OLIGOPOLY AND THE INCENTIVE FOR HORIZONTAL MERGER

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ABSTRACT

In order to talk about merger, one needs some notion of assets or capital which can be combined, and one must allow for asymmetry in the equilibrium to reflect such. Using a simple notion of capital with linear marginal cost and linear demand, we show in two types of models when there is and when there is not an incentive to merge. Merger results in an increase in the equilibrium price to the benefit of all firms. However, this price increase arises primarily because the output of the merged firm is lower after the merger than the combined output of its partners prior to merger. We show how the profitability of merger depends upon both the structural and behavioral parameters of the model.

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A principal concern of the Federal Trade Commission and the Antitrust Division of the Justice Department is the enforcement of merger policy under Section 7 of the Clayton Act. Since 1968, the Antitrust Division has published merger guidelines, which serve as the standards for deciding whether to challenge a merger. In June 1982, the guidelines were substantially revised.[1] In addition, the 1976 amendments to the Clayton Act created Section 7A which requires advance notice to the FTC and the Justice Department of large mergers. Section 7A prevents consummation of the merger for at least 30 days so that the FTC or the Justice Department can evaluate the competitive impact and decide whether to seek an injunction against the proposed merger.

In simple industry models, economists have articulated the important factors in evaluating the welfare implications of mergers. For example, mergers can reduce competition and result in a higher price to the detriment of consumers. However, mergers could increase total welfare if they created efficiency gains from scale economies.[2] The welfare tradeoffs are often clear, but the net result is usually a difficult empirical question.

Although we are ultimately concerned about the welfare implications of merger, we focus in this paper upon the incentives for firms to merge. We suggest that very little is known about the incentive to merge. Moreover, we believe the reason for this is that existing models have not captured the conceptual essence of a merger. In the context of a simple model, we propose a solution and examine the implications for the incentive to merge. We then briefly discuss the implications of these results for public policy toward merger.

It is often assumed that firms always have an incentive to merge. This misconception derives from at least two sources. First, the wave of merger activity at the turn of the century is thought to have been undermined by the Sherman and Clayton Acts, rather than a lack of profitable opportunities. Periodic merger waves since then have often been attributed to reduced enforcement activities. Second, industry models, such as the symmetric Cournot model, exhibit higher prices and profit per firm as the number of firms in the industry is reduced.[3] Thus, there would seem to be gains from merger through a reduction in the number of firms.

On the contrary, Stigler (1950) and others have argued that firms which do not participate in a merger may benefit more than the participants, and that the incentive to not participate in mergers may prevent the formation of tight oligopolies. When a merger occurs, the new firm will typically reduce its output from the combined levels of its constituent firms, and so price will increase. (This assumes that there are not significant cost reductions associated with merger.) Nonparticipants will then expand output and so benefit from both the higher price and their increased production. Thus merger participants do not capture all the profits which result from the merger. Because of this externality, mergers which increase total industry profits may not be consummated.

In a recent paper, Salant, Switzer, and Reynolds (SSR) (1983) reexamined the incentive to merge in a static model. They employed the Cournot model with linear final demand, constant marginal costs, and a fixed cost for each of the n firms. Even though profits per firm are higher with fewer firms, SSR demonstrated that the profits of one firm in an n-firm oligopoly are lower than the profits of two firms in an (n+1)-firm oligopoly when fixed costs are not large. The exception to this finding is when duopolists merge into monopoly. From this, they concluded that merger was generally unprofitable in a Cournot model.

However, the SSR model severely understates the incentive to merge. The problem is that merger is not well-defined. Merger of two firms from a symmetric equilibrium of (n+1) firms should produce (n-1) of the old firms and one new firm "twice the size" of the old firms. In particular, the new firm should be able to employ the combined productive capacity of both merger partners. However, the merged firm in the SSR model does not differ from the other firms. Thus, rather than yielding no incentive to merge, the SSR model produces no incentive for "lock-up". So viewed, it is clear why the incentive does not exist. The so-called merger increases price somewhat to the external benefit of all firms, but the merged firm sacrifices the profits of one of the two original firms. This can only be profitable when two firms merge to create an industry monopoly. Otherwise, the price increase will be swamped by the output reduction from shutting down one merger partner. Models with constant marginal cost invite this conceptual fallacy by preventing one from dealing with assets or firm size.

After noting the surprising results of SSR, Davidson and Deneckere (1983) (DD) reexamine the incentives to merge in a model where the products of firms are differentiated. With Nash behavior in prices, they uniformly find incentives to merge. But with Nash behavior in quantitites (as in SSR), the incentive to merge again disappears when the products are not sufficiently differentiated. With the DD model, product differentiation avoids the "lock-up" problem from the demand side. The merged firm continues to produce all the products of the merger partners. Thus, symmetry with respect to the products leaves multi-product firms inherently larger than firms with only one product. Although product differentiation can reverse the SSR results, we instead propose to deal with the notion of merger directly from the cost side. In this way, we can reexamine the incentive to merge for the case of a homogeneous product. Unlike SSR, we can find many circumstances in which an incentive to merge exists even though the product is homogeneous. Moreover, we can articulate the behavioral and structural circumstances which will or will not give rise to the incentive.[4]

Our formulation of mergers has the following features. First, we explicitly specify a tangible asset which the merged firm aggregates from its two partners, expanding the output it can produce at a given variable cost. Second, our model addresses the industry asymmetries caused by the merger of a subset of firms. The merged firm faces a different maximization problem because of its altered cost function and new strategic considerations. Third, the notion of assets is uniform and fungible enough to discuss combinations of firms in a meaningful and convenient way.

In section I we define the simplest such model. Assets are captured by specifying a factor which has fixed supply for the industry. In section II, we examine merger in the context of a dominant oligopoly model. There is an amorphous competitive fringe from which mergers occur to create oligopolists of a particular size. Like the traditional dominant firm model, the oligopolistic equilibrium conditions take into account the supply of the competitive fringe. With linear demand and quadratic cost, this model yields cases in which incentives to merge may or may not exist. Since the model of section II does not quite have the flavor of the SSR model in which two smaller firms merge into one larger firm, section III examines a more difficult model in which there are m small firms and n large firms twice the size of the small firms. Thus, two small firms can merge into one large firm. Again with linear demand and quadratic costs, we find that there may or may not be incentives to merge.

I. Cost and Demand Structure

The cost structure will be a key feature of the models discussed below. We assume that there is a factor whose total supply to the industry is fixed, say capital. It is a necessary input in the production process. What will distinguish firms is the amount of capital which they own. For convenience, we normalize the total quantity of capital to be unity. Obviously, this suppresses de novo entry into the industry. With entry, the gains to merger diminish in that the price increases would be dampened by the new competition. To capture this effect, it might be preferable to specify a rising supply curve for the fixed factor. We could then examine the impact of entry, which becomes increasingly costly, on the ability of existing firms to raise price through merger and therefore on their incentive to merge. However, this would substantially complicate the model with little gain in insight. The fixed factor assumption is a limiting case of an upward sloping factor supply. Its convenience derives from fixing the size of the industry so that we can readily examine changes in the number of firms and their sizes.

The cost function of a firm which owns a fraction s of the capital stock and produces output x is denoted $C(x,s)$. We assume that x is produced by a combination of the fixed factor and a vector of variable inputs z, according to a smooth concave production function, $x = f(z,s)$. Then $C(x,s)$ is dual to $f(z,s)$, where the cost function implicitly subsumes the factor prices corresponding to the input vector z. We further assume that f is linearly homogeneous in z and s. This implies that the cost function will be linearly homogeneous in x and s. Thus, any proportionate increase in output and capital will result in an equiproportionate increase in production costs. Because of the presence of a fixed factor of production, the marginal cost function of any single firm is increasing.[5] Furthermore, since $C_1(x,s)$ is homogeneous of degree zero, the competitive industry supply curve is fixed at $C_1(x,1)$, irrespective of the distribution of capital among firms. Thus, the competitive industry supply curve is rising, and the marginal cost function of each firm mirrors this industry marginal cost function. In particular, a firm's marginal cost curve is obtained by horizontally shifting the industry marginal cost curve inward, proportionately by the firm's share s of the industry capital stock. Thus, we can arbitrarily reconstitute industry structure without altering overall size of the industry.[6]

This cost structure specifically rules out the realization of scale economies as a motive for merger. Rather, we wish to focus on the incentives to merge which arise solely from firm size and behavior in an imperfectly competitive environment. In particular, we ask what are the gains from merger absent internal efficiencies. If no such incentives to merge existed in a specific context, then the observation of merger activity would lead to the presumption of nontrivial synergetic economies. However, when these incentives do exist, observed mergers may not be socially beneficial. The welfare losses of consumers from higher prices after merger may not be offset by the combination of higher firm profits <u>and</u> production economies.

In order to illustrate the gains and losses associated with merger, we specify particular cost and demand functions. Both demand and marginal cost are assumed to be linear functions of output. The cost function for a firm with capital stock s is

$$C(x,s) = s \cdot f + d \cdot x + \frac{e}{2 \cdot s} \cdot x^2. \tag{1}$$

Clearly, $C(x,s)$ is linearly homogeneous. Industry fixed costs, f, are distributed in proportion to holdings of capital. The resulting marginal cost function is linearly increasing with an intercept of d:

$$C_1(x,s) = d + \frac{e}{s} \cdot x. \tag{2}$$

The marginal cost function rotates about its intercept, d, with movements in s.

Similarly, the industry inverse demand function is linear:

(3) $$P(Z) = a - b \cdot Z.$$

We are now ready to discuss mergers in the context of two different models.

II. Oligopoly with a Competitive Fringe

In this section, we consider an industry with n oligopolists, each of which own a fraction s of the industry capital, and a competitive fringe which owns the remaining capital, 1-ns.[7] We then examine the incentives of some of the fringe firms to coalesce and form a new oligopolist with capital share s. Enough capital must be held by the fringe for this to be feasible, so that $1-sn \geqq s$. If this inequality is strict, then the fringe will contract but not vanish when the merger occurs.

Let V be the quantity supplied by the fringe firms, and X the total quantity supplied by the oligopolists. Fringe firms produce at output levels which equate price and marginal costs. With the horizontal summation property of the marginal cost function, this fringe equilibrium condition can be expressed simply as:

$$P(X+V) = C_1(V,1-sn). \tag{4}$$

The quantity supplied by the fringe is implicitly defined by (4) as a function of the output of the oligopolist, V(X).

The inverse residual demand function facing the oligopolists is then P(X+V(X)). They behave as a Stackelberg group with respect to the competitive fringe. This is a simple generalization of the dominant firm model to dominant oligopolies. The equilibrium condition for the oligopolists equates perceived marginal revenue to marginal cost, given a capital stock s for each.

The imperfectly competitive interaction of the oligopolists will be summarized by a constant conjectural variation. Let δ be the conjectured output response of the remaining oligopolists to a unit change in own output. If $\delta = -1$, then the oligopolists behave competitively. Therefore, we assume $\delta > -1$. If $\delta = 0$, we have the Cournot model in which each oligopolist assumes that the others will not respond to his output changes. Finally, if $\delta = (n-1)$, the n oligopolists act collusively to maximize joint profits. The existence of the competitive fringe does not eliminate the potential ability of the oligopolists to earn profits, since the fringe supply is constrained by their fixed capital stock, 1-sn.

The symmetric oligopoly equilibrium output level is then given by

$$P + (1+\delta)\cdot(1+V')\cdot\left(\frac{X}{n}\right)\cdot P' = C_1\left(\frac{X}{n},s\right) \tag{5}$$

where $P = P(X+V(X))$ and prime denotes the first derivative. Given the cost and demand specifications of section I, we can solve (4) and (5) to obtain the aggregate quantities of the oligopolists and the fringe,

(6) $$X(n) = \frac{(a-d)sn}{e+b[1+s(1+\delta)]}$$

(7) $$V(n) = \frac{(a-d)(1-sn)\{e+b[1-sn+s(1+\delta)]\}}{\{e+b[1+s(1+\delta)]\}\cdot[e+b(1-sn)]}$$

As a consequence of the linearity, the output produced by a single oligopolist, $X(n)/n$, is independent of n. Industry output $Z(n)$ is the sum of $X(n)$ and $V(n)$, so that

(8) $$Z(n) = \frac{(a-d)\cdot K(n)}{e+b\cdot K(n)}$$

where $$K(n) = \frac{e+b(1-sn)[1+s(1+\delta)]}{e+b[1-sn+s(1+\delta)]} .$$

Total output $Z(n)$ is an increasing function of $K(n)$, which is decreasing in n. Therefore, price rises as the number of oligopolists increases. This occurs because on balance the industry is behaving less competitively, with a lower capital stock held by the competitive fringe. Without this price effect, there would be no incentive for fringe firms to merge. A new oligopolist supplies less than did the component share s of the fringe prior to the merger:

(9) $$\frac{sV(n)}{1-sn} - \frac{X(n+1)}{n+1} = \frac{(a-d)s}{\{e+b[1+s(1+\delta)]\}}\cdot\frac{sb(1+\delta)}{e+b(1-sn)} > 0.$$

Therefore, the profits of the merged firm can increase only if the merger results in a price rise sufficient to offset the lower output level. Similarly, fringe firms would perceive no incentive to merge if they myopically failed to recognize that their merger would contribute to an increase in industry price.

In order to evaluate the incentive to merge, we compare the profits of fringe firms owning a fraction s of the capital stock, $\pi_c(n)$, when there are n oligopolists, with the profits of one of n+1 oligopolists, $\pi_o(n+1)$. A fraction $\frac{s}{1-sn}$ of the fringe earns

$$(10) \qquad \pi_c(n) = \frac{s}{1-sn} \cdot [P(Z(n)) \cdot V(n) - C(V(n), 1-sn)]$$

while the merged firm would earn

$$(11) \qquad \pi_o(n+1) = P(Z(n+1)) \cdot \frac{X(n+1)}{n+1} - C\left(\frac{X(n+1)}{n+1}, s\right).$$

Since both entities incur fixed costs of sf, f clearly has no impact on the incentive to merge. Moreover, we also discover that the comparison of profits is independent of the scaling parameters a and d. But otherwise, the incentive to merge depends upon the slope parameters b and e, and more importantly, the conjectural variation δ and the number of firms n.

Consider the first merger. When a single oligopolist forms, we restrict $\delta = 0$ since the oligopolist has no direct competitors. After substituting the appropriate expressions into (10) and (11), we find that $\pi_o(1) > \pi_c(0)$ for all parameter values. In this case, there is always an incentive to merge. However, when we discuss any further merger activity, so that $n \geq 1$, any δ less than or equal to n-1 is conceptually feasible.

When n = 1, a merger involves a movement in δ from zero to a value in the interval (-1,1), as unity is now the collusive conjecture. Then $\pi_o(2) > \pi_c(1)$ if

$$(1-\delta)^2(e+b)^3 + 2(1+\delta^2)(e+b)^2bs + (e+b)b^2s^2 + 2\delta b^3s^3 > 0.$$

This expression is positive for all nonnegative values of δ. However, if δ is sufficiently close to -1, this expression will be negative. Thus, a second firm will form from the fringe unless the resultant equilibria is too competitive. In general, if oligopoly behavior becomes more competitive <u>as a result of the merger</u>, merger will be less profitable. But for $n \geq 2$, a comparison of equations (10) and (11) is computationally cumbersome if δ changes with n. Thus, hereafter, we assume that δ remains unchanged after a merger occurs.

Consider now the third and subsequent mergers. Substituting from the functional specifications of Section I and the equilibrium quantities (6)-(8), we find that the incentive to merge can be simplified to:

(12) $$\pi_o(n+1) \gtrless \pi_c(n) \quad \text{as}$$

$$I(\delta,n) = -sb(1-\delta)n + [(1-\delta)(e+b)+sb(1+\delta)] \gtrless 0$$

Consider first cases where $\delta < 1$ (which includes the Cournot case where $\delta = 0$). For these cases, the term $I(\delta,n)$ is decreasing in n. Since $I(\delta,n)$ is positive at the largest possible value of $n = (1/s)-1$ (where the next merger eliminates the fringe), $I(\delta,n)$ must be positive for all relevant n. And $I(\delta=1,n) > 0$ clearly. Therefore, for $\delta \leq 1$, there is always an incentive for an additional merger from the fringe.

For $\delta > 1$, $I(\delta,n)$ is now increasing in n. Evaluating $I(\delta,n)$ at the relevant minimum of $n = 2$, we find that it is positive for $\delta < (e+b-sb)/(e+b-3sb)$. For any such δ, there is again an incentive to merge for all n which exceed $1+\delta$ (recall that $\delta \leqq n-1$). And since $(e+b-sb)/(e+b-3sb) > 1$ this case subsumes the previous case of $\delta \leq 1$.

Relatively competitive behavior gives rise to an incentive to merge from the fringe. But when δ is less competitive, we obtain cases in which no such incentive exists. In particular, when $I(\delta,n)$ is evaluated at the maximum $n = (1/s)-1$, we see that there can never be an incentive to merge at any $\delta > (e+2sb)/e$ with $n \geq 1+\delta$. This leaves a range of intermediate cases (when $s > 1/3$) for which there is an incentive to merge when n is large but not when n is small. This arises because the term in (12) is increasing in n. We can summarize the results for $n \geq 2$ as follows:

(13a) $\pi_o(n+1) > \pi_c(n)$ for $\delta < (e+b-sb)/(e+b-3sb)$,

(13b) $\pi_o(n+1) \gtrless \pi_c(n)$ as $n \gtrless \dfrac{(\delta-1)(e+b)-sb(1+\delta)}{sb(\delta-1)}$

for $\dfrac{e+b-sb}{e+b-3sb} < \delta < \dfrac{e+2sb}{e}$ and $n \geq 1+\delta$,

(13c) $\pi_o(n+1) < \pi_c(n)$ for $(e+2sb)/e < \delta < n-1$ and $n \geq 1+\delta$.

The intuition for these results can best be understood by reference to Figure 1. We have assumed that $b = 1$, $d = 0$, and $e = s$. And we consider the incentive for the next to last merger from the fringe. The output OA is the equilibrium output of the $n = \frac{1}{s} - 2$ firms in the dominant oligopoly. Since output per firm is independent of n in this linear case, OA is also independent of the merger choice of the (n+1)st firm examined here. This leaves a residual demand curve of QT facing the fringe. The marginal cost curve for the fringe firms considering merger is AR ($C_1(x,s) = \frac{e}{s}\cdot x = x$). This is also the supply curve of the fringe that would remain after such a merger. Thus, the residual demand facing the firms in the fringe considering merger is ET. Without merger, these firms would continue to behave competitively, produce BH at a price AB, and earn profits ABH (before netting out the fixed costs of $s \cdot f$). However, if these firms merged into the dominant oligopoly having the Cournot conjecture $\delta = 0$, they would produce CG at a price AC, and earn profits ACGM. This is simply the monopoly outcome with respect to the residual demand, and thus profits are clearly higher with merger when $\delta = 0$. The merger would also be profitable if $\delta = 1$. (Note that total output OA of the other oligopolists would vary with the conjecture. But Figure 1 remains an accurate depiction because we need modify only the scaling of the output axis.) If $\delta = 2$, the merged firms would produce DF at a price AD and earn profits of ADFN. But for the chosen parameters, the profits ADFN are now equal to the competitive profits ABH, and merger becomes a matter of indifference. Thus, for even less competitive conjectures $\delta > 2$ there is no incentive to merge. In particular, the merger into the dominant oligopoly requires such a large contraction in output that profits are less than the competitive rents. The conjecture greatly overshoots the monopoly output contraction and reduces profits. This clarifies the results (13a) and (13c).

The interesting aspect about the intermediate case (13b) is that there exists an incentive to merge for large n, but not for smaller $n \geq 2$ (recall that there is always an incentive for the first merger and generally for the second merger). When the number of merged firms is small, the competitive fringe looms larger and results in a relatively flatter residual demand curve facing the firms in the fringe considering merger. Thus, it takes a larger output contraction to increase the price and the profitability of merger is reduced for any conjecture. In particular, for this example in which $e/s = 1$, the incentive to merge exists only for $\delta < 2\beta+1$ where β is the absolute value of the slope of the residual demand curve facing those considering merger.[8] Thus, $\beta = 1$ for the last merger and $\delta < 3$ would generate an incentive. However, $\beta = 1/2$ for the next to last merger shown in Figure 1, and thus as we previously observed, $\delta < 2$ is required for an incentive to merge. Similarly, β is lower for earlier mergers, so the conjecture must be smaller yet to generate the same incentive (e.g., $\delta < 5/3$ for the third from last merger). This then accounts for the result (13b).

One last special case remains for this model. Consider the situation in which the oligopolists behave collusively both before and after each merger. This requires that δ change with n. In particular, the conjecture is $\delta = n-1$ for the pre-merger oligopoly equilibrium since there are n firms. But the conjecture becomes $\delta = n$ for the post-merger equilibrium. After simplification, the incentive to merge arises as follows:

(14) $$\pi_o(n+1) \gtrless \pi_c(n) \quad \text{as}$$

$$(e+b)^2[1+2n-n^2] + n^4(sb)^2 \gtrless 0.$$

Clearly, there are sufficient incentives for n = 1 and 2, so that three mergers will take place. Whether a fourth merger would be profitable depends upon the parameters. In particular, for small enough values of s, a fourth merger will not occur (for $s < \sqrt{2}\cdot(e+b)/9b$). But for such a merger to be feasible, s cannot exceed 1/4, since $(n+1)s$ must be less than or equal to unity. If $e = kb$ for some positive constant k, a fourth merger will not occur if $k \geq 0.59$ (for then $\sqrt{2}\cdot(e+b)/9b > 0.25$).

For n greater than 2 (and s small enough to consider large n), we can rewrite (14) so that $\pi_o(n+1) > \pi_c(n)$ if $s > (n^2-2n-1)^{\frac{1}{2}}(e+b)/n^2b$. If $e = kb$, then no feasible merger will be profitable if the right-hand side of this expression exceeds $1/(n+1)$, that is, if $(k+1)$ exceeds $n^2/[(n+1)(n^2-2n-1)^{\frac{1}{2}}]$. As n increases this expression approaches unity. Therefore, no feasible merger will be profitable if k is positive. Thus, as long as e is positive, merger will be unprofitable in the collusive case for large enough n.

The collusive case of our dominant oligopoly model is related to recent models of cartel stability by d'Aspremont et. al. (1983), Donsimoni et. al. (1981) and Donsimoni (1982). These are models of coalition formation in which firms choose to be part of a price-taking competitive fringe or to join a perfectly collusive cartel which takes the supply function of the fringe into account when it chooses the output which maximizes joint profits. A cartel is said to be stable if no competitive firm could gain by joining the cartel, and no cartel firm chooses to defect to the fringe, given that firms accurately compute the equilibrium profits that accrue to them following their actions. Our incentive condition for merger is similar to these equilibrium conditions. But there is an important conceptual difference. The firms in our model leave the fringe by coalescing into a single entity. While a coalition has been formed to create a new oligopolist, this new firm has only one owner, who has acquired the capital stock of the other participants in the merger. Thus, a merger is consummated only if the profits are greater than those which could be earned by behaving competitively. And once merger has occurred, defection is not an issue. The merged firm is a new single agent, not a coalition.

III. Oligopoly of Small and Large Firms

In this section, we consider an industry with n "large" oligopolists and m "small" oligopolists. The large oligopolists own s of the fixed factor as in section II, while the small oligopolists own only s/2 of the fixed factor. The constraint $s \cdot n + (s/2) \cdot m = 1$ must now hold, and this defines a linear relationship between n and m for given s. We can thus examine the incentive for two small firms to merge into one large firm. For example, the case discussed by SSR requires the comparison of $(n=0, m=2/s)$ with $(n=1, m=2(1-s)/s)$.

Let V be the quantity supplied by the small firms, and X the quantity supplied by the large firms. Rather than the small firms behaving competitively as in section II, we now assume that all firms have the same conjectural variation δ despite their differing sizes. Thus, the industry equilibrium can be defined by the two symmetric first-order conditions for the two firm sizes.

$$P(X+V) + (1+\delta) \cdot \frac{V}{m} \cdot P'(X+V) = C_1\left(\frac{V}{m}, \frac{s}{2}\right) \tag{15a}$$

$$P(X+V) + (1+\delta) \cdot \frac{X}{n} \cdot P'(X+V) = C_1\left(\frac{X}{n}, s\right) \tag{15b}$$

Given the cost and demand specifications of section I, we can solve this system for the output of the two groups of firms as a function of the number of large firms n:

(16a) $$X(n) = \frac{(a-d)\cdot[b(1+\delta)+2e/s]\cdot n}{\Delta(n)}$$

(16b) $$V(n) = \frac{2(a-d)\cdot[b(1+\delta)+e/s]\cdot[1/s-n]}{\Delta(n)}$$

where $\Delta(n) = [b(1+\delta)+e/s]\cdot[b(1+\delta)+2\cdot(e+b)/s] - b^2(1+\delta)\cdot n$. Industry output $Z(n)$ is the sum of $X(n)$ and $V(n)$, so that

(17) $$Z(n) = \frac{(a-d)\cdot[b(1+\delta)(2/s-n)+2e/s^2]}{\Delta(n)}$$

It is a simple matter to show that total output is a decreasing function of the number of large firms n. Thus, mergers again result in an increase in price to consumers. But unlike the previous model, firm behavior remains the same and price increases simply because there are now fewer firms in the industry.

The increase in price again provides the possibility for an incentive to merge. As in the prior model, the price increase must more than compensate for the reduction in output by the merged firm. The output of two small firms prior to merger is greater than output of the one post-merger firm:

(18) $$\frac{2\cdot V(n)}{m} - \frac{X(n+1)}{n+1} = (a-d)\cdot\left\{\frac{2\cdot[b(1+\delta)+e/s]}{\Delta(n)} - \frac{[b(1+\delta)+2e/s]}{\Delta(n+1)}\right\} > 0$$

Despite the fact that $\Delta(n) > \Delta(n+1)$, the inequality (18) holds if $\Delta(n) > 2b\cdot[b(1+\delta)+e/s]$ and this condition is satisfied even for $\min_n \Delta(n) = \Delta(1/s-1)$.

Thus, the incentive to merge again involves a balance of the price increase relative to the reduction in output. To evaluate this incentive, we simply compare the profits of one post-merger large firm, $\pi_\ell(n+1)$, with the profits of two pre-merger small firms, $2\cdot\pi_s(n)$. The profits of each small firm before merger are

$$\text{(19)} \qquad \pi_s(n) = P(Z(n))\cdot\left[\frac{s\cdot V(n)}{2\cdot(1-sn)}\right] - C\left(\frac{s\cdot V(n)}{2\cdot(1-sn)},\frac{s}{2}\right)$$

while the profits of each large firm after merger are

$$\text{(20)} \qquad \pi_\ell(n+1) = P(Z(n+1))\cdot\frac{X(n+1)}{n+1} - C\left(\frac{X(n+1)}{n+1},s\right)$$

Using the demand and cost assumptions of section I and substituting from (15) and (16), we find that

$$\text{(21)} \quad \pi_\ell(n+1) \gtrless 2\cdot\pi_s(n) \quad \text{as}$$

$$\{[4b(1+\delta)+2e/s]\cdot[b(1+\delta)+2e/s]^2 - 8\cdot[b(1+\delta)+e/s]^3\}\cdot[\Delta(n)]^2$$
$$+ 16b^2(1+\delta)\cdot[b(1+\delta)+e/s]^3\cdot\Delta(n)$$
$$- 8b^4(1+\delta)^2\cdot[b(1+\delta)+e/s]^3 \gtrless 0$$

This expression is a quadratic in $\Delta(n)$ which is in turn a function of the number of firms. But since $\Delta(n)$ is a simple decreasing function of n, we shall first characterize the incentive to integrate in terms of Δ. The quadratic is negative at $\Delta = 0$, then increases with Δ reaching a root at $\underline{\Delta}$. The quadratic then attains a maximum and declines thereafter to the second root at $\overline{\Delta}$. Thus, there is an incentive to merge when $\underline{\Delta} < \Delta(n) < \overline{\Delta}$, but not otherwise:

$$\text{(22)} \qquad \pi_{\ell}(n+1) > 2 \cdot \pi_s(n) \quad \text{as} \quad \underline{\Delta} < \Delta(n) < \overline{\Delta}$$

where

$$\text{(23a)} \qquad \underline{\Delta} = \frac{2b^2 \cdot [b(1+\delta)+e/s]}{2 \cdot [b(1+\delta)+e/s] + [b(1+\delta)+2e/s] \cdot [2b(1+\delta)+e/s]^{1/2} \cdot [b(1+\delta)+e/s]^{-1/2}}$$

$$\text{(23b)} \qquad \overline{\Delta} = \frac{2b^2 \cdot [b(1+\delta)+e/s]}{2 \cdot [b(1+\delta)+e/s] - [b(1+\delta)+2e/s] \cdot [2b(1+\delta)+e/s]^{1/2} \cdot [b(1+\delta)+e/s]^{-1/2}}$$

Figure 2 depicts the quadratic equation in (20) and the ranges of Δ for which there is an incentive to merge. An easy first result is that $\min_n \Delta(n) = \Delta(1/s-1)$ is greater than $\underline{\Delta}$. This means that the region $0 \leq \Delta \leq \underline{\Delta}$, in which there is no incentive to merge, is not relevant. Thus, the two questions which need to be examined are (1) when is $\max_n \Delta(n) = \Delta(0) \leq \overline{\Delta}$ so that there is always an incentive to merge, or on the other hand (2) when is $\min_n \Delta(n) \geq \overline{\Delta}$ so that there is never an incentive to merge. In answering these questions, we discover the intermediate cases in which $\min_n \Delta(n) < \overline{\Delta} < \max_n \Delta(n)$. These cases produce an incentive to merge when there are many large firms but no such incentive when there are only a few large firms. As one can readily see, these comparisons are algebraically non-trivial. Moreover, there are no general one-sided results. Thus, we restrict our analysis to the Cournot case where $\delta = 0$.

Consider when there would always be an incentive to merge. In particular, when is $\max_n \Delta(n) = \Delta(0) < \overline{\Delta}$?[9] First, we find this to be the case for all s when $e \geq 3b$. Figure 2 illustrates this case. However, when $e < 3b$, $\Delta(0)$ can exceed $\overline{\Delta}$ for s sufficiently small. For example, when $e = 2b$, $\Delta(0) < \overline{\Delta}$ for $s \geq 2/7$ (so that 7 small firms or as many as 3 large firms can exist); when $e = b$, $\Delta(0) < \overline{\Delta}$ for $s \geq 2/3$ (3 small firms or only one large firm); and when $e = b/2$, $\Delta(0) < \overline{\Delta}$ for $s = 1$ (2 small firms or only one large firm). Thus, for e much below b/2, $\Delta(0) > \overline{\Delta}$ for almost all values of s and there is no incentive for the first merger to occur.

Now consider when there would never be an incentive to merge. In particular, when is $\min_n \Delta(n) = \Delta((1/s)-1) > \overline{\Delta}$?[10] Obviously $\Delta((1/s)-1) < \Delta(0) < \overline{\Delta}$ for $e \geq 3b$. This is the case where there is always an incentive to merge. So we examine our previous examples. When $e = 2b$, $\Delta((1/s)-1) > \overline{\Delta}$ for

$s \geq 2/9$; when $e = b$, $\Delta(1/s-1) > \bar{\Delta}$ for $s \geq 2/5$, and for $e = b/2$, $\Delta(1/s-1) > \bar{\Delta}$ for $s \geq 1/2$. Thus, for all of these cases, there is never an incentive to merge. Figure 2 illustrates the case of $e = b$ with $s \geq 2/5$ (at least 5 small firms or as many as 2 large firms). A value of e significantly below b/2 will insure that $\Delta(1/s-1) > \bar{\Delta}$ for most values of s, and thus no incentive to merge at any n.

Finally, the mixed cases arise between the crevices of the "always" and "never" cases. For example, when $e = 2b$, $s = 1/4$ gives rise to a mixed case as illustrated in Figure 2. Four mergers are possible here, and we have shown the first would not be profitable, but that the fourth would be profitable. By calculating $\Delta(n)$, we find that the second and third are also profitable.[11] These mixed cases may not be very common, but they are similar to the mixed cases (13b) which arose in our prior model. When the industry has no large firms, there is no incentive for a first merger; but if there are a number of large firms the remaining small firms have an incentive to merge.

These results seem to differ from those of the dominant oligopoly model in that the Cournot conjecture always generated an incentive to merge in that model. With respect to a given residual demand facing two small firms, merger would always be profitable under the Cournot conjecture because it achieves the monopoly profits. This was the insight in the dominant oligopoly model. However, in the model of this section, the residual demand contracts with the merger of two small firms. This occurs because when the merged firm contracts its output, the other oligopolists notice a higher price and an increase in demand. As a result, they expand output,[12] and thereby reduce the residual demand facing the merged firm. For there to then be an incentive to merge, the monopoly profits on the lower demand must exceed the combined duopoly profits on the higher demand. This comparison depends upon how much the other oligopolists expanded.

Recall that the mixed cases yield the interesting result that an incentive to merge only arises when there are already several large firms. Consider our mixed case example of e = 2b. It is relatively easy to show that the output expansion of the other firms (note 12) decreases with the number of large firms n. As a result, the contraction of the residual demand facing the merger partners is less severe at high n and thus an incentive to merge can arise.

Finally, this model allows us to explicitly consider the results of SSR. In effect, they found that there would be no incentive for the <u>first</u> merger if the industry behaves in Cournot fashion (except for merger of duopoly into monopoly). To reexamine this in our model, we let $n = 0$ and $m = 2/s$ initially, and then consider the merger of two small firms into the first large firm. An incentive to merge exists if $\Delta(0) < \bar{\Delta}$ (note 9). This condition can be expressed in terms of m (substitute $s = 2/m$ in the condition in note 9) so as to yield an incentive to merge whenever

$$(24) \qquad e(e+b)(e-3b)\cdot m^3 + 2b[5e^2+be-2b^2]\cdot m^2 + b^2(8b+17e)\cdot m + 4b^3 > 0$$

Again, if $e \geq 3b$, merger will be profitable for all m. Thus, the first merger is always profitable. However, if $e = 2b$, merger is profitable only for $m < 8$; if $e = b$, only if $m \leq 3$; and if $e = b/2$, only if $m = 2$. Thus, if demand is steep relative to marginal cost, the first merger will be profitable only when there are few firms. This is because the price increase from merger is diminished when m is large. Only merger to monopoly (m=2) is always profitable.

In conclusion, the incentive to merge depends upon a complex resolution of two forces. First, a merger results in a price increase. But second, the output of the merged firm declines relative to that of its partners prior to the merger. The price increase benefits all firms, but unlike SSR and DD, the increase can still be enough to compensate for the output reduction of the merged firm and result in an increase in its profits. This is because once one allows a merged firm to be twice as "large" as each partner, the output reduction is not nearly as severe as in the SSR and DD models. However, we also find that there need not always be an incentive to merge. In the dominant oligopoly/competitive fringe model, the conjectural variation alters this tradeoff. More competitive conjectures yield incentives to merge while less competitive conjectures create incentives not to merge (to cheat). In the large-small oligopoly model, we have also illustrated that even with Cournot behavior, the incentive to merge depends in a complex way upon the relationship between the underlying demand and cost parameters.

What do these results mean for merger policy? First, some industries may be such that incentives to merge would never arise. These industries would merit little supervision. Second, other industries may be such that incentives to merge are strong. If so, special attention may be warranted in evaluating the welfare implications of mergers in such industries. Welfare considerations could then supplement the general guidelines in passing upon a given merger proposal under Section 7A.[13] Finally, our results lend some credence to the argument that early mergers, despite their seemingly small effect on competition at the time, should be scrutinized carefully in that they beget further mergers which may then have unfortunate welfare consequences. In our models, early mergers often enhance the incentive for subsequent mergers. Moreover, we found specific examples in which no incentive to merge exists until there are already a substantial number of large firms. Thus, concentration, arising initially for technological or random reasons and surviving as a result of loyalty or inertia, can generate current incentives for merger even though these incentives are derived from imperfectly competitive behavior rather than synergetic economies.

Footnotes

* We have benefited from the comments of John Panzar. This work was initiated while Porter was a postdoctoral fellow at Bell Laboratories. This article represents the views and assumptions of the author and not necessarily those of the Bell System.

1. Justice Department Merger Guidelines (1982). See Ordover and Willig (1983) for an assessment of these changes.

2. Williamson (1968) documents these effects.

3. For example, see Perry (1984) and Seade (1980).

4. In DD (1983), each product is initially assigned to one firm. Thus, merger increases the number of products for the new firms and creates an automatic asymmetry. But if the number of products is a choice variable for each firm, then merger need not create a "larger" firm unless it alters the costs of producing more than one product. Thus, one would again have to deal with the notion of merger from the cost side.

5. If $C(x,s)$ is linearly homogeneous, $C_1(x,s)$ will be homogeneous of degree zero, where $C_i(\cdot)$ is the derivative of $C(\cdot)$ with respect to its i^{th} argument. Then $C_{12}(x,s) < 0 < C_{11}(x,s)$, i.e. marginal costs decrease as capital increases, and so marginal costs must increase with output levels. (Recall, from Euler's Theorem, that $xC_{11}(x,s) + sC_{12}(x,s) = 0$.)

6. See Perry (1978) for a more complete derivation of this construction and a use of it to examine backward integration by a monopsonist into a competitive upstream industry.

7. The maximum size of an oligopolist could be thought of as arising from the Justice Department antitrust guidelines. For example, a guideline which triggered a challenge for mergers which put the industry over a 60% four firm concentration ratio would imply $s = .15$ under the assumption of symmetric oligopolists. Similarly, a Herfindahl index limit of 1600 would imply $s = .20$, if there were four oligopolists, as $4\cdot(100\cdot s)^2 = 1600$.

8. For linear demand $p(x) = \alpha-\beta\cdot x$ with costs $c(x) = x^2/2$, profits at the competitive output $x_c = \alpha/(\beta+1)$ are $\alpha^2/2(\beta+1)^2$. If output is contracted, profits increase down to the monopoly output $x_m = \alpha/(2\beta+1)$ and decline until at $x_\ell = \alpha/(\beta+1)(2\beta+1)$ they are again equivalent to the competitive profits. Thus, for the merger to be profitable, the conjecture δ must be such that post-merger equilibrium x_e of these fringe firms $(p+(1+\delta)\cdot x_e\cdot p' = c')$ yields an output greater than x_ℓ. Since $x_e = \alpha/[(2+\delta)\beta+1]$, we find that $x_e > x_\ell$ and an incentive to merge when $\delta < 2\beta+1$.

9. After algebraic simplification, we find that

$$\Delta(0) < \bar{\Delta} \quad \text{as} \quad -2\cdot b^3-8\cdot b^3/s-17\cdot b^2 e/s$$
$$-4\cdot(b/s^2)[5e^2+be-2b^2]$$
$$-4\cdot(e/s^3)(e+b)(e-3b) < 0$$

10. Again, $\Delta(1/s) < \bar{\Delta}$ as

$$-4\cdot b^3-8\cdot b^2 e/s$$
$$-6\cdot b\cdot e^2/s^2+2\cdot b^3/s^2$$
$$-(e/s^3)(e+b)(e-3b) < 0$$

11. We find that $\Delta(n) = 225-n$ and $\bar{\Delta} \approx 224.16$.

12. The expansion of the non-merging firms, large and small, can be expressed as

$$n\cdot\left\{\frac{X(n+1)}{n+1} - \frac{X(n)}{n}\right\} + [1-s(n+1)]\cdot\left\{\frac{V(n+1)}{1-s(n+1)} - \frac{V(n)}{1-sn}\right\}$$
$$= (a-d)\cdot\left\{\frac{1}{\Delta(n+1)} - \frac{1}{\Delta(n)}\right\}\cdot\left\{\frac{2(1-s)}{s}\cdot\left[b + \frac{e}{s}\right] - b\cdot n\right\} > 0$$

where Δ defined at (16).

13. The general standards of the Justice Department Merger Guidelines are found in section III(A)(1). But other factors are considered as discussed in section III(C).

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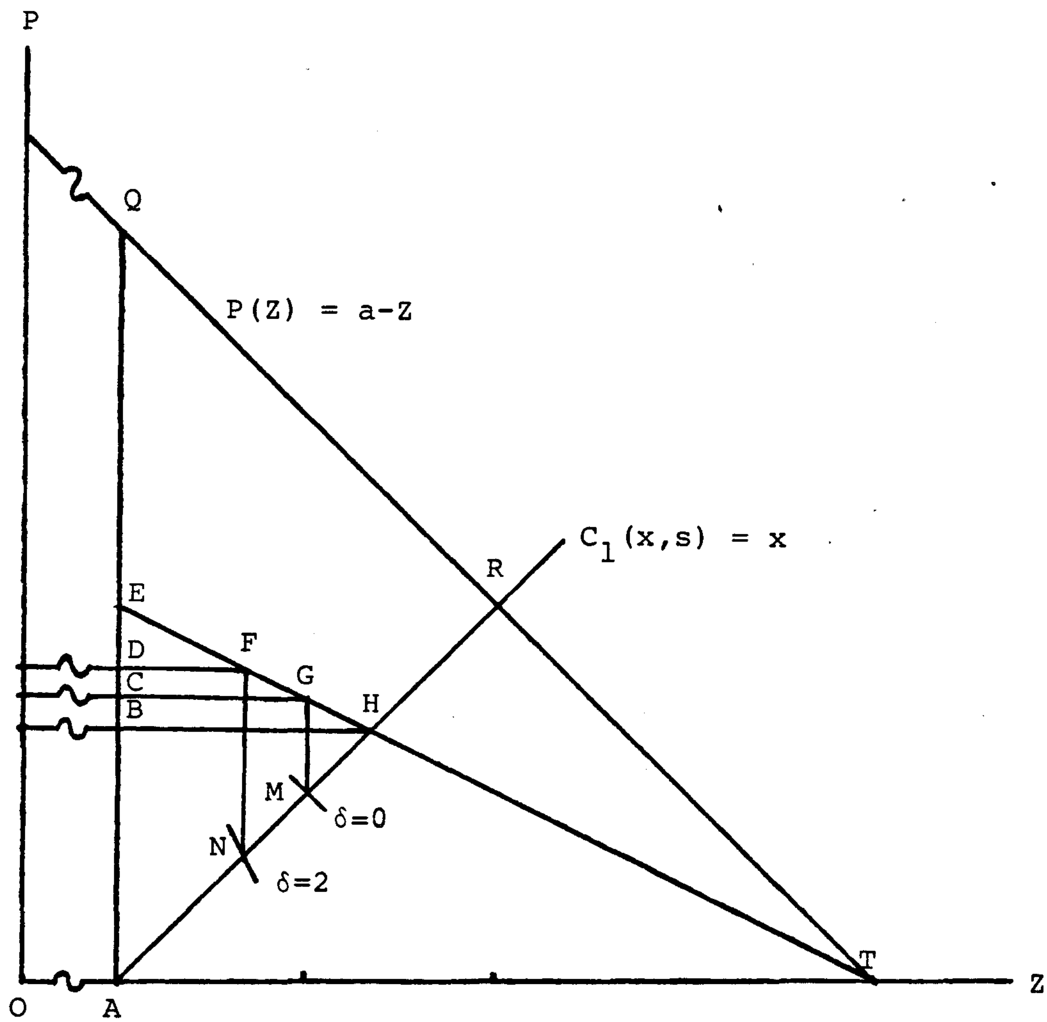

Figure 1: Incentive to Merge in the Dominant Oligopoly Model

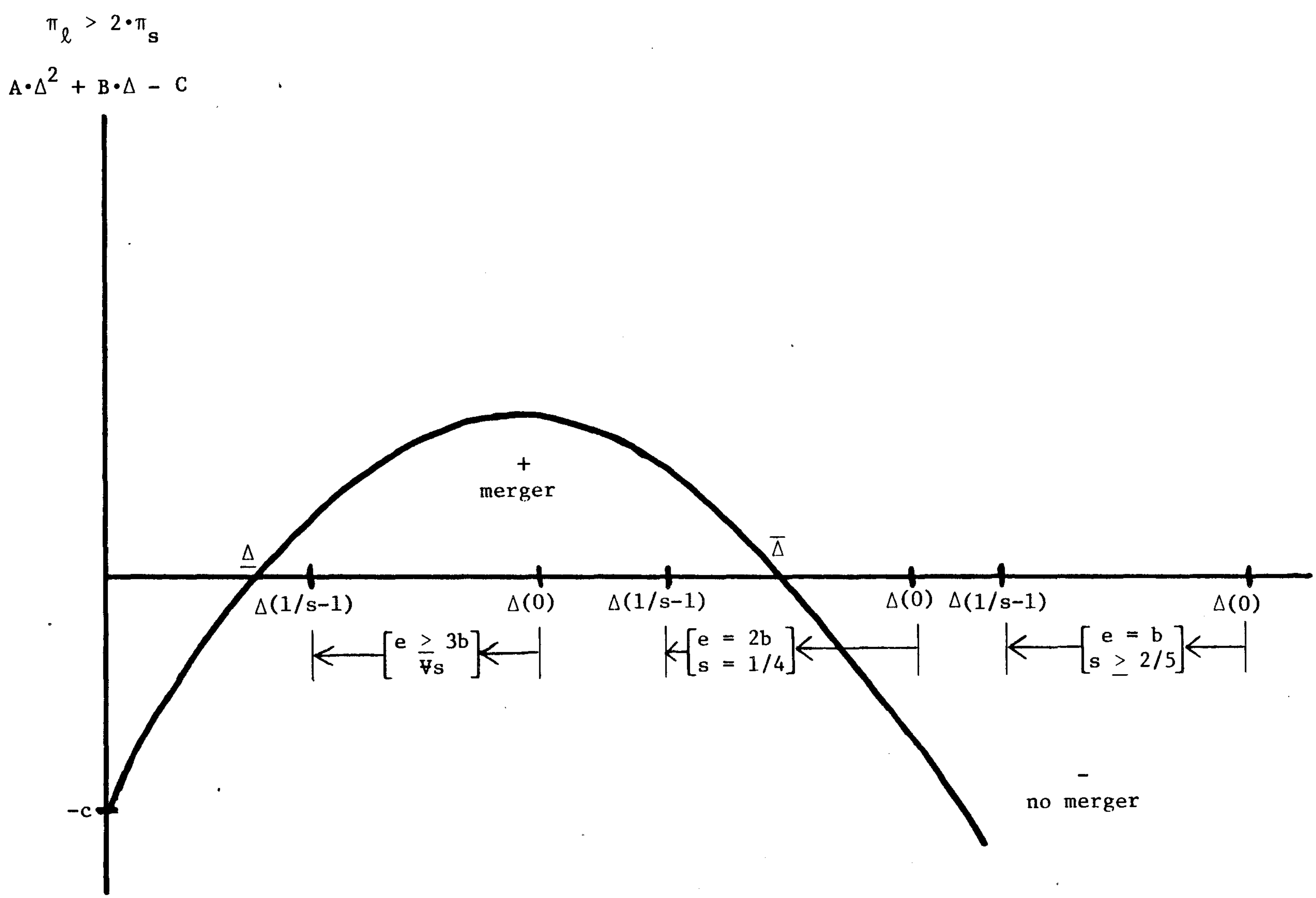

Figure 2: Incentive to Merge in the Large-Small Oligopoly Model