OPTIMAL CHOICE OF MONETARY POLICY INSTRUMENTS: AN OPTIMAL CONTROL SIMULATION WITH VAR FOR JAPAN

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Abstract

The paper analyzes the effective method of monetary control by estimating a vector autoregressive model of three variables, GNP, money supply, and short-term interest rates, and conducting dynamic optimal control simulation for Japan. The result indicates that active monetary control with any monetary policy instruments is better than passive monetary control which keeps certain monetary instruments constant. Among active monetary policy, money supply control turns out to be better than interest rate control. Yet, it appears to be more feasible to follow active interest rate control.

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Optimal Choice of Monetary Policy Instruments: An Optimal Control Simulation with VAR for Japan

Experiencing a period of monetary control under a flexible exchange rate regime, it would be worthwhile investigating what could have been done to stabilize variability in GNP. This paper addresses the issue by estimating a vector autoregressive model (VAR) for three variables, GNP, money supply, and short-term interest rates, and conducting dynamic optimal control simulation for Japan. The result indicates that an active monetary control with any monetary policy instruments is better than passive monetary control which keeps certain monetary instruments constant. Among active monetary policy, money supply control turns out to be better than interest rate control. Yet, it appears to be more feasible to follow active interest rate control.

The paper is organized as follows. Section I summarizes the past theoretical analysis on optimal monetary policy instruments. Section II describes data and analyzes relative power contributions (RPC) among three variables. Section III compares the result of dynamic control simulation. Finally, Section IV discusses the feasibility of control and warns against blind adherence to the result.

I. Choice of Optimal Monetary Policy Instruments

Based on the IS and LM framework, William Poole discussed that without any uncertainty in goods and money markets it would not matter which monetary instruments were chosen: money supply or interest rates. However, under uncertainties in both markets, the choice of monetary instruments would affect the variability of income. If the uncertainty in the goods market exceeds that in the money market, then it is better to choose money supply as a monetary instrument rather than interest rates.

Canzoneri, Henderson and Rogoff (CHR) incorporated the Lucas-type supply function and rational price expectations into the model framework. If the wage setters do not form price expectations rationally, the active monetary policy, which specifies money supply as a function of nominal interest rates, would reduce the variability of income. As in Poole, if the uncertainty in the goods market predominates, the money supply would be stabilized as a result of monetary policy.

The theoretical investigations so far all suggest that it is an empirical question which monetary policy instrument is to be used. It wouls be important to capture a dynamic structure in an empirical analysis since the adjustment process plays an important role in reality, though both theoretical frameworks discussed above are static in nature. Therefore, a vector autoregressive model is estimated to capture both a feedback and adjustment process and also to exploit optimal control simulation.

II. Data and Relative Power Contribution

Our VAR consists of just three variables: nominal GNP, the call rate, and money supply M_2+CD, since these correspond to the target variable and two monetary policy instruments Poole considered. Nominal GNP is used since it is a single variable which represents the whole economic state and a product of price and real output, either of which could be affected by monetary policy. To make the time series data of nominal GNP stationary, the first difference of the rate of change over the same quarter of the previous year is taken. The call rate is the interest rate on the overnight loan among banks. It is a convenient monetary policy instrument for the Bank of Japan (BOJ) since the BOJ can interfere in the market anytime to affect the rate. The first difference in the annual rate is incorporated. Money supply M_2+CD is used since it is a monetary measure that the BOJ watches carefully as an instrument of monetary control. It covers the currency in circulation held by the public, demand deposits, and time deposits inclusive of certificates of deposit (CD). The first difference of the rate of change over the same quarter of the previous year is also employed.

The sample period of the original data is 1971:3Q-1983:1Q, which covers the period of the flexible exchange rate regime. Since two of the series utilize the first difference of the rate of change over the previous year, the actual sample period of the transformed data becomes 1972:4Q-1983:1Q.[1] Figure 1 illustrates the historical movement of the data used in the estimation. Table 1-A shows the coefficients of VAR when all three variables are taken to be endogenous. The lag length is chosen so as to minimize the final prediction error which is defined by

$$\text{(1)} \qquad FPEC(M) = \Big((N+Mk+1)/(N-Mk-1) \Big)^r \, || d_{r,M} ||$$

where N is the number of observations, M is the number of lags employed, k the number of explaining variables, r the number of endogenous variables, $||d_{r,M}||$ the determinant of $d_{r,M}$ which is the estimates of the variance-covariance matrix of white noise, obtained from the residual matrix of the model. Since all three variables are endogenous here, r is equal to k. The minimum of FPEC(M) is achieved at M=1. All the coefficients are positive except one for the lagged call rate in explaining money supply. The same coefficients apply even when either one of the monetary variables is chosen to be a policy variable, since the same lag structure happens to apply for all of the model. According to Table 1-B the correlation coefficient between the residuals of M_2CD and the call rate is -0.27 and is the largest. It implies that 7 percent of the respective residuals is explained by the other residual. Figure 2 illustrates the accumulative relative power contribution (RPC) of the noise of each variable to the respective variable. The noise of a certain variable X_i is such a part of X_i that is not attributable to the past information of relevant variables. The movement of X_i can be decomposed into the distributed lags of the noise of the respective variables. Then the variance of X_i is expressed as the sum of variances of these noises assuming that the noises are uncorrelated each other. Therefore, X_i's power spectrum, which is a Fourier transform of x_i's variance, can be expressed as the sum of power spectrums of noise. The relative contribution to x_i's power spectrum of each noise is defined as RPC. The mathematical specification is given in Appendix 1.

The horizontal axis scales frequency or period which is an inverse of frequency. The most right hand side is 1/2 of the frequency or the two periods of the cycle. The mid-point measures 1/4 the frequency or a one year period. The left hand side represents zero frequency or an infinite period, which corresponds to a constant of the data. The vertical axis measures the sum of RPC. Figure 2-A indicates that the RPC of monetary variables to nominal GNP is at most 20 percent for the period around three years of which 17 to 18 percent is atrributed to M_2CD. The RPC of the call rate is at most 6 percent for the period around 1.5 to 2 years. Figure 2-B suggests that the RPC of the call rate to M_2CD is relatively large, exceeding 40 percent for a four year period and 8 percent for short waves of less than three quarters, while the RPC of GNP is minimal. Figure 2-C shows that the RPC of other variables than call rate is minor, the maximum RPC of GNP being 6 percent for periods over three years. The overall analysis suggests that the call rate is relatively independent of other variables, having a large impact on the long waves of money supply which in turn affects GNP's long waves.

III. Optimal Control Simulation

A VAR can be exploited for optimal control once some of the variables are identified as policy variables. This is because the future value of target variables are dependent on the present and past values of themselves and of policy variables and such a relationship can be exploited to reduce the variance of the future value of target variables by appropriately setting the policy variables. For that purpose, the VAR needs to be rewritten in a state space representation and the loss function needs to be specified to evaluate the fluctuation of both target and policy variables as discussed in Appendix 2. The value of policy variables is set so as to minimize the following loss function. That is,

$$(2) \qquad J_I = EK_I = E\sum_{t=1}^{I}\{Z_t'Q_tZ_t + Y_{t-1}'R(t)Y_{t-1}\}$$

which becomes the weighted sum of the variance of target variables and policy variables over the simulation period I with the Q matrix being the weight given to target variables Z_t and the R matrix being the weight given to policy variables Y_{t-1}. The reaction function of policy variables against target variables is derived through the optimizing principle of dynamic programming as discussed in Appendix 2. The practical procedure of optimal control simulation is summarized as follows.

1) Estimate a VAR excluding a particular monetary policy instrument from dependent variables, determining the appropriate lag structure by FPEC (M).

2) Supply arbitrary Q and R matrices and a simulation period to obtain a gain matrix, which specifies the reaction function of the monetary authority, that is, provides it with the value of the policy variable against the observed value of target variables so as to minimize the above loss function over the simulation

period under given weights matrices Q and R and the coefficient matrix of the VAR.[2]

3) Generate a series of white noises of two endogenous variables for the simulation period.

4) Inputing white noise series from 3) and the gain matrix from 2), perform optimal control simulation to get the means and variances of the respective variables over the simulation period.

5) Repeat 2) to 4) with different sets of Q and R matrices to find the best result within a loose constraint so that the variance of target and policy variables does not exceedingly outperform the historical performance.

6) Get a result under a fixed control using a zero gain matrix and the same white noises, and compare the result with one under the feedback control.

7) Repeat 1) to 6) with a different instrumental variable and compare the results with each other.

Table 2 reports the results of the feedback and fixed control with the call rate as a monetary policy instrument as well as the historical performance during the sample period. Under the feedback control the variance of nominal GNP (VGNP) is reduced to 2.29 which is 1.08 times the variance of white noise (VWN), compared to the historical variance (HV) of 3.52 which is 1.14 times the variance of residuals (VR). Further, the variance of M_2CD (VM_2CD) is reduced to 1.21, 1.01 times VWN from 1.94 HV and 1.14 VAR while the variance of the call rate (VCR) remains 1.27 which is about the same as 1.28 HV. On the other hand, under the fixed control, VGNP is 2.41 and 1.14 times VWN which is as good as HV, whereas VM_2CD is 0.96 times VWN, which is much better than HV.[3] Thus, the result suggests that the fixed control is as good as the historical performance in terms of the variance of nominal GNP and that the feedback control can smooth fluctuations of both nominal GNP and money supply.

Table 3 summarizes the results when M_2CD is used as a policy instrument. Under a feedback control VGNP is reduced to one equal to VWN and VCR is made smaller than VWN, whereas under a fixed control VGNP is reduced to 1.11 times VWN and VCR to 0.95 times VWN. The results are better than those when the call rate is used as a policy instrument since VGNP is smaller in either the feedback or fixed control case. Thus it appears that money supply control is better than short term interest rate control in either feedback or fixed control as suggested by Poole. Such a result conforms to the fact that the VGNP is larger than the variance in the money market whether it is measured with call rate or M_2CD. Further, comparing the result under feedback control to those under fixed control, it can be said that feedback control with call rate as a policy instrument is better than fixed control with money supply M_2CD as a policy instrument.

Similar results are confirmed with a five variables model where the nominal asset transactions and private inventory-output ratio are added to the three variables model. One exception is that a combination policy which utilizes both call rate and M_2CD gives an even better result in terms of VGNP. The results are reported in Appendix 3.[4]

IV. Feasibility of Control and Concluding Remarks

Optimal control simulation assumes that any selected control variable is under perfect control by a monetary authority. But in reality money supply M_2CD and the call rate are not controllable in the same magnitude. As discussed in Section II, M_2CD has a broad coverage including currency in circulation held by the public and time deposits, which are a major form of saving in Japan. Then it could be the case that it is not an easy task for the monetary authority to control M_2CD contemporaneously as nominal GNP and the call rate change. On the other hand, the call rate is considered to be a handy monetary policy instrument which the monetary authority can affect relatively easily by interfering with the call market through dealers. Then, feedback control with the call rate as a monetary policy instrument could be a more practical control.

According to the gain matrix the first difference of the call rate needs to be inversely adjusted to the first difference of the rates of change in both nominal GNP and M_2CD, the latter being the dominant factor. Since money supply and call rates are inversely related as illustrated in Figure 1 and as shown by a negative coefficient in Table 1-A, it indicates that call rates need to be reduced (raised) further when they are declining (rising), money supply is increasing (decreasing), and GNP starts increasing (decreasing). It would mean that the monetary authority can depend on the tendency of money supply to lead GNP in manipulating call rates and it should further strengthen the countercyclical monetary policy to smooth fluctuations of GNP than ever realized during the sample period.

In summary, the analysis in this paper shows that any active monetary policy is better than any passive monetary policy to smooth fluctuations of GNP and that feedback control with money supply M_2CD as a monetary policy instrument is better than that using call rates as an instrument as suggested by Poole and others. Nevertheless, it appears more feasible to manipulate call rates as a monetary policy instrument reacting mainly to money supply than to employ money supply as a direct instrument.

Appendix 1

When $x_i(t)$ is explained by x_j's, and the residual $u_i(t)$, that is,

(3) $$x_i(t) = \sum_{j=1}^{k} \sum_{m=1}^{M} a_{ij}(m) x_j(m) + u_i(t)$$

$u_i(t)$ is said to be the noise of x_i's since it is the part of x_i's which is not explained by the past information of relevant variables (Akaike, p. 68).

Relative power contribution is defined as follows (Akaike, pp. 69-70). Equation (3) can be expressed in a matrix form as follows. That is,

(3′) $$X(t) = \sum_{m=1}^{M} A(m) X(t-m) + U(t)$$

where $X(t)$ is a vector $(x_1(t), x_2(t), \ldots, x_k(t))'$ in which k is the number of variables and $'$ indicates a transpose; $A(m)$ is a $(k \times k)$ matrix an (i,j) element of which is $A_{ij}(m)$; $U(t)=(\epsilon_1(t), \epsilon_2(t), \ldots, \epsilon_k(t))'$ with $E[\epsilon_j(t)]=E[\epsilon_i(t)\epsilon_j(t)]=0$, $(i \neq j)$. Then, a matrix of the power spectrum function for $x_i(t)$ $(i=1,2,\ldots k)$ at frequency f is given by

(4) $$P(f) = (A(f))^{-1} \Sigma(\overline{(A(f))'})^{-1}$$

whose (i,i) element is $x_i(t)$'s power spectrum function $p_{ii}(f)$ and (i,j) element is a cross spectrum function $p_{ij}(f)$ between $x_i(t)$ and $x_j(t)$. Σ is a variance-covariance matrix among $\epsilon_i(t)$ and $\epsilon_j(t)$ whose (i,i) element is σ_i^2 and (i,j) element is σ_{ij}. $\overline{A(f)}'$ is a matrix of a complex conjugate of a respective element of a transpose of A(f), which is given by

(5) $$A(f) = (I - \sum_{m=1}^{M} A(m) \exp(-i2\pi fm))$$

where I is a (k×k) unit matrix, $i^2=1$, and π is measured in radian. If $\epsilon_i(t)$ and $\epsilon_j(t)$ $(i\neq j)$ are uncorrelated, i.e., $\sigma_{ij}=0$ $(i\neq j)$, then the power spectrum function of $x_i(t)$ is given by

(6) $$p_{ii}(f) = \sum_{j=1}^{k} |A(f)^{-1}{}_{ij}|^2\sigma_j^2$$

where $A(f)^{-1}{}_{ij}$ is an (i,j) element of $(A(f))^{-1}$ and $|\ |$ absolute value. Now, a part of the power spectrum of $x_i(t)$ attributable to the noise of $x_j(t)$ can be expressed as a function of frequency f as follows.

(7) $$p_{ij}(f) = |A(f)^{-1}{}_{ij}|^2\sigma_j^2$$

Its relative share of $x_i(t)$'s power spectrum is called a relative power contribution and is given by

(8) $$r_{ij}(f) = \frac{p_{ij}(f)}{p_{ii}(f)} = \frac{|A(f)^{-1}{}_{ij}|^2\sigma_j^2}{\sum_{j=1}^{k}|A(f)^{-1}{}_{ij}|^2\sigma_j^2}$$

Its accumulative share is

(9) $$R_{ij}(f) = \sum_{h=1}^{j} r_{ih}(f) \quad (j=1,2,\ldots,k)$$

The vertical scale of Figure 2 plots $R_{ij}(f)$ for respective frequency f.

Appendix 2

Since the variables in a VAR are classified into target and policy variables, it can be rewritten as follows.

$$(10) \qquad X(t) = \sum_{m=1}^{M} A(m)X(t-m) + \sum_{m=1}^{M} B(m)Y(t-m)+U(t)$$

where X(t) is a $(r\times1)$ matrix of target variables, A(m) a $(r\times r)$ coefficient matrix, $Y(t-m)(\ell\times1)$ policy variables, $B(m)(r\times\ell)$ a coefficient matrix, and U(t) an $(r\times1)$ disturbance matrix.

Then, a state space expression can be derived as follows. Here, a state space is defined as the minimum present and past information which is required to forecast the future. Replacing t in equation (10) with t+s and rewriting the right hand side, we obtain

$$(11) \qquad X(t+s) = \sum_{h=1}^{M-s} A(s+h)X(t-h) + \sum_{h=1}^{M-s} B(s+h)Y(t-h) + \sum_{n=0}^{s-1} A(s-n)X(t+n) + \sum_{n=0}^{s-1} B(s-n)Y(t+n)+U(t+s)$$

The first terms are factors dependent on the observations till t-1 in forecasting X(t+s), and the last two terms are forecasts for periods after t. Let $Z_s(t)$ represent the first two terms where the subscript s indicates the lead before the forecast period.

$Z_s(t)$ can be rewritten as follows.

$$(12) \qquad Z_s(t) = \sum_{h=1}^{M-s-1} A(s+h+1)X(t-1-h) + \sum_{h=1}^{M-s-1} B(s+h+1)Y(t-1-h) + A(s+1)X(t-1)+B(s+1)Y(t-1)$$

Now, considering the moment a new observation X(t) is obtained, set

(13) $$Z_0(t) = X(t)$$

Then the following set of equations will hold

(14)
$$
\begin{aligned}
Z_0(t) &= Z_1(t-1)+A(1)Z_0(t-1)+B(1)Y(t-1)+U(t) \\
Z_1(t) &= Z_2(t-1)+A(2)Z_0(t-1)+B(2)Y(t-1) \\
&\vdots \\
Z_k(t) &= Z_{k+1}(t-1)+A(k+1)Z_0(t-1)+B(k+1)Y(t-1) \\
&\vdots \\
Z_{M-1}(t) &= A(M)Z_0(t-1)+B(M)Y(t-1)
\end{aligned}
$$

Now, define Z_t as follows.

(15)
$$
Z_t = \begin{array}{c|c|c}
 & \leftarrow 1 \rightarrow & \\
\updownarrow r & & \\
\updownarrow r & \begin{bmatrix} Z_0(t) \\ Z_1(t) \\ \vdots \\ Z_{m-1}(t) \end{bmatrix} & M\times r \\
\vdots & & \\
\updownarrow r & &
\end{array}
$$

Then (14) can be expressed in the following state space expression. That is,

(16)
$$
\begin{aligned}
Z_t &= \Phi Z_{t-1}+\Gamma Y_{t-1}+W_t \\
X(t) &= HZ_t
\end{aligned}
$$

where

$$
\Phi = \begin{array}{c} \\ \updownarrow r \\ \updownarrow r \\ \vdots \\ \updownarrow r \\ \updownarrow r \end{array}
\begin{array}{c}
\begin{array}{ccccc} \leftarrow r \rightarrow & \leftarrow r \rightarrow & \leftarrow r \rightarrow & \cdots & \leftarrow r \rightarrow \end{array} \\
\begin{bmatrix}
A(1) & I & 0 & \cdots & 0 \\
A(2) & 0 & I & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A(M-1) & 0 & 0 & \cdots & I \\
A(M) & 0 & 0 & \cdots & 0
\end{bmatrix}
\end{array}
$$

$$\Gamma = \begin{array}{c} \\ r \\ r \\ \vdots \\ r \\ r \end{array} \begin{array}{c} \ell \\ \begin{bmatrix} B(1) \\ B(2) \\ \vdots \\ B(M-1) \\ B(M) \end{bmatrix} \end{array} \qquad W_t = \begin{array}{c} \\ r \\ r \\ \vdots \\ r \\ r \end{array} \begin{array}{c} 1 \\ \begin{bmatrix} U(t) \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \end{array}$$

$$H = r\begin{array}{c} \begin{array}{ccccc} r & r & & r & r \end{array} \\ \begin{bmatrix} I & 0 & \ldots & 0 & 0 \end{bmatrix} \end{array} \qquad Y_t = Y(t)$$

Now, it is possible to design such a control system as to minimize the following loss function, That is,

$$J_I = EK_I = E\sum_{t=1}^{I}\{Z_t'Q_tZ_t+Y_{t-1}'R(t)Y_{t-1}\} \tag{17}$$

where Q(t) is a Mr×Mr nonnegative matrix, R(t) is a $\ell\times\ell$ positive matrix, ′ represents a transpose, and E stands for expectation. The first term of the right hand side of equation (17) is the weighted average of the variance-covariance of the present value and the portion of future forecast value that is dependent on past observations. Q_t is the weight utilized. The second term is the weighted sum of the variance-covariance of policy variables, with R(t) being the weight. Such a term is included since the increase in the variance-covariance of policy variables would increase the loss of resources and it is unrealistic to allow too large a variance for policy variables. We will focus only on the movement of X_t and therefore use a diagonal matrix of the form

$$\begin{array}{cc} & \begin{array}{cc} \leftarrow r \rightarrow & \leftarrow (M-1)r \rightarrow \end{array} \\ \begin{array}{c} \uparrow \\ r \\ \downarrow \\ \uparrow \\ (M-1)r \\ \downarrow \end{array} & \begin{bmatrix} \begin{matrix} q_1 & 0 & \\ & q_2 & \\ 0 & & q_r \end{matrix} & 0 \\ 0 & 0 \end{bmatrix} \end{array}$$

The weight matrix $R(t)$ for policy variables is also taken as a diagonal. Then the loss function becomes the sum of a weighted average of variances of the target variables at period t and a weighted average of the variances of policy variables at period t-1.

Once the loss function is specified, the value of the policy variables can be determined so as to minimize the loss. Such values are obtained through the optimizing principle of dynamic programming and called an optimum control solution. The gain matrix G_i, which specifies the optimum input Y_{I-i} against Z_{t-i} for the forecast horizon I, is given by the following equation.

(18) $$G_i = -(R+\Gamma' P_{i-1}\Gamma)^{-1}\Gamma P_{i-1}\Phi$$

where G_i is a $\ell \times Mr$ matrix and P_{i-1} is calculated recursively by the following procedure. That is,

(19) $$\begin{aligned} P_0 &= Q \\ M_i &= P_{i-1}-P_{i-1}\Gamma(R+\Gamma' P_{i-1}\Gamma)^{-1}\Gamma' P_{i-1} \\ P_i &= \Phi' M_i \Phi + Q \quad (i=1,2,\ldots I) \end{aligned}$$

Then, the optimum input Y_{I-i} is given by

(20) $$Y_{I-i} = G_i Z_{I-i}$$

when $i=I$,

$$G_I = -(R+\Gamma' P_{I-1}\Gamma)^{-1}\Gamma' P_{I-1}\Phi \tag{21}$$

and

$$Y_0 = G_I Z_0.$$

If G_I is fixed and used for all Z_t to set $Y_t=G_I Z_t$, Y_t becomes the optimum input of policy variables which takes into account the movement of the system until I period ahead. The gain matrix used in the text is such that G_I for $I = 16$ quarters.

Appendix 3

In the five variables VAR model two more variables are added: the first difference of the rate of change in asset trading over the same quarter of the previous year, and the first difference of the ratio of nominal private inventory investment to nominal GNP. The first variable is included since money demand arises from asset trading as well as goods transactions and the second is included since the inventory investment is a close real substitute to money. The results for optimal control simulation are summarized in Tables 4 to 6. Now, instead of just one variable i.e., nominal GNP, three variables, i.e., nominal GNP, nominal asset transactions, and inventory-output ratios are always included as target variables regardless of whichever monetary variable is used as a policy instrument. The results are very similar to those of the three variables models. Looking at both the variance of nominal GNP and nominal asset transactions, it is said that any feedback is better than passive fixed control and that a feedback control with money supply M_2CD is more effective than one with a call rate. Besides, it is further confirmed that if both M_2CD and the call rate are used as control instruments, then even better results could be achieved. However, it is noted that the variance of inventory-output ratio is consistently larger than the historical performance in terms of the ratio to the variance of residuals or white noise and that it is smaller in fixed control than in feedback control although such dominance is almost negligible when both the call rate and M_2CD are controlled.

Footnotes

[1]The end point is determined by the end point of the available asset transaction data which are published by the Economic Planning Agency. The asset transaction is incorporated in Appendix 3 where two variables are added to the model.

[2]The variance of the first difference is a linear transformation of the variance of the original series: $Var(Y_t - Y_{t-1}) = 2(1-\rho_1)\sigma_y^2$, where ρ_1 is an autocorrelation coefficient between Y_t and Y_{t-1}, and σ_y^2 is the common variance of Y_t and Y_{t-1}.

[3]In terms of the variance of M_2CD the fixed control gives the better result. However, by allowing more VGNP it is possible to reduce VM_2CD further to make both variances smaller than those under the fixed control.

[4]An attempt to control both monetary instruments in the three variables model failed since the final prediction error indicated no need to include lagged variables. This suggests that the first difference of the rate of change in nominal GNP over the same quarter of the previous year is close to the random disturbance and the three lagged variables cannot improve predictability as much as their joint inclusion is justified.

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Table 1-A

Three Variables Autoregressive Model (1972IV-83I)

	DGGNP*	DGM_2CD	DCALLR
DGGNP(-1)	0.0814	0.0201	0.1197
$DGM_2CD(-1)$	0.5262	0.3057	0.0285
DCALLR(-1)	0.3674	-0.5127	0.5614

Table 1-B

Correlation Among Residuals

	DGGNP	DGM_2CD
DGM_2CD	-0.22	-
DCALLR	-0.06	-0.27

*DGGNP = First difference of the rate of change in nominal GNP over the same quarter of the previous year.

DGM_2CD = First difference of the rate of change in M_2+CD over the same quarter of the previous year.

DCALLR = First difference of the call rate.

Table 2

Optimal Control with Call Rate as an Instrument

		Nominal GNP	M_2CD	Call Rate
1) Historical Performance[a]	Mean	-0.2565	-0.3646	0.0517
	Variance	3.5237	1.9416	1.2780
		(1.14)	(1.63)	
2) Feedback Control[b]	Mean	0.0151	1.2589	-1.1897
	Variance	2.2856	1.2050	1.2721
		[1.08]	[1.01]	
3) Fixed Control[c]	Mean	0.0030	0.4164	0.0
	Variance	2.4141	1.1531	0.0
		[1.14]	[0.96]	
4) White Noise	Mean	-0.2036	0.2951	-
	Variance	2.1202	1.1951	-
		[1.00]	[1.00]	
5) Residuals[d]	Variance	3.0980	1.1924	-
		(1.00)	(1.00)	

[a]The number in parentheses is a ratio of variance to that of residuals.

[b]Weight Matrices are $Q = 100 \; {}_1$ R=[1]. Gain matrix is G=[-0.1565 -0.9432].

The number in brackets is a ratio of variance to that of white noises.

[c]To fix the instrumental variable at the mean of the past sample period, set G=[0,0].

[d]A correlation coefficient between the residuals of nominal GNP and M_2CD is -0.22.

Table 3

Optimal Control with Money Supply M_2CD as an Instrument

		Nominal GNP	Call Rate	M_2CD
1) Historical Performance	Mean	-0.2565	0.0517	-0.3646
	Variance	3.5237	1.2780	1.9416
		(1.14)	(1.50)	
2) Feedback Control[a]	Mean	-0.2143	0.4558	-0.3047
	Variance	2.1200	0.9037	0.6330
		[1.00]	[0.98]	
3) Fixed Control[b]	Mean	-0.0077	0.5374	0.0
	Variance	2.3523	0.8765	0.0
		[1.11]	[0.95]	
4) White Noise	Mean	-0.2036	0.2377	-
	Variance	2.1202	0.9236	-
		[1.00]	[1.00]	
5) Residuals[c]	Variance	3.0980	0.8505	-
		(1.00)	(1.00)	

[a] $Q = \begin{bmatrix} 300 & \\ & 100 \end{bmatrix}$ R=[1]. G=[-0.1647 -0.7458].

[b] G=[0 0].

[c] A correlation coefficient between the residuals of nominal GNP and the call rate is -0.06.

Table 4

Variances under Optimal Control with Call Rate as an Instrument

(5 Variables Model)

	Nominal GNP	Nominal Assets Transations	M_2CD	Inventory Output Ratio	Call Rate
1) Historical Performance	3.5237 (1.21)	497.82 (1.31)	1.9416 (1.63)	4.0591 (1.73)	1.2780
2) Feedback Control[a]	3.0453 [1.13]	721.27 [1.33]	0.6892 [2.27]	2.4382 [2.67]	1.8243
3) Fixed Control[b]	3.7108 [1.38]	827.88 [1.52]	0.2923 [0.96]	1.5410 [1.69]	0.0
4) White Noise	2.6858 [1.00]	543.31 [1.00]	0.3031 [1.00]	0.9140 [1.00]	-
5) Residuals[c]	2.9223 (1.00)	380.45 (1.00)	1.1918 (1.00)	2.3491 (1.00)	-

[a]Weight matrices are $Q = \begin{bmatrix} 100 & & & \\ & 5 & & \\ & & 5 & \\ & & & 5 \end{bmatrix}$ $R = [2]$.

Gain matrix is $G = [0.7149 \quad -0.0077 \quad -0.6066 \quad -0.0724]$.

[b]$G = [0 \quad 0 \quad 0 \quad 0]$.

[c]Correlation coefficient matrix among residuals is given by the table below.

	Nominal GNP	Nominal Asset Transactions	M_2CD
Nominal Asset Transactions	0.12		
M_2CD	-0.22	0.33	
Inventory Output Ratio	0.06	0.13	0.14

Table 5

Variances under Optimal Control with M_2CD as an Instrument

	Nominal GNP	Nominal Assets Transations	Call Rate	Inventory Output Ratio	M_2CD
1) Historical Performance	3.5237 (1.21)	497.82 (1.31)	1.2780 (1.57)	4.0591 (1.73)	1.9416
2) Feedback Control[a]	2.6912 [1.00]	647.09 [1.19]	0.2430 [1.12]	2.4747 [2.31]	0.7579
3) Fixed Control[b]	3.2374 [1.21]	773.04 [1.42]	0.2707 [1.25]	2.1318 [1.99]	0.0
4) White Noise	2.6858 [1.00]	543.31 [1.00]	0.2171 [1.00]	1.0728 [1.00]	-
5) Residuals[c]	2.9223 (1.00)	380.45 (1.00)	0.8130 (1.00)	2.3491 (1.00)	-

[a] $Q = \begin{bmatrix} 100 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$ R = [1]. Gain matrix G = [0.1657 -0.0329 -0.8114 -0.1531].

[b] G = [0 0 0 0]

[c] Correlation coefficient matrix among residuals is given by the table below.

	Nominal GNP	Nominal Asset Transactions	Call Rate
Nominal Asset Transactions	0.12		
Call Rate	-0.08	-0.15	
Inventory Output Ratio	0.06	0.13	-0.12

Table 6

Variances under Optimal Control with Call Rate and M_2CD as an Instrument

(5 Variable Model)

	Nominal GNP	Nominal Assets Transations	Inventory Output Ratio	M_2CD	Call Rate
1) Historical Performance	3.5237 (1.21)	497.82 (1.31)	4.0591 (1.73)	1.9416	1.2780
2) Feedback Control[a]	3.2991 [0.99]	235.21 [0.94]	4.8697 [2.77]	2.0565	0.1871
3) Fixed Control[b]	3.7477 [1.12]	276.63 [1.11]	479.17 [2.72]	0.0	0.0
4) White Noise	3.3409 [1.00]	249.76 [1.00]	1.7630 [1.00]	-	-
5) Residuals[c]	2.9223 (1.0)	380.45 (1.0)	2.3491 (1.0)	-	-

[a] $Q = \begin{bmatrix} 10 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} R = \begin{bmatrix} 5 & \\ & 1 \end{bmatrix}$. Gain matrix $G = \begin{bmatrix} -0.8039 & 0.0063 & -0.0482 \\ 0.1408 & -0.0263 & 0.0041 \end{bmatrix}$

[b] $G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

[c] Correlation coefficient matrix among residuals is given by the table below.

	Nominal GNP	Nominal Asset Transactions
Nominal Asset Transactions	0.12	
Inventory Output Ratio	0.06	0.13

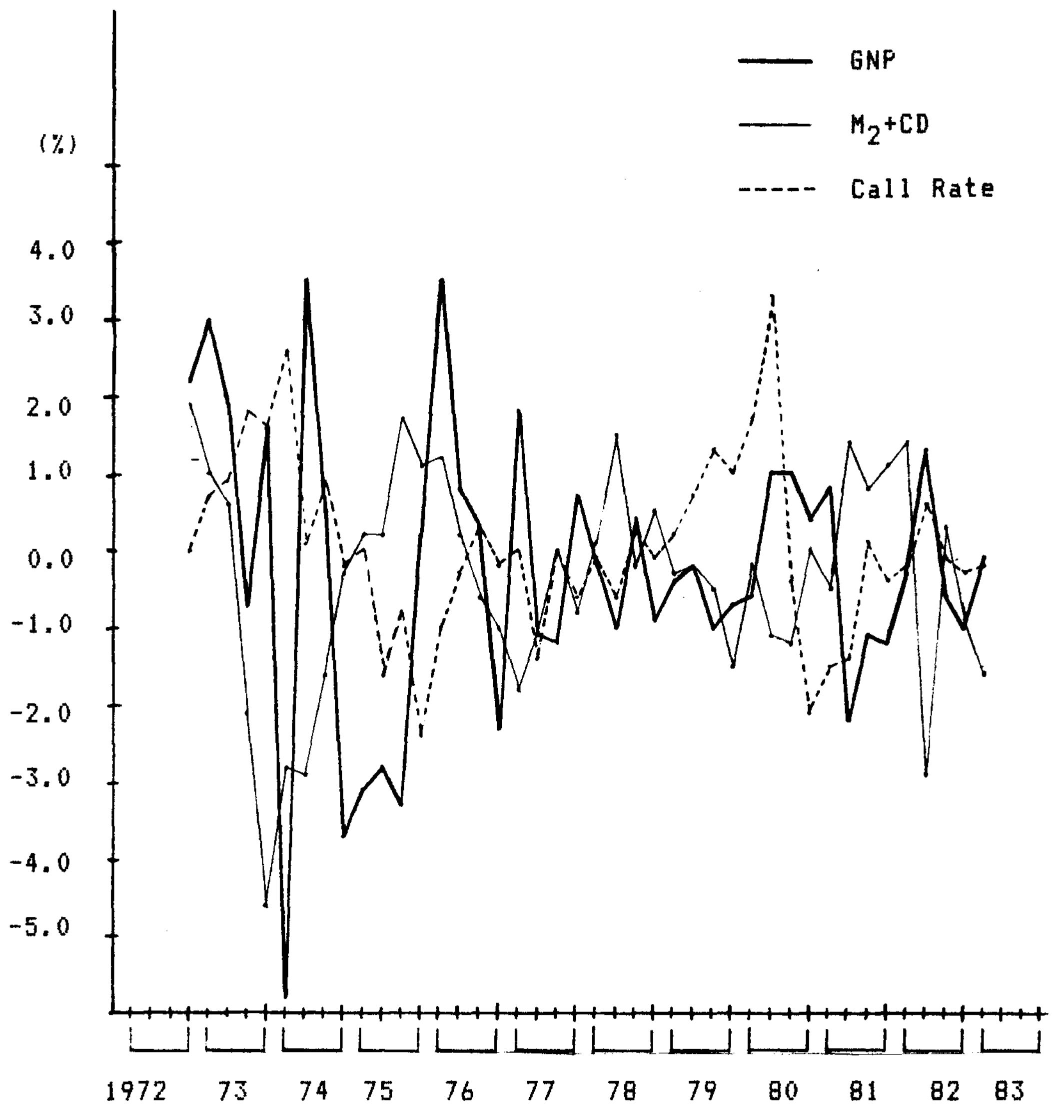

Figure 1

Historical Movement of Three Variables

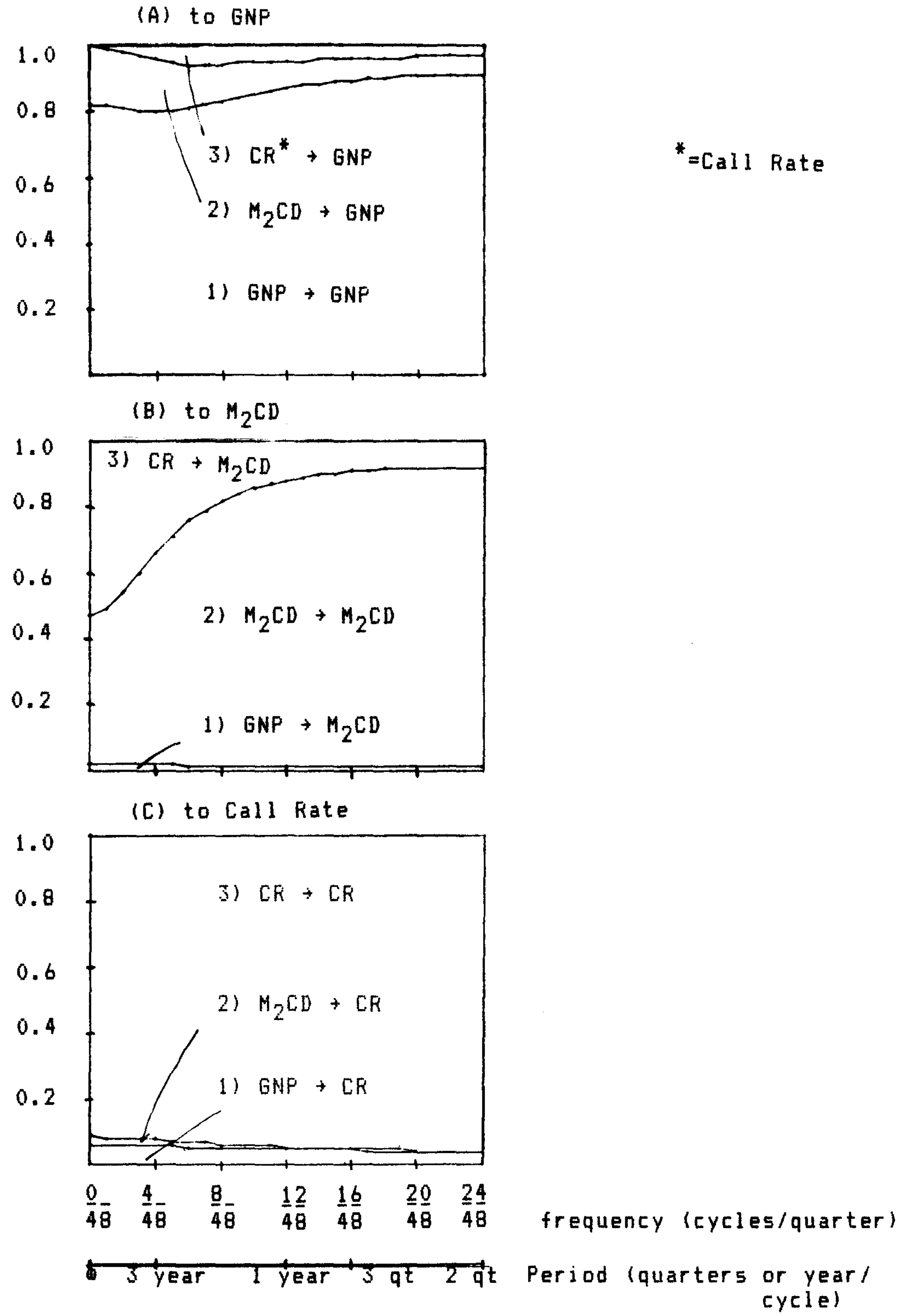

Figure 2
Accumulative Relative Power Contribution