ACTIVITY-BASED COSTING FOR

ECONOMIC VALUE ADDED

by

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Abstract

Economic value added (EVA), which is the currently popular term for the traditional accounting concept of residual income, subtracts from operating income an interest charge for invested capital. This paper provides both a normative justification for EVA and an activity-based cost system that supports EVA maximization. We construct a model of participatory budgeting for a multi-activity firm in which we show that, if investment decisions are made myopically each period to maximize EVA, the resulting path of plant and equipment vectors asymptotically approximates the path that maximizes discounted cash flows. The cost system allocates plant and equipment cost to products using a formula that includes the interest charge.

1. Introduction

The allocation of fixed cost, especially plant and equipment cost, is one of the most troublesome problems in all of managerial accounting. If allocated to products, it has the potential to mislead numerous decisions to which it is economically irrelevant. If it is not allocated or charged to users in some fashion, management loses an essential tool for determining the profitability of maintaining or expanding capacity. Two recent trends in managerial accounting, the widespread adoption of activity-based cost systems and the increasing popularity of "economic value-added" as a performance measure, have made the resolution of this traditional dilemma more imperative.

Activity-based costing (ABC) relies on the careful tracing of resource flows within the firm, but its product cost calculations make no distinction between variable and fixed resources. Distinctions among resource costs, if any, are made according to the nature of the demand for resources, rather than the nature of the supply. Thus, if the cost driver for a drill-press is machine hours, for example, then each unit of product will absorb drill-press depreciation in the same linear fashion that it absorbs material and labor cost. As a result, the unit cost of a product may overstate the savings that could be realized by producing fewer units. Firms that replace a simplistic traditional cost system with a more elaborate and costly ABC system in the hope of identifying and dropping unprofitable products may make decisions that cause greater reductions in revenue than cost.

Economic value added (EVA), the currently popular name for the traditional accounting concept of residual income, subtracts from operating income an interest charge for invested capital (Stewart, 1991). When different performance centers share common plant and equipment, some method must be found to allocate the common capital costs. More specifically, the cost system needs to allocate plant and equipment costs to products in a way that embeds the interest charge in the product cost. Otherwise, a performance center manager will not be able to determine whether any given decision will increase or decrease EVA.

The present paper constitutes an ambitious two-front attack on the problem of allocating plant and equipment cost in a general product cost system. First, we provide a normative justification for the use of EVA as a performance measure from the viewpoint of discounted cash flow maximization. Second, we describe an ABC system that includes specific rules for allocating plant and equipment costs, and show how the resulting costs support the maximization of EVA.

Before going further, we should clarify what we mean by "fixed", or "plant equipment" cost. Capital goods are distinguished from other inputs by the fact that they are not consumed by use, but deteriorate over time. A machine that is consumed by use, such as a printer that has a useful life measured in total pages printed, is conceptually no different than an inventory of raw materials, and thus does not require a conceptually different accounting treatment. Inputs that are consumed by the passage of time, independently of use, require a very different method of cost allocation to avoid distorting the causal relation between a product and its cost. Such inputs are the focus of this paper. We will use the term "investment activity" to denote an activity that purchases and installs such an input, say drill-presses. If the use of the input necessitates maintenance expense, or the input requires labor to operate, we will suppose that there is another activity that is responsible for operating the input and incurring those expenses that are caused by its use. Thus the only expense incurred by an investment activity is the purchase and installation cost. It is precisely this cost, or its accounting incarnation as depreciation expense, that is the fixed cost of this paper. In justification of the absence of any distinction between fixed and variable costs in ABC systems, Cooper (1992, p. B1-10) writes, "Expenses are not intrinsically either fixed or variable." We take the opposing view that the expenses of investment activities are intrinsically fixed.

Section 2 gives the formal model of a firm as a collection of productive activities, extending the model in Jordan (1994) to include investment activities. Section 3 provides a normative justification for EVA. Since the term residual income (RI) is more common in the accounting literature, we will refer to RI rather than EVA in the remainder of this paper. If the firm makes investment decisions each period to maximize RI, the resulting sequence of cash flows will typically fail to maximize present value (PV). However, Anctil (1993), shows, in a single-good model, that the sequence of investments converges to the same capital level that is the limit of the PV-maximizing path. Thus residual income maximization is a myopic period-by-period decision rule that asymptotically approximates the outcome of the more complex and farsighted present value maximization rule. Section 3 generalizes Anctil's result to the multi-activity setting of the present model. The specific assumptions needed are discussed at the beginning of Section 3, but one assumption should be mentioned here. In order for RI maximization to approximate PV maximization, it is essential that for each capital input, the depreciation rate used in measuring RI is equal to the rate at which the input deteriorates over time. If the depreciation rate overstates the "deterioration rate", for example, then RI overstates the cost of the input, which can permanently hold investment in the input below the PV-maximizing level.

Section 4 adds investment activity costs to the ABC model constructed by Jordan (1994). This model is significantly different from others in the small but growing theoretical ABC literature, so we should mention its main features. As Noreen (1991) and Christensen and Demski (1992) have argued, ABC costs are true marginal costs if and only if the firm has a Leontief, linear fixed-coefficient technology. In this case, as Banker and Hughes (1994) state, ABC costs provide information that is "economically sufficient" for the complete description of the production technology. In contrast, we do not assume linearity, so ABC costs are not an economically sufficient representation of the firm's technology, and do not equal or reveal the true marginal costs. The imperfection of accounting information forces the consideration of "second-best" optimality criteria, and also leads us to ensure that the model is compatible with the use of any additional information that managers might possess. The ABC costs are used to parameterize performance measures which activity managers, who know the technology of their own activity but know nothing about other activities, seek to maximize in a participatory budgeting process. A plan that maximizes residual income is a budget equilibrium (Proposition 4.15), but there are many other less desirable budget equilibria as well. However, every budget equilibrium has the "second-best" property that residual income could not be improved by dropping any product or out-sourcing any internally produced input (Proposition 4.19 and Remarks 4.20). Moreover, the activity performance measures are "residual income seeking" in the sense that if activity managers possess additional information, engage in ad hoc communication with other managers, or coordinate their budget proposals with other managers, their measured performance will increase if and only if the residual income of the firm increases (Theorem 4.16). This is the "open architecture" property that allows and encourages the use of additional information. Section 4.20 contains a specific discussion of the Banker and Hughes (1994) model and its relation to the present paper.

The asymptotic equivalence of residual income and discounted cash flow maximization and the result that an RI maximum is a budget equilibrium rely on the assumption that, except for fixed costs, the firm's technology is convex. The "second-best" property of budget equilibria uses the weaker assumption that, except for fixed resources, activities can be scaled down linearly without violating technological feasibility. Convexity is a pervasive assumption in managerial accounting theory, but it rules out "step" costs and increasing returns to scale. In the present paper, speaking very loosely, convexity is limited to long-run cost and short-run variable cost. Of course, since the calculation of cost is endogenous to the model, assumptions are stated in terms of technological possibilities.

The specific rule for allocating investment cost is one that we have not seen elsewhere, so we will devote some discussion to it here. The traditional method of fixed cost allocation consists of allocating the total cost over some base that corresponds, at least roughly, to usage of the resource. The cost of a machine, for example, might be allocated over hours of usage. An unfortunate effect of this procedure is that the budgeted "burden rate", or cost per hour, depends on the budgeted hours of use. A modification recommended by Cooper and Kaplan (1992) to remove this effect is to average the cost over the practical capacity, and leave the cost of the unused capacity unallocated. While this modification prevents product managers from being charged for unused resources, it does not go far enough. In a given period, a capital input is available in a fixed amount at a cost that is not merely fixed but sunk (previously paid or obligated). The maximization of residual income, or any other economically natural measure of profit, requires that such an input be utilized up to the point where its marginal contribution is zero, or its practical capacity, whichever is smaller. The allocation of any of the fixed investment cost could artificially hold its utilization below capacity. Accordingly, we will calculate a budgeted "burden rate" based not on the historical cost per unit of practical capacity, but on the unit cost of budgeted gross investment. If there is no gross investment, which means that the capacity is being reduced through depreciation, the burden rate is zero, in order to encourage the full utilization of an economically costless resource.

For example, suppose the investment activity consists of the purchase and installation of drill-presses, and suppose the firm has chosen a 10% (declining balance) depreciation rate for this activity. If the capacity available in this period (in hours of machine time) is k, then the firm estimates that $0.9k$ will be available next period in the absence of any purchases and installations this period. In particular, suppose that 10% of the machines wear out and are scrapped each period. If the usage of machine hours budgeted for next period, say k', exceeds $0.9k$, then some gross investment, say \$I, must be incurred this period. If ρ is the interest rate at which the firm discounts future cash flows, then the hourly charge for using a drill-press will be budgeted at $(\rho + 0.1)[I/(k' - 0.9k)]$ (plus maintenance, operator labor and other usage related expenses). The second factor, $I/(k' - 0.9k)$, is the budgeted purchase and installation cost per unit of added capacity. This is multiplied by 10% because it is depreciated rather than expensed, and also multiplied by ρ to add the interest charge used in calculating residual income. If the usage budgeted for next period is equal to $0.9k$ or less, then no gross investment is budgeted and the hourly charge for using a drill-press is budgeted at zero (plus usage related expenses).

Section 5 discusses the special case of a "stationary state", in which the usage of each capital input is constant over time. and gross investment each period equals depreciation. In the drill-press example, $k = k'$, so $I/(k' - 0.9k) = I/0.1k$. Since gross investment equals depreciation, $I = 0.1K$, where K is the total book-value of capacity. Hence the hourly charge for using a drill-press is budgeted at $(\rho + 0.1)(K/k)$, and since budgeted usage equals k, the budgeted total allocation is $(\rho + 0.1)K$.

Before turning to the formal model, it may be useful to close with two observations on the scope of the concept of investment activity. First, it will *not* be assumed that the firm's technology imposes a fixed ratio between outputs and inputs, which permits considerable flexibility in defining the units of capital goods. In the drill-press example, a one-year old drill-press, after depreciation, constitutes 0.9 equivalent units of a new drill-press. However, with additional maintenence expense, it may produce the same hourly output as a new machine. In other words, maintenence, or some other input, might serve as a substitute for the 0.1 equivalent machine units.

Second, fixed resources other than plant and equipment can also be treated as investment activities. In principle, any resource that, independently of use, is committed for a length of time exceeding the budget period can be capitalized, and the appropriate interest and depreciation charges against residual income can be calculated. A natural example is labor in a firm that has an explicit or implicit long-term committment to its employees. As a more specific illustration, suppose the firm has a pool of employees engaged in order-processing. Each employee is paid $\$25,000$ per year in salary, including benefits. When a new employee is hired, the firm pays $\$500$ to an employment agency and sends the employee to a software training program that costs the firm $\$1300$ in training expenses, including the employee's salary during the training program. For simplicity, suppose the budget period is one-year. If 15% of the order-processing employees leave the firm or are promoted to other positions each year, then 0.15 is an appropriate depreciation rate for the employment fee and training expense, making the annual charge against residual income equal to $(\rho + 0.15)1,800$ in the first year, $(\rho + 0.15)1,530$ in the second year; etc. Since the salary is paid contemporaneously with the usage of the resource, the annual interest charge on the capitalized salary expense is simply $\$25,000$. Depreciation, which in this case is attrition, should not be charged because it reduces the salary liability by the same amount that it reduces the capitalized value. Instead, the "book-value" of the salary committment can be recalculated each period by counting the employees who remain. Gross investment corresponds to hiring new employees. Suppose the firm hires one new employee, who is expected to provide $2,000$ hours of order-processing capacity. Then, by the same method as in the earlier drill-press example, one obtains a budgeted hourly charge for order-processing equal to $[25,000 + (\rho + 0.15)1,800]/2,000$.

2. The Firm This Section models the firm as a collection of productive activities. As in Jordan, (1994), the model includes *cost activities*, which incur cash outflows in order to produce inputs for other activities, and *revenue activities*, which use the goods or services produced by the cost activities in order to produce cash inflows. In addition, the present model includes a special class of cost activities, termed *investment activities*, which incur cash outflows in order to provide capital goods such as those that typically appear in plant and equipment accounts. Since the basic activity model is described and illustrated by examples in Jordan (1994), most of the expository comments in the present paper will be limited to investment activities.

2.1 Plans: There are n activities. The first m activities are *cost activities*, of which the first ℓ are investment activities. The remaining $n-m$ activities are *revenue activities.* An *action* for an activity is a vector $y_i \in \Re^{m+1}$, with coordinates $(y_{i0}, \ldots, y_{im})$. The first component, y_{i0}, denotes a cash flow. If $i \neq j$, $-y_{ij}$ denotes the amount of the output of activity j that activity i uses as an input. For a cost activity i, y_{ii} denotes the amount of output. Each activity i has a *proposed action set* $P^i \subset \Re^{m+1}$. The proposed action sets for the different types of activities are as follows:

i) for $i = 1, \ldots, \ell$, $P^i = \{y_i \in \Re^{m+1} : y_{i0} \leq 0, \quad y_{ii} \geq 0,$ and $y_{ij} = 0$ for all j other than 0 and $i\}$;

ii) for $i = \ell + 1, , \ldots, m$, $P^i = \{y_i \in \Re^{m+1} : y_{i0} \leq 0, \quad y_{ii} \geq 0,$ $y_{i0} < 0$ if $y_{ii} > 0$, and $y_{ij} \leq 0$ for all $j \neq i\}$; and

iii) for $i = m + 1, \ldots, n$, $P^i = \{y_i \in \Re^{m+1} : y_{i0} \geq 0, \quad y_{ij} \leq 0$ for all $j \geq 1$, and $y_{ij} < 0$ for some $j \geq 1$ if $y_{i0} > 0\}$.

Let $P = \Pi_{i=1}^{n} P^i$. A *plan* is an n-tuple of proposed actions $(y_i)_i \in P$. A *sequence of plans* is a sequence $\{(y_i^t)_i\}_{t=1}^{\infty} \subset P$.

2.2 Feasible Plans:

Each activity $i = \ell + 1, \ldots, n$ has a *feasible action set* $Y^i \subset P^i$. Each investment activity $i = 1, \ldots, \ell$ has an *investment outlay function* $I_i : \Re^2_+ \to \Re_+$. Each investment activity i has a *feasible action correspondence* $Y^i : \Re_+ \rightarrow\!\!\!\rightarrow P^i$ defined by:

$$Y^i(x) = \{y_i \in P^i : y_{i0} \leq -I_i(x, y_{ii})\}.$$

A *firm* f consists of investment outlay functions $(I_i)_{i=1}^{\ell}$ and feasible actions sets $(Y^i)_{i=\ell+1}^{n}$. A plan $(y_i)_{i=1}^{n} \in P$ is *feasible for* f *given* $k^o \in \Re^{\ell}_+$ if

i) for each $i = 1, \ldots, \ell, \quad y_i \in Y^i(k^o_i)$;

ii) for each $i = \ell + 1, \ldots, n, \quad y_i \in Y^i$; and

iii) for each $j = 1, \ldots, m, \quad \sum_{i=1}^{n} y_{ij} \geq 0.$

A sequence of plans $\{(y^t_i)_i\}_{t=1}^{\infty}$ is *feasible for* f *given* k^o if for each t, $(y^t_i)_i$ is feasible for f given k^{t-1}, where

iv)
$$k^{t-1} = \begin{cases} k^o & \text{if} \quad t = 1; \quad \text{and} \\ (y^{t-1}_{11}, \ldots, y^{t-1}_{\ell\ell}) & \text{if} \quad t > 1. \end{cases}$$

2.3 Remarks: Capital goods are distinguished from other inputs to by the fact that they are consumed through depreciation rather than use. For this reason, we will refer to the output of an investment activity as *capacity.* For example, the capacity provided by a building might be measured as floor space, while the capacity of a Xerox machine might be measured in pages per month.

Since capacity is reduced by depreciation rather than use, the cash outlay needed by investment activity i to provide y^{t+1}_{ii} units of capacity in period $t+1$ depends on the amount of capacity that was provided in period t. For example, the investment outlay function might take the form

$$I_i(y^t_{ii}, y^{t+1}_{ii}) = h_i[y^{t+1}_{ii} - (1 - d_i)y^t_{ii}],$$

where $h_i > 0$ represents the unit cost of purchasing and installing capacity, and $d_i y_{ii}^t$ measures the amount of capacity lost through depreciation, so that $y_{ii}^{t+1} - (1 - d_i)y_{ii}^t$ is the amount of capacity that must be purchased and installed in order to provide y_{ii}^{t+1}, assuming that $y_{ii}^{t+1} \geq (1 - d_i)y_{ii}^t$.

If y_i is a proposed action by activity i, then for any cost activity $j \neq i$, $-y_{ij}$ is the amount of the output of activity j that activity i proposes to use as input. We will often refer to the output of cost activity j as "commodity j". The sum $\sum_{i=1}^{n} y_{ij}$ is the net supply of commodity j, so the feasibility condition 2.2(iii) states that the total usage of commodity j cannot exceed the amount produced. For an investment activity i, every proposed action $y_i \in P^i$ has $y_{ij} = 0$ for every cost activity $j \neq i$. The need for this restriction is explained in 2.8 below.

The cash flow, y_{i0}, generated by a revenue activity i is not explicitly represented as revenue from the sale of a good or service. While the revenue, y_{i0}, and the cost of the inputs, $-y_{i1}, \ldots, -y_{im}$, used to produce the revenue are economically relevant, the decomposition of revenue into price and quantity of sales is irrelevant. However, in order to obtain a unit cost of the commodity sold by revenue activity i, the quantity of sales must be represented somewhere. For this purpose, one can model the production of the commodity as a cost activity j, so that $-y_{ij}$ is the quantity of sales.

The firm must specify a discount rate $\rho > 0$ in order to evaluate the present value of a sequence of plans. Since residual income involves a charge for depreciation, the firm must specify a depreciation policy for each capital good i. We will assume that the declining balance method is used, with a depreciation rate $\delta_i > 0$. It should be emphasised that except for explicit assumptions stated below, δ_i is an accounting depreciation rate applied to the book-value, K_i, of the capacity k_i, and need not bear any relation to the physical deterioration of capacity reflected by the investment outlay function I_i. In addition, there is an initial book-value, K_i^o, which may reflect historical purchase and installation costs. The following definitions formalize these specifications.

2.4 Capital Policy Parameters: The capital policy parameters for a firm consist of a *discount rate* $\rho > 0$, a *depreciation rate* $\delta_i \geq 0$ for each $i = 1, \ldots, \ell$, and an *initial book-value vector* $K^o \in \Re^{\ell}_{+}$.

2.5 Present Value: Given a discount rate $\rho > 0$, the *present value* of a sequence of plans $\{(y_i^t)_i\}_{t=1}^{\infty}$ is:

$$\sum_{t=1}^{\infty}(1+\rho)^{-t}\left[\sum_{i=1}^{\ell} y_{i0}^{t+1} + \sum_{i=\ell+1}^{n} y_{i0}^{t}\right] + \sum_{i=1}^{\ell} y_{i0}^{1}.$$

A sequence of plans is *present value maximizing for a firm* f *given* $k^o \in \Re^{\ell}_{+}$ if it is feasible for f *given* k^o and if there is no other sequence of plans that is feasible for f *given* k^o and has a higher present value.

2.6 Remarks: The definition of present value indicates our timing convention that the investment outlay occurs in the period before the capacity is available for use. Thus, if $y_i^t \in Y_i(k_i^{t-1})$ then the cash outlay $-y_{i0}^t$ occurs in period $t-1$, and the capacity supply y_{ii}^t occurs in period t. Of course, if the committed cash outlays span several periods, then $-y_{i0}^t$ represents the period $t-1$ discounted value of the cash outlay stream. For all activities $i \geq \ell$, the cash and commodity flows specified by a proposed action y_i^t all occur in period t. This timing convention is also reflected in the following definition of residual income.

2.7 Residual Income: Given a discount rate $\rho > 0$, a depreciation rate δ_i for each $i = 1, \ldots, \ell$, and *initial book-value vector* $K^o \in \Re^{\ell}_{+}$, the *residual income* of a plan is defined as

$$\sum_{i=\ell+1}^{n} y_{i0} - \sum_{i=1}^{\ell}(\rho + \delta_i)K_i$$

where

$$K_i = (1-\delta_i)K_i^o - y_{i0} \quad \text{for each} \quad i = 1, \ldots, \ell.$$

A plan $(y_i)_i$ is *residual income maximizing for a firm* f *given* $k^o \in \Re^\ell_+$ if it is feasible for f given k^o and there is no other plan that is feasible for f given k^o and has a higher residual income. A sequence of plans $\{(y_i^t)_i\}_{t=1}^\infty$ is *residual income maximizing for* f *given* k^o if it is feasible for f given k^o and for each $t = 1, \ldots,$ the plan $(y_i^t)_i$ is residual income maximizing for f given k^{t-1}, as defined in 2.2(iv), and given the initial book-value vector K^{t-1}, where, for each $t \geq 0$ and each $i = 1, \ldots, \ell$,

$$K_i^t = \begin{cases} K_i^o & \text{if} \quad t = 0; \quad \text{and} \\ (1-\delta_i)K_i^{t-1} + y_{i0}^t & \text{if} \quad t \geq 1. \end{cases}$$

2.8 Remarks: It is not possible to define the present value of a plan $(y_i^t)_i$ without reference to future plans $(y_i^{t+s})_i$, $s = 1, \ldots$. In contrast, the residual income of a plan $(y_i^t)_i$ requires no forecast of future actions, and no knowledge of future business conditions. However, the myopic nature of residual income depends on our assumption that investment activities do not use the outputs of any other cost activities. For example, a natural violation of our assumption would be a construction firm that constructs its own warehouse. In this case, a decision in period t to construct additional warehouse space for period $t+1$ must be anticipated in period $t-1$ in order to provide the equipment needed in period t. The same problem can arise indirectly if an investment activity uses a cost activity that uses an investment activity. Our definition of the investment outlay function I_i implicitly assumes that the capacity to be made available in period t can be decided in period $t-1$. This assumption can be interpreted as an implicit definition of the minimal budget period for which residual-income is defined. Such an assumption is needed because residual income maximization is obviously inconsistent with any expenditure that produces no benefits until after the residual income horizon.

2.9 Other Fixed Costs: Many costs that are typically viewed as fixed are not associated with committed resources. If lights can be turned off in unused areas of a factory, for example, then factory lighting is not a committed resource, even though it does not vary directly with production volume. In this case, factory lighting can be treated as a noninvestment cost activity that, along with the investment activity, "available factory floor space", serves as an input to the noninvestment cost activity, "useable factory floor space". Thus, the inclusion of investment activities in the model of the firm also makes possible the inclusion of a broad spectrum of noncapital fixed costs that are incurred along with capital investments in the provision of useable capacity.

3. The asymptotic equivalence of residual income and present value maximization

This section provides the normative justification for residual income maximization (for brevity, the acronym PVM (resp. RIM) will be used for both the noun, present value maximization (resp. residual income maximization) and the adjective, present value maximizing (resp. residual income maximizing)). PVM requires infinite foresight, while RIM looks but one period ahead. As a result, the two objectives typically produce different investment decisions in every period. Nonetheless, under the assumptions described below, RIM and PVM paths approach identical long-run limits. This result, which is the main objective of this section, justifies the interpretation of RIM as a practical approximation to the PVM ideal.

Residual income involves a charge for accounting depreciation, so we will need to assume that accounting depreciation is an accurate measure of the physical deterioration of capacity. More precisely, we will assume that for each investment activity i, if k_i^t is the available capacity in period t, then $(1-\delta_i)k_i^t$ will be costlessly available in period $t+1$. To simplify the analysis, we will ignore the possibility of scrapping capacity and limit disinvestment to depreciation by imposing, as a feasibility condition, that $k_i^{t+1} \geq (1-\delta_i)k_i^t$. We will also assume that the purchase and installation of additional capacity, above $(1-\delta_i)k_i^t$, requires an outlay of h_i dollars per unit. The unit cost, h_i, which will be assumed positive, affects the desired capacity, but does not prevent the firm from moving to the desired capacity at once, in which case the main result of this Section holds trivially. In order to make the result nontrivial, we will include in the investment outlay a cost of expansion, $e_i(k_i^{t+1} - k_i^t)$, so that the investment outlay function takes the form

$$I_i(k_i^t, k_i^{t+1}) = h_i\left[k_i^{t+1} - (1 - \delta_i)k_i^t\right] + e_i(k_i^{t+1} - k_i^t).$$

The expansion cost function, which was introduced in an optimizing model of business investment by Eisner and Strotz (1963) (see also Sargent (1979, Chapter VI)), represents the premium paid to increase capacity by the amount $k_i^{t+1} - k_i^t$ in a single period, rather than spreading the increase over the indefinite future. For example, let $e_i(k_i^{t+1} - k_i^t) = a_i(k_i^{t+1} - k_i^t)^2$ for $k_i^{t+1} \geq k_i^t$, where a_i is a positive constant. Since we have limited disinvestment to depreciation, we will assume that $e_i(k_i^{t+1} - k_i^t) = 0$ if $k_i^{t+1} \leq k_i^t$, so that there is no premium for adjusting capacity downward. This specification of the investment outlay function is stated formally as Assumption I (3.2), and the requirement that $k_i^{t+1} \geq (1 - \delta_i)k_i^t$ is imposed in the definitions of RIM (3.10) and PVM (3.16).

Anctil (1994) proves the asymptotic equivalence of PVM and RIM in a two-activity model with a single capital good. Anctil also assumes that net revenue is a strictly concave differentiable function of the single capacity level, and that the expansion cost function is differentiable and strictly convex. Anctil's proof of asymptotic equivalence exploits the one-dimensional nature of the investment dynamics in her model. In the present multi-activity model, the number of capital goods is arbitrary, and the net revenue obtainable from any vector of capacities is not a primitive of the model, but must be derived as the solution of a constrained maximization problem involving the feasible sets $Y^i, i = l+1, \ldots, n$. Thus the proof given below is necessarily quite different from Anctil's. The result proved here is much more general, but is weaker in one important respect. Anctil proves that an RIM path converges to the long-run steady-state at an exponential rate, assuming that the net revenue function has a strictly negative second derivative. The more abstract setting of the present model does not seem conducive to rate of convergence results. This issue is discussed in more detail at the end of this section.

The proof of asymptotic equivalence is quite lengthy, so it may be useful to begin with an informal overview. Both PVM and RIM require, within each period, that cost and revenue activity vectors be chosen to maximize short-run net revenue subject to the current capacity vector. Accordingly, the first order of business is to obtain a well-behaved net revenue function, r, that gives the maximum short-run cash flow, $r(k^t)$, obtainable from any capacity vector k^t. This is accomplished by Proposition 3.5 and Lemma 3.6.

The assumptions used to prove that r is well-defined and continuous (Assumptions 3.2) suffice to carry us almost to the final result. We assume that each activity set Y^i is closed (Assumption C), that each Y^i satisfies the down-scaling assumption of Jordan (1994) (Assumption D), and that no cost activity can produce positive output without some cash expenditure (Assumption N). The down-scaling assumption, which states that any activity vector that is technologically feasible remains feasible if it is reduced in scale, is included here largely because of its pervasive role in later sections of this paper. However, by including it here, we are able to weaken the boundedness assumption needed to ensure the existence of a short-run profit maximum. In particular, we assume only that for each revenue activity, average revenue approaches zero at infinite scale (Assumption B). We also include in Assumption C the assumption that each expansion cost function is continuous, which is needed to prove the existence of present value and residual income maxima.

The next stage of the proof consists of parallel but separate analyses of RIM and PVM paths. Taking RIM first, we show that there is a well-behaved function v_r with the Lyapunov property that along an RIM path, $v_r(k^t)$ is monotonically increasing. The function v_r plays the role of a Lyapunov function in the proof that $k^t \to S_r$, where S_r denotes the set of capacity vectors that are steady-states (remain stationary) under RIM (Proposition 3.15). (Like many convergence proofs in economic theory, our proof roughly follows Lyapunov's Second Method (e.g., Lefschetz (1977) and Lucas and Stokey (1989), from which the term *Lyapunov function* originates.) The analysis of PVM paths is more intricate because the object of choice is the entire infinite sequence of capacity vectors. Nonetheless, there is a function v_p that serves as a Lyapunov function proving that $k^t \to S_p$, where S_p is the set of steady-states under PVM (Proposition 3.30).

The final stage consists of the proof that $S_r = S_p$. It is shown that $\rho v_p(\cdot) \geq v_r(\cdot)$ (Proposition 3.31) and that equality holds on S_p, which helps to prove that $M_r \subset S_p \subset S_r$ (Propositions 3.32 and 3.35), where M_r denotes the set of capacity vectors that maximize v_r. The final step, showing that $S_r \subset M_r$ (Proposition 3.37), requires us to assume that the sets Y^i are convex and that the marginal expansion cost is zero at zero $(De_j(0) = 0)$. The need for these two additional assumptions is discussed following Proposition 3.35. Theorem 3.39 is the formal statement of asymptotic equivalence.

The mathematical tools used below are conventional in the economic theory of optimization (e.g., Debreu (1959, Chapter 1) or Border (1985, Chapters 11 and 12)). As in Debreu (1959), the maximum norm is the most convenient for calculations.

3.1 Definitions: On any Euclidean space $\Re^p$, the norm $||\cdot|| : \Re^p \to \Re_+$ that will be used throughout this paper is the *maximum norm*, $||x|| = \max_j |x_j|$. For each $x \in \Re^p$ and each nonempty set $S \subset \Re^p$, define $d(x, S) = \inf\{||x - y|| : y \in S\}$.

3.2 Assumptions: The following assumptions are imposed throughout this section.

B (Boundedness): For each revenue activity $i = m+1, \dots, n$, and each sequence $\{y_i^s\}_{s=1}^{\infty} \subset Y^i$, if $||y_i^s|| \to \infty$ then $\limsup_{s \to \infty} y_{i0}^s \,/\, ||y_i^s|| = 0$.

C (Continuity): For each activity $i = \ell+1, \dots, n$, Y^i is closed, and for each investment activity $i = 1, \dots, \ell$, I_i is continuous.

D (Down-scaling): For each activity $i = \ell+1, \dots, n$, $\lambda y^i \in Y^i$ for every $y^i \in Y^i$ and every $0 \leq \lambda \leq 1$.

N (No free production): For each cost activity $i = \ell+1, \dots, m$, if $y_i \in Y^i$ and $y_{ii} > 0$ then $y_{i0} < 0$. For each revenue activity $i = m+1, \dots, n$, if $y_i \in Y^i$ and $y_{i0} > 0$, then $y_{ij} < 0$ for some $j = 1, \dots, m$.

I (Investment outlay specification): For each investment activity $i = 1, \ldots, \ell$, $\delta_i > 0$ and there is a number $h_i > 0$ and an *expansion cost function* $e_i : \Re \to \Re^+$ such that $e_i(x) = 0$ for all $x \leq 0$, and for each $(k_i, k_i') \in \Re^2_+$,

$$I_i(k_i, k_i') = h_i\left[k_i' - (1 - \delta_i)k_i\right]^+ + e_i(k_i' - k_i),$$

where $[\cdot]^+$ denotes $max\{[\cdot], 0\}$.

The following lemma establishes the boundedness needed to ensure that solutions exist to the residual income and present value maximization problems.

3.3 Boundedness Lemma: Let $\{k^s\}_{s=1}^{\infty} \subset \Re^{\ell}_+$ and let $\{(y_i^s)_{i=\ell+1}^{n}\}_{s=1}^{\infty} \subset \Pi_{i=\ell+1}^{n} P^i$ satisfy, for each s,

i) $\quad y_i^s \in Y^i$ for each i;

ii) $\quad k_j^s + \sum_{i=\ell+1}^{n} y_{ij}^s \geq 0$ for each $j = 1, \ldots, \ell$; and

iii) $\quad \sum_{i=\ell+1}^{n} y_{ij}^s \geq 0$ for each $j = \ell + 1, \ldots, m$.

For each s, let $z^s = \sum_{i=\ell+1}^{n} y_{i0}^s$. If for some cost activity i^0, $y_{i^0 i^0}^s \,/\, \max\{1, ||k^s||\} \to \infty$, then $z^s \to -\infty$.

Proof: Suppose that for some i^0, $y_{i^0 i^0}^s \,/\, \max\{1, ||k^s||\} \to \infty$. The feasibility condition (ii) implies that, taking a subsequence if necessary, we can choose i^0 so that, $y_{i^0 i^0}^s \geq |y_{ij}^s|$ for all $i > \ell$ and all $j \geq 1$. Reunumbering activities if necessary, suppose that $i^0 = m$. By Assumption B, $y_{i0}^s \,/\, y_{mm}^s \to 0$ for each $i > m$. Therefore $\limsup z^s \,/\, y_{mm}^s = \limsup(\sum_{i=\ell+1}^{m} y_{i0}^s) \,/\, y_{mm}^s \leq \limsup y_{m0}^s \,/\, y_{mm}^s$. If $\limsup y_{m0}^s \,/\, y_{mm}^s < 0$, then since $y_{mm}^s \to \infty$, $z^s \to -\infty$. Suppose by way of contradiction that $\limsup y_{m0}^s \,/\, y_{mm}^s = 0$.

For each s, let $y'^s_m = (1/y^s_{mm})y^s_m$. By Assumption D, $y'^s_m \in Y^m$ for each s. Taking a subsequence if necessary, $y'^s_m \to y^*_m$ for some $y^*_m \in \Re^{m+1}$. By Assumption C, $y^*_m \in Y^m$. However, by the definition of y'^s_m, $y^*_{mm} = 1$ and $y^*_{m0} = 0$, which contradicts Assumption N. ∎

3.4 Definition: Let $r : \Re^\ell_+ \to \Re$ be defined as $r(k) = \max\{\sum_{i=\ell+1}^n y_{i0}$:

i) $\quad y_i \in Y^i$ for each i;

ii) $\quad k_j + \sum_{i=\ell+1}^n y_{ij} \geq 0$ for each $j = 1, \ldots, \ell$; and

iii) $\quad \sum_{i=\ell+1}^n y_{ij} \geq 0$ for each $j = \ell + 1, \ldots, m\}$.

3.5 Proposition: Under Assumptions 3.2, r is a well-defined continuous function.

Proof: Let $a > 0$ and let $K^a = \{k \in \Re^\ell_+ : ||k|| \leq a\}$. We will show that r is well-defined and continuous on K^a. Let $Y = \Pi_{i=\ell+1}^n Y^i$, and define the correspondence $f : K^a \to\to Y$ by $f(k) = \{y \in Y$:

i) $\quad k_j + \sum_{i=\ell+1}^n y_{ij} \geq 0$ for each $j = 1, \ldots, \ell$;

ii) $\quad \sum_{i=\ell+1}^n y_{ij} \geq 0$ for each $j = \ell + 1, \ldots, m$; and

iii) $\quad \sum_{i=\ell+1}^n y_{i0} \geq 0\}$.

By Assumption D, with $\lambda = 0$, $0 \in Y^i$ for each i, so $f(k) \neq \emptyset$ for all k. Therefore $r(k) = \max\{\sum_{i=\ell+1}^n y_{i0} : y \in f(k)\}$ for each $k \in K^a$. By Assumption C, f has a closed graph. Because of the constraint (iii) in the definition of f, Assumption B and the

Boundedness Lemma imply that there is a compact subset $Y^c \subset Y$ such that the graph of f is contained in $K^a \times Y^c$. Therefore f is upper semi-continuous (Debreu (1959, p. 17)). In particular, for each $k \in K^a$, $f(k)$ is compact, so $r(k)$ is well-defined. It remains to show that f is lower semi-continuous (Debreu (1959, p. 17)). Let $\{k^s\}_{s=1}^{\infty}$ be a sequence in K^a converging to some k, and let $y \in f(k)$. For each s, define $\lambda^s = \max\{\lambda : \lambda \leq 1$ and $\lambda y \in f(k^s)\}$, and let $y^s = \lambda^s y$. By Assumption D, λ^s is well-defined for each s. For each s, $\lambda y \in f(k^s)$ if and only if λy satisfies (i) above for k^s. Since $k^s \to k$, $\lambda^s \to 1$, so $y^s \to y$. Thus f is lower semi-continuous; therefore r is continuous on K^a by the Berge Maximum Theorem (Debreu (1959)). Since a is arbitrary, the result follows. ∎

3.6 Lemma: For any sequence $\{k^s\}_{s=1}^{\infty} \subset \Re_+^{\ell}$ with $||k^s|| \to \infty$, $r(k^s) \;/\; ||k^s|| \to 0$.

Proof: Assumption D with $\lambda = 0$ implies that $r(k) \geq 0$ for all $k \in \Re_+^{\ell}$. If $\limsup r(k^s) < \infty$ the result is immediate, so suppose that, taking a subsequence if necessary, $r(k^s) \to \infty$. For each s, let $(y_i^s)_{i=\ell+1}^{n}$ achieve the maximum that defines $r(k^s)$. Assumption B, together with the feasibility constraint (iii) in the definition of $r(k^s)$ (Definition 3.4), implies that for some cost activity i, $r(k^s) \;/\; y_{ii}^s \to 0$. The Boundedness Lemma implies that $\limsup y_{ii}^s \;/\; ||k^s|| < \infty$, so $r(k^s) \;/\; ||k^s|| \to 0$. ∎

We now turn to the analysis of RIM. Residual income is simply net revenue minus a charge for capital. The capital charge is based on the book-value of capital, which involves past prices of capital goods. The Lyapunov function v_r mentioned above is obtained by valuing previously acquired capital at the current prices h_j rather than past prices. The valuation of previously acquired capital at current prices is necessary to ensure that the maximized value of residual income is increasing along an RIM path. Otherwise, since initial book values are arbitrary, the depreciation of previously acquired capital can have an arbitrary effect on residual income. Proposition 3.8 shows that, since the revaluation pertains only to previously acquired capital, it does not affect the RIM capacity vectors.

3.7 Definitions: Define $I : \Re_+^{2\ell} \to \Re$ by $I(k,k') = \sum_{j=1}^{\ell} I_j(k_j, k_j')$. Also define $w : \Re_+^{2\ell} \to \Re$ by $w(k,k') = r(k') - \sum_{j=1}^{\ell}(\rho + \delta_j)\left[h_j(1-\delta_j)k_j + I_j(k_j,k_j')\right]$, and define $v_r : \Re_+^{\ell} \to \Re$ by $v_r(k) = \max\{w(k,k') : k_j' \geq (1-\delta_j)k_j \text{ for all } j = 1,\ldots,\ell\}$.

3.8 Proposition: Let $k, k^*, K \in \Re_+^{\ell}$. Then k^* maximizes

$$(*) \qquad r(k') - \sum_{j=1}^{\ell}(\rho+\delta_j)\left[(1-\delta_j)K_j + I_j(k_j,k_j')\right]$$
$$\text{subject to} \quad k_j' \geq (1-\delta_j)k_j \quad \text{for all} \quad j$$

if and only if k^* maximizes $w(k,k')$ subject to the same constraint.

Proof: The expressions $(*)$ and $w(k,k')$ are identical except that the former contains the term $-\sum_{j=1}^{\ell}(\rho + \delta_j)(1 - \delta_j)K_j$ while the latter contains the term $-\sum_{j=1}^{\ell}(\rho+\delta_j)h_j(1-\delta_j)k_j$, neither of which depend on k'. ∎

3.9 Proposition: The function w is continuous and v_r is well-defined and continuous.

Proof: The continuity of w follows from Proposition 3.5 and Assumption C, which ensures the continuity of each I_j.

Let $a > 0$ and let $K^a = \{k \in \Re_+^{\ell} : k_j \leq a \text{ for all } j\}$. We will show that v_r is well-defined and continuous on K^a. For each $k \in K^a$, $v_r(k) \geq w(k,k) = r(k) - \sum_{j=1}^{\ell}(\rho+\delta_j)h_jk_j \geq -a\sum_{j=1}^{\ell}(\rho+\delta_j)h_j$, where the last inequality follows from the fact that $r(k) \geq 0$. Also, for each j, $I_j(k_j,k_j') \geq h_j \max\{0, k_j' - k_j\}$, so Lemma 3.6 implies that there is some $a' > a$ such that for every $k \in K^a$, $\left\{k' \in \Re_+^{\ell} : w(k,k') \geq -a\sum_{k=1}^{\ell}(\rho+\delta_j)h_j\right\} \subset \{k' \in \Re_+^{\ell} : k_j \leq a'\}$. Therefore, for every $k \in K^a$, $v_r(k) = \max\left\{w(k,k') : (1-\delta_j)k_j \leq k_j' \leq a' \text{ for all } j\right\}$. The Berge Maximum Theorem (Debreu, (1959, p. 19)) then implies that v_r is well-defined and continuous on K^a. Since a is arbitrary, the result follows. ∎

3.10 Definition: A sequence $\{k^t\}_{t=0}^{\infty} \subset \Re_+^{\ell}$ is a *residual income maximizing* (RIM) sequence if for each $t \geq 1$, $k_j^t \geq (1-\delta_j)k_j^{t-1}$ for all j, and $w(k^{t-1},k^t) = v_r(k^{t-1})$.

3.11 Lemma: For every $k \in \Re^{\ell}_{+}$ and every $k' \in \Re^{\ell}_{+}$ satisfying $k'_j \geq (1 - \delta_j)k_j$ for each j, $w(k,k') \leq r(k') - \sum_{j=1}^{\ell}(\rho + \delta_j)h_j k'_j = w(k',k')$.

Proof: For each j, $e_j(k'_j - k_j) \geq 0$, so $I_j(k_j, k'_j) \geq h_j \max\{0, k'_j - k_j\}$. The result now follows from the definition of w. ∎

The following proposition establishes the Lyapunov property of v_r.

3.12 Proposition: If $\{k^t\}_{t=0}^{\infty}$ is an RIM sequence then for each t, $v_r(k^{t+1}) \geq v_r(k^t)$.

Proof: For each t, $v_r(k^{t+1}) \geq w(k^{t+1}, k^{t+1}) \geq w(k^t, k^{t+1}) = v_r(k^t)$, where the first inequality follows from the definition of v_r, the second from Lemma 3.11, and the final equality from the definition of an RIM sequence. ∎

3.13 Lemma: For every real number a, the set $\{k \in \Re^{\ell}_{+} : v_r(k) \geq a\}$ is compact.

Proof: That the set is closed follows from the continuity of v_r (Proposition 3.9). Lemmas 3.11 and 3.6 imply that the set is bounded. ∎

3.14 Definitions: Let $S_r = \{k \in \Re^{\ell}_{+} : v_r(k) = w(k,k)\}$.

3.15 Proposition: If $\{k^t\}_{t=0}^{\infty}$ is an RIM sequence then $d(k^t, S_r) \to 0$.

Proof: By Lemma 3.13 and Proposition 3.12, the sequence $\{k^t\}_{t=0}^{\infty}$ is bounded. Therefore it suffices to show that if k^* is a cluster point of the sequence then $k^* \in S_r$. Choosing a subsequence $\{k^s\}_{s=0}^{\infty}$, if necessary, we can assume that $k^s \to k^*$. By Lemma 3.11, for each s,

1) $$v_r(k^{s-1}) \leq w(k^s, k^s) \leq v_r(k^s).$$

By the continuity of v_r and w (Proposition 3.9), $w(k^s, k^s) \to w(k^*, k^*)$ and $v_r(k^s) \to v_r(k^*)$. By Lemma 3.13, there is some real number v^* such that $v_r(k^{s-1}) \to v^*$ and $v_r(k^s) \to v^*$. Hence by (1), $v^* = w(k^*, k^*) = v_r(k^*)$, which proves that $k^* \in S_r$. ∎

Proposition 3.15 states that all RIM paths converge to the set of RIM-stationary capacity vectors, which completes our analysis of RIM dynamics. We now turn to the parallel, but unfortunately somewhat more intricate analysis of PVM dynamics. We first define the Lyapunov function v_p mentioned above by subtracting the value of previously acquired capital, at current prices, from the net present value of current and future cash flows. As in the case of residual income maximization, this transformation eliminates the valuation effect of a large initial endowment which is not profitable to maintain.

PVM requires choosing the entire sequence of capital vectors at once, which makes the existence and good behavior of maxima dependent on ruling out unbounded sequences. We will take the approach of imposing an artificial upper bound, a, on the size of the capital vector that can be chosen for any period, and showing that the bound can be made large enough to make the set of artificially constrained PVM paths identical to the set of true PVM paths.

3.16 Definitions: For each $k^0 \in \Re^\ell_+$, define $v_p(k^0) = \max\left\{ \sum_{t=1}^{\infty} (1+\rho)^{-t}\left[r(k^t) - I(k^t, k^{t+1})\right] - I(k^0, k^1) - \sum_{j=1}^{\ell} h_j(1-\delta_j)k_j^0 : k_j^{t+1} \geq (1-\delta_j)k_j^t \text{ for each } t \geq 1 \text{ and each } j\right\}$. For each $a > 0$, define $K^a = \left\{k \in \Re^\ell_+ : k_j \leq a \text{ for all } j\right\}$, and for each $k^0 \in K^a$, define $v_p^a(k^0)$ by imposing the additional constraint that $k^t \in K^a$ for all $t \geq 1$ on the maximum that defines $v_p(k^0)$. A sequence $\{k^t\}_{t=0}^{\infty} \subset \Re^\ell_+$ is a *present value maximizing* (PVM) *sequence* if the sequence $\{k^t\}_{t=1}^{\infty}$ solves the constrained maximization problem that defines $v_p(k^0)$. Given $a > 0$, if $||k^0|| \leq a$ and $\{k^t\}_{t=1}^{\infty}$ solves the constrained maximization problem that defines $v_p^a(k^0)$, then $\{k^t\}_{t=0}^{\infty}$ is called a *PVM^a sequence.*

3.17 Proposition: For each $k^0 \in \Re^\ell_+$, a sequence $\{k^{*t}\}_{t=1}^{\infty}$ maximizes

$$(*) \qquad \sum_{t=1}^{\infty} (1+\rho)^{-t}\left[r(k^t) - I(k^t, k^{t+1})\right] - I(k^0, k^1)$$

$$\text{subject to} \quad k_j^t \geq (1-\delta_j)k_j^{t-1} \quad \text{for all} \quad t \geq 1 \quad \text{and all} \quad j$$

if and only if the sequence achieves the constrained maximum that defines $v_p(k^0)$. Moreover, if $a > 0$ and $||k|| \leq a$, then $\{k^{*t}\}_{t=1}^{\infty}$ maximizes (*) subject to $k_j^t \geq (1-\delta_j)k_j^{t-1}$ for all $t \geq 1$ and all j and the additional constraint that $k^t \in K^a$ for all $t \geq 1$ if and only if the sequence achieves the constrained maximum that defines $v_p^a(k^0)$.

Proof: The maximand in the definitions of $v_p(k^0)$ and $v_p^a(k^0)$ differs from (*) only by including the term $-\sum_{j=1}^{\ell} h_j(1-\delta_j)k_j^0$, which is independent of k^t for all $t \geq 1$. ∎

3.18 Lemma: For each $a > 0$ and each PVM^a sequence $\{k^t\}_{t=0}^{\infty}$, the sequence $\{k^{t+s}\}_{t=0}^{\infty}$ is also a PVM^a sequence for every positive integer s.

Proof: This follows directly from Proposition 3.17. ∎

3.19 Proposition: For each $a > 0$, the function $v_p^a : K^a \to \Re$ is well-defined and continuous.

Proof: By Proposition 3.5, r is continuous, and Assumption C ensures that each I_j is continuous. Therefore the function $(k, k') \mapsto r(k) - I(k, k')$ is a continuous function on the compact set $K^a \times K^a$. The result now follows from the fact that $(1+\rho) > 1$ and the Berge Maximum Theorem (Debreu (1959, p. 19)). ∎

3.20 Lemma: For each $a > 0$, let $r_a = r(a, \ldots, a)$. Then for each $a > 0$ and each $k \in K^a$,

$$v_p^a(k) \leq r_a \,/\, \rho - \sum_{j=1}^{\ell} h_j(1-\delta_j)k_j.$$

Proof: Since r is nondecreasing, $r(k) \leq r_a$ for all $k \in K^a$. The result now follows directly from the definition of $v_p^a(k)$. ∎

We want to establish the Lyapunov property of v_p^a. In order to do this we define a function v_{p-}^a and show that its value at k^{t+1} always falls between $v_p^a(k^t)$ and $v_p^a(k^{t+1})$ along the PVM path. The function v_{p-}^a is the same as v_p^a except that it restricts the firm to remain at its initial capacity level for one period rather than moving to k^1 on the PVM path. The effect of this restriction is that each step along that path occurs one period later. The proof of the Lyapunov property follows from the fact that if the firm were to have k^1 on the PVM path initially, then staying at k^1 an extra period is better (weakly) than going from k^0 to k^1, but not as profitable as being allowed to move to the PVM k^2.

3.21 Definition: For each $a > 0$ and each $k \in K^a$, define $v^a_{p^-}(k) = \sum_{t=1}^{\infty} (1+\rho)^{-t}$ $[r(k^{t-1}) - I(k^{t-1}, k^t)] - \sum_{j=1}^{\ell} h_j k_j$, for any PVM^a sequence $\{k^t\}_{t=0}^{\infty}$ with $k^0 = k$.

3.22 Lemma: Let $a > 0$ and let $\{k^t\}_{t=0}^{\infty}$ be a PVM^a sequence. Then for each t

$$v^a_p(k^t) \leq v^a_{p^-}(k^{t+1}) \leq v^a_p(k^{t+1}).$$

Proof: By Lemma 3.18, it suffices to prove the result for $t = 0$. Since $\sum_{j=1}^{\ell} h_j k^1_j \leq \sum_{j=1}^{\ell} h_j (1-\delta_j) k^0_j + I(k^0, k^1)$, it follows from the definitions of v^a_p and $v^a_{p^-}$ that $v^a_p(k^0) \leq v^a_{p^-}(k^1)$. By Lemma 3.18, the sequence $\{k^{t+1}\}_{t=1}^{\infty}$ achieves the constrained maximum that defines $v^a_p(k^1)$. However, $v^a_{p^-}(k^1)$ is obtained by evaluating the maximand in the definition of $v^a_p(k^1)$ at the sequence $\{k^t\}_{t=1}^{\infty}$, so $v^a_{p^-}(k^1) \leq v^a_p(k^1)$. ∎

3.23 Lemma: Let $a^o > 0$. Then there is some $a^* > a^o$ such that for every $k \in K^{a^o}$, $v^a_p(k) = v^{a^*}_p(k)$ for every $a > a^*$.

Proof: If not, there is an increasing sequence $\{a_s\}_{s=1}^{\infty}$ with $a_1 > a^o$ and $a_s \to \infty$, and a sequence $\{k^s\}_{s=1}^{\infty} \subset K^{a^o}$ with the property that $v^{a_s}_p(k^s) > v^{a_s - 1}_p(k^s)$ for each s. It follows from the definition of v^a_p that for each s, there is a PVM$^{a_s+1}$ sequence $\{k^{st}\}_{t=0}^{\infty}$ with $k^{s0} = k^s$ and for some t_s, $||k^{st_s}|| > a_s - 1$. By Lemma 3.22, $v^{a_s}_p(k^{st_s}) \geq v^{a_s}_p(k^s) \geq v^{a^o}_p(k^s)$. Since $v^{a^o}_p$ is continuous and K^{a^o} is compact, it follows that $\liminf_{s\to\infty} v^{a_s}_p(k^{st_s}) \geq \liminf_{s\to\infty} v^{a^o}_p(k^s) > -\infty$. However, Lemmas 3.20 and 3.6 imply that $v^{a_s}_p(k^{st_s}) \to -\infty$, a contradiction. ∎

3.24 Lemma For each $a > 0$ and each $k \in K^a$,

$$v_{p^-}^a(k) = (1+\rho)^{-1} v_p^a(k) + (1+\rho)^{-1} r(k) + (1+\rho)^{-1} \sum_{j=1}^{\ell} h_j(1-\delta_j)k_j$$
$$- \sum_{j=1}^{\ell} h_j k_j$$

In particular, $v_{p^-}^a : K^a \to \Re$ is a well-defined continuous function.

Proof: The first assertion follows directly from the definitions of v_p^a and $v_{p^-}^a$. The second follows from the first and the continuity of v_p^a and r.

3.25 Lemma Let $a^o > 0$ and let a^* be given by Lemma 3.23. Then for every $k \in K^{a^o}$, $v_p^{a^*}(k) = v_p(k)$.

Proof: Suppose by way of contradiction that there is some $k \in K^{a^o}$ with $v_p^{a^*}(k) \neq v_p(k)$, including the possibility that $v_p(k)$ is undefined. By Lemma 3.23, $v_p^{a^*}(k) = v_p^a(k)$ for all $a > a^*$, so there must be an unbounded sequence $\{k^t\}_{t=0}^{\infty} \subset \Re_+^{\ell}$ with $k^0 = k$, $k_j^{t+1} \geq (1-\delta_j)k_j^t$ for all $t \geq 1$ and each j; and

$$v_p^{a^*}(k) < \sum_{t=1}^{\infty} (1+\rho)^{-t}\left[r(k^t) - I(k^t, k^{t+1})\right] - I(k^0, k^1)$$
$$- \sum_{j=1}^{\ell} h_j(1-\delta_j)k_j^0.$$

Then for T sufficiently large,

$$v_p^{a^*}(k) < \sum_{t=1}^{T} (1+\rho)^{-t}\left[r(k^t) - I(k^t, k^{t+1})\right] - I(k^0, k^1)$$
$$- \sum_{j=1}^{\ell} h_j(1-\delta_j)k_j^0.$$

Define the bounded sequence $\{k'^t\}_{t=0}^{\infty}$ by

$$k'^t = \begin{cases} k^t & \text{if } t \leq T+1 \\ \left((1-\delta_1)k_1'^{t-1}, \ldots, (1-\delta_\ell)k_\ell'^{t-1}\right) & \text{if } t > T. \end{cases}$$

Then

$$\sum_{t=1}^{\infty}(1+\rho)^{-t}\big[r(k'^t)-I(k'^t,k^{t+1})\big]$$

$$=\sum_{t=1}^{T}(1+\rho)^{-t}\big[r(k^t)-I(k^t,k^{t+1})\big]+\sum_{t=T+1}^{\infty}(1+\rho)^{-t}r(k'^t)$$

$$\geq\sum_{t=1}^{T}(1+\rho)^{-t}\big[r(k^t)-I(k^t,k^{t+1})\big].$$

Then for $a=\max\{||k^t||:t\leq T\}$, this implies that $v_p^a(k)>v_p^{a^*}(k)$, which contradicts the choice of a^*.

3.26 Proposition: The function $v_p:\Re_+^\ell\rightarrow\Re$ is well-defined and continuous.

Proof: This follows directly from Lemma 3.25 and Proposition 3.19. ∎

Lemma 3.25 above shows that PVM can be achieved with bounded sequences. The following Proposition states that only bounded sequences are PVM.

3.27 Proposition: Every PVM sequence is bounded.

Proof: Let $\{k^t\}_{t=0}^{\infty}$ be a PVM sequence. Lemma 3.25 implies that for each t there is some $a_t\geq||k^t||$ with $v_p(k^t)=v_p^{a_t}(k^t)$. In particular, each a_t can be chosen so that there is a PVM^{a_t} sequence $\{k^{st}\}_{s=0}^{\infty}$ with $k^{0t}=k^t$ and $\sup\{||k^{st}||:s\geq 0\}>a_t-1$. Then for each t, Lemmas 3.20 and 3.22 imply that

$$v_p^{a_t}(k^t)\leq r_{a_t}/\rho-\min\{h_j(1-\delta_j):1\leq j\leq\ell\}(a_t-1). \tag{2}$$

Suppose by way of contradiction that the sequence $\{k^t\}_{t=0}^{\infty}$ is unbounded. Then (2) and Lemma 3.6 imply that $\liminf_{t\rightarrow\infty}v_p^{a_t}(k^t)=-\infty$. However, Lemma 3.22 implies that $v_p^{a_t}(k^t)\geq v_p(k^0)$ for all t, a contradiction. ∎

3.28 Definition: Let $S_p=\{k\in\Re_+^\ell:$ the constant sequence $k^t=k$ for all t is a PVM sequence$\}$.

3.29 Proposition: $S_p = \{k \in \Re_+^\ell : v_p(k) = (1/\rho)[r(k) - \sum_{j=1}^\ell (\rho + \delta_j) h_j k_j]\}$.

Proof: For any PVM sequence $\{k^t\}_{t=0}^\infty$, the definition of v_p and Lemma 3.18 imply that

$$(3) \qquad v_p(k^0) = (1+\rho)^{-1} v_p(k^1) + (1+\rho)^{-1} r(k^1) + (1+\rho)^{-1} \sum_{j=1}^\ell h_j (1-\delta_j) k_j^1 - I(k^0, k^1) - \sum_{j=1}^\ell h_j (1-\delta_j) k_j^0.$$

By the definition of S_p, $k^0 \in S_p$ if and only if the constant sequence $k^t = k^0$ for all t is a PVM sequence, which by (3) is equivalent to

$$v_p(k^0) = (1+\rho)^{-1} v_p(k^0) + (1+\rho)^{-1} r(k^0) + (1+\rho)^{-1} \sum_{j=1}^\ell h_j (1-\delta_j) k_j^0 - \sum_{j=1}^\ell h_j k_j^0,$$

which further reduces to

$$(1+\rho) v_p(k^0) = v_p(k^0) + r(k^0) - \sum_{j=1}^\ell (\rho + \delta_j) h_j k_j^0. \quad \blacksquare$$

3.30 Proposition: Let $\{k^t\}_{t=0}^\infty$ be a PVM sequence. Then $d(k^t, S_p) \to 0$.

Proof: By Proposition 3.27, the sequence is bounded, so it suffices to show that any cluster point k^* lies in S_p. Let $a^o > \sup\{||k^t|| : t \geq 0\}$, and using Lemma 3.25, let $a > a^o$ such that $v_p(k) = v_p^a(k)$ for all $k \in K^{a^o}$. Choosing a subsequence $\{k^{t_s}\}_{s=1}^\infty$ if necessary, we can assume that $k^{t_s} \to k^*$ and that $k^{t_s - 1} \to k'$ for some $k' \in K^{a^o}$. By Lemma 3.22, $v_p^a(k^{t_s - 1}) \leq v_{p-}^a(k^{t_s}) \leq v_p^a(k^{t_s})$ for all s. By Lemma 3.22 and Proposition 3.19, the sequence $\{v_p^a(k^t)\}_{t=0}^\infty$ converges monotonically to some real number v^*, so by the continuity of v_p^a and v_{p-}^a, $v_{p-}^a(k^*) = v_p^a(k^*) = v^*$. Since $v_{p-}^a(k^*) = v_p^a(k^*)$, Lemma 3.24

implies that

$$v_p^a(k^*) = (1+\rho)^{-1} v_p^a(k^*) + (1+\rho)^{-1} r(k^*) + (1+\rho)^{-1} \sum_{j=1}^{\ell} h_j (1-\delta_j) k_j^*$$

$$- \sum_{j=1}^{\ell} h_j k_j^*,$$

which reduces to $v_p^a(k^*) = \left[r(k^*) - \sum_{j=1}^{\ell} (\rho + \delta_j) h_j k_j^* \right] / \rho$. By the choice of a, $v_p^a(k^*) = v_p(k^*)$, so the result follows from Proposition 3.29. ∎

Proposition 3.30 states that all PVM paths converge to the set of PVM-stationary capital vectors, which completes our analysis of PVM dynamics. It only remains to show that the limit set for PVM and RIM are identical.

3.31 Proposition: For each $k \in \Re_+^{\ell}$, $\rho v_p(k) \geq v_r(k)$.

Proof: Let $k \in \Re_+^{\ell}$, and let $k' \in \Re_+^{\ell}$ such that $v_r(k) = w(k, k')$. Then by the definition of w,

4) $$v_r(k) = r(k') - \sum_{j=1}^{\ell} (\rho + \delta_j) \left[h_j (1-\delta_j) k_j + I_j(k_j, k_j') \right].$$

By the definition of v_p,

$$\begin{aligned} v_p(k) &\geq \sum_{t=1}^{\infty} (1+\rho)^{-t} \left[r(k') - I(k', k') \right] \\ &\quad - \sum_{j=1}^{\ell} \left[h_j (1-\delta_j) k_j + I_j(k_j, k_j') \right] \\ &= (1/\rho) \left[r(k') - \sum_{j=1}^{\ell} \delta_j h_j k_j' \right] \\ &\quad - \sum_{j=1}^{\ell} \left[h_j (1-\delta_j) k_j + I_j(k_j, k_j') \right]. \end{aligned}$$

Since $h_j(1-\delta_j)k_j + I_j(k_j, k'_j) \geq h_j k'_j$ for all j, it follows that

$$\begin{aligned} v_p(k) \geq &(1/\rho)\Big\{r(k') - \sum_{j=1}^{\ell} \delta_j \big[h_j(1-\delta_j)k_j + I_j(k_j, k'_j)\big]\Big\} \\ &- \sum_{j=1}^{\ell} \big[h_j(1-\delta_j)k_j + I_j(k_j, k'_j)\big]. \end{aligned}$$

The result now follows from (4). ∎

3.32 Proposition: $S_p \subset S_r$.

Proof: Let $k \in S_p$. By Proposition 3.31, $v_p(k) \geq v_r(k)/\rho$. By the definition of v_r, $v_r(k) \geq w(k,k) = r(k) - \sum_{j=1}^{\ell}(\rho + \delta_j)h_j k_j$. However, since $k \in S_p$, Proposition 3.29 implies that $v_p(k) = w(k,k)/\rho$, so we have

$$v_p(k) \geq v_r(k)/\rho \geq w(k,k)/\rho = v_p(k).$$

Hence, $v_r(k) = w(k,k)$, so $k \in S_r$. ∎

3.33 Lemma: Let $k \in \Re^{\ell}_{+}$ and let $a > ||k||$ such that $v^a_p(k) = v_p(k)$. Then $k \in S_p$ if and only if $v^a_p(k) = v^a_{p^-}(k)$.

Proof: This follows directly from Lemma 3.24 and Proposition 3.29. ∎

3.34 Definition: Let $M_r = \{k \in \Re^{\ell}_{+} : k$ maximizes $v_r\}$.

3.35 Proposition: $M_r \subset S_p$.

Proof: Let $k \in M_r$, and let $a > ||k||$ such that $v^a_p(k) = v_p(k)$. By Lemma 3.33, it suffices to prove that $v^a_{p^-}(k) = v^a_p(k)$. By the definitions of v^a_p and $v^a_{p^-}$, $v^a_{p^-}(k) \leq v^a_p(k)$. Suppose by way of contradiction that $v^a_p(k) > v^a_{p^-}(k)$. Then it follows directly from Lemma 3.24 that

5) $$v_p(k) > (1/\rho)\big[r(k) - \sum_{j=1}^{\ell}(\rho + \delta_j)h_j k_j\big].$$

Since $k \in M_r$, the right hand side of (5) is equal to $(1/\rho)v_r(k)$, so

6)
$$v_p(k) > (1/\rho)v_r(k).$$

By following a PVM sequence from k we can, by Lemma 3.25, 3.22 and Proposition 3.30 find some $k' \in S_p$ with $v_p(k') \geq v_p(k)$. Since $S_p \subset S_r$ (Proposition 3.32), Proposition 3.29 implies that $v_p(k') = (1/\rho)v_r(k')$. Hence (6) implies that $v_r(k') > v_r(k)$, which is contrary to the fact that $k \in M_r$. ∎

Propositions 3.32 and 3.35 imply that any limit point of a PVM path is a long-run equilibrium for both residual income and present value maximization. However, there remains the possibility that RIM may remain stationary at capacity vectors that are not PVM. There are two reasons for this. First, we have not assumed that $De_j(0) = 0$, so the marginal cost of expansion at a stationary point can be positive. Residual income charges depreciation on the entire book-value of investment, including the expansion cost, so a positive marginal expansion cost induces a positive marginal depreciation charge that is not a true cost of investment. Because of this artificial impediment to investment, S_r may include S_p as a proper subset.

Second, we have made no convexity assumptions. The myopic nature of RIM makes it more local than PVM. If the net revenue function is not concave, PVM might result in a path to a distant but much more desirable capital vector, while RIM would remain stationary if every initial investment large enough to produce an immediate increase in net revenue has prohibitive adjustment costs. The following two assumptions are necessary to ensure that PVM and RIM have the same stationary sets, and thus the same limit sets.

3.36 Additional Assumptions:

Assumption K (Convexity): For each $i \in \ell+1, \ldots, n$, the set Y^i is convex.

Assumption Z (Zero marginal expansion cost): The function e_i is differentiable at 0 for each $i = 1, \ldots, \ell$. In particular, $De_i(0) = 0$ for each $i = 1, \ldots, \ell$.

3.37 Proposition: Under Additional Assumptions 3.36, $S_r \subset M_r$.

Proof: Let $k^o \in S_r$. Suppose by way of contradiction that $k^o \notin M_r$. Then there is some $k \in \Re_+^\ell$ with $v_r(k) > v_r(k^o)$. By following an RIM sequence from k, Propositions 3.12 and 3.30 imply that we can find $k' \in S_r$ with $v_r(k') > v_r(k^o)$. Since k^o, $k' \in S_r$, we have

$$v_r(k') = r(k') - \sum_{j=1}^{\ell}(\rho + \delta_j)h_j k'_j$$

$$> r(k^o) - \sum_{j=1}^{\ell}(\rho + \delta_j)h_j k^o_j = v_r(k^o).$$

For each $0 < \lambda < 1$, let $k^\lambda = \lambda k' + (1-\lambda)k$. Assumption K implies that r is concave, so $r(k^\lambda) \geq r(k^o) + \lambda\big[r(k') - r(k^o)\big]$ for all $0 < \lambda < 1$. For each j, if λ is sufficiently small that $k_j^\lambda \geq (1-\delta_j)k_j^o$ then

7) $$I_j(k_j^o, k_j^\lambda) = \delta_j h_j k_j^o + h_j(k_j^\lambda - k_j^o) + e_j(k_j^\lambda - k_j^o).$$

By (7) and Assumption Z,

$$\lim_{\lambda \to 0}\left[I_j(k_j^o, k_j^\lambda) - \delta_j h_j k_j^o\right] / \lambda \ = \ h_j(k'_j - k_j^o).$$

Therefore,

$$\liminf_{\lambda \to 0}\big[w(k^o, k^\lambda) - v_r(k^o)\big] / \lambda \ \geq \ r(k') - r(k^o)$$

$$- \sum_{j=1}^{\ell}(\rho + \delta_j)h_j(k'_j - k_j^o) \ = \ v_r(k') - v_r(k^o).$$

It follows that for λ sufficiently small, $w(k^o, k^\lambda) > v_r(k^o)$, which is contrary to the definition of $v_r(k^o)$. ∎

3.38 Proposition: Under Additional Assumptions 3.36, $S_p = S_r$.

Proof: This follows directly from Propositions 3.32, 3.35 and 3.37. ∎

3.39 Theorem: Assumptions B, C, D, and N imply that

a) for every residual income maximizing sequence $\{k^t\}_{t=0}^{\infty}$, $d(k^t, S_r) \to 0$; and

b) for every present value maximizing sequence $\{k^t\}_{t=0}^{\infty}$, $d(k^t, S_p) \to 0$.

Under the additional assumptions K and Z,

c) $S_r = S_p$.

In the absence of strict convexity assumptions, the common limit set need not be a singleton. Under the assumption that the second derivative of the net revenue function is strictly negative, Anctil (1994) shows that an RIM path converges to the unique stationary capital level at an exponential rate. The multidimensional analog of Anctil's assumption would be negative definiteness of D^2r, which might be ensured by imposing smoothness and curvature assumptions on the sets Y^i. It would also be necessary to assume that the expansion cost functions are smooth and convex. The smoothness and curvature assumptions would have to include boundaries in order to avoid the implausible assumption that every activity requires the use of every internal commodity. In fact, the model should allow the possibility that some activities are inactive at the stationary capital vector. In any case, the assumptions needed to ensure the existence and negative definiteness of D^2r will be cumbersome at best in any model that, like the present model, strives for generality in the class of allowable activity networks.

4. Activity-Based Costing and Residual Income Maximization

We now turn to the design of a cost accounting system to support residual income maximization. In contrast to the previous section, we focus on the choice of a single-period plan, taking the previous capacity vector as given. The accounting system has two principal elements. First, there are rules for computing the unit cost of the output of each cost activity, including investment activities. Second, there are performance measures that tell managers how to use the unit costs in selecting proposed actions. The specific accounting system we will construct, termed the *ABC mechanism*, is an extension of the

ABC mechanism constructed by Jordan (1994). The ABC mechanism is a special case of a general class of mechanisms called *budget mechanisms.* Several examples and a more extensive discussion of budget mechanisms are given in Jordan (1994).

4.1 Definitions: Fix an initial capacity vector $(k_i^o)_{i=1}^{\ell}$, an initial book-value vector $(K_i^o)_{i=1}^{\ell} \in \Re_+^{\ell}$, and capital policy parameters $\left(\rho, (\delta_i)_{i=1}^{\ell}\right) \in \Re_{++} \times (0,1)^{\ell}$. A firm, f, is then specified by the remaining characteristics $(I_i)_{i=1}^{\ell}$ and $(Y^i)_{i=\ell+1}^{n}$. For an investment activity i, in the interest of notational simplicity we will sometimes write Y^i to mean the set $Y^i(k_i^o)$ defined in 2.2 above.

4.2 Budget Mechanisms: For each activity i, let A^i be a set, with generic element a_i, and let $A = \Pi_{i=1}^n A^i$. For each i, let $\alpha_i : \Pi_{j \neq i}(A^j \times P^j) \rightarrow A^i$, let $\beta_i : A^i \times P^i \rightarrow P^i$, and let $\pi_i : A^i \times P^i \rightarrow \Re$. The function α_i is a *reporting function,* β_i is a *budget function,* and π_i is a *performance measurement function.* A *budget mechanism* is an n-tuple $(A^i, \alpha_i, \beta_i, \pi_i)_i$ such that the reporting functions satisfy:

Assumption A: For each plan $(y_i)_i \in P$ such that for each $j = 1, \ldots, m$, $y_{jj} > 0$ and $\sum_{i=1}^n y_{ij} \geq 0$, there is at most one $(a_i)_i \in A$ that satisfies $a_i = \alpha_i\left((a_j, y_j)_{j \neq i}\right)$ for all $i = 1, \ldots, n$.

4.3 Budget Equilibrium: Given a budget mechanism $(A^i, \alpha_i, \beta_i, \pi_i)_i$, a budget equilibrium for a firm $f = \left((I_i)_{i=1}^{\ell}, (Y^i)_{i=\ell+1}^{n}\right)$ is an n-tuple $(a_i^*, y_i^*)_i$ satisfying, for each $i = 1, \ldots, n$

i) $\quad a_i^* = \alpha_i\left((a_j^*, y_j^*)_{j \neq i}\right)$;

ii) $\quad y_i^* = \beta_i(\alpha_i^*, y_i^*)$; and

iii) $\quad y_i^*$ maximizes $\pi_i(a_i^*, \cdot)$ on $\beta_i(a_i^*, P^i) \cap Y^i$.

4.4 Remarks: Assumption A embodies the restriction that the reports a_i contain accounting information only. Since the reports are determined by the proposed actions alone, they are precluded from conveying information about marginal costs, marginal revenues, or opportunity costs except to the extent that such information can be inferred from proposed actions.

A budget equilibrium is formally a static concept, but it can be interpreted as an outcome of a participatory budgeting process. Central management sends a report a_i to manager i, containing information about actions proposed by other managers. Manager i then proposes an action y_i, which is modified by central management to $\beta_i(a_i, y_i)$. Manager i, knowing a_i and the budget function β_i, anticipates this modification by proposing an action y_i such that $\beta_i(a_i, y_i) \in Y^i$, so the action is feasible, and y_i maximizes the performance measure $\pi_i\big(a_i, \beta_i(a_i, \cdot)\big)$ among such proposals. Central management, in response to such proposals, sends new reports as dictated by the reporting functions α_i, activity managers send new proposals in response to the new reports, and so on. A budget equilibrium is a fixed-point of this process.

One could model a budget process explicitly as an iterative or continuous-time adjustment process of this sort, but such a model would be artificially restrictive. In actual participatory budgeting, there can be communication, perhaps informal or ad hoc, among managers as well as between managers and central management, so manager i may obtain information beyond the limits of the report a_i, and may even coordinate proposals with some other managers. We now define properties of budget mechanisms motivated by the need to make the model robust to additional information and coordination.

4.5 Definitions: Let $\mathcal{F}$ denote a set of firms, that is, a set of n-tuples $f = \big((I_i)_{i=1}^{\ell},\ (Y^i)_{i=\ell+1}^{n}\big)$. A budget mechanism $(A^i, \alpha_i, \beta_i, \pi_i)_i$ is *RI-compatible* on $\mathcal{F}$ if for each $f \in \mathcal{F}$, and each plan $(y_i^*)_i$ which is residual income maximizing for f given k^o, there is some $(a_i^*)_i \in A$ such that $(a_i^*, y_i^*)_i$ is a budget equilibrium for f.

For each plan $(y_i)_i \in P$, let $\pi_0\big((y_i)_i\big)$ denote the residual income,

$$\pi_0\big((y_i)_i\big) = \sum_{i=\ell+1}^{n} y_{i0} - \sum_{i=1}^{\ell}(\rho + \delta_i)\big[-y_{i0} - (1-\delta_i)K_i^o\big].$$

Given a budget mechanism $(A^i, \alpha_i, \beta_i, \pi_i)_i$, define $\mathcal{E}(\mathcal{F}) = \{(a_i, y_i)_i : (a_i, y_i)_i$ is a budget equilibrium for some $f \in \mathcal{F}\}$. The budget mechanism is *RI-seeking* on $\mathcal{F}$ if for each $(a_i, y_i)_i$ and $(a'_i, y'_i)_i \in \mathcal{E}(\mathcal{F})$, $\pi_0((y_i)_i) \geq \pi_0((y'_i)_i)$ if and only if $\pi_i(a_i, y_i) \geq \pi_i(a'_i, y'_i)$ for all $i = 1, \ldots, n$.

4.6 Remarks: RI-compatibility means that if a residual income maximizing plan were somehow proposed, it would be accepted as the outcome of the budgeting process. Unfortunately, the informational limitations of the accounting reports $(a_i)_i$ cause many other plans to be equilibria as well. If the budget mechanism is RI-seeking, managers who acquire additional information or who coordinate their proposals with others in order to improve their measured performance will also raise the firm's residual income. The RI-seeking property is an "open architecture" property that makes the budget mechanism compatible with additional information and communication flows.

The concepts RI-compatible and RI-seeking are defined with reference to a set of firms, $\mathcal{F}$, in order to formalize an information constraint on the budget mechanism. The size of the set $\mathcal{F}$ represents the extent of central management's ignorance of the characteristics $f = \left((I_i)_{i=1}^{\ell}, (Y^i)_{i=\ell+1}^{n}\right)$. If central management knows these characteristics, then $\mathcal{F}$ should be taken to be a one-point set $\{f\}$. In this case, the design of a budget mechanism that supports residual income maximization is trivial. Let $(y^{*i})_i$ be a residual income maximizing plan, and for each i, let β_i be the constant function $\beta_i(\cdot) = y_i^*$. If the set $\mathcal{F}$ is large, however, the inability to tailor the budget mechanisms to specific elements in $\mathcal{F}$ will typically force an RI-compatible budget mechanism to admit many budget equilibria that do not maximize residual income. The maximization condition 4.3 (iii) implicitly assumes that Y_i is known by manager i, but we assume that these characteristics are not known by other managers or by central management beyond the knowledge that $f \in \mathcal{F}$.

The initial capacity vector, k^o, the initial book-value, K^o, and the capital policy parameters, $(\rho, (\delta_i)_{i=1}^{\ell})$ are fixed over $\mathcal{F}$ and thus assumed to be known by central management, and may be used as parameters in the budget mechanism, as in the ABC mechanism defined below.

4.7 The ABC Mechanism: Let $B^i = \{b_i \in \Re^m : \text{ for some } x \in \Re,\ (x, b_i) \in P^i\}$, for $i = 1, \ldots, n$. Let $C^i = \{c_i \in \Re^m : c_{ii} = 0\}$ for $i = 1, \ldots, m$, and $C^i = \Re^m$ for $i = m+1, \ldots, n$. Define the report space $A^i = \Re \times B^i \times C^i$, and $A = \Pi_{i=1}^n A^i$.

For each $i = 1, \ldots, n$, the reporting function $\alpha_i = \Pi_{j \neq i}(A^j \times P^j) \to A^i$ is given by

$$\alpha_i\big((a_j, y_j)_{j \neq i}\big) = (a_{i0}, b_i, c_i), \quad \text{where}$$

$$a_{i0} = \sum_{\substack{j=\ell+1 \\ j \neq i}}^{n} y_{j0} - \sum_{\substack{j=1 \\ j \neq i}}^{\ell} (\rho + \delta_j) K_j, \quad \text{where}$$

$$K_j = (1 - \delta_j) K_j^o - y_{j0} \quad \text{for each } j = 1, \ldots, \ell;$$

$$b_{ik} = \begin{cases} -\sum_{\substack{j=1 \\ j \neq i}}^{n} y_{jk} & \text{if } k = i; \\ \min\{0, -\sum_{\substack{j=1 \\ j \neq i}}^{n} y_{jk}\} & \text{if } k \neq i \text{ and } i > \ell; \text{ and} \\ 0 \text{ otherwise.} & \end{cases}$$

$$c_{ij} = \begin{cases} -\dfrac{(\rho + \delta_j) y_{j0}}{y_{jj} - (1 - \delta_j) k_j^o} & \text{if } j \leq \ell,\ j \neq i, \text{ and } y_{jj} > (1 - \delta_j) k_j^o; \\ -\dfrac{y_{j0} + \sum_{k=1}^{m} c_{kj}\, y_{jk}}{y_{jj}} & \text{if } \ell < j \leq m,\ j \neq i, \text{ and } y_{jj} > 0; \text{ and} \\ 0 \text{ otherwise.} & \end{cases}$$

The budget functions $\beta_i : A^i \times P^i \to P^i$ are given by $\beta_i\big((a_{i0}, b_i, c_i), y_i\big) = y_i^b$, where

$$y_{i0}^b = y_{i0}; \quad \text{and}$$

$$y_{ij}^b = \max\{b_{ij}, y_{ij}\}, \quad j = 1, \ldots, m.$$

The performance functions $\pi_i : A^i \times P_i \to R$ are given by

$$\pi_i\big((a_{i0}, b_i, c_i), y_i^b\big) \;=\; \begin{cases} a_{i0} - (\rho + \delta_i)K_i, & i = 1, \ldots, \ell; \quad \text{and} \\ a_{i0} + y_{i0}^b + \sum_{j=1}^m c_{ij}(y_{ij}^b - b_{ij})^+, & i = \ell + 1, \ldots, n, \end{cases}$$

where $(x)^+$ denotes $\max\{x, 0\}$.

4.8 Remarks: The report, a_i, to manager i consists first of a_{i0}, which is the residual income of the firm less the contribution of activity i. The m-vector b_i tells manager i the net supply and demand of the rest of the firm. For $i \neq j$, b_{ij} is the net amount of commodity j available for use by activity i, and b_{ii} is the total demand for commodity i. The budget functions use the b_i vectors to force aggregate feasibility. The vector c_i communicates to each manager i the unit cost, c_{ij}, of commodity j. Although $c_{ii} = 0$ for each i, the other components of c_i are the same for all i, that is, if $i \neq j$ and $i' \neq j$, then $c_{ij} = c_{i'j}$; so all managers $i \neq j$ face the same cost for commodity j. For $j = \ell + 1, \ldots, m$, the unit cost, c_{ij}, of commodity j is computed in the obvious way. The only subtlety is that reciprocal flows among cost activities may cause c_{ij} to be included in computing c_{ji} and vice versa. This means that the equilibrium condition 4.3 (i) determines equilibrium unit costs as the solution to a system of linear equations just as in the well-known reciprocal method of service center cost allocation.

The computation of unit cost for investment activities differs in one important respect. Instead of computing the average capital cost, $(\rho + \delta_i)K^i/y_{ii}$, the unit cost of capacity i is computed as a unit cost of gross investment, if gross investment is positive, and the unit cost is taken to be zero otherwise. This is done to ensure RI-compatibility, which requires this complication for two reasons. First, recall that $K_i = (1 - \delta_i)K_i^o - y_{i0}$, of which $(1 - \delta_i)K_i^o$ is sunk cost. If, for historical reasons, K_i^o is artificially high, including the sunk cost $(1 - \delta_i)K_i^o$ in the computation of the unit cost might depress the usage of capacity i below the residual income maximizing level. Second, if there is no gross investment, the capacity that remains is free and must be fully utilized if it is at all productive. Therefore activities that use capacity i must not be quoted a positive unit cost when capacity i is freely available. For these two reasons, the unit cost of capacity i is computed as the unit cost of gross investment. Unfortunately, this computation presents a new difficulty because the denominator is problematic. The depreciation rate δ_i is a capital policy parameter chosen by central management. Ideally, as in Assumption I (3.2), δ_i is also the rate of physical deterioration of capacity, but this ideal is rarely met in practice. If δ_i is too small, the denominator $y_{ii} - (1 - \delta_i)k_i^o$ will be too small, again raising the possibility that an overstated unit cost could depress capacity usage. For this reason, in proving that the ABC mechanism is RI-compatible (Proposition 4.15) we will need to assume that δ_i is sufficiently large that $I_i\left(k_i^o, (1 - \delta_i)k_i^o\right) = 0$ so that $(1 - \delta_i)k_i^o$ would be freely available.

Contrary to what one might expect, the performance measures π_i do not use the unit costs to compute a separate net income for each activity. Instead, the unit cost c_{ij} enters π_i as a bonus for using less of commodity j than the available amount, y_{ij}^b. This provides exactly the same incentive to economize as would charging activity i the amount $c_{ij}|y_{ij}|$ for the usage $|y_{ij}|$. In addition, it avoids a potential conflict of interest among managers. Suppose managers i and j discover that residual income would increase if some capacity that i was proposing to use was instead diverted to j. If i and j have separate income statements, the result might be a reduction in i's net income and an increase, by a greater amount, in j's net income. In this case, the budget mechanism might fail to be RI-seeking.

The following proposition records the fact that the ABC mechanism satisfies Assumption A, and thus the definition of a budget mechanism.

4.9 Proposition: The ABC mechanism is a budget mechanism (4.2).

Proof: Let $(y_i)_i \in P$ with $y_{ii} > 0$ for all $i = 1, \ldots, m$, and $\sum_{i=1}^n y_{ij} \geq 0$ for each $j = 1, \ldots, m$. It is clear that for each i, a_{i0} and b_i are uniquely determined by the reporting functions. Also, for each i, the unit cost c_{ij} of each investment activity $j = 1, \ldots, \ell$ is uniquely determined. If one replaces y_{i0} by $y_{i0} + \sum_{j=1}^{\ell} c_{ij} y_{ij}$ for each cost activity $i = \ell + 1, \ldots, m$, then the unique determination of the unit cost c_{ij} for each i and each $j = \ell + 1, \ldots, m$ follows exactly as in the proof of Proposition 5.3 in Jordan (1994). ∎

4.10 Remarks: The following Proposition derives an equivalent but more transparent statement of the ABC equilibrium conditions.

4.11 Proposition: Let $(a_i^*, y_i^*)_i \in \Pi_{i=1}^n (A_i \times P_i)$ satisfy $a_i^* = \alpha_i\big((a_i^*, y_i^*)_{i \neq j}\big)$ for all $i = 1, \ldots, n$. Then (a_i^*, y_i^*) is an ABC equilibrium given an initial capacity vector k^o if and only if

i) $(y_i^*)_i$ is feasible given k^o;

ii) there is a vector p^* satisfying $p_j^* = c_{ij}^*$

for each i, $j = 1, \ldots, m$ and $i \neq j$,

and for each $i = 1, \ldots, \ell$,

$$(\rho + \delta_i) y_{i0}^* \big(y_{ii}^* / (y_{ii}^* - (1 - \delta_i) k_i^o)\big) + p_i^* y_{ii}^* = 0$$

$$\text{if} \quad y_{ii}^* > (1 - \delta_i) k_i^o, \quad \text{and}$$

$p_i^* = 0$ otherwise,

and for each $i = \ell + 1, \ldots, m$,

$$y_{i0}^* + \sum_{j=1}^{m} p_j^* y_{ij}^* = 0; \quad \text{and}$$

$p_i^* = 0$ if and only if $y_{ii}^* = 0$; and *iii)* for each $i = 1, \ldots, \ell$, y_i^* maximizes

$$(\rho + \delta_i) y_{i0}$$

subject to $y_{i0} \leq -I_i(k_i^o, y_{ii})$

and $y_{ii} + \sum_{i' \neq i} y_{i'i}^* \geq 0$; and

for each $i = \ell + 1, \ldots, n$, y_i^* maximizes

$$y_{i0} + \sum_{\substack{j=1 \\ j \neq i}}^{m} p_j^* \, y_{ij}$$

subject to $y_i \in Y^i$ and

$$y_{ij} + \sum_{i' \neq i} y_{i'j}^* \geq 0 \quad \text{for all} \quad j = 1, \ldots, m.$$

Proof: Feasibility is equivalent to the equilibrium condition $\beta_i(a_i^*, y_i^*) = y_i^*$ for all i (4.3 (ii)) together with the requirement in 4.3 (iii) that $\beta_i(a_i^*, y_i^*) \in Y^i$ for all i. Condition (ii) follows from the assumption that $a_i^* = \alpha_i\big((a_j^*, y_j^*)_{j \neq i}\big)$ for all i, which is equilibrium condition 4.3 (i). Condition (iii) is equivalent to equilibrium condition 4.3 (iii). ∎

4.12 Remarks: In 4.11 (ii), p_i^* is the unit cost of commodity i. The notation p_i^* is used because, as 4.11 (iii) indicates, p_i^* provides the same incentive effect as a transfer price. However, the constraints in 4.11 (iii) are much more restrictive than the decision set typically found in transfer price mechanisms. Each cost manager i must choose y_{ii} large enough to meet demand, and no manager i can use more than the available quantity of any commodity j. For this reason, there are many ABC equilibria at volume levels far below residual income maximization. In particular, the zero plan, $y_i = 0$ for all i, is always an ABC equilibrium. Since $c_{ii} = 0$ for every cost activity i, cost managers have no incentive to exceed demand. Thus any increase in volume requires some coordination among managers, which makes the RI-seeking property indispensable.

We now proceed to prove that the ABC mechanisms is RI-seeking. In addition to some of the assumptions used in Section 3, we will need a version of downscaling for investment activities. In addition, we will make a simplifying assumption that all cost activities are active at a residual income maximum. When this assumption is not satisfied, the proofs can be revised by dropping the inactive activities.

4.13 Assumptions:

AA (Active activities): If $(y_i^*)_i$ is a residual income maximizing plan given k^o then $y_{ii}^* > 0$ for all $i = 1, \ldots, m$.

ID (Investment downscaling): For each $i = 1, \ldots, \ell$, $I_i(k_i^o, 0) = 0$ and if $x, x' \geq 0$ satisfy $I_i(k_i^o, x) = 0$ and $I_i(k_i^o, x') > 0$ then $x' > x$ and for each $0 \leq \lambda \leq 1$,

$$I_i\big(k_i^o, \lambda x' + (1-\lambda)x\big) \leq \lambda I_i(k_i^o, x').$$

4.14 Remarks: Assumption ID implies that for each investment activity i, there is some $x_i \geq 0$, possibly infinite, such that $I_i(k_i^o, x) = 0$ for all $x \leq x_i$, and $I_i(k_i^o, \cdot)$ is strictly increasing on the interval $[x_i, \infty)$. Thus $k_i^o - x_i$ is, implicitly, the amount of physical depreciation, although we will not need to make the natural assumption that $x_i \leq k_i^o$.

As mentioned above, in order to prove that an RIM is an ABC equilibrium we rely on some of the results of Jordan (1994). These results show that, given any allocation, a proposed reduction in any input usage or output by any cost or revenue activity, leads to determinable (perhaps zero) reductions in input use by all cost activities.

4.15 Proposition: Assume D, N (3.2), K (3.36), AA and ID. Let $(y_i)_i$ maximize residual income given k^o. Then there is a unique $(a_i^*)_i \in \Pi_i A^i$ that satisfies $a_i^* = \alpha_i\big((a_j^*, y_j^*)_{j \neq i}\big)$ for all i. If $I_i\big(k_i^o, (1-\delta_i)k_i^o\big) = 0$ for all $i = 1, \ldots, \ell$, then $(a_i^*, y_i^*)_i$ is an ABC equilibrium given k^o.

Proof: To prove the first assertion, for each i let a_{i0}^*, b_i^*, and c_{ij}^* for $j = 1, \ldots, \ell$ be given by the definition of the reporting functions α_i (4.2). To obtain c_{ij}^* for $j = \ell+1, \ldots, m$, replace y^*_{i0} with $y^*_{i0}+\sum_{j=1}^{\ell} c^*_{ij}\, y^*_{ij}$ for each $i = \ell+1, \ldots, m$. Since $(y_i)_i$ is residual income maximizing, Assumptions N and D imply that $\sum_{i=\ell+1}^{n} y^*_{ij} = 0$ for all $j = \ell+1, \ldots, m$, and that there are no *redundant* activities (Jordan (1994, A.2)), so the required unit costs exist by Lemma A.6 of Jordan (1994).

We now show that conditions (i) - (iii) of Proposition 4.11 are satisfied. The fact that $(y_i)_i$ is residual income maximizing implies 4.11 (i) and that part of 4.11 (iii) pertaining to investment activities. Proposition 4.9 implies 4.11 (ii), since $y^*_{ii} > 0$ for all $i = 1, \ldots, m$ by Assumption AA. Hence, it suffices to show that for each $i = m+1, \ldots, n$, y^*_i solves the problem

(*)
$$\text{maximize} \quad y_{i0} + \sum_{j=1}^{m} p^*_i\, y_{ij}$$
$$\text{subject to} \quad y_i \in Y^i$$
$$\text{and} \quad y_{ij} + \sum_{i' \neq i} y^*_{i'j} \geq 0, \quad j = 1, \ldots, m.$$

By Assumption ID, $y^*_{i0} = 0$ for each $i = 1, \ldots, \ell$ such that $\sum_{i'=1}^{n} y^*_{i'i} > 0$. Hence, without loss of generality, we can assume that for each $i = 1, \ldots, \ell$, $y^*_{ii} = -\sum_{i' \neq i} y^*_{i'i}$. Let $i \geq m+1$, and let y_i satisfy the constraints in (*). By Assumption K, for each $0 \leq \lambda \leq 1$, $\lambda y_i + (1-\lambda) y^*_i$ also satisfies the constraints in (*). Let $y^\lambda_i = \lambda y_i + (1-\lambda) y^*_i$ for some $0 < \lambda \leq 1$. First suppose that $i > m$. Then by Jordan (1994, Lemma A.11), there exist numbers λ_j, $j = 1, \ldots, m$ satisfying

8)
$$\sum_{j=1}^{m} (1-\lambda_j) y^*_{jj'} = y^\lambda_{ij'} - y^*_{ij'} \quad \text{for all} \quad j' = 1, \ldots, m.$$

Therefore, by Assumption D, the plan $(y'_i)_i = (\lambda_1 y^*_1, \ldots, \lambda_m y^*_m, y^*_{m+1}, \ldots, y^*_{i-1}, y^\lambda_i, y^*_{i+1}, \ldots, y^*_n)$ is feasible. by 4.11 (ii) and (8), we have

9)
$$y^\lambda_{i0} - y^*_{i0} + \sum_{j=1}^{m} p^*_j (y^\lambda_{ij} - y^*_{ij}) = \sum_{j=\ell+1}^{m} (y'_{j0} - y_{j0})$$

$$+\sum_{j=1}^{\ell}(\rho+\delta_j)\big[y^*_{j0}/(y^*_{jj}-(1-\delta_j)k^o_j\big](y'_{jj}-y^*_{jj}).$$

Choosing λ sufficiently small will make y^λ_i sufficiently near y^*_i and each λ_j sufficiently near 1 that for each $j=1,\dots,\ell$ with $y^*_{jj}>(1-\delta_j)k^o_j$ we also have $y^*_{jj}\geq y'_{jj}>(1-\delta_j)k^o_j$. Then for each $j=1,\dots,\ell$, Assumption ID and the fact that $I_j\big(k^o_j,(1-\delta_j)k^o_j\big)=0$ imply that

10) $$I_j(k^o_j,y'_{jj})\leq y^*_{j0}-(y^*_{jj}-y'_{jj})y^*_{j0}/(y^*_{jj}-(1-\delta_j)k^o_j).$$

Consider the plan $(y''_j)_j$ obtained by replacing y'_{j0} by $I_j(k^o_j,y'_{jj})$ for each $j=1,\dots,\ell$ in the plan $(y'_j)_j$. Then (10) implies that $(y''_j)_j$ is feasible, and together with (9), that if $y^\lambda_{i0}-y^*_{i0}+\sum_{j=1}^{m}p^*_j(y^\lambda_{ij}-y^*_{ij})>0$ then the residual income from the plan $(y''_j)_j$ exceeds the residual income from $(y^{*k})_j$. Since $y^\lambda_i=\lambda y_i+(1-\lambda)y^*_i$, this contradiction implies that $y_{i0}+\sum_{j=1}^{m}p^*_j y_{ij}\leq y_{i0}+\sum_{j=1}^{m}p^*_j y^*_{ij}$. This proves 4.11 (iii) for $i\geq m+1$. The case $\ell+1\leq i\leq m$ is nearly identical except that one uses Lemma A.13 of Jordan (1994) instead of A.12 and obtains

8′) $$\sum_{\substack{j=1\\ j\neq i}}^{m}(1-\lambda_j)y^*_{jj'}=\lambda_i y^\lambda_{ij'}-y^*_{ij'},\quad j'=1,\dots,l$$

instead of (8), and uses $\lambda_i y^\lambda_i$ instead of y^λ_i in the comparison plan $(y'_j)_j$. ∎

4.16 Theorem: Let $\mathcal{F}^*$ denote the set of firms satisfying Assumptions D, N, K, AA, and ID, and for which $I_i(k^o_i,(1-\delta_i)k^o_i)=0$ for all $i=1,\dots,\ell$. Then the ABC mechanism is RI-seeking on $\mathcal{F}^*$.

Proof: Proposition 4.15 establishes that the ABC mechanism is RI-compatible on $\mathcal{F}^*$. Let $(a^*_i,y^*_i)_i$ be an ABC equilibrium for some firm in $\mathcal{F}^*$. Then Assumption N and D, together with 4.11 (iii) imply that $y^*_{ij}=b^*_{ij}$ for all i and all $j=\ell+1,\dots,n$. Assumption ID and 4.11 (iii) imply that for each $j=1,\dots,\ell$ and each i, either $y^*_{ij}=b^*_{ij}$ or $c^*_{ij}=0$. Therefore

$$\pi_i(a^*_i,y^*_i)=\begin{cases}a^*_{i0}+(\rho+\delta_i)y^*_{i0} & \text{if}\quad i\leq\ell;\quad\text{and}\\ a^*_{i0}+y^*_{i0} & \text{otherwise.}\end{cases}$$

Hence $\pi_i(a_i^*, y_i^*) = \pi_0\big((y_j^*)_j\big)$ for all i. ∎

4.17 Remarks: The downscaling assumptions, D and ID, are indispensable to the *RI*-compatibility of the ABC mechanism. More generally, the feasibility of down-scaling is the kernel of linearity that is essential to the usefulness of activity-based costs. For example, suppose that activity-based unit costs p_j, $j = 1, \ldots, m$ indicate that revenue activity i, operating at the action y_i, is unprofitable. That is, $y_{i0} + \sum_{j=1}^{m} p_j y_{ij} < 0$. The implication is that the residual income of the firm would increase if revenue activity i were dropped or at least down-scaled. Dropping activity i will actually save the cost $\sum_{j=1}^{m} p_j |y_{ij}|$ if each cost activity j can be down-scaled in response to the reduced demand for commodity j. Otherwise actual cost savings may be less than the revenue, y_{i0}, that is lost by dropping activity i.

An investment activity j cannot be down-scaled beyond the capacity that would remain in the absence of gross investment, say $(1-d_j)k_j^o$, where d_j is the rate of physical depreciation. Assumption ID implies that p_j, the unit cost of gross investment, represents a lower bound on the savings that could be achieved by scaling capacity down to any level between y_{jj} and $(1-d_j)k_j^o$. The additional assumption that $I_j(k_j^o, (1-\delta_j)k_j^o) = 0$ implies that $\delta_j \geq d_j$ (book depreciation does not understate physical depreciation), and that p_j is a lower bound on the potential cost reduction from cutting back to any level between y_{jj} and $(1-\delta_j)k_j^o$.

The convexity assumption, K, is needed in Proposition 4.14 to ensure that if manager i can increase measured performance by moving from y_i^* to some other feasible action y_i, then any move from y_i^* in the direction of y_i is also feasible, as well as performance improving. Small moves preserve the inequality $y_{jj} > (1-\delta_j)k_j^o$ for investment activities, so that the performance improvement indicated by the unit costs p_j^* implies an actual improvement in residual income. Hence, at a residual income maximum, the unit costs p_j^* are sufficiently accurate to avoid signalling false opportunities for improvement.

Unfortunately, the signals provided by the ABC mechanism are sufficiently coarse to avoid signalling many real opportunities for improvement as well. However, there is a "second best" optimality criterion that is satisfied by every ABC equilibrium. A plan that cannot be improved by down-scaling any activity or activities was termed a *semi-linear optimum* in Jordan (1994, 3.3), where it was shown that, in the absence of fixed costs, every ABC equilibrium is a semi-linear optimum. It will be shown below that, despite the presence of fixed costs, an analogous result holds in the present model.

The definition of semi-linear optimality with respect to residual income (RISLO), which is stated formally below, requires some care. The ABC costs give activity managers the illusion that if capacity usage is reduced, capacity costs will be reduced proportionately. Accordingly, RISLO is based on a definition of downscaling that incorporates this illusion. In particular, the definition permits capacity downscaling, with capacity valued at its ABC cost, rather than historical cost. That is, capacity is valued the unit cost of gross investment, or zero in the absence of gross investment. Proposition 4.19 states that every ABC equilibrium is an RISLO. Thus, even though potential cost savings over stated by including illusory reductions in the interest and depreciation charges for capital, the residual income of an ABC equilibrium cannot be increased by downscaling.

4.18 Definition: Given a plan $(y_i^*)_i$, define $K^* \in \Re_+^\ell$ as

$$K_j^* = \begin{cases} \dfrac{-y_{j0}^* y_{jj}^*}{y_{jj}^* - (1-\delta_j)k_j^o} & \text{if} \quad y_{jj}^* > (1-\delta_j)k_j^o, \quad \text{and} \\ 0 \quad \text{otherwise} \text{ , for all } j = 1, \dots, l. \end{cases}$$

The plan $(y_i^*)_i$ constitutes a semi-linear optimum with respect to residual income (RISLO) if the problem

$$\begin{aligned} &\text{maximize} \quad \sum_{i=\ell+1}^{n} \lambda_i\, y_{i0}^* - \sum_{i=1}^{\ell}(\rho+\delta_i)\lambda_i K_i^* \\ &\text{subject to} \quad 0 \le \lambda_i \le 1 \quad \text{for all} \quad i=1,\ldots,n, \quad \text{and} \\ &\qquad\qquad \sum_{i=1}^{n} \lambda_i\, y_{ij}^* \ge 0 \quad \text{for all} \quad j=1,\ldots,m \end{aligned}$$

has $\lambda_i = 1$ for all $i = 1, \ldots, n$ as a solution.

4.19 Proposition: Assume N, D and ID, and let $(a_i^*, y_i^*)_i$ be an ABC equilibrium given an initial capacity value vector k^o. Then $(y_i^*)_i$ is an RISLO.

Proof: For each $i = 1, \ldots, \ell$, Proposition 4.11 (iii) and Assumption ID imply that either $y_{i0}^* = 0$ or $y_{ii}^* + \sum_{i' \neq i} y_{i'i}^* = 0$. Assumptions N and D imply that for each $i = \ell+1, \ldots, m$, $y_{ii}^* + \sum_{i' \neq i} y_{i'i}^* = 0$. Hence, if the constrained maximization problem that defines RISLO has a solution $(\lambda_i)_i$ with $\lambda_i = 1$ for $i = m+1, \ldots, n$, then $\lambda_i = 1$ for $i = 1, \ldots, m$ also. Let $(\lambda_i)_{i=m+1}^{n} \in (0,1]^{n-m}$. Then Jordan (1994, Lemma A.6) implies that there exists a unique m-tuple $(\lambda_i)_{i=1}^{m} \in [0,1]^m$ satisfying $\sum_{i=1}^{n} \lambda_i\, y_{ij}^* = 0$ for all $j = 1, \ldots, m$. Then by Proposition 4.11 (ii), $\sum_{i=\ell+1}^{n} \lambda_i\, y_{i0}^* + \sum_{i=1}^{\ell}(\rho+\delta_i)\lambda_i K_i^* = \sum_{i=m+1}^{n} \lambda_i (y_{i0}^* + \sum_{j=1}^{m} p_j^*\, y_{ij}^*)$. For each $i = 1, \ldots, n$, Assumption D implies that $0 \in Y^i$, so Proposition 4.11 (iii) implies that $y_{i0}^* + \sum_{j=1}^{m} p_i^*\, y_{ij}^* \ge 0$. Hence the constrained maximization problem that defines RISLO has a solution with $\lambda_i = 1$ for all $i = m+1, \ldots, n$. ∎

4.20 Remarks: Proponents of activity-based costing claim that it improves decisions about which products to keep and which to drop (e.g. Cooper (1990)). Proposition 4.19 formalizes this claim, since dropping a product corresponds to setting $\lambda_i = 0$ for a revenue activity i. Cooper (1990) also claims that ABC is helpful in product sourcing decisions. In particular, ABC costs should be helpful in deciding whether to replace an internal activity with an external supplier. For example, suppose that some manager j' has the opportunity to purchase commodity j^o from an outside supplier at a price p_{j^o}

per unit. If the ABC unit cost of activity j^o, $p^*_{j^o}$, exceeds p_{j^o}, then Propostion 4.11(iii) implies that manager j' will switch to the outside supplier. Formally, manager j' will change his or her proposed activity vector from $y^*_{j'}$ to

$$y_{j'} = (y^*_{j'0} + p_{j^o}y^*_{j'j^o},\ y^*_{j'1}, \ldots,\ y^*_{j'(j^o-1)},\ 0,\ y^*_{j'(j^o+1)}, \ldots,\ y^*_{j'm}).$$

Proposition 4.21(i), below, states that the ABC unit cost, $p^*_{j^o}$, accurately represents the cost savings that would be achieved by downscaling activity j^o, subject to the same qualification about capacity downscaling as in Proposition 4.19. Assertion (ii) records the fact that at an ABC equilibrium, only unattractive outsourcing opportunities can remain untaken.

4.21 Proposition: Let $(a^*_i, y^*_i)_i$ be an ABC equilibrium, and for each $i = 1, \ldots, m$, let p^*_i be given by Proposition 4.11(ii) and assume that $y^*_{i0} < 0$. For some $j^o \leq m$, let $p_{j^o} \geq 0$, and for some $j' > \ell$ with $y^*_{j'j^o} < 0$, let

$$y_{j'} = (y^*_{j'0} + p_{j^o}y^*_{j'j^o},\ y^*_{j'1}, \ldots,\ y^*_{j'(j^o-1)},\ 0,\ y^*_{j'(j^o+1)}, \ldots,\ y^*_{j'm}).$$

For each $i \neq j'$, let $y_i = y^*_i$. For each $i = 1, \ldots, m$, let $0 \leq \lambda_i \leq 1$ be the unique coefficient such that for all $j = 1, \ldots, m$, $\sum_{i=1}^m \lambda_i y_{ij} + \sum_{i=m+1}^n y_{ij} = 0$. Let $z = \sum_{i=\ell+1}^m \lambda_i y_{i0} + \sum_{i=m+1}^n y_{i0} - \sum_{i=1}^\ell (\rho + \delta_i)\lambda_i K^*_i$, where K^*_i is given by Definition 4.18, $i = 1, \ldots, \ell$. Let $z^* = \sum_{i=\ell+1}^m y^*_{i0} - \sum_{i=1}^\ell (\rho + \delta_i)K^*_i$. Then

i) $z - z^* = (p^*_{j^o} - p_{j^o})|y_{j'j^o}|$; and

ii) if $y_{j'} \in Y^{j'}$ then $p_{j^o} \geq p^*_{j^o}$.

Proof: Assertion (i) follows directly from Jordan (1994, Lemma A.11) if $j' > m$, and from Jordan (1994, Lemma A.13) if $j' \leq m$. Assertion (ii) follows directly from Proposition 4.11(iii). ∎

4.22 Remarks: The outsourcing result is subject to a subtle but substantive limitation in the presence of reciprocal relationships among cost activities. Suppose that activity j'

is a cost activity. Then, in the notation of the Proposition, it can happen that $\lambda_{j'} < 1$. This means that the downscaling of the outsourced activity, j^o, has indirectly lessened the demand for commodity j'. In particular, the amount of commodity j^o that will be purchased externally is not $|y^*_{j'j^o}|$ but the lesser quantity $\lambda_{j'}|y^*_{j'j^o}|$. Thus the definition of z embodies the assumption that the external price p_{j^o} per unit of commodity j^o is unaffected by downscaling the quantity purchased.

Thus the ABC mechanism provides good support for weeding out unprofitable products and outsourcing inputs that can be obtained externally at lower cost. Of course, in evaluating these decisions, only the implications of downscaling the existing plan are considered. If some cost activity has sharply decreasing returns to scale, a reduction in the scale of output would result in more than proportional cost reduction. Such nonlinear opportunities are not revealed by ABC unit costs.

4.23 Relation to recent literature: An important recent addition to the theoretical literature on the activity-based costing of fixed resources is given by Banker and Hughes (1994). Banker and Hughes formulate a two-period model in which the firm chooses fixed levels of certain inputs in the first period, and produces to meet demand in the second period. The production technology is Leontief up to capacity levels set by the fixed inputs, beyond which costs increase at a constant but higher marginal cost. The demand curve facing each of the firm's products is linear, so revenue functions are quadratic. The production technology is known by a production manager, and the demand functions are known by a marketing manager. These aspects of the Banker and Hughes model can be included in the activity model of the present paper as a special case (Banker and Hughes point out that their demand linearity assumption can be dropped, but their use of first-order conditions would presumably still require the concavity of revenue functions). However, the focal feature of their model is the assumption that each demand curve is perturbed by an additive stochastic shock that is not observed until after the fixed inputs are in place and the product prices have been set. They assume that the demand and cost parameters are such that the firm will always find it profitable to meet the entire realized demand. Nonetheless, they obtain the surprising result that if the fixed inputs are chosen to maximize *ex ante* expected profit, then the ABC costs generated by the Leontief technology in the region below the fixed-input capacities represent "economically sufficient" information about the production technology for the marketing manager to choose the expected-profit maximizing product prices.

The Banker and Hughes result suggests the possibility that the nonstochastic model of the present paper might be extended to allow the firm's characteristics to include random disturbance terms, and to replace the residual income performance measure with expected residual income. Unfortunately, Banker and Hughes note that the production manager's choice of the expected-profit maximizing fixed-input levels requires the knowledge of the probability distribution of the demand shocks. Work currently underway by Jordan and Dingbo Xu indicates that even in a two-activity, two-period model, expected-profit maximization can require an infinite amount of communication between managers.

Accounting Horizons has published a series of papers for educators and practitioners on activity-based costing, some of which explicitly address the treatment of fixed costs. Malcom (1991) and Mak and Roush (1994) recommend that the costs of individual activities be decomposed into variable and fixed (Malcom uses the term "sticky") components, which Kaplan (1994) regards as a potentially useful refinement. The present paper, in contrast, posits a sufficiently fine-grained activity model that fixed and variable costs are separated into different activities. In particular, the acquisition of each fixed resource is a separate investment activity. Activities that use fixed resources are treated as though their costs were entirely variable, although our downscaling assumption (Assumption D) is much weaker than the proportionality assumption that accompanies most analyses of variable cost. Indeed, a major goal of this paper is to show that activity-based costs can be useful even when the proportionality assumption is violated.

Cooper and Kaplan (1992) and Kaplan (1994) recommend a specific calculation of the unit cost of a fixed resource. Their examples involve only contemporaneous expenses, so their cost calculations do not involve interest charges. More importantly, they recommend that the budgeted unit cost of a fixed-resource be computed by dividing the total budgeted cost of the resource by its "practical capacity" (e.g., Kaplan (1994, p. 106)). In the notation of the present model (including the interest charge), this would yield :

$$CK) \qquad \frac{(\rho + \delta_j)K_j}{max\{y^*_{jj},\ (1-\delta_j)k^o_j\}},$$

taking $(1-\delta_j)k^o_j$ as a measure of practical capacity in the absence of gross investment. The unit capacity cost in the present model (see 4.11 (ii)) is:

$$AJM) \qquad \begin{cases} \dfrac{-(\rho+\delta_j)y^*_{j0}}{y^*_{jj} - (1-\delta_j)k^o_j} & \text{if} \quad y^*_{jj} > (1-\delta_j)k^o_j, \quad \text{and} \\ 0 \quad \text{otherwise.} \end{cases}$$

The major distinction between the two formulae lies in the treatment of excess capacity ($y^*_{jj} \leq (1-\delta_j)k^o_j$). In this case, the capacity expense, $(\rho+\delta_j)K_j$, which is charged against residual income, is a purely sunk cost, and the economic cost of capacity is zero. Nonetheless, (CK) is positive in this case. If the formula (CK) were used in the ABC mechanism, Proposition 4.15 and Theorem 4.16 would no longer hold. A residual income maximum could fail to be an ABC equilibrium because activity managers could be discouraged from fully utilizing an economically costless resource. Any cost system that allocates the cost of fixed resources risks overstating the potential cost savings from dropping products or outsourcing activities. As mentioned in Section 4.17, the ABC mechanism of the present paper is no exception. The capacity cost given by (AJM) keeps the overstatment of cost savings below the point of discouraging the optimal use of the resource.

4.24 Example: The relation between ABC equilibrium and residual income maximization can be illustrated by a simple quadratic example. Suppose the firm consists of a single investment activity and a single revenue activity. Where there is no risk of confusion, we will suppress the activity subscripts. The investment function I is given by

$$I(k^o, k) = 120[k - (1-\delta)k^o]^+ + 5[(k - k^o)^+]^2,$$

which satisfies Assumptions C, I, Z and ID. Hence the set of feasible actions for the investment activity is given by

$$Y^1(k^o) = \{(y_{10}, y_{11}) : y_{11} \geq 0 \quad \text{and} \quad y_{10} \leq -I(k^o, y_{11})\}.$$

For the revenue activity, the feasible action set is given by

$$Y^2 = \{(y_{20}, y_{21}) : y_{21} \leq 0 \quad \text{and} \quad y_{20} \leq \gamma(-y_{21}) - (y_{21})^2\},$$

where γ is a positive parameter that will be varied to create different versions of the example. Hence Y^2 satisfies Assumptions B C, D, N and K. As long as $k^o > 0$, Assumption AA is satisfied, so the firm satisfies all of the assumptions of Sections 3 and 4. A plan $(y_{10}, y_{11}; y_{20}, y_{21})$ is feasible for the firm given an initial capacity k^o if $(y_{10}, y_{11}) \in Y^1(k^o)$, $(y_{20}, y_{21}) \in Y^2$ and $y_{11} + y_{21} \geq 0$.

Given an initial book-value K^o, a plan $(y_{10}, y_{11}; y_{20}, y_{21})$ results in the new book value

$$K = (1-\delta)K^o - y_{10},$$

and the residual income

$$RI = y_{20} - (\rho + \delta)K.$$

Since $\gamma > 0$, the choice of a residual income maximizing plan reduces to a single variable problem, because an RI maximizing plan must satisfy $y_{11} + y_{21} = 0$, $y_{10} = -I(k^o, y_{11})$ and $y_{20} = \gamma(-y_{21}) - (y_{21})^2$, provided that k^o is not so large that excess capacity is optimal (i. e., $(1-\delta)k^o \leq \gamma/2$). Let k denote the common value of y_{11} and $-y_{21}$. Viewing RI as revenue minus cost, we can express the marginal revenue as

$$MR(k) = \gamma - 2k,$$

and the marginal cost as

$$MC(k) = \begin{cases} (\rho+\delta)[120+10(k-k^o)] & \text{if } k > k^o; \\ (\rho+\delta)120 & \text{if } (1-\delta)k^o < k \le k^o; \\ 0 & \text{if } k < (1-\delta)k^o. \end{cases}$$

In this example, Proposition 3.35 implies that the stationary capacity, k^s can be obtained as the unique solution to the equations $k^o = k^s$ and $MR(k^s) = MC(k^s)$, which yield

$$k^s = \frac{\gamma - (\rho+\delta)120}{2}.$$

Given an initial capacity k^o, the residual income maximizing capacity satisfies

$$\hat{k} = sup\{k : MR(k) \ge MC(k)\},$$

which can be expressed more explicitly by distinguishing three cases depending on the size of the initial capacity relative to the stationary optimum:

$$\hat{k} = \begin{cases} k^o + \frac{(k^s - k^o)}{[1+5(\rho+\delta)]} & \text{if } k^0 < k^s; \\ k^s & \text{if } (1-\delta)k^o \le k^s \le k^o; \\ (1-\delta)k^o & \text{if } k^s < (1-\delta)k^o \le \gamma/2. \end{cases}$$

In the first case, the firm is growing toward k^s; in the second case, the firm has reached the stationary optimum; and in the third case, the firm is contracting toward k^s via depreciation while continuing to produce at full capacity.

The ABC mechanism in this example can be described as follows. Given a proposed plan $(\tilde{y}_{10}, \tilde{y}_{11}; \tilde{y}_{20}, \tilde{y}_{21})$, manager 1 receives the report $(a_{10}, b_{11}, c_{11}) = (\tilde{y}_{20}, -\tilde{y}_{21}, 0)$, which parameterizes the budget function $\beta_1((a_{10}, b_{11}, c_{11}), (y_{10}, y_{11})) = (y^b_{10}, y^b_{11})$ as

$$(y^b_{10}, y^b_{11}) = (y_{10},\ \max\{b_{11}, y_{11}\}),$$

and the performance function

$$\pi_1((a_{10}, b_{11}, c_{11}), (y^b_{10}, y^b_{11})) = a_{10} - (\rho+\delta)K,$$

where $K = (1-\delta)K^o - y^b_{10}$. Manager 1 is constrained to choose a proposal (y_{10}, y_{11}) that satisfies the feasibility requirement that $(y^b_{10}, y^b_{11}) \in Y^1(k^o)$. As a result, Manager 1 will respond by simply meeting Manager 2's proposed capacity demand $(y_{11} = -\tilde{y}_{21})$ and reporting the minimal investment cost needed to achieve that capacity $(y_{10} = -I(k^o, y_{11}))$. Manager 2 receives the report (a_{20}, b_{21}, c_{21}), given by $a_{20} = -(\rho+\delta)K$, $b_{21} = -\tilde{y}_{11}$ and

$$c_{21} = \begin{cases} \dfrac{(\rho+\delta)\tilde{y}_{10}}{\tilde{y}_{11} - (1-\delta)k^o} & \text{if} \quad \tilde{y}_{11} - (1-\delta)k^o \geq 0; \quad \text{and} \\ 0 \quad \text{otherwise.} \end{cases}$$

which parameterizes the budget function $\beta_2((a_{20}, b_{21}, c_{21}), (y_{20}, y_{21})) = (y^b_{20}, y^b_{21})$ as

$$(y^b_{20}, y^b_{21}) = (y_{20},\ \max\{b_{21}, y_{21}\}),$$

and the performance function

$$\pi_2((a_{20}, b_{21}, c_{21}), (y^b_{20}, y^b_{21})) = a_{20} + y^b_{20} + c_{21}(y^b_{21} - b_{21})^+.$$

Manager 2 must choose (y_{20}, y_{21}) so that $(y^b_{20}, y^b_{21}) \in Y^2$. As a result, Manager 2 will respond by choosing $-y_{21} = \max\{k : k \leq -b_{21}$ and $MR(k) \geq c_{21}\}$, and reporting $y_{20} = \gamma(-y_{21}) - (y_{21})^2$. Hence, any proposed plan $(\tilde{y}_{10}, \tilde{y}_{11}; \tilde{y}_{20}, \tilde{y}_{21})$ that satisfies $\tilde{y}_{11} = -\tilde{y}_{21}$, $\tilde{y}_{10} = -I(k^o, \tilde{y}_{11})$, $\tilde{y}_{20} = \gamma(-\tilde{y}_{21}) - (\tilde{y}_{21})^2$ and $MR(-\tilde{y}_{21}) \geq c_{21}$ is an ABC equilibrium. Let k^* denote that common value of $\tilde{y}_{11}$ and $-\tilde{y}_{21}$. Then the key equilibrium condition, $MR(k^*) \geq c_{21}$, reduces to

$$ABC) \qquad MR(k^*) \geq \begin{cases} (\rho+\delta)[120 + \frac{5(k^*-k^o)^2}{(k^*-(1-\delta)k^o)}] & \text{if } k^o < k^*; \\ (\rho+\delta)120 & \text{if } (1-\delta)k^o < k^* \leq k^o; \\ 0 & \text{if } k^* \leq (1-\delta)k^o. \end{cases}$$

Any k^* that satisfies (ABC) is an ABC equilibrium level of capacity usage.

The inequality (ABC) allows a wide range of equilibria. Since the budget functions do not allow Manager 2 to increase capacity unilaterally above the level proposed by Manager 1, the mechanism can get stuck at low output levels. However, in this case, $MR(k^*)$ is well above the reported capacity cost c_{21}, so one might expect Manager 2, in the interest of increasing π_2, to communicate directly with Manager 1 and attempt to elicit an increase in $\tilde{y}_{11}$. Since the ABC mechanism is RI-seeking (Theorem 4.16), such an attempt can succeed. There are also ABC equilibria with capacity levels above the residual income maximum $\hat{k}$. The investment function $I(k^o, \cdot)$ is strictly convex above k^o, reflecting expansion costs, so the average cost of gross investment in this region is less than the marginal cost. Therefore there can be ABC equilibria satisfying $MC(k^*) > MR(k^*) \geq c_{21}$. Although Manager 1 knows $MC(k^*)$, the report (a_{10}, b_{11}, c_{11}) contains no revenue information, so neither manager has any information suggesting that k^* is too large. This problem can be mitigated by extending the budget mechanism to report average revenue, $\tilde{y}_{20}/k^*$, to Manager 1, and allowing Manager 1 to reduce capacity below $-\tilde{y}_{21}$ subject to a performance penalty equal to average revenue times the amount of the shortfall. This will prevent $MC(k^*)$ from exceeding average revenue. However, if revenue is a strictly concave function of k^*, as in the present example, average revenue exceeds marginal revenue, so there will continue to be a range of ABC equilibria with $MC(k^*) > MR(k^*)$, that is, $k^* > \hat{k}$. Such an extension of the ABC mechanism is constructed in Jordan(1994, Section 6) in a single-period model without fixed costs.

Suppose that c_{21} is calculated not as the unit cost of gross investment, but as the unit cost of practical capacity, as suggested by Cooper and Kaplan (1992) and Kaplan (1994). The formula 4.23 (CK), applied to the present example, yields

$$c'_{21} = \frac{(\rho+\delta)K}{max\{\tilde{y}_{11},\ (1-\delta)k^o\}},$$

and the equilibrium condition becomes $MR(k') \geq (\rho+\delta)K/max\{k',\ (1-\delta)k^o\}$. Assume that $K^o = 120k^o$, i. e., that the initial book value is not distorted by previous expansion costs. Then the equilibrium condition can be written

$$ABC') \qquad MR(k') \geq \begin{cases} (\rho+\delta)[120 + \frac{5(k'-k^o)^2}{k'}] & \text{if } k^o < k'; \\ (\rho+\delta)120 & \text{if } k' \leq k^o. \end{cases}$$

Comparing the equilibrium conditions (ABC) and (ABC′) indicates that c'_{21} is lower than c_{21} if the firm is expanding, higher if the firm is contracting, and the same if capacity is stationary. In general, c_{21} approximates MC as well or better than c'_{21}.

The comparison of equilibria is illustrated in the following two numerical cases. In both cases we retain the assumption that $K^o = 120k^o$, and also set $\delta = 0.4$ and $\rho = 0.1$. First, let $\gamma = 150$, and $k^o = 100$. These values specify a contractionary case, with $k^s = 45 < 60 = (1-\delta)k^o$. The residual income maximum is $\hat{k} = 60$, with $MR(60) = 30$. The residual income maximum is the largest ABC equilibrium, but since $c'_{21} = 120$, the largest ABC$'$ equilibrium is the stationary optimum, k^s. Thus if the unit cost of practical capacity is used in place of c_{21}, which is zero in this case, then 15 units of costless capacity will remain unused.

To specify an expansionary case, set $\gamma = 300$ and $k^o = 50$. Then $k^s = 120 > 30 = (1-\delta)k^o$, and the residual income maximum is $\hat{k} = 70$. In this case, $MR(\hat{k}) = 160 > c_{21} = 85 > c'_{21} \approx 74$, so the residual income maximum is supported as an equilibrium by both costing methods. The largest ABC equilibrium is $k^* \approx 88.4$, at which $MR(k^*) = c_{21} \approx 123 > c'_{21} \approx 102$. It follows from the inequality that (ABC$'$) permits larger equilibrium capacities with correspondingly lower residual income. The largest ABC$'$ equilibrium is $k' \approx 94.1$, at which $MR(k') = c'_{21} \approx 112 < c_{21} \approx 136$. Again, c_{21} provides the better approximation to marginal cost.

4.25 Asset Sales: The ABC mechanism directs cost activity managers to meet the demand for their output at the minimum cost. As illustrated by the previous example, the manager of an investment activity receives no revenue information, and has no means to compare the cost of providing capacity with the revenue generated by its use. Only revenue activity managers are in a position to make such comparisons. As a result, the sale of capital goods by investment managers is outside the scope of the present model. Of course, such sales as arise incidentally in the course of replacing old equipment with new create no conceptual or formal difficulties. Any revenue realized in the sale of an old drill-press is subtracted from the cost of the new drill-press in calculating y_{i0}, in accordance with the conventional accounting treatment of similar asset exchanges.

The sale of assets for the purpose of earning revenue, as opposed to minimizing replacement cost, conflicts with our sign convention that $y_{i0} \leq 0$. The sign convention and the more explicit restriction in Section 3 that capacity can decrease only through depreciation are imposed largely for simplification. The opportunity to sell an asset, if anticipated at the time the asset is acquired, reduces the extent to which its cost is fixed and can affect its capitalization. We have avoided such complications in order to focus on fixed costs. It is likely that only a minor modification of the model would be required to encompass the unanticipated sale of unused capacity. However, the sale of capacity that would otherwise be used requires an opportunity cost calculation that is not supported by the ABC mechanism.

The asset that seems least suited to our model of investment activities is land. Since land does not depreciate, gross investment occurs only when new land is acquired. Hence, under the ABC mechanism, the unit cost of land is positive only when new acquisitions are budgeted. The zero depreciation rate also excludes land from the dynamic analysis in Section 3. If the firm's initial land holdings exceed their stationary optimal level, the stationary optimum cannot be achieved in a model without asset sales.

The ABC mechanism is refined in Jordan (1994, Section 6) by using budgeted revenues to construct a measure of opportunity cost. The refinement, called the *ABOC mechanism*, improves performance by enabling managers to reject many of the suboptimal ABC equilibria. However, the cost calculations are considerably more intricate and, needless to say, unconventional. An extension of the ABOC mechanism to the present model could provide capacity costs in the absence of gross investment that are positive but not disfunctional, and might provide at least limited support for the disposal of assets that can be sold for greater revenue than is generated by their use.

5. Stationary Equilibria

A stationary ABC equilibrium $(a_i^*, y_i^*)_i$ is an equilibrium with the additional property that $k_i^o = y_{ii}^*$ for each investment activity i, so the plan remains constant over time. Stationary equilibria deserve special attention for two reasons. First, Section 3 showed that a sequence of residual income maximizing plans is asymptotically stationary. Second, the capital valuation equation, $K_i^t = (1-\delta_i)K_i^{t-1} - y_{i0}^t$, drives K_i^t to the stationary book-value, $K_i^* = -y_{i0}^*/\delta_i$, if $\delta_i > 0$ and $y_{i0}^t = y_{i0}^*$ for all t. This implies that the average cost of gross investment is equal to the average book-value of capacity, so the ABC unit cost p_i^* is given by $p_i^* = (\rho + \delta_i)K_i^*/y_{ii}^*$. This fact, which is recorded as Proposition 5.2, is independent of any relation between the depreciation rate δ_i and the physical depreciation of capacity.

Proposition 5.3 states that, under the hypothesis of Proposition 4.15, a stationary plan that maximizes residual income is also a stationary ABC equilibrium. Proposition 5.5 states that a stationary equilibrium, together with the corresponding stationary book value vector, constitutes a semi-linear optimum.

5.1 Definitions: An n-tuple $(a_i^*, y_i^*)_i$ is a *stationary ABC equilibrium* if $(a_i^*, y_i^*)_i$ is an ABC equilibrium for the initial capacity vector $(y_{ii}^*)_{i=1}^{\ell}$. A plan $(y_i^*)_i$ is a *stationary residual income maximizer* if it maximizes residual income given the initial capacity vector $(y_{ii}^*)_{i=1}^{\ell}$.

5.2 Proposition: Let $(a_i^*, y_i^*)_i$ be a stationary ABC equilibrium. For each investment activity $i = 1, \ldots, \ell$ let p_i^* be given by 4.11 (ii), and let

$$K_i^* = \begin{cases} -y_{i0}^*/\delta_i & \text{if} \quad \delta_i > 0; \quad \text{and} \\ 0 & \textit{otherwise}. \end{cases}$$

Then $p_i^* y_{ii}^* = (\rho + \delta_i)K_i^*$ for each $i = 1, \ldots, \ell$.

Proof: The result follows from 4.11 (ii) and the definition of K_i^*, together with the fact that $k_i^o = y_{ii}^*$. ∎

5.3 Proposition: Assume D, N, K, AA, and DI. Let $(y_i^*)_i$ be a stationary residual income maximizer. If, for each $i = 1, \ldots, \ell$, $I_i(y^*_{ii}, (1-\delta_i)y^*_{ii}) = 0$, then there is a unique $(a^*_i)_i \in \Pi^n_{i=1} A^i$ such that $(a^*_i, y^*_i)_i$ is a stationary ABC equilibrium.

Proof: This follows immediately from Proposition 4.15 and Definition 5.1. ∎

5.4 Definition: A plan $(y^*_i)_i$ is a *long-run semi-linear optimum* (LRSLO) if $(y^*_i)_i$ is an RISLO for the initial capacity vector $k^o = (y^*_{11}, \ldots, y^*_{\ell\ell})$.

5.5 Proposition: Assume N, D, and ID, and let $(a^*_i, y^*_i)_i$ be a stationary ABC equilibrium. Then $(y^*_i)_i$ is an LRSLO.

Proof: This follows directly from Proposition 4.19 and Definition 5.4. ∎

6. Conclusion

This paper provides an integrated model of activity-based costing and performance measurement in an informationally decentralized participatory budgeting process. Fixed costs are included by incorporating investment decisions in the budgeting process. The objective of the budgeting process each period is the maximization of residual income (economic value added). Residual income maximization is rigorously justified as an asymptotic approximation to discounted cash flow maximization, extending an earlier result of Anctil (1994).

The ABC costs support residual income maximization in the sense that a residual income maximum is an equilibrium of the budget process. However, since the model permits nonlinear production technologies, ABC costs are not generally equal to the true marginal costs, so there are also budget equilibria that are not residual income maxima. For this reason, the ABC budget mechanism is shown to have two additional properties. First, the mechanism has the "open-architecture" property that if activity managers use additional information or collude in the budgeting process to increase their measured performance, then they will also increase the firm's residual income. Second, although the budget mechanism does not enforce residual income maximization, it has the "second-best" property of enforcing the dropping of unprofitable products, in the following sense. Suppose that dropping a particular product would result in a revenue loss that is less than the cost savings that would be obtained by linearly downscaling the expenses and activities formerly needed by that product to the maximum extent consistent with meeting the needs of the remaining products. Then that product will not be produced in a budget equilibrium. In the same sense, the budget mechanism also enforces the outsourcing of activities that can be more cheaply obtained externally. Of course, the ABC costs are not sufficiently accurate to reveal opportunities for super-linear cost savings. These results extend analogous results obtained by Jordan (1994) in a single-period model that excludes fixed costs.

The principal remaining limitation of the model is that it is entirely nonstochastic. Uncertainty in the sense of ignorance is plausibly represented by our assumption that an activity manager has no direct knowledge of the characteristics of other activities. The open architecture property ensures that additional knowledge, if acquired, will be used in the interest of the firm. However, uncertainty in the sense of known probability distributions governing market or technological characteristics is excluded from the model. Even in a two-period model with stochastic activity characteristics, the complexity of expected profit maximization makes a theory of accounting support problematic. It appears unlikely that a model along the present lines could be attempted unless a practical heuristic can be found to play the same role for expected profit maximization that residual income plays for present value maximization.

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