KREP'S "THREE ESSAYS ON CAPITAL MARKETS"

ALMOST TEN YEARS LATER

by

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ABSTRACT

A short overview of research on incomplete financial markets. I follow the development of the themes discussed in Kreps's "Three Essays..." from 1979 to 1987.

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FOREWORD

"Three Essays..." were written in 1979 as a survey on the themes of research in incomplete markets that had taken place over the preceding ten or so years (see Author's Preface). In active fields of research, however, most ten-year old papers have only an archeological interest as papers that enhance our understanding of the history of economic thought. This is particularly true of survey papers. Kreps's "Three Essays..." is an exception to the rule, not because the field has not advanced since 1979; incomplete markets has been one of the most active areas of research in economic theory in the last few years. "Three Essays..." stands apart as an excellent guide for the research that followed it.

In this article I give short descriptions of the places that one should visit to follow Kreps's 1979 guide up to 1987.[1] For this, I follow the evolution of each Essay separately and I leave a more personal point of view for the concluding remarks.

ESSAY I. Radner Equilibria in the Context of Pure Exchange

From an abstract point of view, the Arrow-Debreu complete contingent claims model is a *special* case of a model with a more general financial structure. This point is made clear in Essay I when the Arrow-Debreu and the Radner economies are related (Sections 3 and 4). This Essay and all previous and subsequent work can be seen as a clarification of the word *special*. In particular, consider the following properties of the Arrow-Debreu model: (1) under suitable conditions (not including bounds on trade) there exists a competitive equilibrium and this result can be proved by means of a standard fixed point theorem [30]; (2) using a differentiable approach, one can show that the set of equilibria is generically finite with respect to endowments and preferences (and possibly technologies) for economies satisfying standard differentiability assumptions, [79], [67]; and (3) under very weak assumptions equilibrium allocations are Pareto optimal [29]. Are these *special* properties of the Arrow-Debreu model? In other words, are these properties robust to changes in the financial structure of the model?

[1] I provide an extensive list of references which covers what, I consider to have been the 'main events'; I have not attempted to offer an exhaustive reference list.

Existence of equilibria

We now have a more refined understanding of *the existence problem* and how it is related to the nature of the securities and their number relative to the number of states of nature. A crucial distinction is whether there is trading of *pure financial assets*, that is, assets whose returns across states of nature are given exogenously in terms of a unit of account (as with Arrow securities [3]), or trading of *real financial assets* such as future commodities or ownership shares (as in the Radner model [89]). When there is trading in *real financial assets*, one has also to distinguish the case in which yields are denominated in a single *numéraire* commodity (e.g., commodity money) from the general case in which there are claims on multiple commodities. These distinctions are important because different asset structures generate different trading possibilities and, therefore, different continuity properties of budget sets and corresponding demands. This is a basic underlying difference between the Arrow-Debreu economy and general economies with possibly incomplete market structures. In an Arrow-Debreu economy the set of feasible trades (the Edgeworth Box of an exchange economy) is defined independently of equilibrium prices. As it should be clear from Essay I, this might not be the case in economies with alternative financial structures. Different types of assets define a different link between equilibrium prices and feasible commodity trades, and it is this link that defines the nature of *the existence problem* and of *the indeterminacy problem*, as we will see.

When there are as many real (pure) financial assets as states of nature one expects that a Radner (resp., Arrow) equilibrium exists, even when there is no artificial bound on trading strategies (Section 5). The idea is that given the existence of an Arrow-Debreu equilibrium, it should be possible to achieve the same allocation as a Radner (Arrow) equilibrium. By number it should be understood *independent number*. For example, in the canonical model with two periods (T= 1), K states of nature in period one and N pure financial assets with return matrix R, with generic element $r_n(\omega)$ denoting the return of asset n in state ω, it is required that rank R $\geq$K. With Arrow securities R = I, where I is the identity matrix, and the rank condition is satisfied. Furthermore, since within the set of K$\times$K matrices the rank condition is a generic property, it follows that when the return matrix is randomly selected it will satisfy the rank condition with probability one. However, financial structures are taken as a *datum* and, hence, might be degenerate. For example, early in the game, Hart provided a counterexample [55] for the case of real financial securities and $N \geq K$. But with real financial securities returns are endogenous and the work of Repullo [90] [91], McManus [81] and Magill and Shafer [73] has shown that Hart's counterexample is not robust to perturbations of the parameters defining the economy, establishing existence as a generic property for this class of economies. While Repullo and McManus study the case of state independent deliveries of commodities, Magill and Shafer study the general-canonical case.

With *pure financial securities and incomplete markets* ($N < K$), the existence of equilibria has been proved by Werner [97], Cass [20] and Duffie [39]. These proofs do not rely on genericity arguments and assumptions and arguments are similar to the ones used in standard (non smooth) equilibrium theory. The starting point of these proofs is the existence of *non-arbitrage asset prices.* An asset price vector $p \in R^N$ is a non-arbitrage price vector if every portfolio θ that yields non-negative returns at every state in period one (i.e. $r(\omega_k)\theta \geq 0$ for $k=1,\ldots,K$), and positive returns at some state in period one (i.e. for some k, $r(\omega_k)\theta > 0$), has a positive value at period zero (i.e. $p \cdot \theta > 0$). A standard separation argument shows that p is a *non-arbitrage asset price vector* if and only if there is a vector $\beta \in R^K_{++}$ such that $p = \sum_{k=1}^{K} \beta_k \cdot r(\omega_k)$, the vector β can be thought as a *present value coefficient.* Cass has shown that there is no loss of generality in assuming that one consumer, say $i=1$, behaves as in the presence of complete markets with properly discounted prices, i.e., his/her budget constraint is $\left\{ x \in X^1 : q(0)x(0) + \sum_{k=1}^{K} \beta_k q(\omega_k) x(\omega_k) \leq 0 \right\}$. With this construction one can easily show that there are no boundary problems and that budgets and corresponding demands satisfy the continuity properties needed for the application of a standard fixed point argument. In particular, there is an equilibrium for every β restriction (see [20], [39], [97] and [98].). These continuity properties are not satisfied in a general Radner model with *real financial assets.*

Geanakoplos and Polemarchakis had proved the existence of equilibria in economies with *real financial assets* that yield payoffs denominated in a single *numéraire* commodity [45]. They also use classical assumptions and methods of general equilibrium theory; however, they are explicit in assuming that with free assets there will be arbitrage. Chae provides another existence proof for the case of *numéraire* [22].

Nevertheless, all these results do not cover the general Radner model where there might be incomplete markets and *real financial securities* with yields denominated in multiple commodities. It is not coincidence that a more general existence result has been more difficult to achieve. In particular, the standard fixed point theorems (Brouwer's and extensions) had been shown to be too weak to handle this problem (see [60]). Duffie and Shafer have provided the first general existence proof [34]. They use the concept of a *pseudo-equilibrium* as an equilibrium in which all income transfers achievable through asset trading are constrained to lie in a particular subspace (this concept already appears in [73]). With full rank conditions a pseudo-equilibrium is a standard equilibrium. If a pseudo-equilibrium it is proved to exist, then by a standard transversality argument it is shown that generically in the space of endowments and asset structures there exist an equilibrium. In proving existence of a pseudo-equilibrium, prices together with N-dimensional subspaces (in our canonical example) are treated as points of a new type of fixed point argument. The natural object of study becomes then *the Grassmanian manifold* $G^{K,N}$, where $L \in G^{K,N}$ if L is a N-dimensional subspace of R^K. The fixed point argument of Duffie-Shafer is based on degree theory. Husseini, Lasry and Magill provide an alternative proof using methods of algebraic topology (cohomology theory) [65]. Their fixed point theorem has the interest that reduces to the standard Brouwer's fixed point theorem when markets are complete, preserving the *special* character of the Arrow-Debreu model. Recently, Hirsch, Magill and Mas-Colell have refined Husseini-Lasry-Magill's argument by following a geometric approach based on intersection theory [60]. Tools that are stronger than Brouwer's fixed point theorem had already been used in equilibrium theory to compute solutions to the walrasian system (see references in [47]). By extending these ideas to a more general framework, Geanakoplos and Shafer have shown that a *generic graph property* of the extended demand function guarantees the existence of a pseudo-equilibrium; the demand function is extended to include prices and N-dimensional subspaces [47].

Even if we have come a long way since Kreps's conjectures on the existence problem to the extend of being able to say that it has been solved (modulo further refinements), there are a couple of problems that seem to deserve further attention. First, the general existence result with *real financial assets* is a *generic* result in the space of endowments and asset structures. One would like to have a generic result in the space of endowments with given financial structures. This result is available only for special cases, e.g., unconditional real returns [34]. Second, while the above results can or have been easily generalized to arbitrary finite horizons (e.g., [38]), one would like to have similar results for models with an infinite horizon and/or continuous time. In an infinite horizon model with *real and purely financial assets*, Levine provides a necessary and sufficient condition for an infinite horizon equilibrium to be the limit of finite horizon truncated equilibria [71]. This condition, called *extensibility*, amounts to the exclusion of Ponzi schemes and gives existence of infinite horizon equilibria as limit of finite horizon truncated equilibria that satisfy a short sales constraint in the final period. In this sense, Levine's approach is closer to the original Radner's approach and extends ideas used in proving the existence of monetary equilibria ([7], [10], [12]) to the general incomplete markets model. Hirsh, Magill and Mas-Colell briefly discuss an infinite-dimensional generalization of their Grassmanian fixed point theorem [60] that might lead to a more direct existence proof for the infinite horizon model.

Indeterminacy of equilibria

By the above results, there exist a competitive equilibrium when standard general equilibrium assumptions are satisfied and asset returns are either *purely financial* or *real* (in this last case if there is no unique *numéraire* one has to include differentiability assumptions and properly qualify the existence result by generic existence). These two models can be viewed as generalizations of intertemporal models with *fiat* money or *commodity* money, respectively. However, a striking difference between the two models (and, hence, of monies) appears in terms of determinacy of equilibria. Geanakoplos and Polemarchakis show that under standard regularity conditions, incomplete market equilibria are generically locally unique in economies with *real financial assets* and a single *numéraire* commodity [45]; a result that extends to the general real assets model. In contrast, economies with incomplete *purely financial structures* show a high degree of indeterminacy.

In the canonical incomplete markets model with *purely financial assets*, we have seen, that p is a *non-arbitrage asset price vector* if and only if $p = \sum_{k=1}^{K} \beta_k r(\omega_k)$ for some $\beta \in R^K_{++}$, that is, and that for every β there is an equilibrium, since we have N non-arbitrage equations and K unknowns generically, there are K - N dimensions of real indeterminacy. This is the result obtained by Cass [21] and Balasko and Cass [6]. But when there is more than one security (i.e. N > 1), the exercise of fixing the *non-arbitrage price vector* is in turn indeterminate and might have more than a nominal effect. More precisely, there are N - 1 dimensions of real indeterminacy due to the fact that there are N - 1 dimensions of indeterminacy of asset prices. Summing both components we obtain that, generically, there are K - 1 dimensions of real indeterminacy of equilibrium allocations. This is the result obtained by Geanakoplos and Mas-Colell [44]. As they say, "if markets are financially complete (in the sense that rank R = K), then the model is determinate. Let just one financial asset be missing and the model becomes highly indeterminate". This manner of splitting the indeterminacy number in its two components is due to Werner [98], who has also extended the result to a multiperiod economy and relaxed a technical condition that was present in the above results (the condition of having the matrix R in *general position*; see references for definitions) [99].

Inefficiency of incomplete markets equilibria

The optimality problem has not been studied as extensively as the existence problems and most of the articles cited above either do not discuss the optimality problem or rely on concepts of constrained optimality similar to the one used by Kreps, that is, with the subspace of income transactions fixed (i.e. M(p,q) given) (see, for example [97]). Younès shows that a notion of optimality based on restricting the allowable directions of trade links the incomplete market literature with the fix-price literature [100]. These concepts of optimality restore the standard duality between equilibrium allocations and (restricted) optimal allocations. However, as Hart's examples have shown ([55], [15]) and as Geanakoplos and Polemarchakis have pointed out, "it seems absurd to say that the economy is using its markets efficiently at one equilibrium when there is another equilibrium in which everyone is better off," [45]. Using a definition of constrained optimality introduced by Stiglitz to analyze efficiency in production economies with incomplete markets [94], Geanakoplos and Polemarchakis have shown that if the *real financial asset* market structure is incomplete and there are at least two assets, then equilibrium allocations are, generically, constrained suboptimal in the sense that "a reallocation of securities alone can lead to a Pareto improvement when prices and allocations in the commodity spot market adjust to maintain equilibrium." The local determinacy of equilibria is needed to make this statement meaningful. Geanakolpos and Polemarchakis as well as Stiglitz show that for exceptional economies (e.g., constant marginal utility of "money") or if there is only one good [31], then incomplete market equilibria can be constrained efficient. The idea behind their constrained suboptimality result is that a reallocations of securities,with the corresponding price adjustment, can lead to "an income redistribution that the market itself could not directly implement. In essence, the central planner has access to a wider class of assets than those directly traded" [45]. Geanakoplos and Polemarchakis have also recently shown that if the planner is constrained to use the information provided by the existing financial market structure to implement any income redistribution then constrained efficiency is being restored[2]. Another limited "positive result" has recently been given by Mas-Colell for a model with a continuum of agents with enough diversity of preferences and a strong monotinicity assumption[2]. The idea is that even with an incomplete market structure diverse enough agents will fill all the directions of trade and restore constrained optimality.

Efficient Radner equilibria, asset pricing models, and efficient funds

Efficiency of Radner equilibria is attained when the market structure is effectively complete (Section 7). For the discrete time model, [70] remains as the basic reference. Kreps' results on dynamic spanning have been extended to the financial securities model by Duffie [39]. Ross' results on spanning by

[2]Communications at the *Workshop on Financial Markets*, Universitat Autònoma de Barcelona, Bellaterra, Barcelona, June 1987.

secondary securities [92] have been extended by Arditti and John [2] and by Green and Jarrow [49]. In particular, Green and Jarrow genaralize Ross' result, that call options are sufficient to complete markets, to economies with an infinite state-space by using the *lattice* structure of the space spanned by options. They also show "that the process of compounding, or writing options on portfolios of options, may eventually become redundant". In fact, only one compound is needed if there exists an *efficient fund*, that is, a portfolio for which the trading of options has the same spanning effect as the trading of options on all existing *real financial assets* (with numéraire payoffs). Arditti and John show that *efficient funds* always exist and are dense in the space of portfolio weights if the state-space is finite [2], as it is in our canonical model. However, for an economy with an infinite state-space, *efficient funds* might be a *rare avis* [49].

An alternative to Radner's formulation, which is widely used in theory of finance and macroeconomics, is to postulate an asset pricing model that does not assume full contingent Arrow-Debreu claim contracts but that it also uniquely defines state prices. Examples are the *Intertemporal Capital Asset Pricing Model* developed by Merton [83] and extended by Breeden [16] and others, and the *Lucas's Asset Pricing Model* [72]. These models are, in general, within the complete markets paradigm. For example, Bewley has shown that in Lucas homogeneous-consumers model a Lucas-temporary equilibrium defines a standard Arrow-Debreu equilibrium, i.e. an efficient allocation [9]. However, for economies with non-homogeneous consumers Lucas equilibria are not necessarily Pareto optimal (for a recent multi-agent extension of the Lucas model see [41]). Part of this literature has focused on the study of *The Martingale Property* of asset prices; now it is well understood that under appropriate price normalizations this property is not *special* of Arrow-Debreu equilibria (see [38] and the discussion of Essay III).

ESSAY II. Production, Stock Markets and the Objectives of the Firm

Essay II is an open discussion on the meaning of *the competitive hypothesis* and *the objectives of the firm* in economies with incomplete markets structures. Today, some new results for production economies have been obtained and the language has become more sophisticated, but the terms of the discussion have remained essentially the sames since Kreps's 'Three Essays...'.[3]

In the study of production economies, the standard Arrow-Debreu-McKenzie general equilibrium model is *special* in several dimensions. Three of them are discussed in Essay II: (a) ownership of firms is given by a fixed ex-ante distribution of firms' shares and shares are not traded; (b) producers behave competitively in that prices are taken parametrically; and (c) production plans do not affect the market structure and, the question, therefore, of whether producers behave competitively with respect to the financial market structure does not arise. It follows from these properties that *value-maximizing* is a well defined objective of the firm that shareholders unanimously agree in their investment plans and that ownership and control of the firm can be identified. However, the standard model is also special in other dimensions; (d) producers together with production sets are given in the model; and (e) the firm does not exist as a specific organization. If, as in Essay I, we want to study the meaning of the word *special* it seems strategic to start by analyzing all these dimensions separately as long as this can be done. The organizational theory of the firm is one of the dimensions along which research has been expanding in the last few years but that I will not discuss here.[4] In the early eighties there was a renewed interest in the theory of imperfect competition in general equilibrium which has lead to some important contributions, but this, says Hart, has "fallen well short of the goal laid down by Triffin" in the forties of building "a general theory of monopolistic competition (...) comparable in scope to the Walrasian theory of general equilibrium under perfect competition" (see [57]). With this limitation in mind, it seems reasonable to focus first on the study of stock (incomplete) market economies in which shareholders behave competitively (with respect to prices and market structures) and discuss separately the social choice problem that arises when shareholders behave strategically. That is, to assume that the objective of the firm is value-maximization and to see how the results described on Essay I on *existence of equilibria, indeterminacy of equilibria and inefficiency of equilibria* extend to production economies.

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[3]See Duffie's recent overview of the theory of the firm in incomplete markets [40]. My discussion of Essay II is partially based on Duffie's survey and I wish to thank him for simplifying my task.

[4]The interested reader is referred to Holmstrom-Tirole's excellent survey [61].

Existence of equilibria

The study of productions economies is especially interesting when production choices affect the span of markets (i.e., change M); otherwise the production sector is a passive component, and the existence and characterization of equilibria are routine extensions of the exchange case previously discussed. In what follows I consider the general case where the possibly incomplete market structure might be affected by production plans. The canonical model is now the one described by Kreps with only two periods (T = 1). For fixed prices q and fixed production plans $(y^f)_{f=1}^F$, trading of shares corresponds to trading of *real financial assets*. Therefore, the difficulties in proving existence of equilibria in the pure exchange model with *real financial assets* are also present in the model with production. Within the tradition of maintaining a short sales restriction, Burke has extended Grossman-Hart's existence result to multiple commodities [19]. Without short sales restrictions, Duffie and Shafer have obtained a *generic* existence theorem that parallels their theorem for exchange economies by "...exploiting the fact that a smooth production economy and its pure exchange version have homotopic excess demand functions (i.e. have the same fixed point index)" [37]. Genericity is defined here with respect to endowments and perturbed production sets. A similar existence result can be found in the recent work of Geanakolpos, Magill, Quinzii and Drèze on generic inefficiency of stock market equilibria discussed below [46].

Indeterminacy of equilibria

Duffie and Shafer introduce *real financial assets* in addition to firms' shares in order to obtain a complete characterization of equilibria. In our notation, with K states of nature in period 1, F firms and N *real financial assets*, their result ([37], Theorem 2) reads as follows: if $F+N \geq K$ or if $F=0$ (i.e. pure exchange), the set of equilibria is finite; however if $F \geq 1$ (i.e. production) and $F+N<K$, there are $S-(F+N)$ dimensions of real indeterminacy of equilibrium allocations. Notice the difference between production and pure exchange. With pure exchange, there is indeterminacy only with *pure financial assets* and the degree of indeterminacy is independent of the number of assets (as long as there are less than states). With production there is indeterminacy with *real financial assets*, but the degree of indeter-

minacy depends on the total number of securities being traded. One could rephrase Geanakoplos and Mas-Colell's comment: "with only *real financial assets* the incomplete markets model is determinate. Let just one firm be created with commonly traded stock and the model becomes highly indeterminate (if N is small compared to K)". What is the source of such indeterminacy? *Firms' shares* are *fixed real securities* only when production plans are being fixed. However, production plans are endogenously defined by solving firms' value-maximization problems. With complete markets, a simple non-arbitrage argument shows that the value-maximization problem is well defined; however non-arbitrage restictions are not enough to uniquely define firms' objectives when markets are incomplete. This is the source of indeterminacy. More precisely, suppose that only shares of F firms are being traded and the initial distribution of shares is given by $\overline{\lambda}_f^i$, f=1,...F, i=1,...I. If production plans are fixed at $y^f \in R^{J\cdot(K+1)}$, k=1,...K, then, given a commodity price vector $q \in R_+^{J\cdot(K+1)}$, prices of shares $r=(r_f)_{f=1}^F$ are *non-arbitrage share prices* if there is no vector of shares λ satisfying $\lambda\cdot q(\omega_k)y(\omega_k) \geq 0$, for k=1,...K, with > for some k, and $\lambda(r - q(0)y(0)) \leq 0$. As with *nominal financial assets*, a standard separation argument shows that r is a *non-arbitrage share price vector* if and only if $r - y(0) = \sum_{k=1}^{K} \beta_k \cdot q(\omega_k)y(\omega_k)$ for some $\beta \in R_{++}^K$. We can apply the *present value coefficient* β to evaluate firms' period one production plans, then a necessary condition for equilibrium is that, for f=1,...,F, $y^f \in argmax\{q(0)y(0) + \sum_{k=1}^{K} \beta_k \cdot q(\omega_k)y(\omega_k) : y \in Y^f\}$ However, we have K - F degrees of freedom in defining β; without a predeterminated choice of β, therefore, generically, there are K - F dimensions of real indeterminacy. (See [37], [40] and [46] for a more detailed discussion of this argument.)

Competitive behavior and shareholders unanimity

The above indeterminacy argument also shows why shareholders would typically disagree if they had to decide on production plans in an economy with an incomplete market structure. If consumer i has to choose production plans for firm f, he/she might as well use his/her own *present value coefficient*, $\alpha_i \in R_+^K$, where $\alpha_i(\omega_k)$ is the (period zero) present value marginal utility of income in state ω_k (see [46]).

With complete markets $\alpha_i = \beta$, for i=1,...I. With incomplete markets, however, ((N + F) < K), "all shareholders (except possibly one) of any firm disagree with maximization of the firm's market value" ([37], Theorem 2).

As it should be clear from Essay II, there are alternative ways to study the problem of shareholders unanimity:

(1) One can simply postulate a particular objective function for the firm and assume that shareholders take such a function as given. This is the approach pioneered by Drèze and followed by Grossman and Hart. The Drèze proposal is to define firm's *present value coefficients* by $\beta^f = \sum_{i=1}^{I} \lambda_f^i \alpha_i$, f=1,...,F, and the Grossman-Hart's proposal is to define $\beta^f = \sum_{i=1}^{I} \bar{\lambda}_f^i \alpha_i$, f=1,...,F, where $(\bar{\lambda}_f^i)_{i=1}^I$ is the initial distribution of firm f's shares and $(\lambda_f^i)_{i=1}^I$ is distribution of firms f's shares after the stock market closes at period zero. Kreps mentions the distinction between a two and a three periods model, but, none of these choices can be made easily consistent with a multiperiod intertemporal model in which production plans have longer time spans than financial transactions. Nevertheless, Geanakoplos, Magill, Quinzii and Drèze use the Drèze criterion to study the efficiency properties of *shareholder equilibria*, which is defined in the standard way with firms maximizing value according to Drèze's criterion [46].

(2) We could also keep the assumption that shareholders take a particular value-maximizing objective as given, but not to place restrictions (other than non-arbitrage) on the choice of the objective function. This is the approach taken by Duffie and Shafer in their analysis of existence and characterization of equilibria. As we have seen, while this approach is consistent with the principle of 'not introducing more restrictions than the ones derived from economic theory', it also shows that the model then is not well specified.

(3) We can incorporate agents' beliefs about changes in prices due to changes in production plans in the competitive paradigm by showing that competitive conjectures can be made consistent. This was

Hart's approach as discussed by Kreps (Section5). Following Hart's tradition [56], Makowski has sharpened his results on perfect competition [74] in [75] and [76]. Makowski postulates the existence of consistent, but not necessarily correct, competitive conjectures; then relates competitiveness with the no surplus condition as defined by Ostroy [86]. Makowski shows that the no short sales restriction plays a more crucial role than the one pointed out by Kreps. In particular, "without short sales the assumption of perfectly elastic demands and no surplus are of equal strength, so a firm's profits measure its surplus. While with short sales, the former assumption is weaker than the latter" [75].

(4) Or we could drop the competitive assumption and study the shareholders' problem as a social choice problem, or as the solution of some noncooperative game. The study of the problem of joint ownership when there is a conflict of interest is as old as economic (and political science). The design of shareholders' decision mechanism can be taken as a particular manifestation of this problem. This is the approach taken by Jordan, he shows that: (i) there is no decision mechanism that is sensitive to shareholdings (e.g., 99% of the equity can determine production activities) that guarantees a (weak) constrained Pareto optimal outcome; (ii) within managerial decision mechanisms, i.e., investors communicate with an impartial manager, there is no mechanism that treats investors symmetrically and achieves Pareto optimal outcomes; and (iii) if the decision process is modeled as a game in which one of the shareholders is elected manager, then for each shareholder there is an equilibrium in which he/she is elected manager [66].

(5) Finally, we can incorporate the shareholders' problem into the more general problem of corporate control in order to characterize shareholders' decision mechanisms according to their performance as mechanisms for corporate control. This is the approach recently taken by Grossman and Hart [50], and Harris and Raviv [51]. For example, Grossman and Hart "show that it is in the shareholders' interest to set the cost of acquiring control to be as large as possible, consistent with a control change occurring whenever this increases the shareholders' wealth. Under certain assumptions, one share/one vote best achieves this goal" [50]. One expects that the threat of a takeover will constrain

the firm towards following an efficient production plan and, therefore, help define the objectives of the firm. This idea has been further studied by Hart [58]. However, as Duffie says, the takeover threat "does not provide a specific objective function for the firm" [40]. Furtermore, as we will see below, a well defined value-maximizing objective does not imply efficiency.

All these approaches follow the general equilibrium tradition of taking firms (production sets) as given. Otherwise, in a competitive world of constant returns to scale and blueprint technologies the number and size of firms would be indeterminate (a continuum of firms each of infinitesimal size?), and the model ill-specified. Nevertheless, if agents realize that they can use blueprint technologies to create firms, they do not have any incentive to do so in an Arrow-Debreu-McKenzie economy. In an economy with incomplete markets this is generally not true. It seems inconsistent that agents as shareholders/managers behave strategically, but that as potential firm (asset) developers they behave as if there were infinite set up costs; unless one is willing to make this transaction costs assumption. Khilstrom and Laffont have developed a model in which enough firms are created so as to resolve any potential conflict arising from joint ownership [69]. It seems that less extreme models are called for. Similarly, it is often accepted that shareholders understand the price effect of changes in production but that they systematically disregard any reaction from other competitive firms. It is not an easy task to incorporate these strategic elements into the model; however, it might help to clarify or even demystify the shareholders' problem.

Inefficiency of incomplete markets equilibria

As should be expected, equilibrium allocations are, generically, suboptimal with incomplete markets ([37]. Theorem 2). On the other hand, with Makowski's 'competitiveness' postulates competitive allocations are (weakly) constrained efficient (in the Diamond sense of keeping M(p,q) fixed). Geanakoplos, Magill, Quinzii and Drèze complete the parallel with the exchange case by showing that with incomplete markets, *shareholder equilibria* are, generically, constrained inefficient (in the Geanakoplos-Polemarchakis sense) [46].

Modigliani-Miller revisited

It is now well understood that the 'Modigliani-Miller Theorem' on financial indeterminacy is also satisfied when the market structure is incomplete; i.e., it is a non *special* result. There is, however, the implicit assumption that firms' portfolios are composed of existing securities, and that if firms issue new securities they only issue redundant securities, i.e., firms do not further span markets by issuing new securities. With this condition, the fact that the financial policy of the firm does not affect its market value is a result independent of the 'dimensionality' of the market (see [37]). Similarly, as DeMarzo has shown, agents' budget feasible consumption sets are invariant to the financial policy of the firms [28] (see [40] for a simple exposition of these results).

On the other hand, if the financial policy of the firms changes the span of the markets, then the Modigliani-Miller neutrality result is no longer satisfied. This strenghtens the previous comment that it is very unsatisfactory to postulate that the shareholders behave strategically with respect to firms' production plans, but that they are unable to create adequate financial instruments (see [93]) (I return to this point in the final remarks). A non-neutrality result is also achieved when there are short sales restrictions that might lead to bankruptcy (see [59]), market frictions (transactions costs, distorsionary taxes, etc.), or agency problems. Holmstrom and Tirole review the three standard arguments for non-neutrality of the financial policy of the firm: incentives, signalling, and control ([61], Section 3).

Efficient production equilibria and the issue of debt and equity

All the results on market completion by spanning that were mentioned in the discussion of Essay I also apply to production economies. The case of firms issuing debt and equity, however, deserves special consideration. Green and Jarrow, extending a result of Hellwig [59], show that if all potential debt and equity claims of a firm are traded and this firm's returns are strictly positive at every state (i.e., $q(\omega_k)y^f(\omega_k) \gg 0$, for k=1,...K), or a riskless asset is traded, then there is market completion. This follows from the fact that the set of debt and equity claims contains all call options written on $q(\omega)\cdot y^f(\omega)$, and the spanning properties of call options mentioned before. Even if they relax their requirements by introducing subordinate debt, their strong result still depends on firms' ability to issue enough types of debt claims (see [49]).

Asset pricing models, by uniquely defining state prices, have well defined firms' objectives and , hence, there is no problem of shareholders unanimity. As we have seen, *Lucas's Asset Pricing Model* with homogeneous consumers has stationary efficient equilibria, this model has been extended to production economies by Brock [17], [18], Prescott and Mehra [82], and others. In particular, Brock uses the existence of stationary optimal programs in the one sector growth model to derive equilibrium asset prices. These results can be extended to a general equilibrium (multisector) growth model by using Marimon's results on existence of stationary equilibria in economies with impatient consumers and complete markets [77]. One can obtain stationary equilibria that are Pareto optimal even when there are no complete contingent markets and firms' shares are not traded. This result follows from the fact that at a stationary equilibria contingent markets for date-event are redundant since prices only depend on the (infinite) past history of the exogenous stochastic process. (See [11] for the no-discounting case.)

ESSAY III. Continuous Time Models

The study of continuous time stochastic economies has been a very active area of research the past few years. Kreps acknowledges Samuelson and Merton as his (and Harrison's) ancestors, but we can already talk of a new generation. One can distinguish this new generation (Duffie, Huang et al.) by their more extensive use of the Strasbourg approach to stochastic integration already used by Harrison and Kreps and by a tighter link with general equilibrium theory (in its infinite dimensional formulation).

As an introduction to continuous time stochastic models there is excellent survey by Harrison and Pliska that expands Kreps' Essay III in several directions , [53]. Together with its sequel [54], this paper helps to clarify some of the issues in Sections 4 and 5 of Kreps' Section III, even though it does not explicitly study the Radner model. More specifically, Harrison and Pliska characterize the set of *admissible* trading strategies, and clarify the relationship between the martingale representation property and market completeness. In order to state these results I must first expand Kreps's notation.

Consider the following variation of Kreps' canonical model. Exogenous uncertainty is represented by an underlying *filtered probability space* $(\Omega, \mathbb{F}, P)$, where $\mathbb{F}$ is a filtration (i.e. increasing family of sub-σ-fields), $\mathbb{F} = \{F_t\,; t \in [0,1]\}$. There are N+1 *long-lived pure financial securities* with $d_n(\omega)$ denoting the dividends paid by security n in state ω at date one. Consumption takes place at dates zero and one. A trading strategy θ is represented by a N dimensional stochastic process $\{\theta_n(t); t \in [0,1],\ n=0,\ldots,N\}$ and prices of these long-lived securities are given by $\{p_n(t)\ F_t$ *measurable* $; t \in [0,1],\ n=0,\ldots,N\}$. It is assumed that one security, say n=0, is *locally riskless* in the sense that the process $p_0(\cdot)$ is continuous and has finite variation (this assumption is not necessary for most results). A present value price system can then be defined by $S_n(t) = p_n(t)/p_0(t)$. This canonical model can be easily generalized to include continuous consumption and securities with returns $d_{n,t}(\omega), t \in [0,1]$, (see [32]).

Let $\mathbb{P}$ be the set of probability measures Q on (Ω, F_1) that are equivalent to P such that S is a (vector) martingale under Q. For a fixed $P^* \in \mathbb{P}$, $E^*(\cdot)$ denotes the corresponding expectation operator, and $L^2[S]$ denotes the class of *predictable* processes[5] θ satisfying the following condition: (I) $E^*\left(\int_0^1 \theta(t)^2 d[S]_t\right) < \infty$ where [S] denotes the quadratic variation process.[6] A trading strategy θ is said to be *admissible* if the *gain process* $\int \theta dS = \sum_{n=0}^{N} \int \theta_n dS_n$ is a well defined stochastic integral and $\theta_n \in L^2[S_n]$. That the *gain process* has to be well defined and that θ has to be *predictable* are very natural restrictions. Condition (I) excludes some 'bad' strategies that might create *free lunches* without unnecessarily restricting the set of admissible strategies, as Harrison and Kreps did by only allowing *simple trading strategies*. However, admissible strategies can be approximated by simple trading strategies. Dybvig and Huang have shown that condition (I) is functionally equivalent to the non-bankruptcy condition $\theta(t) \cdot S(t) \geq 0$ [42]. As Duffie has remarked, when only *admissible* strategies are considered there are *no expected financial gains from trade*, since whenever S is a martingale (with respect to P*) and $\theta \in L^2[S]$ it follows that $\int \theta dS$

[5]Loosely speaking, θ is predictable if, at every t, $\theta(t)$ depends on information up to, but not including, time t.

[6]For definitions of technical terms see [53], [32] or any standard book on stochastic integration.

is also a martingale [32].

Let $\mathbb{M}$ denote the set of all martingales (with respect to P*) and $\mathbb{M}(S;N)$ denote the subset of $\mathbb{M}$ of all martingales X that are representable in the form

$$X_t = X_0 + \sum_{n=1}^{N} \int \theta_n(t)\, dS_n(t), \text{ for some } \theta \text{ with } \theta_n \in L^2[S_n],\ n=1,...N .$$

S is said to have *the martingale representation property* if $\mathbb{M}=\mathbb{M}(S;N)$ for some N (the Kunita-Watanabe theorem cited by Kreps). Harrison and Pliska show the equivalence between: (a) *market completion*, in the sense that every integrable claim is attainable; (b) *the martingale representation property*; and (c) the existence of *a unique martingale measure* P* in $\mathbb{P}$. This last result, proven in Harrison and Kreps [52] for the Black-Sholes model, is the key element of Kreps' informal proof that Black-Sholes prices can be Radner equilibrium prices (Section 4). Harrison and Pliska's result can be used to give a shorter and more rigorous proof of Kreps' argument. Similarly, [53] Section 3, provides a nice complement to Kreps' Section 5.

Complete spanning

The above equivalence between market completion and *the martingale representation property* leads to the following remarkable result: the allocation of any Arrow-Debreu equilibrium can be implemented as a Radner (Arrow) equilibrium with continuous trading of N+1 securities, where the number N is the *martingale multiplicity* of the underlying filtration, that is $\mathbb{M}=\mathbb{M}(S;N)$, and, as in our canonical model, the 'plus one' is required to transfer income across time. This result, generalizing Kreps's results for the discrete time (tree) model, has been obtained by Duffie and Huang [33] and further refined by Duffie [32], in particular it is shown in [32] that the *martingale multiplicity* plus one is the smallest number of securities needed to implement Arrow-Debreu equilibria and that this "number is independent of the probability assessments under which expected gain from trade is zero".

The implementation result is meaningful if Arrow-Debreu equilibria are shown to exist in these continuous time stochastic economies, that is if a general existence proof is available for economies with L^2

commodity spaces (a consumption point at one is assumed to be a square-integrable function). Duffie and Huang [33] could not rely on such an existence result since it was not available at that time. Fortunately, infinite-dimensional general equilibrium theory has also reached its maturity in the past few years. Mas-Colell, for example, partially motivated by Duffie-Huang's results, obtains a general existence result theorem that covers these financial economies [80], and Duffie uses this result to show existence of an Arrow-Debreu equilibrium together with its implementation as an Arrow equilibrium [32]. These results have been extended to continuous time production economies by Duffie and Huang [36].

Incomplete markets and approximation results

As I mentioned in the discussion of Essay I, there is no satisfactory result on existence of Radner (Arrow) equilibria for economies with an infinite dimensional commodity space (as in our canonical model) and an incomplete market structure. Nevertheless, as Kreps remarks, one would like to know whether the above results are robust in the sense that economies in which trade can take place 'almost' continuously, can achieve an 'approximate' completion of markets. Kreps addresses more explicitly this question in [70] with limited results. Recently, Ketterer has provided approximation results by defining the degree of incompleteness of an 'approximate' economy, e.g., an economy where trade takes time at small intervals of time by the size of the transfers that a planner would need if he/she had to implement an Arrow-Debreu equilibrium of the continuous trading economy [68].

In the context of 'complete' markets, redundant securities such as an European call option can be priced, e.g., by using the Black-Sholes' formula. However, of greater interest is to price non-redundant securities. Recently, Föllmer and Sonderman [43] and Müller [85] [84] have made a first step in the direction of extending the martingale approach of Harrison and Kreps to contingent claims which are non-redundant. In the context of the Radner model (with the no-bankruptcy rule of constraining consumption to be non-negative), Back asks whether all claims can be *rationally priced* in the sense of preserving the *no expected gain from trade* property. He shows that while this is true for bounded claims it is not generally true, e.g., for claims with finite variance [4].

Information structures and asset prices

The integration of general equilibrium theory and asset pricing theory has been developed in continuous time by Cox, Ingersoll and Ross ([25], [24]) and more recently by Huang [64] and Chamberlain [23]. While Cox, Ingersoll and Ross use dynamic programming techniques to derive existence and characterization of agents' optimal portfolio policies, Huang and Chamberlain follow the *martingale representation* approach discussed above (for a more complete treatment of the single agent problem see [26], [27] and [87]). The *martingale representation* approach is not only more elegant and widely applicable than the Markovian stochastic dynamic programming approach but it provides a sharper result (lower dimension) when there is a finite dimensional Markov system and both approaches are applicable. Chamberlain's derivation of a single-beta intertemporal capital asset pricing model [23] is a good example of the strenght of this approach. As in the discrete case, these asset pricing models are within the paradigm of 'complete' markets.

It has been common practice in asset pricing theory to assume that markets are efficient in the sense that prices rapidly adjust to new information (i.e., prices have continuous sample paths). For example, a standard primitive assumption in many continuous-time asset pricing models is that prices can be represented as Itô integrals. Huang has shown how these "assumptions" can be derived from assumptions on agents' information structures (continuous revelation) and preferences (continuous in the appropiate topology) [62], [63]. For example, he shows that when the information is generated by a Brownian motion, equilibrium asset prices are then Itô integrals.

In all these models agents have symmetric information. Duffie and Huang study a model with differential information in which one agent is better informed than another. They show that both agents must agree on the *resolution times* of all commonly marketed events when there are *no free lunches*. The *resolution time* of one event is the first time that uncertainty regarding such event is resolved, i.e., the first time one knows with probabilty one whether such event is to ocurr or not. It would be interesting to generalize this work on *fully revealing* equilibria to a more general version of a rational expectations model with assymetric information in continuous time.

CONCLUDING REMARKS

Although I have left aside several areas of research on capital markets not included in "Three Essays...", in following Kreps's 1979 guide up to 1987, e.g., further developments of asset pricing models, econometric models, theory of the firm, etc. this has been a long trip in which we have seen how capital markets theory has reached maturity. Many open problems have been solved and the theory of finance and general equilibrium theory have been more fully integrated. As a result, we now not only have a better understanding of incomplete markets but also a better understanding of equilibrium theory. We also have better tools with which to study continuous time stochastic economies beyond financial models. While many contributions have been technical in nature or have provided the necessary partial answers to open problems, there are two results that I think, summarize what we have learned since "Three Essays..." were first written: first and foremost, the existence and characterization of incomplete market equilibria; and second, the implementation of Arrow-Debreu equilibria in continuous time stochastic economies. The germ of these results already appears in "Three Essays...".

As I mentioned, incomplete markets theory shows how *special* the Arrow-Debreu model is, and as long as market completeness is an idealization of real financial structures, models of incomplete markets might provide a better framework for the study of real economies. However, it seems to me that the evaluation of the incomplete financial markets model as a model that enhances our understanding of real economies is far from clear.

A mechanism design approach

There are many arguments that have been made to explain why there are no markets for complete contingent Arrow-Debreu contracts: the existence of environmental contraints such as limited communication, incentive constraints such as the presence of moral hazard and adverse selection problems, technological constraints such as limitations on the ability to write and/or enforce contracts, etc.. However, the starting point of the incomplete markets model is not a specific (perhaps non classical) environment but a limited set of financial instruments (in an otherwise classical environment) and, in this sense, the equilibrium map is not a well defined map from environments to outcomes. That is, the problem is not only that the equilibrium outcomes might be indeterminated but also that the domain of the map (e.g., the return matrix) is not the original environment. For example, the Overlapping Generations model also suffers from indeterminacy problems and its financial structure is usually also taken as given (e.g., *money*); nevertheless, the constraints that the geographical environment places on the set of feasible contracts are made explicit.

From the prespective of mechanism design, the literature on capital markets presents a peculiar discontinuity. With a small number of securities it is possible to implement Arrow-Debreu competitive allocations. This greatly simplifies the competitive mechanism; a mechanism designer, and maybe some real economies, therefore might prefer to implement Arrow-Debreu allocations with financial securities rather than with complete contingent contracts. If the designer is one security short, however, then the competitive mechanism might present serious inefficiency and indeterminacy problems. A mechanism designer who has this information and fails to create the *missing securities* would probably loose his/her job. For societies, markets are public goods and it is not easy to explain why the *missing markets* are not created, or why Coase's Theorem is not at work. One possible answer is that there are costs involved in creating securities/markets, we are not, however, in the exact context of the incomplete markets model. In particular, the results on constrained suboptimality are based on the assumption that a planner can freely reallocate securities. It is not clear to me weather indeterminacy of equilibria is preserved when new financial assets can be created at some cost. Nevertheless, we know of economies where assets are created at some cost and equilibria might still be constrained suboptimal and locally non-unique (see [78]).

In a non-classical environment, the designer might have further restictions. For example, contracts might have to be incentive compatible, and the corresponding exercise of creating enough *financial instuments* might not be trivial. However, we now have a better understanding of how the Arrow-Debreu paradigm can be extended to some non-classical environments: e.g., to economies with externalities, and more recently, to economies with private information, moral hazard, etc. as studied by Prescott, Townsend and others (see [88], [95]). These non-classical environments require more sophisticated contractual arrangements, such as contracts on lotteries, but as with classical environments one can ask whether (incentive compatible) efficient allocations might be achieved with simpler market structures. Townsend for example, shows that efficient allocations might be implemented in simple economies with limited communication/participation with few financial instuments, such as I.O.U.'s (see [95], [96], and [8] for a generalization). As in the model with *financial securities*, one might postulate a model where *limited participation* prevents agents from achieving Arrow-Debreu allocations. This is the approach of Balasko, Cass and Siconolfi, who prove that *limited participation* competitive equilibria exist and have the same indeterminacy properties as *incomplete markets* competitive equilibria with *pure financial securities* [5]. With the *limited participation model* we are a step closer to the environment and to being able to ask in a meaningful way: why are no more financial instruments created?. That is, we are a step closer to the integration of the mechanism approach to financial intermediation and the incomplete markets approach.

Indeterminacy and rational expectations

Indeterminacy of equilibrium allocations is a generic property in economies where *pure financial securities* and/or *stocks* are traded and the market structure is incomplete. These are rational expectations equilibria, in the sense that every point of the equilibrium manifold is supported as an equilibrium by having all agents belief that that particular point (e.g., price) is the equilibrium. It seems inconsistent to assume that agents can behave this way in an economy where agents cannot learn a particular price vector from past experience, This has been often seen as a critique of the rational expectations equilibrium concept. However, if one takes seriously the fact that agents might not behave this way, then what is needed is to explore other behavioral assumptions (see, for example, [13]).

Stationary equilibria

Two-period models are very useful in studying canonical properties of equilibria. Inherent to these models, however, is the problem that agents have to cleverly form their expectations. In a dynamical model, the rational expectations hypothesis is justified if stationary equilibria are shown to exist. For example, suppose that uncertainty is generated by a finite-state Markovian chain with N possible values. The market structure then can be simplified by introducing trading of N long-lived securities, which makes possible the implementation of any Arrow-Debreu equilibrium allocation (see [33]). In a non-stationary equilibrium, however, the portfolio decision problem is still highly dimensional. In a strictly stationary equilibrium, prices recover the Markovian structure of the exogenous process and agents' decisions problems are also simplified.

Similarly, agents might be able to almost self-insure themselves if equilibria are stationary and agents patient enough even when markets are incomplete. An extreme case of market incompleteness is when money is the only available asset. If consumers' pure rate of time preference are small enough and there is no aggregate uncertainty then consumers' marginal utilities of money stay nearly constant over time at a stationary equilibrium which exists when the money supply is large enough (see [14]). Green and Spear study an overlapping generations model with an incomplete financial structure, and by restricting their attention to a special class of stationary equilibria (prices can only depend on the immediate payoff-relevant parameters), they show that incomplete stationary markets equilibria are locally unique and constrained optimal [48]. It would be interesting to know if similar results extend to more general definitions of stationarity (e.g., prices as stationary processes).

Financial innovation

In the discussion of the shareholders' problem, the issue of why shareholders do not act as financial innovators arises naturally. This question is not specific of production economies, and it must arise again in a dynamical context where the market costs of creating new financial instruments might be negligible. Except for the tivial --but not irrelevant-- case where all the necessary securities to achieve spanning are created, this is not a simple modeling problem. Attempts to study it have been made by Duffie and Jackson, who analyze a model with strategic choice of contracts [35]; and by Anderson and Harris, who study a game where offers of contracts can be made at chosen times [1]. Takening into account all the financial innovations that have taken place in the past few years, I might conclude that economic theory in this area is far behind reality.

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