AN AGGREGATE MODEL OF FIRM SPECIFIC CAPITAL WITH AND WITHOUT COMMITMENT

by

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ABSTRACT

This paper studies the implications of an agency problem on the equilibrium outcome of an intertemporal model. The model considered is a two-period lived overlapping generations model with an aggregate productivity shock. In each generation, a subset of the agents, the entrepreneurs, choose the asset specificity of their projects. An agency problem exists because the entrepreneurs cannot commit to supplying their human capital which is essential to the project. I compare equilibria with and without commitment. The main result is that in the long run, the equilibrium without commitment has lower asset specificity and per capita output, and the productivity shocks have more lasting effects. However, it need not have larger aggregate fluctuations.

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I. INTRODUCTION

In various agency settings, it has been shown that the presence of agency problems can exacerbate and propagate an aggregate productivity shock through the amount of investment undertaken in an economy[1]. The objective of this paper is to study if the amplification effect is robust to the class of agency problems considered and to the assumption about the entrepreneur's ability to adjust asset specificity[2].

This paper considers the class of agency models in which entrepreneurs cannot commit to their investment projects[3]. That is, they can withhold their human capitals after the projects are started. Since the human capital of the entrepreneur is crucial to the project, the entrepreneur can re-negotiate for a more favorable sharing rule after the project is started. Recent literature on bargaining shows that the entrepreneur might be able to negotiate the payoff of outside investors down to the project's liquidation value[4]. Anticipating the ex post opportunistic behavior of the entrepreneur, investors are willing to invest at most the present value of the project's liquidated assets ex ante, thereby limiting the amount of external funds that an entrepreneur can raise.

In this paper, entrepreneurs can adjust their liquidity positions by varying the degree of specificity of their investment projects. Asset specificity refers to the degree that the capital goods are specifically adapted to a project. An increase in the degree of asset specificity raises the productivity of the capital goods within the project and reduces their outside values. Therefore, the choice of asset specificity entails a trade-off between the project's on-going value and liquidation value. An entrepreneur who has few internal funds can increase the liquidity of an investment project by using less specific assets. As the net worth of the entrepreneur increases, less external fund is required to start the project and therefore more specific capital goods can be used. Thus, the net worth of an entrepreneur is positively related to the specificity of the capital goods used.

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[1]For example, Williamson (1987) and Bernanke and Gertler (1989).
[2]Bernanke and Gertler(1989) suggest that issues raised by the ability of the borrowers to adjust their liquidity positions deserve further research.
[3]This class of agency problems is similar to the ones considered in Hart and Moore (1991).
[4]See, for example, Sutton (1986) and Binmore, Shaked and Sutton (1989)

To study the implications of the agency problem on aggregate fluctuations, this paper adopts a two-period lived overlapping-generations model with an aggregate productivity shock. Preferences, endowments and technology are such that when entrepreneurs are able to commit, then regardless of their net worth positions, they can guarantee outside investors their opportunity cost of capital and choose the degree of asset specificity that maximizes the expected value of their investment projects. Therefore, the aggregate productivity shock does not affect asset specificity and hence has no persistent effect in the economy. In contrast, if entrepreneurs cannot commit, then the current aggregate shock that determines the current labor income can affect the asset specificity of the new investment projects which, in turn, affects the productivity (and therefore the labor income) of future entrepreneurs. Thus, the aggregate shock has a persistent effect in the economy.

The rest of this paper is organized as follows: section II describes the model, section III defines the equilibrium concept, section IV and V characterize the equilibrium in the environments with and without commitment, section VI compares the long-run behavior of the two economies and section VII summarizes the paper.

II. THE MODEL

This section first describes the physical environment of the model and then the agency problem due to the lack of commitment.

II.1. Physical Environment

This paper considers an infinite time horizon overlapping generations model in which each generation lives for two dates. At each date t, t = 0, 1, 2, ..., the population consists of N old agents and N young agents. Each agent cares only about the expected value of consumption when old. They have access to a constant return to scale storage technology that yields R units of date t+1 consumption good for every unit of date t consumption good stored. In each generation, a fraction $(1-\alpha)$ of the agents are "investors" and the rest (α) are "entrepreneurs". The investors do not have any labor endowment nor any investment projects but each of them is endowed with 1 unit of consumption good when young. Each of the entrepreneurs when young is endowed with 1 unit of labor and has access to an investment project.

At date 0, we assume that each of the existing old enteprerneurs is endowed a capital good with z_0 of specificity and they observe an aggregate productivity shock θ_0. At each date t, $t \geq 1$, if the entrepreneurs born at date t-1 have started their projects at date t-1, then they have capital goods with z_t degree of specificity and the aggregate productivity shock θ_t is publicly observed. Together with the old investors, they decide if the projects should be continued or liquidated. If the investment project is continued at date t, the output of each project (in terms of date t consumption good) is $f(\ell_t, \theta_t, z_t)$ where ℓ_t is the amount of labor hired from the young entrepreneurs born at date t. If the project is liquidated, the output (in terms of date t consumption good) is $d(z_t)$. Once the capital good is used at date t, either because the project is continued or liquidated, it is fully depreciated. After working for the old entrepreneurs, the young entrepreneurs can start their investment projects. Each of the projects requires one unit of date t consumption good. The consumption good can be developed into a capital good with z_{t+1} degree of asset specificity. Thus, the young entrepreneur has to choose the amount of internal investment (I_t), the amount to borrow from the young investors $(1-I_t)$, the degree of asset specificity of their investment projects (z_{t+1}), a labor input rule $(\ell_{t+1}(\theta_{t+1}))$ and a sharing rule

($x_{t+1}(\theta_{t+1})$: payoff to investors). The sequence of events that take place at date t is depicted at the following diagram:

z_t developed, θ_t publicly observed

Project continued and ℓ_t labor hired from young entrepreneur: output is $f(\ell_t, \theta_t, z_t)$.	Project liquidated: liquidation value is $d(z_t)$.

↓

Old agents consume outputs from investment projects and from storage.

↓

After working for old entrepreneurs, young entrepreneurs start their projects and choose:

$$\begin{cases} \text{the amount of internal investment}(I_t), \\ \text{the amount of loan from young investors } (1 - I_t), \\ \text{asset specificity level } (z_{t+1}), \\ \text{labor input rule } (\ell_{t+1}(\theta_{t+1})), \text{ and} \\ \text{sharing rule}(x_{t+1}(\theta_{t+1})). \end{cases}$$

The degree of specificity of the capital good z can be chosen from an interval $\mathcal{Z} = [\underline{z}, \bar{z}] \subset \Re_+$. An increase in z refers to an increase in the specificity of the capital good. The productivity of each young entrepreneur at each date t, $t \geq 0$, is subject to a common productivity shock θ_t which is identically and independently distributed through time. For all $t \geq 0$, $\theta_t \in \Theta$ where $\Theta = [\underline{\theta}, \bar{\theta}] \subset \Re_+$, $\bar{\theta} > \underline{\theta}$ and $(\Theta, \mathcal{F}, \pi)$ is a probability space. The probability measure π assigns strictly positive probability measure to all non-degenerate subintervals in $\Theta = [\underline{\theta}, \bar{\theta}]$. The productivity shock θ_t is publicly observable and verifiable at the beginning of date t.

The production function $f(\cdot, \cdot, \cdot): [0, \infty) \times \Theta \times \mathcal{Z} \to \Re_{++}$ is twice differentiable, strictly increasing in each of its arguments, has strictly positive cross partial derivatives and strictly diminishing marginal product of labor. Furthermore, for all $\ell \in [0, \infty)$, $\theta \in \Theta$ and $z \in \mathcal{Z}$,

$$\lim_{\ell \to 0} f_\ell(\ell, \theta, z) = +\infty, \qquad \text{and} \qquad \lim_{\ell \to +\infty} f_\ell(\ell, \theta, z) = 0.$$

The liquidation function $d(\cdot): \mathcal{Z} \to \Re_{++}$ is differentiable with $d_z(z) < 0$ for all $z \in \mathcal{Z}$.

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For all $\theta \in \Theta$ and $z \in \mathcal{Z}$, let $v(\theta, z) \stackrel{\text{def}}{=} f(1, \theta, z) - f_\ell(1, \theta, z)$. In a competitive market, labor is paid according to its marginal product. Therefore, if 1 unit of labor is hired for each project in equilibrium, then $v(\theta, z)$ can be interpreted as the per capita (per young labor hired) profit of the investment project when the productivity shock is θ and the asset specificity adopted is z. The following assumptions are made for all $z \in \mathcal{Z}$ and for all $\theta \in \Theta$.

Assumption A1. $v(\theta, z) > d(z)$.

Assumption A2. $\int_\Theta v(\theta, z)\pi(d\theta) > R \geq d(z)$.

Assumption A3. $f_\ell(1, \bar{\theta}, \underline{z}) > f_\ell(1, \underline{\theta}, \bar{z})$.

Assumption A4. $f_\ell(1, \underline{\theta}, \underline{z}) + \frac{d(\underline{z})}{R} \geq 1 \geq f_\ell(1, \bar{\theta}, \bar{z}) + \frac{d(\bar{z})}{R}$.

Assumption A1 says that ex post, regardless of the productivity shock, the project always has a higher on-going value than liquidation. This assumption guarantees that liquidation will never occur in equilibrium and therefore simplifies the analysis. The first inequality of assumption A2 says that the per capita expected profit of the invesment project as an on-going concern is higher than that of the storage technology. It implies that the projects are worth investing in. The second inequality of assumption A2 implies that ex ante, if the project is liquidated for sure, then it is not worth investing in. This assumption implies that investors will never fully finance the investment projects when there is no commitment (except when the asset specificity level is the lowest). Assumption A3 says that the marginal product of 1 unit of labor is higher with the best aggregate shock and the lowest asset specificity than with the worst aggregate shock and the highest asset specificity. Assumption A4 rules out two extreme and un-interesting equilibria under no commitment. The first inequality of assumption A4 implies that young entrepreneurs can always raise sufficient funds to start their investment projects and, therefore, rules out an equilibrium in which no projects are ever financed. The second inequality of assumption A4 implies that young entrepreneurs cannot always adopt the most specific assets and therefore rules out a degenerate equilibrium with $z_t = \bar{z}$.

II.2. Commitment

In this paper, all variables are publicly observable and verifiable, i.e., there is no asymmetric information. However, an agency problem exists due to the lack of commitment: entrepreneurs cannot commit to supplying their human capital after the capital good is developed. More specifically, suppose that at date t-1, the young investors agree to lend to a young entrepreneur to start an investment project. By date t, the capital good is developed with specificity z_t and the productivity shock θ_t is observed (refer to the diagram that illustrates the sequence of events that take place at date t). Since the project has a higher on-going value than liquidation, the entrepreneur can withhold his/her human capital and bargain for a larger share of the project's output. If an agreement is reached, the entrepreneur (who is now old) will hire labor from the young entrepreneurs and put the capital good into production. Alternatively, if agreement cannot be reached, the project will be liquidated and the liquidation value will go to the investors. In this case, the entrepreneur will get nothing[5].

To study the implications of the agency problem, this paper also considers the environment in which entrepreneurs are able to commit their human capital to the projects. In this environment, investors and entrepreneurs can sign complete state contingent sharing rules ex ante and no negotiation will take place after the project is started.

[5]For simplicity, this paper assumes that the entrepreneur's outside wage is zero at date t.

III. EQUILIBRIUM

At each date t, $t \geq 0$, given (θ_t, z_t), let $w_t(\theta_t, z_t)$ denote the labor income of the representative entrepreneur born at date t. After earning the labor income $w_t(\theta_t, z_t)$, the young entrepreneur starts his/her own investment project and decides on:

(i) how much to invest in his/her own project (I_t),

(ii) the asset specificity to adopt (z_{t+1}),

(iii) the labor-input rule $\ell_{t+1}(\theta_{t+1})$ and

(iv) the profit-sharing rule $x_{t+1}(\theta_{t+1})$ where $x_{t+1}(\theta_{t+1})$ is the amount of the date t+1 consumption good that the investor gets when the productivity shock at date t+1 is θ_{t+1}.

Let $w_{t+1}(\theta_{t+1})$ denote the labor wage rate at date t+1 when the productivity shock is θ_{t+1}[6]. Facing the state-contingent wages $w_{t+1}(\theta_{t+1})$, the optimal choice of the representative entrepreneur is to choose $\{I_t, z_{t+1}, \ell_{t+1}(\theta_{t+1}), x_{t+1}(\theta_{t+1})\}$ to maximize[7]

$$\int_{\Theta}[f(\ell_{t+1}(\theta_{t+1}), \theta_{t+1}, z_{t+1}) - w_{t+1}(\theta_{t+1})\ell_{t+1}(\theta_{t+1}) - x_{t+1}(\theta_{t+1})]\,\pi(d\theta_{t+1}) + [w_t(\theta_t, z_t) - I_t]R$$

subject to

$$w_t(\theta_t, z_t) \geq I_t \geq 0 \tag{C1}$$

$$\int_{\Theta} x_{t+1}(\theta_{t+1})\pi(d\theta_{t+1}) \geq (1 - I_t)R \tag{C2}$$

For all $\theta_{t+1} \in \Theta$, $x_{t+1}(\theta_{t+1})$ is the equilibrium sharing rule of the bargaining game at date t+1. (C3)

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[6]In equilibrium, $w_{t+1}(\theta_{t+1})$ will be functions of z_{t+1} in general. That is why the labor income at date t is expressed as a function of the aggregate productivity shock at date t and the degree of asset specificity adopted by date t. However, because the labor market is competitive, when the entrepreneurs make their investment decisions at date t, they take the future wage rates as given. Therefore, in order to emphasize that the entrepreneurs do not perceive that they can affect the future wage rate through their choice of asset specificity, z_{t+1} is suppressed from the expression of the future wage rate. Of course, in order to forecast the future state contingent wage rates, the entrepreneurs have to forecast the state variable z_{t+1}.

[7]The non-negative consumption constraints and the state contingent liquidation rules are omitted from the optimal choice problem. It can be easily shown that non-negative consumptions are satisfied in equilibrium. Assumption A1 implies that liquidation will never occur in the economies considered in this paper (with or without commitment). The proof of this result will be given in the Appendix.

Constraint (C1) is the constraint on the amount that the entrepreneur can invest in his/her own project. Constraint (C2) is the constraint that the investors have to earn at least a gross rate of return of R on their investments[8]. Constraint (C3) says that the sharing rule has to be consistent with the agency problem in the environment. When the entrepreneur cannot commit to the project, the sharing rule ($x_{t+1}(\theta_{t+1})$) has to be an equilibrium outcome of the bargaining game that takes place at date t+1. Sutton (1986) and Binmore, Shaked and Sutton (1989) show that in a bargaining game of alternating offers, if the liquidation value of the project (which is the outside option of the investors in this model) is larger than the subgame perfect equilibrium payoff of the investors in the same game without outside option, then the unique subgame perfect equilibrium of the game is that the investors get the project's liquidation value. This paper assumes that for all realizations of the aggregate shock θ_t and for all asset specificity level z_t, $t \geq 0$, the parameters are such that the unique subgame perfect equilibrium of the bargaining game is that the investors get the liquidation value of the project, i.e., constraint (C3) implies that [9]

$$x_{t+1}(\theta_{t+1}) = d(z_{t+1}). \tag{1}$$

Let (P1) denote the above optimal choice problem when there is no commitment.

On the other hand, if entrepreneurs can commit to their projects, then no negotiation will occur after the project is started. Ex ante, the investor and the entrepreneur can sign complete state contingent sharing rules which do not have to satisfy constraint (C3). Let (P2) denote the above optimal choice problem without constraint (C3) when there is commitment.

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[8]While the demand for investment fund is at most $N\alpha$, the supply of fund is perfectly elastic up to the amount $N(1-\alpha)$ at the gross rate of return R. Since $1-\alpha>\alpha$ by assumption, the equilibrium in the market for investment fund has the property that the entrepreneur's expected payoff is maximized subject to the condition that the investors break even in their investments.

[9]Sutton (1986) and Binmore, Shaked and Sutton (1989) consider the class of two-person bargaining game with a shrinking pie. In order to apply their bargaining game in this paper, we consider the case that (1) the investors organise in such a way that they have a representative to bargain with the entrepreneur and (2) the productivity of the capital good, continued or liquidated, will depreciate as the bargaining drags on. More specifically, after each round of negotiation, the output of the project, continuted or liquidated, will depreciate by a factor $\delta \in [0,1)$. A sufficient condition that the investor gets the project's liquidation value in the unique subgame perfect equilibrium for all realizations of θ and for all $z \in \mathbb{Z}$, is that $d(z) > \delta v(\theta, z)$. If δ is close to zero (i.e., the capital good depreciates very quickly), then the sufficient condition can be satisfied easily.

In both environments, at date t+1, when the productivity shock is θ_{t+1}, an interior optimal choice of labor is characterized by the following equation:

$$f_\ell(\ell_{t+1}, \theta_{t+1}, z_{t+1}) = w_{t+1}(\theta_{t+1}) \tag{2}$$

Also, the market clearing condition in the labor market implies that

$$\ell_{t+1}(\theta_{t+1}) = 1. \tag{3}$$

Therefore, equations (2) and (3) imply that in equilibrium,

$$f_\ell(1, \theta_{t+1}, z_{t+1}) = w_{t+1}(\theta_{t+1}). \tag{4}$$

At date 0, the equilibrium in the labor market is characterized by[10]:

$$f_\ell(1, \theta_0, z_0) = w(\theta_0, z_0). \tag{4'}$$

Thus, a rational expectation equilibrium given z_0 and the stochastic process $\{\theta_t\}_{t=0}^{\infty}$ is a collection of stochastic processes $\left\langle \{I_t, w_t\}_{t=0}^{\infty}, \{\ell_t, x_t, z_t\}_{t=1}^{\infty} \right\rangle$ that solve equations (3), (4) and the optimal choice problem: (P1) in the environment without commitment and (P2) in the environment with commitment . Due to the recursive nature of the model, the equilibrium in each of the environments is necessarily a recursive equilibrium.

III. 1. Equilibrium without commitment

In equilibrium, $\ell_{t+1}(\theta_{t+1})$ is fixed by equation (3). Therefore, once we know the equilibrium z_{t+1}, we can use equation (1) to solve for $x_{t+1}(\theta_{t+1})$ and equation (4) to solve for $w_{t+1}(\theta_{t+1})$. Hence, we can characterize the equilibrium at date t given (θ_t, z_t) by the

[10]In the Appendix, it will be shown that in equilibrium the investment projects are always continued and that the optimal choice of labor input is interior solution. Therefore, the market clearing condition in the labor market must be characterized by equations (2), (3), (4) and (4').

pair of $\{I_t, z_{t+1}\}$ that solve (P1). Let $\hat{I}_t = \hat{g}_t(\theta_t, z_t)$ and $\hat{z}_{t+1} = \hat{h}_t(\theta_t, z_t)$ be the solutions of $\{I_t, z_{t+1}\}$ given (θ_t, z_t). Given the recursive nature of the model, the functions $\hat{g}_t$ and $\hat{h}_t$ are time independent.

Definition 1. A recursive equilibrium in the environment without commitment is a pair of functions $\hat{g}(\cdot,\cdot): \Theta \times \mathcal{Z} \to \Re_+$ and $\hat{h}(\cdot,\cdot): \Theta \times \mathcal{Z} \to \mathcal{Z}$ such that for any $(\theta_t, z_t) \in \Theta \times \mathcal{Z}$, $\hat{I}_t = \hat{g}(\theta_t, z_t)$ and $\hat{z}_{t+1} = \hat{h}(\theta_t, z_t)$ are the optimal choice of I_t and z_{t+1} in problem (P1) where $w_t(\theta_t, z_t) = f_\ell(1, \theta_t, z_t)$, $x_{t+1}(\theta_{t+1})$ is given by equation (1) and $\langle w_{t+1}(\theta_{t+1}), \hat{z}_{t+1} \rangle$ satisfy equation (4) for all $\theta_{t+1} \in \Theta$.

III. 2. Equilibrium with commitment

In the economy with commitment, assumption A2 implies that for any $I_t \in [0, w_t(\theta_t, z_t)]$, for all $z_{t+1} \in \mathcal{Z}$, there exists state contingent sharing rules $x_{t+1}(\theta_{t+1})$ such that

$$\int_\Theta x_{t+1}(\theta_{t+1})\pi(d\theta_{t+1}) = (1 - I_t)R. \tag{5}$$

Substituting equations (5) into the objective function of problem (P2), problem (P2) reduces to choosing $\{I_t, z_{t+1}, \ell_{t+1}(\theta_{t+1})\}$ to maximize

$$\int_\Theta \left[f(\ell_{t+1}(\theta_{t+1}), \theta_{t+1}, z_{t+1}) - w_{t+1}(\theta_{t+1})\ell_{t+1}(\theta_{t+1})\right]\pi(d\theta_{t+1}) + [w_t(\theta_t, z_t) - 1]R$$

subject to

$$w_t(\theta_t, z_t) \geq I_t \geq 0. \tag{C1}$$

Let (P2') denote the above optimal choice problem. Since I_t does not enter the objective function and the choice of $\{z_{t+1}, \ell_{t+1}(\theta_{t+1})\}$ does not affect constraint (C1), any I_t that satisfies (C1) is a solution to the above optimal choice problem. Thus, the real choice variables in problem (P2') are $\{z_{t+1}, \ell_{t+1}(\theta_{t+1})\}$. Again, since $\ell_{t+1}(\theta_{t+1})$ is fixed by equation (3), once we know the equilibrium z_{t+1}, we can use equation (4) to solve for $w_{t+1}(\theta_{t+1})$. Thus, we can characterize the equilibrium at date t given (θ_t, z_t) by the z_{t+1} that solves (P2'). Let $z^*_{t+1} = h^*_t(\theta_t, z_t)$ be the solution of z_{t+1} given (θ_t, z_t). Again, the recursive nature of the model implies that the functions h^*_t must be time independent.

Definition 2. A recursive equilibrium in the environment with commitment is a function $h^*(\cdot,\cdot): \Theta \times Z \to Z$ such that for any $(\theta_t, z_t) \in \Theta \times Z$, $z^*_{t+1} = h^*(\theta_t, z_t)$ is the optimal choice of z_{t+1} in problem (P2') where $w_t(\theta_t, z_t) = f_\ell(1, \theta_t, z_t)$, $x_{t+1}(\theta_{t+1})$ is given by equation (5) and $\langle w_{t+1}(\theta_{t+1}), z^*_{t+1} \rangle$ satisfy equation (4) for all $\theta_{t+1} \in \Theta$[11].

In order to study the implications of the agency problem, the following two sections characterize the recursive equilibrium in the environment without commitment and in the environment with commitment.

[11] Obviously, implicit in our definition, the equilibrium choice of I_t must satisfy constraint (C1).

IV. WITHOUT COMMITMENT

In the environment without commitment, the sharing rules have to satisfy equation (1). Equation (1) renders the class of sharing rule characterized by equation (5) infeasible, for example, when $I_t = 0$ and $x_{t+1}(\theta_{t+1}) = d(z_{t+1})$, equation (5) does not hold because assumption (A2) says that $R \geq d(z)$. Thus, even though the project yields an ex ante expected return higher than R, the entrepreneur cannot pre-commit to paying the investors R units of date t+1 consumption good. As a result, ex ante, investors will not fully finance the projects. They are willing to invest at most $d(z_{t+1}) / R \leq 1$ units of date t consumption good. Since $d(z)$ is strictly decreasing in z, the degree of asset specificity determines the amount of external fund that the entrepreneur can raise. As a result, internal fund and external fund are not perfect substitutes in this environment. The following lemma establishes this observation by showing that the entreperneur will try to minimize the amount of external fund by investing all of his/her income to the projects.

Lemma 1. Constraints (C1) and (C2) must be binding at the optimal choice of the entrepreneur's maximization problem (P1') in equilibrium.

Proof. We can use equation (1) to re-write the optimal choice problem (P1) as choosing $\{I_t, z_{t+1}, \ell_{t+1}(\theta_{t+1})\}$ to maximize

$$\int_{\Theta}[f(\ell_{t+1}(\theta_{t+1}), \theta_{t+1}, z_{t+1}) - w_{t+1}(\theta_{t+1})\ell_{t+1}(\theta_{t+1})]\,\pi(d\theta_{t+1}) - d(z_{t+1}) + [w_t(\theta_t, z_t) - I_t]R$$

subject to

$$w_t(\theta_t, z_t) \geq I_t \geq 0 \tag{C1}$$

$$d(z_{t+1}) \geq (1 - I_t)R \tag{C2'}$$

Let (P1') denote the above optimal choice problem. Since the objective function is strictly decreasing in $d(z_{t+1})$, constraint (C2') must be binding for any given I_t at the optimum, i.e.,

$$d(z_{t+1}) = (1 - I_t)R. \tag{6}$$

Substituting equation (6) into the objective function, we can re-write problem (P1') as choosing $\langle z_{t+1}, \ell_{t+1}(\theta_{t+1})\rangle$ to maximize

$$\int_{\Theta} \left[f(\ell_{t+1}(\theta_{t+1}), \theta_{t+1}, z_{t+1}) - w_{t+1}(\theta_{t+1})\ell_{t+1}(\theta_{t+1})\right] \pi(d\theta_{t+1}) + \left[w_t(\theta_t, z_t) - 1\right]R$$

subject to

$$w_t(\theta_t, z_t) \geq 1 - \frac{d(z_{t+1})}{R}. \tag{C1'}$$

By the envelope theorem, the optimal choice of z_{t+1} is the one that maxmizes

$$\int_{\Theta} f(\ell_{t+1}(\theta_{t+1}), \theta_{t+1}, z_{t+1})\, \pi(d\theta_{t+1})$$

subject to

$$w_t(\theta_t, z_t) \geq 1 - \frac{d(z_{t+1})}{R} \tag{C1'}$$

where $\ell_{t+1}(\theta_{t+1})$ is characterized by equation (2).

Since $d(z_{t+1})$ is strictly decreasing in z_{t+1}, constraint (C1') puts an upper limit on z_{t+1}. In equilibrium, the wage rate in the labor market at date t is given by equation (4). Thus, we can rewrite constraint (C1') as:

$$f_\ell(1, \theta_t, z_t) \geq 1 - \frac{d(z_{t+1})}{R}. \tag{C1''}$$

Since the objective function is strictly increasing in z_{t+1}, the optimal choice of z_{t+1} is the highest z_{t+1} that constraint (C1") allows. Since $f_\ell(1, \theta_t, z_t)$ is strictly increasing in z_t, assumption (A4) implies that $z_{t+1} = \bar{z}$ is infeasible except possibly when $\theta_t = \bar{\theta}$ and

$z_t = \bar{z}$. Thus, the optimal choice of z_{t+1} occurs when constraint (C1") (and therefore constraint (C1)) is binding. Q.E.D.

An implication of lemma 1 is that in the environment without commitment, internal fund and external fund are not perfect substitutes. The amount of external fund that the entrepreneur has to raise constrains the asset specificity that the entrepreneur can adopt. Thus, the entrepreneur will minimize the amount of external fund that he/she has to raise by investing all of his/her income into the project. As a result, the income of the entrepreneur will be positively related to the asset specificity adopted in their projects. Since the labor income of the entrepreneur is determined by the current productivity shock and the existing degree of asset specificity, there is a relationship between the current and future asset specificity which is summarized in the following proposition.

Proposition 1. There exists a unique recursive equilibrium where the function $\hat{h}(\cdot,\cdot): \Theta \times \mathbf{Z} \to \mathbf{Z}$ is continuous and is strictly increasing in its arguments.

Proof. At any date t, $t \geq 0$, given any $(\theta_t, z_t) \in \Theta \times \mathbf{Z}$, the Appendix shows that the only equilibrium that can exist is that investment projects are continued and that the optimal choice of labor is an interior solution. Since $f_\ell(\ell, \theta, z)$ is strictly increasing in ℓ and since $\lim_{\ell \to 0} f_\ell(\ell, \theta, z) = +\infty$ and $\lim_{\ell \to +\infty} f_\ell(\ell, \theta, z) = 0$, $w_t(\theta_t, z_t) = f_\ell(1, \theta_t, z_t)$ is well-defined and is the unique wage rate that clears the labor market given (θ_t, z_t). Lemma 1 says that given $w_t(\theta_t, z_t) = f_\ell(1, \theta_t, z_t)$, the optimal choice of z_{t+1} must have the property that (C1") is binding, i.e.,

$$\frac{d(z_{t+1})}{R} = 1 - f_\ell(1, \theta_t, z_t). \tag{7}$$

Assumption (A4) implies that there exists a unique $\hat{z}_{t+1} \in \mathbf{Z}$ satisfying equation (7). Since the above argument hold for any $(\theta_t, z_t) \in \Theta \times \mathbf{Z}$, $t \geq 0$, it holds for all $(\theta_t, z_t) \in \Theta \times \mathbf{Z}$, $t \geq 0$.

Furthermore, as a result of assumption (A4) and the properties that $f_{\ell\theta}(1, \theta, z) > 0$, $f_{\ell z}(1, \theta, z) > 0$ and $d_z(z) < 0$ for all $\theta \in \Theta$ and $z \in \mathbf{Z}$, equation (7) defines a function

$\hat{h}(\cdot,\cdot):\Theta\times\mathcal{Z}\rightarrow\mathcal{Z}$ for all $t\geq 0$ where $\hat{z}_{t+1}=\hat{h}(\theta_t,z_t)$, $\hat{h}_\theta(\theta_t,z_t)>0$ and $\hat{h}_z(\theta_t,z_t)>0$.

Q.E.D.

Proposition 1 implies that in the environment without commitment, a temporary aggregate shock has a lasting effect in the economy. The temporary productivity shock at date t affects not only the output of the investment projects at date t but also the labor income of the young entrepreneurs at date t. The labor income of the young entrepreneurs determines the asset specificity to be developed by date t+1 which in turn determines the output and the productivity (and therefore the labor income) of the young entrepreneurs at date t+1. Thus, the shock at date t affects future outputs through the degree of asset specificity adopted. The following proposition shows that the degree of asset specificity will not converge to a single value in the long run.

Proposition 2. Let $\mathcal{B}$ be the $\sigma-$field of Borel sets of $\mathcal{Z}$. $(\Theta\times\mathcal{Z},\ \mathcal{F}\times\mathcal{B})$ is the product space of $(\Theta,\ \mathcal{F})$ and $(\mathcal{Z},\mathcal{B})$. Let $\hat{H}((\cdot,\cdot);(\cdot,\cdot)):(\Theta\times\mathcal{Z})\times(\mathcal{F}\times\mathcal{B})\rightarrow[0,1]$ be the transition function defined by π and the function $\hat{h}$ as follows:

$$\hat{H}((\theta,z);(A,B))=\begin{cases}\pi(A) & \text{if } \hat{h}(\theta,z)\in B\\ 0 & \text{otherwise.}\end{cases}\tag{8}$$

For any probability measure λ_0 on $(\Theta\times\mathcal{Z},\ \mathcal{F}\times\mathcal{B})$, let $\{\lambda_t\}_{t=1}^{\infty}$ be the sequence of probability measure on $(\Theta\times\mathcal{Z},\ \mathcal{F}\times\mathcal{B})$ defined by $\lambda_{t+1}=\int\hat{H}d\lambda_t$. λ_t converges weakly to a unique invariant probability measure $\hat{\lambda}$ on $(\Theta\times\mathcal{Z},\ \mathcal{F}\times\mathcal{B})$. $\hat{\lambda}$ can be written as $\pi\hat{\mu}$ where $\hat{\mu}$ is a non-degenerate probability measure on $(\mathcal{Z},\mathcal{B})$.

Proof. Let $f(\cdot,\cdot):\Theta\times\mathcal{Z}\rightarrow\Re$ be a $(\mathcal{F}\times\mathcal{B})$-measurable function and let $(Tf)(\theta,z)\overset{\text{def}}{=}\int_{\Theta\times\mathcal{Z}}f(\theta',z')\hat{H}((\theta,z);d\theta'\times dz')$. Using the definition of $\hat{H}$ in equation (8),

$$(Tf)(\theta,z)=\int_{\Theta}f\left(\theta',\hat{h}(\theta,z)\right)\pi(d\theta')\text{ where } z'=\hat{h}(\theta,z).\tag{9}$$

To prove proposition 2, we have to establish that the transition function $\hat{H}$ has three properties:

(i) $\hat{H}$ is monotone, i.e., (Tf) is non-decreasing if f is non-decreasing;

(ii) $\hat{H}$ has the Feller property, i.e., (Tf) is bounded and continuous if f is bounded and continuous, and

(iii) there exist an element $(\tilde{\theta}, \tilde{z}) \in (\Theta \times \mathcal{Z})$, an $\varepsilon > 0$ and an $N \geq 1$ such that $\hat{H}^N\left((\underline{\theta}, \underline{z}); [\tilde{\theta}, \bar{\theta}] \times [\tilde{z}, \bar{z}]\right) \geq \varepsilon$ and $\hat{H}^N\left((\bar{\theta}, \bar{z}); [\underline{\theta}, \tilde{\theta}] \times [\underline{z}, \tilde{z}]\right) \geq \varepsilon$.

(i) Since h is strictly increasing in its arguments, if f is non-decreasing, then it is easy to see from equation (9) that (Tf) is also non-decreasing. That is, $\hat{H}$ is monotone.

(ii) Since $(\Theta \times \mathcal{Z})$ is a compact set in $\Re^2$, if (Tf) is continuous, then it must be bounded. Therefore, to prove that (Tf) satisfies the Feller property, it suffices to show that (Tf) is continuous if f is continuous and bounded. Let (θ, z) be an element in $(\Theta \times \mathcal{Z})$. Let $\{(\theta_n, z_n)\}$ be a sequence of elements in $(\Theta \times \mathcal{Z})$ that converges to (θ, z). Since f and $\hat{h}$ are continuous, the sequence of functions $f\left(\theta', \hat{h}(\theta_n, z_n)\right)$ converges a.e. to $f\left(\theta', \hat{h}(\theta, z)\right)$. Let $c = \max\limits_{(\theta', z') \in \Theta \times \mathcal{Z}} |f(\theta', z')|$. Such a c exists and is finite $(\Theta \times \mathcal{Z})$ is compact and f is continuous. Since $\hat{h}(\theta_n, z_n) \in \mathcal{Z}$ for all n, $c \geq f\left(\theta', \hat{h}(\theta_n, z_n)\right)$ for all n. Thus, by Lebesque Dominated Convergence Theorem,

$$\lim_{n \to \infty} \int_{\Theta} f\left(\theta', \hat{h}(\theta_n, z_n)\right) \pi(d\theta') = \int_{\Theta} f\left(\theta', \hat{h}(\theta, z)\right) \pi(d\theta'),$$

i.e.,
$$\lim_{n \to \infty} (Tf)(\theta_n, z_n) = (Tf)(\theta, z).$$

Since the above argument holds for any element in $(\Theta \times \mathcal{Z})$, it follows that (Tf) is continuous on $(\Theta \times \mathcal{Z})$.

(iii) Let $\tilde{\theta} = E(\theta)$ and $\tilde{z} \in \left(h(\underline{\theta}, \bar{z}), h(\bar{\theta}, \underline{z})\right)$. Suppose that $(\theta_0, z_0) = (\underline{\theta}, \underline{z})$. Then $z_1 = h(\underline{\theta}, \underline{z})$. Let $A_1 = \left\{\theta_1 \in \Theta: z_2 \in [\tilde{z}, \bar{z}]\right\}$. Assumption (A4) implies that $\pi(A_1) > 0$. Let $A_2 = \left\{\theta_2 \in \Theta: \theta_2 \in [\tilde{\theta}, \bar{\theta}]\right\}$. $\pi(A_2) > 0$. Since θ_1 and θ_2 are independent,

$$\hat{H}^2\left((\underline{\theta}, \underline{z}); [\tilde{\theta}, \bar{\theta}] \times [\tilde{z}, \bar{z}]\right) = \pi(A_1)\pi(A_2).$$

Let $\varepsilon' = \pi(A_1)\pi(A_2)$, $\varepsilon' > 0$.
Similarly, we can find an $\varepsilon'' > 0$ such that $\hat{H}^2\left((\bar{\theta}, \bar{z}); [\underline{\theta}, \tilde{\theta}] \times [\underline{z}, \tilde{z}]\right) \geq \varepsilon''$. Let $\varepsilon = \min\{\varepsilon', \varepsilon''\}$, $\varepsilon > 0$. Thus, property (iii) is satisfied for N=2, $\tilde{\theta} = E(\theta)$ and some $\tilde{z} \in \left(h(\underline{\theta}, \bar{z}), h(\bar{\theta}, \underline{z})\right)$.

Given that $\Theta \times \mathcal{Z}$ is a compact set in $\Re^2$, (i), (ii) and (iii) guarantee that any probability measure λ_0 on $(\Theta \times \mathcal{Z}, \mathcal{F} \times \mathcal{B})$ converges weakly to a unique invariant probability measure $\hat{\lambda}$ on $(\Theta \times \mathcal{Z}, \mathcal{F} \times \mathcal{B})$[12]. Since θ_t and z_t are independent, $\hat{\lambda}$ can be written as $\pi\hat{\mu}$ where $\hat{\mu}$ is the limiting probability measure on $(\mathcal{Z}, \mathcal{B})$. Since $\hat{h}$ is strictly increasing in its arguments, the only possibility that $\hat{\mu}$ is degenerate is that $\hat{z} = \bar{z}$ with probability 1. However, assumption (A4) and equation (7) imply that $\hat{h}(\theta, \bar{z}) \leq \bar{z}$ for all $\theta \in \Theta$. Since $\hat{h}(\theta, \bar{z}) = \bar{z}$ only at $\theta = \bar{\theta}$. Thus, it cannot be the case that $\hat{z} = \bar{z}$ with probability 1. Q.E.D.

It follows from proposition 2 that:

$$E(\hat{z}) = \int_{\mathcal{Z}} z\hat{\mu}(dz) < \bar{z}, \qquad \text{and}$$

$$\operatorname{var}(\hat{z}) = \int_{\mathcal{Z}} (z - E(z))^2 \hat{\mu}(dz) > 0.$$

That is, in the long run, on average, entrepreneurs do not adopt assets that are as specific as possible and the asset specificity levels varies over time.

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[12]See, for example, Theorem 12.12 of Chapter 12 in *Recursive Methods in Economic Dynamics* by Nancy Stokey, Robert Lucas and Edward Prescott, 1989.

V. WITH COMMITMENT

When entrepreneurs can commit to supplying their human capitals, then they can sign complete state contingent sharing rules that guarantee the investors an expected rate of return R on their investments. As a result, regardless of the the entrepreneur's labor income (and therefore regardless of the aggregate productivity shock), the highest degree of asset specificity will be adopted to maximize the expected value of the project (i.e., $z = \bar{z}$). This observation is summarized in the following proposition.

Proposition 3. In the environment with commitment, there exists a unique recursive equilibrium which has the property that $\bar{z} = h^*(\theta, z)$ for all $(\theta, z) \in \Theta \times \mathcal{Z}$.

Proof.
Since R and $w_t(\theta_t, z_t)$ are given at date t, the optimal choice problem (P2') is equivalent to choosing $\langle z_{t+1}, \ell_{t+1}(\theta_{t+1})\rangle$ to maximize

$$\int_\Theta \left[f(\ell_{t+1}(\theta_{t+1}), \theta_{t+1}, z_{t+1}) - w_{t+1}(\theta_{t+1})\ell_{t+1}(\theta_{t+1})\right] \pi(d\theta_{t+1})$$

By the envelope theorem, we can see that the optimal choice of z_{t+1} is one that solves the following problem:

$$\underset{z_{t+1}}{\max\text{imize}} \int_\Theta f(\ell_{t+1}(\theta_{t+1}), \theta_{t+1}, z_{t+1})\, \pi(d\theta_{t+1})$$

where $\ell_{t+1}(\theta_{t+1})$ is characterized by equation (2).

Since $f(\ell_{t+1}, \theta_{t+1}, z_{t+1})$ is strictly increasing in z_{t+1} for all $\theta_{t+1} \in \Theta$ and $\ell_{t+1} \in [0, \infty)$, the optimal choice of z_{t+1} is characterized by:

$$z_{t+1} = \bar{z}. \tag{10}$$

Since the above argument holds for any given (θ_t, z_t), $t \geq 0$, it holds for all (θ_t, z_t), $t \geq 0$. Thus, $\bar{z} = h^*(\theta, z)$ is the unique recursive equilibrium in the environment with commitment. Q.E.D.

In this environment, I_t can be any value in the interval $[0, w_t(\theta_t, z_t)]$. That is, internal fund and external fund are perfect substitutes. Therefore, the labor income of the entrepeneur does not affect the degree of asset specificity adopted. Consequently, a productivity shock that affects the labor income of the young entrepeneurs has no persistent effect in the economy and the only source of output fluctuations is the exogenous aggregate productivity shock.

Finally, since the equilibrium asset specificity level is always $\bar{z}$, the recursive equilibrium is also a stationary equilibrium, i.e., the stationary probability measure μ^* has the property that $\mu^*(\bar{z}) = 1$.

VI. COMPARISONS

To study the implications of the agency problem on the equilibrium outcome, this section compares the long-run behavior of aggregate outputs in the environments with and without commitment. At each date t+1, $t \geq 0$, let y_{t+1} denote the per capita (per member in the generation) output. It consists of outputs from the storage technology and from the investment projects. Given the state variables (θ_t, z_t) at date t and the state variables (θ_{t+1}, z_{t+1}) at date t+1, the per capita output from storage at date t+1 is:

$$y^s_{t+1} = [(1-\alpha) + \alpha f_\ell(1, \theta_t, z_t) - \alpha]R,$$

and the per capita output from investment project at date t+1 is:

$$y^i_{t+1} = \alpha f(1, \theta_{t+1}, z_{t+1}).$$

Thus, the per capita output is:

$$\begin{aligned} y_{t+1} &= y^s_{t+1} + y^i_{t+1} \\ &= [(1-\alpha) + \alpha f_\ell(1, \theta_t, z_t) - \alpha]R + \alpha f(1, \theta_{t+1}, z_{t+1}). \end{aligned} \tag{11}$$

Let $\hat{y}_{t+1}$ denote the per capita output in the environment without commitment and y^*_{t+1} denote the per capita output in the environment with commitment. First, it is easy to see that $E(\hat{y}_{t+1}) < E(y^*_{t+1})$ because $E(\hat{z}) < \bar{z}$. That is, in the long run, the lack of commitment reduces the average per capita output. Second, the propagation effect of the agency problem follows directly from the fact that the equilibrium function $\hat{h}(\cdot,\cdot): \Theta \times Z \to Z$ in the environment without commitment is strictly increasing in its arguments while the equilibrium function $h^*(\cdot,\cdot): \Theta \times Z \to Z$ is independent of its arguments. Third, if the production function $f(\ell,\cdot,\cdot)$ is linear in z, then it is easy to see from equation (11) that the agency problem exacerbates aggregate fluctuations because compared to the environment with commitment, the fluctuations in z in the environment without commitment adds as another source of output fluctuations. However, whether the agency problem exacerbates aggregate fluctuations in general is not directly observable from equation (11).

To study the variance and serial correlation of y_{t+1}, we linearize equation (11) around some $\hat{\theta} \in \Theta$ and $\bar{z} \in Z$[13]. Let $\hat{y}'_{t+1}$ be the linear approximation of y_{t+1} around $(\hat{\theta}, \bar{z})$. To simplify the expressions, the arguments of f, $(1, \hat{\theta}, \bar{z})$, are suppressed in the following equations.

$$\begin{aligned} \hat{y}'_{t+1} &= \left[(1-\alpha) + \alpha f_\ell - \alpha\right]R + \alpha f \\ &+ \alpha\left[(\theta_t - \hat{\theta})f_{\ell\theta} + (z_t - \bar{z})f_{\ell z}\right]R \\ &+ \alpha\left[(\theta_{t+1} - \hat{\theta})f_\theta + (z_{t+1} - \bar{z})f_z\right]. \end{aligned} \tag{12}$$

Since asset specificity varies over time in the environment without commitment, equation (12) is a linear approximation of the per capita output in the environment without commitment. Using the probability measures $(\pi, \hat{\mu})$, the variance and serial covariance of $\hat{y}'_{t+1}$ can be expressed as:

$$\begin{aligned} \frac{\mathrm{var}(\hat{y}'_{t+1})}{\alpha^2} &= \left[R^2 f_{\ell\theta}^2 + f_\theta^2\right]\mathrm{var}(\theta) + \left[R^2 f_{\ell z}^2 + f_z^2\right]\mathrm{var}(\hat{z}) \\ &+ R f_{\ell\theta} f_z \,\mathrm{cov}(\theta_t, z_{t+1}) + f_{\ell z} f_z \,\mathrm{cov}(z_t, z_{t+1}), \end{aligned} \tag{13}$$

$$\frac{\mathrm{cov}(\hat{y}'_{t+1}, \hat{y}'_{t+1-j})}{\alpha^2} = A + \begin{cases} R f_{\ell\theta} f_\theta \,\mathrm{var}(\theta) + f_z f_\theta \,\mathrm{cov}(\theta_t, z_{t+1}) & \\ + R f_z f_{\ell z} \,\mathrm{var}(\hat{z}) & \text{if } j = 1 \\ & \\ R f_{\ell z} f_\theta \,\mathrm{cov}(z_t, \theta_{t+1-j}) + f_z^2 \,\mathrm{cov}(z_{t+1}, z_{t+1-j}) & \\ + f_z f_\theta \,\mathrm{cov}(z_{t+1}, \theta_{t+1-j}) & \text{if } j > 1. \end{cases} \tag{14}$$

where $A = R^2 f_{\ell\theta} f_{\ell z} \,\mathrm{cov}(\theta_{t-j}, z_t) + R f_{\ell\theta} f_z \,\mathrm{cov}(\theta_{t-j}, z_{t+1})$
$\quad + R^2 f_{\ell z}^2 \,\mathrm{cov}(z_t, z_{t-j}) + R f_{\ell z} f_z \,\mathrm{cov}(z_{t+1}, z_{t-j})$.

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[13] We linearize the equation (9) around $\bar{z}$ so that we can compare the results in the two economies (with and without commitment).

Since $z_{t+1} = \hat{h}(\theta_t, z_t)$ where h is strictly increasing in its arguments, all the covariance terms in equations (13) and (14) are strictly positive. Equation (13) shows that the fluctuations in aggregate outputs come from both the productivity shock and the fluctuations in asset specificity. The fact that the aggregate shock has a persistent effect in the economy can be seen from equation (14): the aggregate outputs are serially correlated with various lags.

In the environment with commitment, the degree of asset specificity is constant at $\bar{z}$, therefore, equation (12) collapses to:

$$\begin{aligned} y'^*_{t+1} &= [(1-\alpha) + \alpha f_\ell - \alpha]R + \alpha f \\ &\quad + \alpha R(\theta_t - \hat{\theta}) f_{\ell\theta} + \alpha(\theta_{t+1} - \hat{\theta}) f_\theta \end{aligned} \tag{15}$$

Therefore, the variance and serial covariance of y'^*_{t+1} are given by:

$$\frac{\text{var}(y'^*_{t+1})}{\alpha^2} = \left[R^2 f^2_{\ell\theta} + f^2_\theta\right] \text{var}(\theta), \tag{16}$$

$$\frac{\text{cov}(y'^*_{t+1}, y'^*_{t+1-j})}{\alpha^2} = \begin{cases} R f_{\ell\theta} f_\theta \,\text{var}(\theta) & \text{if } j = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

Equation (16) shows that the only source of aggregate output fluctuations is the aggregate productivity shock. The fact that the aggregate shock has no persistent effect in the economy can be seen from equation (17): the serial correlation of aggregate output exists for a one-period lag only[14].

[14]The productivity shock at date t affects the output of the investment projects at date t and the labor income of the young entrepreneurs at date t. Even though the labor income of the young entrepreneurs does not affect their choice of asset specificity, it does affect the amount of saving through the storage technology. Thus, the productivity shock at date t affects the aggregate outputs of date t and date t+1 only, i.e., has no persistent effect in the economy.

It follows from equations (13), (14), (16) and (17) that $\mathrm{var}(\hat{y}'_{t+1}) > \mathrm{var}(y'^{*}_{t+1})$ and $\mathrm{cov}(\hat{y}'_{t+1-j}, \hat{y}'_{t+1}) > \mathrm{cov}\,(y'^{*}_{t+1-j}, y'^{*}_{t+1})$ for all $j \geq 1$. That is, the presence of the agency problem exacerbates and propagates the aggregate shock. However, since $\hat{y}'_{t+1}$ and y'^{*}_{t+1} are linear approximations, the accuracy of these observations depends on the parameters of the model and the class of functions considered. The following two examples satisfy all of the assumptions of the model and they illustrate that the amplification effect of the agency problem depends on the class of function considered.

Example 1 ($f(\ell, \cdot, \cdot)$ is linear in θ and z)

If the production function $f(\ell, \cdot, \cdot)$ is linear in θ and z, then all the partial derivatives in equations (10)-(17) are independent of θ and z. Hence, equation (13), (14), (16) and (17) gives us the equations to find out the exact values of the moments of the variables. The following is an example where $f(\ell, \cdot, \cdot)$ is linear in θ and z.

$\alpha = 2/5$,

$R = 1$,

θ is uniformly distributed over $[0, 1]$,

$z \in [0, 1]$,

$$f(\ell, \theta, z) = 1 + \ell^{1/2}\left(\frac{1}{2} + \theta + \frac{1}{2}z\right), \qquad \text{and}$$

$$d(z) = \frac{4}{5}(1 - z).$$

In the environment without commitment, for all $t \geq 0$,

$$\hat{z}_{t+1} = \frac{5}{8}\theta_t + \frac{5}{16}\hat{z}_t + \frac{1}{16},$$

$$\hat{y}^s_{t+1} = \frac{1}{5}\theta_t + \frac{1}{10}\hat{z}_t + \frac{3}{10},$$

$$\hat{y}^i_{t+1} = \frac{2}{5}\theta_{t+1} + \frac{1}{5}\hat{z}_{t+1} + \frac{3}{5}, \qquad \text{and}$$

$$\hat{y}_{t+1} = \hat{y}^s_{t+1} + \hat{y}^i_{t+1}.$$

In the environment with commitment, for all $t \geq 0$,

$$z^*_t = 1,$$

$$y^{s*}_{t+1} = \frac{2}{5} + \frac{1}{5}\theta_t,$$

$$y^{i*}_{t+1} = \frac{4}{5} + \frac{2}{5}\theta_{t+1}, \qquad \text{and}$$

$$y^*_{t+1} = y^{s*}_{t+1} + y^{i*}_{t+1}.$$

In both models, the means, variances and co-variances of the variables can be calculated[15].

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[15]In the environment without commitment, by successive substitution of the stochastic difference equation of z_t, we get $z_t = \left(\frac{5}{16}\right)^t z_0 + \frac{5}{8}\sum_{i=0}^{t-1}\left(\frac{5}{16}\right)^i \theta_i + \frac{1}{16}\sum_{i=0}^{t-1}\left(\frac{5}{16}\right)^i$. Taking the limit of the equation, we get $\hat{z} = \lim_{t\to\infty} z_t = \frac{5}{8}\sum_{i=0}^{\infty}\left(\frac{5}{16}\right)^i \theta_i + \frac{1}{16}\sum_{i=0}^{\infty}\left(\frac{5}{16}\right)^i$. Since the stochastic process $\{\theta_t\}_{t=0}^{\infty}$ is i.i.d., $E(\hat{z}) = \frac{5}{8}\sum_{i=0}^{\infty}\left(\frac{5}{16}\right)^i E(\theta_i) + \frac{1}{16}\sum_{i=0}^{\infty}\left(\frac{5}{16}\right)^i$ and $\mathrm{var}(\hat{z}) = \left(\frac{5}{8}\right)^2 \sum_{i=0}^{\infty}\left(\frac{5}{16}\right)^{2i} \mathrm{var}(\theta_i)$. Since we can compute $E(\theta)$ and $\mathrm{var}(\theta)$, we can calculate $E(\hat{z})$ and $\mathrm{var}(\hat{z})$ using the last two equations. Once we know $E(\theta)$, $\mathrm{var}(\theta)$, $E(\hat{z})$ and $\mathrm{var}(\hat{z})$, we can calculate the moments of all the variables.

The results are presented in the following table.

	mean	variance	serial correlation (1-period lag)	serial correlation (2-period lag)
$\hat{z}$	0.5454	0.0361	0.3125	0.0977
z^*	1.0000	0.0000	0.0000	0.0000
$\hat{y}^s$	0.4545	0.0037	0.3108	0.0973
y^{s*}	0.5016	0.0033	0.0000	0.0000
$\hat{y}^i$	0.9091	0.0147	0.3129	0.0977
y^{i*}	1.0033	0.0133	0.0000	0.0000
$\hat{y}$	1.3636	0.0207	0.6711	0.2080
y^*	1.5049	0.0166	0.3916	0.0000

From the above table, we can see that the lack of commitment reduces the average output and exacerbates and propagates aggregate fluctuations.

Example 2 (Cobb-Douglas Production Function)

The following is an example that the presence of the agency problem does not induce a larger variance of the per capita output even though it does propagate the aggregate shock and reduces the average output.

$\alpha = 2/5$,

$R = 3/4$,

θ is uniformly distributed over $[1, 2]$,

$z \in [1, 2]$,

$f(\ell, \theta, z) = \theta \ell^{1/5} z^{4/5}$, and

$d(z) = \frac{3}{4}(2 - z)$.

In the environment without commitment, for all $t \geq 0$,

$$\hat{z}_{t+1} = 1 + \frac{1}{5}\theta_t \hat{z}_t^{4/5},$$

$$\hat{y}^s_{t+1} = \frac{3}{20} + \frac{1}{50}\theta_t \hat{z}_t^{4/5},$$

$$\hat{y}^i_{t+1} = \frac{2}{5}\theta_{t+1}\hat{z}_{t+1}^{4/5}, \qquad \text{and}$$

$$\hat{y}_{t+1} = \hat{y}^s_{t+1} + \hat{y}^i_{t+1}.$$

In the environment with commitment, for all $t \geq 0$,

$$z^*_t = 2,$$

$$y^{s*}_{t+1} = \frac{3}{20} + \frac{2^{4/5}}{50}\theta_t,$$

$$y^{i*}_{t+1} = \frac{2^{9/5}}{5}\theta_{t+1}, \qquad \text{and}$$

$$y^*_{t+1} = y^{s*}_{t+1} + y^{i*}_{t+1}.$$

The moments of the variables in the environment with commitment can be calculated directly. Those in the environment without commitment is obtained by simulating the model with 10,000 observations. The sample statistics are presented in the following table[16].

	mean	variance	serial correlation (1-period lag)	serial correlation (2-period lag)
$\hat{z}$	1.3890	0.0061	0.2459	0.0656
z^*	2.0000	0.0000	0.0000	0.0000
$\hat{y}^s$	0.1889	0.0001	0.0000	0.0000
y^{s*}	0.2024	0.0001	0.0000	0.0000
$\hat{y}^i$	0.7781	0.0245	0.2367	0.0571
y^{i*}	1.0447	0.0404	0.0000	0.0000
$\hat{y}$	0.9670	0.0251	0.2829	0.0677
y^*	1.2471	0.0405	0.0499	0.0000

From the above table, we can see that while the lack of commitment reduces average output and propagates fluctuations, it does not exacerbate output fluctuations.

[16]The environment with commitment is also simulated with 10,000 observations and the statistics are very close to the ones calculated directly.

VII. SUMMARY

This paper considers an intertemporal model in which entrepreneurs can vary asset specificity to adjust the liquidity of their investment projects. The objective is to study how the presence of an agency problem due to the lack of commitment can affect the equilibrium outcome. The main result of the paper is that the lack of commitment can reduce aggregate output and propagate aggregate shocks through asset specificity. However, whether the agency problem exacerbates output fluctuations depends on the parameters of the model and the class of production functions considered. While existing literature has shown that the presence of agency problems can amplify aggregate shocks through the amount of investment undertaken in an economy, this paper shows that the propagation effect of agency problems is robust to the assumption about the entrepreneur's ability to adjust the liquidity of an investment project and to the class of agency problems considered.

APPENDIX

Claim. In both economies (with and without commitment), the equilibrium must have the property that the optimal choice of labor input is an interior solution.

Proof. In the economy with commitment, assumption (A1) implies that liquidation never happens. In the economy without commitment, liquidation never occurs either. If liquidation occurs, then the old entrepreneur gets nothing, the old investor gets the liquidation value of the project, the demand for labor is zero and the equilibrium wage rate must be zero. However, for any given (θ, z), if the project is continued and if 1 unit of labor is hired at the zero wage rate, the profit of the project is $f(1, \theta, z)$ and

$$f(1, \theta, z) > v(\theta, z) > d(z).$$

Then regardless of the negotiation outcome, either the entrepreneur or the investor must be strictly better off while the other is at least as well off as when the project is liquidated. Thus, liquidation cannot be an equilibrium outcome.

Similarly, if the project is continued and if no labor is hired, then the equilibrium wage rate must be zero. Since $f(\ell, \theta, z)$ strictly increasing in ℓ, $\ell = 0$ cannot be an optimal choice in both economies. Q.E.D.

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