RATIONAL EXPECTATIONS VS. ADAPTIVE BEHAVIOR IN A HYPERINFLATIONARY WORLD: EXPERIMENTAL EVIDENCE

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ABSTRACT

We study an overlapping generations (OLG) economy where the government finances a fixed real deficit through seignorage, the structure of the economy and the level of deficit is common knowledge, and agents observe past prices. In this model there is a continuum of nonstationary equilibria and two stationary equilibria. When the rational expectations hypothesis is satisfied, a continuum of equilibria have paths converging to the stationary equilibrium with a higher inflation; conversely, when adaptive behavior is shown by agents, a continuum of inflation paths converge to the lower inflation --Pareto superior-- stationary equilibrium (see [11] and [8]). We test these contrasting hypotheses in an experimental environment.

We find that inflationary paths lie close to the lower inflation stationary equilibrium and even when agents start at inflation rates that diverge from it, they do not follow rational expectations noninflationary paths, but paths that converge to a neighborhood of the lower inflation stationary equilibrium; furthermore, agents do not react to advance knowledge of future changes in parameters. The data, however, show a bias towards the Pareto optimal path of constant consumption and the corresponding stationary inflation rate. This bias does not seem to be caused by a deviation from perfect competition, i.e., as a noncooperative solution to a market game. Our subjects gain enough experience to be able to forecast prices with great accuracy within a stationary environment, but are not able to foresee the effects of announced changes in parameters, or to specify sophisticated supply schedules. This factor may explain the bias towards constant consumption.

1. INTRODUCTION

We can think of resource allocation mechanisms, such as the price mechanism, as defining a map from economic environments to final allocations. In economies with limited exchange possibilities, however, the price-monetary mechanism usually assigns a continuum of equilibria to any specific environment. An example of such indeterminacy problem appears in the overlapping generations environment with fiat money as the unique financial asset.

While a serious theoretical effort has been made in recent years to study and characterize the indeterminacy problem, we can say little about its empirical relevance. Time series from *real economies* may shed no light on this problem since ex-post realizations can come from a large (indetermined) or small (determined) set of possible equilibria. Cross-sectional evidence of multiplicity is equally uninformative since it may come from different environments. Even if, by using equilibrium restrictions, we can characterize a given realized path, e.g., a stationary path, the observed data and the available theory do not provide a satisfactory explanation of why the economy has settled in a particular path. In fact, as long as equilibrium restrictions are satisfied, economic theory only says that the economy followed an observed path because it was one of the possible equilibria. Perhaps this is an accurate picture and we should not try to narrow the set of equilibria.

Using experimental data we can observe human behavior in a controlled economy for which we have a complete characterization of the set of equilibria. If we systematically observe that some of the possible equilibria are disregarded, we must either conclude that our theory (if it is claimed to be general enough so the experimental environment is included in the range of its applicability) is in need of further refinement. Otherwise, we must argue that our experimental environment does not capture some critical features of the theoretical models which predict such equilibria and indeterminacy problems. Similar conclusions will have to be reached if we observe systematic patterns of behavior that can only be described as disequilibrium phenomena by our existing theories.

In mapping environments to outcomes, it is necessary to make behavioral assumptions about how agents learn and make decisions. The description of the set of equilibria is conditional on a particular set of behavioral assumptions. For example, under the rational expectations hypothesis agents exploit all available information; under an adaptive hypothesis agents may not use only a part of the available information to guide their future actions. The rational expectations hypothesis is, essentially, an equilibrium hypothesis that does not specify behavior outside equilibrium paths. It does not tell us how an intelligent agent forecast prices when the known past history does not conform to a rational expectation equilibrium path. Alternatively, adaptive hypotheses do provide guidance on how agents may behave under **any** observed history. Such hypotheses, however, may have the inconsistent feature that the agents, after they gain enough experience, may want to deviate from parsimoniously adapting to the past. If the adaptive paths converge fast enough, this inconsistency may not be observable because they can only converge to stationary rational expectations paths.

In [7] Lim, Prescott and Sunder (L.P.S.) offer the first experimental evidence on OLG economies. They study a simple OLG model with two-period lived agents, stationary endowments patterns and preferences and no government demand. Fiat money is the unique financial asset. While the set of rational expectations equilibria includes a continuum of equilibria converging to the autarkic solution where money has no value, none of these equilibrium paths emerges in the experimental environment. Trading patterns and prices converge to the Pareto optimal rational expectations equilibrium of constant consumption (prices show an upward bias that may be explained as an outcome of imperfect competition).

The L.P.S. experiment can be thought as a sophisticated version of the experiment proposed by Lucas in order to identify adaptive behavior versus rational expectations [8]. As Lucas argues, it is implausible that in a long but finite horizon economy with zero terminal value of money, agents will solve by backwards induction the *perfect foresight* equilibrium as to be reduced to the autarkic equilibrium, and *"how much less plausible is it that similar subjects, situated in an infinite stage version of this same economy, should unanimously hit on the identical, wholly arbitrary value of* q_0 $(=1/p_0$, *the inverse of the price level) that sets them and (somehow) their successors off on one of this economy's unstable equilibrium paths?"* [8] {p. 238}. As we discuss in Section 4 the last two experiments in L.P.S. (where they use *the forecasting game*) are a close approximation to the infinite horizon model.

One would like to isolate the test of adaptive versus rational expectations from other extraneous factors. In an economy where the continuum of rational expectations paths converges to a long run equilibrium of zero value of money, one might argue that agents are simply avoiding this bad outcome, in the same sense that we observe cooperation in finitely repeated Prisoner's Dilemma games played in the laboratory. It is not clear, however, how agents can reach any form of coordination in an OLG structure (even if our subjects re-enter the game with new random groupings). In order to focus more closely on the issue of adaptive versus rational expectations, it seemed to us that more refined experiments were in order.

We study an OLG economy where the government finances a fixed real deficit through seignorage, the structure of the economy and the level of deficit is common knowledge, and agents observe past prices. This is a version of Cagan's model of hyperinflation [5] as studied by Sargent and Wallace [12, 13]. In this model there is a continuum of non-stationary equilibria and two stationary equilibria. When the rational expectations hypothesis is satisfied, a continuum of equilibria have paths converging to the stationary equilibrium with a higher inflation; conversely, when adaptive behavior is shown by agents, a continuum of inflation paths converge to the lower inflation --Pareto superior-- stationary equilibrium. Furthermore, only the low inflation stationary equilibrium is *classical* in the sense that lowering the government deficit forever lowers the stationary inflation rate.

Alternative forms of adaptive learning have been studied by Bray, Blume, Easley and others (see [3, 4], [1, 2]). More recently, Sargent and Marcet characterized *least squares learning* as a form of adaptive behavior [9-11]. In particular, in [11] they analyzed Cagan's model of hyperinflation and provided conditions under which least squares learning converges to the low inflation stationary equilibrium. In our experimental work we take *least squares learning* as a reference even though it is only a specific form of adaptive learning. What is the general feature of adaptive learning is the direction in which agents adapt to past histories and prices, rather than the specific path postulated by a leasts squares learning rule.

In our experiments the structure of the economy and the level of deficit is common knowledge, and agents observe past prices. We implement indefinitely-lived economies of OLG models in laboratory environment by promising (and keeping the promise) to convert the money balances held by the participants at the end of the game into consumption good at a price equal to the average prediction of price for that period (see [7]). Price predictions for the following period are solicited from nonparticipating subjects in every period of the economy.

We find that inflationary paths lie close to the stable adaptive equilibrium and even when agents start at inflation rates that diverge from it, they do not follow rational expectations noninflationary paths, but paths that converge to a neighborhood of the lower inflation stationary equilibrium; furthermore, agents do not react to advance knowledge of future changes in parameters. The data, however, show a bias towards the Pareto optimal path of constant consumption and the corresponding stationary inflation rate. This bias does not seem to be caused by a deviation from perfect competition, because it is opposite in direction to the noncooperative solution to a market game. Our subjects gain enough experience to be able to forecast prices with great accuracy within a stationary environment. In reacting to changes in parameters announced in advance, they wait until the changes are actually implemented. They do not specify sophisticated supply schedules. This factor may explain the bias towards constant consumption.

The rest of the paper is organized as follows. In Section 2 we introduce the competitive model with its two alternative behavioral assumptions; in Section 3 we analyze a version of the market games implemented in the experiment; in Sections 4 and 5 we describe the experimental environment and the four experimental economies, respectively; in Sections 6 and 7 we discuss our results and some further research.

2. THE COMPETITIVE MODEL

This is a simple OLG model with n agents in each generation and all generations born after time zero live for two periods. Two period-lived agents are homogeneous with endowments $(\omega_t^1, \omega_t^2) = (\omega^1, \omega^2)$ and preferences represented by $u(c_t^1, c_t^2) = \alpha(c_t^1 \cdot c_t^2)^{1/2}$, where c_t^n denotes the consumption of an agent born in period t at the n^{th} period of his life, and $\alpha > 0$ (in experiments α is the transformation factor from experimental commodity to dollars).

The initial old generation has an endowment of $\omega_0^2 = \omega^2$ and preferences represented by $u(c_0^2) = \alpha(c_0^2)^{1/2}$.

Rational Expectations

The representative consumer of generation $t \geq 1$ solves

$$\max \quad \alpha(c_t^1 \cdot c_t^2)^{1/2}$$

$$\text{s.t.} \quad p_t(c_t^1 - \omega^1) + p_{t+1}(c_t^2 - \omega^2) \leq 0$$

$$p_t(c_t^1 - \omega^1) \leq 0$$

Let $\pi_{t+1} = \dfrac{p_{t+1}}{p_t}$, then if $\omega^1 - \omega^2$ is large enough, the solution to this problem satisfies $\dfrac{c_t^1}{c_t^2} = \pi_{t+1}$. That is,

$$s_t = (\omega^1 - c_t^1) = 1/2\,(\omega^1 - \pi_{t+1}\omega^2) \geq 0 \tag{1}$$

Let H_t be the money supply at time t, $h_t = \dfrac{H_t}{n}$ the per capita nominal money supply and $m_t = \dfrac{h_t}{p_t}$ the per capita real money supply. Then the following equilibrium condition must be satisfied

$$m_t = s_t = 1/2\,(\omega^1 - \pi_{t+1}\omega^2) \tag{2}$$

The government has to finance a constant deficit D through seignorage. Let $d = D/n$, then the monetary policy is given by

$$d = \frac{h_t - h_{t-1}}{p_t} = (m_t - m_{t-1}/\pi_t) \tag{3}$$

Substituting (2) into (3) we obtain

$$d = \frac{\omega^1}{2} - \pi_{t+1}\frac{\omega^2}{2} - \frac{\omega^1}{2\,\pi_t} + \frac{\omega^2}{2} \tag{4}$$

That is, under the rational expectations hypothesis, paths of equilibrium inflation rates are given by

$$\pi_{t+1} = \frac{\omega^1}{\omega^2} + 1 - \frac{2d}{\omega^2} - \frac{\omega^1}{\omega^2}\frac{1}{\pi_t} \tag{5}$$

$$\text{i.e., } \pi_{t+1} = a - \frac{b}{\pi_t}, \tag{5'}$$

where $b = \dfrac{\omega^1}{\omega^2}$ and $a = b + 1 - \dfrac{2d}{\omega^2}$. If $a^2 > 4b$ (5') has two stationary solutions $(\underline{\pi}, \bar{\pi}) = ½\,(a - (a^2 - 4b)^{½}, a + (a^2 - 4b)^{½})$, and $\bar{\pi}$ is the stable solution characterizing the asymptotic behavior of a continuum of rational expectations equilibrium paths.

Adaptive Expectations (Least Squares Learning)

Equations (2) and (3) define the evolution of prices and money holdings as

$$p_t = b^{-1}p_{t+1} + ch_t \tag{6}$$

$$\text{and } h_t = h_{t-1} + dp_t \tag{7}$$

where $c = 2/\omega^1$.

Suppose agents postulate a linear relation between prices of the form $p_{t+1} = \beta p_t \ \ \forall_t$, then from (6) and (7) we obtain

$$p_t = \frac{c}{(1 - b^{-1}\beta)}\, h_t \tag{6'}$$

$$\text{and } h_t = \frac{1 - b^{-1}\beta}{1 - b^{-1}\beta - cd}\, h_{t-1} \tag{7'}$$

$$\text{or } p_{t+1} = S(\beta)\, p_t, \quad \text{where } S(\beta) = \frac{1 - b^{-1}\beta}{1 - b^{-1}\beta - cd} \tag{8}$$

Solutions for prices and money balances exist as long as $\beta < (1 - dc) \cdot b$. The map $S(\beta)$ has two fixed points, provided that $a^2 > 4b$, which are the stationary solutions obtained through (5). Notice that the quadratic form is $\beta^2 - a\beta + b = 0$, where $a = 1 + b - bcd$.

With *least squares learning* (see [11]), the evolution of inflation rates is given by

$$\beta_t = (1 - g_{t-1})\, \beta_{t-1} + g_{t-1} \frac{(1 - b^{-1}\beta_{t-2})}{(1 - b^{-1}\beta_{t-1})} S(\beta_{t-1}) \tag{9}$$

or approximately by,

$$\beta_t = (1 - g_{t-1})\beta_{t-1} + g_{t-1}\, S(\beta_{t-1}) \tag{10}$$

where $g_{t-1} = \dfrac{p_{t-2}^2}{R_0 + \Sigma_{j=1}^{t-1}\, p_{s-1}^2}$, and R_0 is an initial value.

3. MARKET GAMES

With a small number of agents in each generation we must characterize the set of Nash equilibria of the corresponding noncooperative game. In the experiments reported here, young agents submit their supply schedules, as in a Shubik game (see [6]). More precisely, they submit a list of reservation prices for all the integer units that they can sell, and individual supply functions are constructed by interpolation.

The game where the set of strategies are all possible supply schedules is fairly complex. As a reference, we analyze a simpler quantity game. In this game, an agent's strategy is the quantity that he offers when he is young. A Nash equilibrium of this game is a Nash equilibrium of the game with supply schedules as strategies.

The Quantity Game When Agents Are Fully Rational

Suppose for $i = 1, \ldots, n$, $s_{it} = \bar{s}_t$, $s_{it+1} = \bar{s}_{t+1}$, and $d_t = d$, then p is an equilibrium price if it satisfies $\bar{s}_{t+1} = \dfrac{h_t}{p} + d$. That is, in equilibrium,

$$p_{t+1} = \frac{h_t}{\bar{s}_{t+1} - d} \quad \text{and} \quad \frac{p_t}{p_{t+1}} = \frac{p_t(\bar{s}_{t+1} - d)}{h_t}$$

Given that $h_t = h_{t-1} + p_t d$,

$$\frac{p_t}{p_{t+1}} = \frac{p_t(\bar{s}_{t+1} - d)}{h_{t-1} + p_t d}$$

$$= \frac{(\bar{s}_{t+1} - d)}{\dfrac{h_{t-1}}{p_t} + d} = \frac{(\bar{s}_{t+1} - d)}{\bar{s}_t}.$$

Agent i's best response to the symmetric strategies $\bar{s}_t$ and $\bar{s}_{t+1}$ is the solution to

$$\max\ (\omega^1 - s)\left[\omega^2 + \frac{s\,(\bar{s}_{t+1} - d)}{\bar{s}_t - \frac{1}{n}(\bar{s}_t - s)}\right] \tag{11}$$

The interior first order condition to this problem is given by

$$-\omega_1 - (\bar{s}_{t+1} - d) + (\omega_0 - s_t)\left[\frac{n-1}{n}\right]\frac{(\bar{s}_{t+1} - d)}{s_t} = 0 \tag{12}$$

Using the fact that $\pi_{t+1} = \bar{s}_t/(\bar{s}_{t+1} - d)$ we obtain the following supply schedule

$$s_t = \left[2 - \frac{1}{n}\right]^{-1}\left[\left[\frac{n-1}{n}\right]\omega^1 - \pi_{t+1}\,\omega^2\right] \tag{13}$$

As before, $\quad d = s_t - s_{t-1}/\pi_t$

$$= \left[2 - \frac{1}{n}\right]^{-1} \left[\left[\frac{n-1}{n}\,\omega^1 - \pi_{t+1}\,\omega^2\right] - \left[\frac{n-1}{n}\right]\frac{\omega^1}{\pi_t} + \omega^2\right]$$

That is,

$$\pi_{t+1} = \left[\frac{n-1}{n}\right]\frac{\omega^1}{\omega^2} + 1 - \left[2 - \frac{1}{n}\right]\frac{d}{\omega^2} - \left[\frac{n-1}{n}\right]\frac{\omega^1}{\omega^2}\,\frac{1}{\pi_t} \tag{14}$$

$$i.e., \pi_{t+1} = a_n - \frac{b_n}{\pi_t},$$

where $b_n = \frac{\omega^1}{\omega^2} - \frac{1}{n}\frac{\omega^1}{\omega^2}$, and $a_n = b_n + 1 - \left[2 - \frac{1}{n}\right]\frac{d}{\omega^2}$.

It should be clear that, whenever they exist, the two stationary solutions of this game converge to the stationary rational expectations equilibria as $n \to \infty$. Furthermore, the curve described by (14) lies below the curve described by (5), which means that the two steady states of the non-cooperative game converge to the steady states of the competitive model *from inside*, e.g., the non-cooperative game has a lower stationary inflation rate characterizing the asymptotic behavior of a continuum of adaptive paths, and a higher stationary inflation rate that defines an adaptive path only if the economy starts at this particular inflation rate.

The Quantity Game with Partially Adaptive Agents

Now suppose agents understand the equilibrium equation

$$p_t = \frac{h_{t-1}}{(\bar{s}_t - d)}$$

but that they do not endogenize the monetary policy rule $h_t = h_{t-1} + p_t d$. Instead, they simply postulate that $h_t = \beta h_{t-1}$, then

$$\frac{p_t}{p_{t+1}} = \frac{h_{t-1}/(\bar{s}_t - d)}{h_t/(\bar{s}_{t+1} - d)} = \frac{\bar{s}_{t+1} - d}{\beta(\bar{s}_t - d)} \tag{15}$$

If agent i plays against the symmetric strategies $\bar{s}_t = \bar{s}$, then his best response is a solution to

$$\max\ (\omega_t^1 - s)\left[\omega^2 + s \cdot \beta^{-1} \cdot \frac{\bar{s} - d}{\bar{s} - d - \frac{1}{n}(\bar{s} - s)}\right] \tag{16}$$

The interior first order condition is given by the following quadratic equation

$$\left[\frac{2n-1}{n}\right]\bar{s}^2 - \left[2d - (\omega^1 - \beta\omega^2) - \frac{\omega^1}{n}\right] s + d(\omega^1 - \beta\omega^2) = 0 \tag{17}$$

Let

$$A_n(\beta) = 2a_n[b_n(\beta) + (\,b_n(\beta)^2 - 4a_n de(\beta)\,)^{1/2}\,]^{-1}, \tag{18}$$

where $e(\beta) = \omega^1 - \beta\omega^2$, $b_n(\beta) = 2d + e(\beta) - \frac{\omega^1}{n}$ and $a_n = 2 - 1/n$. It follows that

$$P_t = A_n(\beta)h_t \tag{19}$$

and given (6),

$$h_t = [\,1 - dA_n(\beta)\,]^{-1} h_{t-1}$$

$$i.e., P_{t+1} = S_n(\beta)\,P_t,$$

where

$$S_n(\beta) = [\,1 - dA_n(\beta)\,]^{-1} \tag{20}$$

It can easily be shown that $S_n(\beta) \to S(\beta)$ as $n \to \infty$. Furthermore, as in the *quantity game with fully rational players*, the two fixed points of $S_n(\cdot)$ converge to the fixed points of $S(\cdot)$ *from inside*, e.g., a continuum of adaptive paths of the non-cooperative game converge to a stationary inflation rate that is higher than the corresponding stationary inflation rate of the competitive model.

4. THE EXPERIMENTAL ENVIRONMENT

Four experimental economies, numbered chronologically for reference in this paper, were conducted in three sessions of approximately 3 hours each. A fixed number N of subjects participate in each session. Subjects know the approximate length of the session but not the duration of a particular economy. For each period of an economy, agents are assigned specific roles: n subjects act as young consumers, n as old consumers, and the remaining $(N-2n)$ wait as interested onlookers in the market. At the beginning of each period the identity of n subjects who enter as young consumers is announced. These n members of the new generation are randomly selected from the $(N-2n)$ players who were outside the market in the previous period. Once an agent enters as a young consumer, he/she stays next period as an old consumer. Consumers receive a higher endowment of chips (ω^1 units) when young and they may offer to sell all or a part of it to the old consumers. To do this they must submit a supply schedule; a reservation price for each integer quantity i, $i = 0, 1, \ldots, \omega^1$. A continuous supply schedule is computed by linear interpolation, and once the market clearing price is determined they automatically sell the quantity defined by their continuous supply. In exchange for the chips they sell, young consumers receive francs that they carry to their old age in the next period. Old consumers receive an endowment of chips ($\omega^2 < \omega^1$) and they simply offer all their money holdings (francs) in exchange for more chips. The number of chips held at the end of young period, c^1 and at the end of old period, c^2 determine the dollar amount $\alpha \cdot \sqrt{c^1 \cdot c^2}$ which is earned by the subject when he/she leaves the market at the end of the old period. This dollar amount is accumulated

Most of the subjects participated in all three sessions; only three new subjects were added in the third session (Economies 3 & 4).

separately and the total is paid to subjects at the end of the experiment. If they reenter the market as young in a subsequent generation they can not use dollars from this account. That is, they reenter as new subjects. The total number of subjects (N) is chosen to be sufficiently large ($N > 3n$) to ensure that each subject sits out for 1 or more periods between leaving and re-entering the economy.

It is *common knowledge* that the experimenter buys D chips every period at the market clearing price and that, therefore, the amount of money (francs) in circulation grows. The market clearing price is computed and announced each period. The past history of prices is displayed on the chalkboard.

The Terminal Condition

The OLG model requires an infinite horizon and in this sense can not be cast in an experimental environment. The experimenter has to choose a form to end the experiment which may affect the set of equilibria. We use a procedure introduced in [7] to end each experiment. During the experiment, players outside the market play a *forecasting game*: at the beginning of each period, they are asked to forecast the equilibrium price for the period. The player(s) whose prediction turns out to be the best ex-post receive(s) a prize (in dollars) that it is added to his/her dollar account. This *forecasting game* has no direct effect on the evolution of the market. Winning forecast is announced and displayed on the chalkboard.

Without any previous announcement, and after forecasts for the following period have been submitted, the experimenter declares that the period just finished is the last period of the economy. It is then that the *forecasting game* plays a role. Money holdings by agents at the end of their entry period are converted into chips using *the average of predicted market prices by outside-market participants*. This procedure for ending the game is announced and explained to all subjects at the outset as a part of the instructions.

It should be clear that this mechanism does not reduce the set of competitive equilibria. It does, however, extend the set of rational expectations equilibria. Paths for which the value of money grows unboundedly can be monetary equilibria of any finite economy if outside and inside participants forecast such values. In this sense, our terminal condition makes the indeterminacy problem more acute by introducing a coordination game between outside and inside participants. With an unknown terminal period, however, beliefs supporting equilibria that are not equilibria of the infinite horizon model might seem unreasonable. But we do not have at this point an argument to rule out such equilibria.

Since we are studying how equilibria may be selected in naturally occurring economies, if our less predictable experimental economy gives rise to empirically predictable behavior we would have made an even stronger argument. As we will see, our data shows that outside participants tend to adapt to the market and their final prediction does not affect equilibrium paths.

5. DESCRIPTION OF THE EXPERIMENTAL ECONOMIES

Remark: To simplify notation, we will call the *high inflation stationary rational expectations equilibrium* the *long-run REE,* since it characterizes the asymptotic behavior of a continuum of nonstationary rational expectations equilibrium paths. In contrast, we will call the *low inflation stationary rational expectations equilibrium* the *long-run AEE,* since it characterizes the asymptotic behavior of a continuum of nonstationary paths when agents behave adaptively. Notice that the long-run Adaptive Expectations Equilibrium is, in fact, a stationary rational expectations equilibrium when agents follow an equilibrium path that starts at the low stationary inflation rate, i.e., at the "right" initial price.

Key design features of the four experimental economies reported here are given in Table 1 and the equilibrium predictions of the theoretical models discussed in Sections 2 and 3 about the performance of these economies are given in Table 2.

In Economy 1 (N=14, n=4, ω^1=7, ω^2=1, d=0.5, T=19), conducted in the first experiment, subjects had no prior history and the initial conditions were determined endogenously. This economy was designed to obtain a large separation between the asymptotic predictions of inflation under the rational expectations and adaptive expectations models. These predicted rates are $\pi = 5.79$ and 1.21 respectively. The economy was terminated (five minutes before the previously announced length of the experiment) at the beginning of Period 20 after price forecasts for that period had been gathered.

Subjects in Economy 1 were inexperienced and had no expectations about inflation that we (as experimenters) can have any reliable knowledge of. The same subjects (with one exception) participated in Economy 2 and it was designed to exploit our knowledge of the expectation they may have formed about inflation (N=13, n=4, ω^1=7, ω^2=1, d=0.25 *or* 1.25, T=13+4+16). The long-run REE prediction of inflation was higher than the long-run REE predictions of Economy 1; it was also higher than the inflation the subjects had experienced in Economy 1. The long-run AEE prediction of Economy 2 was lower than the long-run AEE prediction of Economy 1 as well as the experienced rate in that economy. While Economy 1 was designed to gather evidence on the *level* of the rate of inflation, Economy 2 was designed to see if the *change* in the rate of inflation occurs in the direction indicated by one model or the other.

A second consideration in designing Economy 2 was to make the best use of one surprise "change" we could spring on the subjects. This consisted of a single change, with a four period advance warning before it took effect, in per capita government deficit from 0.25 to 1.25. This change, announced at the beginning of period 14 and implemented in period 18, increased the long-run AEE equilibrium rate of inflation from $\pi = 1.09$ to 2 and lowered the long-run REE equilibrium rate from $\pi = 6.41$ to 3.5. Changes predicted by these two models in rates of inflation had opposite signs. This design also permitted us to test the predictive validity of the two sets of assumptions about human behavior along another dimension. A strict form of the adaptive expectations hypothesis predicts that no change in behavior should be observable until the change in parameters is actually implemented in period 18. The REE model, on the other hand, predicts that the effect of change in parameters should become observable immediately after the announcement of the proposed change in period 14; it does not say, however, that there is a well defined path from one long-run REE to another long-run REE.

Economies 3 and 4 were run consecutively in a single session with 9 subjects drawn from those who participated in Economies 1 and 2 and three new subjects to make up a total of 12. Economy 3 (N=12, n=3, ω^1=7, ω^2=1, d=1.25, T=17) retained the parameters of Economy 2b to replicate its results with a smaller number of players (n=3) and an assurance to the subjects that there will be no changes in parameters between the beginning and the end of the economy. The initial per capita money holdings, h_0, was set at 3.722 which was 1/10,000 of the per capita money holdings at the end of Economy 2. The history of prices from Economy 2 was publicly displayed at the beginning of Economy 3.

Since the results obtained from the first three economies strongly supported the long-run AEE as a clearly superior predictor of data, Economy 4 was designed to give the REE model its "best shot." We did so by retaining the parameters of Economy 3 for the first 7 periods and announcing at the outset that, beginning period 8, we shall change the endowment of the young (ω^1 from 7 to 3) and the government deficit (d from 1.25 to 0.25). With this shift in parameters, the long-run AEE rate of Economy 4a changed from $\pi = 2$ to 1.5 and the long-run REE rate changed from $\pi = 3.5$ to 2. In other words, the long-run REE rate shifted to the long-run AEE rate and in the neighborhood of the inflation rate experienced by the subjects during Economies 2b, 3 and 4a. Our purpose in designing this economy was to find out if long-run REE might be sustained in an environment where the past experience has been consistent with its predictions.

6. DISCUSSION OF THE RESULTS

Even if more experimental data is needed, some empirical regularities seem to arise from our experimental economies.[1]

Adaptive Behavior vs. Rational Expectations

Figure 1 clearly shows that *the continuum of equilibria converging to the long-run REE equilibrium of high inflation are not observed in our experimental economies*. Economies 1 and 2a are unusually clean pictures of the stability of the AEE of low inflation. Data from economies 2b, 3 and 4 tend to cluster close to, but slightly below, the long-run AEE equilibrium. Economy 4b, while showing more dispersion around the long-run AEE, has the interesting feature that convergence is achieved even from a distant initial condition -- in fact from a point that is unstable under least squares learning.

The evolution of inflation paths can be closely seen in Figure 2 (Notice that, given the scale chosen, some of the predicted equilibria are not in the figures). In Economies 1 and 2a we observe rapid convergence and not much oscillation around the long-run AEE. In contrast, Economies 2b and 4 show more fluctuations and a less clear tendency to converge to the long-run AEE. Economy 3, a replica of 2b, shows convergence but to an inflationary rate below the

[1] Detailed data is available from the authors upon request.

long-run AEE.

Two related features distinguish Economies 1 and 2a from the remaining economies. First, the high rate of convergence for an adaptive rule, such as least squares learning, and second, the *closeness* of the long-run AEE to the stationary rate of inflation supporting the Pareto optimal constant consumption path. Economies 1 and 2a have a higher rate of convergence than the other economies; this can be seen from simple inspection of the graph of S(β) in Fig. 1 and it is also given by the coefficient k in Table 2 (a lower k corresponds to a higher rate of convergence). That we may observe a tighter convergence in economies with low rates of convergence when they are allowed to operate for more periods is an open possibility. Economies 1 and 2a, however, also have a long-run AEE inflation rates that are very close to the corresponding stationary PO inflation rates, and it is not possible to statistically reject the hypothesis that inflation rates converge to the stationary PO inflation rate in Economies 1 and 2a. The inflationary path of economy 3 seems to reinforce the existence of a bias, already detected in Economy 2b, towards the PO point.

The Bias Towards Constant Consumption

Figure 3 shows the evolution of prices (the log of the price level) over time. We can clearly see the bias towards the stationary inflation rate supporting the Pareto optimal path of constant consumption (after the deficit is being paid). This bias in inflation rates corresponds to an excess volume of trade (see Fig. 4) and results in an extremely good performance of these economies in terms of efficiency (as measured by dollar earnings) (see Fig. 5). It is a common feature of experimental markets that they tend to arrive, especially experienced subjects, at competitive equilibrium and the subjects extract the largest possible pay off from the experimenter. In our case, however, the Pareto optimal allocation of constant consumption is not an equilibrium allocation. Given the PO inflationary path, if young agents behave either competitively or strategically, there are individual gains from defecting by reducing the individual supply (see Table 2). As we have seen in Section 3, the the long-run Adaptive Nash Equilibrium of the quantity game has a higher rate of inflation than the competitive long-run AEE; this result seems to extend to other more general offer games as long as supplies are well behaved increasing functions.

The bias towards efficiency is suggestive of a more cooperative solution of the dynamic (repeated) game. There are two problems with this interpretation. First, agents cannot coordinate their actions through some form of pre-play communication. Similarly, opportunities for endogenous self-enforcement through reputation/punishment (folk theorems) are very limited since individual actions are not observed and even if agents reenter the economy, they are randomly selected from the group of outsiders. Second, if agents follow strategies oriented to support an optimal path of constant consumption, then we should expect very inelastic supply curves around the point of PO supply. Figure 6 shows that this has not been the case. On the contrary, Fig. 6 shows that supply curves have been *too elastic* even compared with the theoretical competitive supplies. The latter have been computed using the price forecasts for the following period submitted by the outsiders, under the assumptions that (1) insiders' predictions of future prices are not too different from the predictions made by the outsiders, and (2) forecasts for p_{t+1} conditional on p_{t-1} are closely approximated by the submitted forecasts which are conditional on p_t. The three curves represent the three theoretical supplies according to the cross-sectional minimum, mean and maximum of the price forecasts made by the outside subjects.

Even if at the individual level we observe some supplies which are inelastic around the point of PO supply, in the aggregate and through the experiment, supplies are similar to the ones depicted in Fig. 6. Even if we have induced specific preferences on agents, one could try to recover preferences from the observed supplies (a non trivial exercise on Richter's *rationalizability*). This exercise is meaningful only locally around the equilibrium. Fig. 6 suggests that supplies may have been generated from relatively *thick* indifference curves. If this were the case (and, in strict sense, we know it is not) then we could explain the high volume and the volatility of trade (see Fig. 4). It is important to notice that for somebody who is unfamiliar with *first order conditions* it is difficult to submit a competitive supply and the relatively easy to accept to sell *around the constant consumption supply.* We think that this factor may be a better explanation for the observed bias, than a conscious effort on the part of the agents to support a Pareto optimal allocation. More refined experiments and models are needed, however, to clarify this issue.

Learning to Live and Forecast in an Inflationary World

This lack of sophisticated supplies might suggest that our agents cannot be compared to *experienced traders outside the laboratory.* In fact, the same Fig. 6 also shows that our agents were skilled traders. Even though supplies were too elastic compared with competitive supplies at the predicted prices, the observed market clearing prices were close to the theoretical equilibria. The ability of our agents to predict prices, almost from the outset, can be seen in Figure 7 where we show the time path of cross-sectional mean, the maximum and the minimum of the predictions submitted for each period, plotted along with the actual prices. In Figure 8 we show the time paths of the *least squares predictions* made on the basis of the past observed prices together with the cross-sectional mean of price predictions made by the outsiders and actual

The fact that agents had to submit reservation prices for all (and only for) integer units may have had an effect on flattening supplies computed by interpolation.

As a curiosity, we ex-post asked the agent responsible for the extreme low predictions in Economy 2 why did he submit such low prices. One can hardly think of a better fit !. Two facts should be noticed. First, the mean predicted price in the terminal period seems an excellent approximation to the price that would have been realized if the economy had not been truncated at this period. Second, after midstream changes in regime, for example after Period 17 in Economy 2 or after Period 7 in Economy 4, mean predictions are closer to actual prices than leasts squares predictions are. Our agents are more sensitive to realized changes than the leasts squares automata. The tracking sensitivity of human subjects to changes can also be seen in Figure 9 where inflation rates implied by mean predictions of price are plotted against the realized rate of inflation in the preceding period.

Learning to Live in a Changing Environment

At two occasions we made advance announcements of a change of regime to the agents. In Economy 2 we announced at the end of Period 13 that the deficit was going to be increased in Period 18 and maintained at the higher level until the end of the economy. As can be seen from the figures and from Table 3, agents *did not* react to the announced change until it was implemented. Similarly, at the beginning of Period 1 in Economy 4 we announced that (1) the deficit was going to be reduced; (2) the endowment of the young was also going to be reduced; (3) both changes were to take effect at the beginning of Period 8; and (4) all parameters were to be maintained at those levels until the end of the economy. Again (see Table 3 and Figs. 2 & 4), agents did not fully anticipate the change; only in Period 7 there is a drastic contraction of the supply which creates a momentary outburst of inflation. In both economies, agents adapted rapidly to the new situation after changes were implemented.

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predictions and he answered that he remembered what happened on October 19, 1987 !.

One could use our data, because it shows a lack of adjustment to a change announced in advance, as evidence in favor of adaptive and against rational expectations (see Appendix I). We do not share this point of view. Our agents have gained experience with inflation, but their experience with announced changes has been limited (this might also be the case in naturally-occurring economies). It is plausible to think that more experienced agents, even if they follow paths converging to the long-run AEE , might still learn to adjust to announced but unimplemented changes, and the reaction in Period 7 in Economy 4 may be a weak indication of this type of learning. More experimental data would have to be generated to test this conjecture.

7. EPILOGUE

These experiments have been some first steps towards a more systematic study of modes of behavior in dynamic environments. The task is not simply *to generate experimental data* but to better understand the direction in which our theoretical models have to be modified to encompass empirical regularities. Learning is a delicate issue. Our equilibrium theories seem to do a good job in explaining long run dynamics; except, perhaps, in choosing the wrong long-run equilibrium. Adaptive learning algorithms seem to provide a good approximation to short run dynamics and to provide a robust criterion for stability; however they may be too insensitive and/or irrational to effectively describe the behavior of professional decision makers. In contrasting these alternative approaches, we think that experimental evidence is a very valuable reference.

It has been argued that experimental evidence might lack the necessary *projectability* to be a guide for real economic phenomena. This is an open question that can only be answered with specific experiments and historical events in mind. Our experiments were partially motivated by Lucas's discussion on *Adaptive Behavior and Economic Theory* [8]. Lucas starts by discussing the empirical relevance of *The Quantity Theory of Money*. We reproduce in Figure 10 two of his figures: non-smoothed data can hardly be explained by the quantity theory, and yet, smoothed data come very close to the theoretical predictions. As we can see in Figure 11, certain key characteristics of our experimental economies, even if they are very simple deterministic economies constructed according our models, do not look very different from the characteristics of Lucas's naturally occurring economy of the United States. Perhaps, something **can be** *projected* from laboratory experience.

Table 1

Design of Experimental Economies							
Economy No.	Number of Subjects in the Economy and in each Generation (N,n)	Experience	Chip Endowment: Young ω^1	Chip Endowment: Old ω^2	Per Capita: Init. Money h_0	Per Capita: Govt. Demand d	Periods T
1	(14,4)	None	7	1	10	0.5	1-19
2	(13,4)	Economy 1	7	1	10	0.25 * 1.25	1-17 * 18-33
3	(12,3)	3 inexperienced 9 from Econ. 1 or 2	7	1	3.722	1.25 **	1-17 **
4	(12,3)	Same as Econ. 3	7 3	1 1	3.722	1.25 *** 0.25	1-7 *** 8-20

* At the end of Period 13, experimenter announced a parameter change in d from 0.25 to 1.25 to become effective the beginning of Period 18.

** At the outset of this economy, participants were informed that there will be no parameter changes between the beginning and termination.

*** At the outset of this economy, experimenter announced that a change in ω^1 from 7 to 3 and a change in d from 1.25 to 0.25 will be effective beginning Period 8 and that there will be no further changes until termination.

Table 2

Model Predictions

	Economy 1			Economy 2a			Economy 2b, 3 and 4a			Economy 4b		
	Infl.	Volume	Payoff	Infl.	Volume	Payoff	Infl.	Volume	Payoff	Infl.	Volume	Payoff
	π	s	$\sqrt{c^1 \cdot c^2}$	π	s	$\sqrt{c^1 \cdot c^2}$	π	s	$\sqrt{c^1 \cdot c^2}$	π	s	$\sqrt{c^1 \cdot c^2}$
Long-run R.E.E.	5.79	0.60	2.66	6.41	0.30	2.65	3.50	1.75	2.81	2.00	0.50	1.77
Long-run A.E.E.	1.21	2.89	3.73	1.09	2.95	3.87	2.00	2.50	3.18	1.50	0.75	1.84
Long-run R.N.E *	4.09	0.66	2.71	4.73	0.32	2.67	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
Low R.N.E. *	1.28	2.27	3.62	1.118	2.36	3.80	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
Long-run A.N.E *	1.33	2.00	3.54	1.123	2.27	3.78	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
High A.N.E. *	-						n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
Pareto Optimal	1.18	3.25	3.75	1.087	3.13	3.88	1.53	3.63	3.38	1.29	1.13	1.88
k **		.066			.030			.300			.555	

* $n = 4$ for Economies 1 and 2, $n = 3$ for Economies 3 and 4.

R.E.E. = Rational Expectations Equilibrium;
A.E.E. = Adaptive Expectations Equilibrium;
R.N.E. = Nash Equilibrium with fully rational agents;
A.N.E. = Nash Equilibrium with partially adaptive agents;
n.s. = Non Sustainable

** Under Leasts Squares Learning, converging paths are monotone and satisfy either $\pi_t - \pi_{t-1} \geq k \cdot (\pi_{t-1} - \pi_{t-2})$ or $\pi_{t-1} - \pi_t \geq k \cdot (\pi_{t-2} - \pi_{t-1})$, depending whether the path converges from below or above, respectively. If $k > 1$ then $\{\pi_t\}$ does not converge. (Proposition 3 in [11]).

Table 3

Predictions vs. Realizations

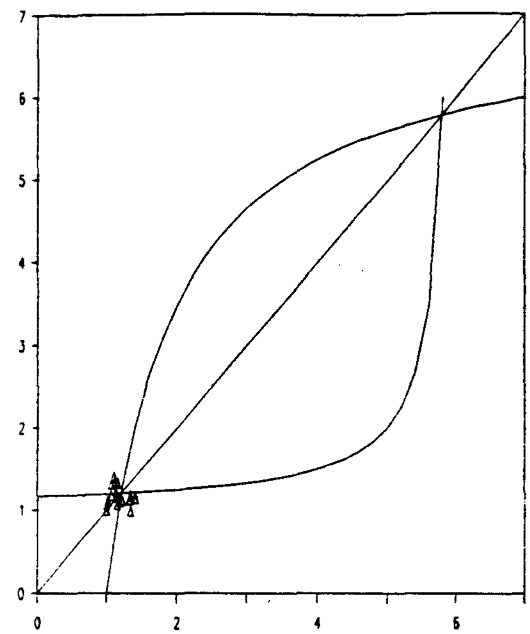
ECONOMY 1			
PREDICTIONS	INFLATION π	IND. SUPPLY S	UTILITY $\sqrt{c^1 \cdot c^2}$
Long-run R.E.E.	5.791	.604	2.658
Long-run A.E.E.	1.209	2.895	3.733
P.O.	1.182	3.25	3.75
DATA ECONOMY 1			
$\frac{1}{5}\Sigma_{n=15}^{19} x_n$	1.201	3.44	3.586

ECONOMY 2a			
PREDICTIONS	INFLATION π	IND. SUPPLY S	UTILITY $\sqrt{c^1 \cdot c^2}$
Long-run R.E.E.	6.407	.296	2.649
Long-run A.E.E.	1.092	2.954	3.871
P.O.	1.087	3.125	3.875
DATA ECONOMY 2			
$n = 2$	1.190	2.109	3.462
$\frac{1}{4}\Sigma_{r=10}^{13} x_n$	1.073	3.521	3.805
$\frac{1}{4}\Sigma_{n=14}^{17} x_n$	1.088	3.528	3.794

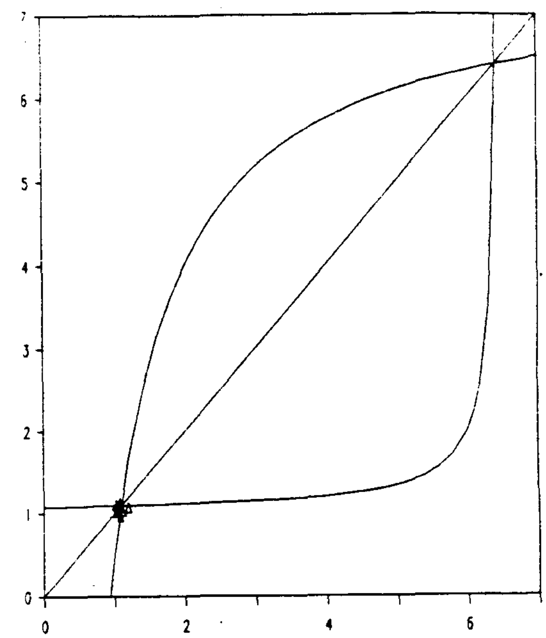
ECONOMY 2b, 3, & 4a			
PREDICTIONS	INFLATION π	IND. SUPPLY S	UTILITY $\sqrt{c^1 \cdot c^2}$
Long-run R.E.E.	3.50	1.75	2.806
Long-run A.E.E.	2.00	2.50	3.182
P.O.	1.526	3.625	3.375
DATA ECONOMY 2b			
$\frac{1}{4}\Sigma_{n=18}^{21} x_n$	1.364	4.217	3.281
$\frac{1}{5}\Sigma_{n=29}^{33} x_n$	1.664	3.281	3.251
DATA ECONOMY 3			
$\frac{1}{5}\Sigma_{n=2}^{6} x_n$	1.309	4.054	3.210
$\frac{1}{5}\Sigma_{n=13}^{17} x_n$	1.561	3.517	3.316
DATA ECONOMY 4			
$\frac{1}{5}\Sigma_{n=3}^{7} x_n$ (*)	1.657	3.506	3.277

ECONOMY 4b			
PREDICTIONS	INFLATION π	IND. SUPPLY S	UTILITY $\sqrt{c^1 \cdot c^2}$
Long-run R.E.E.	2.00	.50	1.768
Long-run A.E.E.	1.50	.75	1.837
P.O.	1.285	1.125	1.875
DATA ECONOMY 4			
$\frac{1}{5}\Sigma_{n=8}^{12} x_n$	1.406	1.501	1.703
$\frac{1}{5}\Sigma_{n=16}^{20} x_n$	1.297	1.186	1.760

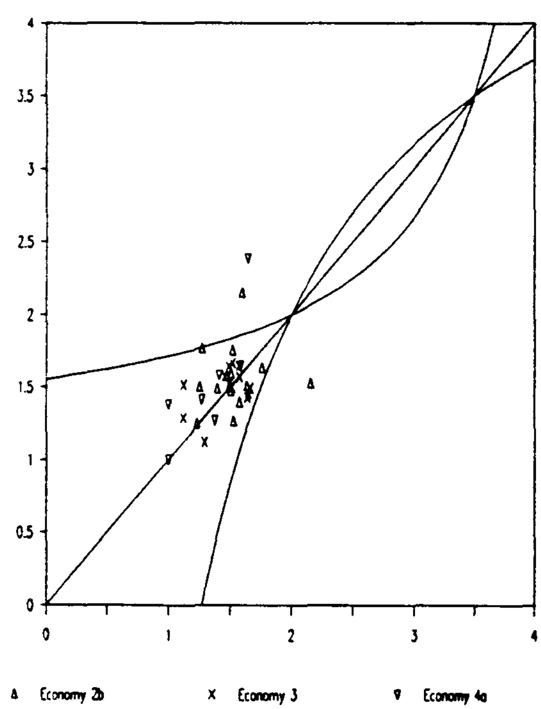
Figure 1



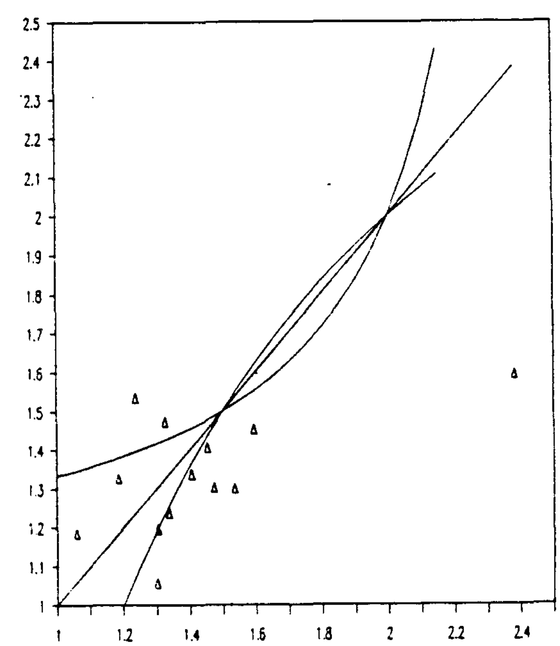
Economy 1



Economy 2A



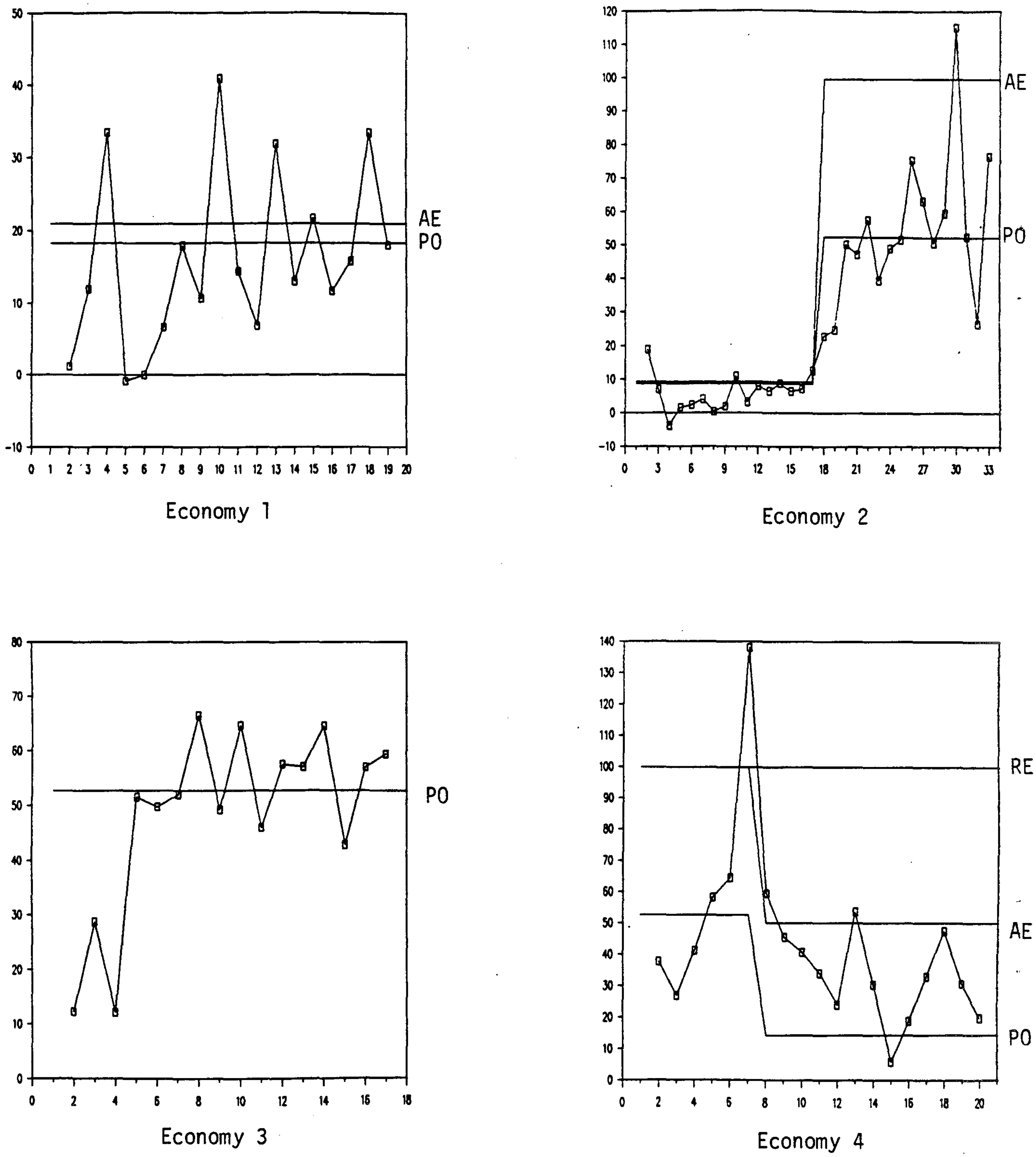
Economies 2B, 3 and 4



Economy 4B

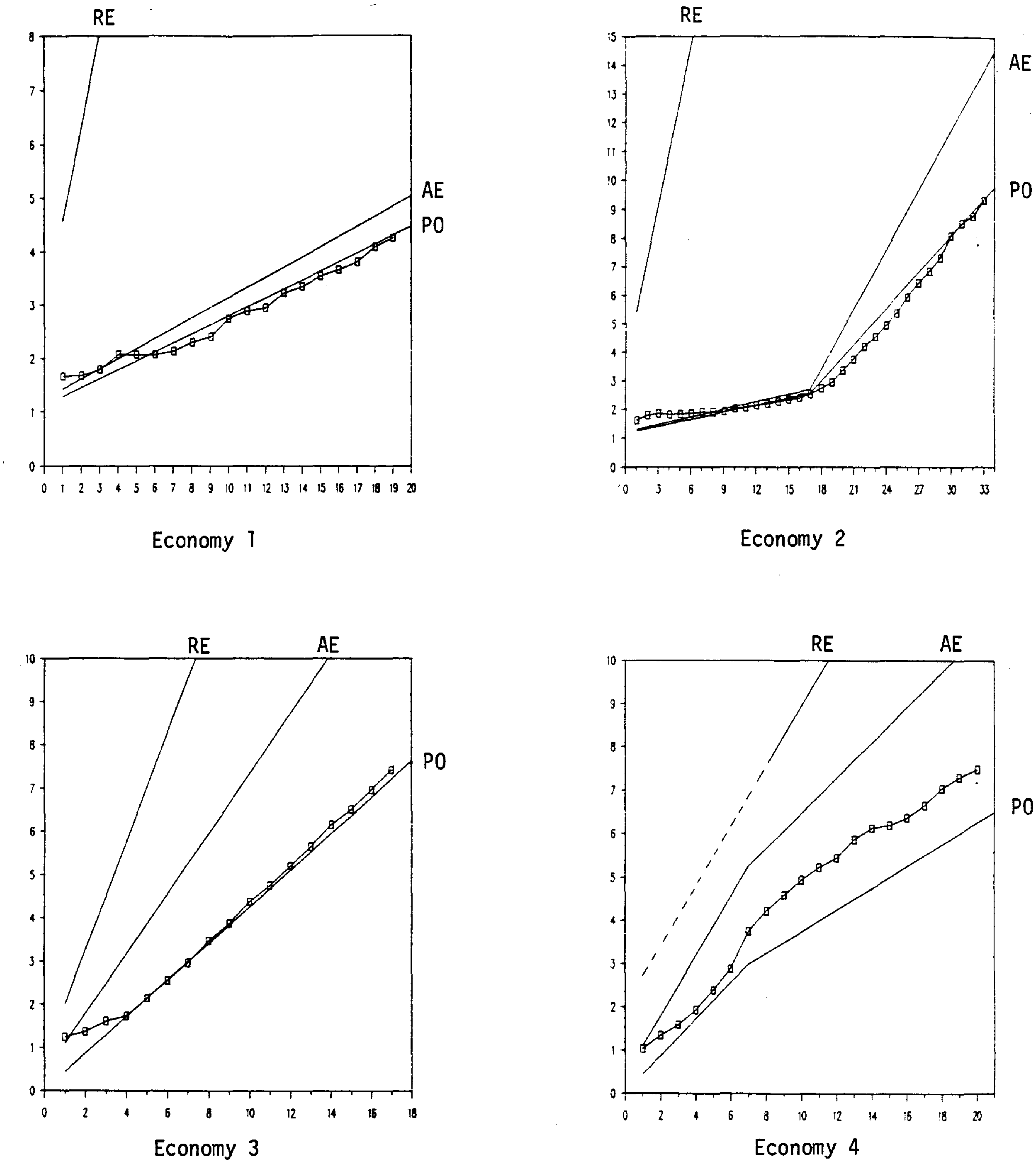
Convergence of Inflation Path
(π_t vs. π_{t+1})

Figure 2



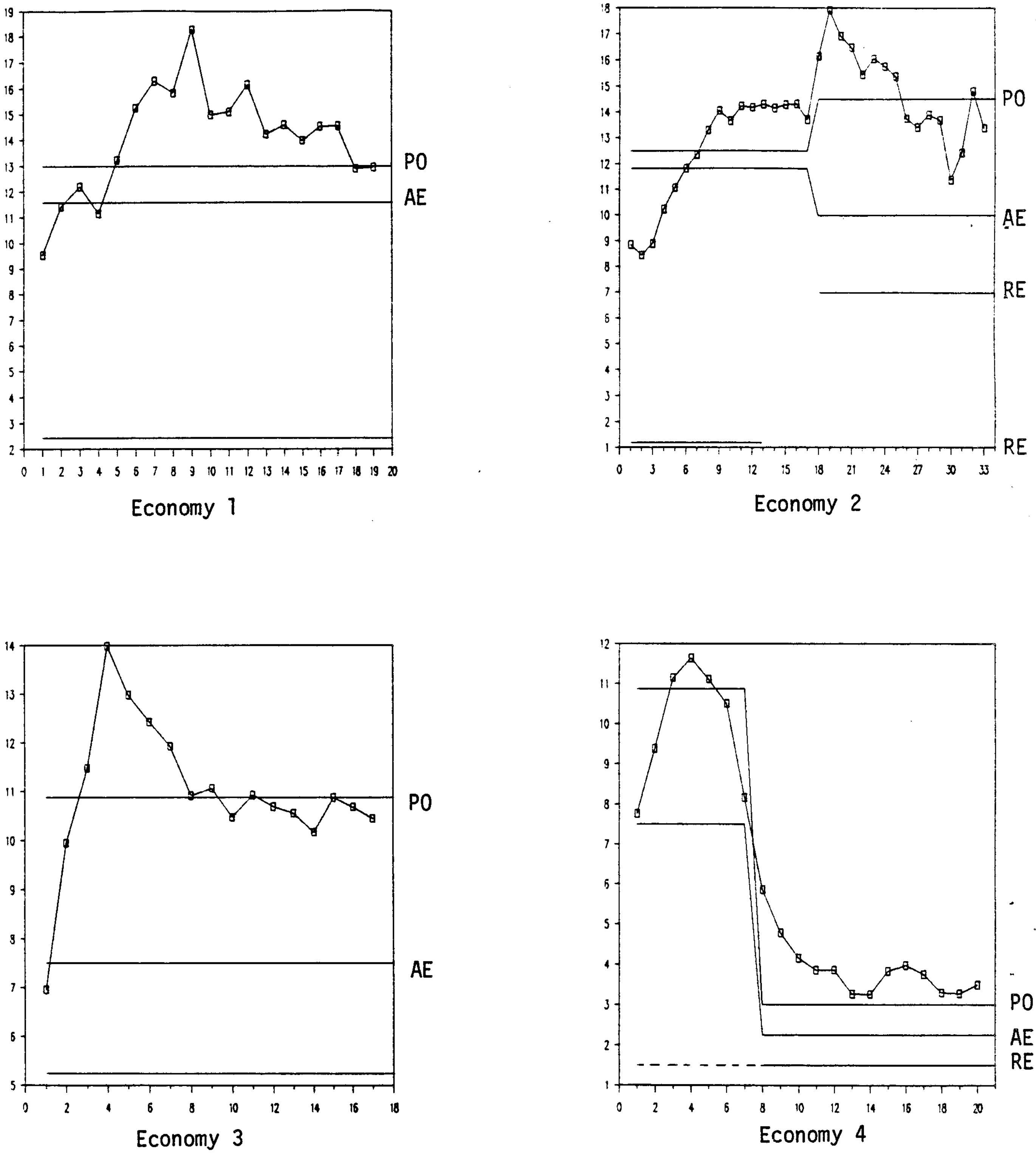
Inflationary Time Paths
(Percent Per Period)

Figure 3



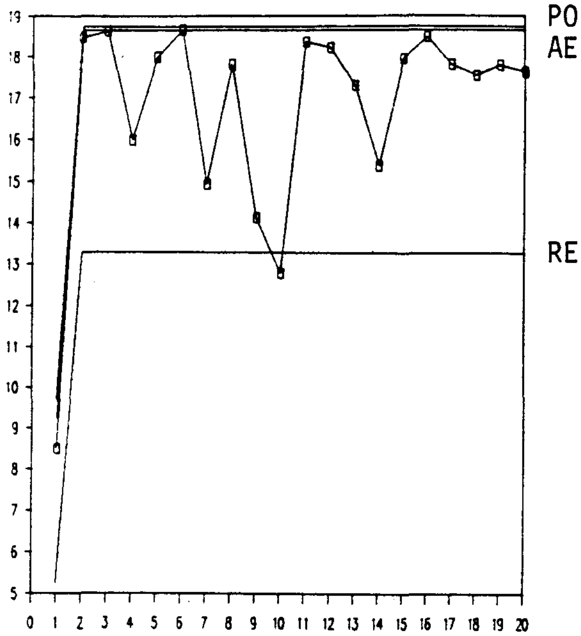
TIME PATHS OF LOG PRICE LEVEL (Francs/Chip)

Figure 4

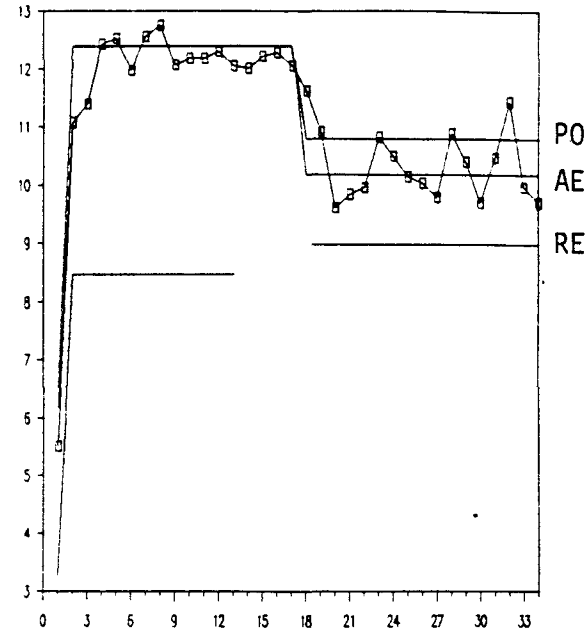


Time Path of Volume of Goods Traded (s_t)

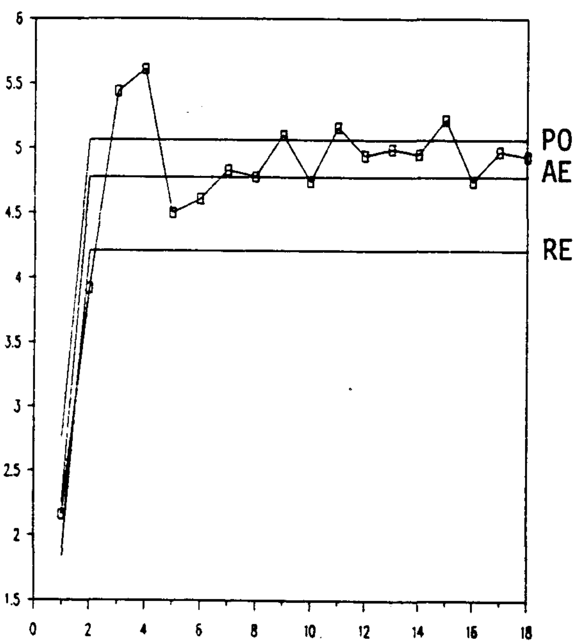
Figure 5



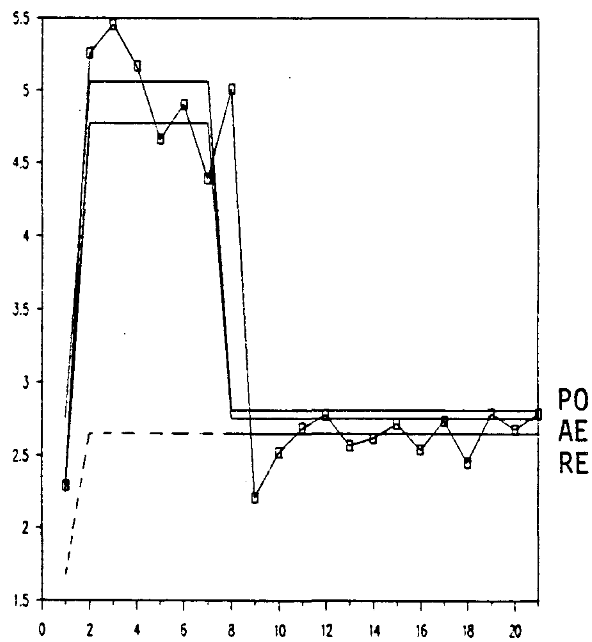
Economy 1



Economy 2



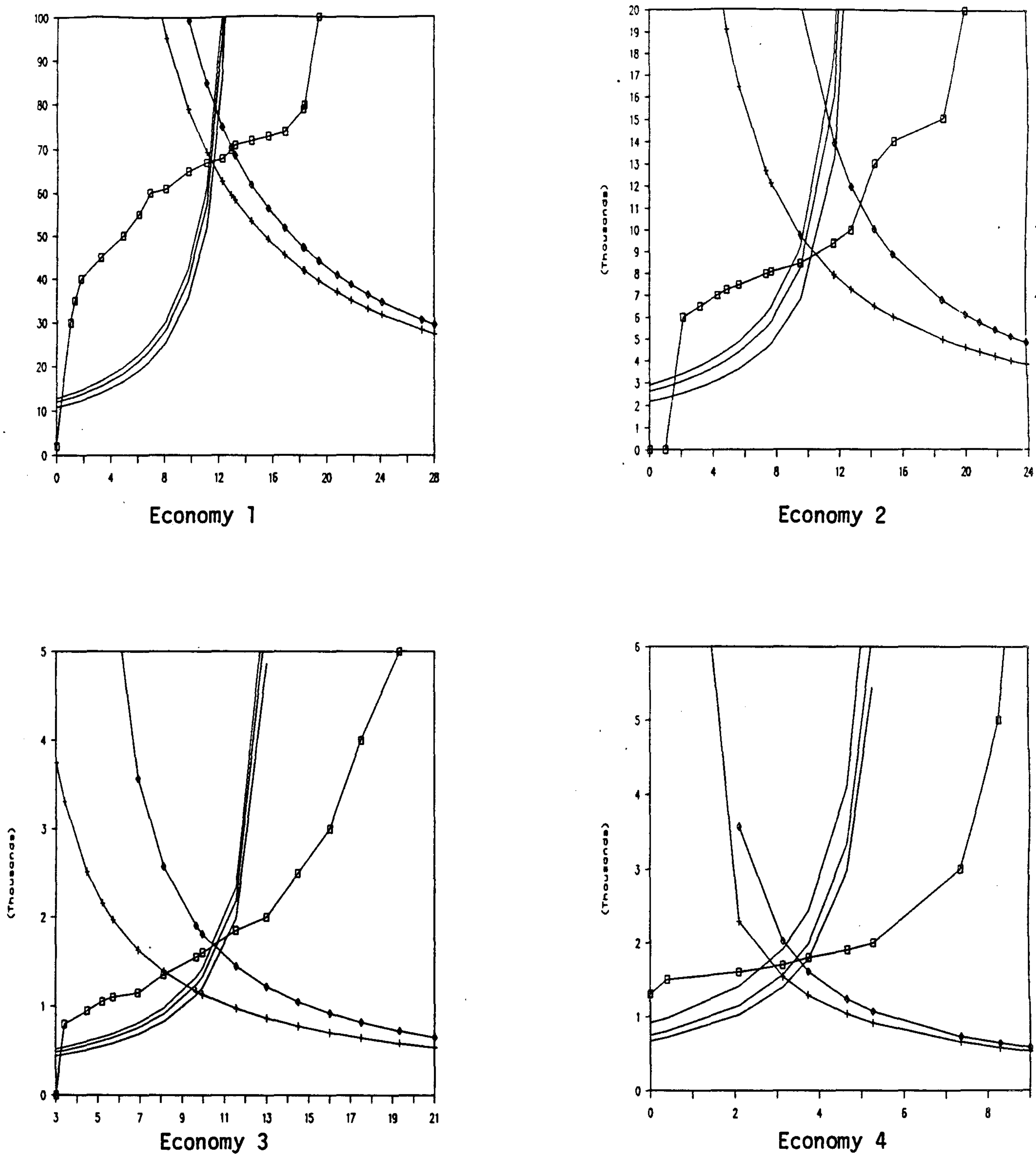
Economy 3



Economy 4

Time Path of Efficiency
(Dollars Earned)

Figure 6

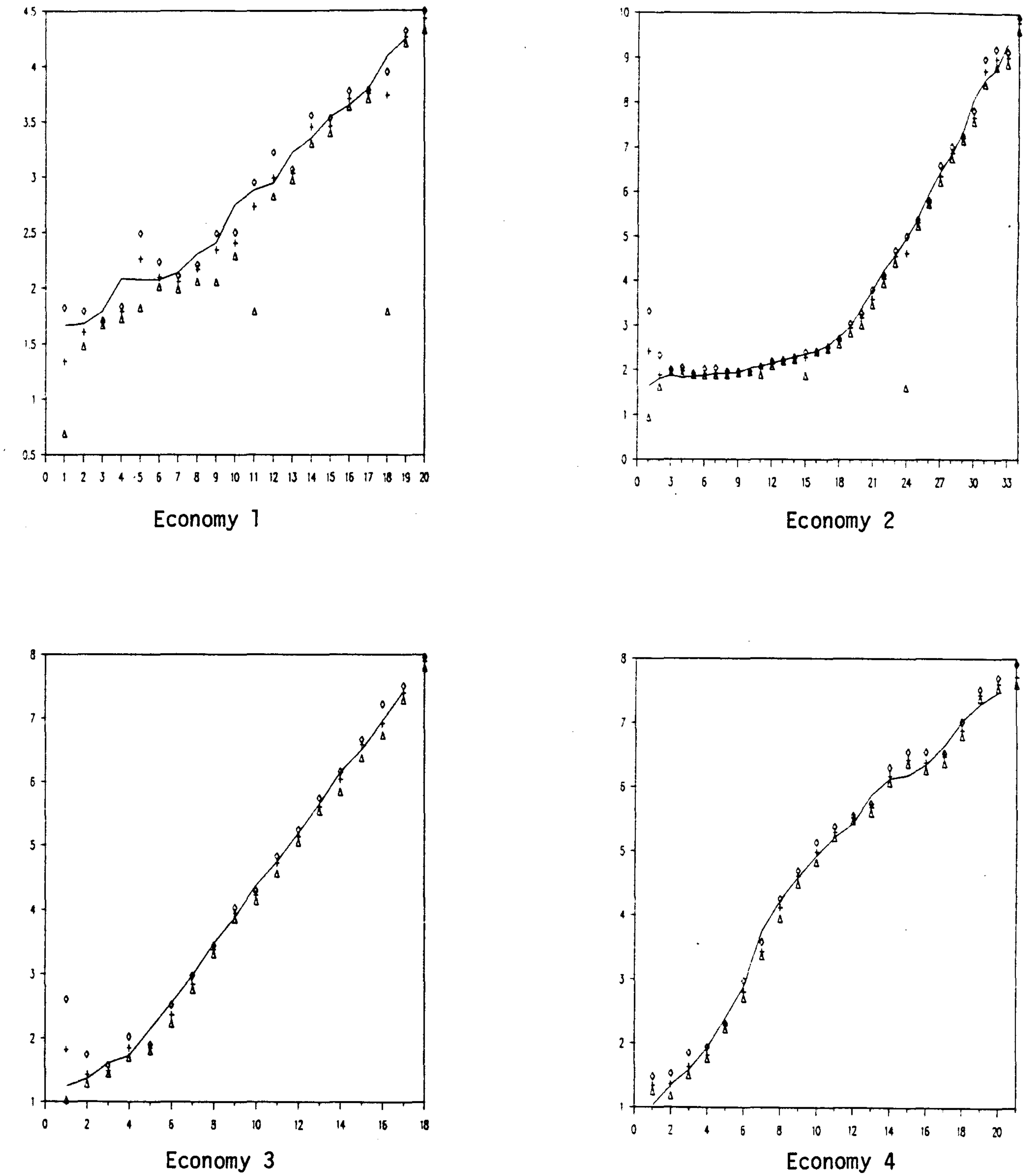


Demand and Supply Functions for the Last Period

☐ Actual Supply ___ Theoretical Supply

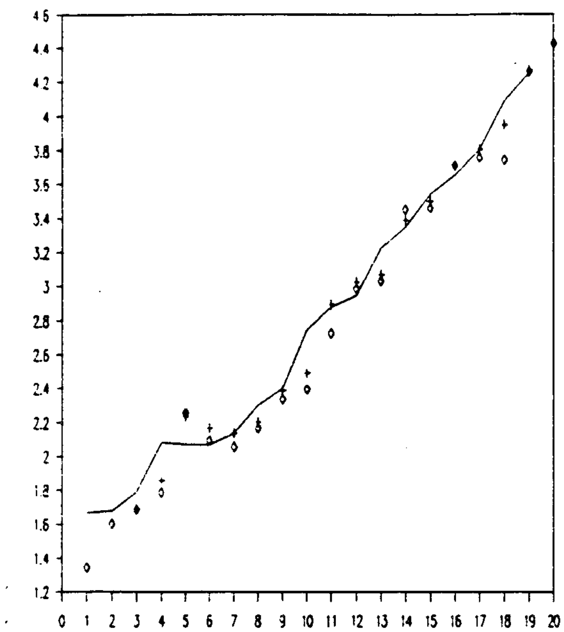
\+ Private Demand ◇ Total Demand

Figure 7

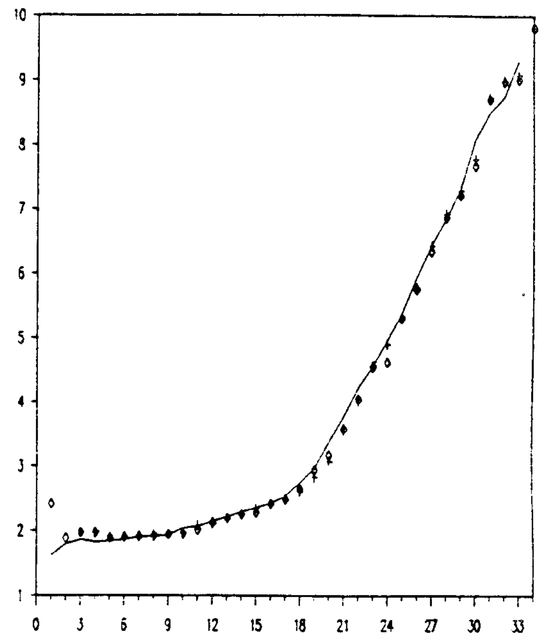


Time Path of Mean and Range of Price Predictions and Actual Prices
(+ Mean ◊ Max Δ Min)

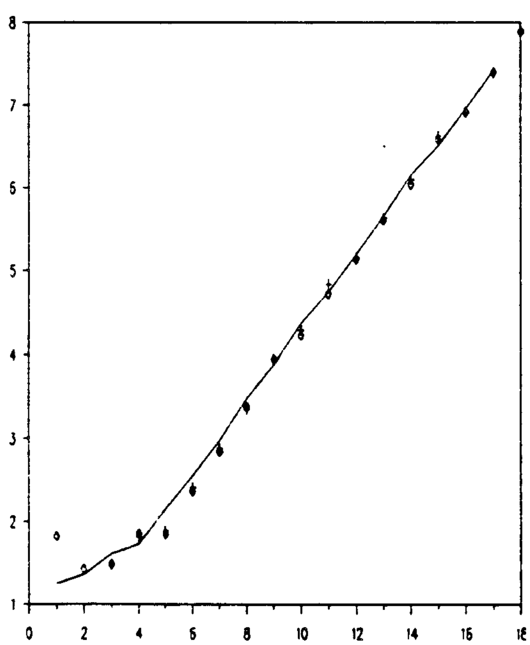
Figure 8



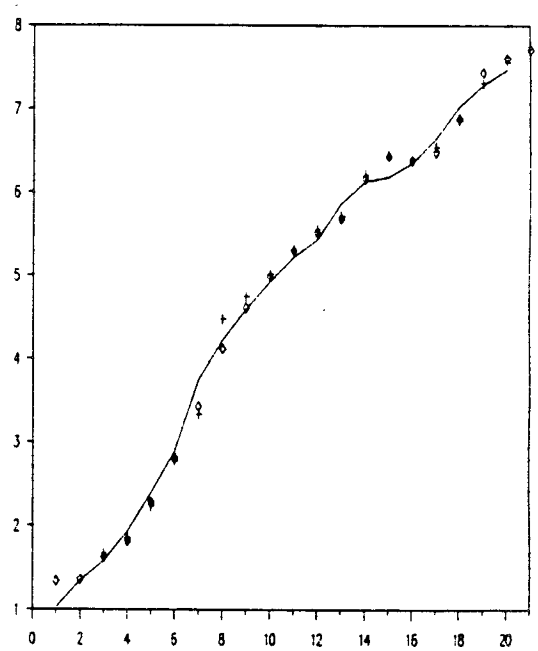
Economy 1



Economy 2



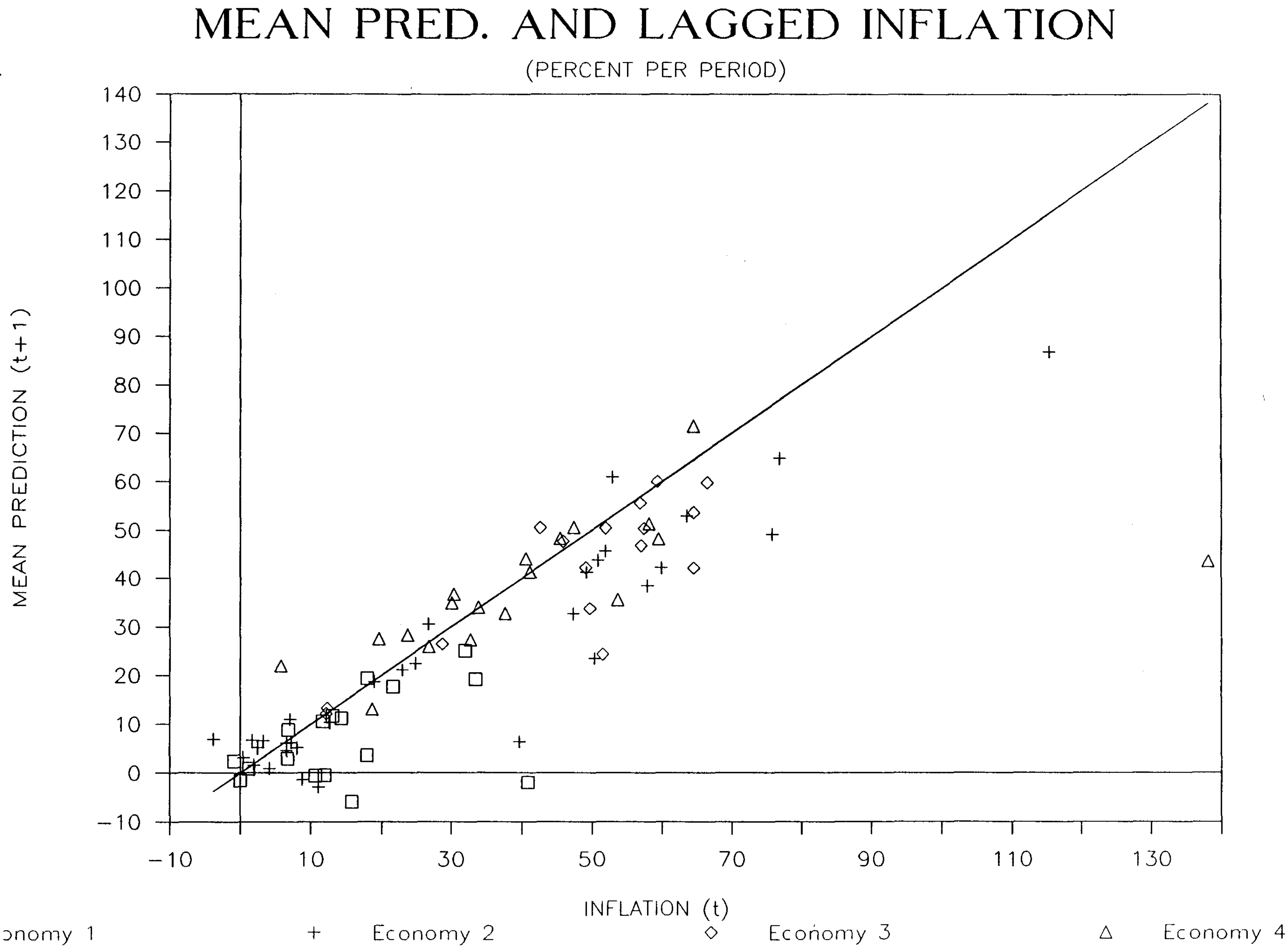
Economy 3



Economy 4

Actual, LS and Mean Predicted Prices
(__ Actual + LS ◊ Mean Prediction)

Figure 9



MEAN PRED. AND LAGGED INFLATION
(PERCENT PER PERIOD)
MEAN PREDICTION (t+1)
INFLATION (t)
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
−10
−10
10
30
50
70
90
110
130
onomy 1
Economy 2
Economy 3
Economy 4

Figure 10
Inflation and Growth of Money in Real Economies
(From [8])

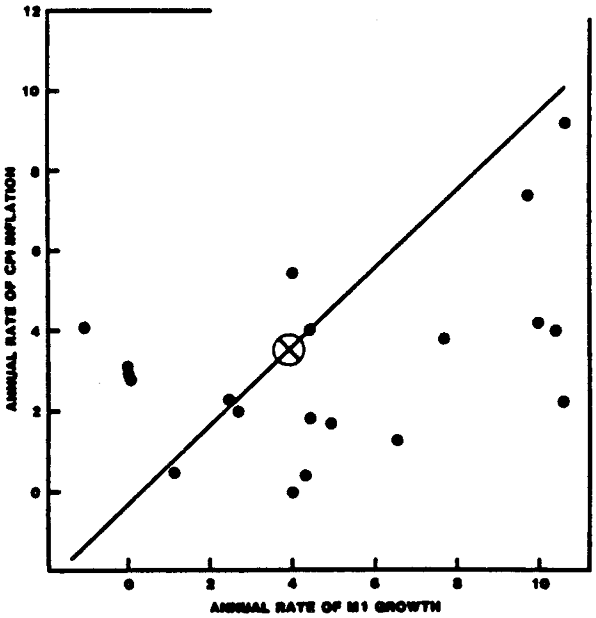


FIG. 2.—Original data for second quarters, 1955–75

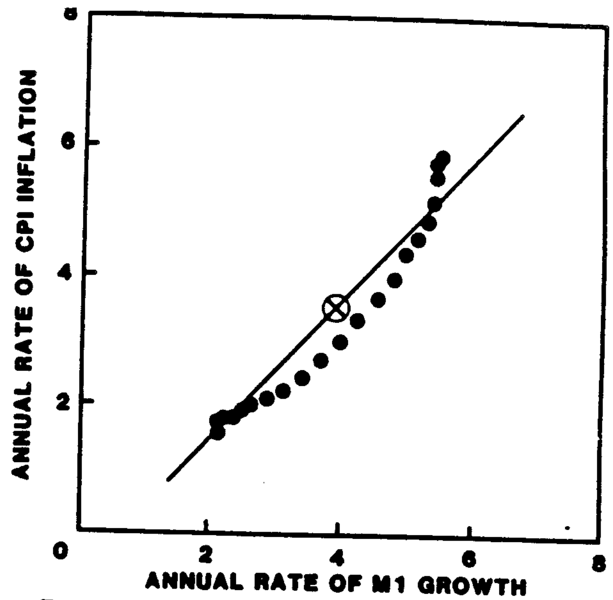
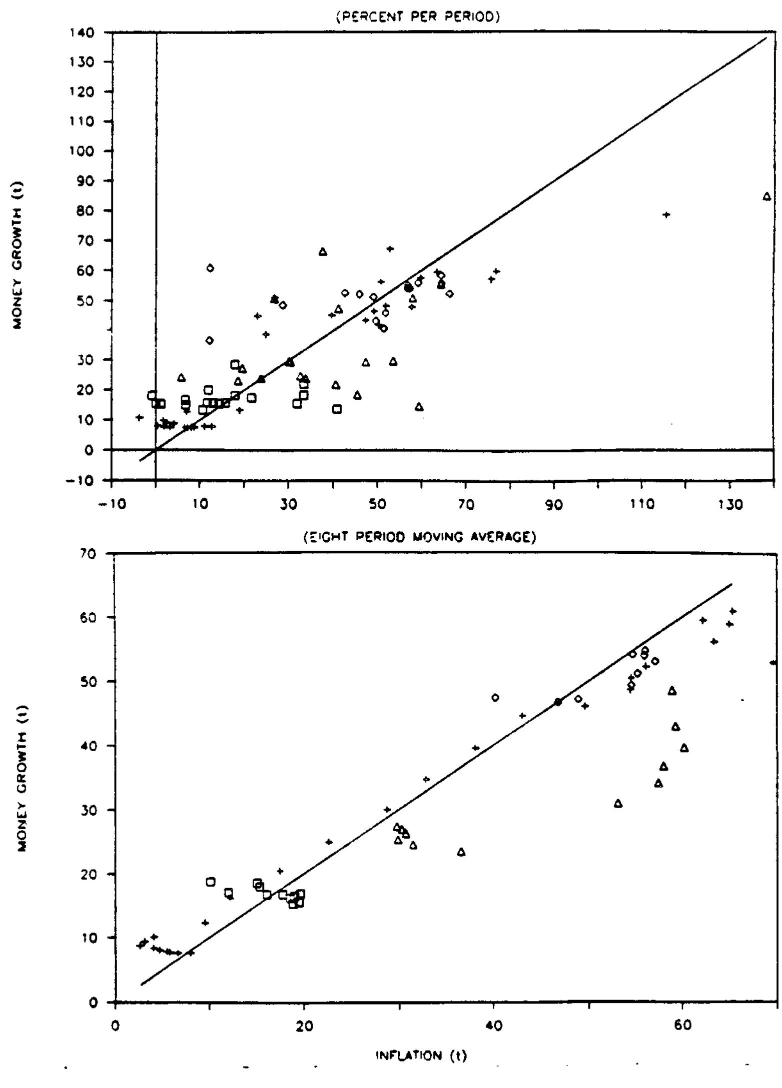


FIG. 3.—Smoothed data for second quarters, 1955–75

Figure 11

INFLATION AND GROWTH OF MONEY



APPENDIX I

Rational Expectations Paths with Announced Changes

In Economies 2 and 4 changes of parameters were announced several periods before they were to take place. As a reference, we analyze the set of rational expectations equilibria when changes are announced in advance. The notation is as in Section 2. Recall that the evolution of prices and money holdings is given by

$$p_t = b_t^{-1} p_{t+1} + c_t h_t \quad \text{and} \tag{6}$$

$$h_t = h_{t-1} + d_t p_t \tag{7}$$

If there are no changes in endowments, $b_t = b$ and $c_t = c$, equation (6) takes the form

$$p_t = c(1 - b^{-1} L^{-1})^{-1} h_t + k(b)^t \tag{21}$$

where L is the *lag operator* and k is a constant. That is,

$$p_t = c \sum_{n=0}^{\infty} b^{-n} h_{t+n} + k(b)^t \tag{21'}$$

If the level of deficit is also unchanged, then equation (7) takes the form

$$p_t = d^{-1}(1 - L) h_t \tag{22}$$

Combining (21) with $k = 0$ and (22) we obtain the equilibrium equation for money holdings,

$$(1 + b^{-1} - dc)h_t - b^{-1} h_{t+1} - h_{t-1} = 0 \tag{23}$$

where h_{-1} is given. To obtain stationary solutions we must solve the quadratic equation

$$b^{-1} L^{-1} \left[-1 + (1{+}b - bdc) L - bL^2 \right] h_t = 0 \tag{24}$$

Notice that equations (24) and (5') have the same stationary solutions.

Suppose now that at the end of period $T - \tau$ it is announced that a new set of parameters, e.g., a new level of deficit, will describe the economy from period T on, and that no further changes will take place.

If $\tau < T$, then for $n = 0, \ldots, T - \tau - 1$, the economy can be considered an economy where there are no changes in parameters, assuming that agents believe this to be the case. This may not be an adequate assumption if agents have experienced unexpected announcements in the past. As a standard procedure we take $\tau = T$ and in what follows we simplify notation by assuming $\tau = T$.[1] Denote by $\tilde{b} = \frac{\tilde{\omega}^1}{\tilde{\omega}^2}$, $\tilde{c} = \frac{2}{\tilde{\omega}^1}$, and $\tilde{d}$ the underlying parameters at $t = 0, \ldots, T - 1$, and b, c, d, the corresponding parameters at $t = T, \ldots$.

For $t \geq T$, equation (23) describes the evolution of money balances. For $t < T$, we see from (6) that

$$p_t = (\tilde{b}^{-1})^{T-t} \cdot p_T + \tilde{c} \sum_{n=0}^{T-1-t} \tilde{b}^{-n} h_{t+n} \tag{25}$$

and substituting (21') p_t can be expressed as

$$p_t = \tilde{b}^{-(T-t)} \cdot c \sum_{n=0}^{\infty} b^{-n} h_{T+n} + \tilde{c} \sum_{n=0}^{T-1-t} \tilde{b}^{-n} h_{t+n} \tag{26}$$

[1] Only in Economy 2 $\tau < T$, but it seems reasonable to assume that agents believed that such announcement had zero probability.

Let $R = c \cdot \sum_{n=0}^{\infty} b^{-n} h_{T+n}$, then substituting (26) in (7) we obtain

$$(1 - \tilde{d}\,\tilde{c})h_t - h_{t-1} - \tilde{d}\,\tilde{c} \sum_{n=1}^{T-1-t} \tilde{b}^{-n} h_{t+n} - d\tilde{b}^{-(T-t)} \cdot R = 0 \tag{27}$$

and, similarly,

$$(1 - \tilde{d}\,\tilde{c})\, h_{t+1} - h_t - \tilde{d}\,\tilde{c} \sum_{n=1}^{T-1-(t+1)} \tilde{b}^{-n} h_{t+1+n} - d\tilde{b}^{-(T-t-1)} \cdot R = 0 \tag{28}$$

multiplying (28) by $\tilde{b}^{-1}$ and subtracting from (27) results in

$$(1 + \tilde{b}^{-1} - \tilde{d}\,\tilde{c})h_t - \tilde{b}^{-1}\, h_{t+1} - h_{t-1} = 0 \tag{29}$$

Notice that for $t = T-1$ (27) takes the form

$$(1 - \tilde{d}\,\tilde{c})\, h_{T-1} - h_{T-2} - \tilde{d}\, c\, \tilde{b}^{-1} \cdot \sum_{n=0}^{\infty} b^{-n} h_{T+n} = 0 \tag{27'}$$

and for $t = T$, (6) and (7) give

$$(1 - dc)h_T - h_{T-1} - dc \sum_{n=1}^{\infty} b^{-n} h_{T+n} = 0 \tag{30}$$

Now multiplying (30) by $\tilde{b}^{-1}$ and substracting from (27') results in

$$(1 + \tilde{b}^{-1} - \tilde{d}\,\tilde{c})h_{T-1} - \left[\tilde{b}^{-1} + (\tilde{d} - d)c\, \tilde{b}^{-1} \right] h_T - h_{T-2} - (\tilde{d} - d)c\tilde{b}^{-1} \cdot \sum_{n=1}^{\infty} b^{-n} h_{T+n} = 0 \tag{31}$$

That is, a sequence of per capita money holdings $\{h_t\}_{t=0}^{\infty}$ defines a rational expectations equilibrium with initial money holdings h_{-1} when there is a change of parameters from $(\tilde{\omega}^1, \tilde{\omega}^2, \tilde{d})$ to (ω^1, ω^2, d) at period T and announced at the beginning of period zero, if it satisfies:

For $t \geq T$

$$h_{t+1} - ah_t + bh_{t-1} = 0 \tag{23'}$$

for $t = 0, \ldots, T-2$

$$h_{t+1} - \tilde{a}h_t + \tilde{b}h_{t-1} = 0 \tag{29'}$$

$$(1 - c \cdot \Delta d)h_T - \tilde{a}h_{T-1} + \tilde{b}h_{T-2} - c \cdot \Delta d \sum_{n=1}^{\infty} b^{-n} h_{T+n} = 0 \tag{31'}$$

and $h_{0-1} = h_{-1}$,

where $b = \frac{\omega^1}{\omega^2} \cdot a = 1 + b \cdot \frac{2d}{\omega^2}$, similarly for $\tilde{b}$ and $\tilde{a}$, and $\Delta d = (d - \tilde{d})$.

By repeatedly substituting (23') into (31') we can express (31') in terms of h_T, h_{T-1}, h_{T-2}.

Notice that

$$\sum_{j=0}^{n} b^{-j} h_{T+j} = \tag{32}$$

$$h_T \sum_{m=1}^{n} i \sum_{r=\max\{0,\, m-(n-m)\}}^{m} \begin{bmatrix} m \\ r \end{bmatrix} (-1)^{(m+r)} \cdot b^{-m} a^r -$$

$$h_{T-1} \sum_{m=0}^{n-1} \sum_{r=\max\{0,\, m-(n-1-m)\}}^{m} \begin{bmatrix} m \\ r \end{bmatrix} (-1)^{(m+r)} b^{-m} a^r$$

or

$$\sum_{j=0}^{n} b^{-j} h_{T+j} = \tag{32'}$$

$$(h_T - h_{T-1}) \sum_{m=1}^{n-1} \sum_{r=\max\{0,\, m-(n-1-m)\}}^{m} \begin{bmatrix} m \\ r \end{bmatrix} (-1)^{(m+r)} \cdot b^{-m} \cdot a^r +$$

$$h_T \sum_{m=1}^{n} \chi_{\{m-(n-m) \geq 0\}} \begin{bmatrix} m \\ 2m-n \end{bmatrix} (-1)^{(3m-n)} b^{-m} a^{2m-n} - h_{T-1},$$

where $\chi_{\{m-(n)\geq 0\}}$ is one if $m-(n-m)\geq 0$ and it is zero otherwise.

Provided that (32′) has a well defined limit as $n\rightarrow\infty$, we can write $\sum_{j=0}^{\infty} b^{-j}h_{T+1}=Ah_T-Bh_{T-1}$, that is,

$$\left[1-c\cdot\Delta d(1+A)\right] h_T-(\tilde{a}-c\cdot\Delta d\,B)h_{T-1}+\tilde{b}h_{T-2}=0 \qquad (31'')$$

For given equilibrium paths for $t\geq T$, for example constant inflation, (31'') and (29') characterize the "anticipated reaction" to the announced changes. Of particular interest, given our empirical results, is whether there are equlibrium paths with constant inflation for $t=0,\ldots,T-1$ that after the change has taken place converge to a new level of constant inflation.

When $\Delta d>0$ as in Economy 2 if the path of constant inflation for $t=0,\ldots,T-1$ corresponds to the lower root of (29') then there is no converging path after T. More in general, (31'') shows that if, for $t=0,\ldots,T-1$, $h_t=\pi^t\cdot h_0$ then $h_T\neq\pi^T h_0$ whenever $\Delta d\neq 0$. That is, in Economy 4 even if 2.00 is the lower root of (23') and the higher root of (29') there is no equilibrium path of constant inflation through the change in parameters.

APPENDIX II

Instructions

This is an experiment in decision-making. Various research foundations have provided funds for this research. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money which will be paid to you in cash.

In this experiment, we are going to have a market in which you may buy and sell chips in a sequence of market periods. Attached to these instructions you will find sheets labeled Information and Record Sheet, Selling Offer Sheet and Market Price Prediction Sheet which help you record your decisions and determine their value to you.

The type of currency used in this market is francs. The only use of this currency is to buy and sell chips. It has no other use. The money you take home with you is in dollars. The procedures for determining the number of dollars you take home with you is explained later in these instructions.

You will participate in the market for two consecutive periods at a time. Let us call the first of these periods *your* entry period (because you begin your participation then) and the second of these periods *your* exit period (because you end your participation in the market). Different individuals may have different entry and exit periods and the experimenter will inform you about when you will enter and exit the market. You may be asked to enter and exit more than once depending on the number of periods for which the market is operated.

At the beginning of your entry period, you will receive 7 chips from the experimenter and at the beginning of your exit period you will receive 1 chip. In your entry period, you may keep these chips or sell chips to others. In your exit period, you can buy more chips from others but you cannot sell. Buying and selling of chips will occur in francs according to the rules to be explained later.

The product of the number of chips you hold at the end of trading each period determines the amount of money you earn for that pair of entry-exit periods. The experimenter will calculate the square root of the product and multiply it by \$1.25 to calculate the amount of dollars you earn. Thus, suppose you hold 5 chips at the end of your entry period and 3.5 chips at the end of your exit period. The product of these two numbers is 5 x 3.5 = 17.5. The square root of 17.5 is 4.18 which is multiplied by \$ 1.25 to yield \$ 5.22 as your earning in these two periods. Note that the higher the product of the numbers of chips held by you at the end of entry and exit periods, the higher is the profit you earn. Also note that if you hold zero chips at the end of either period, your profits will be zero because the product of zero with any other finite number is zero. All chips are returned to the experimenter at the end of each period.

The first period of the market will be an entry period for some of you (as described above). For some of you, however, this first period itself will be an exit period and you will receive the exit period endowment of 1 chip at the beginning of this period. In addition, each of you for whom the first period is an exit period will receive 10 francs from the experimenter at the beginning of this period. You have to use all these francs to buy chips during the exit period because the francs you hold at the end of an exit period are worthless; they cannot be converted into dollars directly.

When you sell chips, your holding of chips decreases and your holding of francs increases by the amount of the price of the chips. Similarly, when you buy chips, your holding of chips increases and your holding of francs vanishes.

At the end of each period, all your chips on hand are used up to earn profits in dollars and thus returned to the experimenter. The francs you have on hand at the end of the entry period are carried over to the exit period and used to buy chips in this latter period.

All outside-market players participate in the market indirectly. At the beginning of each period, each outsider-market player predicts the market price of the period. The average of the predicted price will be used to convert the francs held by the entry-period players to chips at the end of the experiment. A $2.00 prize will be given to the player whose prediction is the closest to the actual market price. If there is a tie, the prize will be split.

Trading and Recording Rules

(1) All entry-period players are sellers and all exit-period players are buyers.

(2) Every exit-period player must pay all his francs to entry-period players in exchange for chips at a *market price* determined below.

(3) At the beginning of each period, every entry-period player must state the following prices on the Selling Offer Sheet and submit it to the experimenter. If the prices you submit are not nondecreasing in the number of chips offered, we shall make them so.

Price below which you don't want to sell any chips _____ francs/chip:

Price at which you are willing to sell up to 1 chip _____ francs/chip:

Price at which you are willing to sell up to 2 chips _____ francs/chip:

Price at which you are willing to sell up to 3 chips _____ francs/chip:

Price at which you are willing to sell up to 4 chips _____ francs/chip:

Price at which you are willing to sell up to 5 chips _____ francs/chip:

Price at which you are willing to sell up to 6 chips _____ francs/chip:

Price at which you are willing to sell up to 7 chips _____ francs/chip:

(4) The experimenter collects the Selling Offer Sheets from all entry-period players and buys **2** chips for himself each period. After considering the amount of francs available from the exit-period players, offers made by the entry-period players and his own need for 2 chips each period, he computes and announces the market clearing price. Exit-period players and the experimenter pay this price for each chip they buy. Each entry-period player will be informed of the number of chips he/she has been able to sell at the market price, and each exit-period player will be told of the number of chips that he/she has been able to buy with his/her francs on hand.

Note that if you (entry-period player) do not specify a price for zero chip, up to one chip of yours may be sold at zero francs. If you do not want to sell more than a specified number of chips under any circumstances, specify a very high price. This is the only way you have of not wanting to sell. The actual number of chips you sell will almost always be in fractions, depending on the market clearing price. The way the market clearing mechanism works, if you are willing to sell, say two units at unit price x and 3 units at unit price y, you may end up selling, say 2.4 units at a price between x and y.

(5) At the beginning of each period, each outside-market player writes down a predicted market price on the Market Price Prediction Sheet which is collected by the experimenter. At the end of each period, the experimenter announces average predicted market price and the winner(s) - the outside-market players whose prediction was the closest to the actual market price. This player records **$2.00** prize on the Market Price Prediction sheet. But, when there is more than one winner, the prize is split. All other outside-market players record **$0** prize on the sheet.

(6) After the transaction information is received from the experimenter, each entry-period

player computes the chips remaining on hand and the francs received from sale and records them on the Information and Record Sheet.

(7) Each exit-period player records the number of chips purchased on the Information and Record Sheet. Then the experimenter computes the product of the number of chips held by each exit-period player at the end of entry and exit periods respectively, takes the square root of the product and multiplies by **$1.25.** This amount is the profit of the exit-period player who records this profit on the Information and Record Sheet. At the conclusion of the experiment, the experimenter will pay each player the total amount of profits made.

(8) The francs received by the entry-period players in the entry period will be used to buy chips in the exit period which follows immediately. So, carry your francs on hand forward to the exit period by entering them in the column Beginning-Francs on Hand on the Information and Record Sheet.

(9) At the end of the experiment, francs held by *all entry-period* players are converted into chips using the average of predicted market prices by outside-market players.

(10) At the end of the experiment, add up the profit column of your Information and Record sheet. The experimenter will pay you this amount of money.

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