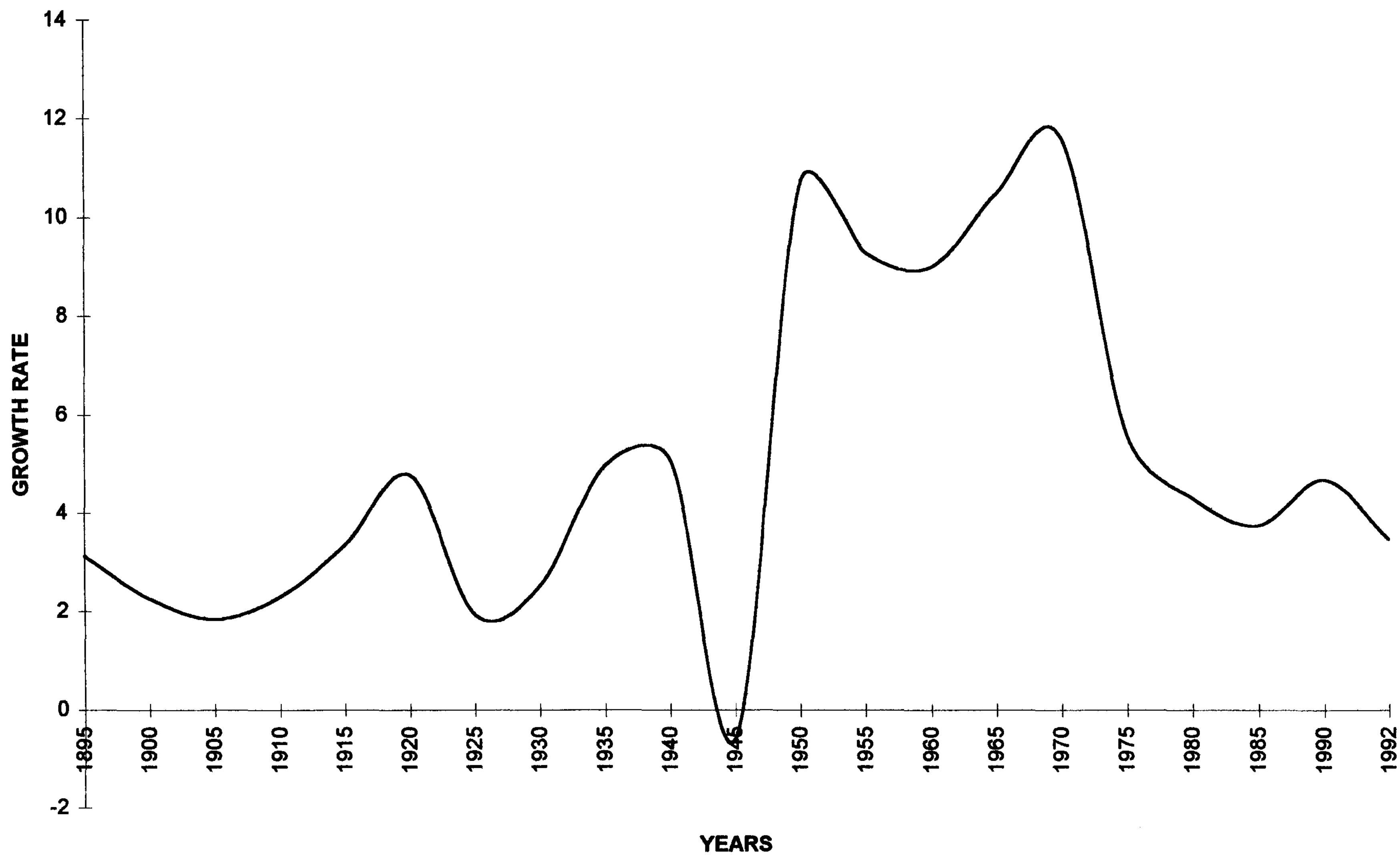
It is easy to see that for any $w \in [w1,w2]$ there exist $N \geq 1$ and $\varepsilon > 0$:

Prob $[\Gamma^{t+N}(w_2,a^*,r^*,v^*) < w \mid w_t = w_2] > \varepsilon$, Prob $[\Gamma^{t+N}(w_1,a^*,r^*,v^*) < w \mid w_t = w_1] > \varepsilon$

and since Γ is monotone, then the Theorem 2 in Hopenhayn and Prescott (1992) can be applied.

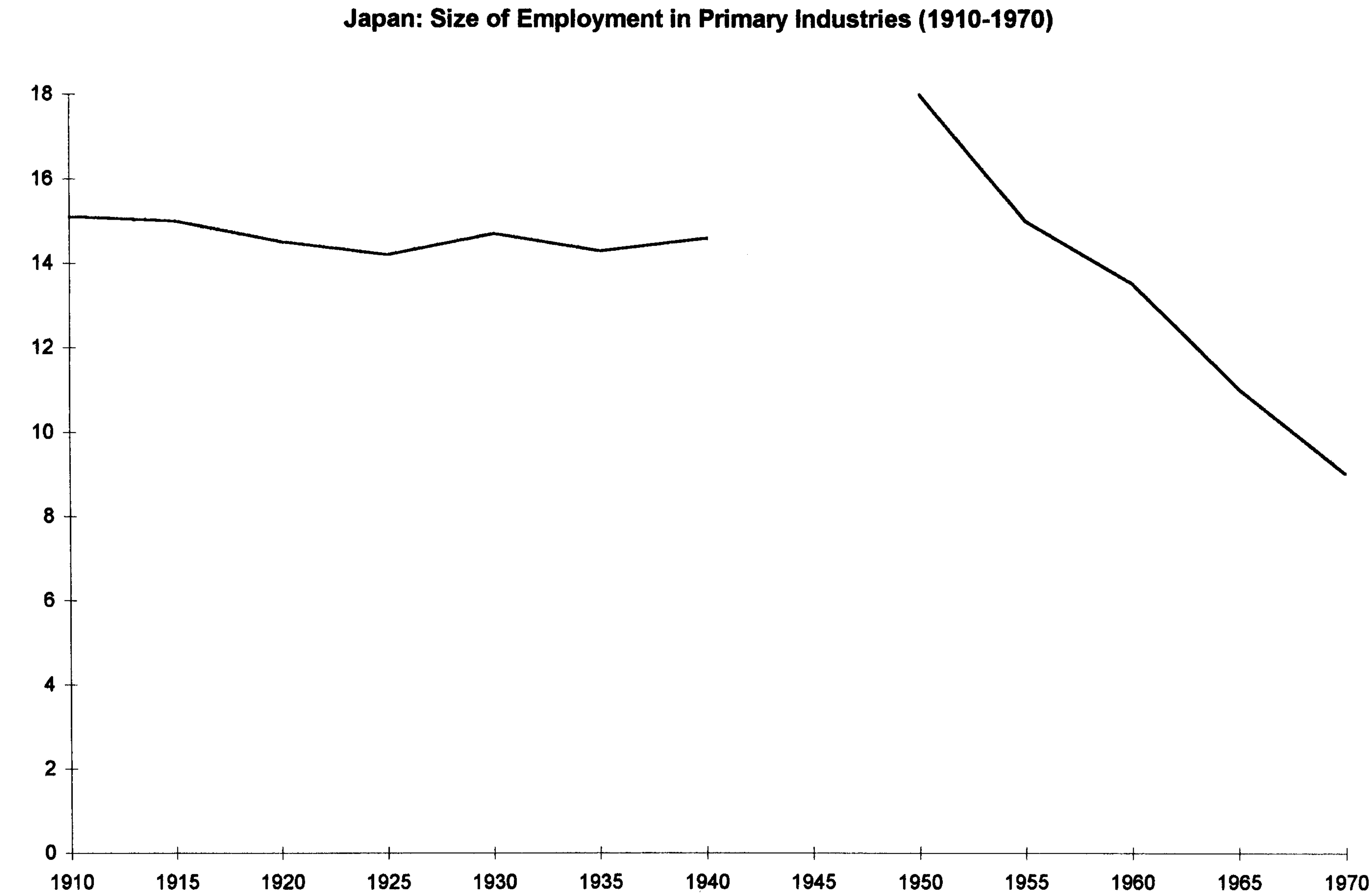
FIGURE 1

PERIOD AVERAGES OF ANNUAL GROWTH RATES: JAPAN 1890-1992



Source: Ohkawa, et. al. (1988) and World Bank Reports

FIGURE 2



Source: Minami (1973)

FIGURE 3

TIMING

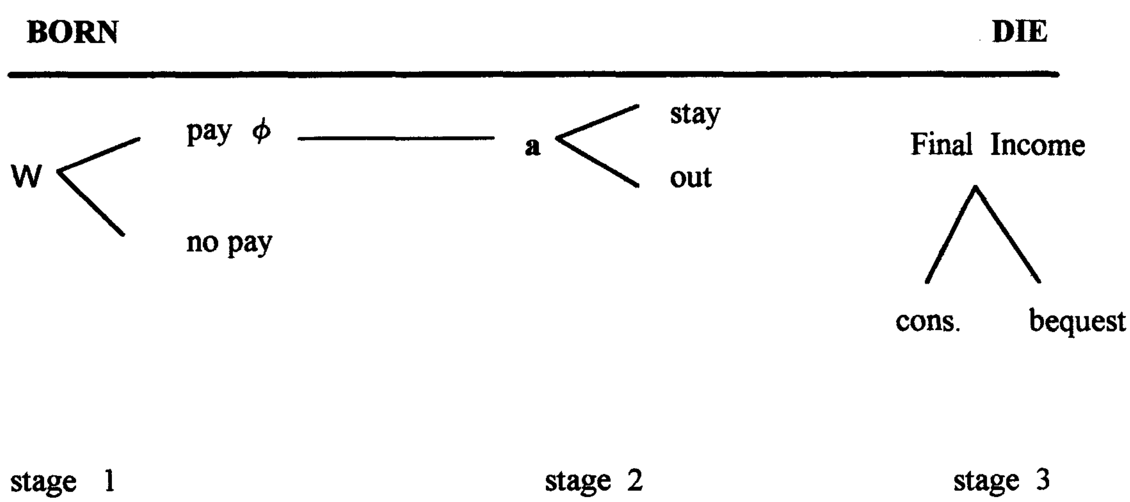


FIGURE 4

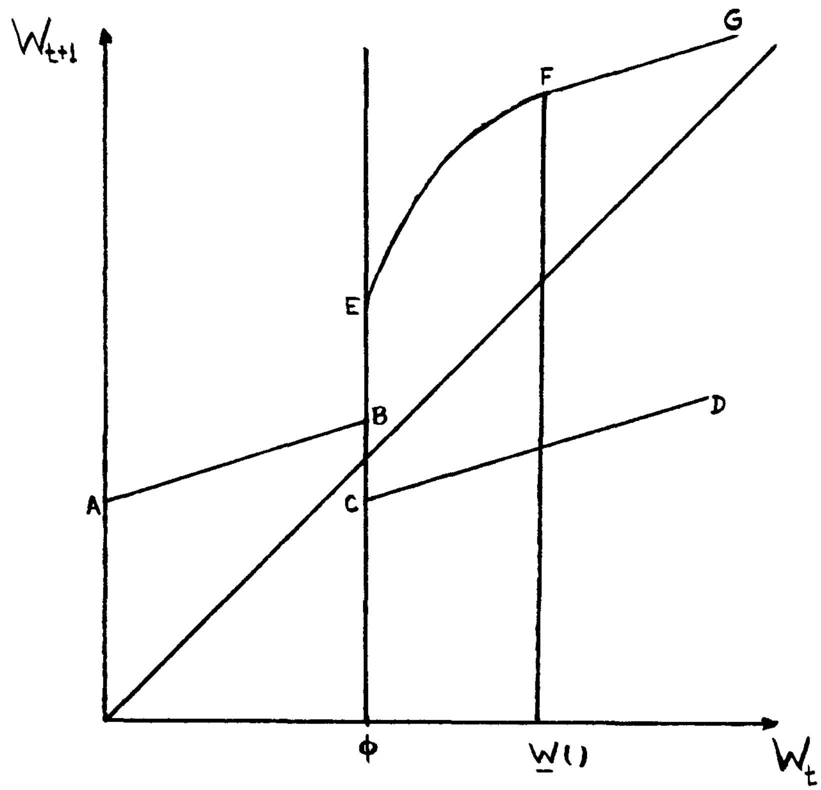


FIGURE 5

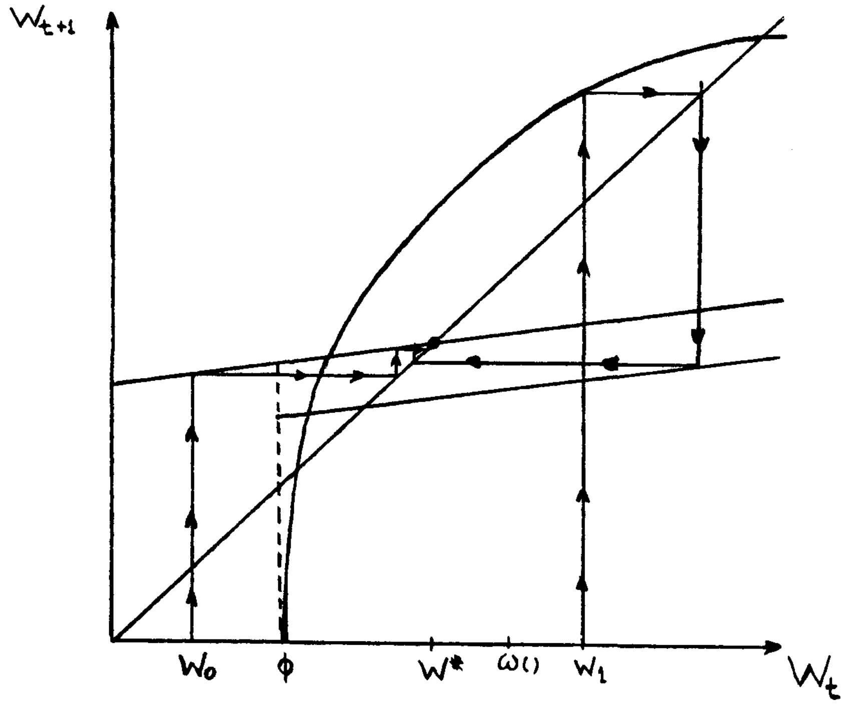


FIGURE 6

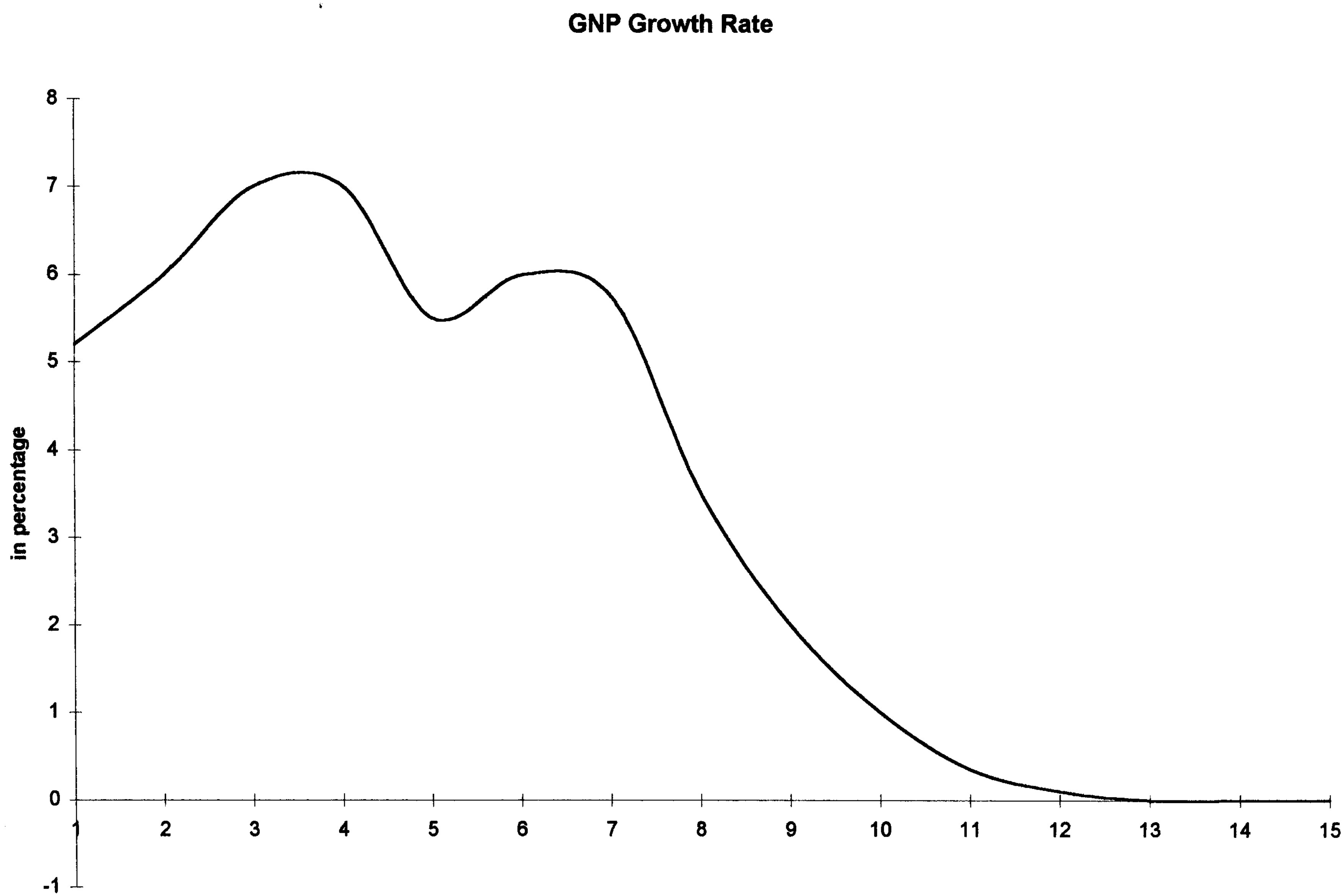
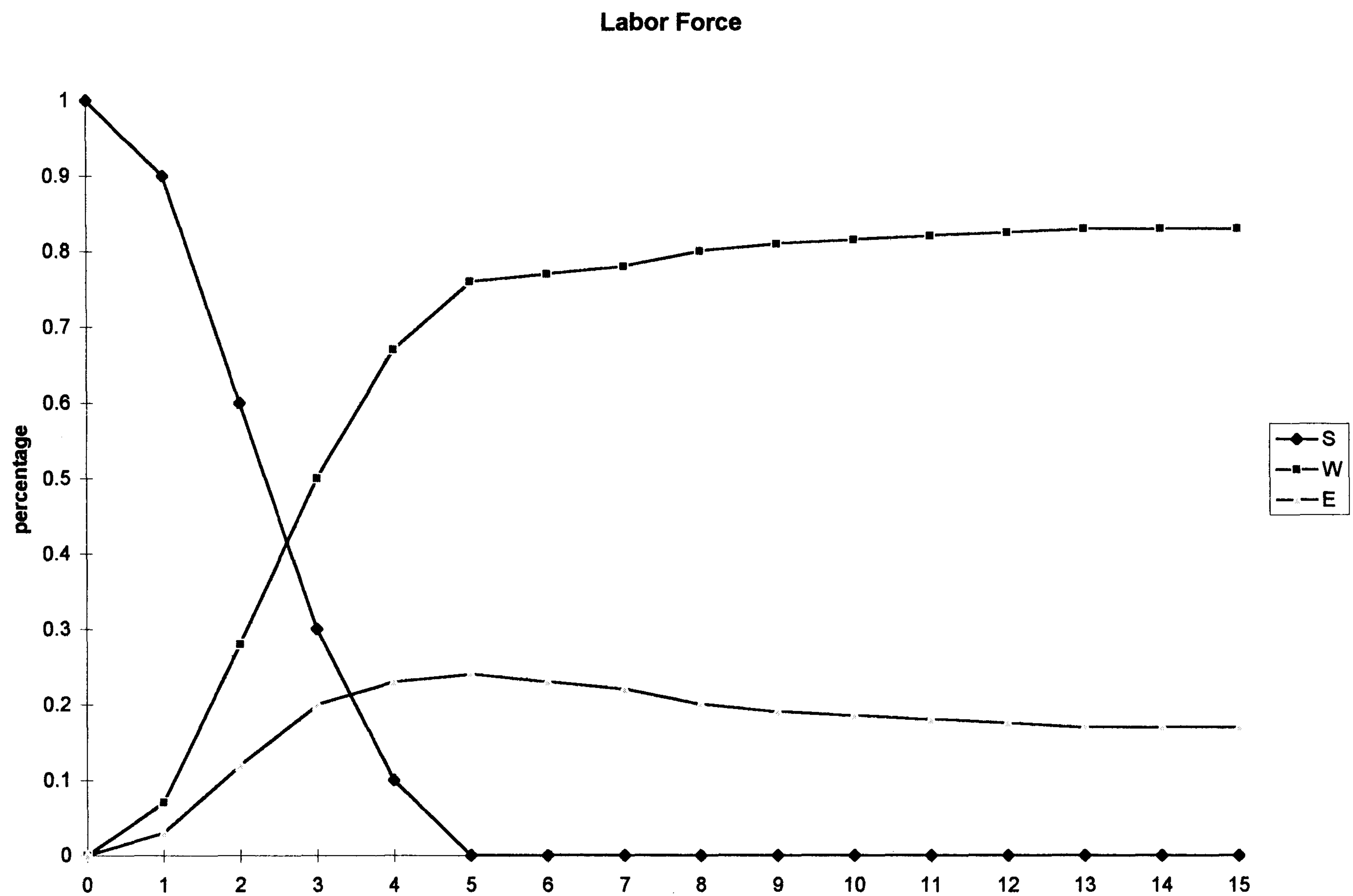

GNP Growth Rate
in percentage
8
7
6
5
4
3
2
1
0
-1
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

FIGURE 7


Labor Force
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
percentage
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
S
W
E