ENDOGENOUS ASSET SPECIFICITY IN A PRINCIPAL-AGENT MODEL: AN INTERPRETATION OF MANAGERIAL MYOPIA

by

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ABSTRACT

This paper studies the implications of an agency problem for the optimal asset specificity of a firm. The main result is that there exists a region in the parameter space such that the optimum in the presence of an agency problem has less specific assets and more frequent liquidation than in the absence of an agency problem. This result provides an interpretation of so-called managerial myopia. That is, the firm seems to be so concerned about the liquidation value of its investment that it is willing to sacrifice the expected output of its investment by using a less specific asset.

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I. INTRODUCTION

This paper studies the implications of an agency problem for the optimal degree of asset specificity. Asset specificity refers to the degree to which an asset is specifically adapted to a particular project. An increase in the specificity level of the asset raises its productivity within the project and reduces its value outside the project. Therefore, the choice of an asset specificity level entails a trade-off between the on-going value of the project and the liquidation value of the project.

Asset specificity has so far received only limited attention. Williamson (1988) is the first study that explores the potential role of asset specificity. In an agency setting, Williamson studies how asset specificity determines the capital structure of corporations. However, no one has studied how the optimal asset specificity level is determined in an environment where an agency problem exists. This paper endogenizes asset specificity in a principal-agent model and studies how the existence of an agency problem can affect the optimal degree of asset specificity.

The agency problem considered in this paper falls within the class of principal-agent models in which the agent's labor input (effort) and the productivity shock are private information to the agent. The model extends Sappington (1983) by allowing asset specificity to be a choice variable. The main result of this paper is that there exists a region in the parameter space such that the optimal allocation in the presence of the agency problem has more frequent liquidation and less specific assets than the one in the absence of the agency problem. This result provides an interpretation of so-called managerial myopia because an outside observer could infer that individuals in the environment with the agency problem are more willing to liquidate and they are so concerned about the liquidation value of their investments that they are willing to use less specific assets.

Stein (1988, 1989), Bebchuk and Stole (1993) have attempted to explain managerial myopia. They show that myopic behavior can be an equilibrium outcome if the allocation of resources between long-term and short-term projects is only privately observerable to the manager and if the manager cares directly about the short-term stock price performance of his company. This class of models begs the question of why managers have such short-term objectives and it is questionable why the allocation of resources is not publicly observable. On the other hand, this paper shows that even without these questionable assumptions, the existence of an agency problem can give rise to an allocation that resembles a myopic outcome. Furthermore, while managerial myopia has been blamed for the relatively poor growth and productivity performance of the United States in the last few decades, this paper shows that it could indeed be an optimal arrangement in an evnvironment where agency problem exists.

The rest of the paper is structured as follows. Section II describes the model. Section III states the main proposition of the paper and outlines the proof of the proposition. Sections IV and V provide the proof of the proposition. Section IV shows that in the absence of a liquidation technology, the agency problem reduces the on-going values of the project in bad states and has no effect on the optimal asset specificity level. Section V incorporates the liquidation technology into the model and shows that there is a range of liquidation values such that the existence of the agency problem will increase the optimal frequency of liquidation and reduces the optimal asset specificity level. Section VI discusses how the main result of this paper can provide an interpretation of so-called managerial myopia. Section VII concludes with some suggestions for future research.

II. THE MODEL

There are two individuals in the environment : the principal and the agent. The principal is endowed with one unit of indivisible capital good and the agent is endowed with $\bar{a}$ unit of labor where $\bar{a} > 0$. There is a consumption good x in the environment. The principal's utility function is $U^P(x) = x$ for all $x \in \Re_+$. The agent's utility function is $U^A(x,a) = x - h(a)$ for all $x \in \Re_+$ and for all $a \in [0,\bar{a}]$. The function $h(\cdot)$ is twice differentiable on $[0,\bar{a})$ where $h(0) = h'(0) = 0$, $\lim_{a \to \bar{a}} h(a) = \lim_{a \to \bar{a}} h'(a) = +\infty$ and for all $a \in (0,\bar{a})$, $h'(a) > 0$, and $h''(a) > 0$.

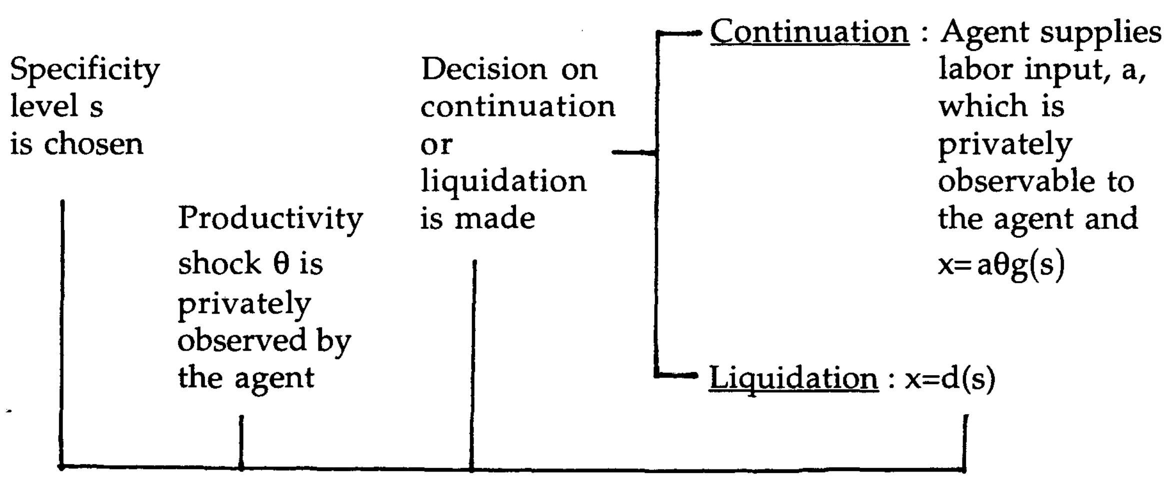
In order to transform the principal's capital good into the consumption good, the capital good has first to be adapted to a project. The production function of the project is given by $x = a\theta g(s)$ where a is the agent's labor input, θ is a productivity shock and s corresponds to the extent that the capital good is adapted to the project, i.e., the asset specificity level. The degree of specificity can be chosen from a closed interval $[\underline{s},\bar{s}] \subset \Re$ where $\underline{s}$ is the lowest specificity level possible, $\bar{s}$ is the highest specificity level possible and for all $s \in [\underline{s},\bar{s}]$, increasing s means increasing the asset specificity level. For simplicity, it is assumed that the cost of adapting the capital good to the project is zero. The function $g(\cdot)$ is a mapping from $[\underline{s},\bar{s}]$ to $\Re_{++}$. It is twice differentiable on $[\underline{s},\bar{s}]$ where $g'(\bar{s}) = 0$. And for all $s \in [\underline{s},\bar{s})$, $g'(s)>0$ and $g''(s)<0$.

After the capital good has been adapted to the project, the agent privately observes the productivity shock of the project. The sample space of θ, the productivity shock, is a two-element set $\{\theta_1,\theta_2\} \subset \Re$ where $\theta_2 > \theta_1 > 0$ and the probability measure on the sample space is π. For i=1,2, $\pi(\theta = \theta_i) > 0$. Throughout this paper, state i refers to the event that $\theta = \theta_i$ and $\pi_i \equiv \pi(\theta = \theta_i)$ for i=1,2.

After the agent observes the productivity shock, the project can either be continued or liquidated. The decision on continuation or liquidation is denoted by *l* where *l* = 0 means continuation and *l* = 1 means liquidation. The decision on liquidation or continuation is publicly observable and verifiable. If continuation occurs, the agent will supply his labor input and the output will be given by $x=a\theta g(s)$. While the output of the project is publicly observable and verifiable, the agent's labor input is private information to the agent. On the other hand, if liquidation occurs, the output is independent of the productivity shock and does not require the agent's labor input. More specifically, the production function is given by $x = d(s)$ where $d(\cdot)$ is a mapping from $[\underline{s},\bar{s}]$ to $\Re_{++}$. And for all $s \in [\underline{s},\bar{s}]$, it is twice differentiable with $d'(s)<0$ and $d''(s) \leq 0$.

In the above formulation, the functions $g(\cdot)$ and $d(\cdot)$ capture the idea that increasing the asset specificity level will increase the spread between the values of the asset within the project and outside the project . For instance, as s increases, the productivity of the asset within the project will increase (because $g'(s) \geq 0$ for all $s \in [\underline{s},\bar{s}]$ with $g'(s)=0$ only at $s=\bar{s}$) and yet the value of the asset outside the project will decrease (because $d'(s)<0$ for all $s \in [\underline{s},\bar{s}]$).

The following timeline summarizes the production process and the information structure :



On the above timeline, all variables are publicly observable and verifiable unless specified otherwise.

Sappington (1983) studies how the existence of limited liability constraints on the agent's compensation may affect the optimal contractual arrangement in a principal-agent model. This paper adopts the same approach and assumes that if the agent chooses to renege on the ex ante arrangement after observing the productivity shock, the minimum consumption that has to be allocated to the agent is L unit of consumption good, $L \geq 0$. Therefore, no matter what the productivity shock turns out to be, if the agent chooses not to supply any labor input, his ex post utility level is $U^A(L,0) = L - h(0) = L$. Hence, in any state, if x is the amount of consumption good allocated to the agent and a is the labor input, they must satisfy the agent's ex post individually rational constraint :

$$U^A(x,a) \geq U^A(L,0) = L\text{-}h(0) = L.$$

Finally, since this paper studies the implication of an agency problem on the optimal degree of asset specificity, two sets of optimal allocations have to be considered. One is in the complete information environment where no agency problem exists and one is in the incomplete information environment where the agency problem exists. The complete information environment refers to the environment in which all the information is publicly observable and verifiable. In this model, it means that both the productivity shock and the agent's labor input are also publicly observable and verifiable. The incomplete information environment refers to the environment in which the agent possesses some private information that cannot be observed publicly. In this model, the agent's private information is the productivity shock and his own labor input. And by revelation principle, only the class of incentive compatible allocations is considered in this paper. The effect of the agency problem on the optimal allocations is studied by comparing the optimal allocations in the two environments.

III. MAIN RESULT

This section presents the main result of the paper and provides an outline of the major arguments that establish it. The main result of this paper is summarized in the following proposition.

Main Proposition .

There exists a region in the parameter space such that :
(1) in the complete information environment, it is optimal to
  (a) continue the project in both states, and
  (b) choose an asset specificity level $s = \bar{s}$;
(2) in the incomplete information environment, it is optimal to
  (a) liquidate at least in state 1, and
  (b) choose an asset specificity level $s < \bar{s}$.

That is, the optimum in the presence of the agency problem has lower specificity and more frequent liquidation than in the absence of the agency problem.

The above proposition is established in two major steps. The first step is to show that without a liquidation technology, the existence of the agency problem reduces the optimal consumption of the principal in both states and has no effect on the optimal asset specificity level. The second step is to show that there is a range of liquidation values such that the result of the above proposition follows.

On the one hand, if the liquidation value is lower than the on-going values of the project in both states in the complete information environment, then it is optimal to continue with the project in both states. Consequently, the optimal asset specificity level remains at the same level when the liquidation technology does not exist .

On the other hand, in the incomplete information environment, if the agency problem is so bad that the liquidation value is higher than the on-going value of the project in state 1, then liquidating in state 1 is a better alternative. First, it can increase the net suplus of the economy in state 1 simply because the liquidation value is higher than the on-going value of the project. Second, it can increase the principal's state-2 consumption because liquidating in state 1 relaxes the incentive compatibility constraint and subsequently reduces the agent's compensation in state 2. Therefore, it is optimal to liquidate at least in state 1. And if liquidation occurs at least in state 1, the asset specificity level will be chosen to balance the tradeoff between the liquidation value of the project and the on-going value of the project. As a result, the optimal asset specificity level will be lower than the one when the liquidation technology does not exist.

Hence, if the liquidation values satisfy the conditions described above, then the optimal allocation in the incomplete information environment will have more frequent liquidation and a lower asset specificity level than the one in the complete information environment.

The next two sections provide the formal proof of the above proposition. Section IV establishes the first step of the proof and section V establishes the second step of the proof .

IV. OPTIMAL ALLOCATIONS WITHOUT A LIQUIDATION TECHNOLOGY

This section considers an environment identical to the one described in section II except that the liquidation technology does not exist. Its purpose is to establish three lemmas to be used in section V. In particular, it shows that without the liquidation technology, the existence of the agency problem reduces the optimal consumption of the principal in both states. Consequently, it is plausible to find a range of liquidation values such that the existence of a liquidation technology can increase the ex ante expected utility of the principal in the incomplete information environment but not in the complete information environment.

When there is no liquidation technology, an allocation[1] is a 7-tuple $\left\langle s,\left(a_i,x_i^A,x_i^P\right)_{i=1,2}\right\rangle$ where s is the asset specificity level, a_i is the agent's labor input in state i , x_i^A is the agent's consumption in state i and x_i^P is the principal's consumption in state i, i=1,2. The optimal allocations in both the complete information environment and the incomplete information environment are characterized by maximizing the principal's ex ante expected utility level subject to a given ex ante expected utility level of the agent and the feasibility constraints. That is, the social planner's problem is to choose an allocation $\left\langle s,\left(a_i,x_i^A,x_i^P\right)_{i=1,2}\right\rangle$ to solve the following maximization problem (P1) :

$$\text{Maximize} \quad \sum_{i=1}^{2}\pi_i x_i^P$$

subject to

$$\sum_{i=1}^{2}\pi_i\left[x_i^A - h(a_i)\right] \geq U^0 , \tag{C1}$$

$$a_i\theta_i g(s) \geq x_i^A + x_i^P \text{ for i=1,2,} \tag{C2}$$

$$x_i^A - h(a_i) \geq L \text{ for i=1,2,} \tag{C3}$$

[1] For each asset specificity level and each state, there is a one-to-one relationship between the labor input and the output of the project. Therefore, specifying a state-contingent output can be formulated as specifying a state-contingent labor input. This paper adopts the latter approach for convenience.

$$x_i^A - h(a_i) \geq x_j^A - h\left(a_j \cdot \frac{\theta_j}{\theta_i}\right) \text{ for } j \neq i \text{ , } i=1,2. \qquad \text{(C4)}$$

C1 is the constraint that the agent's ex ante expected utility level has to be at least U^0. C3 is the agent's ex post individually rational constraint in each state. In this paper, the special case $L=U^0=0$ is considered. The second equality is merely a simplification. The first equality implies that the agent is protected from suffering any loss even if he walks away from the project after observing the productivity shock[2]. C2 is the resource constraint for consumption in each state. C4 are the incentive compatibility constraints. The left hand side of C4 is the agent's ex post utility if he truthfully reports the underlying state i. The right hand side of C4 is the agent's ex post utility if he lies and reports that the state is $j, j \neq i$. If the agent reports that the state is j, he has to produce an output $x=a_j\theta_j g(s)$. Therefore, when the true state is i, if the agent reports that the state is j, $j \neq i$, he has to supply a labor input $\left(a_j \cdot \frac{\theta_j}{\theta_i}\right)$ to produce the output $x=a_j\theta_j g(s)$ because $\left(a_j \cdot \frac{\theta_j}{\theta_i}\right)\theta_i g(s) = a_j\theta_j g(s)$.

In the complete information environment, since the productivity shock and the agent's labor input are publicly observable and verifiable, constraint C4 can be dropped from the maximization problem. In the incomplete information environment, however, all constraints C1-C4 are have to be considered. Let $\left\langle s^*, \left(a_i^*, x_i^{A*}, x_i^{P*}\right)_{i=1,2}\right\rangle$ denote the solution to the above maximization problem in the complete information environment and $\left\langle \hat{s}, \left(\hat{a}_i, \hat{x}_i^A, \hat{x}_i^P\right)_{i=1,2}\right\rangle$ denote the solution to the above maximization problem in the incomplete information environment[3] .

In order to establish the three lemmas in this section, it is useful to characterize the solutions of the above maximization problem in two steps.

2 In a contractual context, Sappington (1983) refers to this restriction as "zero-liability ".

3 From now on, it is understood that "the maximization problem in the complete information environment" refers to the social planner's problem where constraint C4 can be dropped from the maximization problem and "the maximization problem in the incomplete information environment" refers to the social planner's problem where constraint C4 has to be considered.

The first step is to characterize the optimal choice of $\left(a_i, x_i^A, x_i^P\right)_{i=1,2}$ in the above maximization problem for each $s \in [\underline{s}, \bar{s}]$. Let $\left(a_i^*(s), x_i^{A*}(s), x_i^{P*}(s)\right)_{i=1}^2$ denote the solution of $\left(a_i, x_i^A, x_i^P\right)_{i=1}^2$ of this sub-problem in the complete information environment and let $\left(\hat{a}_i(s), \hat{x}_i^A(s), \hat{x}_i^P(s)\right)_{i=1}^2$ denote the solution of $\left(a_i, x_i^A, x_i^P\right)_{i=1}^2$ of this sub-problem in the incomplete information environment. Given the solutions in the first step, the optimal asset specificity level is an $s \in [\underline{s}, \bar{s}]$ that maximizes $\sum_i \pi_i x_i^{P*}(s)$ in the complete information environment and $\sum_i \pi_i \hat{x}_i^P(s)$ in the incomplete information environment. Let $(s^*, \hat{s})$ be the optimal asset specificity level in the complete and incompete information environment respectively. Hence, in the complete information environment, the optimal allocation $\left\langle s^*, \left(a_i^*, x_i^{A*}, x_i^{P*}\right)_{i=1}^2 \right\rangle$ is given by $\left\langle s^*, \left(a_i^*(s^*), x_i^{A*}(s^*), x_i^{P*}(s^*)\right)_{i=1}^2 \right\rangle$. In the incomplete information environment, the optimal allocation $\left\langle \hat{s}, \left(\hat{a}_i, \hat{x}_i^A, \hat{x}_i^P\right)_{i=1}^2 \right\rangle$ is given by $\left\langle \hat{s}, \left(\hat{a}_i(\hat{s}), \hat{x}_i^A(\hat{s}), \hat{x}_i^P(\hat{s})\right)_{i=1}^2 \right\rangle$.

In the complete information environment, for each $s \in [\underline{s}, \bar{s}]$, the solutions $\left(a_i^*(s), x_i^{A*}(s), x_i^{P*}(s)\right)_{i=1}^2$ are characterized by the following conditions[4] :

$$h'\left(a_i^*(s)\right) = \theta_i g(s), \tag{1}$$

$$x_i^{A*}(s) = h\left(a_i^*(s)\right), \tag{2}$$

$$x_i^{P*}(s) = a_i^*(s)\theta_i g(s) - x_i^{A*}(s) \tag{3}$$

for i=1,2. Equation (1) implies that the marginal product of the agent's labor input is equal to the marginal disutility in each state. Equation (2) implies that the agent is paid the amount that is just sufficient to keep him in the project in each state. Finally, it follows from equations (2) and (3) that

[4] Details of derivations of equations (1) - (9) are given in the Appendix.

$$x_i^{P*}(s) = a_i^*(s)\theta_i g(s) - h\left(a_i^*(s)\right). \tag{3'}$$

Equation (3') means that the principal gets all the net surplus of the project in each state.

In the incomplete information environment, for each $s \in [\underline{s}, \bar{s}]$, the solutions $\left(\hat{a}_i(s), \hat{x}_i^A(s), \hat{x}_i^P(s)\right)_{i=1}^2$ are characterized by the following conditions :

$$\frac{h'(\hat{a}_1(s))}{\pi_1} - \left(\frac{\pi_2}{\pi_1}\right)\left(\frac{\theta_1}{\theta_2}\right)h'\left(\hat{a}_1(s)\left(\frac{\theta_1}{\theta_2}\right)\right) = \theta_1 g(s), \tag{4}$$

$$h'(\hat{a}_2(s)) = \theta_2 g(s), \tag{5}$$

$$\hat{x}_1^A(s) = h(\hat{a}_1(s)), \tag{6}$$

$$\hat{x}_1^P(s) = \hat{a}_1(s)\theta_1 g(s) - \hat{x}_1^A(s), \tag{7}$$

$$\hat{x}_2^A(s) = h(\hat{a}_2(s)) + [h(\hat{a}_1(s)) - h(\hat{a}_1(s)\cdot\frac{\theta_1}{\theta_2})], \tag{8}$$

$$\hat{x}_2^P(s) = \hat{a}_2(s)\theta_2 g(s) - \hat{x}_2^A(s). \tag{9}$$

Equations (4) and (5) characterize the optimal labor input in state 1 and state 2 respectively. Equations (6) and (8) characterize the optimal consumption for the agent in state 1 and state 2 respectively. Finally, equations (6) and (7) imply that

$$\hat{x}_1^P(s) = \hat{a}_1(s)\theta_1 g(s) - h(\hat{a}_1(s)). \tag{7'}$$

And equations (8) and (9) imply that

$$\hat{x}_2^P(s) = \hat{a}_2(s)\theta_2 g(s) - h(\hat{a}_2(s)) - [h(\hat{a}_1(s)) - h(\hat{a}_1(s)\cdot\frac{\theta_1}{\theta_2})]. \tag{9'}$$

Equations (7') and (9') characterize the optimal consumption for the principal in state 1 and state 2 respectively.

The crucial difference between the solutions $\left(a_i^*(s), x_i^{A*}(s), x_i^{P*}(s)\right)_{i=1}^2$ in the complete information environment and the solutions $\left(\hat{a}_i(s), \hat{x}_i^A(s), \hat{x}_i^P(s)\right)_{i=1}^2$ in the incomplete information environment is summarized in Lemma 1.

Lemma 1. For each $s \in [\underline{s}, \bar{s}]$ and for i=1,2, $x_i^{P*}(s) > \hat{x}_i^P(s)$.

Proof :

(i) Since $h''(a) > 0$ for all $a \in (0, \bar{a})$, it is easy to see from equations (1) and (3') that for each $s \in [\underline{s}, \bar{s}]$ and for i=1,2, $a_i^*(s) = \arg\max_{a_i \in [0, \bar{a}]} \{a_i \theta_i g(s) - h(a_i)\}$.

(ii) Equation (4) implies that

$$\pi_1[\theta_1 g(s) - h'(\hat{a}_1(s))] = \pi_2\left[h'(\hat{a}_1(s)) - \left(\frac{\theta_1}{\theta_2}\right) h'\left(\hat{a}_1(s)\left(\frac{\theta_1}{\theta_2}\right)\right)\right].$$

The RHS of the above equality is strictly positive because $\pi_2 > 0$, $\theta_2 > \theta_1$, $\hat{a}_1(s) > 0$ and $h''(a) > 0$ for all $a \in (0, \bar{a})$. Since $\pi_1 > 0$, it follows that $\theta_1 g(s) > h'(\hat{a}_1(s))$.

(iii) For each $s \in [\underline{s}, \bar{s}]$, $\hat{a}_1(s) < a_1^*(s)$ follows from (ii), equation (1), and the assumption that $h''(a) > 0$ for all $a \in (0, \bar{a})$.

(iv) Equations (3') and (7') imply that in both environments, the prinicpal's state-1 consumption has the form $a_1 \theta_1 g(s) - h(a_1)$. Hence, (i) and (iii) imply that for each $s \in [\underline{s}, \bar{s}]$, $x_1^{P*}(s) > \hat{x}_1^P(s)$.

(v) While equation (1) for i=2 and equation (5) imply that the labor inputs in state 2 are the same in both environments, equation (9') differs from equation (3') for i=2 by the term $[h(\hat{a}_1(s)) - h(\hat{a}_1(s) \cdot \frac{\theta_1}{\theta_2})]$. This term is strictly postive for every $s \in [\underline{s}, \bar{s}]$ because $h'(a) > 0$ for all $a \in (0, \bar{a})$ and $\hat{a}_1(s) > 0$ for each $s \in [\underline{s}, \bar{s}]$. Therefore, $x_2^{P*}(s) > \hat{x}_2^P(s)$ for each $s \in [\underline{s}, \bar{s}]$. ‖

It follows trivially from Lemma 1 that for i=1,2, $x_i^{P*} > \hat{x}_i^P$ where $x_i^{P*} = x_i^{P*}(s^*)$ and $\hat{x}_i^P = \hat{x}_i^P(\hat{s})$. That is, the existence of the agency problem reduces the optimal consumption of the principal in both states.

The next lemma identifies and compares the values of s^* and $\hat{s}$.

Lemma 2 . $s^* = \hat{s} = \bar{s}$.

Proof :

(i) For i=1,2, $\frac{d}{ds}x_i^{P*}(s) = \left(\frac{d}{ds}a_i^*(s)\right)\left[\theta_i g(s) - h'(a_i^*(s))\right] + a_i^*(s)\theta_i g'(s) = a_i^*(s)\theta_i g'(s)$

where the last equality follows from equation (1).
Since for all $s \in [\underline{s}, \bar{s}]$ and for i=1,2, $a_i^*(s) > 0$ and $g'(s) \geq 0$ with equality at $s = \bar{s}$ only, it follows that $\frac{d}{ds}x_i^{P*}(s) \geq 0$ for all $s \in [\underline{s}, \bar{s}]$ with equality at $s = \bar{s}$ only.

(ii) It follows from (i) that for i=1, 2, $x_i^{P*}(s)$ has a unique maximum at $s = \bar{s}$ and therefore $\sum_i \pi_i x_i^{P*}(s)$ has a unique maximum at $s = \bar{s}$, i.e., $s^* = \bar{s}$.

(iii) $\frac{d}{ds}\left[\pi_1 \hat{x}_1^P(s) + \pi_2 \hat{x}_2^P(s)\right]$

$$
\begin{aligned}
&= \left\{\pi_1\left(\frac{d}{ds}\hat{a}_1(s)\right)\left[\theta_1 g(s) - h'(\hat{a}_1(s))\right]\right\} + \pi_1\left[\hat{a}_1(s)\theta_1 g'(s)\right] \\
&\quad + \left\{\pi_2\left(\frac{d}{ds}\hat{a}_2(s)\right)\left[\theta_2 g(s) - h'(\hat{a}_2(s))\right]\right\} + \pi_2\left[\hat{a}_2(s)\theta_2 g'(s)\right] \\
&\quad + \left\{\pi_2 \cdot \frac{d}{ds}(\hat{a}_1(s)) \cdot \left[-h'(\hat{a}_1(s)) + \frac{\theta_1}{\theta_2}h'\left(\hat{a}_1(s) \cdot \frac{\theta_1}{\theta_2}\right)\right]\right\}
\end{aligned}
$$

$$= \left[\pi_1 \hat{a}_1(s)\theta_1 + \pi_2 \hat{a}_2(s)\theta_2\right] g'(s)$$

where the last equality follows from equations (4) and (5).
Since for all $s \in [\underline{s}, \bar{s}]$ and for i=1,2, $\hat{a}_i(s) > 0$ and $g'(s) \geq 0$ with equality at $s = \bar{s}$ only, it follows that $\frac{d}{ds}\left[\pi_1 \hat{x}_1^P(s) + \pi_2 \hat{x}_2^P(s)\right] \geq 0$ for all $s \in [\underline{s}, \bar{s}]$ with equality at $s = \bar{s}$ only.

(iv) It follows from (iii) that $\sum_i \pi_i \hat{x}_i^P(s)$ has a unique maximum at $s = \bar{s}$, i.e., $\hat{s} = \bar{s}$. ‖

Therefore, Lemma 2 shows that without a liquidation technology, the existence of the agency problem has no effect on the optimal asset specificity level : it is optimal to use an asset that is as specific as possible whether the agency problem exists or not. Moreover, it follows from Lemma 2 that the optimal allocation in the complete information environment $\left\langle s^*, \left(a_i^*, x_i^{A*}, x_i^{P*}\right)_{i=1,2}\right\rangle$ is given by $\left\langle \bar{s}, \left(a_i^*(\bar{s}), x_i^{A*}(\bar{s}), x_i^{P*}(\bar{s})\right)_{i=1}^2\right\rangle$ and the optimal allocation in the incomplete information environment $\left\langle \hat{s}, \left(\hat{a}_i, \hat{x}_i^A, \hat{x}_i^P\right)_{i=1,2}\right\rangle$ is given by $\left\langle \bar{s}, \left(\hat{a}_i(\bar{s}), \hat{x}_i^A(\bar{s}), \hat{x}_i^P(\bar{s})\right)_{i=1}^2\right\rangle$.

Finally, an additional observation in the complete information environment is useful in proving the main proposition of this paper. It is summarized in Lemma 3 which shows that in the complete information environment, the principal's optimal consumption is higher in state 2 than in state 1.

Lemma 3 . For each $s \in [\underline{s}, \bar{s}]$, $x_2^{P*}(s) > x_1^{P*}(s)$.

Proof :

For each $s \in [\underline{s}, \bar{s}]$, at $\tilde{\bar{a}}_2 = a_1^*(s)$, $[a_2\theta_2 g(s) - h(a_2)]$ is strictly increasing in a_2 because
$$\frac{d}{da_2}[a_2\theta_2 g(s) - h(a_2)]\Big|_{a_2 = \tilde{\bar{a}}_2} = \theta_2 g(s) - h'\left(\tilde{\bar{a}}_2\right) = \theta_2 g(s) - h'\left(a_1^*(s)\right)$$
$$> \theta_1 g(s) - h'\left(a_1^*(s)\right) = 0$$
where the inequality follows from $\theta_2 > \theta_1$ and the last equality follows fron equation (1) for i=1. Thus, together with equation (3'), it follows that $x_2^{P*}(s) > x_1^{P*}(s)$. ‖

To summarize, this section shows that without a liquidation technology, the existence of the agency problem reduces the optimal consumption of the principal in both states and has no effect on the optimal asset specificity level.

V. OPTIMAL ALLOCATIONS WITH A LIQUIDATION TECHNOLOGY

This section considers the environment described in section II and incorporates the liquidation technology. Based on the results established in section IV, this section shows that there exists a range of liquidation values such that the liquidation technology can be used to increase the ex ante expected utility level of the principal in the incomplete information environment but not in the complete information environment. That is, liquidation will occur in the incomplete information environment but not in the complete information environments. As a result, the optimal asset specificity level will be lower in the incomplete information environment than in the complete information environment.

When there is a liquidation technology, an allocation is a vector $\left\langle s,\left(l_i, a_i, x^A_{il_i}, x^P_{il_i}\right)_{i=1,2}\right\rangle$ where s is the asset specificity level, l_i is the liquidation decision in state i, a_i is the agent's labor input in state i when the project is continued, $x^A_{il_i}$ and $x^P_{il_i}$ are the agent's consumption and the principal's consumption respectively in state i when the liquidation decision is l_i. In both the complete and incomplete information environments, since the liquidation technology $d(\cdot)$ is independent of the agent's labor input, any optimal allocation will require that a=0 when liquidation occurs. Therefore, for simplicity, the agent's labor input (and hence the agent's disutlity of labor input $h(\cdot)$) is set at zero when liquidation occurs. And a_i corresponds to the labor input when continuation occurs in state i.

As in section IV, the optimal allocations in both the complete and incomplete information environment are characterized by maximizing the principal's ex ante expected utility level subject to a given ex ante expected utility level of the agent and the feasibility constraints. That is, the social planner's problem is to choose an allocation $\left\langle s,\left(l_i, a_i, x^A_{il_i}, x^P_{il_i}\right)_{i=1,2}\right\rangle$ to solve the following maximization problem (P1') :

$$\text{Maximize} \quad \sum_{i=1}^{2} \pi_i x^P_{il_i}$$

subject to

$$\sum_{i=1}^{2} \pi_i \left[x^A_{il_i} - (1-l_i) h(a_i) \right] \geq U^0, \tag{C1'}$$

$$l_i d(s) + (1-l_i) a_i \theta_i g(s) \geq \left(x^A_{il_i} + x^P_{il_i} \right) \text{ for i=1,2,} \tag{C2'}$$

$$x^A_{il_i} - (1-l_i) h(a_i) \geq L \text{ for i=1,2,} \tag{C3'}$$

$$x^A_{il_i} - (1-l_i) h(a_i) \geq x^A_{jl_j} - (1-l_j) h\left(a_j \cdot \frac{\theta_j}{\theta_i} \right) \text{ for } j \neq i, \text{ i=1,2.} \tag{C4'}$$

The constraints are analogous to the ones in section IV. C1' is the constraint that the agent's ex ante expected utility level has to be at least U^0. C3' is the agent's ex post indiviually rational constraint in each state. As in the previous section, this section considers the special case U^0=L=0. C2' is the resource constraint for consumption in each state. And C4' is the incentive compatibility constraint. The main difference between problem (P1') and problem (P1) is that in (P1'), there is an additional choice variable l_i in state i for i=1,2. That is, it is possible to stop the project and adopt the liquidation technology in each state. In state i, if continuation of the project occurs (i.e.,l_i=0) and if the agent's labor input is a_i, then the agent's ex post utility is $x^A_{i0} - h(a_i)$ and constraint C2' implies that the consumption good available is $a_i \theta_i g(s)$. On the other hand, if liquidation occurs in state i (i.e.,l_i=1), then the agent's ex post utility is x^A_{i1} (recall that the agent's labor inpur is zero when liquidation occurs) and constraint C2' implies that the consumption good available is d(s). If l_i = 0 for i=1,2, then (P1') reduces to (P1). That is, section IV has already solved for the optimal choice of $\left\langle s, \left(a_i, x^A_i, x^P_i \right)_{i=1,2} \right\rangle$ for the special case when l_i = 0 for i=1,2.

Two observations in the incomplete information environment are useful in proving the main proposition of this paper and they are summarized in Lemmas 4 and 5. Lemma 4 shows that if liquidation occurs in state 1 and continuation occurs in state 2, then the principal can attain the optimal state-2 consumption of the complete information environment described in the previous section.

Lemma 4. In the incomplete information environment, for each $s \in [\underline{s}, \bar{s}]$, if liquidation occurs in state 1 and continuation occurs in state 2, then the optimal consumption for the principal is $d(s)$ in state 1 and $x_2^{P*}(s)$ in state 2 where $x_2^{P*}(s)$ is given by equation (3') for i=2.

Proof :

(i) Liquidation in state 1 and continuation in state 2 imply that $l_1 = 1$ and $l_2 = 0$. Using the same proof as the proof of the claim in the Appendix , it can be shown that at the optimal solution, constraint C3' for i=1 and constraint C4' for i=2 bind, and constraint C3' for i=2 and constraint C4' for i=1 are slack. The equality of constraint C3' for i=1 and the equality of constraint C4' for i=2 imply that for every $s \in [\underline{s}, \bar{s}]$, $x_{11}^A(s) = 0$ and $x_{20}^A(s) - h(a_2(s)) = 0$.

(ii) It follows from $x_{11}^A(s) = 0$ that the optimal consumption for the principal is $d(s)$ in state 1, i.e., all the output is allocated to the principal when liquidation occurs.

(iii) If $x_{20}^A(s) - h(a_2(s)) = 0$, then the principal's consumption is $a_2\theta_2 g(s) - h(a_2)$. It follows from the proof of Lemma 1 that the optimal labor input is $a_2^*(s)$ and the principal's consumption is $x_2^{P*}(s)$. ‖

The second observation is that if liquidation occurs in state 2, it must also occur in state 1.

Lemma 5. In the incomplete information environment, for each $s \in [\underline{s}, \bar{s}]$, $l_2 = 1 \Rightarrow l_1 = 1$.

Proof :

Suppose that liquidation occurs in state 2 and continuation occurs in state 1. The incentive compatibility constraints become :

$$x_{10} - h(a_1) \geq x_{21} \text{ and } x_{21} \geq x_{10} - h\left(a_1 \cdot \frac{\theta_1}{\theta_2}\right).$$

However, the above two inequalities together imply that

$$x_{10} - h(a_1) \geq x_{10} - h\left(a_1 \cdot \frac{\theta_1}{\theta_2}\right).$$

Since $\theta_2 > \theta_1$ and $h'(a) > 0$ for all $a \in (0, \bar{a})$, the above inquality can be satisfied at equality only if a_1=0. Yet a_1=0 neccessarily implies that liquidation occurs because $a_1 = 0 \Rightarrow a_1\theta_1 g(s) = 0$ and yet $d(s) > 0$ for all $s \in [\underline{s}, \bar{s}]$. Thus, a contradiction. ‖

An implication of Lemma 5 is that in the incomplete information environment, the liquidation rule that prescribes liquidation in state 2 and continuation in state 1 is (incentive) infeasible and therefore can be ruled out as an optimal liquidation rule.

The analysis thus far has not made use of the liquidation values except for the fact that they are strictly positive. The following is to show that there exists a range of liquidation values such that the liquidation technology will have different implications on the optimal liquidation rules and the optimal asset specificity levels in the complete information environment and in the incomplete information environment. Assumption (A1) specifies such a range on the liquidation values.

Assumption (A1). For all $s \in [\underline{s}, \bar{s}]$, $d(s) \in [\bar{x}_1^P, x_1^{P*}]$ where $\bar{x}_1^P = \underset{s \in [\underline{s}, \bar{s}]}{\text{maximum}}\ \hat{x}_1^P(s)$, $\hat{x}_1^P(s)$ is given by equation (7').

Lemma 1 and Lemma 2 imply that $x_1^{P*} = x_1^{P*}(s^*) \geq x_1^{P*}(s) > \hat{x}_1^P(s)$ for all $s \in [\underline{s}, \bar{s}]$. Therefore, $x_1^{P*} > \bar{x}_1^P = \underset{s \in [\underline{s}, \bar{s}]}{\text{maximum}}\ \hat{x}_1^P(s)$, i.e., the closed interval $[\bar{x}_1^P, x_1^{P*}]$ is well-defined.

The next two lemmas show that if the liquidation values satisfy (A1), then the presence of the liquidation technology will affect the optimal allocation differently in the complete information environment and in the incomplete information environment.

Lemma 6. For the complete information environment, under assumption (A1), it is optimal to :

(a) continue the project in both states, and

(b) choose an asset specificity level $s = \bar{s}$.

Proof :

(i) In the complete information environment, section IV has shown that if continuation occurs in both states (i.e., $l_i = 0$ for i=1,2), the optimal consumption for the principal is $\left(x_1^{P*}, x_2^{P*}\right)$ in state 1 and state 2 respectively and Lemma 3 shows that $x_2^{P*} > x_1^{P*}$.

(ii) Assumption (A1) implies that $x_1^{P*} \geq d(s)$ for all $s \in [\underline{s}, \bar{s}]$ with equality possibly at $s=\underline{s}$ only.

(iii) The feasibilty condition for consumption implies that for all $s \in [\underline{s}, \bar{s}]$, d(s) is the maximum consumption that the principal can possibly get when liquidation occurs.

(iv) It follows from (i), (ii) and (iii) that no matter in which state liquidation occurs and no matter what asset specificity level is chosen, the principal cannot obtain state-contingent consumptions that provide him a higher level of ex ante expected utility than at $\left(x_1^{P*}, x_2^{P*}\right)$. Therefore, it is optimal to continue the project in both states.

(v) If continuation occurs in both states, then the optimal asset specificity level remains at $\bar{s}$. ‖

Thus, Lemma 6 shows that if the liquidation values satisfy assumption (A1), then the liquidation technology has no effect on the optimal allocation in the complete information environment. That is, the optimal allocation remains the same as when the liquidation technology does not exist. However, the presence of the liquidation technology will have a different implication on the optimal allocation in the incomplete information environment and its effect is summarized in Lemma 7.

Lemma 7. In the incomplete information environment, under assumption (A1), it is optimal to :

(a) liquidate at least in state 1, and

(b) choose an asset specificity level $s < \bar{s}$.

Proof :

(i) Lemma 5 shows that in the incomplete information environment, there are three possible liquidation rules to consider:

(a) never liquidate, i.e., continue with the project in both states,

(b) liquidate in state 1 and continue in state 2 , and

(c) liquidate in both states.

Section IV shows if the liquidation rule is (a), then the optimal consumption for the principal is $\left(\hat{x}_1^P, \hat{x}_2^P\right)$ in state 1 and state 2 respectively. Lemma 4 shows that if the liquidation rule is (b), then the principal's optimal consumption is $(d(s), x_2^{P*}(s))$ in state 1 and state 2 respectively. For all $s \in [\underline{s}, \bar{s}]$, $x_2^{P*}(s) > \hat{x}_2^P(s)$ (by Lemma 1) and $d(s) \geq \bar{x}_1^P \geq \hat{x}_1^P(s)$ (by Assumption (A1)). Therefore, compared to (a), (b) increases the principal's ex post consumption in each state and Consequently his ex ante expected utility level. Hence, the optimal liquidation rule is either (b) or (c).

(ii) If (c) is the optimal liquidation rule, then the optimal asset specificity level is $\underline{s}$, the lowest specificity level possible, because $d'(s) < 0$ for all $s \in [\underline{s}, \bar{s}]$.

(iii) If the optimal liquidation rule is (b), then the optimal asset specificity level is an $s \in [\underline{s}, \bar{s}]$ that maximizes $\pi_1 d(s) + \pi_2 x_2^{P*}(s)$.

Since $\pi_1 > 0$, $d'(s) < 0$ for all $s \in [\underline{s}, \bar{s}]$ and $\frac{d}{ds} x_2^{P*}(s)\Big|_{s=\bar{s}} = 0$ (see (i) of the proof in Lemma 2), $\frac{d}{ds}\left[\pi_1 d(s) + \pi_2 x_2^{P*}(s)\right]\Big|_{s=\bar{s}} = \pi_1 d(s)\Big|_{s=\bar{s}} < 0$. Therefore, the optimal specificity level is strictly less than $\bar{s}$. ‖

Thus, Lemma 7 shows that if the liquidation values satisfy assumption (A1), the optimal liquidation rule in the incomplete information environment will involve liquidation occurring at least in state 1 and the optimal asset specificity level will be lower than $\bar{s}$.

Hence, Lemma 6 and Lemma 7 show that assumption (A1) specifies a range of liquidation values such that :

(I) in the complete information environment, it is optimal to

(a) continue the project in both states, and

(b) choose an asset specificity level $s = \bar{s}$;

(II) in the incomplete information environment, it is optimal to

(a) liquidate at least in state 1, and

(b) choose an asset specificity level $s < \bar{s}$.

Therefore, the main proposition stated in section III is established.

VI. INTERPRETATION OF MANAGERIAL MYOPIA

One widely held view about managerial myopia is that it helps account for the relatively poor growth and productivity performance of the United States for the last few decades. Stein (1988, 1989), Bebchuk and Stole (1993) have attempted to explain managerial myopia. They show that if a manager cares directly about the short-term stock price performance of his company, then he will behave myopically. That is, rather than maximizing the ex ante expected value of the company, the manager will over-invest in short-term projects and under-invest in long-term projects.

Three assumptions are crucial in the models described above. First, the manager has private information about the prospect of the company. Second, the allocation of resources between short-term and long-term investments is not publicly observable. Third, the short-term stock price, i.e., the market's perceived value of the company on the interim date, enters directly into the utility function of the manager. The last two assumptions are unsatisfactory. First, the allocation of monetary and physical resources between short-term and long-term projects is typically publicly observable and verifiable. Second, and perhaps more importantly, these models beg the question of why the manager cares directly about the short-term stock price of the company: it is certainly unsatisfactory to say that the manager derives utility from the stock price performance of the company. Some have argued that the short-term objective arises because the manager is worried about takeovers and some have argued that it is because the manager is compensated according to the stock price performance. However, it is unclear why incentive contracts cannot be used to eliminate the manager's fear of takeover and why it is optimal to tie the manager's compensation to the short-term stock price of the company. In contrast, this paper is free of these questionable assumptions and shows that the so-called managerial myopia can indeed be an optimal arrangement in an environment where agency problem exists.

The main result of this paper is that there exists a region in the parameter space such that the optimum in the incomplete information environment has more frequent liquidation and less specific assets than the one in the complete information environment. A comparison of the optimal arrangements between these two environments can provide an interpretation of so-called managerial myopia.

Consider two economies that have the same physical environment described in section II. However, one has complete information and one has incomplete information. Imagine that there is a very large number of agents in each economy. These agents are ex ante identical and are subject to idiosyncratic shocks described in section II. An outside observer does not know the information structure of the economy nor the preferences of the individuals. However, he can observe the degree of asset specificity adopted, the number of firms that liquidate and the output. Compared to the economy with complete information, the economy with incomplete information has :
(1) lower outputs[5] ,
(2) more liquidation, and
(3) less specific assets.
Consequently, he could infer that individuals in the economy with incomplete information are myopic because they do not seem to be maximizing the expected output of the investment. Rather, they seem to be more ready to liquidate and they seem to be so concerned about the liquidation value of their projects that they are willing to use less specific assets.

Hence, while this paper shows that a myopic outcome can arise in an incomplete information environment, it does not rely on the assumption that the allocation of resources among projects are unobservable nor the ambigous assumption that the manager has so-called short-term objectives.

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[5] If the number of agents is very large in both economies, then in each economy, the fraction of agents having the bad shock is π_1 and the fraction of agents having the good shock is π_2. Moreover,
(a) in the complete information environment, no project is ever liquidated and the average output per project is $\pi_1\left(x_1^{P*}+x_1^{A*}\right)+\pi_2\left(x_2^{P*}+x_2^{A*}\right)$;
(b) in the incomplete information environment, depending on the parameters of the economy, either that a fraction π_1 of projects is liquidated and a fraction π_2 of projects is continued or that all firms liquidate, and the average per project output is either
$\pi_1 d(\tilde{s})+\pi_2\left(x_2^{P*}(\tilde{s})+x_2^{A*}(\tilde{s})\right)$ where $\tilde{s}=\underset{s\in[\underline{s},\bar{s}]}{\operatorname{argmax}}\,\pi_1 d(s)+\pi_2 x_2^{P*}(s)$ or $d(\underline{s})$, either way, the average output is lower than $\pi_1\left(x_1^{P*}+x_1^{A*}\right)+\pi_2\left(x_2^{P*}+x_2^{A*}\right)$ by assumption (A1).

Furthermore, contrary to the popular negative view associated with myopia, this paper shows that the so-called managerial myopia can indeed be an optimal arrangement in an environment where agency problem exists.

VII. CONCLUSION

The objective of this paper is to study the implications of an agency problem for the optimal asset specificity level. The main result of this paper is that there exists a region in the parameter space such that the optimum in the presence of an agency problem has more frequent liquidation and less specific assets than in the absence of an agency problem. Consequently, an outside observer could infer that individuals in the environment with the agency problem are myopic because they seem to be more willing to liquidate and they seem to be so concerned about the liquidation value of their investments that they are willing use less specific assets. Hence, this paper provides an interpretation of so-called managerial myopia : it interprets such outcome as an optimal arrangement in a principal-agent model.

In sum, this paper makes two contributions. First, it is the first study that examines the implications of an agency problem on the optimal asset specificity level. Second, it provides an interpretation of managerial myopia in a primitive environment where an agency problem exists. Two directions for future research are worth pursuing. One is to study how the optimal asset specificity level is determined in other kinds of agency models. And one is to explore the macroeconomic implications of the asset specificity choice made by individual firms.

APPENDIX

This appendix shows how $\left(a_i, x_i^A, x_i^P\right)_{i=1}^{2}$ are chosen to solve the maximization problem (P1) for each $s \in [\underline{s}, \bar{s}]$ in the complete information environment and in the incomplete information environment.

The assumption that $L=U^0=0$ simplifies the maximization problem (P1) because any allocation that satisfies constraint C3 will satisfy constraint C1. Therefore, C1 can be dropped from the maximization problem (P1). In addition, in both environments, it is easy to see that constraint C2 must be binding in both states at the optimal solutions because they simply imply the exhaustion of the consumption goods available. Therefore, for i=1,2, x_i^P can be eliminated by using C2 in equality. Now the maximization problem (P1) can be simplified to the following maximization problem (P2) :

$$\text{Maximize} \quad \sum_{i=1}^{2} \pi_i \left(a_i \theta_i g(s) - x_i^A\right)$$

subject to

$$x_i^A - h(a_i) \geq 0 \text{ for i=1,2,} \tag{C3}$$

$$x_i^A - h(a_i) \geq x_j^A - h\left(a_j \cdot \frac{\theta_j}{\theta_i}\right) \text{ for i=1,2 and j=1,2,} \tag{C4}$$

$$a_i \theta_i g(s) - x_i^A \geq 0 \text{ for i=1,2.} \tag{C5}$$

Constraint C5 is the non-negative consumption constraint for the principal, i.e., $x_i^P = a_i \theta_i g(s) - x_i^A \geq 0$ for i=1,2. The rest of the appendix will show how the optimal solutions of $\left(a_i, x_i^A, x_i^P\right)_{i=1}^{2}$ are derived for each $s \in [\underline{s}, \bar{s}]$.

In the complete information environment, constraint C3 must be binding at the optimal solution. It is because, ceteris paribus, reducing x_i^A for i=1,2 relaxes constraint C5 and at the same time increases the value of the objective function. Therefore, x_i^A can be eliminated by using constraint C3 at equality. Thus, for each $s \in [\underline{s}, \bar{s}]$, problem (P2) reduces to the following :

$$\text{Maximize} \quad \sum_{i=1}^{2} \pi_i \left[a_i \theta_i g(s) - h(a_i) \right]$$

subject to

$$a_i \theta_i g(s) - h(a_i) \geq 0 \text{ for i=1,2.} \tag{C5}$$

Since $\pi_i > 0$ for i=1,2, the above maximization problem is equivalent to a state-by-state maximization. That is, for i=1,2, $a_i \in [0, \bar{a}]$ is chosen to maximize $a_i \theta_i g(s) - h(a_i)$ subject to $a_i \theta_i g(s) - h(a_i) \geq 0$. The strict convexity of $h(\cdot)$ implies for i=1,2, $a_i \theta_i g(s) - h(a_i)$ is strictly concave in a_i. Therefore, the first order conditions of a_i are necessary and sufficient. In addition, the assumptions that $h(0) = h'(0) = 0$ and $\lim_{a \to \bar{a}} h'(a) = +\infty$ guarantee the existence of solutions of $(a_i)_{i=1,2}$ which lie in the interior of $[0, \bar{a}]$. The first order conditions of $(a_i)_{i=1,2}$ are given by equation (1) for i=1,2. Given the solutions of $(a_i)_{i=1,2}$, the optimal $(x_i^A, x_i^P)_{i=1,2}$ are solved by using constraint C2 and C3 at equality and they are given by equations (2) and (3) for i=1,2.

In the incomplete information environment, the maximization problem can be simplified by the following claim.

Claim : For each $s \in [\underline{s}, \bar{s}]$, at the optimal solution of problem (P2), constraint C3 for i=1 and constraint C4 for i=2 must be binding, however, constraint C3 for i=2 and constraint C4 for i=1 must be slack.

Proof : This claim can be proved in two steps. Step 1 is to ignore constraint C3 for i=2 and constraint C4 for i=1 : show that constraint C3 for i=1 and constraint C4 for i=2 must be binding at the optimal solution in the relaxed program and solve for the optimal solution. Step 2 is to verify that at the optimal solution of the relaxed program, constraints C3 for i=2 and C4 for i=1 are satisfied at slack.

Step 1 :

(i) In the relaxed program, for any given levels of labor input (a_1, a_2), reducing x_1^A simultaneously relaxes the constraint C4 for i=2 and increases the value of the objective function. Therefore, constraint C3 for i=1 must be binding at the optimal solution of this relaxed program. And for any given $[x_1^A, (a_1, a_2)]$, constraint C4 for i=2 must be binding at the solution to the relaxed program because the objective function is strictly decreasing in x_2^A.

(ii) Using constraint C3 for i=1 and C4 for i=2 at equality to eliminate $(x_1^A,\ x_2^A)$, for each $s \in [\underline{s}, \bar{s}]$, the maximization problem (P2) can be written as :

$$\text{Maximize} \quad \pi_1[a_1\theta_1 g(s) - h(a_1)] + \pi_2\left[a_2\theta_2 g(s) - h(a_2) - \left(h(a_1) - h\left(a_1 \cdot \frac{\theta_1}{\theta_2}\right)\right)\right]$$

subject to

$$a_i\theta_i g(s) - h(a_i) \geq 0 \text{ for i=1,2.} \tag{C5}$$

The existence of solutions, which will be interior solutions, follows from the same arguments used in the complete information environment. The first order conditions of $(a_i)_{i=1,2}$ are given by equations (4) and (5). Given the optimal $(a_i)_{i=1,2}$, the optimal $(x_i^A)_{i=1,2}$ are solved by using C3 at equality for i=1 and C4 at equality for i=2. The solutions are given by equations (6) and (8). Finally, the $(x_i^P)_{i=1,2}$ are solved by using constraint C2 at equality. And the solutions are given by equations (7) and (9). Therefore, the optimal solution in the relaxed program is characterized by equations (4)-(9).

Step 2 :

To verify that the solutions characterized by equations (4)-(9) satisfy constraints C3 for i=2 and C4 for i=1 at slack.

(a) To show that C3 for i=2 is satisfied at slack.

(i) At equality, constraint C4 for i=2 is $x_2^A - h(a_2) = x_1^A - h(a_1 \cdot \frac{\theta_1}{\theta_2})$.

(ii) $x_1^A - h(a_1 \cdot \frac{\theta_1}{\theta_2}) > x_1^A - h(a_1)$ because $\theta_2 > \theta_1$ and $h(a)$ is strictly increasing for all a>0.

(iii) At equality, constraint C3 for i=1 is $x_1^A - h(a_1)=0$.

(iv) Since equation (4) implies that $\hat{a}_1(s) > 0$ for all $s \in [\underline{s}, \bar{s}]$, it follows that (i), (ii) and (iii) that $x_2^A - h(a_2)>0$.

(b) To show that C4 for i=1 is satisfied at slack.

(i) The equality of constraint C3 for i=1 implies that the LHS of C4 for i=1 is zero.

(ii) The equality of constraint C3 for i=1 and C4 for i=2 implies that

$$x_2^A = h(a_2) + h(a_1) - h(a_1 \cdot \frac{\theta_1}{\theta_2})$$

$$\Rightarrow x_2^A - h\left(a_2 \cdot \frac{\theta_2}{\theta_1}\right) = h(a_2) + h(a_1) - h(a_1 \cdot \frac{\theta_1}{\theta_2}) - h\left(a_2 \cdot \frac{\theta_2}{\theta_1}\right).$$

Hence, to establish that C4 is satisfied at slack for i=1, it suffices to show that at the optimal solution of the relaxed program,

$$h(a_2) + h(a_1) - h(a_1 \cdot \frac{\theta_1}{\theta_2}) - h\left(a_2 \cdot \frac{\theta_2}{\theta_1}\right) < 0.$$

(iii) Let $H(a_2; a_1, \theta_1, \theta_2) = h(a_2) + h(a_1) - h\left(a_1 \cdot \frac{\theta_1}{\theta_2}\right) - h\left(a_2 \cdot \frac{\theta_2}{\theta_1}\right)$.

First, for any given a_1, at $a_2 = a_1 \cdot \frac{\theta_1}{\theta_2}$, $h(a_2) + h(a_1) - h(a_1 \cdot \frac{\theta_1}{\theta_2}) - h\left(a_2 \cdot \frac{\theta_2}{\theta_1}\right) = 0$.

Second, for all $\theta_2 > \theta_1 > 0$, $H(a_2; a_1, \theta_1, \theta_2)$ is decreasing in a_2 because

$\frac{\partial}{\partial a_2} H(a_2; a_1, \theta_1, \theta_2) = h'(a_2) - \frac{\theta_2}{\theta_1} h'\left(a_2 \cdot \frac{\theta_2}{\theta_1}\right)$ where $h'(a_2) - \frac{\theta_2}{\theta_1} h'\left(a_2 \cdot \frac{\theta_2}{\theta_1}\right) < 0$ for all $a_2 > 0$ (by the assumption that $h''(a) > 0$ for all a>0). Therefore, if $a_2 > a_1 \cdot \frac{\theta_1}{\theta_2}$ where $a_1 \geq 0$, then $H(a_2; a_1, \theta_1, \theta_2) < 0$.

(iv) Since $\hat{a}_1(s) > 0$ for each $s \in [\underline{s}, \bar{s}]$, the last property to be established is that $\hat{a}_2(s) > \hat{a}_1(s) \cdot \frac{\theta_1}{\theta_2}$ for each $s \in [\underline{s}, \bar{s}]$.

For each $s \in [\underline{s}, \bar{s}]$, let $\tilde{a}_2(s) = \hat{a}_1(s) \cdot \frac{\theta_1}{\theta_2}$. Substituting $\tilde{a}_2(s)$ into equation (4), we get,

$$\left(\frac{1}{\pi_1}\right)h'\left(\tilde{a}_2(s)\cdot\frac{\theta_2}{\theta_1}\right)-\left(\frac{\pi_2}{\pi_1}\right)\left(\frac{\theta_1}{\theta_2}\right)h'(\tilde{a}_2(s))=\theta_1 g(s)$$
$$\Rightarrow h'\left(\tilde{a}_2(s)\cdot\frac{\theta_2}{\theta_1}\right)-(\pi_2)\left(\frac{\theta_1}{\theta_2}\right)h'(\tilde{a}_2(s))=\pi_1\theta_1 g(s)$$
$$\Rightarrow h'\left(\tilde{a}_2(s)\cdot\frac{\theta_2}{\theta_1}\right)-\left(\frac{\theta_1}{\theta_2}\right)h'(\tilde{a}_2(s))=\pi_1\left[\theta_1 g(s)-\left(\frac{\theta_1}{\theta_2}\right)h'(\tilde{a}_2(s))\right]$$
The LHS of the last equation is strictly positive because $\theta_2>\theta_1$, $h''(a)>0$ for all a>0 and $\tilde{a}_2(s)=\hat{a}_1(s)\cdot\frac{\theta_1}{\theta_2}>0$.

Since $\pi_1>0$, it follows that $\theta_1 g(s)>\left(\frac{\theta_1}{\theta_2}\right)h'(\tilde{a}_2(s))$ which in turn implies that $\theta_2 g(s)>h'(\tilde{a}_2(s))$.

Comparing the last inequality with equation (5), it follows that for each $s\in[\underline{s},\bar{s}]$, $\hat{a}_2(s)>\tilde{a}_2(s)=\hat{a}_1(s)\cdot\frac{\theta_1}{\theta_2}$. Thus, for every $s\in[\underline{s},\bar{s}]$, $H(\hat{a}_2(s);\hat{a}_1(s),\theta_1,\theta_2)<0$.‖

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