

A NOTE ON LONG-TERM CONTRACTS

by

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## ABSTRACT

The purpose of this note is to pinpoint reasons why we should consider long-term contracts and to classify contract configurations depending on assumptions on ex post mobility of workers and on information of observability of the stochastic productivity. Both Holmstrom and Townsend construct models where a single-period contract is not viable, but a long-term contract is. However, reasons responsible for this characteristic in the two models are quite different. Holmstrom considers the case where the ex post mobility of workers destroys complete risk-sharing, while Townsend considers the case that private information limits the extent of risk-sharing.

This note proposes a general framework which nests the two models as special cases. First, Townsend's model is interpreted in the framework of implicit labor contracts. In his model, the productivity is assumed to be the private information of the worker, contrary to the popular assumption in the labor contract literature, adding an insight about a contract configuration for a different environment. Second, I propose to consider the case of declining mobility, i.e., workers have the perfect ex post mobility in the first period but no ex post mobility in the second period. Townsend's solution to the pure borrowing-lending scheme is shown to be an optimal solution in the case of declining mobility. Last, Holmstrom, Townsend, and models with the declining mobility are classified and summarized in a table.

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A NOTE ON LONG-TERM CONTRACTS

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1. INTRODUCTION AND THE MODEL

The purpose of this note is to examine reasons why we observe long-term contracts and to classify configurations of long-term contracts depending on assumptions on ex post mobility and informational observability. Townsend (1982) and Holmstrom (1983) have presented two distinct explanations of why long-term contracts should emerge in an environment where a single-period contract does not exist. Different kinds of incentive-compatibility constraints play an important role in the two models. Holmstrom (1983) considers an implicit labor contract model in the tradition of Azariadis (1975). In Holmstrom's model, a long-term contract is needed to accommodate the possibility of ex post breaching by the worker after the state of nature is revealed. Because of the prohibition of slavery, a contract is enforceable only for the firm. Townsend's long-term contract is a result of incentive compatibility constraints in an environment where states of nature are private information of the worker (risk-averse agent). However, Townsend's did not consider problems associated with the ex post possibility of recontracting on the part of the worker (risk averse agents).

It is of great interest to consider a model which nests both Holmstrom and Townsend, and to investigate how different environments produce different optimal contract configurations.<sup>1</sup> In addition to environments originally considered by Holmstrom and Townsend, I propose to consider the case where worker's mobility declines overtime. Because of developing firm-specific skills and obtaining a home, a move to another firm in a different community

when a worker is older (in the second period) is considerably more costly than when young (in the first period). Optimal contracts are investigated for the case of declining mobility in both Holmstrom-type and Townsend-type environments. It will be shown that the case of declining mobility in the Townsend-type environment is solved in his paper as an example of sub-optimal contract.

Suppose a two-period model with many identical risk-neutral firms and many identical risk-averse workers. The worker inelastically supplies one unit of labor. The stochastic production function at period  $t$  is,

$$Y = y(s_t)L$$

where  $s_t = g$  with probability  $p$  and  $s_t = b$  with probability  $1-p$  independently in each period,  $t = 1, 2$ . Assume, without loss of generality,  $y(g) > y(b)$ . The worker's intertemporal utility function is assumed to be  $Eu(w(s_1)) + Eu(w(s_2))$ , where  $w(s_t)$  is the wage at period  $t$  and  $u' > 0$ ,  $u'' < 0$ .

## 2. Complete Insurance

If the state of nature is publicly observable and if contracts are enforceable without costs, then complete insurance is available between the (representative) firm and the worker. Because of the linear production function of the firm, it is assumed that competition among the firm will force its expected profit from production and from insurance each equal to zero: the firm is willing to employ any number of workers paying each worker at the (constant) marginal product; and the firm is also willing to offer an actuarially fair insurance policy against productivity uncertainty. The optimal actuarially fair coverage  $\{d(g), d(b)\}$  is calculated by solving the following problem:

$$\text{Max}_{\{d(g), d(b)\}} pu(y(g)-d(g)) + (1-p)u(y(b)+d(b))$$

$$\text{subject to } pd(g) = (1-p)d(b).$$

The contract wage is equal to the sum of the marginal product and the net coverage of the actuarially fair insurance policies. It is easy to show that the following contract is achieved through competition in the ex ante contract market. Before the dawn of each period, the firm and the worker will strike a deal specifying the following optimal contract wage:

$$(w^*(g), w^*(b)) = (\bar{y}, \bar{y}),$$

where  $\bar{y} = py(g) + (1-p)y(b)$ . We may regard the stabilized level of contract wage  $\bar{y}$  as the sum of the spot market wage  $y(s)$  and the net coverage of the actuarially fair insurance policies,  $d(s)^* = (\bar{y} - y(s))$ . Thus, all the risks are absorbed by the risk-neutral agent, and the payoff to the risk-averse agent is completely "stabilized" by the complete insurance  $d(s)$ . Note that the optimal long-term (two-period) contract is equivalent to two repeated single-period contracts.

### 3. Holmstrom variations

Let us assume that (i) [No slavery and costless moving] the worker is allowed to breach a contract anytime (ex ante, or ex post, in each period) without penalty and to recontract with another firm without incurring any mobility and transactions costs; and (ii) [public information] the realized state of nature is observable each period to both the firm and workers.

Because of the no-slavery constraint, any single-period contract which specifies an income of the worker in state of nature  $s$  less than  $y(s)$  will be breached in that state of nature. Predicting this breaching possibility, the firm would not offer an actuarially fair single-period contract. Thus

there is no viable single-period contract in the Holmstrom environment. A contribution of Holmstrom (1983) is to point out the possibility of a long-term contract in this environment. The Holmstrom long-term contract will be struck after the state of nature is revealed in the first period, but ex ante to the state of nature in the second period. When  $s = g$  in the first period, the optimal contract is a solution to the following problem.

Holmstrom problem

$$\begin{aligned} \text{Max} \quad & u(y(s_1) - d) + pu(y(g) + d(g)) + (1-p)u(y(b) + d(b)) \\ \text{subject to} \quad & y(s_1) \text{ being known,} \quad \quad \quad [\text{Ex post in } s_1], \\ & d(g) \geq 0, \quad d(b) \geq 0, \quad \quad \quad [\text{No-slavery constraint}], \\ & d = pd(g) + (1-p)d(b) \quad \quad \quad [\text{actuarially fair insurance}]. \end{aligned}$$

Substituting the actuarially fair insurance into the objective function and maximizing it with respect to  $d(g)$  and  $d(b)$  will yield the following optimal long-term contract,  $\{d^*, d(g)^*, d(b)^*\}$ :

Only when  $s_1 = g$ :

$$d^* = (1-p)d(b)^*$$

$$d(g)^* = 0,$$

and  $u'(y(g) - d^*) = pu'(y(g)) + (1-p)u'(y(b) + d(b)^*).$

The optimal contract is insuring the bad state of nature in the second period with a premium paid in the good state of nature in the first period. Note that in the good state of nature in the second period, the payment to the risk-averse agent has to be "bid" up because of the no-slavery constraint, therefore that state is not covered by the insurance purchased in the first period. Holmstrom emphasized this "downward rigidity" as a result of optimal contract in his model. Note also that in the bad state of the first period, the worker does not find it profitable to pay a premium to cover the (equally)

bad state of nature. [In the general case of more than two states of nature, only the worst state of nature has this characteristic of no contract.]

Now we have seen that the ex post mobility and no-slavery constraint plays a major role in determining a contract configuration.<sup>2</sup> The above-mentioned results such as the downward rigidity of the wage crucially depends on the ex post mobility of the worker in both periods. However, it is plausible that the worker's mobility declines overtime. First, having a contract in the first-period with a particular firm develops "firm-specific" skills. Second, the worker becomes "older", he faces higher moving costs, i.e., transactions costs associated with selling and buying houses, psychological adjustment costs of the family members to the new neighbors and school system. Let us assume an extreme case that there is perfect free mobility in the first period, but no mobility in the second period.

The modified Holmstrom model with declining mobility is formulated as follows. Let us denote the  $d(s_1)$  as the premium payment (if  $d(s_1) > 0$ ) in the state of nature  $s_1$  in the first period, and  $d(s, s_2)$ ,  $s_2 = g$ , or  $b$ , as the second period coverage in the  $s_2$  state of nature given the first period state of nature  $s$ . Note that the no-slavery condition for the second period,  $d(s, s_2) \geq 0$ , is dropped. However, the no-slavery constraint is maintained for the first period, so that the actuarially fair insurance does not allow risk sharing over  $s_1$ .

Holmstrom Problem with Declining Mobility

$$\text{Max } u(y(s_1) - d(s_1)) + pu(y(g) + d(s_1, g)) + (1-p)u(y(b) + d(s_1, b))$$

subject to

$$y(s_1) \text{ being known} \quad [\text{Ex post in } s_1],$$

$$d(s_1) = pd(s_1, g) + (1-p)d(s_1, b) \quad [\text{actuarially fair insurance, given } s_1].$$

Substituting the actuarially fair insurance constraint into the objective function and maximizing it with respect to  $d(s_1, g)$  and  $d(s_1, b)$  will yield the following optimal long-term contract  $(d(s_1)^*, d(s_1, g)^*, d(s_1, b)^*)$ :

For  $s_1 = g, b$ , and for  $s_2 = g, b$ ,

$$d(s_1)^* = pd(s_1, g) + (1-p)d(s_1, b)^*$$

$$\text{and } y(s_1) - d(s_1)^* = y(g) + d(s_1, g)^* = y(b) + d(s_1, b)^*.$$

Note that since information on the state of nature is public and there is no mobility, the second-period payment is state contingent. The optimal insurance will stabilize the net income across the two periods and across the state of nature of the second period, given the realization of the first period. A simple inspection will verify that  $d(g)^* > 0$  and  $d(b)^* < 0$ .

Let us illustrate the difference between two Holmstrom variations. Both problems yield a result that a deal is made after the state of nature in the first period is revealed. Suppose that the state of the first period is "good." The Holmstrom model with declining mobility will prescribe the solution where the premium of insurance against the bad state of nature is paid partly in the first period and partly in the good state of nature of the second period. In the original Holmstrom model in which the no-slavery constraint should be satisfied in the second period as well as the first period, the good state of nature in the second period cannot be used for paying a premium, because the ex post mobility of the worker would drive up the wage to the marginal product. Suppose next that the state of the first period is "bad." Then the Holmstrom model with declining mobility still yields the contract in which the "coverage" is paid in the first period counting on that the worker continues to work in the second period, while in the original Holmstrom model, there is no viable contract.

#### 4. Townsend Variations

Townsend (1982) constructed a two-period model with the risk-neutral agent who receives a constant endowment in each period and the risk-averse agent who receives a stochastic endowment in each period. The realizations of endowment to the risk-averse agent is private information, i.e., unobservable by the risk-neutral agent. The Townsend model is interpreted in our framework as follows. Suppose that the wage, which is set equal to the productivity  $w(s)=y(s)$  and received by the worker, is not observable or verifiable by the firm (risk-neutral agent). This may be justified if the firm owns the farm land where workers cultivate and harvest. As the competition forces the rent to be zero, the worker keeps all the proceeds from the land. Although the firm is willing to sell an actuarially fair insurance policy, single-period contracts are not viable because of the unobservability of  $y(s)$ . Townsend's contribution is to show that there is a viable two-period contract even in this private information case.

Townsend considered two problems for this private-information two-period model: First, the pure "borrowing-lending scheme" given the state of nature of  $s_1$  [Townsend (1982): Problem 4, p. 1176]; and second, the "optimal, endowment-contingent transfers" with incentive compatibility constraints [Townsend (1982): Problem 5, p.1177].<sup>3</sup> In the latter problem, although the second period payment cannot be state-contingent, there is a risk-sharing contract over the first-period states as long as the worker has an incentive to-reveal the true state of nature. Townsend claims that the pure borrowing-lending scheme yields the sub-optimal solution, while I will claim that this corresponds to an optimal solution in the environment that worker's mobility declines overtime. Two problems are now stated and solved, and then my claim will be proved.

Townsend's Problem 4 [pure borrowing and lending]

Given  $s_1$  is revealed to the worker,

$$\text{Max}_{d(s_1)} u(y(s_1)+d(s_1)) + Eu(y(s_2)-d(s_1)).$$

This problem is solved after the state of nature  $s_1$  is revealed to the worker. Since the loan received in the first period is paid back in the second period irrespective of the states of nature of the second period. In that sense, the actuarially fair insurance constraint is already substituted in the objective function. In other words, the size of loans  $d(s_1)$  will depend on the state of nature of the first period, while the payment in the second period cannot be dependent on the state of nature in that period because of the private information assumption. The solution,  $d(g)^*$  and  $d(b)^*$ , satisfies the following first order conditions:<sup>4</sup> For  $s_1 = g$  and  $b$ ,

$$u'(y(s_1)+d(s_1)^*) = Eu'(y(s_2)-d(s_1)^*).$$

A simple calculation will reveal that  $d(g)^* < 0$ , and  $d(b)^* > 0$ . Since the firm is indifferent to the size of the loan, the solution can be regarded as the worker's unilateral decision on the size of the loan. The chosen size of the loan reveals the private information on  $s_1$ . This full revelation by the size of the loan naturally motivates a further step toward a better contract, which is the main thrust of Townsend's paper. Since  $d(g)^* < 0 < d(b)^*$ , there is an opportunity of risk sharing before  $s_1$  is revealed. A better contract should be negotiated before the state of nature of the first period is revealed, and should satisfy the incentive-compatibility constraints in that it is not utility-increasing for the risk-averse agent to lie about the state of nature. A key observation is that an incentive compatibility constraint for the first-period is quite different from that for the second-period.

Townsend's Problem 5 [first-period-state-contingent contract]

Suppose that a contract is negotiated before  $s_1$  is revealed. If the worker claims that the  $s_1$  implies the good (bad, resp.) state of nature, then the worker is paid  $d_1(g)$  ( $d_1(b)$ , resp.) in the first period and pays back  $d_2(g)$  ( $d_2(b)$ , resp.) in the second period.  $\delta = (d_1(g), d_1(b), d_2(g), d_2(b))$ ,

$$\text{Max}_{\delta} p[u(y(g)+d_1(g))+Eu(y(s_2)-d_2(g))]+(1-p)[u(y(b)+d_1(b))+Eu(y(s_2)-d_2(b))]$$

subject to

$$(4.1) \quad u(y(g)+d_1(g)) + Eu(y(s_2)-d_2(g)) \geq u(y(g)+d_1(b)) + Eu(y(s_2)-d_2(b)),$$

$$(4.2) \quad u(y(b)+d_1(b)) + Eu(y(s_2)-d_2(b)) \geq u(y(b)+d_1(g)) + Eu(y(s_2)-d_2(g)),$$

$$(4.3) \quad p(d_1(g)-d_2(g)) + (1-p)(d_1(b)-d_2(b)) = 0.$$

The (4.1) ((4.2), resp.) constraint states that when the state of nature in the first period turns out to be good (bad, resp.), then there is no incentive to lie and claim that the state of nature is bad (good, resp.). The fair-insurance constraint does not imply two constraints:  $d(g)=d_2(g)$  and  $d(b)=d_2(b)$ , such as in problem 4. Instead, the actuarially fair insurance policies in this environment is expressed as (4.3), because risk-sharing across the states of nature of the first period is now permitted. Recall that Problem 4 prescribed  $d(b)^* > 0 > d(g)^*$ . It is a hint (as well as another hint in Townsend's footnote 12) that (4.1) becomes binding in the optimum. Let us denote the optimum for Problem 5 by  $[(d_t(s)^*), t=1,2, s=g,b]$ , given that it is an interior solution and (4.1) is binding. The solution is obtained by solving the following equations, where  $\lambda^* > 0$  is a Lagrange multiplier for constraint (4.1):<sup>5</sup>

$$\begin{aligned}
& pu'(\gamma(g)+d_1(g)^*) - \lambda^* [u'(\gamma(g)+d_1(b)^*)] \\
& = pEu'(\gamma(s_2)-d_2(g)^*) - \lambda^* [Eu'(\gamma(s_2)-d_2(b)^*)], \\
& (1-p)u'(\gamma(b)+d_1(b)^*) - \lambda^* [u'(\gamma(b)+d_1(g)^*)] \\
& = (1-p)Eu'(\gamma(s_2)-d_2(b)^*) - \lambda^* [Eu'(\gamma(s_2)-d_2(g)^*)], \\
& u(\gamma(g)+d_1(g)^*) + Eu(\gamma(s_2)-d_2(g)^*) = u(\gamma(g)+d_1(b)^*) + Eu(\gamma(s_2)-d_2(b)^*), \\
& p(d_1(g)^*-d_2(g)^*) + (1-p)(d_1(b)^*-d_2(b)^*) = 0.
\end{aligned}$$

Although Townsend characterizes the pure borrowing-lending scheme (Problem 4) as a suboptimal solution, I claim that this is an optimal solution for the declining-mobility environment. That is, the worker cannot change the firm in the second period, while he may be able to move to another employer without penalty after the state of nature is revealed in the first period. In order to claim the optimality of Problem 4 for such an environment, let us observe that the Townsend optimal solution to the first-period-state-contingent contract (Problem 5) implicitly assumes that the worker does not have any ex post mobility in the first period as well as in the second period. Risk sharing over  $s_1$  would break down if a worker decide to go to another firm (without penalty) ex post in the first period. Suppose that the optimal first-period-state-contingent contract  $[(d_t(s)^*), t=1,2, s=g,b]$  is negotiated, with an implicit assumption that workers do not breach the contract. However, when the state of nature is revealed, the worker may find it beneficial to breach the contract and negotiate a new contract. Recall that an advantage of the solution to Problem 5 over that to Problem 4 is in its risk-sharing over  $s_1$ . In other words, the difference between the two models emerges when  $[d_1(g)^*-d_2(g)^*]$  is not equal to zero. Hence, (4.3) implies that either  $[d_1(g)^*-d_2(g)^*] < 0$  or  $[d_1(b)^*-d_2(b)^*] < 0$ , when Problem 5 is different from Problem 4.

Suppose that  $[d_1(g)^* - d_2(g)^*] < 0$ . That is, ex post with respect to  $s_1$ ,  $[d_1(g)^*, d_2(g)^*]$  is a "less than fair" insurance policy. Recall that Problem 4 solves the expected utility maximization with  $[d_1(g)^* - d_2(g)^*] = 0$ . Therefore, it is easy to establish that when  $[d_1(g)^* - d_2(g)^*] < 0$ ,

$$u(y(g) + d(g)^*) + Eu(y(s_2) - d(g)^*) > u(y(g) + d_1(g)^*) + Eu(y(s_2) - d_2(g)^*),$$

where  $d(g)^*$  is a solution to Problem 4 and  $[d_1(g)^*, d_2(g)^*]$  is a solution to Problem 5 when the state of nature of the first period turns out to be good. This proves that the worker has an incentive to walk out on the contract  $(d_1(g)^*, d_2(g)^*)$ , and seek a new employer to obtain the solution of Problem 4.

Suppose next  $[d_1(b)^* - d_2(b)^*] < 0$ , then an above-mentioned proof replacing  $g$  by  $b$  shows that the worker has an incentive to renege the solution to Problem 5 when  $s_1 = b$ . This completes the proof that if agents have mobility in the first period, and if agents are allowed to renegotiate without penalty ex post with respect to  $s_1$ , then the solution to Problem 5 is not in general feasible, and the solution to Problem 4 becomes an optimum contract.

The different configurations of long-term contracts are summarized in Table 1. Note that both the Townsend's problem 4 and the Holmstrom model with declining mobility yield a characteristic that the contract is struck after the state of nature in the first period is revealed. The difference between the two solutions is whether the second period payoff is state-contingent. Obviously, the Holmstrom model with declining mobility yields a better contract than problem 4 of the Townsend model, because the firm observes  $y(s)$  on which the contract can be contingent. The complete insurance is possible when the state of nature is observable and there is no mobility. It has a characteristic that the long-term contract is equivalent to the two repeated single-period contracts. No long-term contract is viable in an environment where two key ingredients for long-term contracts investigated by

Holmstrom and Townsend are mixed, that is,  $y(s)$  is private information and the worker has the ex post mobility in both periods.

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INSERT TABLE 1 HERE  
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### 5. Concluding Remarks

We have clarified reasons why long-term contracts between the risk-neutral agent (firm) and the risk-averse agent (worker) should be established in different environments where a single-period contract is not viable. Both Holmstrom and Townsend construct models where a single-period contract is not viable, but a long-term contract is. However, reasons responsible for this characteristic in the two models are quite different. Holmstrom considers the case where the ex post mobility of workers destroys complete risk-sharing, while Townsend considers the case that private information limits the extent of risk-sharing. This note contributes to the literature in the following respects. First, Townsend's model is interpreted in the framework of implicit labor contracts. In his model, the productivity is assumed to be the private information of the worker, contrary to the popular assumption in the labor contract literature. Therefore, interpreting his model in the labor contract framework gives an insight about a contract configuration for a different environment. Second, I propose to consider the case of declining mobility, i.e., workers have the perfect ex post mobility in the first period but no ex post mobility in the second period. Townsend's solution to the pure borrowing-lending scheme is shown to be an optimal solution in the case of declining mobility. Last, Holmstrom, Townsend, and models with the declining mobility are classified and summarized in a table.

TABLE 1: CONTRACT CONFIGURATIONS IN DIFFERENT ENVIRONMENT

|   |              | EX POST MOBILITY                                  |   |   |
|---|--------------|---|---|---|
|   |              | FIRST PERIOD<br>NO<br><br>SECOND PERIOD<br>NO     | FIRST PERIOD<br>YES<br><br>SECOND PERIOD<br>NO    | FIRST PERIOD<br>YES<br><br>SECOND PERIOD<br>YES |
| I<br>N<br>F<br>O<br>R<br>M<br>A<br>T<br>I<br>O<br>N | y(s) PUBLIC  | COMPLETE<br>INSURANCE                             | HOLMSTROM<br>WITH DECLINING<br>MOBILITY           | HOLMSTROM                                       |
|   | y(s) PRIVATE | TOWNSEND's<br>Problem 5<br>[state-<br>contingent] | TOWNSEND's<br>Problem 4<br>[pure borrow<br>-lend] | NO<br>CONTRACT                                  |

## FOOTNOTES

1. A simplified Holmstrom-type model is employed here, and Townsend's model is translated into a labor contract framework. Although original Townsend's model is not presented in terms of implicit labor contracts, it is important to cast his model in the labor contract framework by two reasons. First, despite its important implication, Townsend's work is not frequently referred to in the labor contract literature just because its terminology. Second, the nature of private information in Townsend's model is in contrast with that in popular works in the implicit labor contract literature, such as Azariadis (1983), Green and Kahn (1983) and Grossman and Hart (1983). Townsend assumes that the productivity (payoff of the risk-averse agent) is not known to the firm (the risk-neutral agent), while research in the implicit contracts adopted a contrary assumption that the productivity is not known to the worker. On the one hand, it is assumed in these labor contract models that not only workers but the firm are risk averse and that a single-period contract consists of not only the wage but the amount of employment. It is well-known that if the firm is risk-neutral, then the optimal contract becomes a trivial solution. On the other hand, Townsend showed that a long-term is a natural solution in this environment. It is an beneficial for labor contract researchers to understand Townsend's work in the labor contract framework.

2. The ex post mobility is a topic of the study in Akerlof and Miyazaki (1980) and Ito (1984). However, these studies emphasized that the mobility would help the resource allocations if different firms have different realizations in stochastic production functions. Holmstrom is the first to emphasize its implications for long-term contracts.

3. By an appropriate translation in notation, his problems are readily applicable here:  $\Pi$ ,  $y'$ ,  $y''$  in his paper are  $p$ ,  $y(g)$ ,  $y(b)$  in this note; and  $2K$  should be set equal to 0, because of the actuarially-fair assumption in this note.

4. The first order condition states the equalization of the marginal utility of the first period and the expected marginal utility of a group of states in the second period. One might wonder whether the (level of) utility in the first period is higher or lower than the expected (level of) utility in the second period, given the first order condition being satisfied. Applying the theorem proved in Imai, Geanakoplos and Ito (1981), it can be shown that the utility in the first period is higher (lower, resp.) than the expected utility in the second period, if the absolute risk aversion is increasing (decreasing, resp.).

5. The solution is obtained by setting  $\Psi_1 = 0$  in Equations (18)-(21) in Townsend (1982: p. 1177) as well as the fair-insurance constraint and the binding (4.1).

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