AN EQUILIBRIUM QUEUEING MODEL OF BRIBERY

by

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I. Introduction

If prizes are awarded simultaneously at a specified time to the first n customers who queue for them, the arrival times of the customers to the queue can serve the function of prices in the allocation process (Holt and Sherman, 1982, 1983). When prizes are awarded in a continuous stream, as is common in practice, this method does not work too well. Instead, bribes for buying better positions in the queue sometimes give useful signals similar to those of a pricing mechanism.[1]

That bribery may have beneficial effects is not a new idea (e.g., Leff, 1970). It is often argued that bribes serve as "lubricants" in an otherwise sluggish economy and improve its efficiency. However, asides from the undesirable distributional consequences, an important opposing view on efficiency also exists. Myrdal, quoting the Santhanam Report on prevention of corruption by the Indian government, argues that the corrupt officials may deliberately cause administrative delays so as to attract more bribes (Myrdal, 1968, Chap. 20). If this is indeed the case, the efficiency argument will be much less appealing. A serious study on bribery should not leave this problem unanswered.

Whether the server of a queue can increase the bribe revenue by slowing down the service does not have a trivial answer. Several issues are involved. For instance, what will happen to the number of incoming customers who choose not to join the queue because the expected waiting time is too long? For those who stay, do they always want to pay larger bribes? A more fundamental difficulty, however, is that a customer's action affects others. Externality must be incorporated in the behavioral model of the queue.

Several queueing models related to bribery are available in the literature. The Kleinrock model, which will be discussed extensively later in this paper, assumes that a customer paying a bribe will be placed in front of all those who have paid smaller bribes in the queue, but behind all those who have paid larger bribes (Kleinrock, 1967). This model has the desirable feature that it can generate socially optimal results. However, in this model, the amounts of bribes to be paid by the different customers are decided by the server (who acts as if he is also a social planner), rather than the customers themselves. To obtain optimal results, great informational burden is imposed on the server, who is required to know the values of time of all the customers. Some later models have less severe informational requirements. Naor (1969) discusses a queue where a uniform toll is imposed on those who want to join it. Rose-Ackerman (1978) proposes a system such that a customer entering a queue served with greater priority will have to pay a higher bribe than one joining a lower priority queue. However, if customers differ in their opportunity costs of time, it can easily be shown that these two models give suboptimal solutions in the sense that the total value of time spent in waiting by the customers is not minimized.[2].

In this paper, we propose an equilibrium queueing model of bribery with decentralized decision makings. This model has some desirable features. Under some specified conditions, it is capable of giving socially optimal solutions. At the same time, it does not have stringent informational requirements. To obtain optimal results, the model is based on the queueing discipline of Kleinrock. However, the amounts of bribe payments are not decided by the server, but by the customers themselves. As we shall see, this lightens the informational requirements significantly. Another important feature is that the desired socially optimal solution is consistent with individual optimization strategies. In other words, there exists a Nash equilibrium of this noncooperative game such that under some specified conditions, the outcome is also socially optimal. Based on this equilibrium concept, we can assess the validity of Myrdal's hypothesis. It should also be pointed out that the model need not be confined to the study of bribery alone. If bribes are regarded as legitimate payments, the mechanism becomes a useful auctioning procedure when a queue is involved.

In the next section, we shall outline a modified version of the Kleinrock model and derive the expected system time function. In Section III, we derive the optimal bribing function for the customers and show that the implied strategies form a Nash equilibrium. In Section IV, we examine the effects on bribe revenue when the speed of service is varied. In Section V, we consider the welfare implications of the model and examine the requirements for optimality. Section VI is a summary of the results. Finally, the Appendix discusses a mathematical generalization of the model.

II. The Queueing Model

The following assumptions are made for the M/M/1 preemptive queueing model in this paper:

1) Customers arrive at the end of the queue according to a Poisson process at a mean rate of m customers per unit of time.

2) At the other end of the queue, there is one server who distributes prizes of uniform value P to the customers. The service time required for giving out a prize obeys an exponential distribution with a mean service time of $1/u$.

3) Let v represent the value of time of a customer. In general, different customers may have different values of time, so that v is a random variable. The cumulative distribution function of v is represented by $A(v)$. It is assumed that $A(v)$ is known to the customers, and the derivative of $A(v)$ is continuous throughout its domain.

4) When a customer comes to the end of a queue, he can follow either of two strategies:

i) He can decide not to join the queue at all.

ii) He can pay a bribe x to the server before he sees the queue length. He will be placed in front of those whose bribes $x' < x$, and behind those whose bribes $x'' \geq x$. The expected system time (waiting time plus service time) spent in the queue by a customer who pays x is represented by $W(x)$. He cannot revise his bribe.

5) The queue is preemptive. A customer being served will be ejected from service, but not from the queue, if a newly entering customer offers a bribe larger than his.[3]

We also let x^* represent the maximum bribe received by the server. Let the truncated distribution function of x be $B(x)$ such that $B(x^*)$ is the proportion of customers who choose to stay in the queue. In general, $B(x^*) \leq 1$. It is assumed that $B(x)$ is continuous. The variable x^* is endogenous in the model and will be determined in Section III.

<u>Proposition 1</u> (variant of Kleinrock's).

Given the assumptions of the model, the expected system time function is given by

$$W(x) = \frac{r}{m[1 - rB(x^*) + rB(x)]^2} \qquad (1)$$

where the utilization index r is defined by $r = m/u$.

<u>Proof</u>. A customer paying a bribe x has to wait for three things before leaving the system:

i) His own expected service time is $1/u$ because of the exponential distribution of service time assumption.

ii) He must wait until service is given to all those still in the queue and who arrived before him and whose bribes are at least as big as his. Due to Little's result,[4] the expected number of such customers whose bribes lie in the region $(y, y+dy)$ is

$$m[dB(y)/dy]W(y)dy.$$

Since each such customer causes him to wait for $1/u$ units of time, his expected waiting time for them is

$$\int_x^{x^*} (m/u)[dB(y)/dy]W(y)dy.$$

iii) He must wait until service is given to those who come after him while he is still in the system and whose bribes exceed his. The expected number of such people coming per unit of time is

$$m\int_x^{x^*} dB(y).$$

Hence, during his average system time $W(x)$, his expected waiting time for these people is

$$W(x)\int_x^{x*}(m/u)\,dB(y).$$

Adding up, we get

$$W(x) = (1/u) + \int_x^{x*}(m/u)W(y)\,dB(y) + W(x)\int_x^{x*}(m/u)\,dB(y)$$

or $$W(x) = [(1/u) + r\int_x^{x*} W(y)\,dB(y)]/[1 - rB(x*) + rB(x)]. \qquad (2)$$

Replacing $W(x)$ and $W(y)$ in equation (2) by the expression given in equation (1), we can establish the proposition if the following equality is true:

$$\frac{(1/u)}{[1 - rB(x*) + rB(x)]^2} = \frac{(1/u) + r\int_x^{x*} \dfrac{(1/u)\,dB(y)}{[1 - rB(x*) + rB(y)]^2}}{1 - rB(x*) + rB(x)}.$$

By simplifying the expression on the right hand side, we see that this equality is true. Thus, equation (1) is indeed the solution to equation (2). This completes the proof.

We now substantiate in part our earlier claim that this model is capable of yielding socially optimal outcomes. We want to examine how the bribe of a customer should be related to his value of time so that the queue has optimal properties. In other words, we want to know the necessary restrictions on the bribing function $x(v)$ such that the ranking of the customers in the queue is "correct."

Definition. A queue is *socially quasi-optimal* for a given mean service time if the customers are ranked in such a way that for a given fixed number of customers in the queue, the average cost of system time spent in the queue is minimized, that is,

$$\int_0^{v^*} vW(x(v))dA(v)$$ is minimized, where v^* is the maximum value of time among those customers who choose to stay in the queue.

Proposition 2.

For any given $A(v)$, the bribing function $x(v)$ results in a socially quasi-optimal queue if $x(v)$ is a strictly increasing function of v.

Proof. See Kleinrock (1967).

The intuition behind this proposition is simple. To minimize the time costs of the queue, all that is needed is to rank customers according to their values of time so that people with higher values of time are placed in front of those with lower values. Since the queueing rule is to rank customers according to x, it necessarily also ranks them according to v for any $x(v)$ with $x'(v) > 0$.

We have used the term "quasi-optimal" rather than "optimal" because the queue is optimal only when the number of customers is fixed. Nothing has been said about the optimal number of people to join the queue. For example, if nobody joins the queue, the time cost will be zero. But this may not be an optimal outcome. Detailed discussion on optimality is postponed to Section V.

III. The Bribing Function and the Nash Equilibrium

We now turn to the derivation of a bribing function which is both socially quasi-optimal and privately optimal. We proceed in two steps. First, we artificially construct a bribing function which satisfies the socially quasi-optimal outcome requirement. Second, we show that if all other customers follow this bribing function, there is no incentive for any one to depart from it.

To guarantee social quasi-optimality, we impose the restriction that $x'(v) > 0$ on the bribing function we construct. Since the ranking of x is the same as the ranking of v,

$$B(x(v)) = A(v). \tag{3}$$

We also note that given this restriction, the definitions of $x*$ and $v*$ imply that $x* = x(v*)$. Therefore,

$$B(x*) = A(v*). \tag{4}$$

Define $b(x) = dB(x)/dx$.

It follows that

$$b(x) = A'(v)v'(x). \tag{5}$$

The function $v(x)$ in (5) is the inverse function of $x(v)$. Hence, $v'(x)$ must be positive.

Each customer with a given value of time $\bar{v}$ solves the following maximization problem:

$$\max_{x} \quad G = P - (x + \bar{v}W(x)) \tag{6}$$

G is the net gain of joining the queue. The term in parentheses is the total cost of joining it. Because of (1), equation (6) can also be stated as

$$\max_{x} \quad G = P - x - \frac{\bar{v}r}{m[1 - rB(x*) + rB(x)]^2} \tag{6'}$$

The first order necessary condition is

$$\frac{dG}{dx} = -1 + \frac{2r^2\bar{v}b(x)}{m[1 - rB(x*) + rB(x)]^3} = 0 \tag{7}$$

Using (3), (4) and (5) and the differential equation (7), we have

$$A'(v)v'(x) = \frac{m[1 - rA(v*) + rA(v)]^3}{2r^2v} \tag{7'}$$

Solving (7'),

$$x = \int \frac{2r^2vA'(v)dv}{m[1 - rA(v*) + rA(v)]^3} + K \tag{8}$$

where K is a constant to be determined.[3]

It is also necessary to show that (7) is the solution of a maximization problem.

$$\frac{d^2G}{dx^2} = \left(\frac{2r^2\bar{v}}{m}\right)\frac{[1-rB(x*)+rB(x)]^3b'(x) - 3rb^2(x)[1-rB(x*)+rB(x)]^2}{[1 - rB(x*) + rB(x)]^6}$$

Using (7) to get expressions for b(x) and b'(x), this simplifies to

$$\frac{d^2G}{dx^2} = \frac{-v'(x)}{v} < 0 \tag{9}$$

Therefore, (7) is indeed the solution of a maximization problem.

To facilitate our discussion, it is desirable to obtain a more explicit bribing function than (8). Therefore, we make the additional assumption that A(v) is a uniform distribution function from $v = 0$ to $v = v_1$.

$$\begin{aligned} A(v) &= Av \qquad \text{for } v \in [0,v_1] \\ Av_1 &= 1 \\ A'(v) &= A \end{aligned} \tag{10}$$

The results for the general distribution function A(v) will be discussed in the Appendix.

Equation (8) now becomes

$$x = \int \frac{2r^2 vAdv}{m[1-rAv*+rAv]^3} + K.$$

Solving this gives

$$x = \frac{-vr}{m[1-rAv*+rAv]^2} - \frac{1}{mA[1-rAv*+rAv]} + K \qquad (8')$$

A necessary condition for Nash equilibrium is that the customer with the lowest value of time does not pay any bribe. This is because other people with higher values of time always pay higher bribes than he does. If he pays positive bribe, he can always improve his gain by paying less without affecting his expected system time. In (8'), $v = 0$ implies $x = 0$. Using this condition, we can solve for K. The bribing function now becomes

$$x = \frac{1}{mA(1-rAv*)} - \frac{vr}{m(1-rAv*+rAv)^2} - \frac{1}{mA(1-rAv*+rAv)} \qquad (8'')$$

It remains to determine $v*$. Recall that $x*$ is the largest bribe paid by a customer in the queue and $v*$ is his corresponding value of time, which is also the largest among those who join the queue. For this customer, his net gain must be nonnegative. Otherwise, he will not join the queue. Moreover, as long as $v* < v_1$, that is, some people do not join the queue, his gain cannot be positive. Otherwise, people with value of time just above his will also join the queue.[6] Hence, for $v* < v_1$,

$$G(x*) = P - x* - v*W(x*) = 0.$$

Using (1),

$$x* = P - \frac{v*r}{m[1-rB(x*)+rB(x*)]^2}$$

$$= P - \frac{v^* r}{m} \tag{11}$$

By substituting $v = v^*$ into (8"), we also obtain[7]

$$x^* = \frac{1}{mA(1-rAv^*)} - \frac{v^* r}{m} - \frac{1}{mA} \tag{12}$$

Solving (11) and (12),

$$v^* = \frac{mPA}{rA(1+mPA)}. \tag{13}$$

For convenience, define $z = mPA$.

Equation (13) is now

$$v^* = \frac{z}{rA(1+z)}. \tag{13'}$$

An alternative expression for v^* is

$$v^* = \frac{zv_1}{r(1+z)}. \tag{13"}$$

We have utilized the fact that $Av_1 = 1$. The condition that $v^* < v_1$ is equivalent to

$$r > \frac{z}{1+z}. \tag{14}$$

In other words, (13') or (13") hold when (14) is true. From (11) and (13'), we also get

$$x^* = \frac{Pz}{1+z}. \tag{15}$$

If $r \leq z/(1+z)$, $v^* = v_1$. (16)

From (12), correspondingly, $x^* = r^2/mA(1-r)$. (15')

If only some customers join the queue, that is, (14) holds, then from (13) and (8"),

$$x = P + \frac{1}{mA} - \frac{vr}{m\left(\frac{1}{1+z} + rav\right)^2} - \frac{1}{mA\left(\frac{1}{1+z} + rAv\right)}. \qquad (17)$$

If all people join the queue, that is, if (16) holds,

$$x = \frac{1}{mA(1-r)} - \frac{vr}{m(1-r+rAv)^2} - \frac{1}{mA(1-r+rAv)}. \qquad (18)$$

Equations (17) and (18) express the bribe x in terms of the parameters m, r, A, P and the variable v. If the customers know their own values of time, they can compute the optimal bribes they should pay.

<u>Proposition 3</u>.

i) Suppose $r > z/(1+z)$. If customers with $v \leq v*$ follow bribing strategies given by (17) and customers with $v > v*$ do not join the queue, where $v*$ is determined by (13), then these strategies form a Nash Equilibrium which is socially quasi-optimal.

ii) Suppose $r \leq z/(1+z)$. If all the customers follow bribing strategies given by (18), then these strategies form a Nash equlibrium which is socially quasi-optimal.

<u>Proof</u>.

i) Suppose $r > z/(1+z)$. From (17), it can easily be seen that $x'(v) > 0$. From Proposition 2, if everybody with $v \leq v*$ follow (17), the solution is socially quasi-optimal.

Suppose all customers with $v \leq v*$ follow (17) and those with $v > v*$ do not join the queue. From how we construct the bribing function, (17) is clearly the solution of the maximization problem (6'). Hence, there is no incentive for those with $v \leq v*$ to change the bribe. Moreover, for those with $v > v*$, their net gain in joining the queue is negative if other customers follow (17). Thus, these customers have no incentive to join the queue. For the customers with $v = 0$, since $x(0) = 0$, they cannot further improve their gain by paying less. They will not depart from the strategy (17). The strategies outlined above therefore form a Nash equilibrium.

ii) The proof is almost identical and omitted here. (Note that given (16), the net gains for all the customers are nonnegative.)

Insert Figure 1 Here.

The following example illustrates much of what has been discussed in this section. Let $r = 1$, $m = 1$, $P = 1$ and $A = 1$. It follows easily that $x* = 0.5$, $v* = 0.5$ and $B(x*) = 0.5$. Figure 1 plots the net gain G against the bribe x for different values of v, assuming that all other customers follow their own equilibrium strategies. The optimal bribe paid by a customer with v is the x which gives the maximum point on the corresponding curve, if the gain is positive. It can be seen that the maximum gain of a customer is a decreasing function of his value of time v. For $v > 0.5$, the customers cannot have positive gain and do not join the queue. For $x \geq 0.5$, the customer is already at the front of the queue. He cannot improve his position by paying more bribe. The curves therefore become decreasing straight lines.

IV. Changes in Bribe Revenue

The bribing function (17) depends on the parameters r, m, P, and A, while (18) depends on r, m, and A. We now turn to consider the effects of changes in r on the average revenue received by the server per period of time.

Proposition 4.

In the model outlined above, if $r < z/(1+z)$, increasing the mean service time per customer $(1/u)$ will cause the average bribe revenue received by the server per period to go up. If $r \geq z/(1+z)$, increasing $1/u$ will cause the average bribe revenue per period to go down.

Proof.

The average bribe paid to the server by an incoming customer is given by

$$\bar{x} = \int_0^{v_1} x(v, r)A\,dv. \tag{19}$$

Since on the average, there are m customers coming to the queue per period, the average bribe revenue per period is $m\bar{x}$. For $r < z/(1+z)$, all customers join the queue. We want to show that

$$\frac{d(m\bar{x})}{d(1/u)} > 0.$$

Since $r = m/u$, for fixed m, it suffices to show that $d\bar{x}/dr > 0$. From (18),

$$\bar{x} = \int_0^{v_1} \left[\frac{1}{mA(1-r)} - \frac{vr}{m(1-r+rAv)^2} - \frac{1}{mA(1-r+rAv)}\right] A\,dv$$

$$= \frac{v_1}{m(1-r)} + \frac{v_1}{m} + \frac{2\ln(1-r)}{rmA} \tag{20}$$

$$\frac{d\bar{x}}{dr} = \frac{v_1}{mr^2}\left[\frac{3r^2 - 2r}{(1-r)^2} - 2\ln(1-r)\right] \qquad (21)$$

Let J be the term inside the parentheses []. Clearly, $r = 0$ implies $J = 0$. Moreover, $dJ/dr = 2r^2/(1-r)^3 > 0$, since $r < z/(1+z) < 1$. Hence, for $r \geq 0$, the smallest value of J is zero. Thus,

$$\frac{d\bar{x}}{dr} > 0 \qquad \text{if} \quad r < z/(1+z). \qquad (23)$$

We next assume that $r > z/(1+z)$. Customers with $v > v^*$ will not join the queue and do not pay any bribe. Equation (19) then become

$$\bar{x} = \int_0^{v^*} x(v,r)Adv + \int_{v^*}^{v_1} x(v,r)Adv$$

$$= \int_0^{v^*} x(v,r)Adv \qquad (19')$$

since the second term is zero.

We want to show that $d\bar{x}/dr < 0$. From (17),

$$\bar{x} = \int_0^{v^*} \left[P + \frac{1}{mA} - \frac{vr}{m\left(\frac{1}{1+z} + rAv\right)^2} - \frac{1}{mA\left(\frac{1}{1+z} + rAv\right)}\right] Adv$$

$$= \frac{P}{r} + \frac{P}{r(1+z)} - \frac{2\ln(1+z)}{rmA} \qquad (24)$$

$$\frac{d\bar{x}}{dz} = \frac{1}{mAr^2}\left[2\ln(1+z) - \left(z + \frac{z}{1+z}\right)\right] \qquad (25)$$

Let W be the term in the parentheses []. $z = 0$ implies $W=0$. $dW/dz = -z^2/(1+z)^2 < 0$.

Thus, for $r \geq 0$, the largest value of W is zero.

$$\frac{d\bar{x}}{dr} < 0 \quad \text{if} \quad r > z/(1+z) \tag{26}$$

We now consider the situation when $r = z/(1+z)$. In this case, increasing r implies that r will become larger than $z/(1+z)$. Equation (26) will apply. Thus, increasing the mean service time reduces the average bribe revenue when $r \geq z/(1+z)$.[8]

Proposition 4 shows that if the server is free to change the speed of service, he will set $r^* = z/(1+z)$ where revenue is at the maximum. This r^* can also be considered part of the Nash equilibrium strategies of the system. If before any bribing occurs, the initial r is larger than r^*, there is incentive for the server to speed up rather than to slow down when bribery is allowed. The contrary of Myrdal's hypothesis may sometimes be true.

Insert Figure 2 Here.

V. Welfare Implications

A. Rent Comparisons

To study the social optimality problem of the bribing mechanism, it is necessary to look at the rent generated rather than just its system time costs. In each period, m customers come to the end of the queue, but only a proportion of Av^* will join it. For a customer joining the queue, the expected rent generated by him is given by

$$P - vW(x(v))$$

where v is the value of time of that customer. In each period, the average expected rent generated by the system is given by

$$R_b = m\int_0^{v*}[P - vW(x(v))]Adv$$

$$= m\int_0^{v*}[P - \frac{vr}{m(1-rAv*+rAv)^2}] Adv \tag{27}$$

If $r > z/(1+z)$, only some incoming customers join the queue. Using (13'), equation (27) can be simplified to

$$R_b = [z - \ln(1+z)]/rA. \tag{27'}$$

If $r \leq z/(1+z)$, all incoming customers join the queue. $v*=v_1$. The rent per period then becomes

$$R_b' = \frac{1+z}{A} + \frac{\ln(1-r)}{rA}. \tag{27"}$$

In (27"), R_b' is used instead of R_b to indicate the different restrictions on the value of r. Both R_b and R_b' can easily be shown to be nonnegative.

To see whether the bribing mechanism is optimal, it is useful to compare its rent with the rent generated by the usual first-come-first-served queue where bribery is not allowed.

Let T be the average system time of a customer joining the first-come-first-served queue. From a standard result of queueing theory (Kleinrock, 1975, Chap. 3),

$$T = \frac{1/u}{1-(m*/u)}. \tag{28}$$

where m* is the average number of new customers joining the queue in each period. Of the m customers who come to the queue each period, only those who expect to receive positive gains will join the queue. In other words, a customer with v will join the queue only if

$$P - vT \geq 0. \tag{29}$$

Let v" be the largest v among those who join the queue. Suppose only a portion of the customers join the queue. It follows easily that $v" < v_1$, and

$$v" = P/T. \tag{30}$$

It also follows that the proportion of customers actually joining the queue is given by Av". Hence,

$$m* = Av"m \tag{31}$$

Making use of (30) and (31), equation (28) can be simplified to

$$T = r(1+z)/m. \tag{28'}$$

Equation (30) also becomes

$$v" = z/(1+z)rA. \tag{30'}$$

The condition that $v" < v_1 = 1/A$ is equivalent to

$z/(1+z)rA < 1/A$,

or, $r > z/(1+z)$

which is the same as condition (14). In this particular model, as long as $r > z/(1+z)$, only some of the incoming customers actually join the queue, whether bribery is allowed or not.

Assume (14) to hold. The rent generated by the first-come-first-served queue in each period is

$$R_* = m \int_0^{v"} (P-vT)Adv \tag{32}$$

Using (30') and (28'), we also get

$$R_* = z^2/2rA(1+z). \tag{32'}$$

Assume $r \leq z/(1+z)$. Then $v'' = v_1$. The rent generated by the queue in one period is then

$$R_f' = \frac{1}{A}\left(z - \frac{r}{2(1-r)}\right) \tag{32''}$$

We can now compare the rents generated by the two queueing systems. Suppose $r > z/(1+z)$.

$$R_b - R_f = ([z - \ln(1+z)]/rA) - z^2/2rA(1+z).$$

Using simple algebra, the expression on the right hand side can be shown to be nonnegative. Thus,

$$R_b - R_f \geq 0. \tag{33}$$

Now assume $r \leq z/(1+z)$.

$$R_b' - R_f' = [r + \ln(1-r) + r^2/2(1-r)]/rA.$$

Again, the expression on the right hand side can be shown to be nonnegative.

$$R_b' - R_f' \geq 0. \tag{33'}$$

Given the assumptions in the model, the queue with bribery is superior to the first-come-first-served queue in terms of rent generated.[9]

Unfortunately, this result is not a general one. For other distributions of $A(v)$, ambiguous answer may occur. There are several things to consider.

First, the bribing mechanism improves the rent because the customers are ranked optimally.

Second, in the bribing mechanism, the expected private gain for the marginal customer joining the queue is $P-x*-v*W(x*)=0$. However, the rent generated by this marginal customer is $P-v*W(x*)$, which is bigger than zero. Unless all the customers are already joining the queue, there is opportunity to generate more rent by having more customers to join it. In the first-come-first-served queue, however, both the rent generated by the marginal customer and his private gain are zero. The queue guarantees that the number of people joining it is optimal.

Third, if more people are in the queue, the average system time of the customers will be longer, and the expected rent will be affected.

In general, it is necessary to know the distribution function A(v) explicitly in order to decide unambiguously which mechanism generates more rent.

B. Adjusting the Number of Customers

The bribing model proposed in this paper can be used as an allocation mechanism similar to that of auction when a queue is involved. Whether the payments to the server are in the form of illegal bribes or in the form of legal prices is immaterial. We have already seen that the strength of the mechanism lies in its ability to rank customers in the most optimal way. However, it does not guarantee the correct number of people in the queue. The latter can be remedied by imposing a uniform fee or giving a uniform subsidy to each of the joining customers.

Let F be the uniform fee imposed on the joining customers, in addition to the bribes they pay. If F is negative, it means a subsidy is given instead. From the point of view of the customers, imposing the fee is not different from reducing the value of the prize P by F. For illustrative purpose, let us retain the assumption of the uniform distribution function of v. From (13), the value of time of the marginal customer becomes[10]

$$v^* = \frac{mA(P - F)}{rA[1 + mA(P-F)]}$$

$$= \frac{z - mFA}{rA(1+z-mFA)} \tag{34}$$

$$\frac{dv^*}{dF} = \frac{-m}{r(1+z-mFA)^2} < 0. \tag{35}$$

Equation (35) means that we can adjust the number of joining customers by changing the value of F. We can also substitute this new value of v* into the rent equation (27) to obtain

$$R_b = \frac{(1+z)(z-mFA)}{rA(1+z-mFA)} - [\ln(1+z-mFA)]/rA \tag{36}$$

$$\frac{dR_b}{dF} = \frac{-FAm^2}{r(1+z-mFA)^2} < 0 \tag{37}$$

The rent in this case can be increased by decreasing F, that is, by giving greater subsidies to attract customers. However, when all incoming customers join the queue, the rent cannot be further increased by giving more subsidies. This can be seen from the fact that the rent R_b' in (27") does not depend on v* and therefore does not depend on F. To summarize, given a uniform distribution function of A(v), the rent will be maximized when all incoming customers are attracted by the subsidies to join the queue. For other functional forms of A(v), we can also go through the same method to find out the subsidy or fee that will generate a socially optimal outcome.

VI. Concluding Remarks

We have constructed a bribing model in the context of an

M/M/1 queue where the decision makings on bribe payments are decentralized to the customers. This does not impose severe informational requirements on the server. The bribing strategies for the customers have been derived. If the customers know their own values of time and a few other parameters, the strategies lead to a Nash equilibrium which is also socially quasi-optimal.

If the server can control the speed of service, he can take the customers' strategies as given, and solve his own maximization problem. Based on this equilibrium bribing model, the contrary of Myrdal's hypothesis has been shown to be possible. Sometimes, the privately optimal speed of service chosen by the server may be faster than the speed without bribery.

The rents generated by the queue and by the ordinary first-come-first-served queue have been derived. The bribing mechanism ranks the customers optimally, but its rent can sometimes be further increased through imposition of appropriate uniform fees or subsidies.

Appendix

We want to examine how Proposition 4 has to be modified if the distribution function of v is a general function A(v) instead of the uniform distribution function Av assumed in the paper. We require that A(v) is a differentiable function with the lowest v at zero, and A'(v) is continuous. We make use of Leibniz Rule for this purpose.

For $v^* < v_1$,

$$\bar{x} = \int_0^{v^*(r)} x(v,r)A'(v)dv. \tag{A-1}$$

Apply Leibniz Rule to (A-1).

$$\frac{d\bar{x}}{dr} = \int_0^{v^*(r)} \frac{dx}{dr} A'(v)dv + x(v^*(r),r)A'(v^*(r))\frac{dv^*(r)}{dr} \tag{A-2}$$

To determine the sign of $d\bar{x}/dr$, it is necessary to know the signs of dv^*/dr and dx/dr. First consider dv^*/dr.

From (11), $x^* = P - (rv^*)/m$.

Equation (8) and the Nash equilibrium condition imply

$$x^* = \int_0^{v^*} \frac{2r^2vdA(v)}{m[1-rA(v^*)+rA(v)]^3}$$

$$= \frac{-rv^*}{m} + \int_0^{v^*} \frac{rdv}{m[1-rA(v^*)+rA(v)]^2}$$

$$P = \int_0^{v^*} \frac{rdv}{m[1-rA(v^*)+rA(v)]^2}. \tag{A-3}$$

Using Leibniz Rule again,

$$0 = dP/dr$$

$$= \int_0^{v*} \frac{d}{dr}\left[\frac{r\,dv}{m(1-rA(v*)+rA(v))^2}\right] + \frac{r(dv*/dr)}{m[1-rA(v*)+rA(v*)]^2}$$

$$= \int_0^{v*} \frac{dv}{m[1-rA(v*)+rA(v)]^2} + \int_0^{v*} \frac{2r[A(v*)-A(v)+r(dv*/dr)A']dv}{m[1-rA(v*)+rA(v)]^3}$$

$$+ (r/m)(dv*/dr) \qquad \text{(A-4)}$$

Suppose $dv*/dr \geq 0$. Then since $A(v*)-A(v)$ in the second term of (A-4) is always positive because $v<v*$, and $A'(v) > 0$, the second term must be strictly positive. The first and the third terms are also positive. The right hand side of (A-4) must therefore be strictly positive. Contradiction.

$$\frac{dv*}{dr} < 0. \qquad \text{(A-5)}$$

Consider dx/dr:

$$\frac{dx}{dr} = \frac{d}{dr}\int_0^{v} \frac{2r^2 v dA(v)}{m[1-rA(v*)+rA(v)]^3}$$

$$= \int_0^{v} \frac{6r^2 v[A(v*)-A(v)+r(dv*/dr)A']dA(v)}{m[1-rA(v*)+rA(v)]^4}$$

$$+ \int_0^{v} \frac{4rvdA(v)}{m[1-rA(v*)+rA(v)]^3} \qquad \text{(A-6)}$$

The second term is clearly positive. In the first term, since dv*/dr is negative, the sign of the numerator is indeterminate.

From (A-5) and (A-6), the first term of (A-2) is indeterminate, while the second term is negative.

$d\bar{x}/dr \lesseqgtr 0 \quad \text{for } v^* < v_1.$ (A-7)

Now consider $v^* = v_1$. (A-1) then becomes

$$\bar{x} = \int_0^{v_1} x(v,r)A'(v)dv \tag{A-8}$$

$$\frac{d\bar{x}}{dr} = \int_0^{v_1} \frac{dx}{dr} A'(v)dv \tag{A-9}$$

since $dv_1/dr = 0$.

$$\frac{dx}{dr} = \int_0^{v} \frac{6r^2v[1-A(v)+r(dv_1/dr)A']dA(v)}{m[1-r+rA(v)]^4} + \int_0^{v} \frac{4rvdA(v)}{m[1-r+rA(v)]^3} \tag{A-10}$$

Since $dv_1/dr = 0$, the first term on the right hand side must be positive. The second term is also clearly positive. Thus, $dx/dr > 0$. From (A-9),

$d\bar{x}/dr > 0 \quad \text{for } v^* = v_1.$ (A-11)

The first part of Proposition 4 is therefore always true. For the case $v^* < v_1$, slowing down causes less people to join the queue because of (A-5). Whether those who stay will pay more bribe is indeterminate. We cannot know unambiguously the sign of $d\bar{x}/dr$. The second part of Proposition 4 should be modified accordingly. But even in this case, it is quite possible to encounter situations contradictory to Myrdal's hypothesis.

It should be pointed out that the algebraic results in the text can also be obtained by using Leibniz Rule.

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Footnotes

[1]Rigid prices often give rise to queues. The waiting time of queues may be regarded as part of the real resource costs of price rigidity. However, as argued by Alchian (1970), stable prices may sometimes be superior to flexible market-clearing prices because the former reduces the search costs of customers who look for lower prices. Bribery in this paper may be viewed as a means to reduce the resource costs further.

[2]Naor assumes that all customers have equal values of time. Suboptimal results are thus avoided in his model. Rose-Ackerman explicitly shows that her system is not optimal. It should be noted that when people differ in their values of time, the Rose-Ackerman system dominates the Naor system in terms of efficiency because it can differentiate the customers to some extent. However, the former also has stronger informational requirements.

[3]The preemptive assumption is not essential, but it greatly reduces the algebra involved. The model can be modified for the non-preemptive case.

[4]Little (1961) shows that the expected number of people in a system is equal to the product of their arrival rate and the expected time they spend in the system.

[5]In (6'), we use the notation $\bar{v}$ to indicate that this is parametric to the maximization problem. In (7') and (8), we use v rather than $\bar{v}$ to indicate that once the maximization is solved, the bribe x is dependent on the variable v.

[6]The argument follows from the fact that the gain of joining the queue is a decreasing function of v, which can be proved

easily.

[7]The ergodicity condition of the queue implies that $rAv^* < 1$. If this is not satisfied, the number of people who come and stay in the queue per period is larger than the number being served. The queue will get infinitely long.

[8]At $r = z/(1+z)$, if r decreases, equation (23) will apply.

[9]Suppose the speed of service is an endogenous variable, as in Proposition 4. Then if bribery shortens the service time, given the other assumptions in the model, it must be better than the first-come-first-served queue. If bribery makes the server act more slowly, the rent comparison will be ambiguous.

[10]The same result can be obtained by going through the argument in Section III again, but the Nash equilibrium strategy of customers with $v = 0$ requires them to pay F instead of zero bribe.

Figure 1

RELATION BETWEEN NET GAIN AND BRIBE

Note:- The line ab joins the maximum points of the curves.

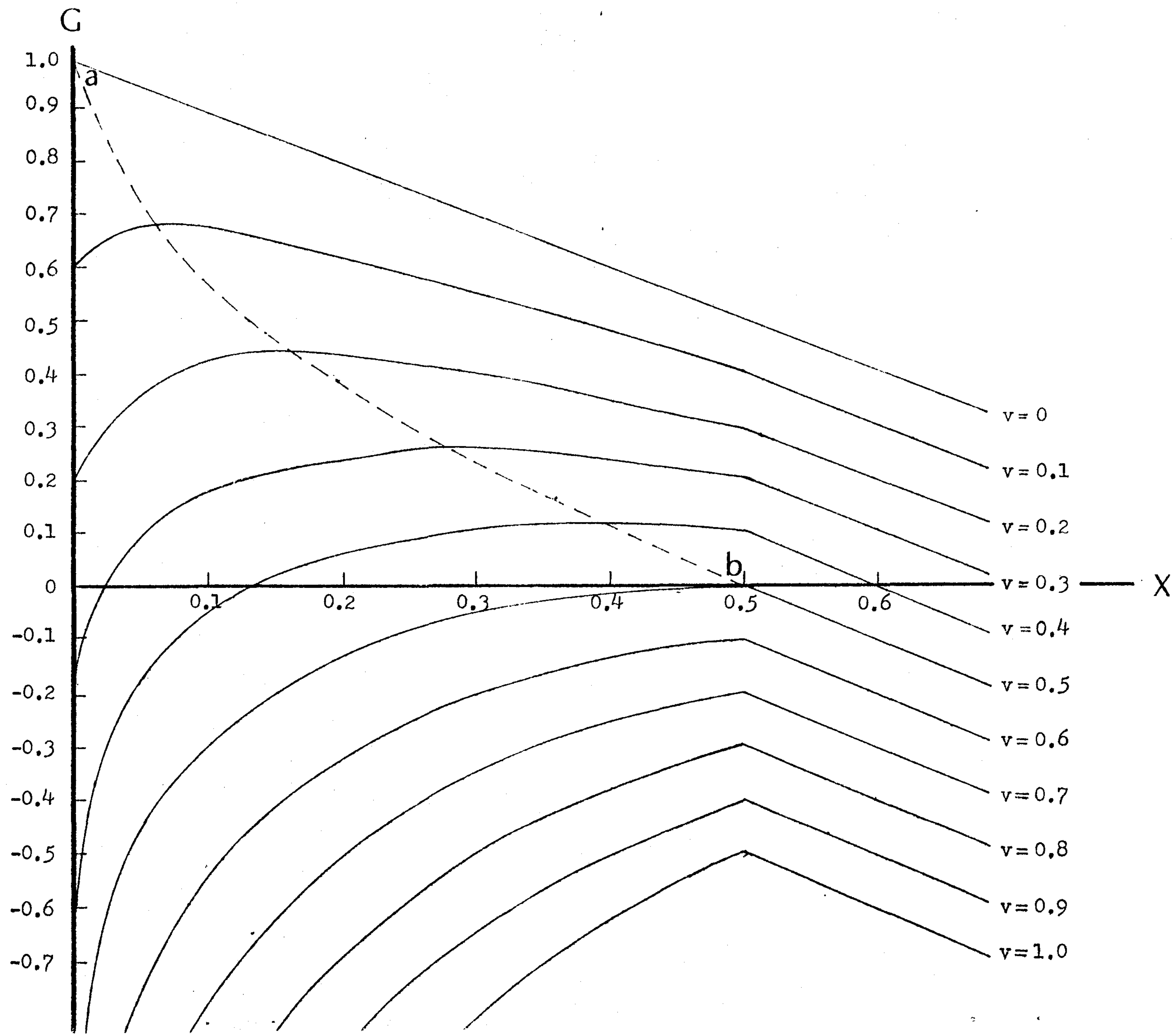
G
X
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0.1
0.2
0.3
0.4
0.5
0.6
a
b
v = 0
v = 0.1
v = 0.2
v = 0.3
v = 0.4
v = 0.5
v = 0.6
v = 0.7
v = 0.8
v = 0.9
v = 1.0

Figure 2

RELATION BETWEEN AVERAGE BRIBE AND THE UTILIZATION INDEX