PRICE DISCRIMINATION ANALYSIS
OF MONETARY POLICY: AN EXTENSION

by

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Abstract

Similar assets with significantly different rates-of-return are observed in many financial markets. This is known as the rate of return paradox in monetary theory, and is explained in this paper as an optimal response by the government to an informational restriction. A general equilibrium model is constructed with heterogeneous agents and a government that must finance an exogenously determined, stationary deficit by issuing bonds or fiat currency. In addition to explaining the paradox, the analysis accomplishes the following: (i) the informational restriction helps justify the ruling out of lump sum taxation, and (ii) the government's financing problem is shown to be formally equivalent to a nonuniform pricing problem.

In a recent paper, Bryant and Wallace (1984) apply price discrimination analysis to the problem of optimally financing an exogenously given government deficit. They suggest (1984, p. 279) that "this type of analysis is a productive way to study the often observed concurrence of legal restrictions on private intermediation and the tailoring of debt issues to the needs of the market." Bryant and Wallace use an identical agents overlapping generations model, and show that there are circumstances under which a prohibition on private intermediation and the issuance of both small denomination fiat currency with a negative real rate of return and large denomination default-free bonds with a higher negative real rate of return give rise to an equilibrium that is Pareto superior to that attainable under financing by currency issue alone. They also show that there is a policy involving the issue of large denomination bonds only which gives rise to a Pareto optimal allocation. In this note, I generalize the Bryant and Wallace model to one with heterogeneous agents. In addition to generalizing the original model, assuming heterogeneity and the usual informational restriction that the monopolist (in the Bryant and Wallace context, the government) knows population characteristics but cannot identify individual types helps justify the ruling out of lump sum taxation, a crucial assumption of the Bryant and Wallace model.[1]

In what follows, I impose heterogeneity in a way that makes the model formally equivalent to Spence's (1978, 1980) price discrimination model. I establish a correspondence between the two models, and then interpret Spence's results in the context of financing a given deficit with bonds of different denominations and returns. The results for the most part are straightforward generalizations of those obtained by Bryant and Wallace. However, in the generalized model, a Pareto optimal (i.e. "first best") allocation cannot be achieved.

I. The Spence Nonuniform Pricing Model

Spence (1978, 1980) studies the situation of a monopolist who wishes to choose a total outlay schedule $P(q)$ such that any agent who purchases quantity q must pay the amount P. As in most price discrimination models, the monopolist is able to prohibit resale of the good (arbitrage among agents). There are N_i type i agents in the economy, with $i=1,\ldots,n$. $R_i(q)$ denotes the reservation outlay of q units of the good to a type i agent (i.e. the maximum amount a type i agent is willing to pay for q units of the good) and $R_i'(q)$ denotes the derivative of $R_i(q)$ (i.e. the inverse demand for q units).

Spence assumes that the following demand and information conditions are satisfied:

A.1: Agent types can be ordered so that for all q,

$R_{i+1}(q) > R_i(q)$ and $R_{i+1}'(q) > R_i'(q)$.

A.2: Agents may purchase nothing, and if they do,

$P(0) = 0$ and $R_i(0) = 0$.

A.3: The monopolist knows $R_i(q)$ and N_i for all i, but does not know the identity of any particular agent.

Assumption A.1 states that a schedule representing $R_{i+1}(q)$ as a function of q lies above a schedule representing $R_i(q)$ and has a steeper slope. Assumption A.2 implies that the consumer surplus of an agent of type i from purchasing quantity $q \geqq 0$, $R_i(q)-P(q)$, is at least as great as the reservation price for purchasing nothing, which is zero. This provides an initial condition for the pricing function and is equivalent to assuming that there are no outside funds available to subsidize the provision of the good. Assumption A.3 is an informational restriction faced by the monopolist which rules out perfect price discrimination. It follows from this assumption that the monopolist must rely on self-selection by agents.

Spence considers an optimization problem of the following form: for $\lambda \epsilon [0,1)$ maximize by choice of q_i and $P(q_i)$, $i=1,\ldots,n$, $\lambda\Sigma_i N_i[R_i(q_i)-P(q_i)] + (1-\lambda)[\Sigma_i N_i P(q_i)-C(\Sigma_i N_i q_i)]$, subject to (the self-selection constraints) $R_i(q_i)-P(q_i) \geqq R_i(q_j)-P(q_j)$. The objective function is a weighted average of consumer surplus and the monopolist's profit, with $C(\Sigma_i N_i q_i)$ denoting the cost of providing the good and q_i denoting the amount of q purchased by a type i agent. The weight λ reflects the amount of redistribution from consumers to the monopolist. A special case of this problem, $\lambda=0$, is profit maximization.

This optimization problem may also be interpreted as the maximization of social welfare subject to a profitability constraint. When the profit constraint is binding, the problem yields the following results (see Spence (1980, p. 823-825)). First, $q_i \geq q_{i-1}$ for all i, must be satisfied for the equilibrium to be sustained by self-selection. This implies that the monopolist should offer weakly larger quantities to higher index groups. Second, the per unit price, $P(q)/q$, is weakly decreasing in q. Thus large quantity purchasers pay lower average prices than small quantity purchasers. Finally, Spence shows that for any uniform price different from marginal cost, there is a non-uniform outlay schedule that weakly benefits all consumers and the monopolist without side payments. Thus, there exists a quantity dependent pricing policy that is Pareto superior to any single price policy except for price equal to marginal cost.

II. The Generalized Bryant and Wallace Model

Bryant and Wallace (1984) consider a stationary, discrete time, overlapping generations economy with one type of two-period lived agent in each generation. The government is the sole supplier of savings instruments in the economy, enforces its monopoly position through a system of legal restrictions, and must finance a constant, exogenously determined real net-of-interest deficit. In this section, I replace the one type of agent in each generation in Bryant and Wallace with n types, but preserve the rest of their structure. Thus, let each generation consist of N_i two-period lived type i agents, $i=1,\ldots,n$, who are characterized by preferences and endowments which satisfy the following assumptions:

B.1: Each type i member of generation $t>0$ has preferences that are representable by a twice differentiable, increasing, and strictly concave utility function, $u[c_t^i(t), c_t^i(t+1)]$, where $c_t^i(t+j)$ is consumption of time t+j good by a type i agent of generation t. Moreover, u is such that second period consumption, $c_t(t+1)$, is a normal good.

B.2: Type i member of generation t is endowed with some time t good, $w_t^i(t)>0$, and some time t+1 good, $w_t^i(t+1)>0$. For all i and t, $w_t^i(t+1) = w_2$ (all agents have the same second period endowment), and $w_t^i(t) = w_1^i$ with $w_1^{i+1} > w_1^i$.

B.3: The government knows u, the endowment pattern, and the N_i, but cannot identify the type of any individual.

I now propose a correspondence between this generalized Bryant and Wallace model and the Spence model described in section I. The monopolist of the Spence model is the government (the no arbitrage assumption is enforced by legal restrictions on financial intermediation). At each date t, the one good being sold is time t+1 good. The demand for this good is the excess demand for second period consumption by generation t (the young at time t). Thus, I let $R_i(q)$ be the reservation outlay of a type i agent for q units of second period consumption in excess of w_2. Letting $C(\Sigma_i N_i q_i) = \Sigma_i N_i q_i$, the monopolist's profit in the Spence model is $\Sigma_i N_i[P(q_i)-q_i]$. This is the government's profit from selling a total of q_i units of excess second period consumption to type i agents (each period) at outlay $P(q_i)$, where $\Sigma_i N_i P(q_i)$ is the revenue obtained from sales to group i in the current period and $\Sigma_i N_i q_i$ is the amount due on liabilities issued to group i last period.

With this identification, the problem: maximize the stationary social welfare of generation t subject to raising a given real net-of-interest deficit becomes a special case of the objective in the Spence model. Assumption A.3, the informational restriction of the Spence model, is imposed directly by assumption B.3. What remains to be shown is that the preference and endowment assumptions, B.1 and B.2, imply $R_i(q)$ functions that satisfy assumptions A.1 and A.2 of the Spence model. This is established by the following proposition:

<u>Proposition</u>: Assumptions B.1 and B.2 imply reservation outlay functions for second period consumption in excess of endowment that satisfy assumptions A.1 and A.2.

Proof: Let $h_i(p)$ denote the excess demand for second period consumption by a type i agent, where p is the price of good t+1 in terms of good t (the inverse of the gross real return at time t). Assumption B.1 implies that $h_i(p)$ is single-valued (by strict concavity) and that it is decreasing in p where excess demand is positive (by the normal goods assumption). Thus, for all $q \geq 0$, the function $h_i(p)$ has an inverse. Let $R_i'(q)$ be this inverse. Since assumptions B.1 and B.2 imply that $h_{i+1}(p) > h_i(p)$, it follows that for all $q \geq 0$, $R_{i+1}'(q) > R_i'(q)$. Further, letting $R_i(q) = \int_0^q R_i'(x)dx$, it follows that $R_{i+1}(q) > R_i(q)$ for all $q \geq 0$, as the ordering of the inverse demand functions is preserved by integration. Thus assumption A.1 is satisfied. Assumption A.2 is also satisfied in the generalized model as agents have the option of consuming their endowment at zero cost and this corresponds to zero demand for excess second period consumption. //

The proposition establishes that the generalized Bryant and Wallace model satisfies the assumptions of the Spence model. Hence the results described in section I have direct analogues in the financing application. In particular, the government offers a different denomination of its bond, q_i, to each group at a different outlay $P(q_i)$.[2] The denomination is weakly increasing in the index ($q_i \geq q_{i-1}$), and the price is decreasing in q. Thus large savers purchase higher denominations than small savers at lower per unit prices (higher real rates of return). In addition, there exists some nonuniform outlay schedule for the government liabilities that Pareto dominates a single price policy, which is what financing by currency alone would imply.[3]

III. Concluding Remarks

This note shows that the price discrimination analysis of monetary policy in Bryant and Wallace (1984) can be extended to an economy with heterogeneous agents and an informational restriction. The generalization helps rationalize the infeasibility of lump sum taxes which is crucial to the Bryant and Wallace analysis.

The nature of agent heterogeneity considered in this model is very special: preferences and second period endowments are identical for all types and only first period endowments differ (assumptions B.1 and B.2). These assumptions are a simple way to satisfy assumption A.1 of the Spence model, and are consistent with the unidimensional distribution of consumer types assumed in most nonuniform pricing models. They correspond to assuming income variation across consumers, no taste variation, and normal goods - assumptions that are common in many price discrimination models. Although this specification of heterogeneity is not necessary to satisfy assumption A.1, it is clear that arbitrary patterns of heterogeneity are not permissible. This suggests that assumption A.1 is quite restrictive, even in this very simple general equilibrium setting. It remains to be determined what sort of price discrimination is desirable in more general settings.

[1] Hammond (1979, p. 274) observes that "it is unlikely to be optimal simply to impose lump-sum taxes, except in the special case where it is endowments which are identifiable and endowments are stochastically independent of preferences for consumption." In the model I propose, the government does not know the endowments of any particular agent and endowment heterogeneity generates heterogeneous demand for the consumption good.

[2] Notice that the optimal solutions in both the Bryant and Wallace analysis and the generalized model require bonds only; agents do not hold a perfectly divisible instrument (fiat currency).

[3] The Spence (1980, p. 825) analysis of Pareto superiority applies directly to the stationary population of two-period lived agents in the generalized model at all dates $t \geq 1$, but there is no one in his model that corresponds to the initial old (generation 0). However, it is easy to see that the result also holds when these people are taken into account. Let p^* denote the uniform price in a currency only solution. When G is positive, this price is <u>not</u> equal to marginal cost (i.e. $p^*>1$). Spence (1980, p. 825) shows that there is some Pareto superior nonuniform price solution p_i' $i=1,\ldots,n$ that satisfies $p^* \geq p_i' \geq 1$ for all i and $p^*>p_i'$ for at least one i. Since $p_i' \leq p^*$ for all i and profits under the p_i' prices are at least as high under p^*, it follows, given

$C(\Sigma_i N_i q_i) = \Sigma_i N_i q_i$, that $\Sigma_i q_i' > \Sigma_i q_i^*$, the latter being total sales under the uniform price p^*. In the context of the Bryant and Wallace model, sales are saving, so the conclusion follows that there exists a discriminatory solution that makes all members of all generations other than the inital old better off and has higher saving than the currency only solution. Since aggregate saving is higher in the discriminatory solution than in the uniform solution, the resource constraint for the initial date implies that more goods are available to the initial old under the nonuniform schedule.

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