

USE OF (TIME-DOMAIN) VECTOR AUTOREGRESSIONS  
TO TEST UNCOVERED INTEREST PARITY

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Abstract

In this paper, a vector autoregression model (VAR) is proposed in order to test uncovered interest parity (UIP) in the foreign exchange market. Consider a VAR system of the spot exchange rate (yen/dollar), the domestic (US) interest rate and the foreign (Japanese) interest rate, describing the interdependence of the domestic and international financial markets. Uncovered interest parity is stated as a null hypothesis that the current difference between the two interest rates is equal to the difference between the expected future (log of) exchange rate and the (log of) current spot exchange rate. Note that the VAR system will yield the expected future spot exchange rate as a k-step ahead unconditional prediction. Hence, the null hypothesis is stated as nonlinear cross-equational restrictions for the three-equation VAR system. Then UIP is tested by the Wald test between the unrestricted and restricted systems. A test of UIP, with a maintained hypothesis of covered interest parity, becomes a hypothesis test of efficiency without risk premium, that is, the forward exchange rate is the unbiased predictor of the future spot exchange rate, and information is efficiently used in its prediction. Our results are compared to the efficiency test with a single equation using the Hansen-Hodrick procedure for the same data set.

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## I. Introduction

In this paper, a vector autoregressions (VAR) model is proposed as a framework conducive to a new way of testing uncovered interest parity in the foreign exchange market. Uncovered interest parity is a hypothesis that the current difference between the interest rates of two countries is equal to the difference between the expected future (log-) exchange rate and the current spot (log-) exchange rate. That is, a gain from investing in an asset denominated in a foreign currency is equal to an expected depreciation loss. A difficulty in obtaining the expected future spot exchange rate is an apparent stumbling block in testing this hypothesis. This paper proposes to use a vector autoregressions model, which includes as endogenous variables at least the interest rates of the both countries and the exchange rate. Considering the interdependence among domestic and international financial variables, the use of a VAR model to describe the dynamic movement of the exchange rate is most appropriate.<sup>1</sup> Note that such a VAR system yields the expected future spot exchange rate as a k-step ahead unconditional prediction. Hence, the null hypothesis of uncovered interest parity can be expressed as nonlinear cross-equational restrictions for the VAR system, if the observational frequency is finer than the length of forward contracts. Uncovered interest parity is tested using the Wald test which is asymptotically the same with the likelihood ratio test of unrestricted and restricted systems.

A test of uncovered interest parity has been a popular topic of investigation, partly because uncovered interest parity (UIP) becomes a hypothesis of market efficiency without risk premium in the foreign exchange market, when covered interest parity (CIP) is holding true. In the absence of capital controls, covered interest arbitrage must equalize net yields of assets denominated in different currencies taking into account the forward premium or discount. The combination of UIP with CIP implies that the forward rate is

the unbiased predictor of the future exchange rate, one of the conditions for the market efficiency.

The efficiency hypothesis, or uncovered interest parity (given covered interest parity), has been studied extensively for many currency and asset combinations. Most existing studies on a test of the efficiency hypothesis use a single equation involving the forward and spot exchange rates. See Cumby and Obstfeld (1982), Frenkel (1981), Gregory and McCurdy (1984), and Hansen and Hodrick (1980), to name a few. The test consists of the unbiasedness of the forward exchange rate as a predictor of the future spot exchange rate and the orthogonality of deviations with other information such as the past forecast errors. It is well-known that when the observational frequency is finer than the length of forward contract, the error term has serial correlation which invalidate the usual t-test. It is notable that Hansen and Hodrick established a method to test the unbiasedness in such a case in the single-equation hypothesis test.

The test proposed in this paper is quite different from existing tests. In this paper, a vector autoregressions model with the spot exchange rate, the domestic interest rate and foreign interest rate is considered. The (rationally) expected future spot exchange rate is calculated as the k-step ahead prediction of the vector autoregressions model, while the forward exchange rate is calculated from the current interest rates and the exchange rate. Thus, equivalence of the forward exchange rate and the future spot exchange rate is stated as cross-equational constraints in the vector autoregressions model. The proposed method has several advantages over existing methods with a single equation.

First, the cross-equational constraints as a testing hypothesis does not suffer from a problem of serially-correlated disturbances even when the obser-

vational frequency is finer than the contract frequency, since the vector autoregressions model is constructed such that it has white noise as disturbances. Second, an information set on which the expectation is conditioned is fully spelled out by the choice of variables for the VAR system. Thus the cross-equational constraint represents not only the unbiasedness of the forward rate but also the efficient use of information in the linear system. Third, the proposed test in this paper is a test of uncovered interest parity which is theoretically separate from a test of unbiasedness of the forward rate, unlike the conventional way of testing uncovered interest parity (c.f. Cumby and Obstfeld (1982)) by the test of unbiasedness of the forward rate. In other words, we can test UIP by the method in this paper even in the absence of CIP. The proposed method could test UIP even without observing forward rate. If financial investments are freely permitted but if formation of the forward market is somehow prohibited or distorted, the traditional method could not test UIP, while the proposed method could.<sup>2</sup>

Remarks on related works are in order. There are a few vector autoregressions studies in the foreign exchange market. Meese and Rogoff (1983) studied predictive powers of various structural models as well as vector regressions models, but did not test the efficiency hypothesis. Baillie, Lippens and McMahon (1983) tested efficiency using a bivariate autoregression model including the forward and spot exchange rates. However, they had to take first differences to obtain covariance stationarity. In this paper, using two interest rates and the spot exchange rate, covariance stationarity is obtained without taking the first differences. This makes the test in this paper stronger than the one by Baillie, et. al (1983).

The proposed test of the efficiency hypothesis as cross-equational restrictions in a VAR model is reminiscent of the test of (rational) expectations hypothesis, proposed by Sargent (1979), in the term structure of

interest rates.<sup>3</sup> The similarity is not surprising considering the common feature in that the expectation is calculated as a k-step ahead prediction in the VAR system. However, the hypothesis of uncovered interest parity is formed quite differently from the expectations hypothesis of the term structure.

Section 2 describes the proposed test. In section 3, we check whether covered interest parity is holding for the data we use, so that whether the test of uncovered interest parity can be regarded as the test of efficiency hypothesis. It will be shown that covered interest parity is holding between in the Euro-yen and Euro-dollar deposit rates for the period from 1975 to 1983, and also between the Japanese domestic interest rate and Euro-dollar deposit rate from 1979 on. Therefore, a test of uncovered interest parity is equivalent to a test of efficiency for those period. The proposed test of uncovered interest parity is carried out in section 4. Monthly data of three-month interest rates of various assets for various subperiods between 1972 to 1984. For the interest rates, both onshore and offshore rates are used. The result is mixed. With a full sample involving the onshore interest rate, the hypothesis is rejected. However, in the Euro market, the hypothesis cannot be rejected. In section 5 results according to the proposed test is compared to results obtained from applying Hansen-Hodrick procedure involving the single-equation of the spot and forward exchange rates. There is no evidence that one test favors the acceptance of the hypothesis.

## II. A Test of Uncovered Interest Parity

It is widely accepted that a VAR model is a powerful method to form forecasts without imposing "incredible" identification restrictions. Although we know that financial variables, in particular the exchange rates and interest rates, are interdependent, the exact "structural" relationship is not

a priori determined. Moreover, there is no evidence that any of the variables is strictly exogenous to the others. In such a framework, the use of a VAR model to obtain the forecast of a variable of a concern is a natural solution.

Consider a system of the logarithm of the spot exchange rate,  $s_t$ , the domestic interest rate,  $RUS_t$ , and the foreign interest rate,  $RJA_t$ , which is estimated to study the interdependence of the domestic and international financial markets:

$$x_t = d + \sum_{j=1}^m A_j x_{t-j} + e_t \quad (2.1)$$

where  $x_t = (s_t, RUS_t, RJA_t)'$ ,  $d = (d_s, d_u, d_j)'$  being a deterministic factor, and  $e_t$  being white noise. Ito (1983b) studied the causality among the variables and dynamic responses to a shock in such a system. In this paper, the same VAR system is used as a framework for testing a hypothesis. In particular, uncovered interest parity will be derived as nonlinear cross-equational constraints in the VAR system.

Now let us take the case of monthly observations of three-month forward exchange and interest rates, and illustrate how to derive the cross-equational constraints.

$$\begin{aligned} E_t x_{t+3} &= d + A_1 E_t x_{t+2} + A_2 E_t x_{t+1} + \sum_{j=1}^{m-2} A_j x_{t-j+1} \\ &= d + A_1 (d + A_1 E_t x_{t+1} + \sum_{w=1}^{m-1} A_{w+1} x_{t-w+1}) + A_2 E_t x_{t+1} + \sum_{j=1}^{m-2} A_{j+2} x_{t-j+1} \\ &= (I + A_1) d + (A_1 A_1 + A_2) E_t x_{t+1} + A_1 \sum_{w=1}^{m-1} A_{w+1} x_{t-w+1} + \sum_{j=1}^{m-2} A_{j+2} x_{t-j+1} \\ &= (I + A_1) d + (A_1 A_1 + A_2) (d + \sum_{z=1}^m A_z x_{t-z+1}) + A_1 \sum_{w=1}^{m-1} A_{w+1} x_{t-w+1} + \sum_{j=1}^{m-2} A_{j+2} x_{t-j+1} \end{aligned}$$

$$= (1+A_1+A_1A_1+A_2)d + \sum_{k=1}^m ((A_1A_1+A_2)A_k + A_1A_{k+1} + A_{k+2})x_{t+1-k} \quad (2.2)$$

where  $A_u = [0]$ , for  $u > m$ , and  $I$  is an identity matrix.

Uncovered interest parity is stated as,

$$E_t s_{t+3} - s_t = (1/400)(RJA_t - RUS_t). \quad (2.3)$$

where the exchange rates are measured in yen per dollar, and the interest rates are expressed in annual yield (%). Therefore, the hypothesis,

$$H_0: E_t s_{t+3} = s_t + (1/400)(RJA_t - RUS_t) \quad (2.4)$$

can be expressed in terms of cross-equational restrictions implied by equating the coefficients of the first row of the right-hand-side coefficient matrix (2.2) to the one in (2.3).

Let us define a coefficient matrix as follows:

$$A_k = \begin{matrix} : a_{sk} & b_{sk} & c_{sk} : \\ : & & : \\ : a_{uk} & b_{uk} & c_{uk} : \\ : & & : \\ : a_{jk} & b_{jk} & c_{jk} : \end{matrix} \quad (2.5)$$

Let us now assume that the deterministic component consists of only constants and that the lag length of the system is 7 ( $m=7$ ). Then we can write explicitly the three-step ahead prediction for the VAR system (2.1) with 7 lags.

$$E_t s_{t+3} = g_d + \sum_{k=1}^7 g_{ak} s_{t-k+1} + \sum_{k=1}^7 g_{bk} RUS_{t-k+1} + \sum_{k=1}^7 g_{ck} RJA_{t-k+1}$$

$$\text{where } g_d = (1+a_{s1})d_s + b_{s1}d_u + c_{s1}d_j + f_a d_s + f_b d_u + f_c d_j,$$

$$g_{hk} = f_a h_{sk} + f_b h_{uk} + f_c h_{jk} + a_{s1} h_{s(k+1)} + b_{s1} h_{u(k+1)} + c_{s1} h_{j(k+1)} + h_{s(k+2)},$$



$$f_h = a_{s1}h_{s1} + b_{s1}h_{u1} + c_{s1}h_{j1} + h_{s2},$$

$$h = a, b, c.$$

Note that  $h_{sk}, h_{uk}, h_{jk} = 0$ , for any  $k > 7$ .

Now the testing hypothesis  $H_0$  becomes the following 22 restrictions.

$$\begin{aligned} H_0: \quad 0 &= g_d - d_s + (d_u - d_j)/400, \\ 0 &= g_{a1} - 1, \\ 0 &= g_{b1} + (1/400), \\ 0 &= g_{c1} - (1/400), \\ 0 &= g_{hk}, \quad h=a,b,c; k=2,3,\dots,7. \end{aligned}$$

Let us now recall that a joint hypothesis of covered and uncovered interest parity is equivalent to the unbiasedness of the forward rate. Covered interest parity states that the difference between the forward and current spot exchange rates is equal to the difference in the interest rates:

$$f_{t,3} - s_t = (1/400)(RJA_t - RUS_t). \quad (2.6)$$

Combining uncovered and covered interest parity, (2.3) and (2.6), the efficiency hypothesis is implied:  $Es_t = f_{t,3}$ . Note that the expectation should be conditional on all the relevant information available in system (2.1) in period  $t$ . In particular, the past interest rates and the spot exchange rate are included in the information set in our system.

Uncovered interest parity will be tested in the yen-dollar relationship for various subperiods between 1972 and 1984. A vector autoregressions model with the spot exchange rate, a yen-denominated interest rate, and a dollar-denominated interest rate has to be constructed. The end-of-month spot exchange rate is defined in terms of yen. For the yen-denominated interest rate, both an onshore rate (three month Gensaki, i.e., repurchase agreement, and an offshore rate (three month Euro-yen deposit rate) are examined. The

choice of the Gensaki rate is a natural one, because the Gensaki rate is one of the few interest rates in Japan which have always been free from various domestic regulations. For the dollar-denominated interest rate, the Euro-dollar deposit rate is chosen. Although this is not the U.S. on-shore rate, it is shown by Kreicher (1982) that covered arbitrage is holding true between the Euro-dollar deposits and the U.S. domestic CDs, taking into account the difference in reserve requirements of the two different CDs. Questions concerning the Granger-causality between the interest rate and the exchange rate, forecast errors of variance decomposition, and the impulse response functions in the three-variable VAR system is found in Ito (1983b).

### III. Covered Interest Parity

As a preliminary investigation, we are interested in whether covered interest parity is holding. If so, our test can be interpreted as the efficiency test. Ito (1983a) and Otani and Tiwari (1981) investigated whether covered interest parity represented by (2.6) was holding for a yen-denominated asset and the Euro-dollar deposit taking into account transactions costs and the bid-ask spread. The following is a summary of Ito's (1983a) findings about covered interest parity.

Let us define the degree of deviations from covered interest parity by

$$\text{Deviation} = \left[ \left(1 + \frac{\text{RUS}}{400}\right) \frac{\text{F}}{\text{S}} - \left(1 + \frac{\text{RJA}}{400}\right) \right] \times 400 \quad (3.1)$$

where F and S is the forward and spot exchange rates: the value of dollar in terms of yen. The Figure 1 shows that the deviation has been decreasing throughout the sample period. In 1973, an investment shift from an onshore yen-denominated asset to Euro-dollar deposits with a covered forward

transaction would have earned a difference up to nearly a 30 percentage point (annualized yields of three-month) in yield. This violation to covered interest parity is due to strict Japanese capital controls then in place. As capital controls being gradually lifted throughout 1970's, the deviation from the parity has decreased. After 1974, there was no deviation more than 5 percentage point in either direction. Most important changes from our viewpoint took place in 1979 and 1980. In May 1979, Gensaki assets became available for non-residents. Moreover, in December 1980, the new Exchange Trade and Foreign Exchange Control law deregulated most capital flows in and out of Japan. The largest deviation since 1981 has not exceeded a one percentage point, within the neutrality band considering the bid-ask spread only.

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INSERT FIGURE 1 ABOUT HERE  
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Covered interest parity between Euro-yen and Euro-dollar was also examined in Ito (1983a). Covered interest parity between Euro-yen and Euro-dollar has been holding since 1975, when the data of Euro-yen became available.

#### IV. Empirical Results

The test is conducted for two combinations of interest rates. First, the three-month Euro-dollar and Euro-yen deposit rates are used as the dollar-denominated and yen-denominated interest rates. Since there is no capital controls, the test of UIP is readily interpreted as the efficiency test. Second, the three-month Gensaki (repurchase agreement) rate is used for the three-month interest rate for an onshore yen-denominated asset. Then the test is conducted between the Gensaki and the Euro-dollar rate. As commented before, the Euro-dollar rate is closely related with the on-shore CD rate. Results of the Wald test are summarized in Table 1.

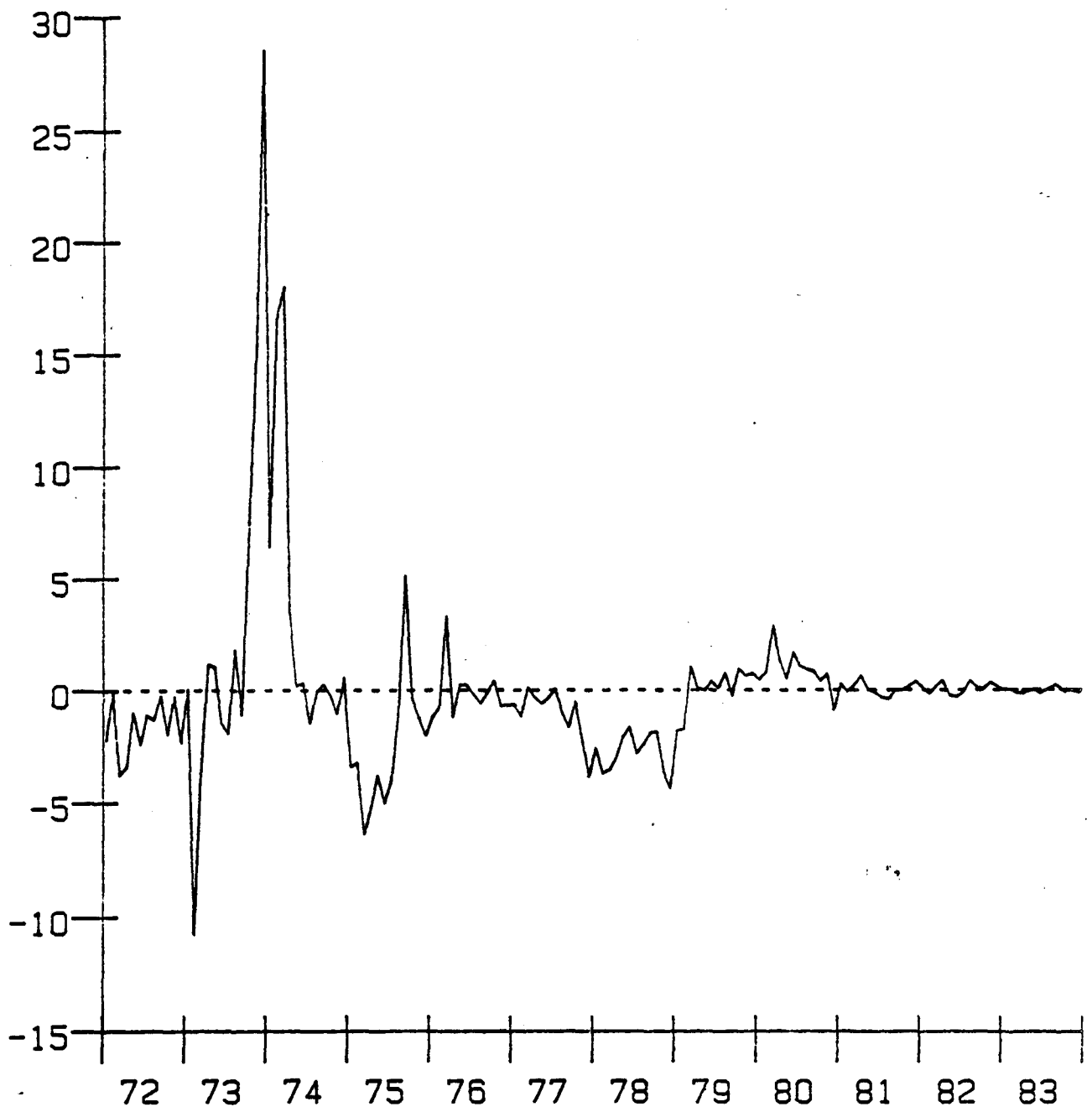


FIGURE 1

Deviations from Covered Interest Parity measured by (3.1)

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INSERT TABLE 1 ABOUT HERE  
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The null hypothesis of uncovered interest parity (efficient market hypothesis with no risk premium) is accepted in all cases except one with a full sample for the onshore-offshore combination. Since results about the hypothesis in the literature (cited in Introduction) are mixed, we did not have any strong prior conjectures. What obtained in this paper should lend support evidence to the efficiency hypothesis with risk premium, or combined covered and uncovered interest parity.

It is natural that the hypothesis with Euro-market rates from 1975 to 1983 is accepted with a lower  $\chi^2$  value than the one with the onshore rates. The Euro-market is believed to require lower transactions costs and assets there are comparable, so that covered interest parity, a part of maintained hypotheses, has been considered to hold very tightly. Since the Japanese capital controls were relatively effective until December 1980, the test is conducted in three separate time periods for the onshore case. The hypothesis with the onshore rates after 1975 was accepted at the level comparable to the Euro-market case.

It is not puzzling that the hypothesis was rejected when the sample period is extended to include periods between 1972 and 1975. Recall that there were dramatic deviations from covered interest parity. The order of deviation in 1973 was in the 30 percentage points in the annual yield, as opposed to the deviation at maximum 6 percentage points between 1975 and 1980, and at maximum a less than one percentage point deviation after December 1980. From 1973 to 1974, Yen was depreciating rapidly, while capital controls preventing foreigners from purchasing Japanese securities and Japanese from purchasing the dollar-denominated assets. With strict controls in financial flows, neither CIP nor UIP did not have a chance to be satisfied during this period.

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TABLE 1

Interest Rates	Sample Period	$\chi^2$
Euroyen, Euro\$	75:8-84:4	12.36
Gensaki, Euro\$	75:8-84:4	16.69
Gensaki, Euro\$	72:8-84:4	48.74*
Gensaki, Euro\$	72:8-80:12	24.93

The degree of freedom is 22 for every case. An asterisk \* denotes a rejection of a hypothesis at the one percent level.

**DATA SOURCE:**

All the data are monthly series observed at the end of month. Time series of the Euro-dollar and Euro-yen three-month deposit rates are taken from Morgan Guaranty Trust, World Financial Markets. The series of three-month Gensaki (Repurchase Agreement) rate in Japan is taken from the Bank of Japan for 1972:1 to 1974:12, and from the Morgan Guaranty Trust for 1975:1 to the present. (Note that the short-term interest rate of Japan in World Financial Markets before 1979:1 was not the Gensaki rate, but a regulated deposit rate. Therefore, the data in this paper are obtained from the Morgan Guaranty Data bank which has backdated the series.) The dollar-yen spot exchange rate is taken by picking the last business day of the month in the DRI daily data series after 1977, and the IMF monthly data from 1972 to 1976.

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#### V. Comparison with the Results obtained by the Conventional Method

If the market is efficient and traders are risk neutral, then the forward exchange rate is equal to an expected future spot exchange rate. This hypothesis has been examined in a series of studies. The test is usually reduced to showing that the deviation of the k-step forward rate at period t,  $F_{t,k}$ , (or its logarithm) from the realized spot rate,  $S_{t+k}$ , (or its logarithm) has mean zero and that the deviation is uncorrelated with information available at the end of period t. Earlier works (for example, Cornell (1977), Frenkel (1977, 1978, 1979, 1981), Geweke and Feige (1979), Levich (1978), and Longworth (1981)) used data with a nonoverlapping frequency (monthly data for one-month forward rates). Frenkel (1981) tested the hypothesis using the following equation.

$$s_t = a + b f_{t,1} + v_{t+1}$$

where  $s_t = \log(S_t)$  and  $f_{t,k} = \log(F_{t,k})$ . The market efficiency with no risk premium is expressed as a test of  $(a, b) = (0, 1)$  and no serial correlation in residuals. However, the test may suffer from the unit root problem, if the exchange rate follows a random walk as suggested in Mussa (1979). Moreover, if the observational frequency is finer than the forward contract length, serial correlations in the residuals would make asymptotic properties of the OLS estimates inappropriate. In order to cope with the unit root problem, Cumby and Obstfeld (1982) proposed to test  $(a, b) = (0, 1)$  in the equation

$$s_{t+k} - s_t = a + b(f_{t,k} - s_t) + v_{t+k}$$

Hansen and Hodrick (1980) proposed to modify the asymptotic covariance matrix to obtain a test of the hypothesis which is applicable when the observational frequency is finer than the forward contract length. Using weekly data of

the three-month forward rate, Hansen and Hodrick estimated, among others,

$$s_{t+13} - f_t = a + b_1(s_t - f_{t-13,13}) + b_2(s_{t-1} - f_{t-14,13}) + e_t. \quad (4.1)$$

The market efficiency hypothesis is  $(a, b_1, b_2) = (0, 0, 0)$ .

Most existing methods, as summarized above, test the market hypothesis by showing that deviations of the forward rate from the actual future spot rate are not correlated with information at period  $t$ . That is,  $s_{t+k} - f_{t,k}$  is uncorrelated with any variables in the information set at period  $t$ ,  $\Omega_t$ . This is implied from  $f_{t,k} = E[s_{t+k} | \Omega_t]$ . However, if we can form the  $k$ -step ahead forecast of the spot rate from the model, we can directly test the hypothesis that the forward rate is equal to the conditional forecast of the future spot rate. This is exactly what achieved in the proposed test in this paper. Note that the information set which the VAR forecast is based on is spelled out by the choices of variables and the lag length in a VAR model.

Being presented the results in Table 1, those who believe in risk premium wonder whether the proposed test is not strong enough to reject the hypothesis of uncovered interest parity.<sup>4</sup> Since a comparison of testing the same hypothesis with the same data set by different econometric methods might reveal that one method is more likely to reject the hypothesis than the other, we tested the efficiency hypothesis by the Hansen-Hodrick procedure, modifying (4.1) to accommodate monthly instead of weekly observations:<sup>5</sup>

$$s_{t+3} - f_t = a + b_1(s_t - f_{t-3,3}) + b_2(s_{t-1} - f_{t-4,3}) + e_t. \quad (4.2)$$

Table 2 summarizes the test statistics. The efficiency hypothesis is not rejected by the Hansen-Hodrick test, either. Findings by the Hansen-Hodrick procedures are in harmony with findings by our test presented in Table 1.

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 INSERT TABLE 2 ABOUT HERE  
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Table 2

Dependent Variable:  $S(t+3) - F(t)$

$$H_0: (a, b_1, b_2) = (0, 0, 0)$$

Sample Period	a	$b_1$	$b_2$	$\bar{R}^2$	D.W.	$\chi^2$	Significance Level
75:8 - 83:10	-.1022 (1.4736)	.2148 (.1514)	.4171 (.1514)	.0418	.626	4.98	.1733
72:5 - 84:1	-.5012 (1.2377)	.1611 (.1230)	-.0712 (.1230)	.0001	.634	5.51	.1382
75:8 - 80:12	-1.9110 (1.6535)	.1910 (.1846)	.2052 (.1842)	.111	.624	7.03	.0708

## VI. Concluding Remarks

The objective of this paper was to propose a new way of testing uncovered interest parity in a vector autoregressions model. The hypothesis of uncovered interest parity is interesting partly because it, combined with covered interest parity, is equivalent to the efficiency of the market, which was extensively studied with a single-equation technique.

The proposed method is used to test uncovered interest parity between yen-denominated assets and dollar-denominated assets between 1975 and 1983. We have accepted the hypothesis both a combination of offshore assets and a combination of onshore-offshore assets. These results are in contrast to some of recent works in the literature on the efficiency market hypothesis.<sup>6</sup> Therefore, it is desirable to carry out testing procedures proposed by others, in particular the Hansen-Hodrick procedure because of the observation interval being finer than the forward-contract interval, for the same data and to compare the results. This was done in the preceding section. Since the Hansen-Hodrick procedure using our data accepted the efficiency hypothesis, there was no evidence that one method is more restrictive than the other regarding the efficiency hypothesis.

An efficient market hypothesis tested as cross-equational restrictions in a VAR model has already been used in the term structure literature. (See, for example, Sargent (1979) and Melino (1980).) Although the basic idea in this paper parallels studies in the term structure literature, the derivation of the cross-equational constraint of a null hypothesis in this paper is quite different from that of the term structure model. In particular, the present technique uses one important side condition (covered interest parity) to derive the efficiency hypothesis. However, the similarity in the frameworks of this paper and of testing expectations hypotheses in the term structure literature suggests that recent developments in the term structure literature

are also applicable to our technique. Computational burden may be reduced by using a frequency domain method proposed by Hansen and Sargent (1981). The frequency domain technique will be especially attractive when the prediction steps become large (such as 13-step ahead prediction of the weekly series against the three-month deposit rates). It is left for future research to translate the time-domain VAR method described in this paper into the frequency domain technique a la Hansen-Sargent method.<sup>7</sup>

## FOOTNOTES

1. Applications of VAR systems to macroeconomic models have been popular since Sims' works (1980, 1982). Recently VAR models have been applied to investigate interdependence and co-movements of international and domestic financial variables (Branson (1983), Baillie et. al. (1983) and Ito (1983b)). Some applications of VAR models, in particular in the direction of forming policy recommendations, are somewhat controversial. (See, for example, a controversy between Sims (1982) vs. McCallum (1983) and Sargent (1983)). However, it is a widely-accepted view among its users and critics that a VAR is a useful tool in the "reduced-form" description, forecasting, and predictions.

2. The forward foreign exchange market in Japan is still regulated in the sense that one has to have "real demands" for the forward exchange in order to participate it. For example, a Japanese resident holding export contracts, or maturing securities or deposits denominated in dollar is permitted to purchase forward yen, so that the exchange rate risk is hedged. However, the resident would not be allowed to "speculate" in the forward market taking a short position.

3. See Melino (1980) and Hansen and Sargent (1981) for further development in the term structure literature. See also Hakkio (1981) for risk premia in the forward market.

4. It has been suggested by Angelo Melino that the proposed test might tend to accept the hypothesis if the true lag length is much shorter than the assumed one. However, if we assumed the lag length shorter than the true lag length, then disturbances would lose whiteness and a framework would fall apart.

5. I have used the RATS program, Doan and Litterman (1981: section 12.12), for calculating relevant covariance matrices based on the General Method of Moments, proposed by Hansen (1979)(1980). It was not necessary to "damp" the covariances.

6. See Hodrick (1981), Hodrick and Srivastava (1983) for models incorporating risk premium.

7. A joint project with Danny Quah in this direction is in process.

## REFERENCES

- Baillie, Richard, Robert E. Lippens, and Patrick C. McMahon, "Testing Rational Expectations and Efficiency in the Foreign Exchange Market," Econometrica, vol. 51, May 1983: 553-563.
- Branson, William H., "A Model of Exchange-Rate Determination with Policy Reaction: Evidence from Monthly Data," NBER w.p. #1178, August 1983.
- Cornell, Bradford, "Spot Rates, Forward Rates and Exchange Market Efficiency," Journal of Financial Economics, vol. 5, August 1977: 55-65.
- Cumby, Robert E. and Maurice Obstfeld, "International Interest-Rate and Price-Level Linkages Under Flexible Exchange Rates: A Review of Recent Evidence," NBER working paper, no. 921, June 1982. To appear in J.F.O. Bilson and R. C. Marston, eds., Exchange Rates: Theory and Practice, University of Chicago Press.
- Doan, Thomas A. and Robert B. Litterman, "User's Manual: RATS," version 4.1, October 1981.
- Frenkel, Jacob A., "The Forward Exchange Rate, Expectations, and the Demand for Money: The German Hyperinflation," American Economic Review, vol. 67, September 1977: 653-670.
- Frenkel, Jacob A., "Purchasing Power Parity: Doctorinal Perspective and Evidence from the 1920's," Journal of International Economics, vol. 8, May 1978: 169-191.
- Frenkel, Jacob A., "Further Evidence on Expectations and the Demand for Money during the German Hyperinflation," Journal of Monetary Economics, vol. 5, January 1979: 81-97.
- Frenkel, Jacob A., "Flexible Exchange Rates, Prices, and the Role of "News": Lessons from the 1970s," Journal of Political Economy, vol. 89, no. 4, August 1981: 665-705.

- Geweke, J. F. and E. L. Feige, "Some Joint Tests of the Efficiency of Markets for Forward Foreign Exchange," Review of Economics and Statistics vol.61, August 1979: 334-341.
- Gregory, Allan W., and Thomas H. McCurdy, "Testing the Unbiasedness Hypothesis in the Forward Foreign Exchange Market: A Specification Analysis," University of Western Ontario, Center for the Study of International Economic Relations, working paper no. 8408C, February 1984.
- Hakkio, Craig S., "The Term Structure of the Forward Premium," Journal of Monetary Economics, vol. 8, 1981: pp.41-58.
- Hansen, Lars Peter, "The Asymptotic Distribution of Least Squares Estimators with Endogenous Regressors and Dependent Residuals," GSIA, Carnegie-Mellon University, Working Paper #38-78-79, March 1979.
- Hansen, Lars Peter, "Large Sample Properties of Generalized Method of Moments Estimators," GSIA, Carnegie-Mellon University, W.P. #56-79-80, March 1980
- Hansen, Lars Peter and Robert J. Hodrick, "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis," Journal of Political Economy, vol. 88, 1980: pp.829-853.
- Hansen, Lars Peter and Robert J. Hodrick, "Risk Averse Speculation in the Forward Foreign Exchange Market: An Econometric Analysis of Linear Models," in Exchange Rates and International Macroeconomics, University of Chicago Press, 1983.
- Hansen, Lars Peter and Thomas J. Sargent, "Exact Linear Rational Expectations Models: Specification and Estimation," Federal Reserve Bank of Minneapolis, Research Department, Staff Report 71, September 1981.
- Hodrick, Robert J. "International Asset Pricing With Time-Varying Risk Premia," Journal of International Economics, vol.11, 1981: pp.573-587.
- Hodrick, Robert J. and Sanjay Srivastava, "An Investigation of Risk and Return in Forward Foreign Exchange," NBER working paper, no. 1180, August, 1983.

- Ito, Takatoshi, "Capital Controls and Covered Interest Parity," NBER working paper no. 1187, August 1983a.
- Ito, Takatoshi, "On the Interdependence of the Exchange Rates and Interest Rates," University of Minnesota, Center for Economic Research, Discussion Paper #83-192, November 1983b.
- Kreicher, Lawrence L, "Eurodollar Arbitrage," Quarterly Review, vol. 7, Federal Reserve Bank of New York, Summer 1982.
- Levich, R. M., "Further Results on the Efficiency of Markets for Foreign Exchange," in Managed Exchange-rate Flexibility: The Recent Experience. Federal Reserve Bank of Boston Conference series no. 20, Boston, 1978.
- Longworth, David, "Testing the Efficiency of the Canadian-U.S. Exchange Market under the Assumption of no Risk Premium," The Journal of Finance, vol.36, no. 1, March 1981.
- McCallum, Bennett T., "A Reconsideration of Sims' Evidence Concerning Monetarism," Economics Letters, vol. 13, 1983: pp. 167-171.
- Meese, Richard and Kenneth Rogoff, "The Out-of-Sample Failure of Empirical Exchange Rate Models: Sampling Error or Misspecification?" in J. A. Frenkel (ed.) Exchange Rates and International Macroeconomics, University of Chicago Press, 1983.
- Melino, Angelo, "Estimation of a Rational Expectations Model of the Term Structure," Essay I in unpublished Ph.D. dissertation, Harvard University, 1980.
- Mussa, Michael, "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Exchange Market," in Policies for Employment, Prices, and Exchange Rates, Carnegie-Rochester Conference Series on Public Policy, vol. 11, supplementary series to the Journal of Monetary Economics, 1979.



- Otani, Ichiro and Siddhart Tiwari, "Capital Controls and Interest Rate Priority: The Japanese Experience 1978-81," Staff Papers, International Monetary Fund, vol. 28, no. 4, December 1981: 793-815.
- Sargent, T. J., "A Note on Maximum Likelihood Estimation of the Rational Expectations of the Term Structure," Journal of Monetary Economics, vol. 5, 1979: pp. 133-143.
- Sargent, T. J., "Autoregressions, Expectations, and Advice," presented at the AEA meeting, San Francisco, December 1983.
- Sims, Christopher A., "Macroeconomics and Reality," Econometrica, January 1980, vol. 48: pp. 1-48.
- Sims, Christopher A., "Policy Analysis with Econometric Models," Brookings Papers of Economic Activity, 1:1982: pp. 57-99.