IMPLICIT CONTRACT THEORY: A CRITICAL SURVEY

by

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1. Introduction

The purpose of this paper is to summarize the state of the art on implicit contract theory by sorting out different models with respect to crucial assumptions. We will start with a very simple framework with straight-forward results and then add complications as we proceed to show more interesting results. It is heuristic to understand an implicit contract model as a general equilibrium model with incomplete information, as well as a macroeconomic model.

Implicit contract theory, originally proposed by Azariadis (1975), Baily (1974), and D. Gordon (1974), regards a wage contract as a form of risk-sharing between a risk-neutral firm (owner) and workers. It will be shown that it is important to view implicit contract theory as a branch of the economic theory of uncertainty, especially with incomplete markets and imperfect information. We know that in an uncertain world (with perfect information about probability distributions), creating markets for contingent claims a la Arrow and Debreu would make it possible for a competitive economy to achieve *ex ante* Pareto optimality. We will keep this fact in mind and will point out what causes deviations from an Arrow-Debreu model, in the discussion of various models with implicit contracts. Thus an Arrow-Debreu model with complete contingent claims markets will serve as a benchmark in implicit contract theory. We will look at non-Pareto optimal allocations, i.e., involuntary unemployment, from the viewpoint that inefficiencies are a result of both restrictions on obtaining information and incentive problems in having contingent claims markets.

The seminal papers by Azariadis, Baily, and Gordon were followed by Negishi (1979), Sargent (1979), Akerlof and Miyazaki (1980), Polemarchakis (1979), Polemarchakis and Weiss (1978), Holmstrom (1980), and Geanakoplos and Ito (1981). These contributed to the literature debating whether "involuntary unemployment" can exist in an economy with implicit contracts. Researchers agree that Azariadis's result of having involuntary unemployment with implicit contracts depends on several assumptions. First, the value of leisure or an alternative opportunity must be high enough (pointed out by Azariadis himself and stressed by Negishi); second, *ex post* mobility should be limited (Akerlof and Miyazaki); and third, no severance payments are available (pointed out by Sargent). If there are no alternative opportunities such as leisure, it is trivial to obtain full employment as an equilibrium with or without contracts. If there is *ex post* mobility, the contract becomes infeasible since competition in the *ex post* spot market forces the wage to be at least equal to the marginal product. Holmstrom solved the difficulty in accomodating *ex post* mobility in implicit contract theory by considering a two-period model. In a sense, he showed that a premium for insurance coverage against lower wages in the second period can be paid in the first period. Working in a dynamic framework similar to Holmstrom's, Arnott, Hosios, and Stiglitz (1980), Hosios (1980), and Geanakoplos and Ito (1981) investigated a model with asymmetric information between firms. When *ex post* mobility is introduced, it is important to know what kinds of wages other firms are offering in the *ex post* spot market. All three papers emphasize how a firm and its workers are restricted in obtaining information on the state of nature of *other* firms. They differ in assumptions on the availability of aggregate information and the equilibrium condition of the spot market.

Another branch of implicit contract theory was developed from the idea that workers do not have complete information about the state of nature of their own firms. This category includes Hall and Lilien (1979), Azariadis (1980), Green (1980) and S. Grossman and Hart (1980). Since Azariadis (1979) surveyed this line of research well, I will not discuss it here. Instead, the rest of this paper is devoted to clarifying questions about hidden assumptions and their validity in the models presented by Azariadis (1975), Polemarchakis and Weiss (1978), Holmstrom (1980), and Geanakoplos and Ito (1981). Since the purpose of this paper is to contrast different results, the simplest possible, instead of the theoretically most general, framework is employed.

It is convenient for later reference to summarize the table of content of the following sections.

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2. Model

2.1 Framework

We consider an economy in which there are many risk-averse workers participating in the labor market. The marginal product of labor varies depending on the state of nature. The uncertainty is technological in our model, rather than uncertainty about the price of the products. Competitive (price-taking) behavior is assumed on the part of both firms and workers. In the spot market for labor without contracts, competitive behavior means that every agent is assumed to regard the wage rate as exogenously given and to ignore the effect of own behavior on the (equilibrium) wage rate. When we consider the labor market with contracts, competitive behavior means that every agent regards as given the value of the contract, which consists of wage offers in different states of nature instead of a single wage rate. In the following, it is assumed that there is no entry into this market, implying that there is a fixed number, say N, of identical firms producing the consumption good. The representative firm hires workers, pays the wage bill and is left with profits. We assume that owners of the firm consume its profits. Assuming technological uncertainty (as opposed to price uncertainty) and firm owners who consume profits, we interpret this model as a simple general equilibrium model, as noted in Holmstrom (1980).

Let us assume that all the workers are identical in tastes and productive ability. The labor supply decision is either all or nothing, abstracting from the leisure-work choice:

1 if working at the firm

0 if staying at home

The representative worker's utility function is denoted by

$$U(x), \qquad U' > 0,\ U'' < 0,$$

where x is the sum of wage and non-wage income. When a worker stays at home, he is assumed to be able to employ himself with a home production technology. We will explain this in detail in the next subsection. We assume that workers are "non-atomic," being uniformly distributed in the interval $[0, L^s]$, and that there are N of these intervals. This is equivalent to saying that there are $N \times L^s$ (discrete) workers and that the integer problem is ignored.[1/] The potential labor force per firm, L^s, will be interpreted as the size of the firm's "labor pool." However, when the workers and firms in the market decide the equilibrium wage rate (or contract value) through a tâtonnement process, the labor pool per firm is not a priori fixed. The labor pool is endogenously determined, simultaneously with the equilibrium wage rate (or contract value).

The representative firm j has a production function which is dependent on its own state of nature:

$$f^j(L;\ s^j), \qquad s^j \in \{G,\ B\}$$

where L is the number (or length of an interval) of workers employed, and the state of nature s^j can be either G (good) or B (bad). Let us denote the marginal product in state of nature s^j by $f^{j\,\prime}_{s^j}$ and assume it is positive and strictly decreasing:

$$f^{j\,\prime}_{s^j} \equiv \frac{\partial f^j}{\partial L}(L,\ s^j) > 0, \qquad \text{for } L \geq 0,$$

$$f^{j\,\prime\prime}_{s^j} \equiv \frac{\partial^2 f^j}{\partial L^2}(L,\ s^j) < 0, \qquad \text{for } L \geq 0.$$

In the following, we omit the superscript j whenever there is no fear of confusion.

We assume that the marginal product is uniformly higher in the good state of nature than in the bad state of nature:

$$f'_G > f'_B \qquad \text{for all} \quad L \geq 0$$

As mentioned before, this is a simple two-commodity general equilibrium model, so the price of the product can be normalized to one. The profit of the firm in the state of nature s is calculated as

$$\pi(s) = f(L, s) - w(s)L(s) - c(s)$$

where $w(s)$, $L(s)$, and $c(s)$ are respectively the wage rate, employment, and the non-wage compensation paid in the state of nature s.

We will now explain the timing of evolving events in one-period models; the framework will later be extended to two-period models. In the <u>first stage</u>, workers and firms participate in the labor contract market. Firms offer contracts, and workers choose from among different contracts the best one. Since we assume that there is arbitrage and recontracting of contracts at this stage, the result will determine endogenously the equilibrium levels of employment and the value of contract.[2/]

In the <u>second stage</u>, the states of nature are revealed. It is assumed that these states can be observed by every agent without cost. The <u>third stage</u> is the decision of employment and layoffs. If workers are physically and legally mobile <u>ex post</u> (after the revelation of states of nature), a spot labor market will open at this stage, too.

Definitions of some implicit contract terminology are introduced in the next subsection.

2.2 Definitions

The state of nature s for an entire economy is a vector of the states of individual firms:

$$s = [s^1, s^2, \ldots, s^N]$$

Since each s^j is assumed to be either G or B, there are 2^N possible events. However, for present purposes it is enough to consider only two possible events: $s = [G, G, \ldots, G]$ and $s = [B, B, \ldots, B]$. Denote the probability of the former event by θ, $0 \leq \theta \leq 1$.

Definition 2.1 (One Industry with Correlated Realizations)

An economy is said to be composed of one industry with correlated realization if either $s^j = G$ for all j or $s^j = B$ for all j.

This assumption is justified, for example, if the state of nature is weather, which affects every agent in the same manner.

Definition 2.2 [The Value of a Contract]

The value of a contract is the expected utility of the contract.

This definition would have to be modified if workers do not have identical utility functions, especially if they have different degrees of risk aversion.

When workers stay at home, they are assumed to engage in a production activity using their own technology. With an input of one unit of labor, a worker is assumed to produce h units of output at home. Therefore, a worker who stays at home enjoys $U(h)$. Hence, $U(h)$ could be interpreted as the value of leisure instead of home production activity. However, with a home production interpretation, we can avoid a problem associated with a change in the rate of marginal substitution between income and leisure.[2a/] For the sake of simplicity, h is assumed to be identical for every worker. Allowing for different h's would not change the qualitative results. Since a worker's reservation wage in the spot market is equal to h, the labor supply curve has a step at h. Therefore, it is convenient to consider two cases: one in which h is relevant in resource allocation and the other in which h is not relevant. The former case arises if the marginal product of the last worker in the labor pool would be less than h should firms hire all the workers in the labor pool.

Definition 2.3 [Home Production]

A worker is said to have a technology of "home production" if $h > f_B'(L^s)$, and of "no home production" if $0 \leq h \leq f_B'(L^s)$.

In an economy with uncertainty, economic agents with different degrees of risk aversion are willing to trade "contingent claims" (side bets) which are monetary transfers contingent on realizations of uncertain events. (See Arrow (1951).) It is known that if such trades are feasible then economic agents will achieve *ex ante* Pareto optimality in competitive eqilibrium. Thus an emphasis in economic theory has been the analysis of the kinds of conditions and restrictions in an economic environment which would prevent the trading of contingent claims. Events may not be observed. (How to verify a worker's effort?) Those who have information may not tell the truth. (Can a manager believe a worker who says he made the best possible effort?) It may not be possible to "enforce" a contingent contract. (That is why insurance premiums are paid in advance.) Moral hazard problems, short of complete swindling, may arise if those who trade contingent claims can affect the probability of certain events.

(A worker may not search hard enough for a new job if a "safety net" provides him with unemployment compensation. Moreover, some workers might collect unemployment compensation before they withdraw from the labor market, even if they never intended to find another job.) There may not be enough information to discriminate between different types of workers in order to avoid adverse selection. (Unhealthy workers would choose firms with a generous sick leave policy and health insurance fringe benefits.) Therefore, it is important to write the contingent claims and contracts so that they depend on what is known to both parties.

Definition 2.4 [Contingent Contracts and Claims]

A contingent contract is defined as an enforceable promise of a wage and employment level (and thus the probability of layoff) contingent upon the information available to both the management of the firm and its workers. A contingent claim is an enforceable agreement of a monetary transfer between the firm and its workers which is contingent upon the information set defined above.

One of the purposes of implicit contract theory is to provide the microeconomic foundations of "involuntary" unemployment. When the market wage is below the home production level, workers withdraw from the labor market voluntarily. (Even if the market wage is equal to h, those workers who stay at home are "voluntarily" unemployed, since they have no incentive to join the labor force.)

Therefore, the mere fact that the employment level is less than the size of labor pool does not necessarily imply the existence of "involuntary" unemployment.[3/] To capture the notion of "involuntary" unemployment, we emphasize the fact that an "involuntarily" unemployed worker cannot find a job despite his willingness to work at a real wage rate infinitesimally below the current going wage.[4/] Therefore, the definition is given for an *ex post* condition, i.e., after the relevation of states of nature. Later, we will see involuntary unemployment as a result of a contract which specifies the possibility of layoff without full compensation. One might contend that since it is *a priori* *voluntarily* agreed to have this kind of contract, the result should be called "voluntary" unemployment. One would then go on to say that an unemployed worker is comparable to a voluntary gambler who happens to lose a bet. However, the important fact here is that the worker is willing to insure his possible loss, but some informational constraints prohibit complete insurance, thus preventing him from obtaining the first-best *a priori contract*. Therefore, the direct comparison of an involuntarily unemployed worker (in our sense) and a gambler is not appropriate. Our interest lies not in the semantics of whether a worker's unemployment is "voluntary" or "involuntary", but rather under what conditions the firm and workers are willing to agree on an *a priori voluntary* contract that leads to the possibility of *ex post* involuntary unemployment. Examining the reasons why resource allocation deviates from the Arrow-Debreu allocation is the key to understanding the contributions of implicit contract theory and involuntary unemployment.

Definition 2.5 [Involuntary Unemployment]

After the revelation of states of nature, if unemployed workers have lower utility than retained workers in some states of nature, then they are said to be unemployed involuntarily.

The next definition concerns the technical and legal feasibility of a worker changing firms after the state of nature is revealed. It may be technically infeasible if production requires some training or labor input before the state of nature is revealed. In such a case we would have to consider a model with decision and production lags. If changing firms *ex post* is technically possible, the firm may not want to lose workers. Therefore firms are willing to propose contracts which prohibit workers from resigning from the firms. However, such contracts are unconstitutional. In this paper, we mostly examine models which allow workers to move after the state of nature is revealed.

Definition 2.6 [Ex Post Mobility]

Workers are said to have ex post mobility if it is technically and legally feasible to quit and be rehired by another firm after the state of nature is revealed by reneging one contract without paying any penalties.

Definition 2.7 [Severance Payment and Quitting Penalty]

If the firm is required to pay compensation to a worker upon its initiation of a separation of a contracted worker, the compensation is called a severance payment. When a worker is required to pay compensation to the firm when he initiates the separation, it is called a quitting penalty.

Since the following analysis is carried out in either a one-period or two-period framework we do not differentiate between the severance payment to a detached worker and private unemployment compensation (in addition to public unemployment compensation) to the temporarily laid off worker. That is, this model does not distinguish between permanent and temporary layoffs.

Definition 2.8 Seniority Wage

The seniority wage system is said to exist if (a) Worker A is older than Worker B; (b) Worker A is equal to or less productive than Worker B; and (c) Worker A is paid more than Worker B, at a point of time in some states of nature.

This is a rather strict definition of seniority. If productivity increases with experience, the definition should be modified. In order to highlight the effect of ex post mobility in an implicit contract model, the productivity of a worker is held constant. Note that an increase in the wage profile over the life of a worker is not sufficient for this definition. A point of the definition is to show that in the same period, workers who are old are paid more than others even if the productivity is the same.

2.3 Numerical Example

In the following analysis, a numerical example will be provided in addition to a formal analysis in each section. The functional forms and values of coefficients for the production and utility functions are the following:

$$f_G'(L) = 800-4L$$

$$f_B'(L) = 200-L$$

$$U(x) = 2000x - x^2 \qquad 0 \leq x \leq 1000$$

$$L^s = 100.$$

$$h = \begin{cases} 140 & \text{if there is "home production"} \\ 0 & \text{if there is "no home production"} \end{cases}$$

$$\theta = 1/2$$

Solutions for L(s) and w(s) will be given at the end of each model in the next section. Diagrams based on these values will also be provided.

3. SOLUTIONS FOR ONE INDUSTRY MODELS

3.1 One Period (Static) Models

MODEL I: SPOT MARKET SOLUTION (EX POST)

First, we consider an economy without any contingent claims or contracts. Since we will emphasize the conceptual similarities between market equilibrium and a contract economy, let us review the logic of competitive equilibrium step by step. We assume that there is *ex post* mobility. After the state of nature is revealed, the labor demand function is derived from the firm maximizing its profit with respect to its labor input:

$$\max_L f^j(L;\, s) - wL.$$

Therefore, the first order conditions are:

$$(3.1)\qquad \begin{aligned} f_G^{j'}(L) &= w_G \qquad \text{if } s = G. \\ f_B^{j'}(L) &= w_B \qquad \text{if } s = B. \end{aligned}$$

Equation (3.1) defines the labor demand function ℓ in each state of nature:

$$(3.2)\qquad L^{dj} = \ell(w_s), \qquad s = G, B.$$

A competitive equilibrium in the labor market is defined by the equality of demand and supply, assuming the wage rate moves infinitely fast to clear the market. We will consider two cases separately: one "without home production" (see Definition 2.3) and the other "with home production."

<u>CASE (IA): No Home production</u>

There are N identical firms and $N \times \bar{L}$ identical workers who would supply one unit of labor as long as $w_s > h$. A competitive equilibrium is defined as the equality of labor demand and supply, given that the resulting wage is higher than home production:

$$N \times \ell(w_s) = N \times \bar{L} \times 1, \qquad s = G, B$$

i.e.,

$$(3.3)\qquad \ell(w_s) = \bar{L}$$

subject to $w_s \geq h$. The assumption of no home production means that

$$(3.4)\qquad f_B^{j'}(\bar{L}) > h.$$

Therefore considering that the ℓ function is implicitly defined by (3.1), assumption (3.4) guarantees that all the workers are working at the firm all the time. Note that the size of the labor pool is endogenously determined. In summary, from equations (3.1), (3.2) and (3.3), the equilibrium wage w_s^* and the equilibrium employment level L_s^* are defined as

$$(3.5)\qquad \begin{array}{lll} w^*_G = f^{j\,'}_G(\bar{L}\ ); & L^*_G = \bar{L} & \text{if } s = G \\ w^*_B = f^{j\,'}_B(\bar{L}\ ); & L^*_B = \bar{L} & \text{if } s = B \end{array}$$

NUMERICAL EXAMPLE

From (3.5) and the functional forms chosen in subsection 2.3, we can calculate the equilibrium values as follows:

$$\begin{array}{ll} w^*_G = 400 & L^*_G = 100 \\ w^*_B = 100 & L^*_B = 100. \end{array}$$

Equilibrium refers to conditions holding across the state of nature. The two equilibria in the ex post spot market are illustrated as Figure 3-1 as E_G and E_B.

Figure 3-1 about here

CASE (IB): Home Production

In the case that home production is productive, the wage calculated from (3.3) would be less than h. That is, if the firms were to hire all the workers, then the marginal product of the last worker in a bad state of nature would be less than what the worker could earn at home. Therefore, the equilibrium employment level is cut back to the level where the marginal product equals h. In the good state of nature, the equilibrium condition (3.3) is applicable. Thus the equation $\ell(w^*_G) = \bar{L}$ defines w^*_G, and $L^*_G = \bar{L}$. In the bad state of nature, $w^*_B = h$ must hold true, and $L^*_B = \ell(h)$. In summary,

$$(3.6)\qquad \begin{cases} w^*_G \equiv f^{j\,'}_G(\bar{L}\ ); & L^*_G = \bar{L}\ , \qquad \text{if } s = G \\ w^*_B \equiv h & L^*_B = \{L \mid f'_B(L) = h\}, \quad \text{if } s = B. \end{cases}$$

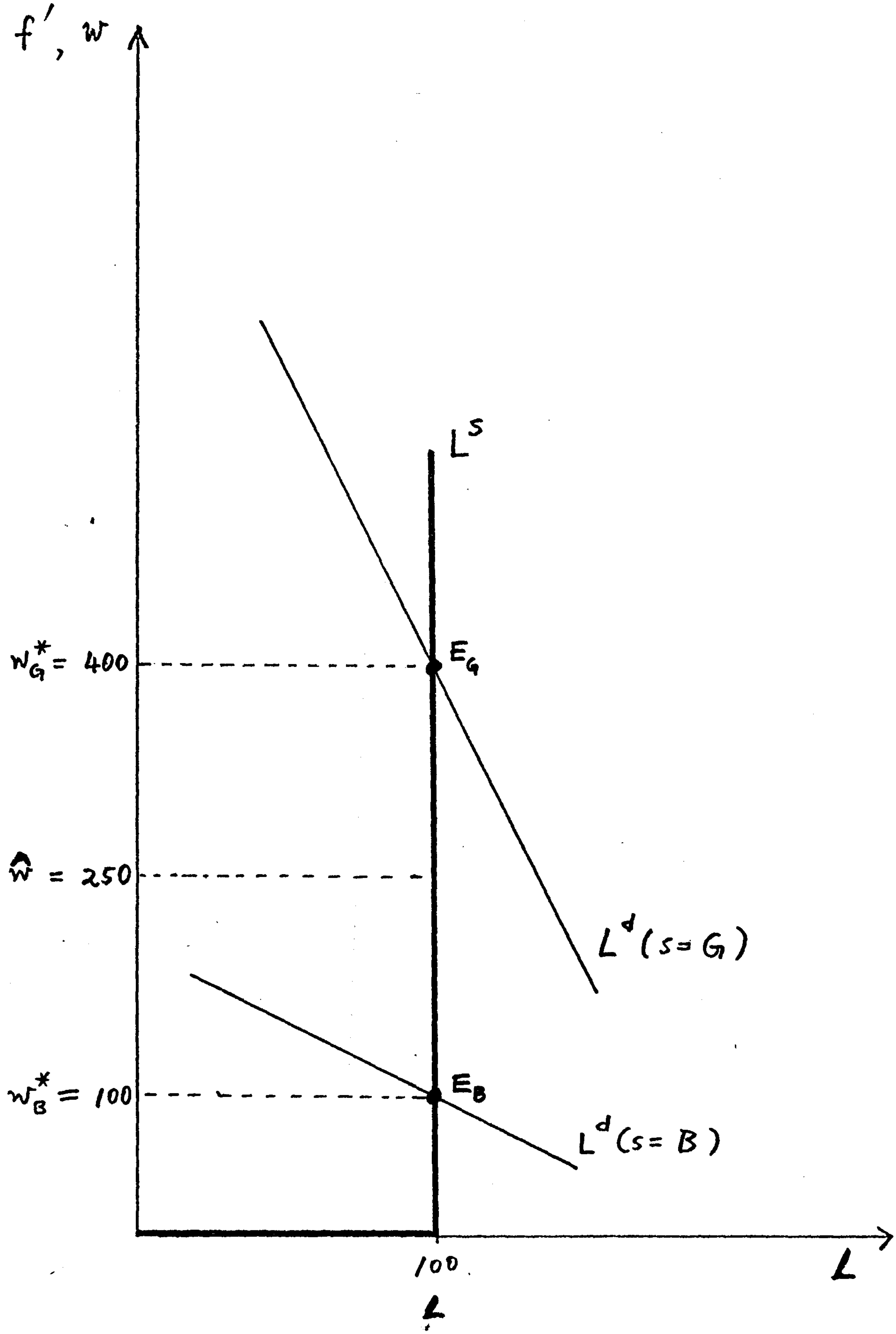


Figure 3-1 $h = 0$

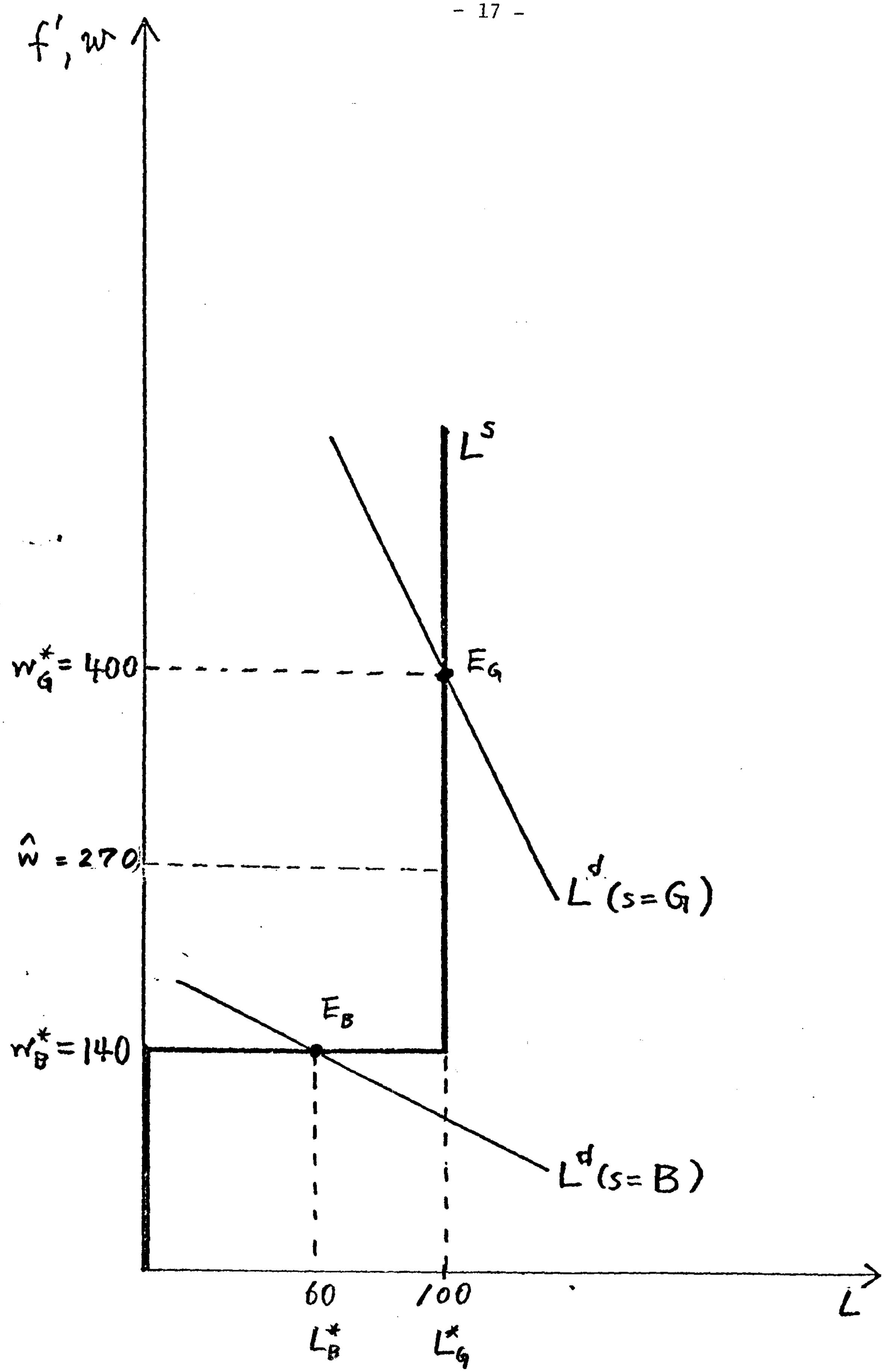


Figure 3-2

NUMERICAL EXAMPLE

From (3.6) and the functional forms chosen in subsection 2.3, the equilibrium values are calculated as

$$w_G^* = 400 \qquad L_G^* = 100$$
$$w_B^* = h = 140 \qquad L_B^* = 60.$$

Points E_G and E_B in Figure 3-2 illustrate the equilibrium in each *ex post* spot market.

Figure 3-2 about here

Note that according to the framework and definition explained in the preceding section, there is "voluntary" unemployment in the bad state of nature. The employment level is L_B^*, while $\bar{L} - L_B^*$ are "self-employed" at home. The workers both in the firm and at home are receiving the same amount of income, i.e., $w_B^* = h$. Therefore, they are indifferent between working in the firm and working at home. Although the employment level is fluctuating, it is the result of the efficient allocation of resources.

MODEL II: ARROW-DEBREU ECONOMY

Let us introduce a contingent claim (see definition 2.4) into Model I. Before the state of nature is revealed (stage I), the firms and workers have a chance to trade a contingent claim (side bet), knowing the solutions of the (ex post) spot market (stage III). The contingent claim we consider takes the following arrangement:

If $s = G$, each worker pays c_G to the firm.
If $s = B$, the firm pays c_B to each worker.

Let us assume that competition among many risk-neutral firms drives the expected payoff down to zero, as is usually assumed in the insurance literature. Recall that θ is the probability of the good state of nature, $0 < \theta < 1$. The expected payoff for the firm is $\theta c_G - (1 - \theta)c_B$. The condition for no expected payoff from this side bet implies $c_B = \theta c_B/(1-\theta)$.

Now consider the consumer's problem:

$$\max_{c_G} \quad \theta U(w_G^* - c_G) + (1-\theta)U(w_B^* + \frac{\theta}{1-\theta} c_G).$$

This has the first order condition:

$$-\theta U'(w_G^* - c_G) + \theta U'(w_B^* + \frac{\theta}{1-\theta} c_G) = 0.$$

Therefore we have $w_G^* - c_B = w_B^* + \frac{\theta}{1-\theta} c_G \equiv \hat{w}$ which implies the complete stabilization of the worker's income, x.

The equilibrium amounts of the side bet, c_G^* and c_B^*, are calculated as

$$\begin{cases} c_G^* = (1-\theta)(w_G^* - w_B^*) \\ c_B^* = \theta(w_G^* - w_B^*). \end{cases}$$

In other words, $\hat{w} = \theta w_G^* + (1-\theta)w_B^*$. This is a confirmation of the well-known result that a risk-neutral agent would absorb all the risk in an economy with contingent claims. Now it is easy to summarize the results of MODEL II for two cases: one without home production and the other with home production.

CASE (IIA): No Home Production

$$(3.7)\quad \begin{cases} w_G^* = f_G^{j\,\prime}(L^S);\ c_G^* = (1-\Theta)(w_G^* - w_B^*); & L_G^* = \bar{L} \quad \text{if } s = G. \\ w_B^* = f_B^{j\,\prime}(L^S);\ c_B^* = \Theta(w_G^* - w_B^*); & L_B^* = L^S \quad \text{if } s = B. \end{cases}$$

This implies that each worker receives, as a sum of wages and payoffs of contingent claims, $\hat{w} = \Theta w_G^* + (1 - \Theta)w_B^*$ in either state of nature, $s = G, B$.

NUMERICAL EXAMPLE

With $\Theta = 1/2$, $c_G^* = c_B^* = 150$. Therefore

$\hat{w} = w_G^* - c_G^* = 250$, $L_G^* = 100$ if $s = G$

$\hat{w} = w_B^* + c_B^* = 250$, $L_B^* = 100$ if $s = B$.

In Figure 3-1, the equilibrium contingent claim implies that the worker pays to the firm $(\hat{w}_G^* - w)$ if $s = G$, and the firms pay the worker $(\hat{w} - w_B^*)$ if $s = B$, so that the worker is always guaranteed a wage $\hat{w}$.

CASE (IIB): Home Production

$$(3.8)\quad \begin{cases} w_G^* = f_G^{j\,\prime}(L^S); \quad c_G^* = (1-\Theta)(w_G^* - h); \quad L_G^* = L^S & \text{if } s = G \\ w_B^* = h;\ c_B^* = \Theta(w_G^* - h); \quad L_B^* = \{L \,|\, f_B'(L) = h\} & \text{if } s = B. \end{cases}$$

This implies $\hat{w} = \Theta w_G^* + (1-\Theta)h$.

NUMERICAL EXAMPLE

Since $\hat{w} = (400 + 140)/2 = 270$

$$\begin{cases} \hat{w} = w_G^* - c_G^* = 270 & L_G^* = 100 & \text{if } s = G \\ \hat{w} = w_B^* + c_B^* = 270 & L_B^* = 60 & \text{if } s = B. \end{cases}$$

In Figure 3-2, the difference $(w_G^* - \hat{w}) = (\hat{w} - w_B^*)$ shows the size of the optimal side bet.

Note that the firm is making the side bet with all the workers in the labor pool whether or not they end up working for the firm. The side bet is assumed to be enforceable independent of the employment relation.

The first theorem of welfare economics gives us the optimality result that a competitive equilibrium is a Pareto optimum. Therefore, the result in this section can be used as a benchmark for the solutions discussed below. Although the above solution concept generates a Pareto optimal outcome, the contingent claim market may not be feasible because of high enforecement costs. Since workers and firms do not necessarily have a binding employment relationship, organizing a market for contingent claims may be tenuous. Workers, who agree to pay $(w_G^* - \hat{w})$ in the event of $s = G$, may just quit the firm rather than make the payment and find a job in the spot market. It may also be difficult to verify whether the state of nature is good or bad, since workers are not necessarily attached to the firm which administers the side bet. In the following, let us assume that there is no contingent claim market in our economy. The basic idea of an implicit labor contract is that the attachment of the labor force to the firm may overcome the difficulty in making side bets. A most important question is to what degree an implicit labor contract can "mimic" the Arrow-Debreu solution.

MODEL III: CONTRACT WITHOUT EX POST MOBILITY

Now let us consider an economy with contingent contracts (Definition 2.4). At the first stage, the firm offers a contract composed of the wages in the good and bad states of nature and the probability of being retained in the bad state of nature: $\{w_G, w_B, r\}$. No severance

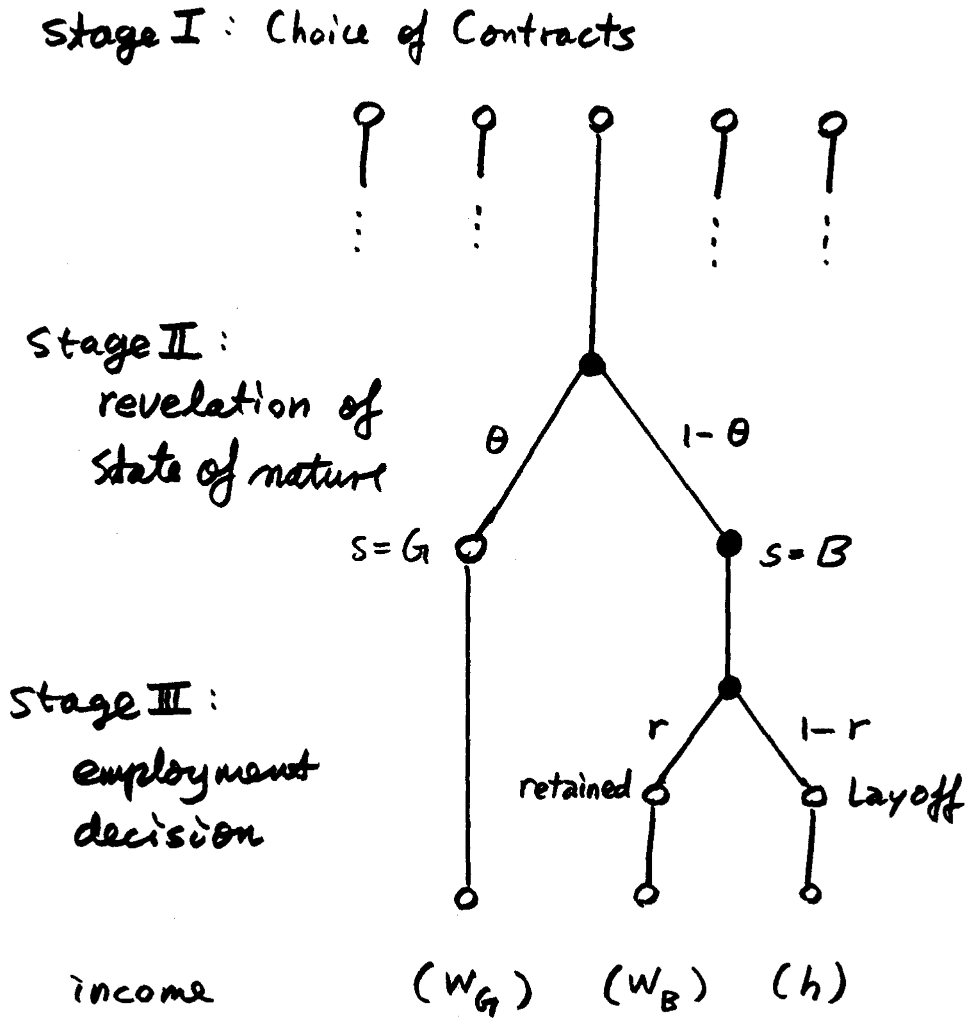

Stage I : Choice of Contracts
Stage II :
revelation of
state of nature
θ
1- θ
s = G
s = B
Stage III :
employment
decision
r
1- r
retained
Layoff
income
(w_G)
(w_B)
(h)

Figure 3-3

payment (Definition 2.7) or *ex post* mobility (Definition 2.6) is assumed. Therefore, the laid-off worker engages in home production. The contract described above yields the following expected utility:

$$(3.9)\quad EU(w_G, w_B, r) \equiv \Theta U(w_G) + (1-\Theta) r U(w_B) + (1-\Theta)(1-r)U(h).$$

In the first stage, if one firm offers a package with a higher expected utility than the others, then that firm would attract all the workers, given that $EU > U(h)$. Of course, the firm can limit how many workers it wants to hire, leaving excess workers to other firms. However, a more profitable policy is to lower the value of contract. By the same token, if the firm offers a contract which has less value than one offered by another firm, then the firm does not attract any workers. Thus, in equilibrium, the value of the contracts offered should be equal for all firms. The value of the contract plays the role of a price in a competitive market. The equilibrium value of the contract will be determined endogeneously through the tâtonnement process in the first stage. The decision tree for a worker is illustrated in Figure 3-3. First, let us derive labor demand as a function of the going market value of contract, $\bar{U}$.

Figure 3-3 about here

$$(3.10)\quad \underset{\{L,\, w_G,\, w_B\, r\}}{\text{Max}} \quad \Theta[f_G(L) - w_G L] + (1-\Theta)[f_B(rL) - w_B rL] \equiv E\pi$$

subject to

$$(3.11)\quad EU(\cdot) \equiv \Theta U(w_G) + (1-\Theta) r U(w_B) + (1-\Theta)(1-r)U(h) \geq \bar{U}$$

$$(3.12)\quad 0 \leq r \leq 1, \text{ and } L \geq 0.$$

First, observe that the constraint (3.11) is always binding, because pushing down the wage is always profitable if it does not make the workers quit voluntarily. Let us construct a Lagrangean, ignoring the constraint (3.12):

$$(3.13) \qquad \pounds = E\pi \quad + \lambda\{EU(\cdot) - \overline{U}\}$$

Constraint (3.12) can be handled as follows. First, if

$$(3.14) \qquad \left.\frac{\partial \pounds}{\partial r}\right|_{r=0} > 0$$

then the optimal r^* should be strictly positive, i.e., $r^* > 0$. It is easy to verify that condition (3.14) holds. Second, a sufficient condition for $r^* < 1$ is the following:

$$(3.15) \qquad \left.\frac{\partial \pounds}{\partial r}\right|_{r=1} = L(1-\Theta)[f_B'(L) - w_B] + \lambda(1-\Theta)[U(w_B) - U(h)] < 0$$

If h is large enough so that (3.15) is satisfied, then there is an interior solution with respect to r. Thus

$$(3.16) \qquad 0 = \frac{\partial \pounds}{\partial r} = L(1-\Theta)[f_B'(rL) - w_B] + \lambda(1-\Theta)[U(w_B) - U(h)]$$

has to be satisfied.

Now, the other first order conditions are:

$$(3.17) \qquad 0 = \frac{\partial \pounds}{\partial L} = \Theta[f_G'(L) - w_G] + (1-\Theta)[rf_B'(rL) - w_B r]$$

$$(3.18) \qquad 0 = \frac{\partial \pounds}{\partial w_G} = \Theta[-L + \lambda U'(w_G)]$$

$$(3.19) \qquad 0 = \frac{\partial \pounds}{\partial w_B} = (1-\Theta)r\left(-L + \lambda U'(w_B)\right)$$

From (3.18) and (3.19),

$$(3.20) \qquad \lambda U'(w_G) = L = \lambda U'(w_B)$$

Therefore,

(3.21) $$w_B = w_G \equiv \hat{w}$$

Equation (3.20) also implies $\lambda = L/U'(w_B)$. Substituting this value of λ and (3.21) into (3.15) and eliminating $L(1-\theta)$ which is strictly positive gives

(3.15') $$[f_B'(L) - \hat{w}] + \frac{U(\hat{w}) - U(h)}{U'(\hat{w})} < 0$$

as a sufficient condition for $r^* < 1$. This corresponds to the condition discussed in Azariadis (1975: p. 1192, equations (18) and (19)) and Negishi (1979: p. 230, equation (16)).

CASE (IIIA): No Home Production

First, consider the case $f_B'(L^S) \geq h$, i.e., no home production. By the concavity of the utility function,

$$U(f_B'(L^S)) < U(\hat{w}) + U'(\hat{w})\cdot[f_B'(L^S) - \hat{w}].$$

Since $f_B'(L^S) \geq h$,

$$U(h) \leq U(f_B'(L^S)) < U(\hat{w}) + U'(\hat{w})\cdot[f_B'(L^S) - \hat{w}].$$

It follows from simple manipulation that

$$\frac{U(h) - U(\hat{w})}{U'(\hat{w})} < f_B'(L^S) - \hat{w}.$$

This contradicts condition (3.15')[5/]. When (3.15') is not satisified and

$$\frac{\partial \pounds}{\partial r} > 0 \text{ for all } 1 \geq r \geq 0,$$

the optimal retained ratio is unity, i.e., there are no layoffs. Then the first order conditions are modified as follows:

(3.17') $\Theta[f_G'(L) - \hat{w}] + (1-\Theta)[f_B'(L) - \hat{w}] = 0.$

(3.11') $\Theta U(\hat{w}) + (1-\Theta)U(\hat{w}) = \bar{U}$

Therefore,

$$U(\hat{w}) = \bar{U}$$

or

$$\hat{w} = U^{-1}(\bar{U}).$$

Now the labor demand function $L^d = \ell(\bar{U})$ can be calculated implicitly from;

$$\Theta[f_G'(L) - U^{-1}(\bar{U})] + (1-\Theta)[f_B'(L) - U^{-1}(\bar{U})] = 0$$

It is easy to verify that labor demand is a decreasing function of the value of the contract:

$$\frac{dL^d}{d\bar{U}} < 0.$$

Therefore, the unique market equilibrium is obtained as an intersection of demand and supply, the latter being exogenously fixed at $\bar{L}$. The equilibrium employment level is determined by $L^*=\bar{L}$ and the equation $\ell(\bar{U})=L^s$. defines the equilibrium value of $\bar{U}$.

NUMERICAL EXAMPLE

Let us solve (3.11') with the given functional forms, noting that $d\hat{w}/d\bar{U} > 0$ is required by the stability of an equilibrium in the labor contract market.

$$\hat{w} = 1000 - \sqrt{(1000)^2 - \bar{U}}.$$

The labor demand function is calculated from (3.17"):

$$L^d = 200 - \hat{w}/2.5$$

$$= -200 + \sqrt{(1000)^2 - \overline{U}}/2.5$$

In equilibrium, $L^d = \overline{L} = 100$, yielding

$\hat{w} = 250$ and $\overline{U} = (1000)^2 - (750)^2$.

CASE (IIIB): Home Production with (3.15') [Azariadis]

As we have demonstrated $h = f_B'(L^s)$ is not enough to guarantee $r < 1$. Let us assume in addition (3.15'). Using (3.21), we can rewrite (3.17) as

$$\text{(3.17")} \qquad \Theta[f_G'(L) - \hat{w}] + (1-\Theta)r[f_B'(rL) - \hat{w}] = 0.$$

Given that (3.15') is true, (3.16) must hold at the optimum, i.e.,

$$\text{(3.16')} \qquad f_B'(rL) - \hat{w} + \frac{U(\hat{w}) - U(h)}{U'(\hat{w})} = 0$$

and (3.11) is now written with an equality:

$$\text{(3.11)} \qquad \Theta U(\hat{w}) + (1-\Theta)rU(\hat{w}) + (1-\Theta)(1-r)U(h) = \overline{U}.$$

Equations (3.17'), (3.16') and (3.11) define L, $\hat{w}$, and r as functions of $\overline{U}$. The value L as a function of $\overline{U}$ can be interpreted as the demand function for labor:

$$L^d = \ell(\overline{U}).$$

By the market equilibrium condition,

$$L^d = L^s.$$

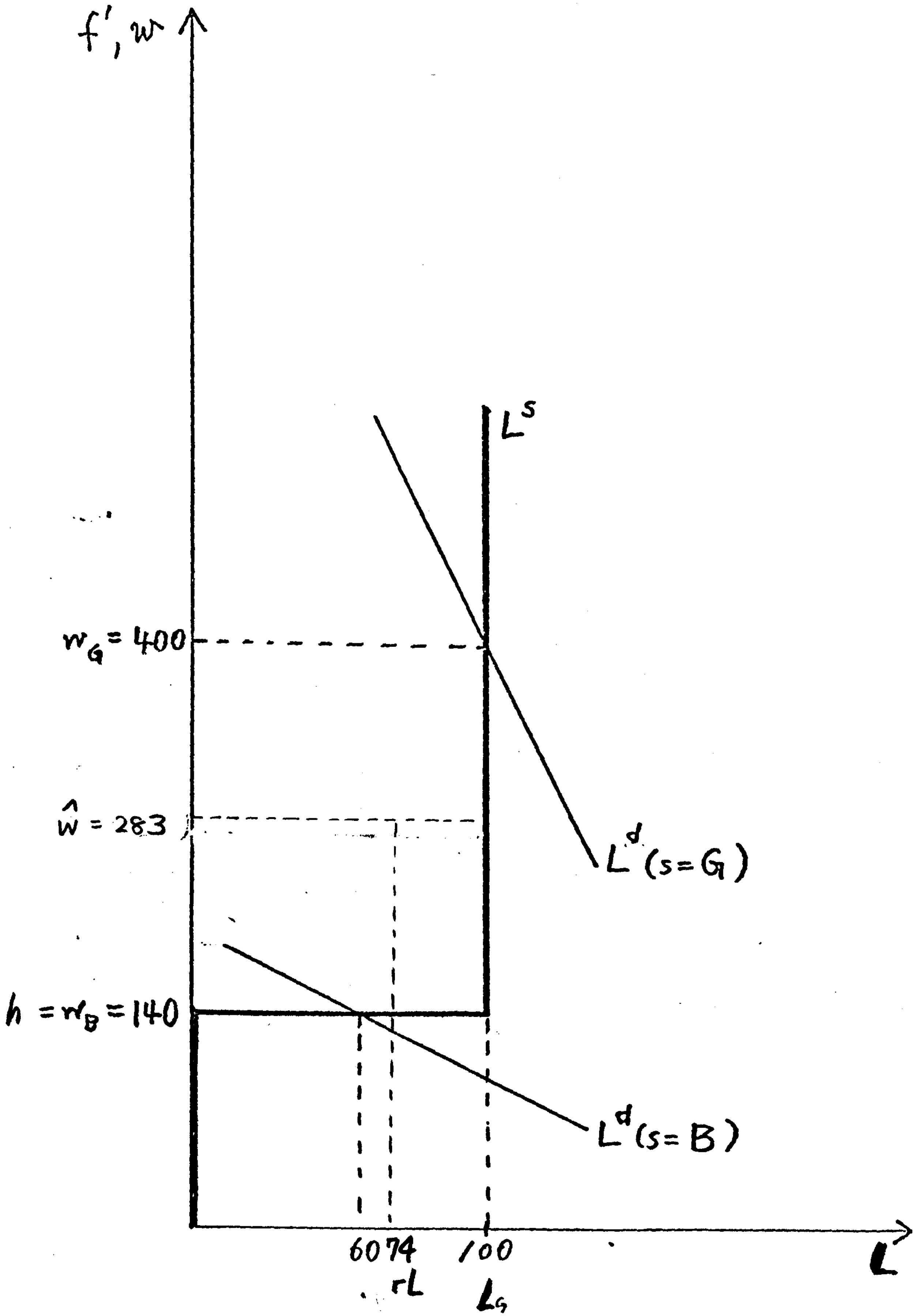


Figure 3-4

Therefore, we can solve (3.17'), (3.16') for $\hat{w}$, r, by substituting $L=L^s$. Then substituting those values into (3.11), we find the value of $\bar{U}$. These values describe a market equilibrium with implicit contracts.

Now let us consider when the condition (3.15') is most likely satisfied. It is apparent that the likelihood of having (3.15') increases as h becomes higher relative to $f_B'(L^s)$, L^s being substituted for L because of the market equilibrium condition.

NUMERICAL EXAMPLE

First order conditions (3.17") and (3.16'), with functional forms defined in section 2.3, give us the following:

$$800 - 4L - \hat{w} + r(200 - rL) - r\hat{w} = 0$$

$$(200 - rL - \hat{w}) + \left(2000\hat{w} - \hat{w}^2 - (140)\cdot(2000) + (140)^2\right)/(2000 - 2\hat{w}) = 0.$$

Substituting the market equilibrium $L^d = L^s = 100$, we find

$r \simeq .743$, $\hat{w} = 283.07$.

The solution is illustrated in Figure 3-4. One hundred workers are attached to this firm in the first stage. In the good state of nature, all of them are hired at the wage of 283. In the bad state of nature, about 74 workers are lucky and are hired at the wage of 283, but the other 26 workers are laid-off so that they go home and are self-employed at the wage of 140.

Figure 3-4 about here

Note that condition (3.16') implies that $f_B'(rL) < h$. Since U is a concave function, $U(\hat{w}) > U'(\hat{w})\{\hat{w} - h\} + U(h)$. This inequality implies $\{U(\hat{w}) - U(h)\}/U'(\hat{w}) > \{\hat{w} - h\}$. Substituting this into (3.16'), $f_B'(rL) < h$ is verified. This means that workers are employed by a firm beyond the point that the marginal productivity is less than the home productivity. In this sense, the solution should be called "overemployment," as well as "involuntary unemployment."

When the state of nature is good, then all workers are employed by the firm and the wage rate is $\hat{w} < f_g'(L^S)$. During a bad state of nature, a worker is retained at the firm with probability r and paid $\hat{w}$, the same wage rate as the one in the good state of nature. With probability $(1-r)$, a worker is unemployed, without any compensation from the firm (by assumption). The unemployed go home and earn h. However, since $h < \hat{w}$, the utility of the unemployed is less than that of the retained; "involuntary" unemployment results (Definition 2.5). Thus, in the Azariadis framework, when condition (3.15') holds, we can show that "involuntary" unemployment exists.

Case (IIIC): Azariadis Model with Severance Payments

In the Azariadis model, the firm is not allowed to arrange severance payments as a part of contracts. However, a clause which specifies payments in the event of severance is not unusual in labor contracts.[6/] Let us assume that the firm arranges to pay c to a worker when it lays him off. Therefore, there are additional costs $c(1-r)L$ incurred in the bad state of nature, while the income of the laid-off worker becomes $c+h$ instead of h. The problem to be solved is the following:

$$(3.22) \quad \underset{\{L,\, w_G,\, w_B,\, r,\, c\}}{\text{Max}} \; \Theta\{f_G(L) - w_G L\} + (1-\Theta)\{f_B(rL) - w_B rL - c(1-r)L\} \equiv E\pi$$

subject to

$$EU \equiv \Theta U(w_G) + (1-\Theta) rU(w_B) + (1-\Theta)(1-r)U(c+h) \geq \overline{U}$$

$$0 \leq r \leq 1.$$

Forming the Lagrangean $\pounds = E\pi + \lambda\{EU(\cdot) - \bar{U}\}$,

we obtain the following first-order conditions.

$$(3.23) \qquad 0 = \frac{\partial \pounds}{\partial L} = \Theta[f'_G(L) - w_G] + (1-\Theta)[rf'_B(rL) - rw_B - c(1-r)]$$

$$(3.24) \qquad 0 = \frac{\partial \pounds}{\partial c} = (1-\Theta)[-(1-r)L] + \lambda(1-\Theta)(1-r)U'(c+h)$$

$$(3.25) \qquad 0 = \frac{\partial \pounds}{\partial w_G} = \Theta[-L + \lambda U'(w_G)]$$

$$(3.26) \qquad 0 = \frac{\partial \pounds}{\partial w_B} = (1-\theta)r[-L + \lambda U'(w_B)].$$

With respect to r, assume h is large enough to have an interior solution (see the argument in Case III-B):

$$(3.27) \qquad 0 = \frac{\partial \pounds}{\partial r} = (1-\Theta)L[f'_B(rL) - w_B + c] + (1-\Theta)\lambda[U(w_B) - U(c+h)].$$

Since $0 < \Theta < 1$, and $1 - r > 0$, equations (3.24), (3.25), and (3.26) yield $w_G = w_B = c+h \equiv \hat{w}$. Since $w_B = c + h$, the second term of (3.27) becomes zero. Therefore, the first term of (3.27) should be equal to zero, too:

$$\begin{aligned} f'_B(rL) &= w_B - c \\ &= h. \end{aligned}$$

The last step comes from $w_B = c + h$. Hence, the marginal product at the firm in the bad state of nature is equal to the marginal product of home production. Notice that this ensures productive efficiency as defined in the Arrow-Debreu economy. Thus the employment level in the bad state of nature in this model, rL, is equal to that in the Arrow-Debreu model. Now it is easy to verify whether not only the employment level but also the utility level for the worker in this model coincide with the ones in the Arrow-Debreu solution. Substituting $f_B'(rL) = h$ and $c = \hat{w} - h$ into equation (3.23), we have

$$0 = \Theta[f_G'(L) - \hat{w}] + (1-\Theta)[rh - r\hat{w} - (\hat{w} - h)(1-r)]$$
$$= \Theta[f_G'(L) - \hat{w}] + (1-\Theta)[-\hat{w} + h].$$

Therefore,

$$\hat{w} = \Theta f_G'(L) + (1-\Theta)h.$$

The "stabilized" wage level is the weighted average of the marginal product in the good state of nature and the level of home production. Since $L^d = \ell^d(\overline{U})$ is defined from the constraint $EU = \overline{U}$,

$$\overline{U} = \Theta U(\hat{w}) + (1-\Theta)r\,U(\hat{w}) + (1-\Theta)(1-r)U(c+h)$$
$$= U(\hat{w}),$$

Use of the labor market equilibrium condition, $L^d = \overline{L}$, confirms that $f_G'(L) = w_G^*$ where w_G^* is the level of wage in the Arrow-Debreu economy. Therefore, the solution in this case is equivalent to that in the Arrow-Debreu economy described in equation (3.8).

It is important to recognize that severance payments serve the role of contingent claims, restoring the full Pareto optimal allocation. This is shown in Figure 3-5.

Figure 3-5 about here

In sum, involuntary unemployment in the Azariadis results from a lack of a contingent commodity for the bad state of nature. Arrow-Debreu contingent claims or severance payments would restore a Pareto optimal allocation.

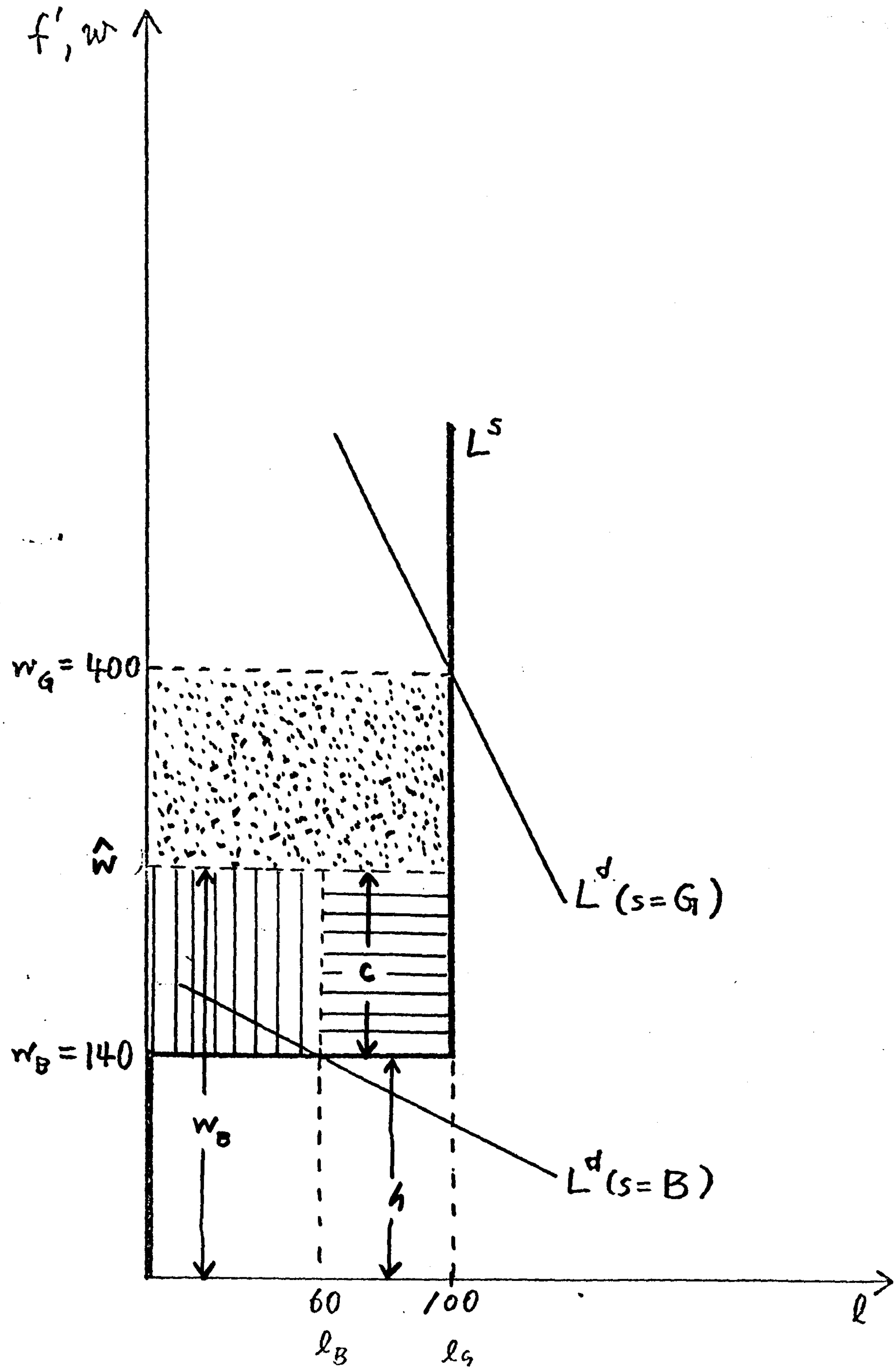


Figure 3 - 5

3.2 Remarks on Azariadis model

Although the Azariadis model shows how involuntary unemployment can exist in a microeconomic model, there are several qualifications. First, the productivity at home or the utility from leisure must be high enough to ensure that the inequality (3.15') holds. Otherwise, the optimal contract would result in a full-employment (i.e., no layoff) solution. Second, it should be emphasized that severance payments are ruled out by assumption. As was shown in (III C), the introduction of severance payments restores Pareto optimality.

Third, involuntary unemployment in the Azariadis model has an unusual feature. The size of involuntary unemployment under the Azariadis-type contract is smaller than the size of voluntary unemployment in the Arrow-Debreu economy. In other words, the employment level in the bad state of nature of the Azariadis model is larger than that of the efficient solution. It is clear that the Azardiadis equilibrium is inefficient. However, it is counter-intuitive that the inefficiency is caused by employment larger than the efficient level in the state in which involuntary unemployment exists. Since there are no severance payments, the only way to eliminate the possibility of layoffs (i.e., to equalize everybody's income) in the bad state of nature is to employ the entire labor pool. However, an efficient allocation of workers (in terms of maximizing profits) calls for the reduction of employment in the bad state of nature. Therefore, the solution of the Azariadis model, given (3.15'), lies between the productively efficient solution and the employment of the entire labor pool. That is, the inefficiency is in the direction of more employment.

Fourth, the assumption of no *ex post* mobility is essential for the existence of any kind of contract in the static model. In order to see this, suppose that there is *ex post* mobility. Then there will be a spot market after the revelation of states of nature but before actual employment and production. In spite of this change, assume that the firm and workers engage in the contract described above. If a state of nature turns out to be good, then the marginal productivity of the last worker in the labor pool is higher than the wage $\hat{w}$ in the *a priori* contract. Therefore, the firm has an incentive to hire additional workers by offering a wage greater than $\hat{w}$, if other firms keep the wage at $\hat{w}$. However, the competitive process of bidding up the spot market wage would result in an equality of the wage and marginal productivity. Thus, the contract stabilizing the wage at $\hat{w}$ is breached in the good state of nature if there is *ex post* mobility. On the one hand, the cost of retaining workers in the good state of nature becomes equal to their marginal product. On the other hand, the wage in the bad state of nature is above the marginal product by the contract, which is honored in this case.[7/] On average, the firm would not be maximizing its profit. Foreseeing this unilateral breakdown of contracts due to *ex post* mobility, the firm is not willing to engage in a contract specifying $w = w^*_G + (1-\theta)w^*_B$ for either state of nature at the first stage. It is apparent from the above argument that there will not be any contracts if an economy is described by a static model with *ex post* mobility. Therefore, in order to obtain any interesting observations associated with contracts in the presence of *ex post* mobility, it is necessary to generalize the framework to a dynamic model.

3.3 Leisure as a Separate Argument in the Utility Function

We have assumed that a worker could engage in the production activity at home if he was voluntarily or involuntarily unemployed. This was a convenient way to avoid problems with having leisure as a separate argument in the utility function. In this subsection, it will be shown that in the Azariadis model with severance payments an involuntarily unemployed worker's level of utility is higher than that of a retained worker, if leisure is a separable argument in the utility function. Suppose that utility depends on income x and leisure z, and that both arguments are separable:

$$U(x, z) = u(x) + v(z)$$

where $u' > 0$, $u'' < 0$ and $v' > 0$. Assume that the worker is not productive at home, and that there are no part-time jobs. If a worker is employed, $z = 0$, and if laid off, $z = 1$. Then the constraint in the maximization problem is modified as follows:

$$EU \equiv \theta\{u(w_G)+v(0)\}+(1-\theta)r\{u(w_B)+v(0)\}+(1-\theta)(1-r)\{u(c)+v(1)\} \geq \bar{U}$$

Solving the first-order conditions of (3.23)-(3.26), we find $u'(w_G) = u'(w_B) = u'(c)$. Therefore, the amount of severance payment is equal to the wage. It follows immediately that the retained workers in both good or bad states of nature have lower utility than the unemployed:

$$U(w_B, 0) < U(c, 1).$$

This result is counter-intuitive. Moreover, it is questionable whether this contract can be implemented, because under the optimal contract all the workers wish to quit and collect the severance payment. It is necessary to limit the severance payment to those who are "assigned" to quit. Suppose that the firms give severance payments only to those who are assigned to be laid off.

Then since the retained have lower utility than the laid-off, we may say that there is involuntary "employment" but not "unemployment."

3.4 Equivalent Problem

In the models above, we have formulated the contract market in a manner parallel to the usual labor market, replacing the wage rate by the value of the contract. Therefore, both firms and workers maximize their profit and utility, taking the value of the contracts as given. Since consumers are supposed to choose the firm which offers the contract with the highest expected value, equilibrium is achieved when all firms offer the same contract value. We know that the risk-neutral firm is indifferent between different wage profiles $\{w_B, w_G\}$, so long as they give the same expected value as the one given by the spot market solution. Therefore a constraint which guarantees that a worker is not bid away from the firm is expressed as:

$$E[f_s'(L) - w_s] = 0.$$

With this constraint, the representative utility can be maximized to obtain a solution for the contract.

$$\max_{\{w_G,\, w_B\}} \ \Theta U(w_G) + (1-\Theta)U(w_B)$$

This maximization problem is much easier to solve than the one in preceding sections.[8/] This formulation will give the same solution as the Arrow-Debreu economy, given that the solution always has full employment. In the case where the optimal solution includes layoffs in some state of nature, this procedure does not yield the same solution.

3.5 Two-Period Models

We have seen that a contract in a static model breaks down if workers are mobile after the state of nature is revealed. In this section, suppose that firms and workers live for two periods. They have an opportunity to make contracts before the state of nature for the first period is revealed. The probability distribution of states of nature in the second period is independent of and identical with that in the first period. Let us assume that there is no time discounting, and that the objective function is additive with respect to time. The representative firm maximizes the sum of expected profit for the first and second period, and the representative worker maximizes the sum of expected utility for the first and second period.

MODEL IV: ARROW-DEBREU ECONOMY

The spot-market wage profile for the two-period problem is $\{w_s(1), w_s(2) | s = G, B\}$ where $w_s(t)$ is the wage at period t in the state of nature s. By introducing contingent claims $\{c_s(1)\}$ and $\{c_s(2)\}$, we would obtain the Arrow-Debreu solution. We need two contingent claims $c_B(1)$ and $c_G(1)$ in the first period, and four contingent claims in the second period (depending on the history of the state of nature in the preceding period and the state of nature in the current period). The solution is to equalize the income of the worker over two periods. Let us take the case with home production. The solution each period is exactly the same as the one with only one period, (3.8). All the workers receive the fixed income $\hat{w}$ as the sum of wages and payoffs of contingent claims each period, where $\hat{w} \equiv \theta w_G^* + (1-\theta)h$.

As in a one-period model, the Arrow-Debreu allocation can be achieved by contracts if one of the following conditions is met: the workers are not <u>ex post</u> mobile, and severance payments can be arranged. We shall not repeat the tedious calculations which are similar to those of Model (IIIC). An interesting case arises when there is mobility but no severance payments.

MODEL V: Contracts without severance payments

Let us consider the two-period model with ex post mobility. At stage I in period one, firms and workers have a chance to make contracts for the wage in the first and second periods. However, the very reason that one-period contracts break down in the presence of mobility is also valid in this model. If the contract was written at this stage, it would be reneged by workers in the event of a good state of nature in the first period.

Contracts should be written only after the state of nature is revealed in the first period. Suppose that the state of nature in the first period is "good". Then it is possible that workers would arrange to receive less than their marginal product in the first period in return of a promise to receive a wage higher than their marginal product in a "bad" state of nature in the second period. Note that ex post mobility again nullifies any contract which pays less than the marginal product in the "good" state of nature in the second period.

Suppose next that the state of nature is "bad" in the first period. Then it is impossible to write a contract because the marginal product in the bad state of nature in the second period is by assumption exactly equal to that in the first period. There is no room for stabilizing income. Of course, if the state of nature in the second period is good, any contract which does not pay at least f'_G will not be honored. Therefore, there is no way to write the contract over two periods, if the state of nature is bad in the first period.

The timing and type of contract are illustrated in Figure 3.6. The bold line indicates the branches covered by a contract. Notation is the same as that introduced in the one-period case. Note that the feasible contract is an insurance policy in which a worker pays the premium in the first period when it is affordable (i.e., when the state of nature is good), so that the possibility of a bad state of nature in the future is covered.

Figure 3-6 about here

MODEL (V.1): Without Severance Payments (Holmstrom)

Suppose that severance payments cannot be arranged in the two-period framework described above. The firm's Lagrangean to be maximized after a good state of nature in the first period becomes the following:

$$\underset{\{L,w_1,w_G,w_B,r\}}{\text{Max}}\ \pounds = [f_1(L,G)-w_1L]+\Theta[f(L,G)-w_GL]+(1-\Theta)[f(rL,B)-w_BrL]+\lambda[U(w_1)+\Theta U(w_G) + (1-\Theta)rU(w_B)+(1-\Theta)(1-r)U(h)-\overline{U}]+\mu[U(w_G)-U(w_G^*)]$$

where w_G^* is the spot market equilibrium wage rate, defined in Model I, and subscript 1 implies the production function and the wage is for the first period after the state of nature is revealed to be good. The last constraint implies that the contract must specify the second-period wage in the good state of nature w_G to be equal to or larger than w_G^*, otherwise competition in the ex post spot market would bid away the worker. Note that w_G^* is equal to $f_G'(\overline{L})$ in model I, so we immediately know $w_G = f_G'(L)$.

The first-order conditions assuming an interior solution are the following:

$$(3.28)\qquad 0 = \frac{\partial \pounds}{\partial L} = f_1' - w_1 + \Theta[f_G'(L) - w_G] + (1-\Theta)[rf_B(rL) - w_Br]$$

$$(3.29)\qquad 0 = \frac{\partial \pounds}{\partial w_1} = -L + \lambda U'(w_1)$$

$$(3.30)\qquad 0 = \frac{\partial \pounds}{\partial w_G} = \Theta[-L + \lambda U'(w_G)] + \mu U'(w_G)$$

$$(3.31)\qquad 0 = \frac{\partial \pounds}{\partial w_B} = (1-\Theta)[-L + \lambda U'(w_B)]$$

$$(3.32)\qquad 0 = \frac{\partial \pounds}{\partial r} = (1-\Theta)L[f_B'(rL) - w_B] + (1-\Theta)\lambda[U(w_G) - U(h)].$$

From (3.29) and (3.31), we have $w_1 = w_B \equiv \hat{w}$. Then $\lambda = L/U'(\hat{w})$. From (3.30), $\mu = L[1 - U'(w_G)/U'(\hat{w})]/U'(w_G)$.

PERIOD 1

Stage (I) Choice of Contracts: But
No contracts because of *ex post* mobility

(II) Revelation of states of nature

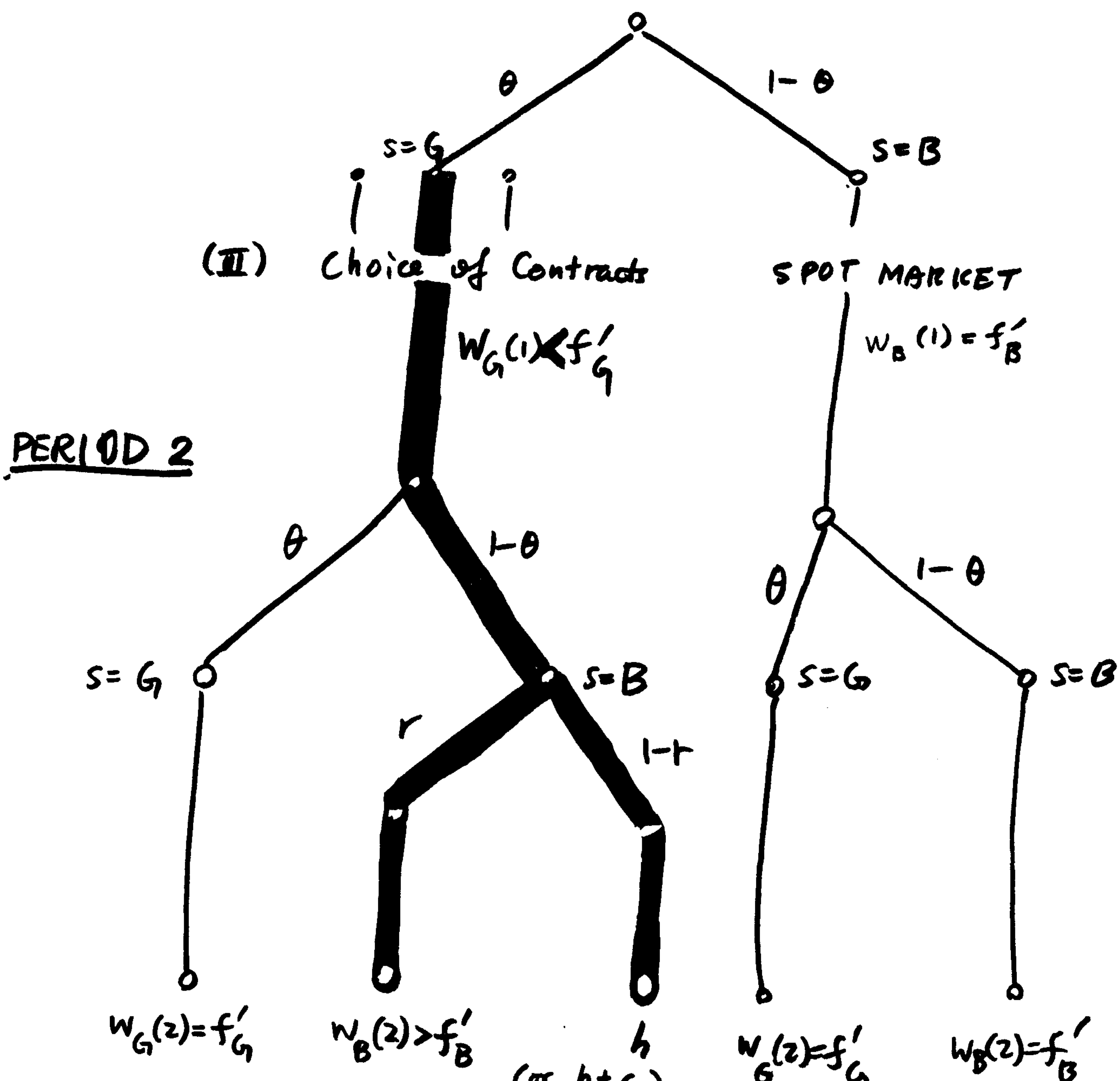


Figure 3-6

An arbitrage condition in the good state of nature implies that $U(w_G) - U(f_G'(L)) = 0$, so that $\mu > 0$. The values of $\hat{w}$, r, L can be solved from

$$(3.28') \qquad f_1'(L) - \hat{w} + (1-\Theta)r(f_B'(rL) - \hat{w}) = 0.$$

$$(3.32) \qquad f_B'(rL) - \hat{w} + \frac{U(\hat{w}) - U(h)}{U'(\hat{w})} = 0$$

$$(3.33) \qquad U(\hat{w}) + \Theta U(w_G) + (1-\Theta)rU(\hat{w}) + (1-\Theta)(1-r)U(h) - \bar{U} = 0.$$

The market equilibrium solution is obtained after substituting $\ell^d(\bar{U}) = L^s$, as was demonstrated in a one-period model. The condition required for an interior solution is that the LHS of (3.32') is strictly negative if $r = 1$. The Holmstrom solution stabilizes the first period wage and the retained workers' wage in the bad state of nature of the second period at the level $\hat{w}$. However, the probability of the bad state of nature is only $(1-\Theta)$, while the first period's state of nature is revealed to be good. This explains why the first term of (3.28') is not weighted by Θ, unlike (3.17") in Azariadis' model.

<u>NUMERICAL EXAMPLE</u>:

Substituting values and function forms of the example, (3.28)' and (3.32) become

$$(3.28'') \qquad (800-4L-\hat{w}) + \frac{r}{2}[200-rL-\hat{w}] = 0.$$

$$(3.32') \qquad (200-rL-\hat{w}) + [2000\hat{w}-\hat{w}^2 + (140)^2 - (2000)\cdot(140)]\,[2000-2\hat{w}]=0.$$

Substituting the equilibrium condition, $L^d = L^s = 100$,

$\hat{w} \simeq 317$

$r \simeq .83.$

Two comments on Holmstrom's model are in order. First, involuntary unemployment in Holmstrom's model has the same undesirable feature that Azariadis' model has. The size of employment in the bad state of nature is larger than the efficient level. In other words, the marginal product of the last worker in the firm is less than home productivity. This can be seem from (3.32') which is identical to (3.16'). Second, the introduction of severance payments into a contract would lead to an efficient solution and no involuntary unemployment. This is shown in the next subsection.

MODEL (V.2): With Severance Payments [Modified Holmstrom]

The maximization problem is modified when severance payments are allowed in the bad state of nature:

$$\max_{\{L,w_1,w_G,w_B,r,c\}} \text{£} = [f_1(L,G)-w_1L]+\Theta[f(L,G)-w_GL]+(1-\Theta)[f(rL,B)-w_BrL-(1-r)cL]$$
$$+\lambda[U(w_1)+\Theta U(w_G)+(1-\Theta)rU(w_B)+(1-\Theta)(1-r)U(h+c)-\overline{U}]$$
$$+\mu[U(w_G)-U(f'(L,G))].$$

The first-order conditions with respect to w_1, w_G and w_B are again (3.29)-(3.31). In addition to these, we have the following.

$$(3.34)\quad 0 = \frac{\partial \text{£}}{\partial L} = (f_1' - w_1)+\Theta[f_G'(L) - w_G] + (1-\Theta)(rf_B'(rL) - w_Br - (1-r)c]$$

$$(3.35)\quad 0 = \frac{\partial \text{£}}{\partial c} = -(1-\Theta)(1-r)L + \lambda(1-\Theta)(1-r)U'(h+c)$$

$$(3.36)\quad 0 = \frac{\partial \text{£}}{\partial r} = (1-\Theta)L\ [f_B'(rL) - w_B + c] + (1-\Theta)\lambda[U(w_B) - U(h+c)]$$

From (3.29), (3.31) and (3.35), we have $w_1 = w_B = h+c \equiv \hat{w}$. From the condition $w_B = h+c$, we know workers are indifferent between being retained and being laid off. It also implies that the second term of (3.36) is zero. That in turn implies that $f_B'(rL) = w_B - c$, provided that $r < 1$. Therefore,

$f'_B(rL) = h$, and productive efficiency is obtained. In this modified Holmstrom model, income of the workers is stabilized for the first period and the bad state of nature, whether or not they are retained. However, since the wage in the good state of nature is different from $\hat{w}$, full Arrow-Debreu Pareto optimality is not obtained. It is easy to see that if a quitting penalty (definition 2.7) is introduced, then full Arrow-Debreu Pareto optimality is restored. In this sense, the inefficiency is caused by a missing market, namely a contingent commodity for $s = G$ in the second period.

NUMERICAL EXAMPLE

The optimality condition $w_1 = w_B = h + c$ obtained from (3.35) and (3.36) implies $f'_B(rL) = h$. In the numerical example, this implies $r = 0.6$, and $f'_B(\cdot) = 140$. The only variable to be calculated is w. Substituting values and functions, equation (3.34) becomes

$$0 = 400 - \hat{w} + (1/2)\cdot\big((0.6)\times 140 - (0.6)\times\hat{w} - (0.4)\cdot(\hat{w} - 140)\big).$$

Solving this equation, we have $\hat{w} = 940/3 \approx 313$. Figure 3-5, which was used to explain a modified Azardias model, summarizes the situation. The only difference is that in the modified Azariadis model, $\hat{w}$ is halfway between w_G and w_B, while in this Holmstrom model, $\hat{w}$ is two-thirds of the way from w_B toward w_G. This is so because in this dynamic problem stabilization takes place over the good-state first-period wage, and the bad-state second-period wage, probability of the latter being only $(1 - \theta) = 1/2$.

Now let us summarize and compare solutions for two-period models. Figure 3-7 shows branches of possible events for a worker in our two-period economy. Numbers in parentheses show the net income of a worker. In the case of

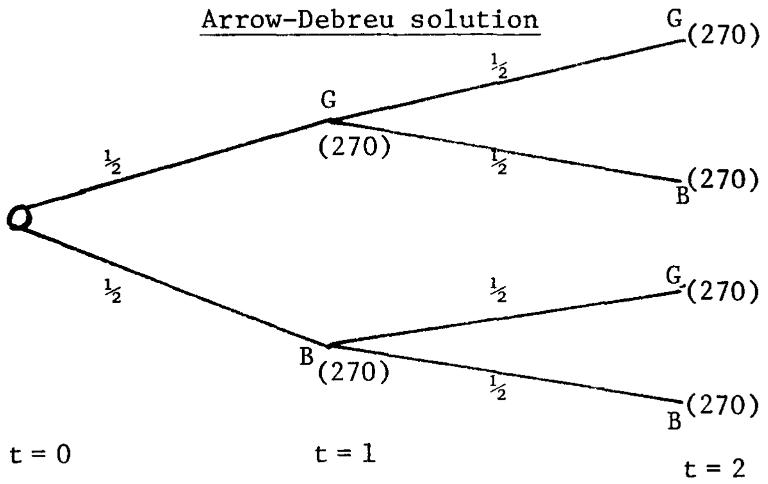
Figure 3-7 about here

an Arrow-Debreu economy, net income is the sum of the spot market wage and the contingent transfer agreed upon at $t = 0$. At the second period in the Holmstrom solution, a layoff without severance-payments occurs with the probability (1-r). Income under contracts are underlined. In the modified Holmstrom solution, laid-off workers at the second period are receiving severance payments which make up the difference between the contract wage (about 313) and home productivity (140). Therefore, although there are layoffs in the bad state of nature in the second period, net incomes for the retained and for the laid off are both 313.

Figure 3-7

Summary of two-period models

Arrow-Debreu solution

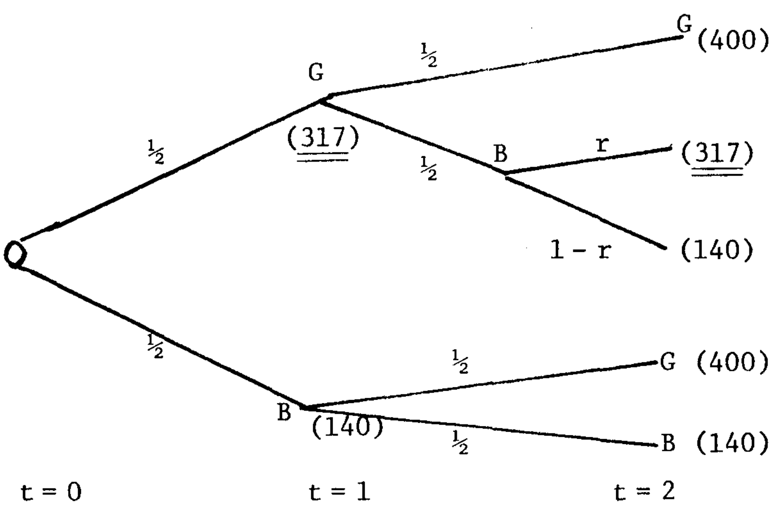


at G: $w_G - c = 270$.

B: $w_B + c = 270$.

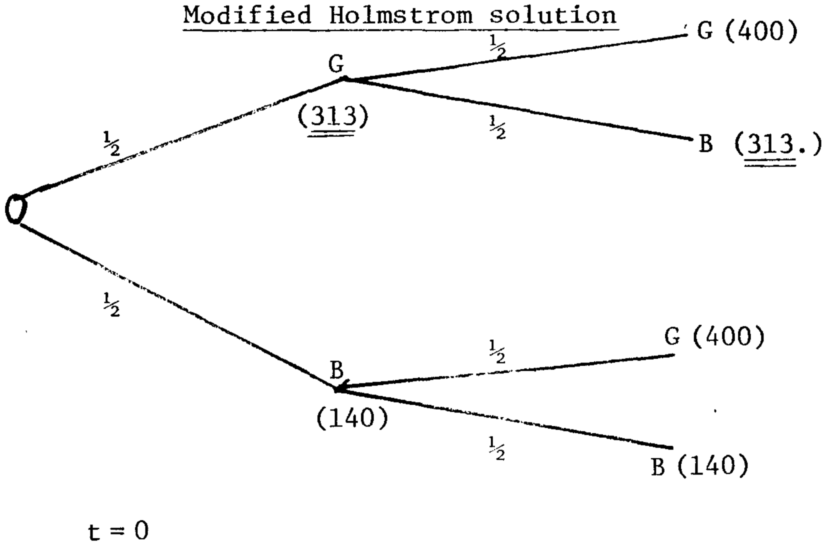
$EU(t{=}0) = U(270) + U(270)$.

Holmstrom solution



r = .83

Modified Holmstrom solution



3.6 Pension, Mobility and Efficiency

In the model above, adding *ex post* mobility always decreases efficiency in the economy, destroying the feasibility of contracts with stabilized wages in the good state of nature. If there is some way to restrict the worker's mobility, it is desirable. We may think of circumstances in which bidding up wages can be avoided in the good state of nature. First, the firms could make a gentleman's agreement (or an explicit cartel agreement) that they will not try to steal workers from each other, even if the state of nature is good. This may be possible if the industry consists of a few companies and workers are traceable. A professional sport industry, such as baseball, may be an example of this.

Second, if a worker can be fined when he quits voluntarily, then full Pareto optimality can be achieved. Suppose that upon his initiating separation from a firm, a worker has to pay a quitting penalty c_W to the firm. Then not only the wage for the good state of nature in the second period but also that in the first period can be stabilized. However, the enforceability of such contracts is questionable.

Third, considering the inefficiency caused by *ex post* mobility, firms and workers may agree that workers can be trained in firm-specific skills. In this way, the firm can offer a wage higher than the marginal product in the bad state of nature and lower than that in the good state of nature. This is an additional argument for firm-specific skills (for the usual argument of firm-specific skill and contractual agreement, see Becker (1962) Hashimoto (1981) and Carmichael (1981)).

Finally, non-refundable pensions and retirement benefits would restrict workers' mobility and thus enhance efficiency. Suppose that a non-productive third period is added to the modified Holmstrom model presented in the preceding subsection (recall Figure 3-7). A worker is productive in the first two periods at the firm or at home. In the third period, a worker retires and earns nothing. Assume also that private saving or private contingent commodities are not available. Optimal contracts are calculated in the same manner as in the previous section. However, in this section let us look at the solution in a heuristic way, which is an interpretation of the "equivalent problem" described in section 3.4. In the first period, workers start accumulating an asset in the pension fund. Suppose that a worker accumulates x_s when the first state of nature is s, s = {B, G}, receiving net wage $w_1 = f_S'(L) - x_s$. In the second period, the amount to be accumulated in the pension fund is x_{ss}, where the first and section subscripts represent the first and second states of nature, respectively. The net wage is $w_{2s} = f_s'(L) - x_{ss}$. In the third period, the firm pays a worker x_{s3} as a pension. Note that the firm is indifferent about managing the pension fund as long as

$$(3.37) \qquad x_s + Ex_{ss} = x_{s3}$$

where the expectation is taken over the second states of nature. Since workers are risk neutral, the best contract would be to stabilize the income stream, given the first state of nature.

$$(3.38) \qquad w_1 = w_{2G} = w_{2B} = x_3.$$

Note that because of the ex post mobility, it is not possible to make a contract before the first period. Given the realized state of nature in the first period, an optimal contract is negotiated. Assuming at this stage that the ex post mobility in the second period does not jeopardize (3.38), we use (3.37) and (3.38) to calculate the optimal income stream

$$(3.39) \qquad \hat{w} = \{2E_s f'_s - f'_1\}/3$$

and the amount of the pension fund

$$(3.40) \qquad x_1 = \{2f'_1 - E_s f'_s\}/3$$

where $E_s f'_s$ is the expected marginal ability in the second period. The optimal solution (3.39) is not viable if the <u>ex post</u> mobility in the second period gives a better opportunity <u>given</u> the realization of the second state of nature. The firm can offer the stabilized wage profile $\{f'_{2s}/2,\ f'_{2s}/2\}$ over the second and third period, given the state of nature in the second period 2s. If $\hat{w} < \{f'_{2s}/2\}$ holds true, then (3.38) cannot be implemented and $w_{2G} \neq w_{2B}$. This possibility is considered in numerical example below.

There are two important observations at this point. First, when there is a non-refundable pension fund, the wage in the good state of nature in the second period may not equal the marginal product, unlike the Holmstrom model. Second, if the pension fund is refundable, the contract which was optimal with non-refundable pensions breaks down and the solution becomes exactly the same as the one without pensions. Let us now explain these observations.

In the case of the modified Holmstrom model, the good state of nature in the second period implies that the wage has to be equal to the marginal product. However, with the pension fund, this may not be the case. If a worker moves to another firm <u>ex post</u> in the second period, the stabilized wage over the second and third (retirement) periods is equal to half of the "good" marginal product, he loses any amount deposited in the pension fund in the first period. If the stabilized wage over three periods given the first state of nature and constraint (3.37) is greater than half of the "good" marginal product, the stabilized wage $w_1 = w_2 = x_3$ prevails.

The second observation is immediately verified. Suppose that a worker in the optimal contract described above quits and collects x_{s1} in the good state of nature in the second period. By doing so his consumption stream becomes $\{f'_{2G}(L) + x_1\}/2$ for both the second and third periods. Withdrawing his part of the pension fund and finding a job in the spot market is better than continuing with the old contract, if the following inequality holds:

$$(3.40) \qquad \{f'_{2G} + x_1\}/2 > \{Ef'_s + f'_1\}/3.$$

In order to prove this, recall that $x_1 = \{2f'_1 - Ef'\}/3$. The left hand side of (3.40) becomes $f'_{2G}/2 + f'_1/3 - Ef'_s/G$. The difference between the left and right hand sides is $\{f'_{2G} - Ef'_s\}/2$ which is always positive, since $Ef'_s = \theta f'_{2G} + (1-\theta)f'_{2B}$, $0 < \theta < 1$, $f'_{2B} < f'_{2G}$.

NUMERICAL EXAMPLE

According to the stabilization plan given the realization of the first state of nature the wage is calculated using (3.39) and (3.40):

$$\begin{cases} \text{If } s = G \text{ in the first period, } \hat{w} = 223. & \text{(Plan G)} \\ \text{If } s = B \text{ in the first period, } \hat{w} = 137. & \text{(Plan B)} \end{cases}$$

The viability of this plan should be examined in the case of the good state of nature in the second period. Firms in the second period are willing to offer the wage profile {200, 200} over the second and third period. This profile is worse than the contract with good first state of nature, but better than the one with the bad first state of nature Competition after the state of nature is revealed in the second period forces a modification of plan B. Therefore, in the case of the bad first state of nature having pensions does not help the stabilized wages. In this case, stabilization over three periods takes place only when the second period also has a bad state of nature:

$$w_1 = f'_B - x_1 = 140 - x_1$$

$$w_2 = E(f'_B - x_2) = \tfrac{1}{2}(140 - x_2).$$

$$Ex_3 = x_1 + Ex_2, \text{ i.e., } x_3 = 2x_1 + x_2.$$

$$w_1 = w_2 = Ex_3.$$

Solving these equations, we find $\hat{x}_1 = \hat{x}_2 = 35$ and $w_1 = w_2 = w_3 = 105$. Results are summarized in Figure 3-8.

Figure 3-8 about here

It is counter-intuitive that ex post mobility has a detrimental effect on the economy. This is a result of an assumption that the realized state of nature is the same for all times. In the general case without this assumption the mobility of workers improves the resource allocation for society because it makes it possible to shift workers from a low-productivity industry (under the bad state of nature) to a high productivity industry (under the good state of nature). This is a general equilibrium effect of the ex post mobility which is missed in all the models with one market, including models of firm-specific skills.

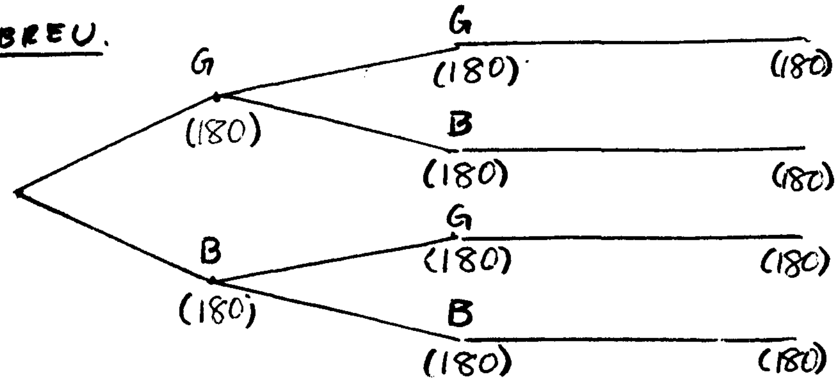
A model to consider two different effects of the mobility of workers should be investigated. A limited attempt is made in the next section of this paper.

So far mobility has been treated exogenously. However, from the literature on search theory, we know how a worker decides to accept or reject the opportunities which he is facing in the presence of incomplete information. The alternative wage may depend on the intensity of search. Both incomplete information about alternative opportunities and also moral hazard in the search process will prevent economic agents from making a contract with an efficient resource allocation. A simple exposition of such a conjecture is acutely needed. The first step in synethesizing implicit contract theory with search theory was presented by Burdett and Mortensen (1980).

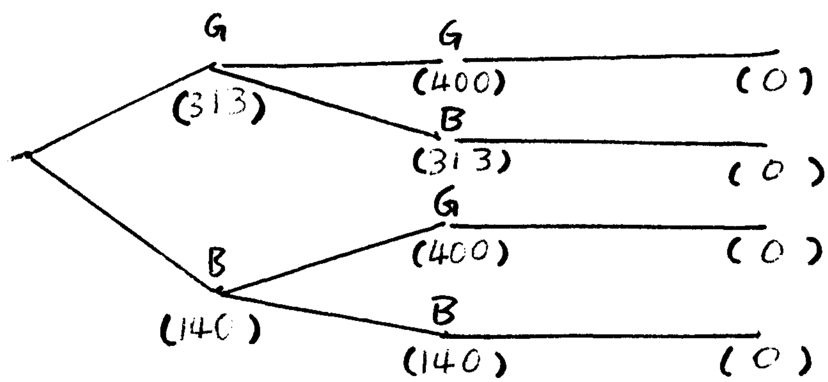
Figure 3-8

Three-period model: $f'_{1G} = f'_{2G} = 400$; $f'_{1B} = f'_{2B} = 140$; $f'_3 = 0$.

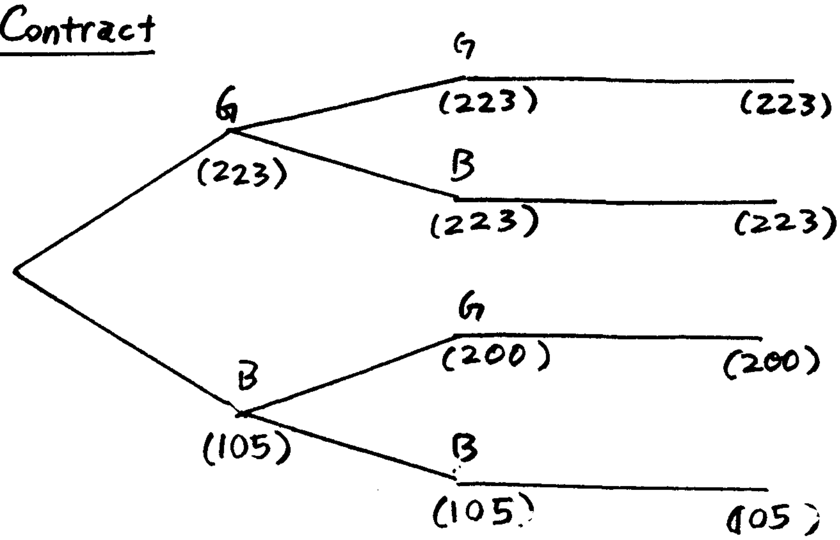
ARROW-DEBREU.



Modified Holmstrom (2 period contract without pension plan) [From Fig. 3.7]



Pension Contract



3.7 Seniority in Many-Period Overlapping Generation Models

Model V can be extended to a many-period model without losing any of the characteristics of the solution (see Harris and Holmstrom (1981)). Since the contract wage in the bad state of nature in the second period following a first period good state of nature is higher than the wage in the bad state of nature in the first period, it can be said that the wage is "downward" rigid. Another way of looking at this phenomenon is an explicit modelling of overlapping generations. Suppose the firm is perpetual, while every worker lives two periods. The firm maximizes the expected profit with respect to each generation. Let us consider the case where the bad state of nature is followed by the good state of nature. The old generation is receiving $w > f'_B$, because it is under the contract written in the preceding period, while the young generation is receiving f'_B as the wage. This is an explanation of seniority wage (Definition 2.8) by an implicit contract theory. The argument, of course, is valid in a multi-period model.

NUMERICAL EXAMPLE

Take the two-period modified Holmstrom model, and interpret it as an overlapping generation model with an equal number of individuals in each generation. The young in the current period are treated as workers in the first period. Thus, in the good state of nature, they are paid at 313, while in the bad state they got 140. The current old are considered as workers in the second period of the modified Holmstrom model. Therefore, their wages depend on the states of nature not only in the current but also in the preceding period. If the preceding period was in the good state of nature, the old generation is under the implicit contracts. Hence, the stabilized wage 313 is paid if the current state of nature is bad. However, if the bad state of nature prevailed in the preceding period, then there is no protection of implicit contracts. Hence the wage is 140 when the current state of nature is also bad. If the current state of nature is good, the old generation is always paid 400, the marginal product. See Figure 3.9 for the summary.

Figure 3.9

Sequence of the states of nature		wage paid to	
previous period	current period	old	young
B	B	140	140
B	G	400	313
G	B	313	140
G	G	400	313

Observe that in the numerical example, the old workers are paid more than the young in three out of four cases. This seniority wage profile is derived from the timing of contracts and *ex post* mobility. Currently young workers were not alive in the previous period in which currently old workers obtained the contract. The *ex post* mobility prevents forming the contract before the first state of nature is revealed.

Nalebuff and Zeckauser (1981) discussed the issues of pensions and seniority in a model similar to one presented here. They discuss relations between different types of contracts and actual institutions we observe in the real world, emphasizing applications of theoretical models. They do not consider an aspect of *ex post* mobility, namely how competition after a state of nature is revealed forces the contract wage to move up if there is a sequence of good states of nature.

4. TWO INDUSTRY MODELS

In this paper we will not fully analyze the multi-industry models, but we will give a concise introduction to other works.

A major difficulty in many-industry models stems from the fact that outside opportunities for laid off workers are endogenous to the model. When some industries are in the good state of nature (i.e., they have higher marginal product schedules) and others are in the bad state, it is efficient to transfer workers from the bad-state industries to those in the good-state until the marginal products of all industries are the same. Outside opportunities are now more complicated than "home production" which has a "fixed" marginal product, but the efficient condition with respect to severance payments does not change. If information is complete in the sense that the states of nature for other industries and the spot market wages in those industries are common information for all firms and workers, then most of the results from the one-industry model carry over. Severance payments will restore the efficient allocation of workers even if there are some mobility costs.

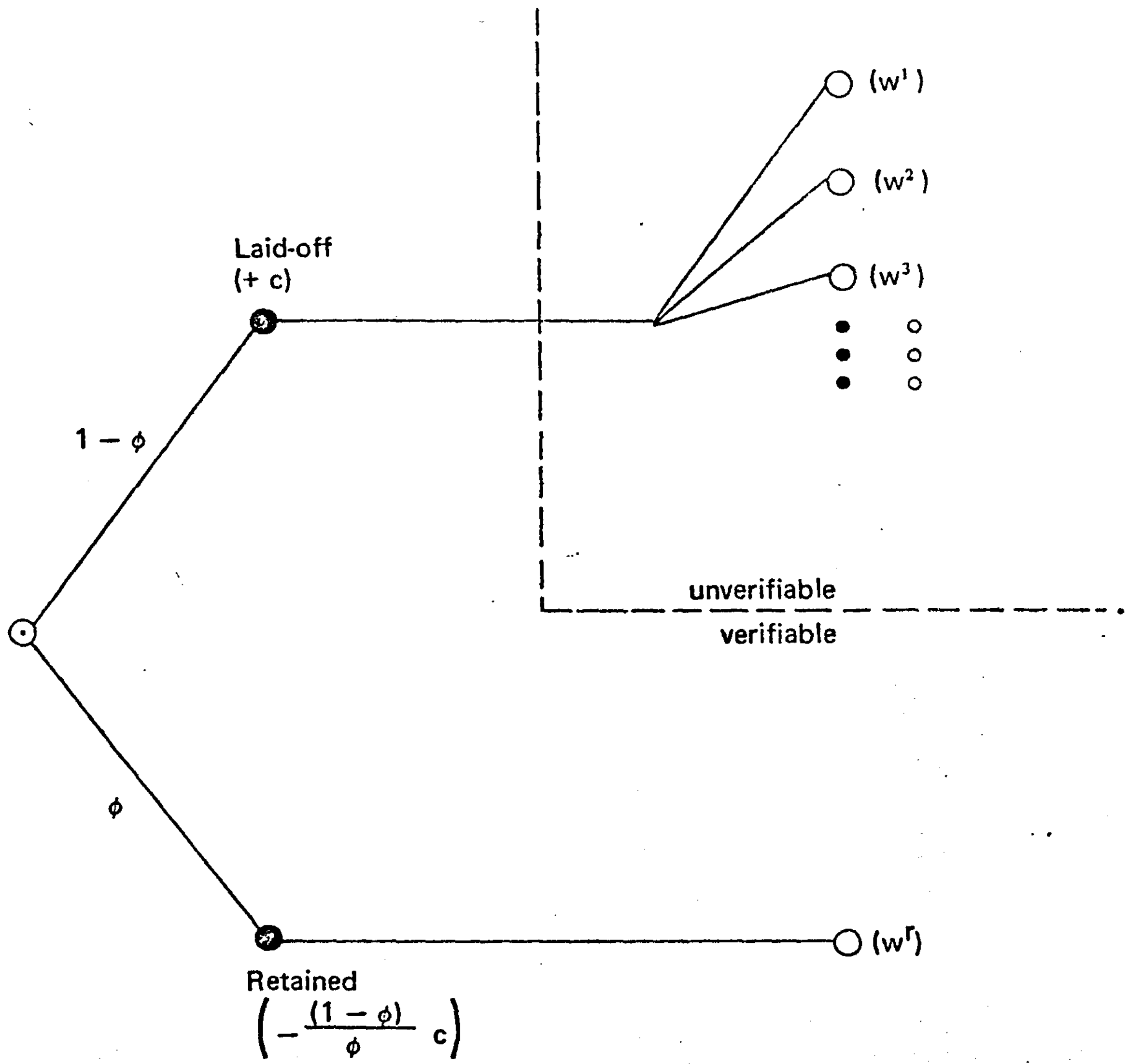
Polemarchakis and Weiss (1978) analyzed such a model with two industries and mobility costs. Consumers' demand for products from these industries fluctuates in a manner such that a decline in demand for one commodity implies an increase in demand for the other. There are moving costs z which are incurred when a worker is transferred from one industry to the other. An efficient solution is to lay off workers from a declining industry and move them to the other industry until the difference in the marginal products of the industries is precisely equal to z. Polemarchakis and Weiss argued that this efficient solution is obtained under the restrictions that the severance payments are equal to z and the initial difference in productivities is less than $2z$. However, it is clear that severance payments need not be restricted to be equal to z. In fact, in the Polemarchakis and Weiss model there is no need to make severance payments, because there is no risk associated with layoff in this model with perfect information on the higher marginal product of the other firm.

Involuntary unemployment arises in the case where information is not complete. Suppose one industry is in the bad state of nature. If the firm and workers do not know the alternative wage a worker can earn after being laid off, it is not possible to write a contract which guarantees full employment. Arnott, Hosios and Stiglitz (1980), Hosios (1980), and Geanakoplos and Ito (1981) explore this framework. These models use different ways of incorporating uncertainty about the wages of outside opportunities. The Arnott, Hosios, and Stiglitz model assumes that outside opportunities are simply unknown. Therefore, severance payments must be determined without any information on the states of nature of other firms. Geanakoplos and Ito (1981) allows economic agents to obtain the aggregate information. However, workers' skills are not compatible with the requirements of all other industries. Thus, the uncertainty about alternative wages is technological.

Imai, Geanakoplos, and Ito (1981) prove an interesting theorem in this kind of incomplete information model. Suppose that a risk-averse agent faces different incomes in different states of nature. It is obvious from the preceding section that optimal risk-sharing between a risk-averse worker and a risk-neutral (insurance) firm results in the equalization of net income (income after premium and coverage) across different states of nature, given that all states of nature are verifiable and severance payments are available. Suppose now that in some subset the states of nature are not verifiable individually, but only as a group of states. See Figure 4-1. Then the optimal insurance arrangement with severance payments implies that the marginal utility of net income at a verifiable state of nature is equal to the expected marginal utility of net income over a group of unverifiable states of nature. Given this optimality condition, Imai, Geanakoplos, and Ito show that the (total) utility of a verifiable state is greater than, equal to, or less than the expected (total) utility of a group of unverifiable states depending on whether the degree of absolute risk aversion is decreasing, constant, or increasing, respectively.

Figure 4-1 about here

If the degree of absolute risk aversion is decreasing, then the theorem has a counter-intuitive implication for the economic example. Given an optimal insurance scheme against layoffs alone, a laid-off worker who does not yet know the rehiring wage elsewhere enjoys a higher level of expected utility than a retained worker. Therefore, every worker wants to be laid off. In order to prevent a worker from voluntarily quitting, severance payments should be paid only in the event of an involuntary layoff.



(w¹)
(w²)
(w³)
Laid-off
(+ c)
1 − φ
unverifiable
verifiable
φ
(wʳ)
Retained
$\left(-\frac{(1-\phi)}{\phi}\ c\right)$

FIGURE 4-1

5. MACROECONOMIC IMPLICATIONS

Implicit contract theory also has interesting macroeconomic implications. With optimal contracts between firms and workers, the amplitude of fluctuations of wages over business cycles should be considerably dampened. It is no longer true when implicit contracts exist that wages paid in any period always equal workers' marginal product. The theory would predict that the wage, nominal or real, is "rigid" in the sense that it does not respond to temporary shocks in technology and taste. At one point, implicit contract theory was thought to provide a micro-foundation for fixed price disequilibrium theory (Gordon (1976)). However, it turned out that the two theories are only remotely related. First, the deviation of the wage from the marginal product is a result of optimization in implicit contract theory, while a standard disequilibrium model interprets the deviation as unsatisfied (excess) demand or supply. Second, it has been shown that implicit contracts stabilize (i.e., give less amplitude of fluctuations over business cycles) the level of employment, as well as the level of the wage. Therefore, a short-side rule of transaction in a simple disequilibrium model is inconsistent with what implicit contracts imply. Some models in the implicit contract literature result in the existence of "involuntary" unemployment as a consequence of an *ex ante* optimal arrangement between supply and demand sides in the labor market, while a disequilibrium model regards it as the result of a discrepancy between demand and supply. Although implicit contract theory is not readily compatible with "disequilibrium" macro models, this does not mean implicit contracts support the results obtained in the school of "equilibrium" business cycles. An important feature of equilibrium business cycles is that nominal shocks to the system cause an unexpected increase or decrease in the nominal wage level which in turn cause an increase or decrease in labor supply through intertemporal leisure substitution. However, if long-term contracts are introduced, the wage rate may be stabilized, undermining this explanation of business cycles.

6. CONCLUDING REMARKS

The state of the art in implicit contract theory along the line of Azariadis, Holmstrom, and Geanakoplos and Ito is summarized in Figure 6.

Figure 6 about here

The following are problems left for the future research. First, we need to study the economic institutions and environmental and legal restrictions which make *ex post* mobility and severance payments possible or impossible, since these restrictions are important in determining equilibrium implicit contracts. For example, it may be interesting to compare the labor market in the U.S. with that of Japan, since the latter is often considered as a society with (implicit) lifetime employment contracts. The idea of international comparisons has been pursued by Hashimoto (1979) and Gordon (1982). Both noticed that wages (including bonuses) in Japan are more flexible than those in the U.S. However, the simple contractual theory in this paper would predict that flexible wages are a sign of mobility in good states of nature, which contradicts lifetime employment. A reconciliation of these two aspects is left for a future research.

Second, the mobility and search process in a multi-industry model should be carefully studied. The alternative wage may depend on the intensity of the search. Both incomplete information of alternative opportunities and also moral hazard in the search process will prevent economic agents from making contracts with efficient resource allocation. A simple exposition of this conjecture is acutely needed. A first step toward synethesizing implicit contract theory and search theory was given by Burdett and Mortensen (1980). This direction should be further explored.

Figure 6

Restrictions on / types	Environment			Labor Contracts			Outcome	
	Home Production	Contingent Claim	horizon	state - contingent wage	state - contingent severence pay	ex post mobility	Type of Unemployment	Worker's Income
Spot Market	No	No	one - period (static)	No	No	Yes	$L = L^s$	variable
	Yes	No		No	No	Yes	$L < L^s$ voluntary	
Arrow-Debreu	No	Yes		No	No	Yes	$L = L^s$	constant
	Yes	Yes		No	No	Yes	$L < L^s$ voluntary	
Azariadis (1975)	Yes	No		Yes	No	No	$L < L^s$ involuntary	constant for retained: variable for laid off
Azariadis with modifications	Yes	No		Yes	Yes	No	$L < L^s$ voluntary	constant
Holmstrom (1980)	Yes	No	2-period	Yes	No	Yes	$L < L^s$ involuntary	variable
Holmstrom with modification	Yes	No	2-period	Yes	Yes	Yes	$L < L^s$ voluntary	variable
Geanakoplos-Ito (1981)	Yes	No	2-period	Yes	Yes	Yes*	$L < L^s$ involuntary	variable

* But uncertainty in skill matching and with search costs.

FOOTNOTES

1/ The assumption of uniformly distributed non-atomic workers is convenient in the sense that competitive behavior is well justified and that the integer problem does not occur. Holmstrom (1980) used this interpretation first. The integer problem means a that the last worker of employment may have to split hours between the firm and home, contrary to an earlier assumption. Of course, our model is then unable to analyze the choice of working hours.

2/ This scenario was not employed in the papers of Azariadis and Baily where they considered the bilateral negotiations between the exogenously fixed labor pool and the firm. Holmstrom was the first to mention this scenario.

2a/ If leisure is an argument separate from income in the utility function of a worker, then unemployment by hours instead of men is a possibility to be studied. It can be shown also that the level of utility of an involuntarily unemployed worker is higher than that of a retained worker in the Azariadis type model with separable leisure in the utility function. See section 3.3 below.

3/ Therefore, the similarities of our labor supply function and the one in a crude Keynesian macro model do not go beyond their appearance. Our definition and explanation of "involuntary"unemployment is totally different from that of a crude Keynesian macroeconomic model.

4/ Recall the definition of involuntary unemployment given by Keynes (1936: p. 15).

5/ This logic is already demonstrated by Negishi (1979: p. 230). However, Negishi's argument against Azariadis, saying "strangely he did not consider (15) [which is (3.15') with a reverse inequality] to be likely and argued for the inferiority of a full employment contract," is pointless, because condition $f_B'(L^s) \leq h$ is as "likely" as (3.15').

6/ Public unemployment compensation should be regarded as a part of home production in our model. Therefore, severance payments in our model corresponds to private (additional) unemployment compensations.

7/ Note that we assume that the firm does not renege the contract in the bad state of nature.

8/ Harris and Holmstrom (1981) is an example where this maximization is employed.

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