A CRITICAL SURVEY OF DISEQUILIBRIUM

GROWTH THEORY

by

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1. Introduction

Disequilibrium growth theory is a study in dynamics of capital accumulation in an economy where price sluggishness causes short-run disequilibrium. Since disequilibrium in the labor or goods market would affect a decision of how much to save or invest, the processes of capital accumulation and dynamic price adjustment should be considered simultaneously. A technique of disequilibrium analysis is carried over from disequilibrium macroeconomics models.[1/] Take the labor market for example. The level of actual transaction is determined at the lesser of demand and supply, given the temporarily fixed wage rate. Growth models are distinguished from the usual disequilibrium macroeconomic model by an explicit consideration of capital accumulation. Two different ways of modeling capital accumulation have been proposed. On the one hand, Ito (1978)(1980) considers a model in which savings behavior determines capital accumulation. Picard (1979), d'Autume (1980), and Malinvaud (1980), on the other hand, emphasize the role of investment in capital accumulation. In the following, I will summarize the results obtained in the works cited above and point out problems to be solved in the future.

2. Disequilibrium Neoclassical Growth Theory

Ito (1978)(1980) introduced (real) wage rigidity into a one-sector neoclassical growth model, with a production function $F(K, L)$ that is homogenous of degree one with respect to labor, L and capital K. The wage rate, w_t, is fixed, and the capital stock, K_t, is historically given at period t. The equality of the marginal product with respect to labor and the wage rate defines the level of labor demand: $L^d = \Omega(w_t) \cdot K_t$, $\Omega'(\cdot) < 0$. The supply of labor, L^s, is exogenously given, and growing at a constant rate, $\dot{L}^s/L^s = n$. An actual employment level, L_t, is the lesser of L^d and L^s: $L_t = \min\,[L^d_t, L^s_t]$. Let us introduce two kinds of capital-labor ratio. The "desired" capital-labor ratio, $k^d_t = K_t/L^d_t(w_t)$ defines the one that capitalists would like to have given the wage rate. The "actual" capital-labor ratio, $k_t = K_t/L^s_t$ measures the natural definition of ratio between the existing capital and labor force. The usual neoclassical growth model would be obtained, if the wage is assumed to adjust to the market clearing level, $\eta(k_t)$, infinitely fast at each moment, where $\eta(k_t) \equiv f(k_t) - k_t f'(k_t)$. An economy is classified into an unemployment regime, if $L^d < L^s$, i.e., the wage rate is higher than the market clearing level, $\eta(k_t)$. If, on the other hand, the current wage rate is lower than $\eta(k_t)$, then the economy is said to be under an overemployment regime because $L^d > L^s$. Production takes place after transaction of labor is carried out on the short side:

$$Y_t = F(K_t, \min\,[L^d_t, L^s_t]).$$

All products are distributed as the wage bill wL, or as profits (Y - wL). Then workers and capital owners decide how much to consume and to save out of their income. Since there are no monetary assets in the model, income has to be spent either as consumption or savings which add up to production. Therefore, there is no disequilibrium in the product market. The budget constraints of workers and capital owners add up to the equality of demand and supply in the product market.

In other words, disequilibrium in the labor market "spills over" to the product market through a reduction in income, but the spill-over is completely absorbed in the product market. Let us assume that saving rates out of both types of income are the same and a constant s.[2] Then, capital accumulation is a constant fraction of income: $\dot{K} = sY$. Along with capital, the wage rate is assumed to be constantly adjusting, according to the pressure of excess demand:

$$\dot{w} = \begin{cases} \xi_1 (L^d - L^s)/L^s & \xi_1 > 0 \text{ if } L^d \geq L^s \\ \xi_2 (L^d - L^s)/L^s & \xi_2 > 0 \text{ if } L^d < L^s. \end{cases}$$

Note that this kind of wage adjustment implies that full employment cannot be maintained unless an economy is in long-run steady state.[3] The neoclassical regimes and the two disequilibrium regimes are considered separately, because the dynamics of capital accumulation and the wage rate differ depending on the regime.

<u>Neoclassical (Full-employment) Regime</u>: $R_f = \{(k, w) | w = f(k) - kf'(k)\}$

$$\begin{cases} \dot{k}_t = sf(k_t) - nk_t \\ \dot{w}_t = 0 \end{cases}$$

The unique long-run steady-state is defined by

$$\begin{cases} \hat{k} = \{k | sf(k) - nk = 0\} \\ \hat{w} = f(\hat{k}) - \hat{k}f'(\hat{k}) \end{cases}$$

It is easy to show that

$$\begin{cases} \dot{k} < 0 & \dot{w} = 0 & \text{if } \hat{k} < k \text{ and } (k, w) \in R_f \\ \dot{k} = 0 & \dot{w} = 0 & \text{if } k = k \text{ and } (k, w) \in R_f \\ \dot{k} > 0 & \dot{w} = 0 & \text{if } \hat{k} < k \text{ and } (k, w) \in R_f. \end{cases}$$

<u>Unemployment Regime</u>: $R_u = \{(k, w) \mid w > f(k) - kf'(k)\}$

$$\begin{cases} \dot{k}_t = k_t \,[sf(k^d(w_t))/k^d(w_t) - n]. \\ \dot{w}_t = \xi_2 \,\{k_t/k^d(w_t) - 1\} \end{cases}$$

It can be shown that

$$\begin{cases} \dot{k} < 0 & \dot{w} < 0 & \text{if } \hat{w} < w \text{ and } (k, w) \in R_u \\ \dot{k} = 0 & \dot{w} < 0 & \text{if } w = \hat{w} \text{ and } (k, w) \in R_u \\ \dot{k} > 0 & \dot{w} < 0 & \text{if } w < \hat{w} \text{ and } (k, w) \in R_u. \end{cases}$$

<u>Overemployment Regime</u>: $R_o = \{(k, w) \mid w < f(k) - kf'(k)\}$

$$\begin{cases} \dot{k}_t = sf(k_t) - nk_t \\ \dot{w}_t = \xi_1\{k_t/k^d(w_t) - 1\} \end{cases}$$

Similarly to the other regime, it can be shown that

$$\begin{cases} \dot{k} < 0 & \dot{w} > 0 & \text{if } k > \hat{k} \text{ and } (k, w) \in R_\theta \\ \dot{k} = 0 & \dot{w} > 0 & \text{if } k = \hat{k} \text{ and } (k, w) \in R_\theta \\ \dot{k} > 0 & \dot{w} > 0 & \text{if } k < \hat{k} \text{ and } (k, w) \in R_\theta. \end{cases}$$

The phase diagram of the above equations are shown as Figure 1.

<u>Figure 1 about here</u>

It is easy to see from the phase diagrams that the long-run steady state is globally asymptotically stable. Any trajectory in the phase diagram converges to the steady state either without switching regimes, or switching into, but not out of, a subregime such that $k < \hat{k}$ in R_u.[4/]

A conclusion obtained from this model is that even if there is a sluggishness in wage adjustment, an economy eventually converges to the long-run full-employment equilibrium. However, during the adjustment process, excess demand or supply will be observed. It is an advantage of this type of model that a neoclassical (equilibrium) growth model is contained as a special case of infinitely fast wage adjustment. The nested equilibrium model gives a benchmark in the sense that a cause of disequilibrium is identified and the inefficiency of disequilibrium can be discussed. However, the tractability of the model is obtained at the cost of several aspects of over-simplification.

First, the savings behavior is described only in an ad hoc manner. It would be desirable to consider agent's maximization problem taking into account that (i) savings behavior would influence the selection of future regimes and (ii) the return from saving depends on which regime an economy finds itself in the future. Although Ito (1978) employs an overlapping generation model which is capable of analyzing this question, the assumption of Cobb-Douglas utility function washes the above-mentioned aspect of the problem.

In this regard, Neary and Stiglitz (1979) provide us with a framework in which an expectation of a future regime matters now. However, Neary and Stiglitz (1979) only analyze a two-period model without capital accumulation, thus, they fall short of full investigation of disequilibrium growth.

Second, the wage adjustment equation is not based on microeconomic behavior. It reflects pure myopic adjustment based on a law of supply and demand. This is a defect common to the usual disequilibrium macroeconomic model. A cure would call for a careful study of pricing behavior in an economy with uncertainties.

Third, as noted above, the model described here does not have any financial asset, thus failing to allow disequilibrium in the product market. Therefore, the usual classification of disequilibrium regimes (see next section) is not available. Since discrepancies between savings and investment give an important implication for capital accumulation, a study of portfolio choice with disequilibrium is a fruitful direction.[5/]

3. Disequilibrium Keynesian Growth Theory

Picard (1979)(1980), d'Autume (1980) and Malinvaud (1980) proposed a model in which the product market is possibly in disequilibrium, and investment determines capital accumulation. We will summarize Picard's (1979) (1980) model in this section.

The production function is of Leontief-type, i.e., capital and labor are not substitutable: $Y^s = \min\ [\beta K, L^s/\alpha]$. Aggregate effective demand output is, like a Keynesian macroeconomic model, composed of consumption, investment and government expenditure: $Y^d = C + I + G$. Actual transaction of output Y is the minimum of Y^d, βK, and L/α: $Y = \min\ [Y^d, \beta K, L^s/\alpha]$. Actual transaction of labor, L, is similarly defined: $L = \min\ [\alpha Y^d, \alpha\beta K, L^s]$. If the first (second, third) entry of the definition of Y is binding, then so is the first (second, third, respectively) of the definition of L. Malinvaud (1980) gave the following classification of regimes:

(i) Keynesian unemployment if $Y = Y^d, \Leftrightarrow L = \alpha Y^d$

(ii) Keynesian unemployment if $Y = \beta K, \Leftrightarrow L = \alpha\beta K$

(iii) Repressed inflation if $Y = L^s/\alpha, \Leftrightarrow L = L^s$.

Capital accumulation is determined by investment behavior.

$$\dot{K} = I(\min\,[\underset{+}{Y^d},\ L^s/\alpha],\ \underset{-}{K},\ \underset{-}{w},\ \underset{-}{r}\}$$

where r is the return to capital. The deficit in the government sector, i.e., expenditures less tax revenue is financed by government money. Private investment is financed exclusively by borrowing funds from the banking sector. The income tax rate is adjusted such that money creation (in real terms) plus the income tax is equal to government expenditures and investment: $\dot{M} = P(G + I - \tau Y)$.

Denote the marginal propensities to consume out of the disposable income by c; and out of the real balance of money by d; and the growth rate of money by Θ: Also define $k \equiv K/L^s$ and $m = M/PL^s$. It is easy to show

$$\dot{k} = [I\ [\min\,[Y^d,\ L^s/\alpha],\ K,\ w,\ r]/K - n]k_t$$

An economy in the regime of Keynesian unemployment,

iff

$$(d + \Theta)M/P \leq \beta K$$

$$(d + \Theta)M/P \leq L^s/\alpha.$$

and $Y = \sigma(d + \Theta)M/P$, where $\sigma = 1/\{1 - c(1 - \tau) - \tau\}$.

Similarly, the regime of classical unemployment is defined by

$$(d + \Theta)M/P \geq \beta K$$

$$\alpha\beta K \leq L^s$$

and $Y = \beta K.$

The repressed inflation regime is the case if,

$$\alpha\sigma(d + \Theta)M/P \geq L^s$$

$$\alpha\beta K \geq L^s$$

and $$Y = L^s/\alpha.$$

The partition of the space into different regime is illustrated by Figure 2, adopted from Picard (1980; Figure 1, p. 12).

Figure 2 about here

The rate of change of the nominal wage s is affected by two different factors: first, the excess demand for labor; and second, the difference between the target real wage rate, w_o, and the current real wage rate w:

$$\dot{s}/s = \lambda(\alpha Y^d - L^s)/\alpha Y^d + \gamma(w_o - w), \; \lambda > 0, \gamma > 0$$

The inflation rate of the price of the product determined by two effects: first, the excess demand for products, and second, the nominal wage inflation:

$$\dot{P}/P = \mu\{Y^d - \min(\beta K, L^s/\alpha)\}/Y^d + \delta\dot{s}/s, \; \mu > 0, \; 0 < \delta < 1$$

Then the dynamics of the real wage is, by definition the difference in $\dot{s}/s$ and $\dot{P}/P$:

$$\dot{w} = [\dot{s}/s - \dot{P}/P]w$$

and $$\dot{m} = [\Theta - \dot{P}/P - n]m$$

The dynamics of the economy are described by the differential equations of w, m, and k, defined above. Picard proves the local stability of the

Walrasian steady state, X, in the case that (i) $\theta = n$, (i.e., the monetary growth rate is equal to the population growth rate); (ii) w_o is equal to "the real wage rate for the investment rate I/K is equal to the natural growth rate n, when desired production $\overline{Y}$ is equal to the production capacity βK."6/ If these two conditions are not quite satisfied, but $\lambda/\mu < 1/(1 - \delta)$, then a locally stable non-Walrasian steady state exists in one of the regimes described above.

Several comments are in order. First, Picard's model is attractive because there are three regimes corresponding to the ones in the usual disequilibrium macroeconomic model. Characteristics of the well-known static models carries over to the growth model. Since the fixed-coefficient production function is assumed, prices are less important in achieving the Walrasian (equilibrium) state. In that sense, this model has inherited characteristics from the so-called Harrod-Domar (Keynesian) growth model. This aspect contrasts to Ito's model which has the neoclassical growth model as a special case where the wage is adjusted infinitely fast. Condition (ii) for the stability of the Walrasian steady state is understood as the well-known formula requiring the equality of the warranted and natural growth rates in the Keynesian growth model. Therefore, a criticism common to all Keynesian growth models apply to Picard's model. In the long-run, the capital-labor ratio <u>can</u> change reflecting the factor price ratio. Since investment determines capital accumulation, it should be justified carefully.7/ Malinvaud's (1980) paper derives the optimal capacity by the profit maximization with an uncertain future demand. Henin and Michel (1981) proposes a derivation of an investment function by intertemporal profit maximization (known as the Jorgenson-type investment function) with a demand constraint. This is a very promising direction.

The assumption that investment always dictates capital accumulation because of unlimited money creation by the banking sector is unrealistic, too. A study of capital accumulation when desired investment differs from desired saving is essential in the future research on disequilibrium growth theory. For that purpose, Neoclassical and Keynesian growth theories presented above should be synthesized.

In the next section, the Neoclassical model presented in section 2 will be modified, so that the different regimes associated with the usual disequilibrium macroeconomic model, like the ones considered in section 3, would arise. However, the degree of modification is minimal and it is not meant to be an ultimate synthesis of two approaches.

4. Disequilibrium Growth Model with Foreign Investment

In this section, we introduce disequilibrium in the goods market within a Neoclassical framework. Let us assume that an economy presented in section 2 is a small country which is open unilaterally to foreigners. Investment and consumption from foreign countries are allowed, but agents in this country cannot invest in foreign countries or consume foreign products. Returns to the portion of capital, θK, which is owned by foreigners are paid out to them, but foreign consumption and investment demand compose a part of aggregate demand. Because of the budget constraints, the domestic part of aggregate demand is always equal to domestic income, i.e., production, as show in section 2. Therefore, a trade deficit (surplus, respectively) implies an excess supply of (demand for, respectively) products of this country. Let us assume that foreigners spend a fixed portion, $1-s$ of their income (i.e., the returns to their capital) on consumption of goods of this country. Investment demand from foreign countries, I^F, depends on the current return on their capital: $I_F = I(q)$, where $q = \{Y - wL\}/K$. Let us assume that (i) The higher the rate of return, the more foreign investment, $I' > 0$; and (ii) At the rate of return realized at the long-run steady state of this country investment is equal to capital income less saving, i.e., trade balances, $I(q^*) = s\,\theta(Y - wL)$ where $q^* = f'(\hat{k})$.

These two assumptions imply that if $q = q^*$, where q^* is the rate of return to capital, then all foreign saving (capital income less consumption, $\theta s\,(Y - wL)$) returns to this country. If $q > q^*$, then a relatively high rate of return attracts more foreign investment than foreign saving, financed by sources other than capital income paid from this country. And if $q < q^*$, then this country looks less attractive for investment, implying some of $\theta s\,(Y - wL)$ is invested in other countries.

Now we are ready to define two regimes with respect to disequilibrium in the goods market.

<u>Excess demand for goods: $Y^d > Y^s$</u>

i.e., $I_F > s\theta(Y - wL)$ if and only if $q > q^*$.

<u>Excess supply for goods: $Y^d < Y^s$</u>

i.e., $I_F < s\theta(Y - wL)$ if and only if $q < q^*$.

Now we have to determine when $q > q^*$ or $q < q^*$. If an economy is in the full employment regime, $q \gtrless q^*$ if and only if $k \lessgtr \hat{k}$, respectively, because $f'' < 0$ and $q = f'(k)$ is the full employment regime.

In the unemployment regime, $K/L = k^d(w_t)$, and $w_t = f(k^d) - k^d f'(k^d)$.

$$q = \{f(k^d(w_t)) - w_t\}/k^d(w_t)$$
$$= f'(k^d(w_t)).$$

Note that $q = q^*$ at $w = \hat{w}$ and $k = \hat{k}$. Since q does not depend on k_t, $q = q^*$ at $w = \hat{w}$ and any value of k_t in this regime. Observe also

$$\frac{dq}{dw} = f''(\cdot)\frac{dk^d}{dw_t} < 0, \quad (k, w) \in R_u$$

Therefore if $w > \hat{w}$ then $q < q^*$, i.e., excess supply of goods, and vice versa.

In the overemployment regime, $q = \{f(k_t) - w_t\}/k_t$. Therefore,

$$\frac{\partial q}{\partial w_t} = -\frac{1}{k_t} < 0, \quad (k, w) \in R_o$$

$$\frac{\partial q}{\partial k_t} = \frac{1}{(k_t)^2}\{kf'(k) - (f(k) - w_t)\}$$

$$< 0, \qquad (k, w) \in R_o .$$

The last inequality is due to the definition of this regime, that is $w_t < f(k_t) - k_t\, f'(k_t)$. Therefore, in the overemployment regime the line where q is equal to q^* is downward sloping. Now define the familiar four regimes as follows.

Keynesian Unemployment (KU):	$L^d < L^s$, $Y^d < Y^s$.
Classical Unemployment (CU):	$L^d < L^s$, $Y^d > Y^s$.
Repressed Inflation (RI):	$L^d > L^s$, $Y^d > Y^s$.
Under-consumption (UC):	$L^d > L^s$, $Y^d < Y^s$.

The result of partioning the (k, w) space is shown in Figure 3.

Figure 3 about here

Now it ias to be investigated how capital accumulation is affected by disequilibrium in the product market. If the product market is in excess demand, some component of aggregate demand has to be rationed. Let us assume that the foreign investment demand is rationed to restore the balance between aggregate demand and supply in this regime.[8/] When aggregate supply exceeds aggregate demand, some ouptut is left unsold, which will further decrease q, the rate of return. This is turn will decrease investment. We assume that this multiplier process converges at some point.

In summary, we assume

$$\dot{K} = s(wL + (1 - \theta)qK) + \min[I(q), s\theta qK]$$

It is easy to verify that in the regime where $q > q^*$, i.e., excess demand for goods, $\dot{K} = sF(K, L)$ so that the differential equation of $\dot{k}$ in section 2 applies. If there is an excess supply of goods, i.e., $q < q^*$, then $\dot{K} < sf(K, L)$. This implies that $\dot{k}$ in this case is less than $\dot{k}$ obtained in section 2. Therefore, the qualitative result of $\dot{k} < 0$ does not change from what we had in section 2.

A complete phase diagram with partitioning of the space into different regimes is shown as Figure 3. The long-run steady state is again globally asymptotically stable.[9/]

5. Productivity Increase and Unemployment

It is well-known that the only way to achieve the wage increase without causing inflation is to enhance the labor productivity. This proposition holds with an assumption of full employment. That is why there is so much concern about a sagging productivity increase in the U.S. compared to Japan and West Germany. One possible explanation to this fact is that the Japanese labor unions are much more cooperative in installing new labor-saving machines in a factory because their employment in a firm, if not in the same division, is guaranteed by the (implicit) life-time employment contract. However, it is a puzzle why the U.S. labor unions would oppose to improving the labor productivity, since it would eventually increase the real wage at full employment. A key to solve this puzzle lies in the distinction between the comparative statics focused only on different steady states and the disequilibrium dynamics from one steady state to another. Take a Neoclassical growth model with the exogenous Harrod-neutral technological progress. At a steady state, the growth rate of per-capita income (i.e., a rate of increase in the real wage rate) is equal to a rate of technological progress. Therefore, the faster technological progress implies the higher wage hike at a new steady state. However, it is important to recognize that an increase in the technological progress rate (say, a spurt in technological progress) would reduce the steady-state capital-labor ratio. Therefore, at the moment of a spurt in technological progress, the old steady state finds its capital-labor ratio too high, and the level of (market-clearing) wage rate too high.
During the transition from the old to new steady state, the wage rate (in terms of the efficiency unit) has to decline. Whether this disequilibrium negative effect overrides a positive effect from technological spurt on the wage increase depends on the elasticity of substitution between capital and

labor. If the production function is sufficiently inelastic, then the wage temporarily has to decrease due to a spurt in the technological progress, a paradox. The following theorem is adopted from Ito (1980, p. 398).

Theorem

When there is a spurt of technological progress from α_1 to α_2:

(i) the real wage in the natural unit decreases to maintain full employment if and only if,

$$\sigma < \left(1 - \frac{\alpha_1}{\alpha_2}\right) \frac{\hat{k}\, f'(\hat{k})}{f(\hat{k})}$$

(ii) the increase of the real wage slows down to maintain full employment if and only if,

$$\sigma < \frac{\hat{k}\, f'(\hat{k})}{f(\hat{k})} ,$$

where σ is the elasticity of substitution; $\hat{k}$ is the level of capital-labor ratio at the old steady-state; and f is the production function in an intensive form.

Instead of a formal proof of this theorem, which is available in Ito [1980; section 5.5], an intuitive analysis is in order. A sudden spurt in the labor-augmenting (Harrod neutral) technological progress should decrease the demand for labor, given the existing capital stock, the price of the products and the factor-price ratio. The relevance of the elasticity of substitution in determining the change of the wage to ensure the full employment is seen with a help of Figure 4.

Figure 4 about here

Suppose that (L_o,K_0) is a steady state at time t=0 with respect to the technological progress rate and the saving rate which are implicit in the diagram. Three different production functions, ordered from inelastic to perfectly elastic in terms of factor substitutability, are represented by isoquants $A_0E_0A_0$, $C_0E_0C_0$, and $B_0E_0B_0$. The spurt in technological progress is represented by the inward shift of isoquants (compare the isoquants with subscript 0 to those with subscript 1 in Figure 4). If the Leontief-type production function is the case, ($A_0E_0A_0$ to $A_1E_1A_1$), then the difference between L_0 and L_1 is the size of the unemployment inevitably caused by technological progress, because the factor prices play no role in selecting the steady state capital-labor ratio or the level of production. This is a well-known result in the Keynesian growth theory (c.f. section 3 of this survey). The other extreme is represented by a case that labor and capital are perfectly substitutable, represented by a shift from $B_0E_0B_0$ to $B_1E_1B_1$. Since the homogeneity (of degree one) of a production function requires parallel isoquants, $B_0E_0B_0$ will remain to be an isoquant but for a higher level of output. Therefore, E_0 stays as a new steady state without altering the factor-price ratio. Between these two extreme cases, a general neoclassical production function is represented by an isoquant $C_0E_0C_0$ at the old steady state. After the spurt of technological progress, isoquants are represented by $C_1E_1C_1$ and $C_2E_0C_2$. If the factor-price ratio remains the same with one at the level of the old steady state, the labor demand becomes L_1 leaving workers with the size of L_1L_0 unemployed. This is understood by recalling K is fixed at the moment and interpreting B_0B_0 and B_1B_1 as the factor-price ratio lines. Unlike the Keynesian case, the change in the wage rate relative to the rental rate of capital would restore full employment. The factor-price ratio has to change to the line to which $C_2E_0C_2$ is tangent at E_0. It is obvious that the wage has to change (decrease) more, the sharper is the curvature of isoquants (close to the case of inelastic isoquants). This is an interpretation of the above theorem.

In this section it has been shown that a spurt in technological progress might cause a transitory effect if the production function is sufficiently inelastic with respect to factor substitutability. This offers an explanation why the labor union without a long-term contract might suffer a temporary set back when technological progress is about to speed up. However, in this model, a technological progress is totally exogenous. It would be desirable to consider a model in which the rate of technological progress is a choice variable in order to investigate causes of the productivity slow-down in the U.S. in connection with its low saving rate.

6. Concluding Remarks

We have reviewed the Neoclassical and Keynesian approaches to disequilibrium growth theory. One of the unsatisfactory aspects of the Neoclassical approach is that the possibility of disequilibrium in the goods market is assumed away. By introducing foreign investment, the model in section 4 incorporates disequilibrium in the goods market.

Problems left for future research are mentioned in the preceding sections. Among others, (i) microeconomic foundations of saving and investment should be carefully constructed; (ii) a process of determining the level of actual capital accumulation should be investigated if *ex ante* saving and investment are different, and (iii) the rate of technological progress should be endogenized. Especially, the effect of the saving rate on technological progress should be modelled properly, unlike the neoclassical model.

FOOTNOTE

[1/]Growth models which try to incorporate unemployment and other disequilibrium phenomena without considering the short-side rule are summarized in Ito (1980: footnote 2).

[2/]Ito (1980) considers the case that saving rates from two different sources are different.

[3/]For an alternative assumption of wage adjustment, see Ito (1980: p. 385, Eq. (2.12)).

[4/]In the case that the savings rate of capital owners is higher than that the wage earners, the stability analysis becomes more complicated. Especially a problem of differential equations a discontinuous right hand side will prevent us from employing the usual stability theorems. See Honkapohja and Ito (1980).

[5/]Sgro (1981) is the first work toward this direction. However, his model fails to incorporate the possibility of disequilibrium in the product market. Moreover, his stability analysis is not complete in the sense that an interaction between the dynamics of the capital-labor ratio and the real wage is ignored.

[6/]Since the system of differential equations has different forms in different regimes, they are continuous over the boundary. Thus, it is possible to use Eckalbar's (1980) result. Problems associated with stability with regime switching would become more complicated if there were different behavioral parameters defined in different regimes.

7/ The investment function would make more sense if it depends on the difference between min[Y^d, L^s/α] and K, instead of each entity separately. Then it has to be explained why investment can be positive even if this difference is negative.

8/ It would not qualitatively change the result, if we assume that every component is proportionally rationed. The boundary of $\dot{k} = 0$ in the unemployment regime would be downward sloping isntead of vertical.

9/ This can be intuitively understood by observing that a subregime of Repressed inflation where $k < \hat{k}$ still is an absorbing regime.

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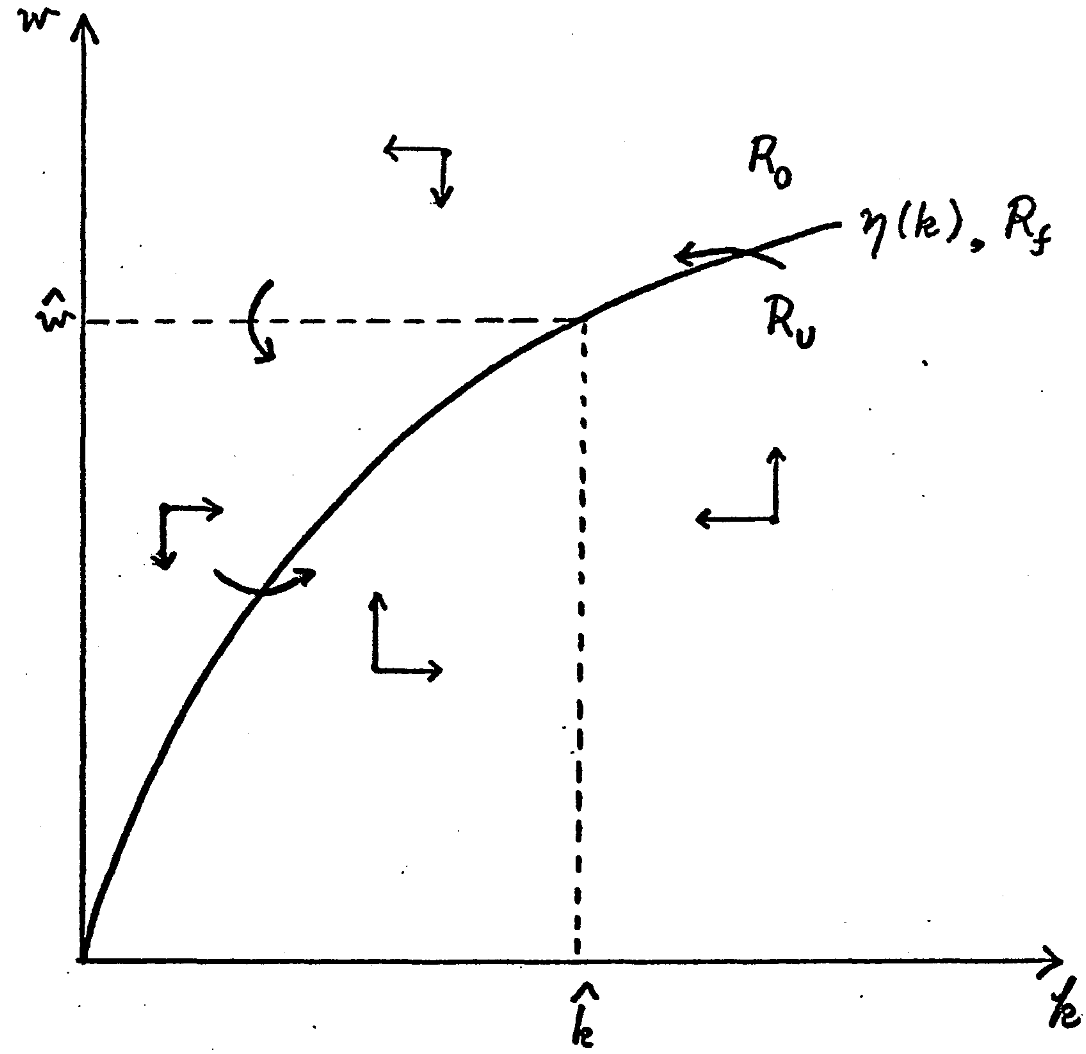

FIGURE II

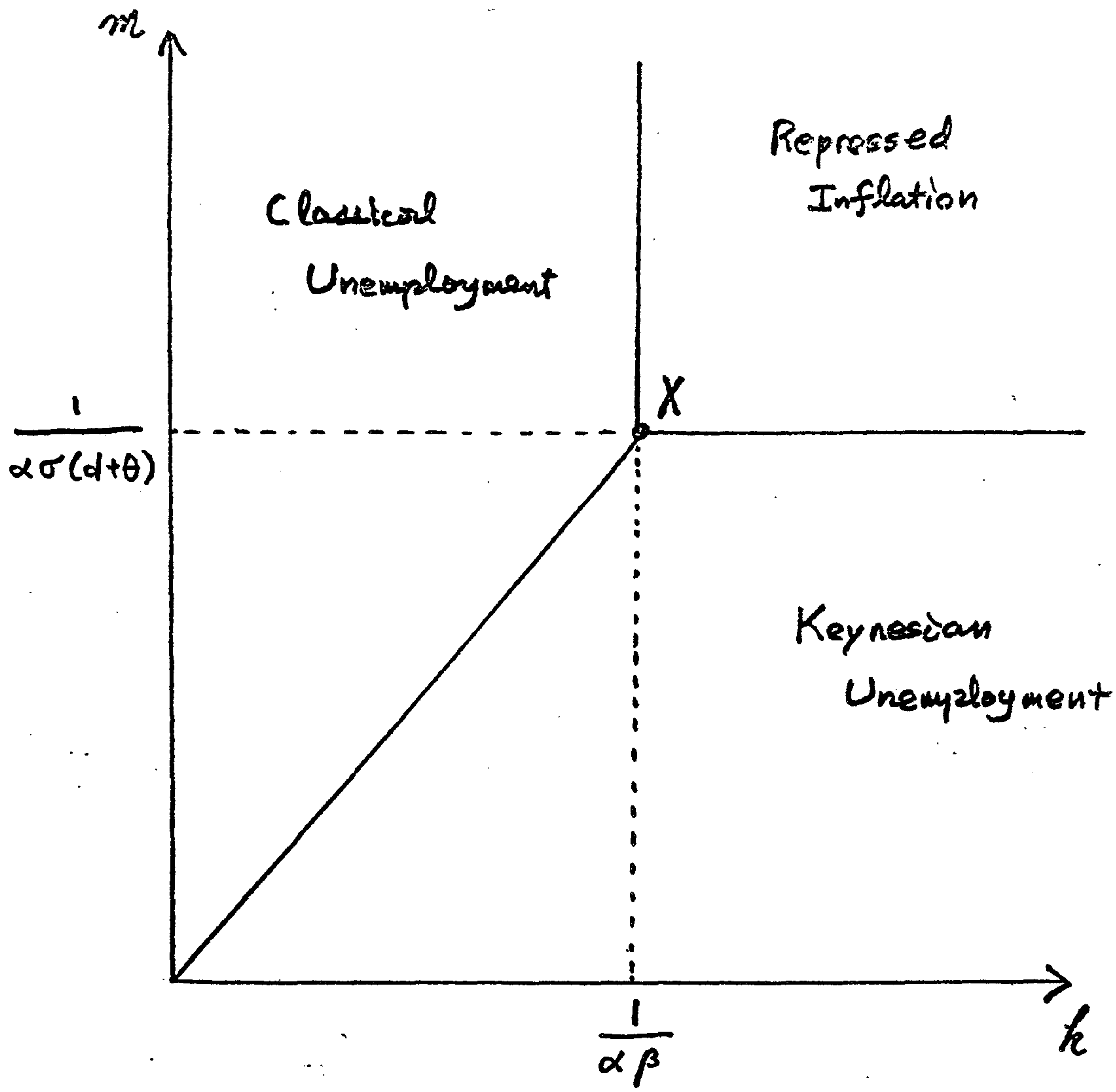

Figure 2 [Picard (1980)]

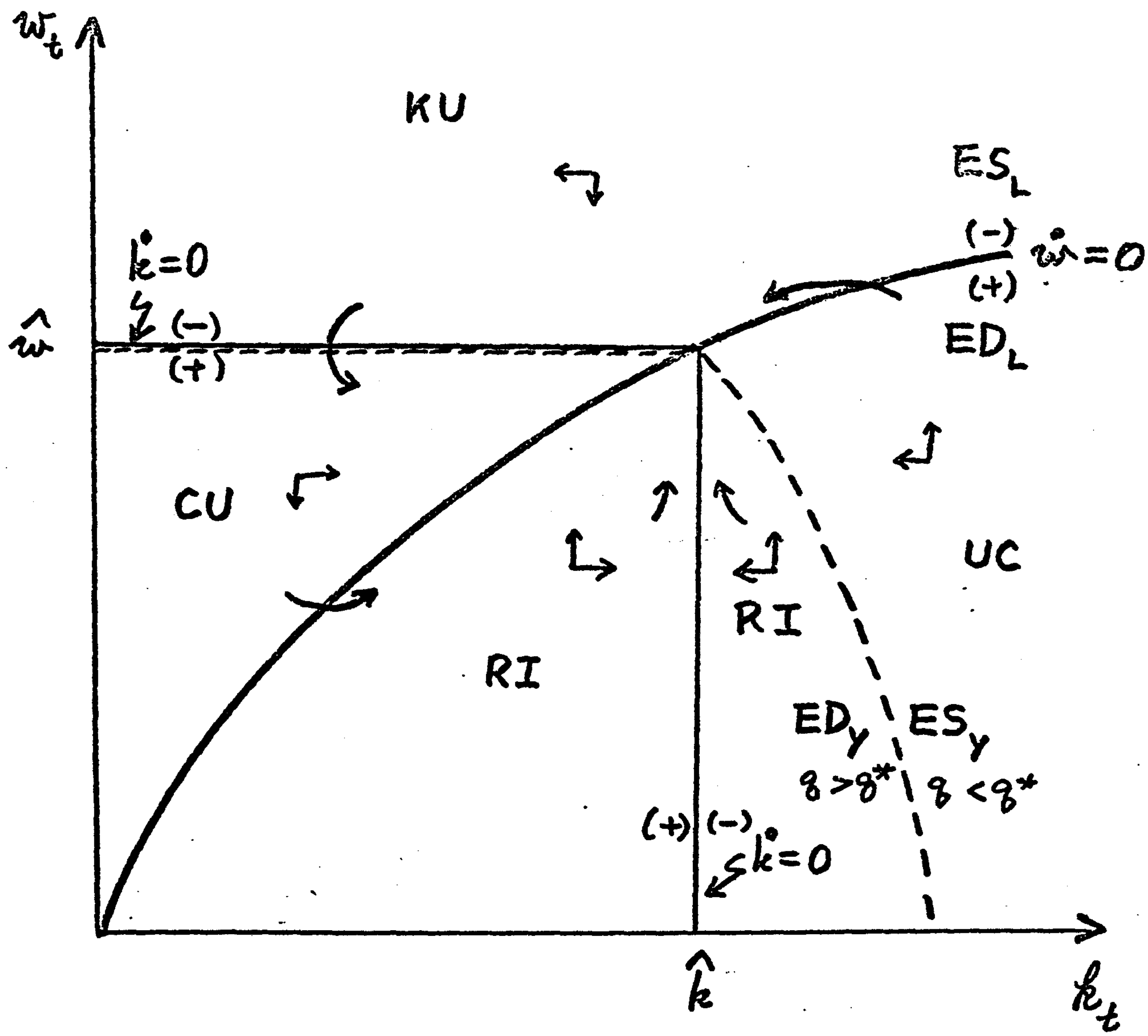
w_t
KU
ES_L
(-)
$\dot{w}=0$
(+)
$\dot{k}=0$
$\hat{w}$
(–)
(+)
ED_L
CU
UC
RI
RI
ED_y
ES_y
$q>q^*$
$q<q^*$
(+)
(-)
$\dot{k}=0$
$\hat{k}$
k_t

Figure 3

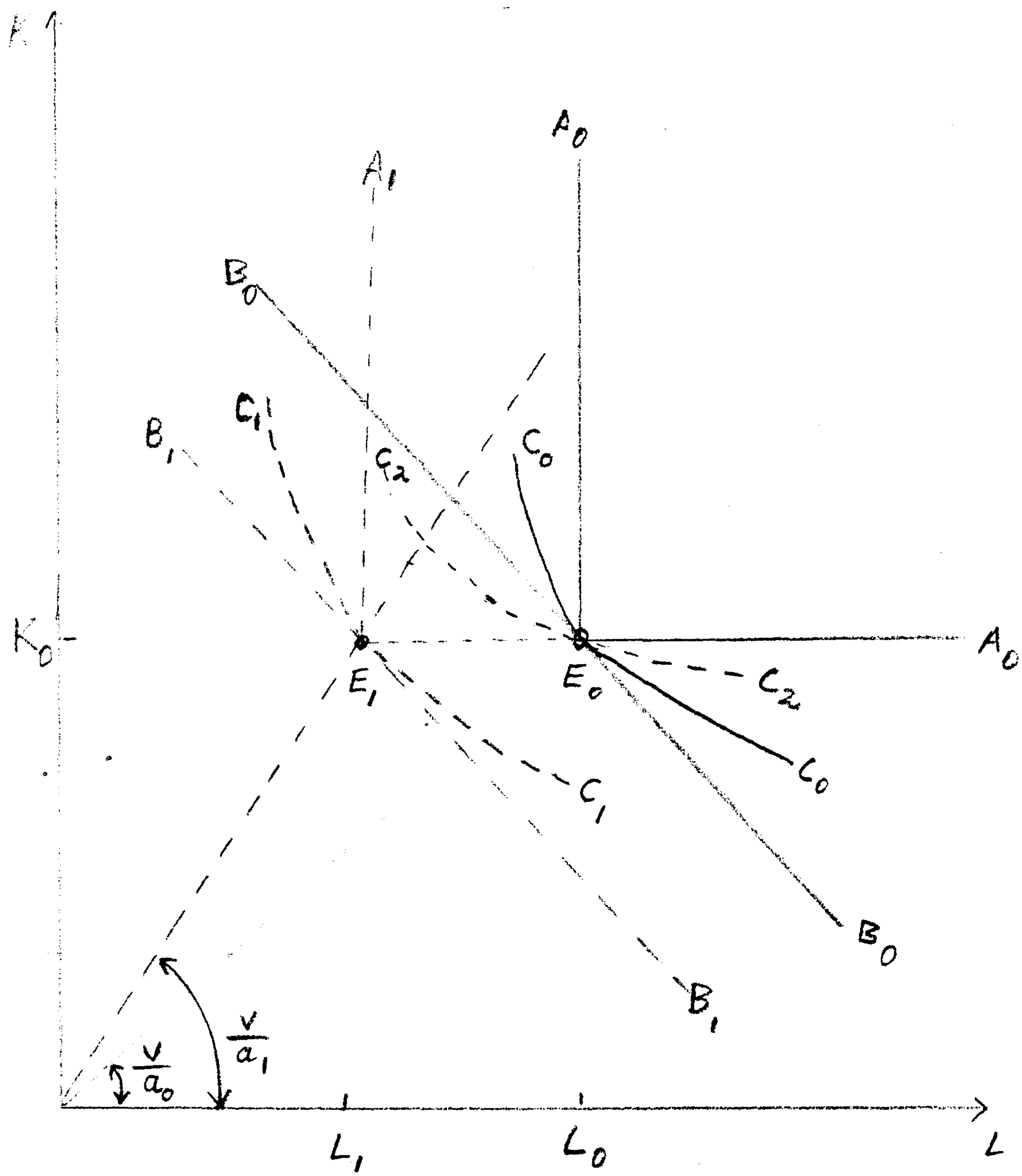
K
A_0
A_1
B_0
C_1
B_1
C_2
C_0
K_0
E_1
E_0
A_0
C_2
C_0
C_1
B_0
B_1
$\frac{v}{a_1}$
$\frac{v}{a_0}$
L_1
L_0
L

Figure 4