OPTIMA AND COMPETITIVE EQUILIBRIA

WITH ADVERSE SELECTION AND MORAL HAZARD

by

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ABSTRACT

This paper explores the extent to which standard, general equilibrium analysis of optima and competitive equilibria in the linear space containing lotteries can be applied to environments with moral hazard and adverse selection problems. Techniques for characterizing optima as solutions to linear programs are found to be useful and nice and appear to be broadly applicable. But existence and optimality of competitive equilibria seem to require that agents with characteristics which are distinct and privately oberved at the time of initial trading enter the economy-wide resource constraints in a homogeneous way; subsequent heterogeneity is not critical. The homogeneity condition is satisfied for a dynamic private-information securities economy and a moral hazard insurance economy, but not for the well-known and interesting signaling and adverse-selection insurance economies. For the latter, heterogeneity introduces an externality of some kind.

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1. Introduction

The purpose of this paper is to explore the extent to which standard general equilibrium analysis of optima and competitive equilibria can be applied to environments with moral hazard and adverse selection problems. In these environments the information structure is explicit but private. Of particular interest are the Rothschild-Stiglitz [1976], Wilson [1978] insurance environment, in which each agent observes a parameter indicating the probability of suffering a loss, that is, whether he is a high risk or low risk agent; the Spence [1974] signaling environment, in which each agent observes a parameter indicating his inherent productivity as well as the disutility of some completely unproductive activity, and the moral hazard insurance environment in which agents can take an unobserved action determining the probability of suffering a loss.

In an earlier paper, Prescott and Townsend [1979], we established that standard general equilibrium analysis of optima and competitive equilibria could be applied to a pure exchange securities economy with private shocks to preferences: in the linear space containing lotteries we established the existence of optima and competitive equilibria, that every competitive equilibrium is an optimum, and, apart from an exceptional case, that every optimum can be supported as a competitive equilibrium. In the present paper we examine the extent to which these results can or cannot be extended. Both the successes and the failures are revealing.

In our approach, and consistent with general equilibrium modeling, we are careful to distinguish the environment itself from the equilibrium notions we employ. We do this notwithstanding the fact that the insurance and signaling environments are intimately associated with specific equilibrium notions. Thus we avoid the use of exogenous, free-entrymotivated, zero-profit conditions and exogenous marginal productivity conditions. The general environment is described in section 2. Consistent with the use of lotteries, the commodity space is a (finite-dimensional) real linear space. A convex subset of this space is the common consumption set for the agents. There are a finite number of agent types, but each agent's type is private information. Each agent type has preferences as defined by a (linear) utility function on the common consumption set. There is also a common endowment. Implementable consumption allocations are defined by a finite set of linear resource constraints and by certain *ex ante* incentive-compatibility conditions. In section 2 it is established that the Rothschild-Stiglitz-Wilson insurance environment, the Spence signaling environment, the Prescott-Townsend private-information securities environment and the moral hazard insurance environment are all special cases of this general structure.

In the context of a tightly specified, general equilibrium environment Pareto optimal allocations can be defined in the usual way. It is established, in section 3, that optima generally do exist and can be determined as solutions to concave programming problems. This technique is used to characterize the set of Pareto optima for both the insurance and signaling environments. In the process it is established that the Rothschild-Stiglitz separating equilibrium is optimal in the space of lotteries under exactly the same conditions which make it optimal in the space of (apparently) deterministic allocations. For the signaling environment it is established, generally, that optima do not involve complete separation of agent types. We also recall that optima for the private-information securities environment involve nontrivial lotteries.

Section 4 begins an attack on questions of equilibrium. In the Spence signaling environment informationally consistent price functions are not unique and in many cases are nonoptimal. Rothschild and Stiglitz adopt a more strategic equilibrium notion, a set of contracts offered by firms such that (i) no contracts make negative expected profits, and (ii) no other potential contract would make a positive profit if offered; the equilibrium is Cournot-Nash in as much as each firm assumes that the contracts its competitors offer are independent of its own actions. Rothschild and Stiglitz give us more nonstandard results--an equilibrium may not exist, and if it exists, it may not be optimal. Wilson's equilibrium is even more strategic, each firm taking into account the reaction of its competitors. This and the equilibrium notions of subsequent authors yield existence in some cases. Riley [1977] offers key insights on the equilibrium notions of this literature.

In our approach we return to standard, general equilibrium, competitive analysis. Thus our approach is nonstrategic. Given the environment, we define a price system, and optimal actions are defined relative to these prices. In equilibrium, (i) each consumer chooses a utility maximizing element in his budget set, (ii) each firm chooses a profit maximizing element in a well-defined production set, and (iii) markets clear. This competitive equilibrium notion is less realistic in a descriptive sense. But the intent of this research is to discover the implications of private information alone for the operation of competitive markets; we seek to do this by employing a standard competitive equilibrium notion rather than equilibrium notions from imperfect competition. This is also the intent of the note by Hahn [1974], but we arrive at somewhat different conclusions.

The natural specification of the competitive equilibrium notion (described in section 4) is a set of prices, one price for each point in the commodity space, a consumption allocation for each agent type, and a production allocation which satisfy the three equilibrium properties of the previous paragraph. This specification was highly successful for the environment of Prescott-Townsend [1979]; both existence and optimality of competitive equilibria were established. And we establish here that these results carry over to the classicial moral hazard insurance environment. But a difficulty emerges in using this specification in environments with a fundamental adverse-selection problem. The difficulty is that a natural specification of the production set involves the proportion of agent types in the population. But market clearing along with this natural production constraint is not sufficient to ensure that the resource constraint is satisfied. For the signaling environment there happens to exist a competitive equilibrium in which resource constraint is satisfied, but it is the optimum without signaling. For the adverse selection insurance environment, such an equilibrium does not exist. This nonexistence is related to the nonexistence of a pooling equilibrium in the Rothschild-Stiglitz analysis.

The general inconsistency of a production set, market clearing, and the resource constraints for standard competitive analysis seems to imply a kind of externality. Section 5 makes precise the nature of this externality for the signaling environment. A competitive equilibrium in an extended commodity space is defined and is shown to involve externalities in production, <u>standard</u> externalities in general equilibrium terms. (In contrast to Hahn, the externality does not seem to involve information transmission.) We also establish that all the Spence signaling equilibria are included in the competitive equilibria of the extended commodity space. These may or may not be optimal.

These results motivate the approach adopted in section 6, that of "stacking" the commodity point. This approach makes the production set consistent with market clearing and the resource constraints and avoids externalities. It is supposed every agent type in the population must choose a vector of allocations, one component per agent type (on an as if basis), subject to an incentive compatibility condition, which makes the ordering operational. The optimality of such an equilibrium can be established. It is also established, apart from an exceptional case, that optima may be supported as competitive equilibria of a certain type, but with some unusual additional constraints on the maximization problem of the consumer. These constraints require that each agent type not choose a vector with a component for some other agent type which makes that agent type worse off than in the optimal allocation. Thus this competitive equilibrium notion fails to decentralize the economy in the usual way; the preferences and assignments of others must be taken into account by the maximizing individual. In some environments it is possible that these additional constraints might be deleted, and so a more standard competitive equilibrium might exist. But in the signaling and adverse-selection insurance environments it is established that no equilibrium without such additional constraints exists, at least if the optimum (in utility space) is not unique. This is due to the fact that in the signaling and insurance environments all the supporting competitive equilibria have the same price system.

We reserve for the concluding section a brief discussion of the major results of this paper.

2. The Economies

Basic Mathematical Structure

There are a finite number of agent types $i=1,\ldots,I$ and a continuum of each type. The fraction of agents of type i is denoted by λ_i. The commodity space is a linear space L and the (common) consumption possibility set for each agent type, $\bar{X}$ L, is closed and convex. The utility function of each agent of type i, $u_i : \bar{X} \to R$, is concave (and frequently linear). The endowment of each agent is $\xi \in L$, the same for all agent types. Each agent's type is private information.

Let $x_i \in L$ be a consumption allocation to each agent of type i. Let r_{ik} be a real-valued linear function on L, k=1,2,...,K, i=1,2,...,I. Then society is subject to resource constraints of the form

$$\sum_i \lambda_i r_{ik}(x_i - \xi) \leq 0 \qquad k = 1, \ldots, K.$$

An I-tuple $x = (x_i)$ of elements belonging to L is _implementable_ if

$$(2.1) \qquad x_i \in \bar{X} \qquad \text{all } i$$

$$(2.2) \qquad u_i(x_i) \geq u_i(x_j) \qquad \text{all } i, j$$

$$(2.3) \qquad \sum_i \lambda_i r_{ik}(x_i - \xi) \leq 0 \qquad \text{all } k.$$

The first requirement is that the consumption vector belong to the individuals' consumption possibility set. The second is that each individual of type i weakly prefer x_i to all the other x_j. Thus it is not in the interest of any agent to claim to be of some other type. These are the _ex ante_ incentive-compatibility constraints. Certain _ex post_ incentive-compatibility constraints are loaded into the common consumption. Justification for restricting attention to the class of allocations satisfying the incentive compatibility constraints can be found in Harris and Townsend [1977], [1981] and Myerson [1979]. The third condition is again the set of resource constraints in this pure exchange economy.

In much of this paper L is assumed to have finite dimension. The assumption that L has finite dimension simplifies the presentation without the loss of anything essential. Then, if both the $u_i(\cdot)$ and $r_{ik}(\cdot)$ are linear, we use the dot product to represent them; that is

$$u_i \cdot x_i \equiv u_i(x_i) = \sum_{\ell} u_{i\ell} x_{i\ell}$$

and similarly

$$r_{ik} \cdot x_i \equiv r_{ik}(x_i) = \sum_{\ell} r_{ik\ell} x_{i\ell},$$

where ℓ indexes components of $x_i \in L$. (Limiting arguments such as those used in Prescott and Townsend [1979] may be used to establish the results of this paper if L is not finite dimensional.)

We now demonstrate how the well-known insurance and signaling environments can be represented in this framework as well as the dynamic securities economy with private information that we considered previously.

<u>Adverse Selection Insurance Economy</u>: E_1

Consider the following insurance environment that was considered by Rothschild-Stiglitz [1976] and Wilson [1978]. There is a continuum of agents, say the set of agents is the unit interval. Each agent of type i receives a random endowment z:

$$z = \begin{cases} z_0 \text{ with probability } \theta_i \\ z_1 \text{ with probability } (1 - \theta_i). \end{cases}$$

Here $0 < z_0 < z_1$, so when $z = z_0$ an agent is said to suffer a loss. This is public information. There are two types of agents by risk class, i=1,2, where $0 < \theta_1 < \theta_2 < 1$. Thus the θ_1-type agents are the low risk people and θ_2-type agents are the high risk people. Each agent's type is private information. Of people of type i, θ_i is also the fraction that will suffer a loss. Thus there is no aggregate uncertainty with the fractions of the various types, λ_i, being known.[1]

Each agent has preferences on C R_+ as defined by the utility function $U : R_+ \to R$ where U is strictly concave, strictly increasing, and continuously differentiable with $U'(0) = \infty$. The points z_0 and z_1 belong to C. Suppose a consumer is assigned c_0 if a loss is suffered and c_1 if one is not. The expected utility for an agent of type i is then

$$\theta_i U(c_0) + (1 - \theta_i) U(c_1)$$

where c_0, $c_1 \in C$.

This environment can be cast in terms of the basic mathematical structure. One approach is to let L be R^2. Then let x_i be a consumption allocation to agents of type i with the first component being c_{i0} and the second c_{i1}. The common endowment is $\xi = (z_0, z_1)$. Then $\bar{X}$ corresponds to C x C and is closed and convex provided C is closed and convex. The single linear resource constraint is

$$\sum_i \lambda_i [\theta_i (c_{i0} - z_0) + (1 - \theta_i)(c_{i1} - z_1)] \leq 0.$$

The incentive compatibility constraints are

$$\theta_i U(c_{i0}) + (1 - \theta_i)U(c_{i1}) \geq \theta_i U(c_{j0}) + (1 - \theta_i)U(c_{j1}), \qquad \text{all } i,j.$$

But the space of consumption allocations (x_i) restricted by such constraints is not convex given the strict concavity of U.

An alternative approach which results in the utility function being linear, and therefore avoids the nonconvexities associated with the incentive compatibility constraints, is to consider lotteries on C. In order that the space L be finite dimensional, the set C is assumed finite with n elements. Then lotteries are n-dimensional vectors specifying the probability of each point in C, say $\mu = (\mu(c))_{c \epsilon C}$ where $\sum_c \mu(c) = 1$ and $\mu(c) \geq 0$ all $c \epsilon C$. Let μ_0 be the lottery if a loss is suffered and μ_1 if one is not. The expected utility of (μ_0, μ_1) for a type i individual is then

$$W_i(\mu_0, \mu_1) = \theta_i \sum_c U(c)\mu_0(c) + (1 - \theta_i) \sum_c U(c)\mu_1(c).$$

It is also assumed here that fraction $\mu_0(c)$ of agents of type i who suffer a loss receive the allocation c and similarly for $\mu_1(c)$, so that lotteries introduce no aggregate randomness. The endowment ξ is a pair of probability distributions on C, the first one of which assigns probability one to $z_0 \epsilon C$ and the second probability one to $z_1 \epsilon C$.

This latter economy can be put into the general mathematical structure as follows. Let $L = R^{2n}$. Let the first n elements of a consumption vector x, denoted x_0, be the $x_{0c} = \mu_0(c)$ defined above for $c \epsilon C$, and let the second n elements of x, denoted x_1, be the $x_{1c} = \mu_1(c)$ for $c \epsilon C$. The consumption possibility set then requires that x_0 and x_1 be probability distributions; that is,

$$\bar{X} = \{x \in L : \sum_c x_{0c} = 1,\ \sum_c x_{1c} = 1,\ x \geq 0\}.$$

Let the first n elements of the utility function u_i be the $\theta_i U(c)$ for $c \in C$ and the second n elements $(1 - \theta_i)U(c)$ for $c \in C$. With these definitions

$$u_i(x) = u_i \cdot x = W_i(\mu_0, \mu_1)$$

which is just the expected utility to an i-type of lottery μ_0 if a loss is suffered and lottery μ_1 if one is not.

The common endowment ξ is an element of L. The resource constraint is that average consumption be less than or equal to the average endowment:

$$\Sigma\lambda_i[\theta_i \sum_c x_{i0c}c + (1 - \theta_i) \sum_c x_{i1c}c]$$

$$\leq \sum_i \lambda_i[\theta_i \sum_c \xi_{0c}c + (1 - \theta_i) \sum_c \xi_{1c}c].$$

As there is a single resource constraint, the k subscript can be dropped on the resource constraint function r_{ik}. The first n components of r_i are the $\theta_i c$ for $c \in C$ and the next n components are $(1 - \theta_i)c$. Thus the resource constraint can be put in the form

$$\sum_i \lambda_i r_i \cdot (x_i - \xi) \leq 0$$

as required by the general formulation.

Signaling Economy: E_2

A particularly interesting class of economic environments are those with signaling opportunities. We consider the following simple signaling economy. The set C is a subset of R^3_+. The first component c_1 is consumption of goods, the second c_2 is the signal, and the third c_3 is consumption of leisure. The utility function for individuals of type i is linear in c, and of the form

$$U_i(c) = \theta_i c_1 - c_2,$$

where the θ_i have been ordered so that $\theta_1 < \theta_2 < \ldots < \theta_I$. The fraction of type i is $\lambda_i > 0$. Again, agent types are private information.

The output of an individual is not observed. A finite fraction, albeit small, of the continuum of individuals is required to produce any output and the resulting productivity of a group is the average of the productivities of the group's members. These assumptions imply it is impossible to deduce anything about an individual's productivity by observing the output of his group. Let π_i denote the output of the consumption good per unit of labor of individuals of type i. Individuals with larger θ_i are more productive so $\pi_I > \pi_{I-1} > \ldots > \pi_1 \geq 0$. The assumption that the signal does not affect output is not crucial and was made for the sake of simplicity. The endowment of leisure time is unity.

Because agents are risk neutral, we need not consider lotteries; that is, the utility functions are already linear. If they were not risk neutral, following the previous example, it would be necessary to consider lotteries on C.

To represent this economy within our basic structure, the commodity space L is R^3. The consumption possibility set is

$$\bar{X} = \{x \varepsilon L: \ x \geq 0\},$$

and the utility function for an i-type is

$$u_i(x) = \theta_i x_1 - x_2.$$

The endowment $\xi \varepsilon L$ is the vector (0, 0, 1).

The resource constraint is

$$\sum_i \lambda_i [x_{i1} + \pi_i (x_{i3} - \xi_3)] \leq 0.$$

This states that the average consumption $\sum_i \lambda_i x_{i1}$ must be less than average product $\sum_i \lambda_i \pi_i (\xi_3 - x_{i3})$. As for the previous example, there is a single resource constraint, characterized by the vectors $r_i \ \varepsilon \ R^3$ where r_i = (1, 0, π_i). With this definition the resource constraint can be put in the form

$$\sum_i \lambda_i r_i \cdot (x_i - \xi) \leq 0$$

as required by the general formulation.

<u>Private Information Securities Economy</u>: E_3

This example is a two-period version of the economy considered in Prescott-Townsend [1979]. The underlying utility function for an agent type i has the form

$$U_i(c_0) + U(c_1, \theta)$$

where c_0 is consumption in the initial period, period zero, made prior to the agent knowing his shock θ to period-one preferences, and c_1 is consumption in period one, made subsequent to the agent knowing θ. The realization of θ is private information as is the specification of agent types i. Both c_0 and c_1 belong to the finite set $C \quad R^{\ell}_{+}$. The fraction of type i people is λ_i and the fraction of <u>any</u> type who have realization θ in period one is π_θ. The π_θ are regarded as probabilities in period zero. There are I possible value of θ. The endowment in both periods is $e \varepsilon C$ for all agents.

Again, this economy can be cast in terms of the basic mathematical structure. Letting n be the number of elements of C, the commodity space L is $R^{n(I+1)}$. The first n components of a consumption vector x, x_{c0} for $c \varepsilon C$, assign probability to the points belonging to C and specify the probabilities of different consumptions c in period zero. The next n components, the x_{c1} for $c \varepsilon C$, specify the probabilities of consumptions c in period one conditional upon $\theta = \theta_1$, the next n components do the same except being conditional upon $\theta = \theta_2$, and so forth. The expected utility of $x \varepsilon L$ for an i-type is

$$u_i(x) = \sum_c x_{c0}\, U_i(c) + \sum_{\theta=1}^{I} \pi_\theta \sum_c x_{c\theta} U(c, \theta)$$

which is linear in x. The consumption possibility set is

$$\bar{X} = \{x \varepsilon L: \ x \geq 0, \ \sum_c x_{cj} = 1 \text{ for } j = 0,1,\ldots,I \text{ and}$$

$$\sum_c x_{c\theta} U(c, \theta) \geq \sum_c x_{c\phi} U(c, \theta) \text{ for } \phi, \theta = 1,\ldots,I\}.$$

This insures both that the $(x_{cj})_{c \varepsilon C}$ are probability measures for each j and that in period one an agent will truthfully reveal his shock θ.

The endowment $\xi \varepsilon L$ is the element for which $\xi_{ej} = 1$ for $j = 0,\ldots,I$ and $\xi_{cj} = 0$ for $c \neq e$. In other words, the endowment is e with certainty. The resource constraint is that average consumption be less than or equal to the average endowment. Letting c_k and e_k be the k^{th} components of the ℓ-dimensional vectors c and e, respectively, the period-zero constraints are

$$\sum_i \lambda_i \sum_c x_{ic0}(c_k - e_k) \leq 0 \qquad \text{for } k = 1,\ldots,\ell.$$

The period-one constraints are

$$\sum_i \lambda_i [\sum_{\theta=1}^{I} \pi_\theta [\sum_c x_{ic\theta}(c_k - e_k)]] \leq 0 \quad \text{for } k = 1,\ldots,\ell.$$

As there are ℓ goods in each period, there are 2ℓ constraints. Unlike the two economies, E_1 and E_2, the resource constraints have the form

$$\sum_i \lambda_i r_k \cdot (x_i - \xi) \leq 0$$

where vectors r_k are not subscripted also by i as in the general formulation (2.3). See Prescott and Townsend [1979]. This turns out to be a crucial difference.

A Moral Hazard Insurance Economy: E_4

The adverse selection insurance economy E_1 is modified as follows. There is only one type of agent, so we can ignore the λ_i in what follows, but the probability of loss depends upon a costly, private action of the agent. More precisely, each agent receives a random endowment $z \in Z = \{z_0, z_1\}$ with $0 < z_0 < z_1$ and the probability of z given the agent's action $a \in A = \{a_1, a_2, \ldots, a_m\}$ is $\theta_{z|a}$. The realization of z is public, the action taken is not. Also, the larger is action a, the smaller is the probability of loss $\theta_{z_0|a}$. The interpretation here is that a larger action corresponds to an agent being more careful. This is a standard set-up.

Each agent has preferences on the finite set C x A, where C is a finite subset of R_+ and has n elements. Preferences are defined by a utility function U(c, a) where U is increasing in c, decreasing in a, and concave. For $(c, a) \in C \times A$, let $U_{ca} = U(c,a)$.

In terms of our basic mathematical structure, let the linear space L be the Euclidean space of dimension n2m. A consumption vector x is a triple indexed element (x_{cza}) for $c \in C$, $z \in Z$ and $a \in A$. The interpretation is as follows. A lottery with probabilities x_a first determines an action a for each agent. Number x_a is also the fraction of agents in the population who are to take action a. Conditional on this action a, a second lottery with probabilities $x_{cz|a}$ determines consumption c and endowment z of the agent. Of course, nature plays a role in this second lottery since the conditional probabilities $\theta_{z|a}$ are technologically determined constants. In fact it is required that

$$\sum_c x_{cz|a} = \theta_{z|a}$$

for consistency. Finally, the marginal and conditional distributions x_a and $x_{cz|a}$ determine the joint distribution x_{cza} specified above.

Agents have preferences on $\bar{X}$ where

$$\bar{X} = \{x \in L_+ : \sum_{c,z,a} x_{cza} = 1;\ \theta_{z|a} \sum_{c,z} x_{cza} = \sum_c x_{cza} \quad \text{all } a, z;$$

$$\sum_{c,z} U_{ca} x_{cza} \geq \sum_{c,z} U_{ca'} x_{cza} \frac{\theta_{z|a'}}{\theta_{z|a}} \text{ all } a, a'\}.$$

The first constraint is that the probabilities sum to one. The second is that the probability distribution of z given a (if defined for that probability distribution) equals the technologically determined probability $\theta_{z|a}$. The third constraint is to ensure incentive compatibility. This is not obvious and is derived as follows. The commodity point x must be structured such that if a occurs, it is not in the interest of the agent to choose some other action a'; that is,

$$x_a \sum_{c,z} U_{ca} x_{cz|a} \geq x_a \sum_{c,z} U_{ca'} Pr\{c,z|a'\}.$$

Here $Pr\{c,z|a'\}$ is the probability of the pair (c,z) given that the agent is subject to lottery x_{cza} but chooses action a'. Thus, under x_{cza}

$$Pr\{c,z|a'\} = x_{c|za}\theta_{z|a'} = \frac{x_{cz|a}}{\theta_{z|a}} \theta_{z|a'}.$$

By substitution, the above expression holds if and only if

$$\sum_{c,z} U_{ca} x_{cz|a} x_a \geq \sum_{c,z} U_{ca'} x_{cz|a} x_a \frac{\theta_{z|a'}}{\theta_{z|a}}.$$

As $x_{cza} = x_{cz|a} x_a$, the third constraint indeed ensures incentive compatibility. This particular representation makes clear that set $\bar{X}$ is convex.

Of course, there is also a resource constraint, that average consumption be no greater than average endowment, or

$$\sum_{c,z,a} x_{cza}(c - z) \leq 0.$$

This constraint corresponds to constraint (2.3) in the basic structure, with endowment ξ being the zero vector in L. As in economy E_3, the single resource constraint takes on a special form,

$$r \cdot (x - \xi) \leq 0$$

with $r_{cza} = c - z$. The vector r is not indexed by i.

3. Pareto Optima

Pareto optimal allocations for the general structure can be obtained by maximizing weighted averages of the agent types' utilities. Let the set of possible weights be

$$\Gamma = \{\gamma \in R^I : \gamma_i \geq 0 \text{ and } \sum_i \gamma_i = 1\}.$$

For $\gamma \in \Gamma$, let $\phi(\gamma)$, denote the set of consumption allocations which are solutions to the program

$$\underset{x=(x_i)}{\text{Max}} \quad \sum_i \gamma_i u_i \cdot x_i$$

subject to $x_i \in \bar{X}$ all i, $u_i \cdot x_i \geq u_i \cdot x_j$ all i and j, and $\sum_i \lambda_i r_{ik} \cdot (x_i - \xi) \leq 0$ all k. Thus $\phi(\gamma)$ is a subset of the I-cross product space of L. Finally, let

$$\Phi = \bigcup_{\gamma \in \Gamma} \phi(\gamma).$$

Lemma 3.1: The set Φ contains all the Pareto optima. If $\gamma > 0$ (i.e., $\gamma_i > 0$ all i), then all allocations belonging to $\phi(\gamma)$ are optima. Finally, if an I-tuple $x = (x_i)$ belongs to $\phi(\gamma)$ and if x is not Pareto dominated by another element belonging to $\phi(\gamma)$, then x is also an optimum.

Proof: The constraints are convex and the objective function linear. Consequently, the utility possibility set is convex. Let γ define a supporting hyperplane at the point on the utility possibilities frontier associated with Pareto optimal allocation x^*. Such a hyperplane exist by the separation theorem. For this $\gamma^* \in \Gamma$, x^* is a solution to the program. This proves the first statement of the lemma.

To prove the second statement consider some $\gamma > 0$ and some allocation $x \in \phi(\gamma)$. Suppose x can be Pareto dominated. Then x cannot be a solution to the γ-program. This contradiction establishes that solutions to such programs are Pareto optima.

To prove the last statement of the lemma, let x be a solution to some γ-program with the specified nondominance property, but suppose x is not an optimum. Then there exists an allocation x' which Pareto dominates x. By assumption, x' does not belong to $\phi(\gamma)$. But the value of x' for the γ-program is at least as great as the value for x, and thus x' must be a solution to the γ-program, the desired contradiction.

Theorem 3.1. If the set $\bar{X}$ is compact and contains ξ, the set of Pareto optimal allocations is nonempty.

Proof: A feasible solution exists namely $x_i = \xi$ for all i so the constraint set is nonempty. The objective function is linear and therefore continuous. As the resource and incentive constraints are closed, a continuous functions is being maximized on a compact set. Consequently for any $\gamma \in \Gamma$, a solution to the program exists. By the lemma, a solution to a γ-program is necessarily an optimum if $\gamma > 0$.

Sometimes $\bar{X}$ will not be compact but can be made compact by imposing additional constraints which are never binding at an optimum. If this is possible and $\xi \in \bar{X}$ the set of Pareto optima is guaranteed to be nonempty.

Pareto Optima for the Adverse Selection Insurance Economy

Rothschild and Stiglitz [1976] demonstrate, under certain conditions, that their separating equilibrium both exists and is an optimum within a more limited class of allocations than the one we consider. One question that will be answered is whether that allocation is an optimum within our larger class of allocations. The principal result, however, is the complete specification of the set of Pareto optima for insurance economy.

Let $\bar{z}$ be the ex post per capita endowment, so

$$\bar{z} = \sum_i \lambda_i [\theta_i z_0 + (1 - \theta_i) z_1].$$

Per capita consumption is constrained by this quantity. With this definition, the program for determining the Pareto optima for $\gamma \epsilon \Gamma$ is the <u>linear</u> program

$$\max \sum_i \gamma_i \sum_c U(c)[x_{i0c}\theta_i + x_{i1c}(1 - \theta_i)]$$

$$x_1, x_2 \geq 0$$

where x_{i0c} is the probability of a type i consuming $c \epsilon C$ conditional on a loss and x_{i1c} is the probability of a type i consuming c conditional on no loss, i = 1, 2. The constraints are

(3.1) $$\sum_c U(c)[x_{10c} - x_{20c})\theta_2 + (x_{11c} - x_{21c})(1 - \theta_2)] \leq 0$$

(Agents of type two weakly prefer x_2 to x_1.)

(3.2) $$\sum_c U(c)[x_{20c} - x_{10c})\theta_1 + (x_{21c} - x_{11c})(1 - \theta_1)] \leq 0$$

(Agents of type one weakly prefer x_1 to x_2.)

(3.3) $$\sum_{i,c} \lambda_i c[x_{i0c}\theta_i + x_{i1c}(1 - \theta_i)] \leq \bar{z}$$

(This is the single resource constraint.)

(3.4) $$\sum_c x_{10c} = 1$$

(3.5) $$\sum_c x_{11c} = 1$$

(3.6) $$\sum_c x_{20c} = 1$$

(3.7) $$\sum_c x_{21c} = 1$$

(Probabilities sum to one.)

Letting μ_k denote the Lagrange multiplier associated with constraint (3.k), differentiating with respect to the x_{10c} yields the first-order conditions

$$(3.8) \qquad \gamma_1 U(c)\theta_1 - \mu_1 U(c)\theta_2 + \mu_2 U(c)\theta_1 - \mu_3 c\theta_1\lambda_1 + \mu_4 \leq 0$$

for all $c \in C$. Analogous first-order conditions hold for the x_{20c}, the x_{11c} and the x_{21c}.

Constraint (3.8) must hold with equality for some $c \in C$. Otherwise all the x_{10c} would be zero and that would violate constraint (3.4). The left-hand side of (3.8) can be viewed as a function of c. Thus, the Lagrange multipliers must be such that this function has a maximum (of zero) at points at which condition (3.8) holds as an equality. Now suppose the set C has a arbitrarily large number of elements, so that the maximal distance between any point and its nearest neighbor is arbitrarily small. Also recall that u(•) is strictly concave. Then if

$$(3.9a) \qquad \gamma_1\theta_1 - \mu_1\theta_2 + \mu_2\theta_1 \leq 0$$

the left-hand side of (3.8) is a strictly decreasing convex function of c and so attains a maximum at c=0. If

$$(3.9b) \qquad \gamma_1\theta_1 - \mu_1\theta_2 + \mu_2\theta_1 > 0$$

the left-hand side of (3.8) is strictly concave function of c and so attains a maximum at a single point (on the assumption that the set C can be made arbitrarily large). In summary the result is that x_{10c} equals one for some $c \in C$ and zero otherwise. By precisely the same argument, probability measures x_{11}, x_{20} and x_{21} place all their mass on single points denoted by c_{11}, c_{20} and c_{21}, respectively. These points depend upon the weights γ chosen.

One implementable allocation is for everyone to consume $\bar{z}$ independent of their realized endowment. The utility for this allocation is $U(\bar{z})$ for everyone. If one agent type realizes expected utility exceeding $U(\bar{z})$, their expected consumption, by Jensen's inequality, must exceed $\bar{z}$. This implies via the resource constraint that the expected consumption of the other type agents is less than $\bar{z}$ and, by Jensen's inequality, their expected utility less than $U(\bar{z})$.

We divide the Pareto optima into three sets. The first is for everyone to consume $\bar{z}$ with certainty. The second set contains those optima for which the expected utility of type one agents exceed $U(\bar{z})$ and the third are those for which the expected utility of type one agents is less than $U(\bar{z})$.

It can be established that for set two there is no uncertainty in consumption for type two agents (i.e., $c_{20} = c_{21} = c_2$). Suppose the contrary. By eliminating uncertainty in the consumption of type two agents (if there is any) while preserving their expected consumption, the utility of type two agents is increased, the resource constraint continues to be satisfied, and slack is introduced into constraint (3.1). Note that constraint (3.2) continues to be satisfied: expected consumption of type two agents is less than $\bar{z}$ in set two so the type one agents strictly prefer their allocation x_1 which yields expected utility greater than $U(\bar{z})$ to the no uncertainty x_2 allocation. This also establishes that $\mu_2 = 0$ in set two. And $c_{10} \neq c_{11}$ as well, for otherwise (3.1) would be violated.

Actually constraint (3.1) is binding in set two for otherwise c_{10} and c_{11} could be made more nearly equal while preserving the expected consumption of type one. This increases expected utility of type one.

Resource constraint (3.3) is also binding for if it were not, by increasing c_2 a little, expected utility of type two agents could be increased without violating any constraint. Since (3.1) and (3.3) are binding, c_{10} and c_{11} must satisfy

$$(3.10) \qquad \theta_2 U(c_{10}) + (1 - \theta_2)U(c_{11}) = U(c_2)$$

and

$$(3.11) \qquad \lambda_1[\theta_1 c_{10} + (1 - \theta_1)c_{11}] + \lambda_2 c_2 = \bar{z}$$

for Pareto optima set two. To find the solution to (3.10) and (3.11), consider the space of (c_{10}, c_{11}) pairs. The point c_2 lies in this space on the 45° line below the negatively sloped line (3.11). (Recall $c_2 < \bar{z}$). Thus the negatively sloped line (3.11) intersects the indifferences curve for which (3.10) is satisfied at two points. The better of the solutions for agents type one is the one for which $c_{10} < c_2 < c_{11}$. Subsequently, c_{10} and c_{11} are functions of c_2 as the better solution to (3.10) and (3.11).

This nearly completes the specification of the allocations in Pareto optima set two. There, however, is the condition that the Lagrange multipliers μ_1 and μ_3 be positive. We exploit this condition with the additional assumption that all the optimal allocations are interior points of the set C and that the set C is sufficiently larger that such allocations satisfy the appropriate conditions for maxima as if C were a continuum. Thus, given that $\mu_2 = 0$ and that $c_{20} = c_{21} = c_2$, from (3.8) and the analogous first-order conditions

$$(3.12)\qquad \begin{aligned} &[\gamma_1 - \mu_1\theta_2/\theta_1]U'(c_{10}) = \mu_3\lambda_1 \\ &[\gamma_1 - \mu_1(1-\theta_2)/(1-\theta_1)]U'(c_{11}) = \mu_3\lambda_1 \\ &[\gamma_2 + \mu_1]U'(c_2) = \mu_3\lambda_2 = \mu_3(1-\lambda_1) \\ &\gamma_1 + \gamma_2 = 1. \end{aligned}$$

Given c_2, these are four linear equations in the unknowns μ_1, μ_3, γ_1 and γ_2. (Remember c_{10} and c_{11} are functions of c_2 being the better solution to (3.10) and (3.11).) An additional requirement is that the solution to (3.12) (which exists) be nonnegative. It is tedious to establish, but this requirement is that

$$(3.13)\qquad \frac{\lambda_2}{\lambda_1}\,\frac{\theta_2-\theta_1}{\theta_1(1-\theta_1)} \geq \frac{U'(c_2)[U'(c_{10})-U'(c_{11})]}{U'(c_{10})U'(c_{11})} \quad .$$

As c_2 approaches $\bar{z}$, the distance between c_{10} and c_{11} goes to zero. Therefore, given that U is continuously differentiable, for c_2 sufficiently near $\bar{z}$, this inequality is satisfied. Thus, Pareto optima set two is nonempty. Finally, let $\bar{c}_2$ be the minimal level for which inequality (3.13) is satisfied. Then it holds for all $\bar{c}_2 \leq c_2 < \bar{z}$. If this were not the case, the utility possibility set would not be convex.

The argument for characterizing optima set three is symmetric with respect to the agents types with some obvious exceptions necessitated by the fact that $\theta_2 > \theta_1$. To characterize optima for set three interchange subscripts for the two agent types with the exception that one uses the solution to (3.10) and (3.11) for which $c_{20} > c_{21}$ and the direction of inequality (3.13) is reversed.

As condition (3.10), (3.11) and (3.13) along with the additional requirement that the contracts be actuarially fair are just those for the optimality of the Rothschild-Stiglitz separating equilibrium allocation, that allocation is Pareto optimal in our larger class of allocations as well.

Randomness in consumption is used to separate the agents. The agent type realizing the higher expected utility incurs the uncertainty. The cost of this uncertainty is less to agents of that type and they are more than compensated for it by higher expected consumption.

Pareto Optima for the Signaling Economy[2] E_2

With this economy there is no need for lotteries as the objective function is already linear, a property which is needed for convexity of the incentive constraints. In this section c_i denotes consumption and s_i the signal for type i agents. Given that productivities are positive and leisure is not valued, for any Pareto optimum the entire time endowment is allocated to production of the consumption good. In the subsequent analysis we consider only allocations for which it is so allocated. For $\gamma \varepsilon \Gamma$, the solutions to the following linear program are the Pareto optima associated with weights γ.

$$\max_{\{c_i, s_i\} \geq 0} \sum_i \gamma_i (\theta_i c_i - s_i)$$

subject to the resource constraint

$$(3.14) \qquad \sum_i \lambda_i c_i \leq \sum_i \lambda_i \pi_i \equiv \bar{\pi}$$

and the incentive constraints

$$(3.15) \qquad \theta_i c_i - s_i \geq \theta_i c_j - s_j \qquad \text{for all } i, j.$$

There is considerable redundancy in constraint set (3.15). Indeed constraints need be satisfied only by adjacent types as summarized by the following result.

Result: Condition (3.15) is satisfied if

(3.16) $$s_{i+1} - s_i \geq \theta_i(c_{i+1} - c_i)$$

(3.17) $$s_{i+1} - s_i \leq \theta_{i+1}(c_{i+1} - c_i)$$

for $i=1,2,\ldots,I-1$.

Proof As $\theta_{i+1} > \theta_i$, (3.16) and (3.17) require $c_{i+1} - c_i$ be nonnegative. If $c_{i+1} - c_i$ is nonnegative, then from (3.16) so must $s_{i+1} - s_i$.

Let i, j and k index agent types and be selected so that $i < j < k$. Suppose type i prefers his allocation to type j's allocation and j prefers his to type k's allocation. It will be established that type i prefers his allocation to type k's allocation. From (3.16) and the hypothesis

$$s_j - s_i \geq \theta_i(c_j - c_i)$$

$$s_k - s_j \geq \theta_j(c_k - c_j).$$

Adding these inequalities

$$s_k - s_i \geq \theta_i(c_j - c_i) + \theta_j(c_k - c_j).$$

But $\theta_j > \theta_i$ and necessarily $c_k \geq c_j$. Thus substituting θ_i for θ_j reduces the value of the right-hand side of the inequality and

$$s_k - s_i \geq \theta_i(c_j - c_i) + \theta_i(c_k - c_j) = \theta_i(c_k - c_i).$$

This is the desired intermediate result.

Now using this intermediate result and constraint (3.16) to begin the induction, constraints (3.16) imply that agents of type i weakly prefer their allocation to those of any agent type j with index $j > i$. A similar argument with $i > j > k$ along with constraints (3.17) to begin the induction can be used to prove that constraints (3.17) imply that agents of type i weakly prefer their allocation to those of agents with index $j < i$. This completes the proof of the result.

For any Pareto optimum, $s_1 = 0$ as otherwise the objective function could be increased without violating any constraint by reducing all the s_i by s_1; note that only differences in signals matter for the constraints. Further, the resource constraint is binding at an optimum. Otherwise the objective function could be increased without violating any constraint in (3.16) and (3.17) by increasing all the c_i by some $\varepsilon > 0$; only differences in consumption matter for the constraints.

If constraint i in (3.16) were not binding, then the s_j for $j > i$ could all be reduced by an $\varepsilon > 0$ without violating any constraint. As this would increase the utility of such agent types, we conclude that constraints in (3.16) are binding at an optimum. If constraints (3.16) are binding, then constraints (3.17) are necessarily satisfied as $\theta_{i+1} > \theta_i$ and necessarily $c_{i+1} \geq c_i$.

Let $y_i = c_i - c_{i-1}$, $i \geq 1$, where $c_0 \equiv 0$. Then

$$c_i = \sum_{j\le i} y_j,$$

$i \geq 1$, and, since (3.16) is binding at optima,

$$s_i = \sum_{j=2}^{i} \theta_{j-1} y_j \text{ for } i \geq 2$$

at optima. (Define $\sum_{j=2}^{1} \theta_{j-1} y_j = 0$ since $s_1 = 0$ at optima.) Thus the optima are solutions to the program

$$\underset{y_i \geq 0}{\text{Max}} \sum_i \gamma_i [\theta_i \sum_{j\le i} y_j - \sum_{j=2}^{i} \theta_{j-1} y_j]$$

subject to

$$(3.18) \qquad \sum_i [\lambda_i \sum_{j\le i} y_j] \leq \bar{\pi}$$

for $\gamma \varepsilon \Gamma$. Letting μ be the Lagrange multiplier for the single constraint and differentiating with respect to y_i yield the first-order conditions

$$(3.19) \qquad \sum_{j\geq i} \gamma_j(\theta_j - \theta_{i-1}) \leq \mu(\lambda_i + \ldots + \lambda_I) \text{ for } i = 1, \ldots I$$

where θ_0 is defined to be zero. The value of the multiplier is

$$\mu = \underset{i}{\text{Max}} \frac{\sum_{j\geq i} (\theta_j - \theta_{i-1}) \gamma_j}{\sum_{j\geq i} \lambda_j} .$$

To be an optimal allocation it is necessary and sufficient that y_i be zero whenever (3.19) holds with a strict inequality, that all the y_i be nonnegative, and that the resource constraint (3.18) be binding. This gives us a procedure for determining Pareto optima: first select any nonnegative, nonzero vector y with $y_i = 0$ if (3.19) does not hold with equality at i, and then scale the vector y so that the resource constraint (3.18) holds with equality. Thus, if (3.19) holds with equality for more than one i, there are multiple optima associated with weights $\gamma > 0$.

We now consider whether there can be complete separation of agent types. It is cumbersome to do this if the distribution of θ's is discrete, so we assume a continuum of θ's on some interval $[\underline{\theta}, \bar{\theta}]$ as a limiting case.

Let $f(\theta)$ be the continuous density of types with $f(\theta) > 0$ for all θ, $\underline{\theta} \leq \theta \leq \bar{\theta}$, $F(x)$ be the fraction of agents with $\theta \geq x$, $G(x)$ be the integral of the weights assigned to agent types with $\theta \geq x$. Defining

$$H(\theta) = \int_{\theta}^{\bar{\theta}} (x-\theta)dG(x) \qquad \text{for } \theta > \underline{\theta}$$

and

$$H(\underline{\theta}) = \int_{\underline{\theta}}^{\bar{\theta}} x\, dG(x) \ ,$$

the first-order conditions corresponding to (3.19) are

$$(3.20) \qquad H(\theta) \leq \mu F(\theta) \qquad \text{for } \bar{\theta} \leq \theta \leq \bar{\theta}.$$

The function H can be shown to be convex and decreasing by differentiation if G(x) has a continuous density. As any G(x) is the limit of sequences of distributions with continuous densities, the results holds in general. At $\underline{\theta}$ the function H will have a discontinuity unless $\underline{\theta} = 0$.

Let $F^*(\theta)$ be the maximal convex function that is less than or equal to the decreasing function $F(\theta)$ (see Figure 1). The convexity of the left-hand side of (3.20) implies that if $F^*(\theta) < F(\theta)$, the constraint cannot hold with equality for such θ and be satisfied at all other points. Thus for such θ the consumption schedule $c(\theta)$ associated with an optimum must be flat; that is, there cannot be a discontinuity nor the derivative be strictly positive at points at which $F^*(\theta) < F(\theta)$.

Quite frequently densities are single-peaked. If the density of types is single-peaked, the function $F(\theta)$ is concave to the left of the peak and convex to the right. Consequently, $F^*(\theta) < F(\theta)$ for θ to left of the peak (see Figure 1) and necessarily for all Pareto optima $c(\theta) = c(\underline{\theta})$ to the left of the peak. Only if the density is decreasing throughout its entire range can a signaling allocation with no flats be an optimum.

Provided $\underline{\theta} > 0$, then for those Pareto optima with $c(\underline{\theta}) > 0$, necessarily $c(\theta) = c(\underline{\theta})$ for some finite interval to the right of $\underline{\theta}$. Function $H(\theta)$ has a discontinuity at $\underline{\theta}$. Thus, for some finite interval to the right of $\underline{\theta}$, $H(\theta)$ must be strictly less than $\mu F(\theta)$ and therefore $c(\theta)$ must be constant over that interval.

Pareto Optima for the Private-Information Securities Economy

The optima in the adverse-section insurance economy E_1 and in the signaling economy E_2 do not involve artificial lotteries. In the adverse-selection insurance economy there is sufficient risk in nature as to whether or not a loss is suffered to separate agent types efficiently. In the signaling economy preferences are linear. But we suspect that in general Pareto optima will involve artificial lotteries. The pure-exchange

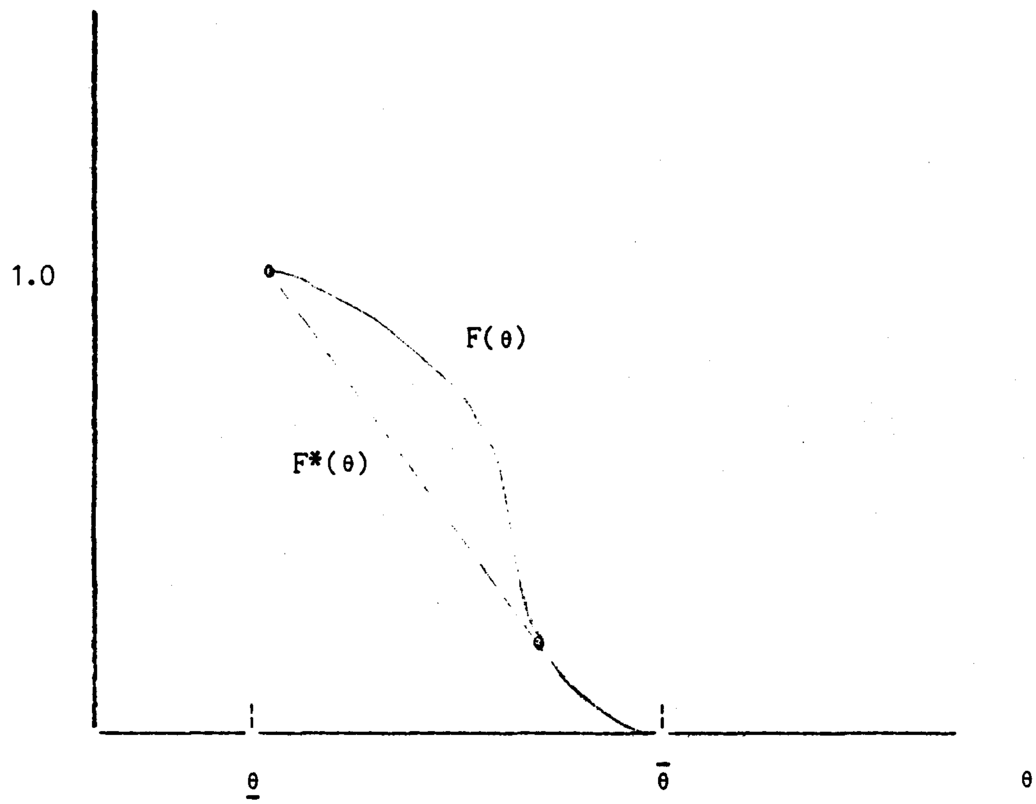

Figure 1

economy E_3 indicates that there need only be some differences in risk preferences at a deterministic (private-information) optimum. A Pareto optimum with artificial lotteries for economy E_3 is given in Prescott and Townsend [1979].

Pareto Optima for the Moral Hazard Insurance Economy

For subsequent reference we note that the Pareto optima for the moral hazard insurance economy E_4 can be obtained by consideration of the following linear program.

$$\max_{\{x_{cza} \geq 0\}} \sum_{c,z,a} x_{cza} U_{ca}$$

subject to

$$\sum_{c,z,a} x_{cza} = 1$$

$$\theta_{z|a} \sum_{c,z} x_{cza} = \sum_{c} x_{cza} \qquad \text{all } a, z$$

$$\sum_{c,z} U_{ca} x_{cza} \geq \sum_{c,z} U_{ca'} x_{cza} \frac{\theta_{z|a'}}{\theta_{z|a}} \qquad \text{all } a, a'$$

$$\sum_{c,z,a} x_{cza}(c - z) \leq 0.$$

We have not investigated the properties of the solution to this program. But we conjecture from the insights of Holmstrom [1979] for a principal-agent environment that there will be no artificial randomness in the action or in consumption c if the utility function U(c,a) is separable (Holmstrom finds that noninformative signals need not be used under the separability condition). We might also conjecture that artificial randomness is needed for some nonseparable utility functions. These are left as open questions.

4. Standard Competitive Equilibria

In this section we define a standard competitive equilibrium in our commodity space. The price system, consumption choices for each agent i, and a production choice are all elements of the linear space L. In equilibrium consumption choices are maximal in the budget set, a production choice maximizes profits in the production possibilities set, and markets clear. We show that such an equilibrium is successful in the moral hazard insurance environment but can have undesirable properties when there is a fundamental adverse selection problem. We make clear what we mean by such a problem.

We might hope to begin by defining a competitive equilibrium directly for the pure exchange economies of the general structure. But the resource constraint (2.3) is not of a form familiar to general equilibrium analysis, that is, not of the form $\sum_i \lambda_i(x_i - \xi) = 0$. In Prescott and Townsend [1979], a production set Y was defined in such a way that $y \in Y$ and a standard market clearing condition $\sum_i \lambda_i(x_i - \xi) = y$ imply the resource constraint (2.3). This production set can be interpreted as an intermediation or exchange technology where negative (positive) components correspond to a commitment to take in (distribute) resources. These commitments can vary across agents with observable characteristics or with unobservable but declared characteristics. But the weights that agent types receive is fixed by the exchange technology Y, beyond the control of the firms. (See Prescott and Townsend [1979] for a more complete interpretation of the exchange technology).

Thus, to proceed we must define the production possibilities set Y. One possible definition, used for the remainder of this section, is

$$(4.1) \qquad Y = \{y \varepsilon L : \sum_i \lambda_i r_{ik}(y) \leq 0, \text{ all } k\}$$

This leads to

<u>Definition 4.1</u>: A <u>competitive equilibrium</u> is an (I+1)-tuple $((x_i^*), y^*)$ of elements of L and a price vector $p^* \varepsilon L$ for which

(i) element x_i^* maximizes $u_i(x)$ over the set $\{x \varepsilon \bar{X} : p^* \cdot x \leq p^* \cdot \xi\}$ for each i;

(ii) element y^* maximizes $p^* \cdot y$ over the set Y;

(iii) $\sum_i \lambda_i x_i^* = y^* + \xi$.

The problem with this definition is that allocations consistent with the production possibilities set and market clearing need not be consistent in general with the resource constraints. We term this a fundamental adverse-selection problem. To see the problem note first that the production possibilities set can be written as

$$(4.2) \qquad Y = \{y \varepsilon L : \bar{r}_k(y) \leq 0, \quad \text{all } k\},$$

where $\bar{r}_k(y) = \sum_i \lambda_i r_{ik}(y)$. Then substituting the market clearing condition (iii) into (4.2) yields

$$(4.3) \qquad \bar{r}_k(\sum_i \lambda_i x_i - \xi) - 0, \qquad \text{all } k.$$

Using the linearity of $\bar{r}_k$, (4.3) yields

$$(4.4) \qquad \sum_i \lambda_i \bar{r}_k(x_i - \xi) \leq 0, \qquad \text{all } k.$$

Now recall the form of the resource constraints

$$(4.5) \qquad \sum_i \lambda_i r_{ik}(x_i - \xi) \leq 0 \qquad \text{all } k.$$

So for consistency, we need

$$(4.6) \qquad \sum_i \lambda_i \bar{r}_k(x_i - \xi) \geq \sum_i \lambda_i r_{ik}(x_i - \xi).$$

Except in special circumstances, (4.6) will not obtain.

There is no problem of this sort in the study of economy E_3 in Prescott-Townsend [1979]. To see why, suppose as in Prescott-Townsend that $r_{ik} = r_k$ for all i, i.e., the agents' types do not influence the coefficients of the resource contraints. Then $\bar{r}_k = r_{ik}$ for all i and (4.6) is trivially satisfied. Thus the production set and market clearing are always consistent with the resource constraints.[3] And we recall that in economy E_3, standard competitive equilibria, as defined in this section, exist and are optimal. The reader is referred to Prescott-Townsend [1979] for the details of the analysis.

There is also no inconsistency problem for the moral hazard insurance economy, E_4. Consistency condition (4.6) is again satisfied. And as with economy E_3, existence and optimality of standard competitive equilibrium are straightforward to establish, as we now indicate.

First, note that the problem confronting the firm in economy E_4 is

$$(4.7) \qquad \underset{\{y_{cza}\}}{\text{Max}} \sum_{c,z,a} p_{cza} y_{cza}$$

subject to

$$(4.8) \qquad r \cdot y = \sum_{c,z,a} (c - z) y_{cza} \leq 0.$$

Necessary and sufficient first-order conditions are

$$p_{cza} - \mu(c - z) = 0 \qquad \text{for all } c,z,a \text{ triples.}$$

Dividing through by the positive Lagrange multiplier μ, let

$$(4.9) \qquad p^*_{cza} = c - z \qquad \text{for all } c,z,a \text{ triples}$$

be the candidate for an equilibrium price system. The problem confronting the consumer under these prices is

$$(4.10) \qquad \underset{\{x_{cza}\} \epsilon \bar{X}}{\text{Max}} \sum_{c,z,a} x_{cza} U_{ca}$$

subject to

$$(4.11) \qquad \sum_{c,z,a} p^*_{cza} x_{cza} \leq 0.$$

Substituting the price system (4.9) into (4.11) results in the resource constraint. Thus system (4.10)-(4.11) becomes the optimum problem of the previous section, and a solution $\{x^*_{cza}\}$ (which exists) to the consumer's problem is optimal. (The budget constraint will be satisfied as an equality.) Thus condition (i) of definition (4.1) is satisfied. Now let $y^*_{cza} = x^*_{cza}$ so that market-clearing condition (iii) is satisfied. Finally, returning to the firm's problem, substitute the price system (4.9) into objective function (4.7). From (4.8), then, profits are nonpositive. With the budget constraint (4.11) as an equality, profits are zero at $\{y^*_{cza}\}$. Thus condition (ii) is satisfied. In summary, a standard competitive equilibrium exists and is optimal for the moral hazard insurance economy.

Returning to the general problem, though, questions concerning the existence and optimality of a standard competitive equilibrium should be treated with some care. Under quite general conditions, as in standard equilibrium analysis, a competitive equilibrium $((x^*_i), y^*), p^*$ exists. And, if one exists, the (x^*_i) satisfy the incentive compatibility constraints of the general structure. But an equilibrium can exist without satisfying the resource constraints, in which case the result is vacuous; the competitive equilibrium allocation is not really feasible after all. The Rothschild-Stiglitz environment produces an example of this, as we show in this section. There is also the possibility that an equilibrium can exist which <u>does</u> satisfy the resource constraints even when the r_{ik} are not equal for all i. The Spence environment produces an example of this. And in addition the equilibrium allocation, in that environment, is optimal. But in other ways it is unsatisfactory. We turn first to the Spence example.

Consider the signaling environment. Consistent with the specification (4.1), let

$$Y = \{y \in R^3 : \sum_i \lambda_i [y_1 + \pi_i y_3] \leq 0\}.$$

The objective of the firm is

$$\underset{y \in Y}{\text{Max}} \sum_{j=1}^{3} p_j y_j ,$$

yielding necessary and sufficient first-order conditions,

$$p_1 - \mu = 0, \; p_2 = 0, \text{ and } p_3 - \mu\bar{\pi} = 0$$

where μ is a (positive) Lagrange multiplier and $\bar{\pi} = \sum_i \lambda_i \pi_i$ is the average productivity across workers. Normalize prices (dividing by μ),

$$p_1^* = 1, \; p_2^* = 0, \; p_3^* = \bar{\pi};$$

the candidate for equilibrium prices has been determined. The problem confronting each agent of type i is

$$\underset{\{x_i\}}{\text{Max}} \; \theta_i x_{i1} - x_{i2}$$

subject to $p^* \cdot x_i \leq p_3^*$.

Clearly in equilibrium no agent will signal, $x_{i2}^* = 0$ for all i, and all income (from labor) will be spent on consumption, $x_{i1}^* = \bar{\pi}$ for all i. That is, $x_i^* = (\bar{\pi}, 0, 0)$. To satisfy the market clearing condition of equilibrium, let $y^* = (\bar{\pi}, 0, -1)$. This is also profit maximizing. Thus, existence of equilibrium has been established. This equilibrium specification _is_ consistent with the resource constraints; everyone receives the economy-wide average product and labor is supplied inelastically.

The general difficulty discussed above is circumvented here by the fact that in _equilibrium_ all agents have the _same_ consumption allocation. That is, setting $x_i^* = x^*$ for all i, (4.6) holds.

Thus, a "standard" competitive equilibrium exists in the signaling environment. Moreover, it is optimal. But it involves no signaling, the phenomenon which the model is intended to explain. In this sense the equilibrium construct is unsuccessful. (It should be noted that if the signaling activity were to use up resources, there would still be no signaling in equilibrium).

It remains to apply the competitive equilibrium concept of this section to the adverse selection insurance environment, E_1. The discussion will prove most revealing if we restrict attention to the underlying commodity space rather than going to the space of lotteries. (Note that this is still consistent with the general structure.) So for the remainder of this section let $\bar{X} = R_+^2$; if $c_i \varepsilon \bar{X}$, then $c_i = (c_{i0}, c_{i1})$, a consumption allocation to agent type i depending on whether or not a loss is suffered. In this space the common endowment is $\xi = (z_0, z_1)$. The resource constraint is

$$\sum_i \lambda_i [\theta_i (c_{i0} - z_0) + (1 - \theta_i)(c_{i1} - z_1)] \leq 0.$$

Consistent with the specification (4.1) of the production set above, let

$$Y = \{(y_0, y_1) \varepsilon R^2 : \bar{\theta} y_0 + (1 - \bar{\theta}) y_1 \leq 0\}$$

where $\bar{\theta} = \sum_i \lambda_i \theta_i$.

To search for a competitive equilibrium, first use the production set to determine prices. The problem of the firm is

$$\max_{y \varepsilon Y} p_0 y_0 + p_1 y_1.$$

As in the Spence structure, examine the first-order conditions for this problem and normalize prices. Thus

$$p_0^* = \bar{\theta} \qquad p_1^* = 1 - \bar{\theta}$$

is the only possible candidate for equilibrium prices. Under these prices, the problem of each agent of type i is

$$\max_{(c_{i0},\, c_{i1}) \geq 0} \theta_i U(c_{i0}) + (1 - \theta_i)U(c_{i1})$$

$$p_0^* c_{i0} + p_1^* c_{i1} \leq p_0^* z_0 + p_1^* z_1.$$

Graphically, the budget constraint as an equality determines a line through the endowment with slope $(1 - \bar{\theta})/\bar{\theta}$. See Figure 2. Noting that agent of type i has marginal rate of substitution

$$\frac{(1 - \theta)U'(c_{i1})}{\theta U'(c_{i0})}$$

with $\theta_1 < \theta_2$, it is clear that the solutions to these problems, c_i^*, i = 1, 2, must differ with a configuration as in Figure 2. These are the candidates for equilibrium consumptions. (In fact, we have established the existence of a standard competitive equilibrium in the space of attainable states.)

It remains to discover whether these equilibrium consumption allocations can be consistent with the resource constraint. To do this, multiply through the budget constraint of agent type i (as an equality) by λ_i and sum over i. This procedure yields

$$\text{(4.12)} \qquad \sum_i \lambda_i[\bar{\theta} c_{i0}^* + (1 - \bar{\theta})(c_{i1}^*)] = \bar{\theta} z_0 + (1 - \bar{\theta})z_1.$$

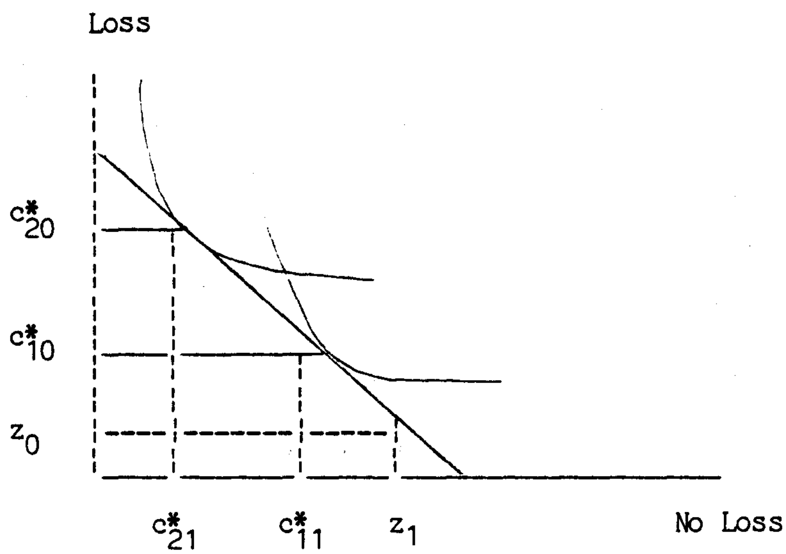

Figure 2

But the resource constraint is

$$\bar{\theta}z_0 + (1 - \bar{\theta})z_1 \geq \sum_i \lambda_i[\theta_i c^*_{i0} + (1 - \theta_i)c^*_{i1}].$$

So the resource constraint and (4.12) yield

$$\text{(4.13)} \qquad \sum_i \lambda_i[\bar{\theta}c^*_{i0} + (1 - \bar{\theta})c^*_{i1}] \geq \sum_i \lambda_i[\theta_i c^*_{i0} + (1 - \theta_i)c^*_{i1}].$$

Careful (if tedious) analysis reveals that the only way for (4.13) to be satisfied is with $c^*_1 = c^*_2$, contrary to their specification. Thus, there does not exist a standard competitive equilibrium which is consistent with the resource constraints.

The nonexistence of such a standard competitive equilibrium here is related to the nonexistence of a pooling equilbrium in the Rothschild-Stiglitz analysis. To be noted is that the candidate for the equilibrium budget line here coincides with pooling zero-profit line (market odds line) in Rothschild-Stiglitz. At any point on this pooling line the marginal rates of substitution differ over types. For Rothschild-Stiglitz this implies the existence of a contract preferred by low-risk types which makes positive profit (if only low risk-types buy it). Here unequal marginal rates of substitution imply unequal consumption choices on the budget line.

We have argued in this section that a standard competitive equilibrium in economies with a fundamental adverse selection problem may have undesirable properties. The natural specification of the production set in the linear space L fails because it, along with market clearing, does not generally imply the economy-wide resource constraints. This specification does not take into account that the contribution of each agent type to the diminution of resources for others varies across agent types. That is, the coefficients in the resource constraints depends on i.

This line of argument suggests that the problem is one of externalities. In the next section we shall make precise the nature of this externality for the signaling environment, economy E_2. An alternative competitive equilibrium concept is proposed in an expanded commodity space. It is established that all the Spence signaling allocations are included in its equilibrium allocations. The externality is shown to be a standard production externality consistent with general equilibrium analysis. In a subsequent section we attempt to remove the externality from the general environment.

5. Competitive Signaling Equilibria

For this section we shall view different signals as different commodities. And for simplicity we shall allow only a finite number n of possible signals s, s=1,2,...,n. As before, there is also a consumption commodity which can take on a continuum of nonnegative values. Finally we note that in economy E_2, labor is supplied inelastically. So we suppress labor supply (leisure consumed) as an explicit point of the commodity space. Apart from this suppression, we have in effect expanded the commodity space of the (previous) signaling environment.

To make this more formal, let x_{i0} denote the consumption of agent type i of the consumption commodity, $x_{i0} \geq 0$, and let x_{is} denote the weight which agent type i assigns to the signal s, where $0 \leq x_{is} \leq 1$ and $\Sigma_{s=1}^{n} x_{is} = 1$. Thus, the signaling part of the commodity point x_i is $\{x_{is}\}_{s=1}^{n}$, a probability measure. We emphasize that the choice is over the probability measure, not over the signal s directly. Also, the use of probability measures here is a convenient way of adding in this expanded commodity space. As argued above, the measures themselves are not really needed; agents are indifferent between a lottery and a degenerate consumption allocation with the same mean.

Thus, for the analysis of this section, the commodity space L is R^{n+1}. The common consumption set $\bar{X}$ L is

$$\bar{X} = \{x = (x_0, x_1, \ldots, x_n) \in R_+^{n+1} : \Sigma_{s=1}^{n} x_s = 1\}.$$

Preferences on $\bar{X}$ for agent type i are defined by

$$u_i(x) = \theta_i x_0 - \Sigma_{s=1}^{n} s x_s.$$

The common endowment in L is defined by $\xi = 0$ (note $\xi \notin \bar{X}$, but this is still consistent with general equilibrium modeling).

Now suppose we are given a fixed specification of signals across agent types, $\{x^*_{is}\}_{s=1}^{n}$ $i=1,2,\ldots,I$, and allow for the fact that even if these measures are degenerate, two or more agent types may put all mass on the same signal. Then let

$$\bar{\pi}_s = \frac{\Sigma_i \lambda_i x^*_{is} \pi_i}{\Sigma_i \lambda_i x^*_{is}}$$

denote the average productivity among all agents who end up making signal s, if the denominator is nonzero in the above expression, and let $\bar{\pi}_s$ equal some specified constant otherwise. Then let the production set of the firm in the space L be defined by

$$Y = \{y \varepsilon R^{n+1} : y_0 - \Sigma_{s=1}^{n} \bar{\pi}_s y_s \leq 0\}.$$

Note that this is a linear constraint of the form $r \cdot y \leq 0$.

We now have a standard competitive equilibrium notion in the expanded commodity space. This leads to

Definition 5.1: A competitive equilibrium is an (I + 1)-tuple $((x_i^*), y^*)$ of elements L and a price system p^* in L for which

(i) x_i^* maximizes $u_i(x)$ over the set $\{x \varepsilon \bar{X}: p^* \cdot x \leq p^* \cdot \xi\}$;

(ii) y^* maximizes $p^* \cdot y$ over the set Y; and

(iii) $\sum_i \lambda_i x_i^* = y^* + \xi$.

Now note we have defined a competitive equilibrium with standard externalities in production, as in Arrow-Hahn [1971]. Though the firm takes the coefficients $\bar{\pi}_s$ as given, in fact they depend on the equilibrium consumption allocation (x_i^*). Thus, we would not be surprised to discover that when such equilibria exist, equilibrium allocations may not be optimal.

We shall argue that among the thus-defined competitive equilibria are the Spence signaling equilibria. To construct such competitive equilibria, begin by specifying the signals of each agent type in a Spence signaling equilibrium. These determine the $\{x_{is}^*\}_{s=1}^{n}$, i=1,2,...,I. Define $\bar{\pi}_s$, s=1,...,n as above. Next, note that in a competitive equilibrium profit maximization on the production set determines prices up to some arbitrary constant. Thus, let

$$p_0^* = 1 \qquad p_s^* = -\bar{\pi}_s \qquad s = 1, 2, \ldots, n.$$

At these prices, the problem of agent of type i then

$$\max_{x_i \geq 0} \quad \theta_i x_{i0} - \Sigma_{s=1}^{n} s x_{is}$$

subject to

$$x_{i0} \leq \Sigma_{s=1}^{n} \bar{\pi}_s x_{is}; \; \Sigma_{s=1}^{n} x_{is} = 1.$$

Also, without loss of generality, restrict attention to solutions $x_s \in \{0, 1\}$, $s=1,2,\ldots,n$, i.e., to degenerate measures. For each signal s not included in the Spence equilibrium, the constant $(-\bar{\pi}_s)$ and hence prices p_s^* can be set sufficiently high that no agent type prefers such a signal and consequent consumption over his choice of signal-consumption pair in the Spence signaling equilibrium. And the remaining set of signals with consequent consumption allocations produces exactly the same set of signal consumption choices for each agent type as in the Spence equilibrium. Hence, all Spence equilibria are competitive equilibria, including the no-signaling equilibrium, minimal signaling equilibria, and over-signaling equilibria. Of course many of these are nonoptimal.

6. Stacked Competitive Equilibria

In this section we attempt to remove externalities from the general environment. This is done by indexing the original commodity point by i, thereby expanding the dimensionality of the commodity space by the factor I. In this space there is a specification of the production set which is always consistent with market clearing and the resource constraints. A competitive equilibrium is defined and the two fundamental welfare theorems are established. The support theorem makes clear the difficulty of decentralization with a price system in these environments with adverse selection. In fact it is established that competitive equilibria in this stacked commodity space generally do not exist for the signaling and adverse selection insurance environments.

The commodity space in this section is the Ith cross-product of L denoted by L^I. The consumption possibility set is

$$X = \{x \epsilon I : x_i \epsilon \bar{X} \text{ and } u_i \cdot x_i \geq u_i \cdot x_j \quad \text{all } i,j\}.$$

The commodity point can be thought to be a set of elements from which the consumer chooses. Element x belonging to X insures that agent i weakly prefers component i of x to all the other components.

The endowment, $\omega \epsilon L^I$, has $\omega_i = \xi$ for all i. Finally, the utility function now defined on L^I, is

$$v_i(x) = u_i \cdot x_i$$

for i = 1,...,I.

The aggregate production possibility set is

$$Y = \{y \epsilon L^I : \sum_i \lambda_i r_{ik} \cdot y_i \leq 0 \qquad \text{all } k\}.$$

With this definition, $y \epsilon Y$ and $y = x - \omega$ (note all agents choose the same x but agent i actually consumes component i of x) imply the resource constraint by the following argument. These conditions imply

$$0 \geq \sum_i \lambda_i r_{ik} \cdot y_i = \sum_i \lambda_i r_{ik} \cdot (x_i - \omega_i) = \sum_i \lambda_i r_{ik} \cdot (x_i - \xi).$$

By indexing the commodity vector by i, a production possibility set can be defined consistent with the resource constraints even when the r_{ik} defining the resource constraints are indexed by i as for the adverse selection and signaling economies.

Let p denote a real-valued linear functional on L^I. As L^I has finite dimension, p is an element of L^I. Consider

Definition 6.1: A competitive equilibrium is a state (x^*, y^*) and a price vector p^* such that

(i) for all i, x^* maximizes $v_i(x)$ subject to $x \epsilon X$ and $p^* \cdot x \leq p^* \cdot \omega$;

(ii) element y^* maximizes $p^* \cdot y$ over the set Y;

(iii) $x^* - y^* = \omega$.

Theorem 6.1: Every competitive equilibrium is an optimum if x^* is not a satiation point for any consumer.

Proof: The set X is convex given that the set $\bar{X}$ is convex and the additional incentive compatibility constraints defining X are linear inequalities. In addition for every i, $x', x'' \epsilon X$ and $u_i \cdot x_i' < u_i \cdot x_i''$ implies $u_i x_i' < u_i \cdot [(1 - t)x_i' + tx_i'']$ for all $t \epsilon (0, 1)$. Finally, note that Y displays constant returns to scale with $0 \epsilon Y$ so $p^* \cdot y^* = 0$, and thus $p^* \cdot \omega = p^* \cdot x^*$. These conditions (see Theorem 1 of Debreu [1954]) establish the theorem. Here after we assume a nonsatiation hypothesis.

The second question is whether any Pareto optimum can be supported by a competitive equilibrium. The following theorem and remark answer this question.

Theorem 6.2: Every optimum (x^*, y^*) under which the set

$$\{x \epsilon X : v_i(x) \geq v_i(x^*) \text{ all } i \text{ and } v_i(x) > v_i(x^*) \text{ some } i\}$$

is nonempty is associated with a nontrivial functional p^* on L^I and weights $\gamma^* \epsilon \Gamma$ such that

(i) x^* minimizes $p^* \cdot x$ over $x \epsilon X$ and $\sum_i \gamma_i^* v_i(x) \geq \sum_i \gamma_i^* v_i(x^*)$,

(ii) y^* maximizes $p^* \cdot y$ over $y \epsilon Y$; and

(iii) $x^* = y^* + \omega$.

<u>Proof</u>: Given an optimum x^*, there is some $\gamma^* \epsilon \Gamma$ for which

(iv) x^* solves

$$\text{Max} \quad \Sigma \gamma_i^* v_i(x)$$

subject to

$$x \epsilon X, \; y \epsilon Y, \; x = y + \omega;$$

see Lemma 3.1. In effect, we have a representative consumer economy with preferences described in the above maximand. All the conditions of Debreu [1954], Theorem 2 are satisfied, and the result follows.

<u>Remark</u>: Let p^*, γ^*, and x^* be as in Theorem 6.2 and suppose $\gamma^* > 0$. If there exists some $x \epsilon X$ such that $p^* \cdot x < p^* \cdot x^*$, then condition (i) of Theorem 6.2 can be replaced with

(v) for all i, x^* maximizes $v_i(x)$ subject to

$$x \epsilon X, \; p^* \cdot x \leq p^* \cdot x^*, \text{ and } v_j(x) \geq v_j(x^*), \; j \neq i.$$

<u>Proof</u>: As x^* does not minimize expenditure on X, by hypothesis, Debreu's [1954] Remark applies to condition (i); thus,

(vi) x^* solves maximize $\sum_i \gamma_i^* v_i(x)$ subject to

$x \in X$ and $p^* \cdot x \leq p^* \cdot x^*$.

By the following argument, condition (v) is also satisfied. Clearly x^* satisfies all the constraints in (v). If x^* were not a maximizer in (v) for some i, it would not be a maximizer in (vi); this completes the proof.

Now note that in Theorem 6.2, y^* maximizes profits relative to p^*. Thus, $p^* \cdot y^* = 0$ and it follows from (iii) that $p^* \cdot x^* = p^* \cdot \omega$. Thus with Theorem 6.2 and the condition of the Remark, that there exists some $x \in X$ with $p^* \cdot x < p^* \cdot \omega$, <u>any optimum can be supported as a restricted competitive equilibrium, without transfers</u>. Here a restricted competitive equilibrium requires in lieu of condition (i) in Definition 6.1 that for all i, x^* maximize $v_i(x)$ not over the set

(A) $\{x \in X : p^* \cdot x \leq p^* \cdot \omega\}$

but over the set

(B) $\{x \in X : p^* \cdot x \leq p^* \cdot \omega \text{ and } v_j(x) \geq v_j(x^*) \quad j \neq i\}$.

Thus, in a restricted competitive equilibrium there is not the usual separation of the decision problems facing different agent types.

It is apparent that if x^* maximizes $v_i(x)$ over A for all i, then x^* maximizes $v_i(x)$ over B for all i. Thus, a competitive equilibrium is a restricted competitive equilibrium. But the converse need not hold, as is now established.

Theorem 6.3: If there is a single resource constraint, if there exist at least two Pareto optima yielding different utility for at least one type, both satisfying the condition of the remark, then no competitive equilibrium exists.

Proof: Suppose a competitive equilibrium did exist. Under the nonsatiation hypothesis it is a Pareto optimum by Theorem 6.1 and also a restrictive competitive equilibrium. The production possibility set Y, being defined by a single linear inequality, determines all the relative prices. Let this vector be p^* and let (x^*, y^*) be the competitive equilibrium allocation. But any Pareto optimum $\bar{x}$ satisfying the condition of the Remark can be supported by a restrictive competitive equilibrium without transfers using price system p^*; that is, $\bar{x}$ solves

$$\max_{x \in X} \; v_i(x)$$

$$p^* \cdot x \leq p^* \cdot \omega \text{ and } v_j(x) \geq v_j(\bar{x}), \; j \neq i.$$

Now pick an agent type i who is worse off under the competitive equilibrium x^* than under some optimum $\bar{x}$ satisfying the conditions of the remark. With $p^* \cdot x^* \leq p^* \cdot \omega$ and $v_i(\bar{x}) > v_i(x^*)$, x^* is not maximal for agent i in the competitive equilibrium. This establishes the theorem.

That the condition of the Remark is satisfied for both the adverse selection insurance economy and the signaling economy is established as follows: First let ℓ be the dimension of the linear space L. Then by normalizing prices, let $p_{i\ell} = \lambda_i r_{i\ell}$, all i and ℓ, where the $r_{i\ell}$ are the coefficients in the resource constraint. In particular, for the signaling environment, let $(p_{i1}, p_{i2}, p_{i3}) = (\lambda_i, 0, \lambda_i \pi_i)$, all i. Thus with $\omega_i = (0, 0, 1)$, $p_i^* \cdot \omega_i = \lambda_i \pi_i$ and $p^* \cdot \omega = \bar{\pi} > 0$. Let $\phi = (0, 0, 0)$ and let x be defined by $x_i = \phi$ all i. Then clearly $p^* \cdot x = 0$, and thus $p^* \cdot x < p^* \cdot \omega$. For the insurance economy, $p^*_{ic} = \lambda_i \theta_i c$ for the first n elements of c and $p^*_{ic} = \lambda_i(1 - \theta_i)c$ for the second n elements of c. Thus $p^*_i \cdot \omega_i = \lambda_i \theta_i z_0 + \lambda_i(1 - \theta_i)z_1 > 0$ and $p^* \cdot \omega > 0$. Let ϕ be the element of L placing all mass at the zero point of the underlying commodity space whether or not the agent suffers a loss. Then let $x_i = \phi$ all i, and note that $p^* \cdot x = 0$. Thus, $p^* \cdot x < p^* \cdot \omega$.

In summary, if the utility possibilities frontier is nondegenerate with positive gradient for the signaling and adverse-selection insurance economies, then no competitive equilibrium in the stacked commodity space exists.

7. Concluding Remarks

As noted in the introduction, the purpose of this paper has been to explore the extent to which standard general equilibrium analysis of optima and competitive equilibria applies to environments with moral hazard and adverse selection problems. In particular we have explored the extent to which the results of Prescott and Townsend [1979] can be extended. We have discovered that techniques for characterizing optima as solutions to concave programming problems in the space of lotteries are useful and nice and seem broadly applicable; lotteries ensure linearity of preferences, a property which is needed to obtain convexity of the incentive compatibility constraints. With respect to existence and optimality of competitive equilibria in the linear space containing lotteries, we have discovered that a certain *ex ante* homogeneity is critical, but that subsequent heterogeneity is not. That is, for standard results on existence and optimality of competitive equilibria to go through, what seems to be needed is that agents with characteristics which are distinct and *privately observed at the time* _of initial trading_ enter the economy-wide resource constraints in a homogeneous way. This homogeneity condition will be satisfied for a broad class of interesting economies, two examples of which we have studied: the moral hazard insurance economy and the dynamic private-information securities economy. On the other hand, this homogeneity condition is not satisfied for the well-known and interesting adverse selection insurance economy and the signaling economy. In these economies _ex ante_ heterogeneity introduces an externality of some kind. Thus nonoptimality typically results when competitive equilibria exist. And with _ex ante_ heterogeneity competitive equilibria frequently do not exist.

Footnotes

[1] As noted in Prescott and Townsend [1979], one cannot go from individual uncertainty to an aggregate distribution with the specified characteristics. We proceed in the other direction.

[2] This subsection has benefited from conversations with V. V. Chari.

[3] To further illustrate the general difficulty, suppose that in the environment of Prescott-Townsend there is some statistical dependence in the θ-shocks and the initial specification of i types; the resource constraints in period one would then be written

$$\sum_i \lambda_i \sum_\theta \pi_{\theta|i}\left[\sum_c x_{ic\theta}(c_k - e_k)\right] \leq 0 \qquad k = 1, \ldots, \ell$$

where $\pi_{\theta|i}$ denotes the fraction of agents of type i (in period zero) who receive shock θ (in period one). These $\pi_{\theta|i}$ enter the components of r_{ik}, which vary over i. Thus the problem reemerges. And there would be a problem in the moral-hazard insurance economy <u>if</u> it were combined with the adverse-selection insurance economy.

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