NONCOOPERATIVE COLLUSION UNDER IMPERFECT

PRICE INFORMATION *

by

Edward J. Green and Robert H. Porter**

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ABSTRACT

Recent work in game theory has shown that, in principle, it may be possible for firms in an industry to form a self-policing cartel to maximize their joint profits. This paper studies the applicability of that work to empirical industrial organization. A parametric model of a noncooperatively supported cartel is presented, and the aspects of industry structure which would make such a cartel viable are discussed. The model is shown to be estimable my means of a multiple-equation switching-regression technique. Thus it may be possible to subject a particular industry to a direct test of collusive conduct. Such a test would complement the reduced-form cross-industry regressions by which hypotheses about collusion (in particular, Stigler's theory of oligopoly) have previously been tested.

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1. Introduction

In his classic paper "A Theory of Oligopoly" [11], George Stigler appealed to dynamic considerations to explain how apparently cooperative industry performance might result from noncooperative motives. According to this theory, the firms of an industry form a cartel, which is designed to enforce monopolistic conduct in a self-policing way. "Self-policing" means precisely that the agreed-upon conduct is noncooperatively viable and that it remains so over time.

Stigler's theory differs markedly from traditional oligopoly theories based on static equilibrium concepts (e.g., Cournot and Stackelberg). This difference is particularly striking in the case of a "mature" industry (i.e. one which because of exogenous considerations has a structure which is essentially immune from entry). The traditional theories would suggest that the performance of such an industry should be largely determined by its degree of concentration -- the number of firms in the industry and their relative sizes -- and by the extent to which substitute goods are available. In contrast, Stigler suggested that the greatest obstacle to collusion in the absence of entry would be what he characterized as "secret price cutting". By informally relating concentration and various other features of industry structure to the immunity of a cartel from entry and to its ability to deter inimical firm behavior, and by assuming that industry profitability reflects successful operation of a cartel, he justified the use of cross-industry regressions to test his theory.

This paper presents a new interpretation of Stigler's theory of cartel enforcement, and formulates an econometric model based on this interpretation which makes possible an empirical test for collusion in a single industry. We will first use Stigler's informal characterization of collusion as the basis for a formal game-theoretic definition of collusive or flagrantly noncompetitive conduct. Next we will focus on a particular industry structure. We will argue that collusive equilibrium in such an industry would have to be of a fairly specific form, and that a collusive industry with this structure would perform in a substantially different way from a quasicompetitive one (e.g. a Cournot oligopoly). Finally, we will argue that this difference in performance is identifiable from time series of industry aggregate data. Thus it should be possible to complement cross-industry regressions with studies to assess the likelihood that collusion has taken place in some particular industries.

Specifically, if Stigler's interpretation of his cross-industry studies is correct, they reveal that collusion characterizes to some extent the conduct and performance of many industries. However, a variety of competing explanations (e.g. that average profitability is positively related to seller concentration because efficient firms grow fastest) have been offered for Stigler's empirical findings. The industry studies that we propose would be of substantial general interest, because they might suggest how cross-industry regressions should be interpreted. Evidence of collusion in a particular industry would support Stigler's contention that he was measuring the profitability of collusion, while failure to discover collusion, in an environment where it should be feasible and statistically detectable, would lend credibility to the alternative explanations of his critics.

2. A Reinterpretation of Stigler's Theory

The obvious interpretation of Stigler [11] is that he made explicit a theory of oligopoly which implicitly conceived of a cartel as a "policeman" which with some frequency is required to punish destabilizing "offenses" of individual cartel members. The somewhat different interpretation of this paper is that Stigler had a view of cartel organization as an instance of an optimization problem: to design an institution which achieves an efficient equilibrium outcome subject to the constraint that agents in the institution behave noncooperatively. On this interpretation, the optimal cartel structure may be one which provides member firms with strong positive incentives which make collusive behavior attractive, rather than one which provides insufficient incentives and which severely punishes defecting firms after the fact.

In fact, two formulations of the cartel problem exist already which treat noncooperative collusion in a rigorous way. Osborne [7] proposes a reaction function equilibrium in which firms respond to changes in output by other firms in order to maintain their proportionate share of industry output. (See also the extensions of Spence [9, 10]). Knowing that other firms will respond in this manner, each firm will realize that it does not pay to deviate from the collusive output level.

Friedman [1], on the other hand, outlines a strategy in which firms respond to suspected cheating, which they infer from a drop in the market price below the price that obtains when all firms produce at agreed-upon levels, by producing at Cournot levels thereafter. If future profit streams are discounted sufficiently slowly, then a firm would reduce the discounted value of its returns by failing to collude. Therefore, for all firms to adopt the collusive strategy would be a noncooperative equilibrium.

The trouble with these formulations, from an applied industrial organization viewpoint, is that incentives in these equilibria are so perfect that the deterrent mechanisms are never observed. The substance of the present contribution is that this perfection is an artifact of the certainty world in which these models are formulated. When the considerations of imperfect information, which played a decisive role in Stigler's theory, are reintroduced, optimal incentive structures may involve episodic recourse to the kind of short-run unprofitable conduct which would have been characterized as "price wars" or "punishment" previously.

We will define collusion as a particular noncooperative equilibrium concept for oligopoly. In order to motivate the definition, an example of an obviously collusive equilibrium will be discussed. This example, due to James Friedman [1], is the limiting case of the equilibrium to be studied in the next section.

Consider a duopoly protected from entry, which produces an undifferentiated product in a deterministic, stationary and time-separable environment. Each firm acts to maximize profits discounted at a rate which is exogenously given. Let (y_1, y_2) be the vector of outputs for the two firms which would maximize joint net returns in any period, and let (z_1, z_2) be the Cournot output vector. Let p denote the inverse demand function. Suppose that each firm i announces to its rival j that it will begin by producing output y_i and will continue to produce y_i as long as the market price is $p(y_1 + y_2)$, but that it will change irrevocably to a Cournot firm producing z_i in every period if it ever observes a lower market price (i.e., if it observes that j has failed to collude by producing y_j). Firm j could raise its return in the initial period by producing more than y_j. However, if it did so, it would face a Cournot rival rather than a cooperative one at every future time and the greatest feasible return for firm j would be less than the cooperative return. Thus, if future returns are discounted sufficiently slowly, firm j would reduce its discounted profits if it were to fail to collude. Since it is to the advantage of neither firm to defect first from the cartel, collusive forever is a noncooperative equilibrium.

We now suggest a distinction between quasicompetitive and collusive industries, based on this notion of noncooperative equilibrium. Intuitively, the idea of this distinction is that a firm is quasicompetitive if it conceives of its competitors' present <u>and future</u> conduct as being a parametric part of its environment (just as a firm in a competitive industry would do), and that it is collusive (or, more generally, flagrantly noncompetitive) otherwise. In other words, a firm is quasicompetitive if it differs from a competitive firm only in its perception of the demand side of its market, and it is flagrantly noncompetitive if it differs also in that it is aware of its interdependence with actual or potential competitors. In some industries, interdependence among firms is a striking fact about the environment, so we do not suggest that collusive equilibrium cannot arise spontaneously. Moreover, a flagrantly noncompetitive equilibrium in an industry may be more efficient than the quasicompetitive equilibrium which would be its alternative -- Milgrom and Roberts [6] have stressed this point in their analysis of limit pricing. The distinction between the two types of industry equilibrium is a positive distinction which concerns firms' expectations, and should not be interpreted as implying any welfare judgment or direct prescription for policy.

By definition a quasicompetitive firm would not perceive any potential gain from attending to the incentives or threats presented by its competitors, because it would perceive those competitors to be acting without any feedback from its own conduct. In contrast, a collusive firm would perceive its competitors to be making plans for future conduct (e.g., for production next period) contingent on events over which the firm in question exercises some control (e.g., the current market price). Thus the collusive firm perceives that it can influence the conduct of its competitors, rather than having to take their conduct as given and having to optimize with respect to it. A firm's competitors' contingent plans have much the same effect as a menu of contracts presented to the firm, in the sense that it will choose which contingency it can most profitably bring about in the same way as it would hypothetically choose among a set of deliberately provided incentives. We suggest that this situation, which is virtually a multilateral agency relationship among the firms in an industry, is what constitutes collusion.

This intuitive characterization of collusion suggests a formal game-theoretic definition. Note that in Friedman's example, an industry has been described as a noncooperative game in extensive form. The players (i.e., firms) have been described in terms of their objectives (i.e., that each firm seeks to induce industry conduct which will maximize its own discounted profit). Their situations have been described by specifying the various actions (i.e., setting production levels) which they must take over time and the sets of options (i.e., production sets) from which they may choose those actions. A comprehensive choice of options for all requisite actions (i.e., a production plan) is a strategy. Industry equilibrium has been presented as an instance of Nash equilibrium, which is the situation in which each player adopts the strategy which is his best response to those adopted by the others.

The intuitive characterization of collusion is based on the recognition that, in an extensive-form game where each player's set of feasible actions remains fixed throughout the game, there are two distinct notions of what is a strategy. A strategy may be thought of as a fixed temporal sequence of actions, to which a player commits himself at the beginning of the game. This will be called a noncontingent strategy. Nash equilibrium in noncontingent strategies is an obvious generalization of Cournot equilibrium. Another possibility is to conceive of a strategy as specifying an initial action and a sequence of rules. At time t a player knows his own past actions and some information about the past history of the market, and the t^{th} rule tells him what action to take at time $t+1$ as a function of this knowledge. This sort of strategy will be called contingent strategy. The announcements made by the duopolists of the example given above describe strategies of this sort, and together they are (if discounting is sufficiently slow) a Nash equilibrium in contingent strategies.

We propose that an industry should be considered collusive or flagrantly noncompetitive if its performance is consistent with the conduct of its actual and potential members being a Nash equilibrium in contingent strategies, but not in noncontingent strategies. This definition is supported by two considerations. First, the example shows that it includes the kind of conduct which Stigler characterized as collusive, but does not include Cournot oligopoly. Second, the definition appears to entail that collusion requires either high concentration or explicit coordination in a market. (Green [2] contains some results which strongly suggest this. However, see Radner [8] which contains a more equivocal result.) Thus, the definition will not lead to categorization as collusive of markets which intuitively would be competitive.

It should be evident that this definition is very general. It applies to price-setting strategies as well as to quantity-setting ones. Furthermore, it applies to attempts to delay or deter entry into an industry (in particular, the limit-pricing equilibrium studied by Milgrom and Roberts [6] is flagrantly noncompetitive, although it is more efficient than the quasicompetitive equilibrium). The collusiveness of an equilibrium is independent of whether firms' conduct results from an explicit agreement or from a tacit understanding. Finally, while an equilibrium involving detection and punishment of defection from a cartel agreement would be collusive, collusion need not take this form in general.

One qualification is in order. The assumption that firms adopting contingent strategies is evidence of collusion presupposes that quantities currently observed by firms are not informative about exogenous features of the future environment. This assumption will be reasonable if stochastic shocks to the steady-state environment are independent over time. We will make this assumption in the present paper. In more complicated environments, quasicompetitive firms would change their plans when their observation of the market leads them to revise their expectations about its exogenous features. We do not pursue the question of exactly how quasicompetitive equilibria would then be characterized.

3. Collusion Under Uncertainty

Collusive equilibria much like the one discussed in the last section may possibly characterize some industries. For instance, a market might be segmented geographically because firms have divided it. As long as this agreement were adhered to, each firm would be a monopolist within its area. Moreover, poaching by one firm in another's territory would be quickly and surely detected, and would invite retaliation. In that situation, no one would poach. All that would ever be "observed" is monopolistic conduct.

Similarly, in an industry in which contracts are awarded by competitive bidding, a scheme to rotate winning bids might be perfectly enforceable. Each firm would act as a monopolist when its turn came, and would clearly see that bidding low out of turn would jeopardize a profitable arrangement. Again, only monopolistic conduct would ever be "observed".

We write "observed" in quotes because it is notoriously difficult in practice to establish that any sort of stable behavior in a single industry is monopolistic. While industry performance is observed, its monopolistic nature is not. This difficulty of characterizing stable performance is the feature of the oligopoly question which is responsible for the emphasis on cross-industry studies. We propose to deal with this feature by another means. Specifically, we propose to look carefully at industries in which performance is unstable to some extent. In this section, a formal model of a collusive equilibrium which yields industry performance of this character will be presented. In the next section, the structural conditions which make this a plausible model for an actual industry will be discussed. (An industry which meets these conditions is briefly described in Appendix A.) Then, the model will be used to formulate an econometric test of the hypothesis that an industry of this type is collusive.

We will study a model in which demand fluctuations not directly observed by firms lead to unstable industry performance. Intuitively firms will act monopolistically while prices remain high, but they will revert for a while to Cournot behavior when prices fall. Specifically, it will be assumed that firms agree (tacitly or explicitly) on a "trigger price" to which they compare the market price when they set their production. Whenever the market price dips below the trigger price while they have been acting monopolistically, they will revert to Cournot behavior for some fixed amount of time before resuming monopolistic conduct. Such an occurrence will be called a "reversionary episode". The considerations which underlie the assumption that firms monitor price (rather than, e.g., market share) and that reversionary episodes are of fixed length, will be discussed in the next section.

Suppose that, at a given time, firms are supposed to be colluding (i.e., they expect one another to collude. They are not in a reversionary episode). If a firm produces more than its share of the monopoly output, its net return at that time will increase. However, by increasing the probability that the market price will fall below the trigger price, the firm incurs a greater risk that the industry will enter a reversionary episode during which profits will be low for everyone. For producing its monopolistic share to be the firms' noncooperatively optimal action, the marginal expected loss in future profits from possibly triggering a Cournot reversion must exactly balance (in terms of present discounted value) the marginal gain from over-producing. Of course, a reversionary episode will sometimes occur without any firm defecting, simply because of low demand. Thus, over a long period, both Cournot behavior and collusive behavior will be observed at various times. In this respect, collusion under uncertainty differs markedly from the collusive equilibria under certainty discussed earlier. The fact that both monopolistic and Cournot performance are observed will make it possible to identify statistically the collusive equilibrium under undertainty.

We now give a formal description of collusion under uncertainty as a Nash equilibrium in contingent strategies. Consider an oligopoly of n firms in an environment like that described in the example of collusion under certainty (in section 2), except that demand is subject to multiplicative uncertainty.

Specifically, i,j range over firms 1,...,n.

$\pi_i : R_+^2 \to R$ is the return function of i. $\pi_i(x_i,p)$ is i's net return from producing x_i units and selling at price p.

β is the discount rate. Firms are risk neutral and maximize $E[\sum_{t=0}^{\infty} \beta^t \pi_i(x_{it},p_t)]$. $p_t = \theta_t\, p(\sum_{i=1}^{n} x_i)$. The random variables θ_t are i.i.d. with c.d.f. F having continuous density f. $E(\theta) = 1$. Each θ_t is a demand shock which firms cannot observe directly.

A contingent strategy for firm i is an infinite sequence $s_t = (s_{i0}, s_{i1},\ldots)$, where s_{i0} is a determinate initial output level x_{i0}, and $s_{it+1} : R_+^{t+1} \to R_+$ determines i's output level at time t as a function of past prices by $s_{it+1}(p_0,\ldots,p_t) = x_{it+1}$. The choice of domain reflects the assumption that firms do not observe rivals' production levels directly.

A strategy profile $(s_1,\ldots s_n)$ determines recursively a stochastic process of prices, which in turn induces a probability distribution on the space of infinite sequences of prices. Expectation with respect to this distribution will be denoted by $E_{s_1\ldots s_n}$.

A Nash equilibrium is a strategy profile $(s_1^*,\ldots s_n^*)$ which satisfies

$$(1) \qquad E_{s_1^*\ldots s_i\ldots s_n^*}\left[\sum_{t=0}^{\infty}\beta^t\,\pi_i(s_{it}(p_0,\ldots,p_{t-1}),p_t)\right]$$

$$\leq E_{s_1^*\ldots s_i^*\ldots s_n^*}\left[\sum_{t=0}^{\infty}\beta^t\,\pi_i(s_{it}^*(p_0,\ldots,p_{t-1}),p_t)\right]$$

for all firms i and feasible strategies s_i.

Now consider how the industry might produce at a monopolistic level most of the time (i.e., except during reversionary episodes) in Nash equilibrium. Firms will initially produce their respective shares of this restricted industry output, and will continue to do so until the market price falls below a trigger price $\bar{p}$. Then they will produce Cournot outputs for the duration (we will specify this to be T-1 periods) of a reversionary episode, regardless of what happens to prices during this time. At the conclusion of the episode, T periods after the price drop, they will resume monopolistic production. This will continue until the next time that $p_t < \bar{p}$, and so forth.

Formally, let $y = (y_1,\ldots,y_n)$ be a profile of restricted outputs, and let $z = (z_1,\ldots,z_n)$ be a Cournot output profile. Choose a price level $\bar{p}$ and a length of time T. Define time t to be <u>normal</u> if (a) $t = 0$, or (b) $t - 1$ was normal and $\bar{p} \leq p_{t-1}$, or (c) $t - T$ was normal and $p_{t-T} < \bar{p}$. Define t to be <u>reversionary</u> otherwise.

Define strategies for firms by

$$x_{it} = \begin{cases} y_i & \text{if } t \text{ is normal} \\ z_i & \text{if } t \text{ is reversionary} \end{cases}$$

These are well-defined policy strategies.

Each firm faces a stationary two-state (normal and reversionary) T-stage Markov dynamic programming problem. Its optimal policy is to produce z_i in reversionary periods, and to produce some fixed quantity r in normal periods. Let $V_i(r)$ be the expected discounted present value of firm i if it sets $x_{it} = r$ in normal periods. Define

$$w_i = \sum_{j \neq i} y_j \qquad \phi_i(r) = E_\theta \pi_i(r, \theta p(r + w_i)) \qquad \psi_i = E_\theta \pi_i(z_i, \theta p(\sum_{j \leq n} z_j))$$

Let Pr(•) denote probability with respect to the distribution of θ. We assume that $\phi_i(y_i) > \psi_i$ for each firm i.

Then V_i satisfies the functional equation

$$V_i(r) = \phi_i(r) + \beta \Pr(\bar{p} \leq \theta p(r + w_i))\, V_i(r) \tag{2}$$

$$+ \Pr(\theta p(r + w_i) < \bar{p})\, [(\sum_{T=1}^{T-1} \beta^t \psi_i) + \beta^T V_i(r)]$$

$\Pr(\theta p(r + w_i) < \bar{p}) = F(\bar{p}/p(r + w_i))$, so (2) is equivalent to

$$V_i(r) = \frac{\phi_i(r) + F(\bar{p}/p(r + w_i))\ ((\beta - \beta^T)/(1-\beta))\psi_i}{1 - \beta + (\beta - \beta^T)\ F(\bar{p}/p(r + w_i))} \tag{3}$$

$$= \frac{\phi_i(r) - \psi_i}{1 - \beta + (\beta - \beta^T)\ F(\bar{p}/p(r + w_i))} + \frac{\psi_i}{1 - \beta}$$

Thus the expected discounted present value of firm i equals what it would be in a Cournot environment, plus the single-period gain in returns to colluding, appropriately discounted. Inequality (1), the defining condition for Nash equilibrium, can now be rewritten

(4) $\quad V_i(r) \leq V_i(y_i) \quad$ for all r and i.

The first-order condition for (4) is

(5) $\quad V'_i(y_i) = 0 \quad$ for all i.

Using the fact that $(f/g)' = 0$ if and only if $f'g - fg' = 0$, (5) is equivalent to

$$0 = [1 - \beta + (\beta-\beta^T)F(\bar{p}/p(\sum_{j\leq n} y_j))]\phi_i'(y_i)$$

(6)

$$+ (\beta-\beta^T)f(\bar{p}/p(\sum_{j\leq n} y_j))]\bar{p}\ p'(\sum_{j\leq n} y_j)/(p(\sum_{j\leq n} y_j))^2](\phi_i(y_i)-\psi_i)$$

for all i.

Equation (6) states that the marginal return to a firm from increasing its production in normal periods ($\phi_i'(y_i)$) must be offset exactly by the marginal increase in risk of suffering a loss in returns ($\phi_i(y_i) - \psi_i$) by triggering a reversionary episode. When this condition holds for all firms, n differential constraints are placed on the n-dimensional vector y of restricted outputs in equilibrium. Thus, the assertion that an equilibrium which satisfies an additional constraint exists will require careful justification. In particular, the output profile which maximizes total returns to the industry may not be supportable in equilibrium. However, we believe it is plausible that a profile close to the return-maximizing one would, in fact, be supportable if firms have reasonably accurate information.

There are two related final observations about the formal model of collusion under uncertainty. First, no firm ever defects from the cartel. More precisely, no firm i has any private information that would lead it to assess its return function π_i more accurately than its competitors do. Thus, every competitor is able to figure out what i will do to maximize profits. The market price reveals information about demand only, and never leads i's competitors to revise their beliefs about how much i has produced. Stigler's theory that firms occasionally defect and are punished entails that such revision of beliefs occurs, and thus requires that private information exist. We suggest that privately held information may be of secondary importance in many industries, and we note that "price wars" may be explained as reversionary episodes without invoking the idea of punishment.

Second, despite the fact that firms know that low prices reflect demand conditions rather than over production by competitors, it is rational for them to participate in reversionary episodes. Basically, a reversionary episode is just a temporary switch to a Nash equilibrium in noncontingent strategies. It doesn't pay any firm to deviate unilaterally from its Nash strategy in this temporary situation, any more than it would if the industry were permanently a Cournot industry. It might be asked why Cournot equilibrium is appropriate at all. If firms know at a particular time that a low price has been observed in the past, and that the cartel has had a perfect record of monopolistic conduct, why do firms not disregard the price and continue to act monopolistically? The answer is that everyone understands the incentive properties of equilibrium. If firms did not revert to Cournot behavior in response to low prices, equation (5) would not hold the rest of the time, so monopolistic behavior would cease to be individually optimal for firms. Adherence to the collusive equilibrium is the only way in which the cartel can realize monopoly prices. Of course, if one wishes, it is possible to make an empirical test of collusive equilibrium against the hypothesis of "altruistic" monopolistic conduct. This will be discussed in section 6.

4. Industry Structure Presumed by the Model.

In the preceding section, a model of collusion under uncertainty was developed and it was suggested that this model might explain the performance of some industries. This section is devoted to specifying exactly what sort of industry the model might appropriately describe, and what features the industry would be required to possess in order for the hypothesis of collusive equilibrium to be statistically testable.

First, the industry structure must be stable over time. Temporal stability is required if the assumption that firms have rational expectations -- an assumption which underlies the use of

Nash equilibrium -- is to be credible. On a more technical level, it justifies the use of stationary dynamic programming to characterize equilibrium. The requirement of temporal stability rules out industries which are rapidly growing or declining, those which experience marked technological change, and those of which capital stock adjustment is a prominent feature.

Second, firms in the industry should not face severe threats of entry. This assumption makes reasonable the imposition of two restrictions on an econometric model. First, the collusive industry output may be taken to be the monopolistic output, rather than a larger output which might be reasonable if it were important to limit the potential gains to entry. Second, firms' expectation that collusive profits can be maintained indefinitely justifies the assumption that profits are discounted in terms of the market interest rate, rather than at some faster (and unspecified) rate which would reflect the hazard that entry would destroy the cartel.

Third, output quantity should be the only decision variable which firms can manipulate. In particular, firms should not be able to engage in product differentiation or have ability to divide their market regionally. With firm decisions so restricted, noncooperative cartel incentive schemes other than Cournot reversion are ruled out. In particular, even if one firm were suspected of violating a cartel agreement, other firms would have no way of isolating it and punishing it differentially.

Fourth, except for each firm's private knowledge about its present and past production, information about the industry and its environment is public. The Nash equilibrium assumption presupposes that firms have an accurate idea of their competitors' cost functions, for example. Also, for firms to coordinate effectively in keeping track of whether the industry is in a collusive or a reversionary state, they must all observe the realization of a common variable. Furthermore, the econometrician must have access to the monitoring variable as well, if the model is to be estimable.

Fifth, the information which firms use to monitor whether the cartel is in a collusive or reversionary state must be imperfectly correlated with firms' conduct. Otherwise, if compliance were optimal for firms in collusive periods, reversion would never occur and the model would not be statistically identifiable. Price is not the only information variable which could be used for monitoring -- price data with correction for a systematic demand component, or market share information, would also be subject to error. However, this assumption of imperfect information is incompatible with transactions in the industry being few and publicly announced, (e.g., with individual contracts being awarded on the basis of sealed-bid auctions) or with completely accurate and current market-share information being available to firms.

Sixth, the industry should not be subject to antitrust laws. A cartel which must evade antitrust enforcement would likely mask its behavior in numerous ways. For instance, it might produce more than the monopoly quantity so that its performance would be less evidently noncompetitive. It might also mask its collusive incentive scheme by such means as randomizing the length of reversionary episodes and incorporating occasional reversionary episodes triggered by totally irrelevant random events. For a cartel which could operate openly, though, the particularly simple Nash equilibrium set forth in the preceding section would be a very natural one to use.

We realize that these assumptions about industry structure are quite restrictive. We emphasize that the particular Nash equilibrium we are studying is not the only sort of Nash equilibrium which would be collusive according to the definition offered in Section 2, and that statistical evidence that this particular Nash equilibrium operates in a specific industry is not the only statistical evidence relevant to forming an opinion about the extent of collusion in various sectors of the economy. What would be special about time-series evidence from an industry of the sort envisioned here is that it would be immune from competing explanations which have been advanced for other types of evidence. Evidence about this special case might make it more or less plausible that other types of evidence, about industries with different structural characteristics, really indicate the presence of collusion and not some other phenomenon. Thus, even though the direct applicability of our model is severely limited, it would be valuable to examine an industry for which it would be appropriate. We believe that the American rail freight industry in the 1880s satisfies our structural conditions quite well. R. Porter is currently pursuing empirical work on this industry. Earlier studies of the industry, by Paul MacAvoy [5] and Thomas Ulen [12, 13] have produced qualitative conclusions which are consistent with our model. While a discussion of the rail freight industry is beyond the scope of this paper, a suggestive quote from Ulen is provided in Appendix A.

A piece of empirical work in a similar spirit is by Kosobud and Stokes [4], who attempt to test whether OPEC has followed an Osborne strategy since the 1973 oil price increases. They test whether market shares have been more stable since 1973 then they were before. The pre-1973 period serves to identify their oligopoly model statistically as the Cournot reversionary episodes will serve to identify ours.

5. Estimation

Except for the work of Kosobud and Stokes just described, quantitative tests of oligopoly theories have generally involved cross-industry tests of reduced-form propositions derived from the model being studied. The time-series estimation and test of a single industry which we propose contrasts sharply with this usual procedure. In this section we will discuss estimation of our model, but we begin by explaining why we have chosen the route we take.

It would be possible, but probably not very persuasive, to interpret cross-industry studies as corroborating our model. The problem is, the structural characteristics implied by our model are sufficiently special that it is doubtful whether any appropriate industry sample of adequate size for reliable cross-industry inference could be found. Our model of collusion leads in principle to a variety of reduced-form cross-industry predictions, but a data base for testing these predictions does not exist. Not only is there a positive advantage to testing our model on the basis of time-series data from a single industry (as was explained at the end of the preceding section), but such a test is the only one available.

This section is then concerned with the possibility of estimating a model of the Nash equilibrium in section 3 using time series data on price and aggregate output levels for a single industry. A simultaneous-equations switching-regressions model is proposed, in which the parameters of the demand and cost functions are estimated, and in which the regime probabilities are unknown and changing endogenously.

We wish to estimate a simultaneous system of two equations corresponding to a demand equation, which will be invariant across regimes, and pseudo-supply function, which will vary across regimes to reflect collusive or Cournot behavior. These two equations are now specified.

Denote the market price by $\hat{p}$, so that $\hat{p} = p\,\theta$. The inverse demand equation is assumed to be a function of national income, I, and aggregate output of the industry, $X = \sum_{i=1}^{n} x_i$. The income variable is exogenous, and is intended to account for any systematic variations in demand according to the business cycle or aggregate growth of the economy. The industry is assumed to produce a good which consumers perceive to be of homogeneous quality across firms. Thus $p = p(I,X)$ for all firms, as only aggregate output affects the price they face. Assume that both income and price elasticity of demand are constant, and so

$$(7)\qquad \hat{p} = p(I,X)\,\theta = dI^{\varepsilon_1} X^{-\varepsilon_2}\theta ,$$

Where $1/\varepsilon_2$ is the absolute value of the price elasticity, and $\varepsilon_1/\varepsilon_2$ the income elasticity. The random disturbance θ is distributed lognormally. Then $\ell n\ \theta$ is distributed normally, here with zero mean and variance σ^2. In this case, $E[\theta] = \exp(\sigma^2/2)$.

Each firm in the industry is assumed to face a one input cost function. If the price of the input is q then the cost of producing output x_i for firm i is

$$(8)\qquad C_i(x_i) = a_i A x_i^{\delta} + F_i, \quad \delta > 1$$

where a_i is a firm-specific shift parameter, δ is the (constant) elasticity of variable costs with respect to output, F_i is the fixed cost faced by firm i, and A varies with factor price according to

$$A = \exp(\alpha_1 \ell nq).$$

Marginal costs faced by firm i at output x_i are

$$(9)\qquad MC_i(x_i) = a_i A \delta x_i^{\delta-1}.$$

Under this specification, different firms can face average cost functions of radically different shape.

Assume that, in the collusive regime, firms choose their output level in order to maximize the expected value of joint net returns in the industry. Denote expected price as a function of output by $p^e(X)$. Then

$$\begin{aligned} p^e(X) &= E[\hat{p}(X,I)] \\ &= p(X,I)E[\theta] , \end{aligned}$$

where for any given period I is exogenously given. Then the expected profits of firm i at output x_i are given by

$$\pi_i(x_i) = p^e(x_i + w_i)x_i - C_i(x_i)$$

(Recall that $w_i = \sum_{j\neq i} x_j$, the total output of the other firms in the industry.) Total industry output is $X = x_i + w_i = \sum_{i=1}^{n} x_i$. Then expected joint net returns are

$$\begin{aligned} \pi(x_1,\ldots,x_n) &= \sum_{i=1}^{n} \pi_i(x_i) \\ &= p^e(X)X - \sum_{i=1}^{n} C_i(x_i) . \end{aligned}$$

If each firm i chooses x_i to maximize $\pi(x_1,\ldots,x_n)$ given w_i, the first order condition is

$$p^e(1-\varepsilon_2) = MC_i(x_i) \qquad \text{for } i=1,\ldots,n\,. \tag{10}$$

Note that we require $\varepsilon_2 < 1$.

If firms are producing at Cournot levels, they choose x_i to maximize their own expected profits, $\pi_i(x_i)$, given w_i, and obtain the first order condition

$$p^e(1-S_i\varepsilon_2) = MC_i(x_i) \qquad \text{for } i=1,\ldots,n\,, \tag{11}$$

where $S_i = x_i/X$, the share of industry output produced by firm i.

For estimation purposes, it is very difficult to obtain a time series of firm-specific output data, so we aggregate. Both sides of equations (10) and (11) are weighted by firm shares S_i and summed over firms, to obtain

$$p^e(X)(1-\varepsilon_2) = \sum_{i=1}^{n} S_i MC_i(x_i) \tag{12}$$

in the collusive regime, and

$$p^e(X)(1-H\varepsilon_2) = \sum_{i=1}^{n} S_i MC_i(x_i) \tag{13}$$

in Cournot periods, where $H = \sum_{i=1}^{n} S_i^2$ is the Herfindahl index.

We can write

$$\sum_{i=1}^{n} S_i MC_i(x_i) = A\delta Q^{\delta-1}[\sum_{i=1}^{n} a_i S_i^{\delta}] .$$

As an approximation, consider the marginal cost of producing HX. (In the case of symmetric firms, with $S_i = 1/n$ for all i, this is the ouput of each firm, as $H = 1/n$.) Here

$$MC(HX) = aA\ \delta H^{\delta-1} X^{\delta-1}$$

$$= \sum_{i=1}^{n} S_i MC_i(x_i)$$

when $a = H^{1-\delta}[\sum_{i=1}^{n} a_i S_i^{\delta}]$. When individual firm output is not observed, then neither are the shares of individual firms. With this specification, however, the optimal share for each firm is constant across regimes and not a function of the exogenous variables, as is demonstrated in Appendix B. Then each firm views its share as constant over time. In this case a will be constant over time. Thus, in this case of constant shares, the approximation is exact if a is interpreted appropriately. Then equations (12) and (13) are replaced for estimation purposes by

$$p^e(1 - \varepsilon_2) = MC(HX) \cdot V$$

in the collusive regime, and

$$p^e(1 - H\varepsilon_2) = MC(HX) \cdot V$$

in the Cournot regime. We have introduced the optimization error, V, which has a lognormal distribution. It is assumed that $\ell n\ V$, which will have a normal distribution, has zero mean and variance τ^2, and that $E[\ell n\theta \cdot \ell n\ V] = \omega$.

Now p^e is not observed, but we know that

$$p^e = \hat{p}E[\theta]/\theta ,$$

and so we observe

$$E[\theta]\hat{p}(1-\varepsilon_2) = MC(HX) \cdot V \cdot \theta$$

in the collusive regime, and

$$E[\theta]p(1-H\varepsilon_2) = MC(HX) \cdot V \cdot \theta$$

in the Cournot regime.

Define the regimes by

$$w_t = \begin{cases} 1 & \text{if } t \text{ is normal (i.e. if } x_t = y) \\ 0 & \text{if } t \text{ is reversionary (i.e. if } x_t = x) \end{cases}$$

Further, denote the probability of observing collusive behavior in period t by λ_t. Then $\lambda_t = \Pr(w_t = 1)$.

As a result, we wish to estimate a simultaneous equation switching regressions model of the following form:

$$(14) \qquad \ell n\hat{p}_t = \varepsilon_0 + \varepsilon_1 \ell nI_t - \varepsilon_2 \ell nX_t + \ell n\theta_t$$

$$(15)\qquad \ell n\hat{p}_t + \ell n(1-\varepsilon_2) = \alpha_0 + \alpha_1 \ell nq_t + \delta_0 \ell nX_t + \ell nV_t + \ell n\theta_t \quad \text{with probability } \lambda_t$$

$$(16)\qquad \ell n\hat{p}_t + \ell n(1-H\varepsilon_2) = \alpha_0 + \alpha_1 \ell nq_t + \delta_0 \ell nX_t + \ell nV_t + \ell n\theta_t \quad \text{with probability } 1-\lambda_t .$$

Here $\varepsilon_0 = \ell nd$, $\delta_0 = \delta - 1$, and $\alpha_0 = \ell na + \delta_0 \ell nH - \sigma^2/2$. Note that the switch only occurs in the constant term of equations (15) and (16). This term is $\alpha_0 - \ell n(1-\varepsilon_2)$ and $\alpha_0 - \ell n(1-H\varepsilon_2)$, respectively. Otherwise, equations (15) and (16) are identical.

The error structure is given by the covariance matrix

$$\Sigma = \begin{bmatrix} \operatorname{var}(\ell n\theta_t) & \operatorname{cov}(\ell n\theta_t + \ell nV_t, \ell n\theta_t) \\ \operatorname{cov}(\ell n\theta_t, \ell n\theta_t + \ell nV_t) & \operatorname{var}(\ell n\theta_t + \ell nV_t) \end{bmatrix}$$

$$= \begin{bmatrix} \sigma^2 & \sigma^2 + \omega \\ \sigma^2 + \omega & \sigma^2 + 2\omega + \tau^2 \end{bmatrix}$$

Then

$$|\Sigma| = \sigma^2\tau^2 - \omega^2 .$$

Equations (15) and (16) can be combined to yield

(17) $$\ell n\hat{p}_t = \gamma_0 + \gamma_1\lambda_t + \alpha_1 \ell n q_t + S_0 \ell n X_t + \ell n\theta_t + \ell n V_t$$

where $\gamma_0 = \alpha_0 - \ell n(1-H\varepsilon_2)$ and $\gamma_1 = \ell n(1-\varepsilon_2) - \ell n(1-H\varepsilon_2)$. Since $\varepsilon_2 < 1$ and $H \leq 1$, $\gamma_1 \geq 0$. This merely reflects the fact that expected price will be higher in collusive regimes, and so price is an increasing function of λ_t, the probability of observing collusive behavior in period t.

We can then summarize equations (14) and (17) by writing

(18) $$By_t = \Gamma Z_t + \Delta\lambda_t + U_t$$

where $y_t' = [\ell n\hat{p}_t, \ell n X_t]$, $Z_t' = [1, \ell n I_t, \ell n q_t]$, $U_t' = [\ell n\theta_t + \ell n V_t]$; and where

$$\Gamma = \begin{bmatrix} \varepsilon_o & \varepsilon_1 & 0 \\ \gamma_o & 0 & \alpha_1 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0 \\ \gamma_1 \end{bmatrix}, \text{ and}$$

$$B = \begin{bmatrix} 1 & \varepsilon_2 \\ 1 & -\delta_o \end{bmatrix}$$

Here U_t is identically and independently distributed $N(0,\Sigma)$.

The probability density function of y_t, given λ_t, is then

$$(19)\quad h(y_t|\lambda_t) = (2\pi)^{-1}|\Sigma|^{-1/2}\|B\|\exp\{-\frac{1}{2}(By_t - \Gamma Z_t - \Delta\lambda_t)'\Sigma^{-1}(By_t - \Gamma Z_t - \Delta\lambda_t)\},$$

and the likelihood function, if there are M observations, is

$$L(\lambda_1,\ldots,\lambda_n) = \prod_{t=1}^{M} h(y_t|\lambda_t)$$

The parameters to be estimated are (i) γ_0, γ_1, γ_1, and δ_0 from the cost function, (ii) ε_0, ε_1 and ε_2 from the demand function, (iii) σ^2, τ^2, and ω from the error structure, and (iv) the regime probability λ_t for each of the M time periods. Thus there are a total of 10 + M parameters to be estimated.

If the sample separation were known, we could estimate by maximum likelihood methods from the two separate data sets, corresponding to $w_t = 1$ and $w_t = 0$ (i.e., $\lambda_t = 1$ and $\lambda_t = 0$, respectively), given the restrictions on parameter values common to both regimes. In this case our concerns would be solely those of identification, requiring that the number of observations in each regime exceed the number of parameters to be estimated. The system will then be identified, by national income in the demand equation, and by the factor price in the pseudo-supply equations.

Since the sample separation is unknown, we must resort to other techniques. Given initial estimates of the regime probabilities, $(\lambda_1^0,\ldots,\lambda_M^0) = \lambda^0$ say, we can obtain initial estimates of Σ, B, Γ and Δ which minimize $L(\lambda^0)$. Denote these estimates of $[\Sigma,B,\Gamma,\Delta]$ by Ω^0.

Kiefer [4] has proposed an E-M algorithm which will provide consistent estimates of $\lambda_1,\ldots,\lambda_M$ and $[\Sigma,B,\Gamma,\Delta] = \Omega$, where the regime probabilities are allowed to vary over the sample period, but ignoring the endogenous nature of their movements. The probability that during a particular period, say t, the behavior of firms is collusive, given λ^0 and Ω^0, is simply

$$\text{(20)} \qquad \lambda_t^1 = \Pr(w_t = 1 | y_t, Z_t, \Omega^0, \lambda^0)$$

$$= \frac{\lambda^0 h(y_t | Z_t, \Omega^0, \lambda_t = 1)}{\lambda^0 h(y_t | Z_t, \Omega^0, \lambda_t = 1) + (1-\lambda^0) h(y_t | Z_t, \Omega^0, \lambda_t = 0)} .$$

With the assumption that the covariance matrix Σ is constant across regimes, this expression is linear logistic in y_t, as is demonstrated in Appendix C. Thus λ_t^1 will lie in the interval [0,1].

Given the estimates $(\lambda_1^1,\ldots,\lambda_M^1)$, new estimates of $[\Sigma,B,\Gamma,\Delta]$, say Ω^1, can be obtained by maximizing

$$L^1 = \prod_{t=1}^{M} h(y_t | Z_t, \Omega, \lambda_t^1)$$

with respect to Ω. This iterative procedure should be continued until convergence occurs, say at $(\hat{\lambda}_1,\ldots,\hat{\lambda}_M)$ and $\hat{\Omega}$.

The Kiefer iterative procedure is computationally simple, and yet the estimates $\hat{\Omega}$ convey whether or not the regimes seem to involve significantly different behavior. In particular, γ_1 is a direct measure of the ratio of the logarithm of prices across regimes. More importantly, the estimated regime probabilities indicate whether the hypothesized behavior of prices and output is correct; that is, whether these probabilities jump discretely from high to low values in some periods, and vice versa, after a "suitable" length of time. The regime probabilities also give an indication of when the industry may have been behaving monopolistically during the sample period.

The probability of observing collusive behavior in period t, λ_t, will satisfy

$$\begin{aligned}\lambda_t &= \Pr(w_t = 1)\\ &= \Pr(w_t = 1 | w_{t-1}=1) \Pr(w_{t-1}=1) + \Pr(w_t = 1 | w_{t-1}=0) \Pr(w_{t-1}=0)\end{aligned}$$

But

$$\Pr(w_t = 1 | w_{t-1} = 0) \Pr(w_{t-1} = 0) = \Pr(w_t = 1, w_{t-1} = 0).$$

Under the null hypothesis, a Nash equilibrium with reversionary episodes consisting of Cournot behavior for T-1 periods is observed. Then $w_t = 1$ and $w_{t-1} = 0$ if and only if $w_{t-T+1} = 0$ and $w_{t-T} = 1$. Hence

$$\begin{aligned}\Pr(w_t = 1, w_{t-1} = 0) &= \Pr(w_{t-T+1} = 0, w_{t-T} = 1)\\ &= \Pr(w_{t-T+1} = 0 | w_{t-T} = 1) \Pr(w_{t-T} = 1)\end{aligned}$$

and so

$$(21) \qquad \lambda_t = \Pr(w_t = 1 \mid w_{t-1} = 1)\lambda_{t-1} + \Pr(w_{t-T+1} = 0 \mid w_{t-T} = 1)\lambda_{t-T}$$

$$= [1 - F(\bar{p}/p_{t-1})]\lambda_{t-1} + F(\bar{p}/p_{t-T})\lambda_{t-T}.$$

Recall that, with the assumption of multiplicative demand uncertainty, $F(p/\bar{p}_t) = \Pr(\theta_t p_t < \bar{p})$, the probability that the market price, $\theta_t p_t$, is less than the trigger price, $\bar{p}$. In this case the firms will revert to Cournot behavior in the next period, $t+1$.

Furthermore, the probability of observing Cournot behavior in period t is

$$(22) \qquad 1 - \lambda_t = \Pr(w_t = 0)$$

$$= \sum_{j=1}^{T-1} \Pr(w_{t-j+1} = 0 \mid w_{t-j} = 1) \Pr(w_{t-j} = 1)$$

$$= \sum_{j=1}^{T-1} F(\bar{p}/p_{t-j})\lambda_{t-j} .$$

The system (21,22) describes the movements of regime probabilities over time. That equation (22) follows from (21) can be confirmed by noting that

$$\lambda_t - \lambda_{t-1} = (1 - \lambda_{t-1}) - (1 - \lambda_t)$$

$$= -F(\bar{p}/p_{t-1})\lambda_{t-1} + F(\bar{p}/p_{t-T})\lambda_{t-T}$$

$$= [\lambda_{t-1} - F(\bar{p}/p_{t-1})\lambda_{t-1} + F(\bar{p}/p_{t-T})\lambda_{t-T}] - \lambda_{t-1}$$

$$= \lambda_t - \lambda_{t-1},$$

as required. The second equality follows from equation (21) and the fourth from (22).

Given the estimates $\hat{\Omega}$ and $\hat{\lambda}$, and given the fact that F is the cumulative distribution function of a lognormal variable, estimates of $\bar{p}$ and T can be obtained via nonlinear methods. If T is taken as given, then an estimate of $\bar{p}$ can be obtained by minimizing the sum of squared deviations in equation (21), which can be rewritten as

$$(23) \qquad (\hat{\lambda}_t - \hat{\lambda}_{t-1}) = F(\bar{p}/p_{t-T})\hat{\lambda}_{t-T} - F(\bar{p}/p_{t-1})\hat{\lambda}_{t-1} \; .$$

Note that here p_{t-1} and p_{t-T} are the expected prices in the collusive regime, and thus should be calculated from equation (17) by setting $\lambda_{t-1} = \lambda_{t-T} = 1$, and employing the appropriate elements of the maximum likelihood estimates of B and Γ. An estimate of T can be obtained by allowing it to vary through some range of feasible integer values, say from 2 to $\bar{T}$, obtaining an estimate of $\bar{p}$ for each value of T, and then choosing that value at which some criterion function (such as average squared derivation from $(\hat{\lambda}_{\bar{T}+1},\ldots,\hat{\lambda}_M)$) is greatest.

Once estimation is completed, several types of tests should be carried out. From $(\hat{\lambda}_1,\ldots,\hat{\lambda}_M)$ we will have estimated the probability of any particular observation belonging to either regime. Their average is an estimate of the proportion of the total sample where collusion has occurred. From the estimated value of γ_1 and its standard error we can test whether or not the two regimes imply significantly different prices for the customers of the industry. At a more basic level, likelihood ratio tests can be used to determine whether or not structural change has in fact occurred, the natural alternative hypotheses being that we only observe either collusive or Cournot behavior, but not both.

We now turn to a discussion of data requirements.

Most importantly, this test requires time series data on some aggregated choice variable (e.g. total quantity X), and some monitored variable (e.g. price p), which firms may use to coordinate their conduct. It is mandatory that these series be long enough to encompass several occasions when reversionary behavior may have occurred, and frequent enough so that we can assume that firms did not have access to much better data during the period under study. This would seem to imply a need for monthly data, preferably (certainly no less frequent than quarterly).

It is important to note that firm specific data is not required for this test procedure. For reserarchers who do not have access to restricted data sets, actual estimation is therefore much more feasible than it would be for a test requiring disaggregate data.

Further variables which would be useful as indicators of the regime and changes between regimes are capacity, inventories, and some profit measure.

Because of the specific functional forms employed in section 5, any industry study following the test procedure should concentrate on an industry which seems to be characterized by fairly stable market shares. Clearly, large scale entry should not occur during the sample period, as the theoretical model has ruled out such a possibility.

6. Conclusion

Stigler and his followers have identified features of an industry which they believe would facilitate enforcement of a cartel agreement (e.g., that the firms produce a standardized product), and have carried out cross-industry regression studies to show that industries which possess these features tend to be more profitable. Skeptics have argued on a variety of grounds that these studies are untrustworthy or inconclusive, and have offered a few alternative cross-industry regression studies of their own.

This particular kind of econometric evidence has not been conclusive for either side in the debate. Cartel theorists can point to documentary evidence of explicit attempts to collude in some particular industries, and perhaps they can exhibit statistical evidence that profits tend to be higher in industries where collusion might be suspected, but they cannot tie these two kinds of evidence together to prove definitely that firms in any particular industry have succeeded in raising joint profits through non-competitive behavior. Skeptics can deny the relevance of documentary evidence and can impugn the validity of the statistical evidence brought forward by the cartel theorists, but they have not succeeded in showing decisively that the maintenance of a stable cartel is generally infeasible.

In particular, cross-industry evidence does not address a crucial issue between the two schools, which is whether or not the limited ability of firms to acquire information about competitor's activities precludes successful enforcement of a cartel agreement.

This paper demonstrates that the enforcement of a cartel agreement has empirically falsifiable consequences, and in so doing attempts to provide a test which can reconcile the conflicts mentioned above.

APPENDIX A

In this appendix we outline an industry which may have witnessed periods when its conduct is best described by the model of section 3.

Our example is the late nineteenth century Joint Executive Committee railroad cartel, which has been studied by MacAvoy [6] and Ulen [12,13]. Consider the following excerpt from Ulen [13].

> The JEC was the successor to a series of informal rate-setting agreements among freight agents of the trunk lines. As the received theory of cartels would predict, these informal arrangements were not successful, being repeatedly broken by secret cheating. The organization which appeared in June, 1879, was a different matter. The colluders had had experience with unsuccessful cartels, knew the shortcomings, and deliberately set out to erect a cartel which would bring them higher profits with more certainty. The key to their organization was the provision of internal enforcement mechanisms, that is, mechanisms which would deter cheating in much the same way that a legal prohibition on cheating (an external enforcement mechanism) might have done. . . .
>
> The effect on cartel stability of these elaborate internal enforcement devices was remarkable. In the

> seven years before the passage of the Interstate Commerce Act in 1887, the cartelists were successful in maintaining near-monopoly rates in more than three-quarters of the 328 weeks surveyed. Cheating and a breakdown of the cartel agreement occurred in fewer than one-quarter of the weeks surveyed. Moreover, cartel success fluctuated with the demand for the transport services of the cartel. Cheating occurred when business was tapering off. . . . Success in jointly monopolizing profits was most likely when demand was expanding. (Ulen [12, p. 308]).

Ulen's description of the operation of the JEC is consistent with the existence of a collusive industry equilibrium such as we have characterized in section 3. In particular, his suggestion that "cheating occurred when demand was tapering off . . ." corresponds to our hypothesis that demand shocks trigger reversionary episodes. Whether the JEC participants explicitly used price as a monitoring variable, or whether they used some imperfect market-share estimator or other monitoring variable prone to error because of demand shocks, the trigger-price strategies we have described are likely to account well for industry performances.

Of the requirements set forth in section 4 for trigger-strategy equilibrium to be plausible, only two raise any problems. Those two, product homogeneity and the restriction of decision variables to quantity-setting, are now discussed.

The principal product which the JEC sold was transportation of wheat from the midwest to various East Coast ports from which it was exported. To the extent that those ports were competitive as ocean-freight terminals, the JEC can be accurately modeled as a cartel controlling a single route. Wheat is not easily perishable, so speed of delivery was probably not an urgent consideration which could have enabled the JEC members to differentiate their products. Thus, the assumption that a homogeneous good was sold seems to have been approximately satisfied.

Price, rather than quantity, has typically been thought to be the strategic variable of firms in the rail-freight industry. In particular, the specification of industry conduct during reversionary episodes as being Cournot might be considered unrealistic. However, price-setting equilibrium has generally been thought plausible on the basis that the rail-freight industry has decreasing long-run average costs. This argument should not be applied to industry conduct during reversionary episodes, which are short-term disruptions of the sort of monopolistic conduct which decreasing LRAC might be expected to engender. It is true that the JEC members set tariffs, but these were often above the prices they actually charged. Thus the hypothesis that the members behaved as quantity setters is at least reasonable, although not evident. In summary, the JEC appears to be an appropriate candidate for the econometric study described in section 6.

APPENDIX B

In this appendix it is shown that the market share for each firm is not a function of the exogenous variables, and is constant across regimes. We deal with each regime in turn.

(1) In the collusive regime, equation (10) is observed, and so

$$p^e(1-\varepsilon_2) = a_i A\delta x_i^{\delta-1} \qquad \text{for } i=1,\ldots,n . \tag{B.1}$$

Then, for each firm i, output follows

$$x_i = a_i^{1/(1-\delta)}[p^e(1-\varepsilon_2)/A\delta]^{1/(\delta-1)} \tag{B.2}$$

and so

$$X = D[p^e(1-\varepsilon_2)/A\delta]^{1/(\delta-1)} \tag{B.3}$$

where $D = \sum_{i=1}^{N} a_i^{1/1-\delta}$. Then the market share of firm i, obtained by dividing (B.2) by (B.3), will be

$$S_i = a_i^{1/1-\delta}/D . \tag{B.4}$$

Solving (B.3) for total output X yields

$$X = [D^{\delta-1} dI^{\varepsilon_1} \mu(1-\varepsilon_2)/A\delta]^{1/\varepsilon_2+\delta-1} \tag{B.5}$$

where $\mu = E(\theta)$.

(2) In the Cournot regime, equation (11) is observed, and so

$$p^e(1-x_i\varepsilon_2/X) = a_i A x_i^{\delta-1} \qquad \text{for } i=1,\ldots,N . \tag{B.6}$$

The solution to (B.6) is of the form $x_i = k^{1/\delta-1} a_i^{1/1-\delta}$. We must then demonstrate that k is common to all firms. If this is indeed the case,

$$X = Dk^{1/\delta-1},$$

where D is defined as above, and then the share of firm i is again given by equation (B.4). But then, for each firm i,

$$p^e(1-a_i^{1/1-\delta}\varepsilon_2/D) = a_i A\delta(k/a_i) . \tag{B.7}$$

Summing equation (B.7) over all i gives

$$p^e(N-\varepsilon_2) = NA\delta k . \tag{B.8}$$

But then

$$k = p^e(1-\varepsilon_2/N)/A\delta , \tag{B.9}$$

which is common to all firms, as required. Note that equation (B.9) implies that total output is given by

$$X = [D^{\delta-1} dI^{\varepsilon_1} \mu(1-\varepsilon_2/N)/A\delta]^{1/\varepsilon_2+\delta-1} . \tag{B.10}$$

Equation (B.10) contrasts with (B.3) in a natural way. In both regimes total output will vary directly with national income, I, and the expected value of θ, μ, and inversely with factor price (of which A is an increasing function). Also, Cournot output exceeds collusive output by a factor of $[(1-\varepsilon_2/N)/(1-\varepsilon_2)]^{1/\varepsilon_2+\delta-1}$.

APPENDIX C

In this appendix we demonstrate that the conditional probability λ_t^1 of $w_t = 1$, given y_t, Z_t, Ω^o and λ^o is linear logistic in y_t.

From equation (22),

$$\text{(C.1)} \qquad \lambda_t^1 = 1/\{1 + [(1-\lambda^o)/\lambda^o](h_0^o/h_1^o)\}$$

where $h_\lambda^o = h(y_t | Z_t, \Omega^o, \lambda_t = \lambda)$. Then

$$h_0^o/h_1^o = \exp[(-1/2)\{(By_t - \Gamma Z_t)'\Sigma^{-1}(By_t - \Gamma Z_t) - (By_t - \Gamma Z_t - \Delta)'\Sigma^{-1}(By_t - \Gamma Z_t - \Delta)\}]$$

$$= \exp[(1/2)\Delta'\Sigma^{-1}\Delta - \Delta'\Sigma^{-1}(By_t - \Gamma Z_t)]$$

$$= \exp\{C + Dy_t\} ,$$

where $C = (1/2)\Delta'\Sigma^{-1}(\Delta + 2\Gamma Z_t)$ and $D = \Delta'\Sigma^{-1}B$.

Then we have

$$\lambda_t^1 = 1/\{1 + \exp\{\ell n[(1-\lambda^o)/\lambda^o] + C + Dy_t\}\} ,$$

which is linear logistic in y_t, as required.

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