

STOCK PRICE VOLATILITY TESTS OF NARROW AND
BROAD DIVIDENDS: A CLOSER EXAMINATION
OF THE RESULTS OF ACKERT AND SMITH

by

Garrett H. TeSelle

Discussion Paper No. 282, September 1995

Center for Economic Research
Department of Economics
University of Minnesota
Minneapolis, MN 55455

Stock Price Volatility Tests of
Narrow and Broad Dividends:
A Closer Examination of the Results of
Ackert and Smith

Garrett H. TeSelle¹
University of Minnesota

February 18, 1995

¹I am indebted to Stephen LeRoy and Ravi Jagannathan for their comments and insights. Thanks also to Lucy Ackert and Brian Smith for their comments and data, and to John Geweke and the U. of M. Econometrics workshop participants who offered valuable input. Any errors or omissions in this paper are my own.

Abstract

Ackert and Smith (1993) suggest that volatility tests of stock prices should be based on a more inclusive definition of dividends than has actually been used in past studies. They perform a West Test using such a dividend definition, and conclude that stock prices are not excessively volatile relative to these dividends. This paper considers their work on two bases: First, it argues that in most cases the expanded dividend definition would not be expected to lead to volatility conclusions that differ from those reached under the narrower definition; second, it presents evidence that Ackert and Smith's empirical results may have been induced by unrecognized test bias created by non-stationarities in their data.

I Introduction

Since the seminal work by LeRoy and Porter (1981) and Shiller (1981), financial economists have repeatedly tested whether stock prices are too volatile to be supported by an economy whose agents' expectations and investments are determined rationally.

The general consensus reached in these tests has been that stock prices appear excessively volatile relative to fluctuations in the stream of dividends that give the stocks value. Introductions to this literature and results can be found in Cochrane (1991), Gilles & LeRoy (1991), and Shiller (1989).

In 1993, Ackert and Smith (AS) reported test results which seemed to challenge both the methods and results of these earlier tests. They argue that the definition of "dividends" in these earlier studies was too strict to properly depict all cash flows relevant to an investor. Indeed, when AS broaden the definition of cash flows, they report a finding that prices are *not* excessively volatile. A strong interpretation of AS's result would be that the "excess" volatility finding of earlier studies was simply an artifact of misspecification, in which economists misspecified the the cash flows relevant to stock prices.

The purpose of this paper is to revisit the approach and results of Ackert & Smith in order to better understand the methodology and results of all volatility tests. Three key points are made:

1. The redefinition of "dividends" made by AS should in theory be irrelevant to the question of whether stock prices are excessively volatile. The redefinition of dividends would not be expected to change the outcome of a well-constructed volatility test except insofar as it changes the size or power of the test.
2. In light of (1), the fact that AS find excess price volatility relative to narrow dividends but not relative to their redefined "broad" dividends does not directly support AS's conclusion that the redefinition of dividends helps to explain stock price volatility. Such a conclusion can only be supported by showing that the volatility test of the broad

dividends is at least as reliable as that of the narrow dividends.

3. Simulations and analysis indicate that the volatility test using broad dividends may contain significant bias that is not present in the test of narrow dividends. Therefore AS's finding of excess volatility (under narrow dividends) is likely more reliable than their finding that prices were not excessively volatile (under broad dividends).

In the end, this paper emphasizes two main results. First, it challenges the interpretation AS gave to their test results; because of potential test bias, we cannot conclude from the AS work that the price volatility question has been in any sense resolved by the redefinition of relevant cash flows. Second, it reemphasizes a central point raised by AS: In constructing volatility tests, the economist must be aware of the various definitions of cash flows that may be reconcilable with rational theory; care must be taken in constructing the test to match data characteristics with the characteristics of the test methods to be employed.

Section II of this paper serves a dual purpose of briefly surveying relevant literature and establishing the role of stock returns in generating the present value hypothesis assumed in AS and previous volatility tests. Section III briefly summarizes the key definitions and results in the work of AS. Section IV discusses the equivalence of returns in the "broad" and "narrow" constructs defined by AS and the resulting suggestion that in the key results of AS, *either* the narrow construct results *or* the broad construct results are misleading. Section V presents intuitive arguments and tests of AS's data as evidence that the "broad" test conducted by AS contained unrecognized bias. Section VI provides a simple simulation which indicates that the direction of the bias in the test of the broad construct is such that it may have generated AS's result regarding the absence of excess volatility in the broad construct. The conclusion summarizes the key points of this paper.

II Background

The stock price volatility literature investigates whether actual stock prices are more volatile than theoretically rational prices. Obviously, then, one must first decide what “rational” prices would be. The workhorse of the price volatility literature has been the “present value hypothesis” (PVH), which states that rational prices are equal to the present value of the expected cash flows that will accrue to an agent holding the stock into the infinite future.

This section derives the PVH from first principles and briefly shows how the PVH has been employed in various tests of price volatility. The reason to present this derivation, well-known to financial economists, is to make a key point. Price volatility tests commonly separate the stream of prices from the stream of dividends and test statistical moments of one stream against those of the other. However, these individual streams are only indirectly related to “rational” price theory. The direct test of price rationality is actually to be found in the behavior of stock *returns*, which are a combination of both prices and dividends. This observation plays a key role in interpreting the “broad” vs. “narrow” tests employed by AS.

To derive the PVH, we start with a simplified version of asset pricing. Consider a representative agent at some time t who plans streams of consumption $\{c_{t+j}\}$ and investment $\{A_{t+j}\}$ ($j = 0, 1, \dots$) to solve

$$\begin{aligned} \text{Max}_{\{c_{t+j}, A_{t+j}\}} \quad & E \left(\sum_{j=0}^{\infty} (1 + \gamma_{t+j})^{-1} u(c_{t+j}) | I_t \right) \\ \text{s.t.} \quad & c_{t+j} + A_{t+j+1} \leq R_{t+j} A_{t+j} \\ & A_t \text{ given} \end{aligned}$$

where

γ_{t+j} = time preference rate in period $t + j$.

A_{t+j} = investment held from period $t + j - 1$ to period $t + j$.

R_{t+j} = gross return earned on assets brought into period $t + j$ (stochastic).

c_{t+j} = consumption in period $t + j$.

I_t = information set available at time t .

$E(\cdot|I_t)$ = expectation operator conditional on I_t .

The necessary first order conditions to this problem include

$$(1) \quad E(\beta_t R_{t+1}|I_t) = 1$$

where β_t is the discount rate reflecting both time preference and the impact of risk aversion in light of uncertainty over future returns.

The PVH is derived by applying additional assumptions to (1). Specifically, if we assume that the discount rate is a constant $\beta_t = \beta$ (as in the case of risk neutrality and constant time preference) (1) gives

$$(2) \quad E(R_{t+1}|I_t) = 1/\beta$$

With the weak assumption that the current asset ex-dividend price is known ($p_t \in I_t$) and that the relationship between ex-dividend prices (p_t, p_{t+1}), dividends (d_{t+1}) and returns are summarized by

$$R_{t+1} = (p_{t+1} + d_{t+1})/p_t$$

equation (2) gives us

$$(3) \quad p_t = \beta E[(p_{t+1} + d_{t+1})|I_t]$$

Finally, assuming no bubbles in prices, we iterate expectations on $\{p_{t+j}\}$ to yield PVH:

$$(4) \quad p_t = E\left[\sum_{j=1}^{\infty} \beta^j d_{t+j} \mid I_t\right]$$

Equation (4) is PVH, the workhorse price model in price volatility literature.

Initial price volatility tests were “Variance Bounds” test of (4). Defining (after Shiller) the “ex-post rational price”

$$p_t^* = \sum_{j=1}^{\infty} \beta^j d_{t+j}$$

gave

$$p_t = E(p_t^* | I_t)$$

and hence

$$p_t^* = p_t - \epsilon_t$$

where ϵ_t is a forecast error. Because ϵ_t must be orthogonal to p_t (and all other components of I_t), we obtain the “Variance Bound”

$$(5) \quad \text{Var}(p_t) = \text{Var}(p_t^*) - \text{Var}(\epsilon_t) \leq \text{Var}(p_t^*)$$

Variance bounds tests then attempted to estimate $\text{Var}(p_t^*)$ and $\text{Var}(p_t)$ by using time series data. This implicitly required that the prices and dividends follow stationary and ergodic processes, so that (5) could be replaced by a testable hypothesis

$$(6) \quad \widehat{\text{Var}}(p) \leq \widehat{\text{Var}}(p^*).$$

These variance bounds tests generally reported that (6) was violated, frequently by a large margin.

The variance bounds tests were criticized on various grounds. Primary criticisms involved:

1. The leap from (5) to (6) required an assumption of stationarity (without differencing); this assumption was criticized on empirical and theoretical grounds.
2. Tests which constructed ex-post rational prices, p_t^* , required the tester to somehow estimate p_t^* without certain knowledge of the future. The estimation process increased potential bias in the variance bounds test which would tend to increase the probability of finding (6) false even if (5) were true.

West (1988) introduced what has become known as a “second-generation” test to overcome these criticisms. Again using the present value relationship (4), he showed that bounds

could be placed on the variance in the *change* in actual prices relative to innovations in price projections based solely on historical dividends. This bound has become known as the “West Inequality”. The West Inequality holds for *any* dividend stream as long as:

1. Stocks are priced as the present value of the expected future dividend stream, and
2. Current and lagged values of the dividends are components of the investors’ information set and all components of the information set follow a joint covariance stationary process after differencing s times ($s \in \{0, 1, 2, \dots\}$).

West (1988) then tested whether this inequality was satisfied in data on prices and dividends. The test procedures have become known as the “West Test”. Importantly, West applied this test only to what AS would subsequently refer to as “narrow” dividends. He found that the West Inequality appeared to be violated; as with variance bounds tests, prices appeared to be excessively volatile.

In ending this brief literature survey, it is important to note that all of these tests assume the PVH model of asset prices of equation (4). This PVH is also employed by AS, who apply the West Test to cash flows measured by what they call “narrow” dividends and “broad” dividends.

However, the use of PVH equation (4) is simply a recasting of the hypothesised relationship between the discount rate and expected returns in equations (1), (2) and (3). It is the behavior of *returns* that is the central object of question in price volatility tests; it is on this ground that we will examine the work of AS.

III Tests & Results of Ackert & Smith

Akert & Smith question the assumptions made in the above literature as to the cash flows that are relevant to rational prices. The above tests all assume that an investor buys and holds a portfolio of stocks infinitely into the future; the cash flows that go to the investor

then are simply the ordinary dividends (“narrow” dividends in AS terminology) that are paid on the stocks. In these tests, the portfolio is never liquidated, either in whole or in part. Therefore, these tests assume that cash proceeds from buyouts and stock repurchases either never occur or else are reinvested in other stocks with no net change in the value of holdings. In short, the tests assume a “self-financed” portfolio.

AS observe that in the late 1970’s and 1980’s buyout and takeover activity seemed to increase relative to historical levels. They observe that the proceeds obtained by the investor from such activities is economically equivalent to a dividend. Therefore AS define “broad” dividends as the sum of narrow dividends plus any proceeds from stock repurchases, takeovers, etc. They then observe that these broad dividends became extremely volatile during the 70’s and 80’s.

AS suggest that these broad dividends are the relevant cash flows for purposes of volatility tests. Therefore, they suggest that the earlier volatility tests, which centered around narrow dividends, may have been missing an important component of dividend volatility when testing for excess price volatility.

To test this possibility, AS perform volatility tests under both the narrow and broad constructs, for data from the Toronto Stock Exchange for the period January 1950 – February 1991. They apply the West Test procedures to both the narrow and broad constructs, on monthly and annual data, using a wide variety of stationarity assumptions (i.e., different levels of differencing assumed necessary to induce stationarity), and test deseasonalized data and data without deseasonalizing.

In West’s original work, the test statistic indicating the presence/absence of detectable excess volatility was the “West Statistic” (WS), with a critical value of zero. A negative WS was an indicator of excess price volatility, while a positive WS would indicate the absence of any detectable excess volatility. He also presents a method of estimating the standard errors of the West Statistic (WSSTD).

AS follow West’s procedure and indicate the calculated WS and WSSTD for each set

of data and stationarity assumptions they test. Overall, most of the WS's estimated were negative. A subset of their results (for monthly data without deseasonalizing, differencing parameters $s = 0, 1$, for both narrow and broad constructs) is presented in Table 1 below.

Table 1

Divided Definition	Differencing		
	Level $s =$	WS	WSSTD
Narrow	0	-1329	365
Broad	0	-921	195
Narrow	1	-863	328
Broad	1	8156	3647

After presenting their results for all cases, AS then suggest how to interpret them. They cite evidence in the literature that stock prices and dividends are not stationary without differencing. Therefore they argue that among their results “the results for the differenced series are more reliable” (p. 1156). In the four monthly differenced cases they examined, the calculated WS using broad dividends was positive (as in the subset of results of Table 1). Similar results were presented for the annual series. Therefore, AS observe that “when the data are differenced [and the broad dividend construct is used]... we cannot reject the hypothesis that stock prices are determined by rational agents as the present value of expected cash flows...” (p. 1156). “Thus there is evidence that stock prices are not too volatile to be determined by rational agents as the expected discounted value of all future cash flows, or broadly defined dividends.” (p. 1159)

Analogously, since the WS for narrow dividends was consistently negative in all cases reported, AS state “we reject the efficient markets hypothesis when dividends are defined narrowly.” (p. 1159)

The apparent implication, then, is that the PVH holds for broad dividends but not for narrow dividends, and that previous price volatility tests detected excess volatility because these earlier tests used a misspecified definition of the relevant cash flows. AS underscore this

with their final comments, “Consequently, not only do these empirical results support the efficient markets model, but they demonstrate the importance of correctly specifying tests of restrictions implied by the efficient markets model. Actual stock prices are determined by rational agents as the expected present discounted value of all future cash flows received by shareholders” (p. 1159).

IV Analysis

One of AS’s central results, then, is that stock prices are excessively volatile relative to narrow dividends but not relative to broad dividends, apparently because stock prices are equal to the present value of broad dividends but not equal to the present value of narrow dividends. This section draws on Section II to demonstrate that this result leads to troubling, seemingly illogical relationships when recast as statements about the returns on stocks.

Consider first the calculation of returns on a portfolio constructed according to the narrow construct when a single asset is held in the portfolio. Since under the narrow construct the shares are held constant, the returns from period t to period $t + 1$ under the narrow construct are

$$(7) \quad R_{t+1}^n = \frac{p_{t+1}^n + d_{t+1}^n}{p_t^n}$$

where

p_t^n = per share ex-dividend price of the underlying stock in period t

d_{t+1}^n = per share dividend paid in period $t + 1$.

Next consider the returns on portfolios constructed according to the broad construct. Analogous to (7) these broad returns would be

$$(8) \quad R_{t+1}^b = \frac{p_{t+1}^b + d_{t+1}^b}{p_t^b}.$$

Since under the broad construct shares can be liquidated, we can specify the components of the broad returns in terms of the per-share prices and dividends above and in terms of changes in equity. Letting s_t^b denote the number of shares held in period t , we have

$$\begin{aligned}
(9) \quad p_t^b &= s_t^b \cdot p_t^n \\
p_{t+1}^b &= s_{t+1}^b \cdot p_{t+1}^n \\
d_{t+1}^b &= s_{t+1}^b \cdot d_{t+1}^n + e_{t+1}.
\end{aligned}$$

Here e_{t+1} denotes the amount of broad dividends obtained through stock repurchases, takeovers, etc. It is the essential difference between narrow and broad dividends.

Since according to AS's results markets are efficient relative to broad dividends, it must be that the equity payment included in the broad dividends accurately reflects the number of shares bought out ($s_t^b - s_{t+1}^b$) and the cum-dividend per-share value of these shares ($p_{t+1}^n + d_{t+1}^n$): i.e.

$$(10) \quad e_{t+1} = (s_t^b - s_{t+1}^b)(p_{t+1}^n + d_{t+1}^n).$$

Substituting (9) and (10) into (8), we see that the returns under both the broad and narrow constructs in theory will be identical:

$$\begin{aligned}
R_{t+1}^b &= \frac{s_{t+1}^b p_{t+1}^n + [s_{t+1}^b d_{t+1}^n + (s_t^b - s_{t+1}^b)(p_{t+1}^n + d_{t+1}^n)]}{s_t^b p_t^b} \\
&= \frac{s_t^b (p_{t+1}^n + d_{t+1}^n)}{s_t^b p_t^b} \\
&= \frac{p_{t+1}^n + d_{t+1}^n}{p_t^n} \\
&= R_{t+1}^n
\end{aligned}$$

This theoretical equivalence of returns under the two constructs was in fact reflected in AS's monthly data. In the 494 months of data (supplied by them), broad returns and narrow returns were equal in 491 months. The three exceptions do not appear to have affected their results or my further analysis. The equivalence of returns did not generally hold in the AS annual data set; these annual series have since been revised by AS.

According to AS, the PVH equation (4) in Section II holds under the broad construct,

i.e.,

$$p_t^b = E \left(\sum_{j=1}^{\infty} \beta^j d_{t+j}^b | I_t \right).$$

With no additional assumptions, we can conclude that equation (2) also holds for the broad construct

$$E \left(R_{t+1}^b | I_t \right) = 1/\beta.$$

The equivalence of returns between the two constructs therefore shows that equation (2) must hold under the narrow construct,

$$(11) \quad E_t \left(R_{t+1}^n | I_t \right) = 1/\beta.$$

But by (11), the PVH hypothesis must¹ also hold for narrow dividends

$$(12) \quad p_t^n = E_t \left(\sum_{j=1}^{\infty} \beta^j d_{t+j}^n | I_t \right)$$

By the above analysis, volatility tests are actually testing the behavior of expectations of returns which are identical under the broad and narrow construct. We therefore conclude the following:

1. The PVH will hold in one construct (narrow or broad) if and only if it also holds in the other;
2. Satisfaction of the PVH for *both* narrow *and* broad constructs are *necessary* conditions for rational stock prices; and
3. A volatility test applied to broad dividends is testing the same null hypothesis as a volatility test applied to narrow dividends.

¹Formally, the equivalence of PVH under the two constructs is not guaranteed if bubbles exist in one construct's price but not the other, or if one of the prices is hidden from the investor's information set. Given the Ackert-Smith conclusion that prices reflect the present value of dividends without bubbles, I exclude bubbles from this analysis. The possibility that prices might be hidden from investors' information sets is not considered realistic and is also excluded here.

Consideration of these points is essential when interpreting AS's dichotomous empirical finding of excess volatility relative to narrow dividends but not relative to broad dividends. Specifically, the above points preclude AS's interpretation that the PVH is satisfied for broad dividends but not for narrow dividends. Instead, the dichotomy would appear to result from sampling error, size/power differences in the narrow and broad tests, or some misspecification or bias in the empirical work.

If the dichotomy results from unspecified size/power differences in the narrow and broad tests, we cannot formally conclude whether the narrow or broad finding is more reliable. However, the results of prior volatility studies of narrow dividends strongly suggest that AS's test of the broad construct simply lacked power to detect excess price volatility that their test of the narrow construct was able to detect. LeRoy and TeSelle (1995) point out that this implies either that the PVH is violated for both the narrow and broad constructs or that investors' information sets implausibly do not contain historical narrow dividend information.

Similar logic applies to the consideration of misspecification or bias as the cause of the dichotomous AS findings. West (1988) tested for potential bias in his volatility test of the narrow construct and found no significant bias. We therefore have no reason to suspect that bias contaminated AS's finding of excess volatility under the narrow construct. The open issue, then, is whether AS's test of the broad construct may have contained bias that prevented detection of excess price volatility.

The remainder of this paper addresses this question of possible bias in the AS test of the broad construct. In brief, it is shown that the broad volatility test of AS likely contains bias of a magnitude and direction that could prevent the detection of excess price volatility.

V Possible Bias in AS Results

This section presents evidence that AS's test of the broad construct may have contained significant unrecognized bias in the WS test statistic; rather than 0, the appropriate critical value may actually have been a large positive number. If so, it may be inappropriate to interpret the positive WS for the broad construct as evidence against excess volatility.

V.a Source of Bias — Intuition

One of the primary steps in performing the West Test is to calculate the AR coefficients of the (differenced) dividend series. An implicit assumption is that this process is stationary and ergodic, so that the coefficient estimates obtained will adequately represent the dividend process into the infinite future.

Now consider the period sampled by AS. In the late 1970's and 1980's, broad dividends appeared to significantly exceed narrow dividends on average, due to the significant increase in buyouts and takeovers during this period.

As discussed in Section II, these buyouts and takeovers increase the broad dividends, but at the same time reduce the holdings of the portfolio.

The application of the West Test assumes, roughly, that this acceleration of broad dividends and portfolio reductions will continue into the infinite future. But clearly this cannot hold; portfolio reductions must terminate when the portfolio is reduced to 0 shares.

Therefore if the sampled period contains broad dividends which on average are increasing relative to narrow dividends, we should expect the AR coefficients on the broad dividends to be different from those of any true underlying stationary process and inappropriate for extrapolating into the infinite future. To the extent that the estimated coefficients differ from the true underlying stationary dividend process, the estimated West Statistic will be biased. The direction of the bias will be considered later in this paper.

V.b Evidence in the AS Data

Examination of the AS dataset suggests that the above effect occurred in their work on broad dividends. The evidence is discussed below. All references are to the AS monthly dataset.

V.b.1 Broad/Narrow Portfolio Value Ratio's

In the first month (January 1950), the broad and narrow portfolios constructed by AS were of equal value, so that the ratio of

$$(\text{broad portfolio value}) : (\text{narrow portfolio value})$$

was exactly equal to one. At December 1976, this ratio was .925. By the end of the data set (February 1991), this ratio had fallen to .581. This indicates that the last 1/3 of the dataset included a large liquidation component that was greater than that of the first 2/3 of the dataset. Therefore the dataset appears on average to include accelerating liquidations which logically could not continue indefinitely into the future.

V.b.2 Split Sample Structure Changes

There is evidence that the accelerated liquidation activity late in the sample period significantly impacted the autoregression coefficients used in the West Statistic for broad dividends.

To determine this, the data for first-differenced dividends were split into two sub-samples separating pre-1977 data from post-1976 data. For each sample, the coefficients of an AR(12) process on first-differenced broad dividends were estimated, as were the estimated standard deviations of disturbances. These estimates are summarized in Table 2 along with estimates of the same parameters for the entire 1950–1991 sample.

Table 2

	Early ('50-'76)	Late ('77-'91)	All ('50-'91)
$\hat{\mu}$ (constant)	.021	-.019	.003
$\hat{\phi}_1$	-.941	-.740	-.761
$\hat{\phi}_2$	-.975	-.607	-.657
$\hat{\phi}_3$	-.781	-.603	-.609
$\hat{\phi}_4$	-.833	-.518	-.565
$\hat{\phi}_5$	-.711	-.463	-.510
$\hat{\phi}_6$	-.397	-.276	-.257
$\hat{\phi}_7$	-.341	-.340	-.346
$\hat{\phi}_8$	-.463	-.332	-.364
$\hat{\phi}_9$	-.410	-.171	-.170
$\hat{\phi}_{10}$	-.497	-.119	-.156
$\hat{\phi}_{11}$	-.452	-.183	-.219
$\hat{\phi}_{12}$	-.049	.007	.036
$\hat{\sigma}_\epsilon^2$.373	11.848	4.309

Estimates of parameters for an AR(12) process
of broad dividends after first differencing:

$$\Delta d_t = \mu + \sum_{j=1}^{12} \phi_j \Delta d_{t-j} + \epsilon_t$$

$$\epsilon_t \sim \text{iid}(0, \sigma_\epsilon^2)$$

As shown, the coefficients and variance estimates are very different between the two sub-samples. The full-sample coefficients are significantly closer in Euclidean distance to the late-sample coefficients than they are to the early-sample coefficients despite the fact that the early sample contains nearly twice as many observations (a standard regression result of the high variance in residuals in the late sample).

These apparent changes in variance and coefficients were tested more formally for

structural shifts. The tests used were a Goldfeld/Quandt test for the variance of disturbances and a Chow-style test for changes in the coefficients after allowing for heteroskedasticity, as outlined by Amemiya (1985) and Pesaran, Smith & Yeo (1985). Both tests assume that disturbances are normally distributed. The two null hypotheses (that the variance was the same in both early and late periods, and that the AR coefficients were unchanged) were *each* rejected with greater than 99.5 % confidence.

V.b.3 Split Sample West Statistics

Because the estimated AR coefficients and AR disturbance variance are key components in the determination of the West Statistic, the West Statistic for the AS data also varies between the early and late subsamples.

The West Statistics were calculated separately for the early (1950–1976) and late (1977–1991) subsamples for monthly first differenced broad dividends. These subsample WS's and their standard errors (WSSTD) are shown in Table 3 along with AS's calculated WS and WSSTD for the full (1950–1991) data sample.

Table 3

	Early ('50 - '76)	Late ('77 - '91)	[AS] All ('50 - '91)
WS	-293	12,523	8156
WSSTD	279	8127	3646

It is apparent that the positive WS obtained by AS for the full dataset is heavily attributed to the activity in the 1977–91 subperiod.

It is interesting to augment the information of Table 3 with additional information. Table 4 repeats the Table 3 information and adds the statistics for the narrow construct as reported by AS. In addition, Table 4 indicates for each case the estimated apparent percent

liquidation of the portfolio that occurred in the subsample time period for each case (based on the discussion in Section V.b.1 above) and the t-value (WS/WSSTD) which gives the number of standard errors the WS differs from AS's critical value of 0:

Table 4

	Narrow	Broad		
		Early	Late	Full
WS	-862	-293	12,523	8156
WSSTD	327	279	8127	3646
t-value	-2.64	-1.05	1.54	2.24
Ending Portfolio Value/ Same-Time Narrow Portfolio Value	100.0%	92.53%	58.07%	58.07%
Implied % Portfolio Liquidation Over Subsample Period	0%	7.5%	37.2%	41.9%

The striking feature of Table 4 is the positive correlation between the percentage of the portfolio liquidated and the t-value. This suggests that liquidations might cause a positive bias in the calculation of the West Statistic. This possibility is explored in the next section.

VI Simulation

The hypothesis that the West Statistic for broad dividends might be positively biased by the amount of liquidations was tested by using the following *deterministic* model:

Consider a firm which earns annual profits of \$1000 with certainty. At time $t = 0$, the firm holds no debt; it has 1000 common shares outstanding, all held in a single portfolio.

An interest rate of 10% applies to all transactions in the economy. The market price of the stock is then \$10 per share. For periods $t = 1, 2, \dots, 100$, the firm distributes all profits in a (narrow) dividend of \$1/share.

In periods $t = 101, \dots, 150$ the firm repurchases outstanding shares from the portfolio, financing each purchase by borrowing at the 10% interest rate. All shares are purchased at the \$10 market price. The number of shares repurchased in period t is denoted by $s(t)$ where

$$s(t) = \begin{cases} 0 & 1 \leq t \leq 100 \\ 2 & t = 101 \\ 1 & t = 102 \\ s(t-2) + 1 & t = 103, \dots, 150 \end{cases}$$

Notice that the share repurchases are simply a debt-for-equity swap in the firm's capitalization. The market price of shares remains \$10/share and dividends remain \$1/share for all periods. Clearly the share price is rational in all periods; there is clearly no excess price volatility.

Using this data, we can calculate West Statistics under the broad construct. Since this is a deterministic model, this is a purely mechanical procedure. By the model's structure, the "correct" West Statistic is exactly 0; any West Statistic obtained different from 0 gives both the magnitude and direction of bias.

Table 5 summarizes the results of the West Test for the broad construct under several differencing specifications, using an AR(1) estimation for the (differenced) dividends and assuming holding periods $\{t = 1, 2, \dots, T\}$ for $T = 120, 130, 140$, and 150. (Cases for T less than 120 are not useful because of multicollinearity effects resulting from constant dividends and prices over the first 100 periods).

Table 5

	WS	WSSTD	t-value
<u>No Differencing:</u>			
Periods 1-120	309	156	1.98
Periods 1-130	2,222	2,288	.97
Periods 1-140	53,226	58,023	.92
Periods 1-150	58,860	26,209	2.25
<u>First Differencing:</u>			
Periods 1-120	263	149	1.77
Periods 1-130	839	355	2.36
Periods 1-140	2,339	889	2.63
Periods 1-150	5,247	1,808	2.90
<u>Second Differencing:</u>			
Periods 1-120	19,120	15,057	1.27
Periods 1-130	17,943	14,064	1.28
Periods 1-140	16,864	13,153	1.28
Periods 1-150	15,892	12,340	1.29

Notice immediately that the WS in each case is positive; this indicates upward bias in the West Statistic for the broad construct in the presence of liquidations. This bias is present under all differencing specifications, and does not appear to be readily resolved when the sample size increases. From the generally large t-values, we conclude that this bias is statistically important in all cases.

For the first-differenced case, the t-values clearly appear to increase as T increases. This indicates that the upward bias in WS becomes statistically more significant as the duration/magnitude of liquidations increase.

From this model, then, we can conclude that in the broad construct the West Test can be significantly biased upward when the portfolio is being liquidated. In the simulation, this bias was statistically most significant in the first-differenced case.

As for the narrow construct, the zero-variance of the narrow price and dividends (i.e. the constant \$10/share price and \$1/share dividend) lead to the “correct” $WS=0$ in this model. Because the narrow-portfolio is by construction “buy and hold”, the liquidations have no impact on the West Test results under the narrow construct.

VII Conclusion

The puzzle addressed by the stock price volatility literature is “Why do prices seem relatively volatile while the underlying dividends seem relatively smooth?” This question is important because of its implications for the rationality of asset markets.

Ackert and Smith have attempted to answer the puzzle by finding a “broad” dividend stream which is more volatile than the “narrow” dividend stream tested previously in the literature. They apply the West Test to both the narrow and broad constructs, and conclude that stock prices are too volatile relative to narrow dividends but not too volatile relative to broad dividends. The apparent implication is that questions of market rationality and excess price volatility have arisen largely because of the unfortunate choice made in previous studies regarding how to define the relevant cash flows.

This paper challenges AS’s results and conclusions. First, it is argued that in general the theoretical returns will be identical under either the broad or narrow construct. As a result, either construct can be used in a volatility test, as long as the test itself is properly tailored to match the construct chosen. Evidence is then presented that the test performed by AS was not properly tailored to match the broad construct. As defined by AS, broad dividends will differ from narrow dividends when portfolios are partially liquidated by stock repurchases and/or takeovers. Yet these liquidations are shown to be capable of generating significant bias in the employed test statistic; this potential bias is not reflected in AS’s choice of a critical value. The direction of this unrecognized bias would be to reduce the power of the test on broad dividends so that excess volatility could exist but not be detected when using the broad construct. This liquidation bias does not appear to be present when

the test is applied to narrow dividends, the dividend stream for which the test was originally created and used by West.

Akert & Smith's conclusion that prices are not too volatile relative to dividends rests primarily on the results obtained when broad dividends are differenced. Interestingly, this case on which their conclusion is primarily based is also the case which a simple simulation suggests may have the most significant unrecognized bias. Their results for broad dividends without differencing and for narrow dividends (differenced or not) are most consistent with the existence of excess volatility.

Considering this analysis, one might interpret Ackert and Smith's results as actually being further evidence that stock prices are excessively volatile.

VIII References

- Ackert, Lucy F. and Brian F. Smith (1993), "Stock Price Volatility, Ordinary Dividends, and Other Cash Flows to Shareholders", *Journal of Finance*, Vol XLVIII, no. 4, September, pp. 1147 - 1160.
- Amemiya, Takeshi (1985), *Advanced Econometrics*, Harvard University Press.
- Cochrane, J. H. (1991), "Volatility Tests and Market Efficiency: A Review Essay", *Journal of Monetary Economics*, 27, pp. 463 - 485.
- Gilles, C. and S. F. LeRoy, 1991, "Econometric Aspects of the Variance Bounds Tests: A Survey", *Review of Financial Studies*, 4, pp. 753 - 791.
- LeRoy, Stephen F. and Richard D. Porter, 1981, "The Present-Value Relation: Tests Based on Implied Variance Bounds", *Econometrica*, 49, pp. 555 - 574
- LeRoy, Stephen F. and Garrett H. TeSelle, (1995), "Stock Price Volatility: A Comment", University of Minnesota Working Paper.

- Pesaran, M. H. , R. P. Smith, and J. S. Yeo, (1985), "Testing for Structural Stability and Prediction Failure: A Review", *Manchester School*, 53, pp. 280 - 295.
- Shiller, Robert J. (1981), "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?", *American Economic Review*, 71, pp. 421 - 436.
- Shiller, Robert J. (1989), *Market Volatility*, MIT Press, Cambridge, MA.
- West, Kenneth D. (1988), "Dividend Innovations and Stock Price Volatility", *Econometrica*, 56, pp. 37 - 61.