OPTIMAL TARIFF EQUILIBRIA WITH CUSTOMS UNIONS

by

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I. Introduction

Optimal tariff retaliation results for the two country-two good case are well known (Johnson 1958). Using simulation techniques we analyze the three country-three good case. With three countries one has to allow for customs unions between any pair of countries. Hence, the examples we construct not only relate to the optimal tariff literature, but the customs unions literature as well. In addition, this paper helps display the relationship between optimal tariffs and customs unions.

We analyze three different examples. Example 1 shows that with three countries, each importing two goods, a tariff externality exists. This occurs because if two countries import the same good, a tariff by one country on this good will benefit the other country which imports it. In a customs union where tariffs to non-members are jointly determined this externality is internalized. This suggests that one reason customs unions exist is that they are a way of internalizing this tariff externality.

The second example shows that all members of a customs union could be better off than at free trade even taking optimal retaliation into account. This is one way the two country retaliation results can be generalized. In addition, we briefly discuss transfer payments between countries in the context of this example.

In the third example one country can benefit from a tariff war even if the other countries form a customs union and optimally retaliate. In fact, under reasonable conditions this would be the equilibrium outcome. This example suggests an alternative explanation of customs unions. They might exist, not because they are welfare improving compared to free trade, but because they are the optimal response to tariffs initiated by a third country.

It is hoped that these simulations not only provide useful insights but point the way to further analytical results.

II. The Model

A three country, three good linear expenditure system is developed. Although each country has some endowment of every good, endowments and preferences are such that each country exports just one good, and imports the other two: country 1 exports good 1, etc. We assume that each country includes a large number of individuals with identical endowments and tastes. The standard individual has a Cobb-Douglas utility function:

$$(1) \qquad U^i = \sum_{j=1}^{3} \beta_j^i \log X_j^i, \; i=1,\ 2,\ 3; \qquad \sum_{j=1}^{3} \beta_j^i = 1,$$

where U^i is utility of country i, X_j^i is consumption of good j in country i and β_j^i is a parameter. Superscripts denote countries and subscripts denote goods. Y_j^i is the aggregate endowment of commodity j in country i and imports are given by

$$(2) \qquad M_j^i \equiv X_j^i - Y_j^i \qquad i,\ j = 1,\ 2,\ 3.$$

Material balance is assumed to hold:

$$(3) \qquad \sum_{s=1}^{3} X_k^s = \sum_{s=1}^{3} Y_k^s, \qquad k = 1,\ 2,\ 3.$$

P_k is the world price of commodity k and W_j^i is the expenditure on commodity j in country i, valued at world prices:

$$(4) \qquad W_j^i \equiv P_j X_j^i.$$

Then the world price of each good is the ratio of world expenditure on that good to the world's endowment of that good:

$$(5) \qquad P_k = \frac{\sum_{s=1}^{3} W_k^s}{\sum_{s=1}^{3} Y_k^s} \qquad k = 1, 2, 3.$$

Aggregate income in country i, I^i, is the sum of the domestic value of production plus tariff proceeds:

$$(6) \qquad I^i = \sum_{k=1}^{3} P_k(1+t_k^i)Y_k^i + P_k t_k^i M_k^i, \qquad i = 1, 2, 3.$$

where t_k^i is the tariff country i charges on imports of commodity k ($t_j^i \geq 0$, $t_i^i = 0$, i, j = 1, 2, 3). Rearranging terms,

$$(7) \qquad I^i = \sum_{k=1}^{3} P_k Y_k^i + t_k^i W_k^i, \qquad i = 1, 2, 3.$$

Each individual in country i faces prices (P_j $(1+t_j^i)$ and maximizes the utility function (1) given an income proportional to I^i. The aggregate demand functions can then be written as

$$(8) \qquad X_j^i = \frac{\beta_j^i I^i}{P_j(1+t_j^i)} \qquad i, j = 1, 2, 3.$$

Rearranging terms and using (5), (8) becomes

$$(9) \qquad (1+t_j^i)W_j^i = \beta_j^i \sum_{k=1}^{3} \left\{ \frac{\sum_{s=1}^{3} W_k^s}{\sum_{s=1}^{3} Y_k^s} Y_k^i + t_k^i W_k^i \right\}$$

Let g_k^i denote the fraction of the world's endowment of good k which is held by country i:

$$g_k^i \equiv \frac{Y_k^i}{\sum_{s=1}^{3} Y_k^s}, \qquad \sum_{i=1}^{3} g_k^i = 1.$$

Let W^i denote i's total expenditure. Then from (9),

$$(10) \qquad W^i = \sum_{j=1}^{3} W_j^i = \sum_{j=1}^{3} \left\{ \frac{\beta_j^i}{1+t_j^i} \sum_{k=1}^{3} \left(g_k^i \sum_{s=1}^{3} W_k^s + t_k^i W_k^i\right) \right\}$$

Define $\theta_j^i \equiv \dfrac{\beta_j^i}{1+t_j^i}$, $\quad \theta^i \equiv \sum_{j=1}^{3} \theta_j^i$, $\quad \lambda_j^i \equiv \dfrac{\theta_j^i}{\theta^i}$; $\quad \sum_{j=1}^{3} \lambda_j^i = 1.$

Note that λ_j^i is expenditure on good j in country i, as a fraction of total expenditure in country i: $W_j^i = \lambda_j^i W^i$. Thus, from (10),

$$(11) \qquad W^i = \theta^i \sum_{k=1}^{3} \left(g_k^i \sum_{s=1}^{3} \lambda_k^s W^s + t_k^i \lambda_k^i W^i\right).$$

Equation (11) can be further simplified by defining

$$\mu^i = \sum_{k=1}^{3} t_k^i \lambda_k^i \qquad \delta_s^i = \sum_{k=1}^{3} g_k^i \lambda_k^s.$$

Then

$$(12) \qquad (1-\theta^i \mu^i) W^i = \theta^i \sum_{s=1}^{3} \delta_s^i W^s \qquad i = 1,2,3.$$

We normalize by letting $\sum_{i=1}^{3} W^i = 1$. This, together with (12), gives

$$(13)\qquad \begin{pmatrix} 1-\theta^1(\mu^1+\delta_1^1) & -\theta^1\delta_2^1 & -\theta^1\delta_3^1 \\ -\theta^2\delta_1^2 & 1-\theta^2(\mu^2+\delta_2^2) & -\theta^2\delta_3^2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} W^1 \\ W^2 \\ W^3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} .$$

Given preferences, endowments and tariffs, the matrix in (13) is determined, so the equilibrium values of W^i can be found by inverting this matrix. Since $W_j^i = \lambda_j^i W^i$, the equilibrium price vector can be found from equation (5), and consumption is then determined by equation (8). Finally, utilities are obtained by inserting the equilibrium consumption values in equation (1).

Optimal Tariffs

Fix endowments and preferences and let $U^i(T)$ denote the utility of country i, in equilibrium, given an arbitrary tariff matrix $T \equiv (t_j^i)$. Let U_j^i be the derivative of this function with respect to t_j^i. Then the first order condition for optimal tariffs in country i is that U_j^i is zero for all $j \neq i$: given the tariffs charged by others, country i varies its tariffs so that the resulting equilibrium yields maximal utility. The effects of tariff changes are complicated, and we do not have closed-form optimal tariff formulae except for the 2 x 2 case with uniform preferences (see Appendix 2). In general, we use numerical approximations of the derivatives $U_j^i(T)$ and $U_{jk}^i(T)$ (the derivative of $U_j^i(T)$ with respect to t_k^i) and compute the optimal tariff by Newton's method.[1]

1. We have verified some of our results by using an alternative grid search procedure.

A Cournot-Nash tariff equilibrium is a tariff matrix T* such that $U^i_j(T^*) = 0$, for all $j \neq i$, and for all i. As usual, this means that no country could gain by changing its tariffs, given the tariffs charged by the other countries. The equilibrium is characterized by a system of six equations in the six off-diagonal elements of T, which can be solved, in principle, by a suitable variant of Newton's method. In practice we have found that a cobweb algorithm works well: the first country finds its optimal tariffs, given the tariffs set by countries 2 and 3, then the second country choses t^2_1 and t^2_3, given the other elements of T, and so on until T repeats itself.

Equilibrium with a Customs Union

A Customs Union is a coalition whose members charge zero tariffs on imports from other member countries, and a common external tariff on imports from non-member countries. In general, there will be a conflict of interest between member countries in choosing the common external tariff. We deal with this by specifying one country as the dominant member of the union: for example, the customs union {1, 2} selects external tariffs to maximize U^1, while {2, 1} maximizes U^2.

In the examples below we explore the implications of a two-person noncooperative game between a two-country customs union and the excluded country. This specification is asymmetric in that the excluded country is not allowed to make binding commitments regarding tariffs, although the customs union members are allowed to do so. In one of our examples, free trade cannot be sustained by a scheme of transfer payments between countries, unless each country is allowed to commit itself to an artificially high (non-optimal) tariff in case the other two countries form a customs union.

III. Results

The results are an extension to three countries of Harry Johnson's original analysis on tariff retaliation. However, with three countries it is possible for customs unions to form, thus complicating the analysis. Our examples illustrate the different kinds of results one can obtain. First, assuming no customs union forms we can analyze a tariff war with three countries. In examples 1 and 2 all countries lose at the tariff retaliation equilibrium. In example 3 one country gains from a tariff war. Our conjecture is that an example can be produced in which two countries gain from a tariff war.

A number of new results arise when we allow customs unions. In a three country model in which each country imports two goods a customs union enables the member countries to internalize an externality that exists with tariffs charged to the third country. This is illustrated in Example 1. Example 2 illustrates a situation in which any customs union improves both members' welfare compared to free trade. Thus, customs unions dominate the tariff retaliation equilibrium and free trade for member countries even allowing for optimal retaliation by the third country.

Example 3 suggests an alternative explanation for customs unions. In Example 3, country 1 benefits from a tariff war. Suppose that countries 2 and 3 form a customs union. It turns out that country 1 still is better off than at free trade. Countries 2 and 3 are better off than at the tariff retaliation equilibrium but worse off than at free trade. Furthermore, the {2, 3} customs union appears to be a stable solution. This suggests that customs unions might exist, not because they improve welfare as compared to free trade, but because they are the best response to a tariff war started by the other country. We next discuss each example in detail.

Example 1. (See Appendix 1 for details of the examples). Countries have symmetric endowments, and they all have a preference for good 1. At the optimal tariff equilibrium, all countries are worse off than at free trade. Now consider a customs union between countries 2 and 3. A customs union means that member tariffs to non-members are the same and members charge no tariffs to each other, $t_1^2 = t_1^3$ and $t_3^2 = t_2^3 = 0$ in this case. One way to compute t_1^2 and t_1^3 is to let each country set their tariffs independently. Since in this example countries 2 and 3 are symmetric it follows that t_1^2 will equal t_1^3 in the final equilibrium. The results of this simulation are listed as CU{2, 3} (no joint strategy). Notice that both countries are worse off than at the optimal tariff equilibrium. How can this surprising result be explained?

The explanation can be seen by considering what happens in country 3 when t_1^2 increases. An increase in t_1^2 will cause the relative price of good 1 to go down with respect to the other two goods. This improves country 3's welfare. If the two countries coordinate their tariffs they can internalize this externality.

In the simulations, customs unions internalize the externality by letting one customs union member choose the tariff for the customs union.[2]

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2. In this case, countries 2 and 3 have no conflict of interest over what the tariff should be. In general, however, there will be a conflict of interest over the external tariff.

These results are reported under CU{2, 3} (joint strategy). Notice that the tariff to the non-member countries is much higher (3.63 compared to .50) and that the member countries are better off than at the optimal tariff equilibrium. In the rest of our examples we use the joint strategy concept in choosing customs union tariffs. Notice that this externality is something that arises only with three or more goods. It appears to be a general phenomenon that exists whenever two countries import the same good. It may also provide an additional motivation for customs union formation.

Wonnacott and Wonnacott (1981) argue that customs unions are different from unilateral tariff reductions because in customs unions member countries not only reduce their own tariffs but require the other members to reduce theirs. The tariff externality illustrated by Example 1 provides a similar motivation for forming customs unions: Not only can a country benefit from tariff reductions within a customs union but by cooperatively setting tariffs to the outside this tariff externality can be internalized.

Example 2. Here the countries are completely symmetric. As must be the case, they all lose from a tariff war. In the optimal tariff equilibrium, prices are the same as at free trade but the volume of trade is reduced. The interesting result from this example is that a customs union by any pair of countries will result in welfare improvement compared to free trade for all member countries. Thus, starting from a completely symmetric situation any pair of countries can improve their welfare by cooperating.

The intuition for this result can be seen by thinking of the customs union as a single country.[3] Consider the {2, 3} customs union. The model reduces to two countries, 1 and {2, 3}, which have identical preferences. Country 1's endowment is (.3, .1, .1) and {2, 3} endowment is (.2, .4, .4). The results is that {2, 3} benefits from a tariff war. Our interpretation of this result is that this occurs because the endowment of {2, 3} is more even than the endowment of country 1. Given that the preferences for all three goods are the same a more even endowment means less dependence on trade and hence a better chance of benefitting from a tariff war. Of course, we have only produced one example and have no proof that evenness of endowment is the cause of the welfare gain. However, this example suggests that one should look for analytical results of this type.

Johnson's results were that in the two country case elasticities determined which country gains from a tariff war. However, without his assumption of constant elasticities it is not clear that this result will generalize.

This example is also useful for discussing the issue of whether allowing transfer payments between countries facilitates attainment of free trade. This is an example in which transfers would not help. First since the countries are completely symmetric, no country would agree to be a net transferer at free trade. Therefore, the only candidate is free trade with zero transfers. But, we know that any two countries will benefit

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3. One has to be careful with this approach. Here it is not misleading to think of the customs union as one country because the countries are symmetric.

from a customs union even after taking account of retaliation, hence free trade is not a stable equilibrium. Other examples can be constructed in which transfers do lead to free trade. Hence, it is of interest to analyze what determines whether or not transfers lead to free trade.

Example 3. All countries' preferences are symmetric over the goods. Countries 2 and 3 have symmetric endowments, but Country 1 has a more even endowment. In the optimal tariff equilibrium country 1 benefits while the others lose. This occurs because the terms of trade shift strongly in Country 1's favor. Now suppose countries 2 and 3 decide to form a customs union. Forming the union makes 2 and 3 better off and 1 worse off than the optimal tariff equilibrium, but country 1 is still better off than at free trade. One can think of this as an extension of Johnson's results in the sense that with three countries a country can benefit from a tariff war even if the other countries react by forming a customs union and setting optimal tariffs.

The intuition for this result is essentially the same as example 2. Treat the {2, 3} customs union as one country. Then country 1 has an endowment of (2, .6, .6) and {2, 3} has an endowment of (.2, 1.2, 1.2). Country 1's endowment is more even in the sense of comparing the endowment of the three goods pairwise. Hence, 1 benefits from the tariff war.

Consider now a customs union between countries 1 and 2 (1 and 3 is symmetric). We now have the problem of a conflict of interest over the union's tariff. In Example 3 we compute each country's desired tariff. It turns out that country 2 desires a larger tariff than country 1. However, for both tariff rates the qualitative results are the same, and they are surprising. Country 1 is worse off in a union with 2 (or 3) than it is when 2 and 3 form a customs union against 1. However, in a customs union with country 2 (or 3), country 1 is still better off than with free trade. Country 2(3) is best off in a customs union with 1. Hence, countries 2 and 3 desire a union with 1, but 1 does best when it refuses to cooperate with either country. It seems under these circumstances country 1 would change tariffs and refuse to cooperate. Given this behavior, the best thing for 2 and 3 to do is to form a customs union. This seems to be a stable equilibrium. Hence this suggests a different way of viewing customs unions. A {2, 3} customs union arises not because it makes its members better off than free trade, but because it is the best response to country 1's behavior.

References

Johnson, H. E. "Optimal Tariffs and Retaliation," Chapter II in _International Trade and Economic Growth_, 1958.

Wonnacott, P. and R. Wonnacott. "Is Unilateral Tariff Reduction Preferable to a Customs Union? The Curious Case of the Missing Foreign Tariffs," _American Economic Review_, September 1981, 71 (4), pp. 704-14.

Appendix 1

Example 1

$\beta = (.9,\ .05,\ .05) \qquad G = \begin{pmatrix} 1.1 & .1 & .1 \\ .1 & 1.1 & .1 \\ .1 & .1 & 1.1 \end{pmatrix}$

	Utility[a]	Tariffs	Prices	Consumption
Free Trade Equilibrium	(3000.00, 2810.29, 2810.29)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	(.69, .04, .04)	$\begin{pmatrix} 1.00 & 1.00 & 1.00 \\ .15 & .15 & .15 \\ .15 & .15 & .15 \end{pmatrix}$
Optimal Tariff Equilibrium	(2999.54, 2798.96, 2798.96	$\begin{pmatrix} 0 & 1.10 & 1.10 \\ 2.80 & 0 & 2.83 \\ 2.80 & 2.83 & 0 \end{pmatrix}$	(.70, .03, .03)	$\begin{pmatrix} 1.05 & .60 & .60 \\ .123 & .56 & .14 \\ .123 & .14 & .56 \end{pmatrix}$
CU{2, 3} (no joint strategy) Equilibrium	(3001.68, 2797.48, 2797.48)	$\begin{pmatrix} 0 & 1.45 & 1.45 \\ .50 & 0 & 0 \\ .50 & 0 & 0 \end{pmatrix}$	(.72, .02, .02)	$\begin{pmatrix} 1.06 & .70 & .70 \\ .121 & .30 & .30 \\ .121 & .30 & .30 \end{pmatrix}$
CU{2, 3} (joint strategy) Equilibrium	(2996.09, 2804.87, 2804.87)	$\begin{pmatrix} 0 & .76 & .76 \\ 3.63 & 0 & 0 \\ 3.63 & 0 & 0 \end{pmatrix}$	(.67, .05, .05)	$\begin{pmatrix} 1.04 & .44 & .44 \\ .126 & .43 & .43 \\ .126 & .43 & .43 \end{pmatrix}$

a. The Utilities we report are actually $3000 + 100\ U^i_j$.

Example 2

$\beta = (.33,\ .33,\ .33)$ $\qquad G = \begin{pmatrix} .3 & .1 & .1 \\ .1 & .3 & .1 \\ .1 & .1 & .3 \end{pmatrix}$

	Utility	Tariffs	Prices	Consumption
Free Trade Equilibrium	(2820.82, 2820.82, 2820.82)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	(.67, .67, .67)	$\begin{pmatrix} .17 & .17 & .17 \\ .17 & .17 & .17 \\ .17 & .17 & .17 \end{pmatrix}$
Optimal Tariff Equilibrium	(2818.52, 2818.52, 2818.52)	$\begin{pmatrix} 0 & .56 & .56 \\ .56 & 0 & .56 \\ .56 & .56 & 0 \end{pmatrix}$	(.67, .67, .67)	$\begin{pmatrix} .22 & .140 & .140 \\ .140 & .22 & .140 \\ .140 & .140 & .22 \end{pmatrix}$
CU{2, 3} Equilibrium	(2814.29, 2821.03, 2821.03)	$\begin{pmatrix} 0 & .46 & .46 \\ .68 & 0 & 0 \\ .68 & 0 & 0 \end{pmatrix}$	(.58, .71, .71)	$\begin{pmatrix} .23 & .128 & .128 \\ .135 & .18 & .18 \\ .135 & .18 & .18 \end{pmatrix}$

Example 3

$\beta = (.33,\ .33,\ .33)$ $\qquad G = (Y^i_j) = \begin{pmatrix} 2 & .6 & .6 \\ .1 & 1.1 & .1 \\ .1 & .1 & 1.1 \end{pmatrix}$

	Utility	Tariffs	Prices	Consumption
Free Trade Equilibrium	(3001.08, 2921.66, 2921.66)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	(.15, .18, .18)	$\begin{pmatrix} 1.16 & .94 & .94 \\ .52 & .43 & .43 \\ .52 & .43 & .43 \end{pmatrix}$
Optimal Tariff Equilibrium	(3004.30, 2902.81, 2902.81)	$\begin{pmatrix} 0 & 2.04 & 2.04 \\ .78 & 0 & .92 \\ .78 & .92 & 0 \end{pmatrix}$	(.214,.146,.146)	$\begin{pmatrix} 1.70 & .82 & .82 \\ .25 & .64 & .37 \\ .25 & .37 & .64 \end{pmatrix}$
CU{2, 3} Equilibrium	(3002.44, 2907.20, 2907.20)	$\begin{pmatrix} 0 & 1.90 & 1.90 \\ .54 & 0 & 0 \\ .54 & 0 & 0 \end{pmatrix}$	(.205,.153,.153)	$\begin{pmatrix} 1.71 & .79 & .79 \\ .24 & .50 & .50 \\ .24 & .50 & .50 \end{pmatrix}$
CU{1, 2} Equilibrium (1 set tariff)	(3002.39, 2929.04, 2891.27)	$\begin{pmatrix} 0 & 0 & 1.40 \\ 0 & 0 & 1.40 \\ .76 & .75 & 0 \end{pmatrix}$	(.176, .21, .12)	$\begin{pmatrix} 1.31 & 1.07 & .77 \\ .63 & .51 & .37 \\ .26 & .22 & .66 \end{pmatrix}$
CU{1, 2} Equilibrium (2 sets tariff)	(3001.73, 2932.31, 2870.39)	$\begin{pmatrix} 0 & 0 & 3.7 \\ 0 & 0 & 3.7 \\ .58 & .57 & 9 \end{pmatrix}$	(.195, .34, .08)	$\begin{pmatrix} 1.34 & 1.09 & .71 \\ .67 & .55 & .36 \\ .18 & .15 & .73 \end{pmatrix}$

Appendix 2

An Optimal Tariff Formula: The 2 x 2 case with Uniform Preferences

In order to provide some analytical understanding of the simulation results, we derive an explicit tariff formula for a simplified case, with two countries and two goods. In this case equation (13) reduces to

$$(1 - \theta^1\mu^1)\, W^1 = \theta^1\delta_1^1\, W^1 + \theta^1\delta_2^1\, (1-W^1) \tag{A1}$$

(using the normalization $W^2 = 1-W$). From equation (5)

$$P_1 = \frac{W_1}{Y_1}\ ;\ P_2 = \frac{W_2}{Y_2} \tag{A.2}$$

where $W_j \equiv W_j^1 + W_j^2$, $Y_j \equiv Y_j^1 + Y_j^2$, $j = 1, 2.$

Then, from equation (4).

$$X_1^1 = \frac{W_1^1}{W_1}\, Y_1,\ X_2^1 = \frac{W_2^1}{W_2}\, Y_2 \tag{A.3}$$

We simplify the optimal tariff problem by assuming $\beta_1^i = \beta_2^i = ½$, $i = 1, 2$ ("uniform preferences"). Then country 1 maximizes $X_1^1 X_2^1$, which is equivalent to:

$$\min_{t_2^1}\ \frac{Y_1}{X_1^1}\ \frac{Y_2}{X_2^1} \tag{A.4}$$

From (A.3),

$$\frac{Y_1}{X_1^1} = \frac{W_1}{W_1^1} = \frac{W_1^2}{W_1^1} + 1 \tag{A.5}$$

Combine this with equation (10) to obtain

$$\frac{Y_1}{X_1^1} = \frac{\theta_1^2 \theta^1}{\theta_1^1 \theta^2} \frac{W^2}{W^1} + 1$$

(A.6) $$= \frac{\theta_1^2}{\theta_1^1} \frac{\theta^1}{\theta^2} \left(\frac{1}{W^1} - 1\right) + 1$$

Then, using (A.1)

(A.7) $$\frac{Y_1}{X_1^1} = \left[\frac{\theta_1^2}{\theta_1^1} \frac{\theta^1}{\theta^2} \frac{(1-\theta^1\mu^1 - \theta^1\delta_1^1)}{\theta^1\delta_2^1} + 1\right]$$

Similarly,

(A.8) $$\frac{Y_2}{X_2^1} = \left[\frac{\theta_2^2}{\theta_2^1} \frac{\theta^1}{\theta^2} \frac{(1-\theta^1\mu^1 - \theta^1\delta_1^1)}{\theta^1\delta_2^1} + 1\right]$$

The optimal tariff problem for country 1 now reduces to the minimization of the product of equations (A.7) and (A.8). Note that θ_1^2, θ_1^1, θ^2 and δ_2^1 are independent of country 1's tariff t_2^1. Also,

(A.9) $$\theta_1^1 = \beta_1^1 = \tfrac{1}{2}; \quad \theta_2^1 = \frac{\beta_2^1}{T_2^1} = \frac{1}{2T_2^1}$$

$$\theta^1 = \frac{1 + T_2^1}{2\, T_2^1}$$

where $T_2^1 \equiv 1 + t_2^1$

(A.10) $$\delta_2^1 = g_1^1 \lambda_1^2 + g_2^1 \lambda_2^2$$

$$= \frac{g_1^1 + g_2^1 T_1^2}{1 + T_1^2}$$

(A.11) $$\delta_1^1 = \frac{g_1}{2\theta^1} + \frac{g_2^1}{2T_2^1\theta^1} \;;\; \theta^1\delta_1^1 = \tfrac{1}{2}(g_1^1 + \frac{g_2^1}{T_2^1})$$

(A.12) $$\mu^1 = t_2^1\,\lambda^1 = \frac{t_2^1}{2(1+t_2^1)\theta^1}$$

(A.13) $$\theta^1\mu^1 = \frac{t_2^1}{2(1+t_2^1)} = \tfrac{1}{2}(1 - \frac{1}{T_2^1})$$

Then, using (A.11) and (A.13),

(A.14) $$\begin{aligned} 1 - \theta^1\,\mu^1 - \theta^1\delta_1^1 &= (\tfrac{1}{2} - \theta^1\mu^1) + (\tfrac{1}{2} - \theta^1\delta_1^1) \\ &= \tfrac{1}{2}\,\frac{1}{T_2^1} + \tfrac{1}{2}\,g_1^2 - \tfrac{1}{2}\,\frac{g_2^1}{T_2^1} \\ &= \tfrac{1}{2}\,(g_1^2 + \frac{g_2^2}{T_2^1}) \end{aligned}$$

Substitute this result in (A.7) to get

(A.15) $$\frac{Y_1}{X_1^1} = [1 + \frac{\theta_1^2}{\theta^2\delta_2^1}\,(g_1^2 + \frac{g_2^2}{T_2^1})]$$

Also, from (A.8)

(A.16) $$\frac{Y_2}{X_2^1} = [1 + \frac{\theta_2^2 T_2^1}{\theta^2\delta_2^1}\,(g_1^2 + \frac{g_2^2}{T_2^1})]$$

Then, according to (A.4), the optimal tariff problem involves choosing T_2^1 to minimize the product of (A.15) and (A.16). This can be written as

(A.17) $$\min_T \quad a\,T + \frac{b}{T}, \text{ where}$$

$$a \equiv (1 + \frac{\theta_1^2 g_1^2}{\theta^2\delta_2^1})\,\frac{\theta_2^2 g_1^2}{\theta^2\delta_2^1}\,, \text{ and}$$

$$b \equiv (1 + \frac{\theta_2^2 g_2^2}{\theta^2\delta_2^1})\,\frac{\theta_1^2 g_2^2}{\theta^2\delta_2^1}$$

The solution of (A.17) is

$$T_2^1 = \sqrt{b/a} \tag{A.18}$$

Finally, from the definitions of a and b

$$b/a = \frac{(\theta^2\delta_2^1 + \theta_2^2\, g_2^2)\ (\theta_1^2\, g_2^2)}{(\theta^2\delta_2^1 + \theta_1^2\, g_1^2)\ (\theta_2^2\, g_1^2)} \tag{A.19}$$

$$= \frac{((1 + T_1^2)\ \delta_2^1 + T_1^2\, g_2^2)\ g_2^2}{((1 + T_1^2)\ \delta_2^1 + g_1^2)\ T_1^2\, g_1^2}$$

Then using (A.10)

$$b/a = \frac{(g_1^1 + T_1^2)\ g_2^2}{(1 + g_2^1\, T_1^2)\ T_1^2\, g_1^2} \tag{A.20}$$

The conclusion of all this is a reasonably simple optimal tariff formula:

$$T_2^1 = \sqrt{\frac{(g_1^1 + T_1^2)\ g_2^2}{(1 + g_2^1\, T_1^2)\ T_1^2\, g_1^2}} \tag{A.21}$$

Nash Equilibrium with Symmetric Endowments

Suppose $g_1^2 = g_2^1 \equiv 1-\gamma$. Then, by symmetry, the Nash equilibrium tariffs satisfy

$$T = \sqrt{\frac{(\gamma + T)\ \gamma}{(1 + (1-\gamma)T)\ (1-\gamma)\ T}} \tag{A.22}$$

Then

(A.23) $T^2 \; (1 + (1-\gamma) \; T) \; (1-\gamma) \; T = (\gamma + T) \; \gamma$

(A.24) $(1-\gamma)^2 \; T^4 + (1-\gamma) \; T^3 - \gamma T - \gamma^2 = 0$

This equation can be factorized as

(A.25) $[(1-\gamma) \; T^2 - \gamma] \; [(1-\gamma) \; T + \gamma] \; (T + 1) = 0$

Since $0 < \gamma < 1$ this equation has just one positive root, so the Nash equilibrium tariff is given by this root, i.e.

(A.26) $T^* = \sqrt{\frac{\gamma}{1-\gamma}}$

Thus the larger is the disparity in endowments, the larger is the equilibrium tariff.