INFORMATION AND INCENTIVES IN LABOR-MANAGED ECONOMIES

by

William Thomson

Discussion Paper No. 80-129, January 1980

Center for Economic Research
Department of Economics
University of Minnesota
Minneapolis, Minnesota 55455

Information and Incentives in Labor-Managed Economies

by

William Thomson

January 1980

1. Introduction

Decentralization of production decisions and egalitarian distributions of income at the level of each firm are usually taken as the defining features of labor-managed economies. This economic structure has been shown in several important studies (Ward [34], Vanek [30], [31], Meade [21]) to result in an inefficient allocation of labor and a number of proposals have been made to correct this problem without destroying the innovative aspects of such economies. The latest such proposal is due to Ireland and Law [17], and it is the purpose of this introduction to present it, to point out a certain number of weaknesses it has, the main body of the paper being devoted to remedying them.

The framework of analysis is the following: firm i $(i = 1,\dots,n)$ hires x_i workers, and, after payment of the services of the fixed factors, receives a net amount $f_i(x_i)$ from the sale of its output. Each firm may also receive an additional (negative or positive) transfer y_i from some government agency (the "incentive fund" of Ireland and Law); its total income is therefore $f_i(x_i) + y_i$, an amount which is distributed equally among its workers. Each worker employed in firm i receives $(f_i(x_i) + y_i)/x_i$ and it is this quantity that firm i will attempt to maximize.

I would like to thank J.-J. Laffont and E. Maskin for helpful comments. Typing assistance from NSF grant SOC 78-25734 is also gratefully acknowledged.

If y_i is zero, such maximization leads the firms to choose levels of employment which are not compatible with overall efficiency; however, Ireland and Law (IL) showed that a judicious choice of y_i could restore efficiency. Specifically, they propose to set y_i equal to $(f_i(x_i) - wx_i)(x_i - \bar{x}_i)/\bar{x}_i$ where $\bar{x}_i$ is the initial amount of labor employed by firm i, and w is a (real number) parameter announced by the center. (The IL side payment to firm i will be denoted y_i^*.) Firm i maximizes the expression $(f_i(x_i) + y_i^*)/x_i$ in x_i, taking w as given, exercise which yields its demand for labor as a function of w, and the center adjusts w so as to achieve equality between the aggregate demand for labor, and the aggregate supply of labor, taken to be a fixed number. IL prove that their choice of y_i^* induces each firm to hire labor until its marginal productivity is equal to w, which ensures an efficient allocation of labor at the equilibrium w.

First, it should be observed that the transfer to firm i involves the term $f_i(x_i)$, and its computation requires that the output of the firm be observable. While this may often be the case in practice, the administrative cost of such a procedure may be prohibitive when firms are numerous and joint production is allowed for. Although the IL scheme mimicks the price mechanism in several important respects, it therefore does not have the informational efficiency of that mechanism. An important question is then whether one can dispense with observability of output in the construction of incentive schemes selecting efficient outcomes.

Next, we are concerned with the behavioral assumptions implicitly made by the authors. In their scheme, each firm is supposed to maximize its utility, taking w as a given parameter. Competitive behavior of this kind may not always be justified. If the number of firms involved is small, if there exist geographical or sociological obstacles to mobility of labor, or if the labor force is divided into pools of workers with different skills, each firm may have a significant impact on the wage rate relevant to its decisions, and strategic consideration may enter in an important way. It is therefore useful to study the construction of schemes leading to efficiency under alternative behavioral assumptions.

Finally, note that in the IL scheme, if the transfers taking place between the center and any given firm may be of either sign, nothing guarantees that their aggregate amount will be zero. This means that the incentive fund may run a deficit or a surplus, implying that deficits are covered from outside sources and that surpluses are allocated to other programs or communities in such a way as not to affect the incentives of the firms involved in the scheme. In a closed system this cannot be done, and the question arises as to the existence of schemes that would satisfy the general equilibrium condition that the budget of the center be always balanced.

The present paper is devoted to answering the questions raised in the last three paragraphs. In Section 2, the framework of analysis is presented. In Section 3 and 4, it is assumed that the center cannot observe output while this assumption is not made in Section 5. In each of these Sections, the existence of satisfactory schemes is examined under several behavioral assumptions. Two classes of schemes are studied, the traditional class of "games" (Section 3); and what is termed here the class of "auctioneer mechanisms" (Section 4): an auctioneer mechanism (the price mechanism, and the IL mechanism are two examples) involves a central agent, the auctioneer, who neither consumes nor produces and whose role is to help in the coordination of economic activities.

The results presented here are mainly negative if observability is not assumed, and positive otherwise. They turn out to be largely independent of the kind of schemes, or of the behavioral assumption under examination.

2. Description of the Framework of Analysis

2.1 Environment

A class of economies sharing particular features will be called an environment. The possibility of constructing mechanisms with certain desirable properties depends in general on the environment over which they are designed to operate. First, we describe the environment with which most of the literature on labor-managed economies (including Ireland and Law) has been concerned:

There are N workers and n firms. Each worker supplies one unit of labor to whichever firm employs him. The firms are indexed by the subscript i, with $i = 1,\dots,n$. Firm i, denoted F_i, is characterized by its production function $f_i\colon R_+ \to R_+$, where labor is the only input. The output can be thought of as being expressed in terms of some unit of account, or evaluated at some fixed prices. Any input other than labor will be held fixed throughout and $f_i(x_i)$ should be considered as the "value" of output after the cost of the inputs other than labor have been substracted. The set of admissible production functions for F_i is denoted $\mathfrak{F}_i$. An environment E is given by a list $\mathfrak{F}_i$, with $i = 1, \ldots, n$. We will write $E = \mathfrak{F}_1 \times \cdots \times \mathfrak{F}_n$. An economy e in E is a list $(f_1, \ldots, f_n)$ in $\mathfrak{F}_1 \times \cdots \times \mathfrak{F}_n$.

Firm i employs x_i units of labor (where, for analytical simplicity, x_i is taken to be a non-negative real number; this means that a worker may be employed part-time by several firms), produces $f_i(x_i)$ units of output and receives a negative or positive transfer payment y_i from the government or center, expressed in the same common unit of account as the various outputs. The y_i will be called monetary transfers for lack of a better term. The total income of F_i is therefore $f_i(x_i) + y_i$. It is assumed that this income is split equally among the workers so that if firm i operates at all, i.e., if x_i is positive, each of its workers will receive the amount $\dfrac{(f_i(x_i) + y_i)}{x_i}$.

We will distinguish two cases, depending as to whether the aggregate transfer Σy_i from the center is zero or not. In partial equilibrium analysis, if the N workers constitute but a small fraction of the total labor force, surpluses may be allocated to other communities and deficits may be covered from outside funds. Then it is legitimate not to impose the condition that $\Sigma y_i = 0$. On the other hand, such a condition is clearly necessary if the present model is seen as a description of a whole economy.

We will always demand full employment of labor, an efficiency requirement if the production functions are non-decreasing in labor input. The set $L = \{x \in R^n_+ | \Sigma x_i = N\}$ represents all the possible allocation of labor among the n firms. An allocation of labor x in L is efficient for the economy $e = (f_1, \ldots, f_n)$ iff

$$\Sigma f_i(x_i) \geq \Sigma f_i(x_i') \quad \text{for all } x' \text{ in } L \, .$$

The vector of monetary transfers may belong to $Y = R^n$ or $Y_0 = \{y \in Y | \Sigma y_i = 0\}$. This permits us to define two sets of feasible allocations:

$$A = L \times Y \quad \text{or} \quad B = L \times Y_0 \, .$$

Given an economy e, two sets of optimal allocations are correspondingly defined:

$$P_A(e) = \{(x,y) \in A | x \text{ is efficient for } e\} \, ,$$

$$P_B(e) = \{(x,y) \in B | x \text{ is efficient for } e\} \, .$$

In keeping with most of the literature on LM economies, we will think of the firms (and not of the workers) as the relevant economic agents. It is, however, easy to find "well-behaved" environments containing economies whose optimal allocations of labor have some coordinates equal to zero, and to deal with such environments, one should adopt a formulation where the number of firms is endogeneously determined.

We will not attempt such a formulation. Instead our model will be of an economy in the short run, where the existence of all firms is economically justified, although some reallocation of labor among them may be needed for purposes of efficiency. If efficiency considerations require that a firm's labor force declines, it is assumed that this can be done and is done at the expense of the junior workers, say, without the management or the senior workers being affected. The latter are, however, constrained to distribute income equally among all those employed (or to distribute income according to some fixed proportions among the various workers' classifications, if such exist; analytically, this would have the same consequences, while it would make the model less tractable). What we take here to be the firm's objective function derives from the common objective of this "core" management workers with a secure employment.

It should be noted, however, that a number of writers have recently questioned this formulation of the objective function of the firms. Furubotn [9] has been mainly concerned with a formulation that would take better account of the long-term effect (through shifts in political power) of hiring decisions. Miyazaki and Neary [23], Ichiichi [16], Greenberg [11], Bennett and Wooders [2] have focussed on the problem of firm formation, letting the self-interest of the individual workers decide which firms would exist in equilibrium. In our context, such a description may not be appropriate, since in these models, workers make decisions with perfect information about the production possibilities of the potential firms. If this information were available to all the workers, there would be no reason to assume that the center would not be able to acquire it. By opposition, it is natural to assume that the core management-workers of each firm has privileged information about it and makes hiring decisions accordingly.

Consequently, and to summarize this discussion, we will restrict our attention to a class of economies for which efficiency always requires that all firms be active. Formally, we set

$$L^+ = \{x \in L | x_i > 0 \quad i = 1,\dots,n\}$$

and define E^+ as the subset of E of economies e such that if (x,y) is in $P_A(e)$ (or $P_B(e)$), then x is in L^+. We correspondingly define $A^+ = L^+ \times Y$ and $B^+ = L^+ \times Y_0$.

Given e in E^+, the preference relation $\succsim_i$ of F_i over A^+ (or B^+) is defined as follows: for (x,y), (x',y') in A^+ (or B^+),

$$(x,y) \succsim_i (x',y') \quad \text{if} \quad \frac{f_i(x_i) + y_i}{x_i} \geq \frac{f_i(x_i') + y_i'}{x_i'} .$$

We will sometimes find it convenient to work with a still more restricted environment, denoted E_F and defined as follows:

There are $N = n + 1$ workers and n firms. Labor can be allocated in discrete units only, and each firm can employ either 1 or 2 workers. The set of feasible allocations of labor is $L_F = \{(x_i = 2 \text{ and } x_j = 1,$ for $j \neq 1) \quad i = 1,\dots,n\}$. Firm i's production function is non-decreasing and can be characterized by the pair $(f_i(1),f_i(2)) \in D_i = [(x,y) \in R_+^2 | y \geq x\}$

that for simplicity we will denote (a_{i1}, a_{i2}) . Finally, we denote $A_F = L_F \times Y$, and $B_F = L_F \times Y_0$.

As far as efficiency is concerned, it is the firm with the greatest difference $a_{i2} - a_{i1}$ who should get a second worker. In case of ties, it is of matter of indifference which of those gets the second worker. Therefore, for each e in E_F , the optimal allocations of e are given by:

$$P_A(e) = \{(x,y) \in A_F | x_i = 2 \text{ for one } i \text{ such that } a_{i2} - a_{i1} \geq \max_{j \neq i}(a_{j2} - a_{j1})\} .$$

Similarly

$$P_B(e) = \{(x,y) \in P_A(e) | \Sigma y_i = 0\} .$$

A <u>performance correspondence</u> Φ: $E \to A$ (or B) specifies a set of socially desired alternatives for each economy e in its domain of definition E . P_A and P_B are two examples.

2.2 Games

If the basic data of each economy e , here the production functions, were known to the center, it would simply compute the desired allocations of e , allocate labor inputs and carry out the corresponding transfer payments. Unfortunately, this knowledge is not in general available, and the center has to rely on decentralized mechanisms. An important class of such mechanisms is the class of games, on which most of this paper will focus.

A <u>game</u> Γ is a pair (M,ℓ) , with $M = M_1 \times \ldots \times M_n$ and M_i designates the i-th agent's strategy space, and ℓ: $M \to A,B$ is the outcome function. Given $m \in M$, $\ell(m)$ is an element of A or B of the form $(g_1(m),\ldots,g_n(m); h_1(m),\ldots,h_n(m))$.

Behavioral assumptions have to be made to describe the kind of maximization exercise that each agent will perform in order to select his message. In what follows, we will examine several such assumptions.

An equilibrium list of messages $m^* \in M$ is such that each agent's message is optimal for the behavioral rule he follows and given the messages of the other agents.

An equilibrium allocation $z^* \in A$ or B is such that there exists an equilibrium list of messages m^* with $z^* = \ell(m^*)$.

A game Γ implements a performance correspondence Φ on E if for every e in E

(i) Γ yields equilibrium allocations

(ii) Every equilibrium allocation of Γ belongs to $\Phi(e)$.

In what follows we are concerned with the existence of games implementing the optimality correspondences P_A and P_B . We will not demand full implementation, i.e., that set of equilibrium allocations of Γ for e coincide with $P_A(e)$ or $P_B(e)$ for all e in E .

We will sometimes deal with direct games, in which the strategy spaces are the spaces of unknown characteristics themselves, i.e., $M_i = \mathfrak{F}_i$ for all i . It is for such games that it makes sense to speak of the truthfulness of an agent.

If output is not observable, we will conclude that there exist no games implementing the optimality correspondences. The proofs will often involve E_F, and it is important to note that working on this restricted environment in fact strengthens any impossibility result. In Section 5 where possibility results are proved, the wider environment E^+ is considered.

For purposes of comparison, we will refer to the situation when each firm is concerned with total revenue instead of revenue per capita. This is the kind of payoff functions that has recently been the object of a considerable amount of work mainly in the context of public goods. The methodology of that literature is used throughout the paper as will be evident to the reader familiar with it.

Notation: $m_{-i} = (m_1,\ldots,m_{i-1},m_{i+1},\ldots,m_n)$. $f_{-i}, x_{-i}, \ldots$, are similarly defined.

3. Non-Observable Output

3.1 Dominant Strategies

We investigate here the existence of games implementing P_A in dominant strategies. A dominant strategy for an agent is a best strategy independently of the strategies chosen by the other agents. Imposing the existence of dominant strategies for every economy is a strong requirement, which is somewhat weakened by not requiring financial feasibility.

Implementation of performance correspondences in dominant strategies has been studied in various contexts and under several assumptions by Clarke [3], Laffont and Maskin [18], Groves [12], Groves and Loeb [14], Ledyard [19], and Vickrey [32]. Green and Laffont [10] provide a complete characterization of the games eliciting true preferences as a dominant strategy when the budget of the center is not required to be balanced, and show the impossibility of balancing the budget. They also prove that nothing is gained by going beyond the class of direct games, a result which appears in Dasgupta, Hammond, and Maskin [5] in the form of the following lemma:

Lemma 1. If a game Γ defined over some environment S implements a performance correspondence Φ in dominant strategies, there exists a direct game Γ' such that:

(i) telling the truth is a dominant strategy for each agent;

(ii) the outcome function of Γ associates to the list of truthful announcements an element of $\Phi(e)$ for every e in S.

(See Dasgupta, Hammond, and Maskin [5], Theorem 4.1.1., p. 194.)

Lemma 2. There exists no direct game defined over E_F and satisfying (i) and (ii) of Lemma 1.

Proof: In a direct game Γ, a strategy for firm i is a pair $\alpha_i = (\alpha_{i1},\alpha_{i2})$ in D_i and the outcome function ℓ of Γ associates to each α in $D = D_1 \times \ldots \times D_n$ an element of A^+ or B^+ such that a firm with the greatest difference $\alpha_{i2} - \alpha_{i1}$ be allocated 2 workers. For efficiency, a firm with the greatest difference $a_{i2} - a_{i1}$ should be allocated 2 workers. Formally, given α in D

$$\ell(\alpha) = (g,h)(\alpha) = (g_1(\alpha),\ldots,g_n(\alpha);\ h_1(\alpha),\ldots,h_n(\alpha))$$

with $g_i(\alpha) = 2$ for one i such that $\alpha_{i2} - \alpha_{i1} \geq \max\limits_{j\neq 1} a_{j2} - a_{j1}$.

Let us focus on firm 1. The payoff to F_1 is equal to:

$$a_{11} + h_{11}(\alpha_1,\alpha_{-1}) \qquad \text{if} \quad \alpha_{12} - \alpha_{11} < \max_{i\neq 1}\ (\alpha_{i2} - \alpha_{i1})$$

$$\frac{a_{12} + h_{12}(\alpha_1,\alpha_{-1})}{2} \qquad \text{if} \quad \alpha_{12} - \alpha_{11} > \max_{i\neq 1}\ (\alpha_{i2} - \alpha_{i1}) \ .$$

If $\alpha_{12} - \alpha_{11} = \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1})$, at least two firms may get the second worker, without a violation of efficiency, but which one gets him need not be specified for the rest of this argument.

<u>Claim 1</u>: <u>For truth-telling to be a dominant strategy, it is necessary that, for all</u> $\alpha_{-1} \in D_{-1}$, h_{11} <u>be independent of</u> α_1 <u>for all</u> α_1 <u>such that</u> $\alpha_{12} - \alpha_{11} > \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1})$.

<u>Proof</u>: Suppose not. Then there exists $\alpha^o_{-1} \in D_{-1}$, $\bar{\alpha}_1, \bar{\bar{\alpha}}_1$ such that

$$\bar{\alpha}_{12} - \bar{\alpha}_{11} > \max_{i\neq 1}(\alpha^o_{i2} - \alpha^o_{i1}) \quad \text{and} \quad \bar{\bar{\alpha}}_{12} - \bar{\bar{\alpha}}_{11} > \max_{i\neq 1}(\alpha^o_{i2} - \alpha^o_{i1})$$

$$\text{with} \quad h_{11}(\bar{\alpha}_1,\alpha^o_{-1}) \neq h_{11}(\bar{\bar{\alpha}}_1,\alpha^o_{-1}) \ .$$

Assume that $h_{11}(\bar{\alpha}_1, \alpha^o_{-1}) > h_{11}(\bar{\bar{\alpha}}_1, \alpha^o_{-1})$ say. Then a firm with true parameter $\bar{\bar{\alpha}}_1$ will be strictly better off announcing $\bar{\alpha}_1$ instead of $\bar{\bar{\alpha}}_1$ if the other firms announce α^o_{-1}. Truthtelling is not a dominant strategy for F_1.

Similarly, it should be the case that for all $\alpha_{-1} \in D_{-1}$, h_{12} be independent of α_1 for all α_1 such that $\alpha_{12} - \alpha_{11} < \max\limits_{i \neq 1} \alpha_{i2} - \alpha_{i1}$.

The payoff to F_1 can then be rewritten as:

$$\begin{cases} a_{11} + h_{11}(\alpha_{-1}) & \text{if } \alpha_{12} - \alpha_{11} < \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1}) \\ \dfrac{a_{12} + h_{12}(\alpha_{-1})}{2} & \text{if } \alpha_{12} - \alpha_{11} > \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1}) \end{cases},$$

where the dependence of h_{11} and h_{12} on α_1 has been suppressed.

Note that for every $\alpha_{-1} \in D_{-1}$ with $\alpha_{-1} \neq 0$, there exist $\bar{\alpha}_1$ and $\bar{\bar{\alpha}}_1$ such that $\bar{\alpha}_{12} - \bar{\alpha}_{11} > \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1})$ and $\bar{\bar{\alpha}}_{12} - \bar{\bar{\alpha}}_{11} < \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1})$. This implies that h_{11} is defined over the whole space D_{-1}, and that h_{12} is defined for all $\alpha_{-1} \in D_{-1}$ with $\alpha_{-1} \neq 0$. Note also that adding to or substracting from both terms of F_1's payoff the same function of α_{-1} will leave its maximization problem unchanged. Let Note also that adding to or substracting from F_1's payoff the same function of α_{-1} will leave its maximization problem unchanged. Let us then add $-h_{11}(\alpha_{-1})$. Calling $k(\alpha_{-1}) = \dfrac{h_{12}(\alpha_{-1})}{2} - h_{11}(\alpha_{-1})$ gives the following expression for F_1's payoff:

$$\begin{cases} a_{11} & \text{if } \alpha_{12} - \alpha_{11} < \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1}) \\ \dfrac{a_{12}}{2} + k(\alpha_{-1}) & \text{if } \alpha_{12} - \alpha_{11} > \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1}) \end{cases} .$$

Claim 2: There exists an economy in E_F where telling the truth is not a dominant strategy for all agents.

Proof: Let $\alpha_{-1} \neq 0$ be given. There exists $\bar{\alpha}_1 \in D_1$ such that $\bar{a}_{12} - \bar{a}_{11} < \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1})$. There also exists $\bar{\bar{a}}_1$ such that $\bar{\bar{a}}_{12} - \bar{\bar{a}}_{11} > \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1})$ and $\bar{a}_{11} > \dfrac{\bar{\bar{a}}_{12}}{2} + k(\alpha_{-1})$. Indeed, given $\bar{a}_{12}$, this means that

$$\frac{\bar{\bar{a}}_{12}}{2} + k(\alpha_{-1}) < \bar{\bar{\alpha}}_{11} < \bar{\bar{\alpha}}_{12} + \max_{i\neq 1}(\alpha_{i2} - \alpha_{i1}) .$$

These two inequalities are consistent iff $\dfrac{\bar{\bar{a}}_{12}}{2} + k(\alpha_{-1}) < \bar{\bar{a}}_{12} + \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1})$, which requires that $\bar{\bar{a}}_{22} > 2[k(\alpha_{-1}) + \max\limits_{i\neq 1}(\alpha_{i2} - \alpha_{i1})]$. After selecting $\bar{a}_1$ and $\bar{\bar{a}}_1$ as just indicated, it is clear that a firm whose true parameter is $\bar{\bar{a}}_1$ is better off announcing $\bar{a}_1$ (since its payoff is then $\bar{\bar{a}}_{11}$) rather than the truth (which would yield the payoff $\dfrac{\bar{\bar{a}}_{12}}{2} + k(\alpha_{-1})$). This completes the proof of Lemma 2.

Proposition 1: There exists no game defined in E_F implementing P_A in dominant strategies.

Proof: It is a straightforward corollary of Lemmas 1 and 2.

(Green and Laffont show the impossibility of devising dominant strategy mechanisms for objective functions of the form $v(x_i, y_i) + y_i$; their theorem does not apply to the situation studied here, with objective functions of the form $v(x_i) + \varpi(x_i)y_i$. I am endebted to J.-J. Laffont for this observation.)

3.2 Nash Strategies

Given the negative results obtained with dominant strategies, a natural next step is to impose the weaker requirement that strategies constitute a (non-cooperative) Nash equilibrium. At an equilibrium, no agent can benefit from departing from his strategy if he believes that the other agents will not change theirs.

Implementation in Nash strategies of the Pareto-correspondence in pure-exchange economies has been shown to be feasible by Hurwicz [15] and Schmeidler [27], and similar conclusions for public good economies have been reached by Groves and Ledyard [13], Hurwicz [15], and Walker [33].

Maskin [20] has shown that for Nash implementation, much more can be accomplished with general games than with direct games, and has derived conditions under which a performance correspondence can be fully implemented.

Definition: A performance correspondence Φ defined on E is monotonic if $\forall$ $e,e' \in E$, $\forall$ $z \in A$, if $z \in \Phi(e)$ and $\forall$ $i \in \{1,\ldots,n\}$, $z \succsim_i z' \Rightarrow z \succsim_i' z'$, then $z \in \Phi(e')$.

Theorem 1. (Maskin) A performance correspondence can be fully implemented only if it is monotonic.

The proof consists in observing that if Φ can be fully implemented in Nash strategies by some game Γ , and if z belongs to $\Phi(e)$, then it is an equilibrium allocation of Γ for some list of strategies m^* . If preferences are changed in such a way that z does not fall in anybody's ordering, then m^* will still be an equilibrium allocation of Γ .

Since we are not concerned with full implementability, we cannot apply the theorem directly. However, the proof of Proposition 2 will rely on exactly the same kind of argument.

Proposition 2: There exists no game implementing P_A: $E \to A$ in Nash strategies.

Proof: Let F_i's production function be given by

$$f_i(x_i) = \begin{cases} \sqrt{x_i} & \text{for } x_i \le 1 \\ 1 + x_i/2 & \text{for } x_i > 1 \end{cases}$$

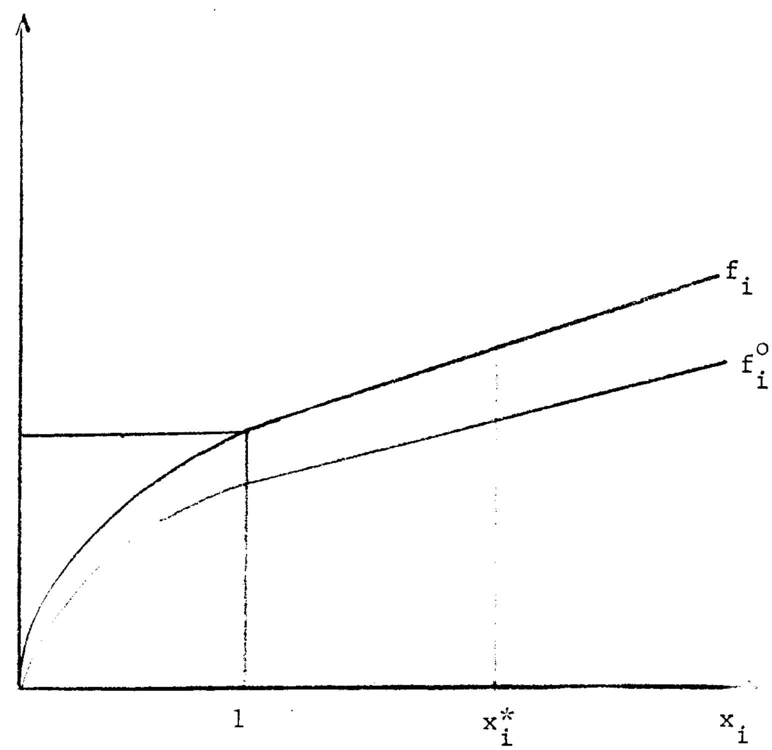

Figure 1

Let x_i^* be such that $1 < x_i^* < N$ (W.L.O.G. assume $N > 1$) and let $e = (f_1,\ldots,f_i,\ldots,f_n) \in E$ containing f_i and such that there exists a unique optimal allocation of labor among the firms x^* with an i-th component equal to x_i^*, and f_j, $j \neq i$, strictly concave, strictly increasing and differentiable.

Assume now that P_A is implementable in Nash strategies. This means that there exists a game $\Gamma = (M,\ell)$ such that e possesses a Nash equilibrium m^* to which is associated an allocation (x^*,y^*) in A.

The set of elements of A to which F_i prefers (x^*,y^*) is equal to

$$L_{f_i}(x^*,y^*) = \bigcup_{x_i \in]0,N]} \{(x,y) \in A \mid \frac{f_i(x_i) + y_i}{x_i} \leq \frac{f_i(x_i^*) + y_i^*}{x_i^*}\} .$$

Let f_i^o be defined by

$$f_i^o(x_i) = f_i(x_i) - x_i/4 .$$

If (x,y) in A is such that $(x^*,y^*) \succsim_i (x,y)$, it is because

$$\frac{f_i(x_i^*) + y_i^*}{x_i^*} \geq \frac{f_i(x_i) + y_i}{x_i} .$$

But then $(x^*,y^*) \succsim_i^o (x,y)$, since

$$\frac{f_i^o(x_i^*) + y_i^*}{x_i^*} = \frac{f_i(x_i^*) - x_i^*/4 + y_i^*}{x_i^*} = \frac{f_i(x_i^*) + y_i^*}{x_i^*} + \tfrac{1}{4} \geq \frac{f_i(x_i) + y_i}{x_i} + \tfrac{1}{4} =$$

$$= \frac{f_i(x_i) - x_i/4 + y_i}{x_i} = \frac{f_i^o(x_i) + y_i}{x_i} .$$

Let e^o be obtained from e by replacing f_i by f_i^o and leaving f_j unchanged for $j \neq i$. Then m^* remains a Nash equilibrium in e^o, since every allocation that each agent prefers to (x^*, y^*) in e^o was also preferred by that agent in e, and was not attainable given the messages of the other agents. However, it is easy to check that x^* is not an optimal allocation of labor in e^o since the slope of F_i's production function at x_i^* is not equal to the common slope of the F_j's production function at x_j^*.

<u>Remark 1</u>: In this proof, we have been concerned with implementing P_A. If we had imposed financial feasibility, i.e., if we had attempted to implement P_B, a negative result would a fortiori been obtained.

<u>Remark 2</u>: On somewhat more restricted environments, possibility results may be proved. Indeed, for the environment E_F, P_A and even P_B can be implemented in Nash strategies as follows from a theorem due to Maskin. To state this theorem, it is necessary to introduce an additional definition.

<u>Definition</u>: A performance correspondence Φ defined on E satisfies <u>no veto power</u> iff $\forall\, z \in A$, $\forall\, i \in \{1,\ldots,n\}$, if $\exists\, e$ such that $\forall\, j \neq i$, $\forall\, z' \in A$, $z \succsim_j z'$, then $z \in \Phi(e)$.

This means that if an alternative is best for $n - 1$ agents, the last agent cannot prevent it from being optimal. As pointed out by Maskin [20], this condition is often vacuously satisfied, and it is the case here. Indeed, the agents cannot agree on the distribution of money.

Theorem. (Maskin) If $n \geq 3$, and $\Phi: E \rightarrow$ is a performance correspondence that satisfies no veto power, then Φ can be fully implemented in Nash strategies if and only if Φ is monotonic.

Lemma 3: P_B can be implemented on E_F in Nash strategies.

To show that P_B is monotonic on E_F, we choose any e in E_F, any $z = (x,y)$ in $P_B(e)$, and we consider any e' in E_F such that for all $z' = (x',y')$ in $P_B(e)$, and for all i,

$$(1) \qquad z \succsim_i z' \Rightarrow z \succsim_i' z' .$$

Then, we want to show that $z' \in P_B(e)$. Without loss of generality, assume that $x_1 = 2$. Then $z \succsim_1 z'$ implies either $[x_1' = 2$ and $y_1 \geq y_1']$ or $[x_1' = 1$ and $a_{12} + y_1 \geq a_{11} + y_1']$. Also, $z \succsim_1' z'$ implies either $[x_1' = 2$ and $y_1 \geq y_1']$ or $[x_1' = 1$ and $a_{12}' + y_1 \geq a_{11}' + y_1']$; (1) is therefore satisfied for agent 1 only if

$$(2) \qquad a_{12}' - a_{11}' \geq a_{12} - a_{11} .$$

A similar reasonong for agent i, $i \neq 1$, would reveal that (1) can be satisfied only if

$$(3) \qquad a_{i2}' - a_{i1}' \leq a_{i2} - a_{i1} , \; i \neq 1 .$$

Since $z \in P_B(e)$, we have $a_{12} - a_{11} \geq \max_{i \neq 1}(a_{i2} - a_{i1})$. (2) and (3) reveal that $a_{12}' - a_{11}' \geq \max_{i \neq 1}(a_{i2}' - a_{i1})$ which in turn implies that $z \in P_B(e')$.

The argument would have been the same if we had assumed $x_i = 2$, $i \neq 1$. This proves the Lemma.

We conclude that P_B can be fully implemented on E_F in Nash strategies. Maskin [20] provides a method of constructing, for each implementable performance correspondence, a game that realizes this implementation. This method can be applied here.

Maximin Strategies

The maximin behavioral postulates embody a strong degree of risk aversion: each firm determines the worst outcome compatible with each of its strategies and selects the one yielding the best of these outcomes.

Implementation in maximin strategies has been studied by Drèze and de la Vallée-Poussin [6], Dubins [8], Green and Laffont [10], Thomson [29], where a characterization result is provided for the case of discrete alternatives, and Crawford [4].

Unfortunately, for the problem at hand, we obtain another impossibility.*

Proposition 3: There is no game implementing P_A on E_F in maximin strategies.

Proof: The proof is not restricted to implementation by direct games. Firm i's message space is denoted M_i . The outcome function can be described by saying that there exists a partition $(S_1,\ldots,S_n)$ of M such that if $m \in S_i$, then $x_i = 2$. Otherwise, $x_i = 1$. The payments to F_i associated with each occurence are denoted $h_{i2}(m)$ and $h_{i1}(m)$. The payoff to F_i is then given by

$$\begin{cases} a_{i1} + h_{i1}(m) & \text{if } m \notin S_i \\ \dfrac{a_{i2} + h_{i2}(m)}{2} & \text{if } m \in S_i \end{cases} .$$

*DHM [5] give sufficient conditions for a single-valued performance correspondence to be implementable in maximin strategies. See their Theorems 6.3 and 6.4.

For every message m_i in M_i , firm i computes the worst outcome that could happen, for each of the cases $m \in S_i$ and $m \notin S_i$, which involves the quantities of:

$$\inf\{a_{i1} + h_{i1}(m) \quad \text{for} \quad m_{-i} \quad \text{such that} \quad (m_i, m_{-i}) \notin S_i\}$$

$$\inf\{\frac{a_{i2} + h_{i2}(m)}{2} \quad \text{for} \quad m_{-i} \quad \text{such that} \quad (m_i, m_{-i}) \in S_i\} .$$

This requires that the functions

$$\tilde{h}_{i1}(m_i) = \inf\{h_{i1}(m) \quad \text{for} \quad m_{-i} \quad \text{such that} \quad (m_i, m_{-i}) \notin S_i\}$$

and

$$\tilde{h}_{i2}(m_i) = \inf\{h_{i2}(m) \quad \text{for} \quad m_{-i} \quad \text{such that} \quad (m_i, m_{-i}) \in S_i\}$$

be well defined ; the payoff expected by F_i can then be written as:

$$\min\{a_{i1} + \tilde{h}_{i1}(m_i) \ , \ \frac{a_{i2} + \tilde{h}_{i2}(m_i)}{2}\} .$$

An element m_i of M_i is a maximin strategy for F_i iff it maximizes this expression. Maximin strategies may not be unique, but the set of maximin strategies for F_i should be non-empty for all a_i ; this set will clearly depend on a_i only. Let us denote it by $X_i(a_i)$.

Maximin implementation of P_A requires that

$$\forall i, \forall a_i \in D_i, X_i(a_i) \neq \emptyset \text{ and } \forall m_i \in X_i(a_i), (m_i(a_i),$$

$$m_{-i}(a_{-i})) \in S_i$$

$$\text{only if} \quad a_{i2} - a_{i1} \geq \max_{j \neq 1}(a_{j2} - a_{j1}) .$$

Note that if for $\bar{a}_1, \bar{\bar{a}}_i \in D_i$, $\bar{a}_{i2} - \bar{a}_{i1} \neq \bar{\bar{a}}_{i2} - \bar{\bar{a}}_{i1}$, then $X_i(\bar{a}_i) \cap X_i(\bar{\bar{a}}_i) = \emptyset$. The reason is that one can find a^o_{-i} such that (assuming that $\bar{a}_{i2} - \bar{a}_{i1} > \bar{\bar{a}}_{i2} - \bar{\bar{a}}_{i1}$ say)

$$\bar{a}_{i2} - \bar{a}_{i1} > \max_{j \neq 1}(a^o_{j2} - a^o_{j1}) > \bar{\bar{a}}_{i2} - \bar{\bar{a}}_{i1} .$$

If then $X_i(\bar{a}_i) \cap X_i(\bar{\bar{a}}_i) \neq \emptyset$, for m^o_i in this intersection, and for m^o_j in $X_j(a^o_j)$, $j \neq i$, the list (m^o_i, m^o_{-i}) is a list of maximin strategies for the economies $(\bar{a}_i, a^o_j)$ and $(\bar{\bar{a}}_i, a^o_j)$, yield a unique outcome, which cannot be optimal for both economies since optimality in the first economy requires $x_i = 2$, while it requires $x_i = 1$ in the second economy.

Let now $a^o_i \in D_i$ be given, and let $m^o_i \in X_i(a^o_i)$. This means that

$$\min\{a^o_{i1} + \tilde{h}_{i1}(m^o_i), \frac{a^o_{i2} + \tilde{h}_{i2}(m^o_i)}{2}\} \geq \min\{a^o_i + \tilde{h}_{i1}(m_i), \frac{a^o_{i2} + h_{i2}(m_i)}{2}\}$$

$$\forall\, m_i \in M_i .$$

Adding $t > 0$ to each of the 4 terms appearing in the above inequality will preserve it, so that

$$\min\{a^o_{i1} + t + \tilde{h}_{i1}(m^o_i), \frac{a^o_{i2} + 2t + \tilde{h}_{i2}(m^o_i)}{2}\} \geq$$

$$\geq \min\{a^o_i + t + \tilde{h}_{i1}(m_i), \frac{a^o_{i2} + 2t + h_{i2}(m_i)}{2}\}$$

$$\forall\, m_i \in M_i .$$

But this means that the same message m_i^o is a maximin message for a firm with true parameters $(a_{i1}^o + t,\ a_{i2}^o + 2t)$, which contradicts the condition derived above that $X_i(a_{i1}^o, a_{i2}^o) \cap X_i(a_{i1}^o + t, a_{i2}^o + 2t) = \emptyset$ since

$$a_{i2}^o - a_{i1}^o \neq a_{i2}^o + 2t - a_{i1}^o - t = a_{i2}^o - a_{i1}^o + t .$$

This completes the proof.

4. Auctioneer Mechanisms

In addition to the n original producers, a mechanism with an auctioneer involves an extra agent who neither consumes nor produces and whose role is purely informational: the auctioneer helps in the coordination of economic activities. The traditional Walrasian mechanism is the prototype of such mechanisms; the Lindahl mechanism also belongs to this family and the mechanism proposed by Ireland and Law is another and new example.

Formally, a <u>mechanism with an auctioneer</u>, or <u>auctioneer mechanism</u>, is a list $(W; (M_i, h_i),\ i = 1,\dots,n;\ G)$ where

W is the message space of the auctioneer;

M_i is the message space of the i-th firm;

$h_i\colon\ M_i \times W \to X_i$ is the i-th firm outcome function;

G is a subset of $W \times M = W \times M_1 \times \dots \times M_n$ specifying the equilibrium list of messages.

For the IL mechanism, we have

$$W = R_+; \; M_i = R_+ \quad \text{for all} \quad i \; , \; h_i(m_i, w) =$$

$$\frac{(m_i, -f_i(m_i) - wm_i)(m_i - \bar{m}_i)}{\bar{m}_i} \quad \text{for all} \quad i \; , \text{ and}$$

$$G = \{(m,w) \in R_+^{n+1} | \Sigma m_i = N\} \; .$$

Auctioneer mechanisms and games are two separate families of mechanisms. They differ in the following ways: firms do not communicate directly among themselves in an auctioneer mechanism, but only through the auctioneer. The outcome is perceived by each firm to vary only with the auctioneer's message and its own message. This means that no consistency condition can be imposed for all lists of messages, by opposition to games, for which such a condition is meaningful; for a mechanism with an auctioneer, consistency is ensured by the specification of G .

The two classes of mechanisms differ in other important respects (descriptive value, dynamics...) but what concerns us is their ability to implement various performance correspondences. The results presented here suggest that what can be achieved by one class of mechanisms is roughly speaking what can be achieved by the other class. These equivalence results have an independent interest and extend beyond the application to labor-managed economies considered in the present paper.

In each of the three subsections below where output is again assumed to be non-observable, an impossibility result is proved, analogous to the one established in the corresponding subsection of Section 3. The proofs overlap to a large extent and are here simply outlined.

4.1 Dominant Strategies

The first lemma states that nothing is lost by restricting one's attention to direct revelation auctioneer mechanisms.

Lemma 4: (This is the counterpart of Theorem 4.1.1 in DHM.) Let $\Gamma = (W;\ (M_i, h_i),\ i = 1,\ldots,n;\ G)$ be a dominant strategy auctioneer mechanism (i.e., for each i, and for each f_i in $\mathcal{F}_i$, there is m_i in M_i which is a dominant strategy). For each strategy selection (i.e., a list of functions s_i from $\mathcal{F}_i$ into M_i associating to each f_i a dominant strategy), there exists a direct revelation auctioneer mechanism such that

(1) Telling the truth is a dominant strategy.

(2) The outcome corresponding to the list of truthful announcements is the one prescribed by the strategy selection composed with h.

Proof: Let $\Gamma = (W;\ (M_i, h_i),\ i = 1,\ldots,n;\ G)$ be a dominant strategy auctioneer mechanism, and $s\colon \mathcal{F} \to M$ be a strategy selection. Given f in $\mathcal{F}$ and $s(f)$ in M, there is one or several w in W such that $(s(f),w) \in G$. Let now $\bar\Gamma = (W; (\mathcal{F}_i, \bar h_i),\ i = 1,\ldots,n;\ \bar G)$ be such that

$$\bar h_i\colon \mathcal{F}_i \times W \to X_i \quad \text{is defined by} \quad \bar h_i(f_i,w) = h_i(s_i(f_i),w) \quad \text{and}$$

$$\bar G \subset \mathcal{F} \times W \quad \text{is defined by} \quad (f,w) \in \bar G \Leftrightarrow (s(f),w) \in \bar G\ .$$

We now show that for $\bar\Gamma$, (i) and (ii) hold. Since $\{m_i \mid m_i = s_i(f_i)\} \subseteq M_i$, and $h_i(s_i(f_i),w) \succsim_{f_i} h_i(m_i,w)$, $\forall\, m_i \in M_i$, $\forall\, w \in W$, then $h_i(s_i(f_i),w) \succsim_{f_i} h_i(s_i(f_i),w)$ $\forall\, f_i \in \mathcal{F}_i$, $\forall\, w \in W$, and therefore $\bar h_i(f_i,w) \succsim_{f_i} h_i(f_i,w)$

$\forall\ f_i \in \mathfrak{F}_i$, $\forall\ w \in W$. This proves (i).

Also, $\bar{h}_i(f_i,w) = h_i(s_i(f_i),w) \quad \forall i$, $\forall\ f_i \in \mathfrak{F}_i$, $\forall\ w \in W$, which proves (ii).

It follows from Lemma 4, that if Γ implements some performance correspondence Φ in dominant strategies, $\bar{\Gamma}$ will be such that

(3) Telling the truth is a dominant strategy.

(4) The outcome corresponding to the list of truthful announcements is one prescribed by Φ .

<u>Lemma 5</u>: If $\Gamma = (W;(\mathfrak{F}_i,h_i),\ i = 1,\ldots,n;\ G)$ is a direct auctioneer mechanism satisfying (3) and (4), there exists a direct game with the same two properties.

<u>Proof</u>: Let $\Gamma = (W;(\mathfrak{F}_i,h_i),\ i = 1,\ldots,n;\ G)$ be given satisfying (3) and (4). Given an economy $f = (f_1,\ldots,f_n)$, there may be several w in G such that $(f,w) \in G$. But there is at least one. Let $w\colon\ \mathfrak{F}_1 \times \ldots \times \mathfrak{F}_n \to W$ be a selection. Now define the game as follows: for each i , choose $M_i = \mathfrak{F}_i$ (since we want a direct game) and $\bar{h}_i(f) = \bar{h}_i(f_i,f_{-i}) = h_i(f_i,w(f))$. This is a well defined game, since for all f , $\bar{h}(f)$ is feasible. Also, telling the truth is a dominant strategy since

$$h_i(f_i,w) \succsim_i h_i(f_i',w) \quad \forall\ f_i' \in \mathfrak{F}_i\ ,\ \forall\ w \in W \ \text{ and}$$

$$\{w \in W | w = w(f) \ \text{ for some } \ f\} \subset W\ .$$

Also, $\bar{h}(f) = (h_i(f_i,w(f)))_i$ is in $\Phi(f)$ by (4).

Proposition 4: There does not exist an auctioneer mechanism implementing P_A on E_F in dominant strategies.

Proof: It follows from Lemma 4, Lemma 5, and Lemma 2, establishing the non-existence of a game satisfying (3) and (4).

Remark: In the above construction, as one goes from Γ to $\bar{\Gamma}$, it is conceivable that new and non-Φ-optimal dominant strategies be introduced. In addition to the fact that such strategies may already exist for Γ, some may be added, since, in the reduction, there may be fewer strategies against which each f_i is required to be best. If we know that for each f_i,

$$\{w \in W | w = w(f_i, f_{-i}) \text{ for some } f_{-i}\} = W ,$$

we could then conclude that a dominant strategy in the game was also a dominant strategy in the auctioneer mechanism. See Dasgupta, Hammond and Maskin [5] for a discussion of this kind of phenomena.

4.2 Nash Strategies

The analogue of Maskin's theorem concerning Nash-implementation by games can easily be proved; as a corollary, it follows that P_A cannot be implemented on E_F in Nash strategies by an auctioneer mechanism.

Theorem 1': A performance correspondence can be fully implemented in Nash strategies by an auctioneer mechanism only if it is monotonic.

Proof: The proof is almost identical to the proof of Maskin's theorem. Let $\Gamma = (W; (M_i, h_i),\ i = 1,\dots,n;\ G)$ be an auctioneer mechanism assumed to fully implement the performance correspondence Φ on the environment F. Given an economy f in F, an outcome x in $\Phi(f)$, let (m^*, w^*) be a list of messages such that $x = h(m^*, w^*)$ and

(1) $h_i(m_i^*, w^*) \succsim_i h_i(m_i, w^*) \quad \forall i,\ \forall m_i \in M_i$ and

(2) $(m^*, w^*) \in G$.

Let f' be obtained from f by changing each agent's preferences in such a way that x does not fall. Then it is the case that

(1)' $h_i(m_i^*, w^*) \succsim_i' h_i(m_i, w^*) \quad \forall i,\ \forall m_i \in M_i$ and

(2)' $(m^*, w^*) \in G$,

so that (m^*, w^*) remains an equilibrium list of messages. x should therefore be an element of $\Phi(f')$.

Proposition 5: There exists no auctioneer mechanism implementing P_A on E_F in Nash strategies.

Proof: We omit it here as it would closely follow the proof of Proposition 2.

4.3 Maximin Strategies

This section is the counterpart of Section 3.3.

Proposition 6: There exists no auctioneer mechanism implementing P_A on E_F in maximin strategies.

Proof: It basically follows the proof of Proposition 3. In the auctioneer mechanism $(W;(M_i,g_i),\ i = 1,\dots,n;G)$, the subset G of $W \times M$ can be partitioned into n non-empty subsets $S_1,\dots,S_n$ and the i-th outcome function $g_i\colon\ M_i \times W \to X_i$ can be specified by a pair $h_{i1},h_{i2}\colon\ M_i \times W \to X_i$ such that F_i's payoff be written as:

$$\begin{cases} a_{i1} + h_{i1}(m_i,w) & \text{if} \quad (m,w) \notin S_i \\ \dfrac{a_{i2} + h_{i2}(m_i,w)}{2} & \text{if} \quad (m,w) \in S_i \ . \end{cases}$$

Agent i computes, for each m_i , the infimum of the first expression over the lists (m_{-i},w) in $M_{-i} \times W$ such that $(m_i,m_{-i},w) \notin S_i$, and the second expression over the lists (m_{-i},w) in $M_{-i} \times W$ such that $(m_i,m_{-i},w) \in S_i$. This yields two functions of m_i, $\tilde{h}_{i1}$, and $\tilde{h}_{i2}$ such that agent i's payoff be evaluated as

$$\min\{a_{i1} + \tilde{h}_{i1}(m_i)\ ,\ \frac{a_{i2} + \tilde{h}_{i2}(m_i)}{2}\}\ .$$

The impossibility proof would proceed from here as in Proposition 3.

5. Observable Output

In this section, it is assumed that the center is able to observe the outputs of the various firms, so that the transfer payments can be made to depend on this knowledge. As noted in the introduction, the scheme proposed by Ireland and Law requires that this be possible; under this condition, they show that P_A can be implemented on E^+ . It is a natural next step to inquire whether P_B itself is implementable, or in other words, whether the side-payments can be made to balance.

We will limit ourselves to implementation by games. As in the case of non-observability, nothing is gained by considering auctioneer mechanisms.

5.1 Dominant Strategies

Proposition 7: When output is observable, implementation of P_A on E^+ is possible.

Proof: It is provided by exhibiting a direct revelation game having the required properties. Let $M_i = \mathcal{F}_i$ for all i, and let $k_i\colon R \to R^+$ for all i be an increasing function. For each list $m = (m_1, \ldots, m_n)$ in $M = M_1 \times \ldots \times M_n$, let $x(m)$ in R_+^n be a maximizer of $\Sigma m_i(x_i)$ subject to the constraint $\Sigma x_i = N$. Finally, for all i, let $h_i\colon M \to R$ be defined by

$$h_i(m) = -f_i(x_i(m)) + k_i(f_i(x_i(m)) + \sum_{j \neq i} m_j(x_j(m)))x_i(m) \ ,$$

for all m in M.

We claim that the pair (M,h) constitutes a game for which the dominance property holds. First, we observe that $h_i(m)$ can actually be computed by the center if $f_i(x_i(m))$ is observable. Second, the payoff to firm i is given by:

$$P_i(m) = k_i(f_i(x_i(m)) + \sum_{j \neq i} m_j(x_j(m))) \ , \quad \text{and}$$

since k_i is increasing, the maximization of P_i in m_i is equivalent to the maximization of the argument of k_i:

However,

$$f_i(x_i(f_i,m_{-i})) + \sum_{j\neq i} m_j(x_j(f_i,m_{-i})) \geq f_i(x_i(m)) + \sum_{j\neq i} m_j(x_j(m))$$

$$\forall\ m_i \in M_{-i}\ ,\ \forall\ m_i \in M_i\ .$$

(We recognize here the maximand of the Groves mechanism.) This inequality shows that telling the truth is a dominant strategy for all agents.

Q.E.D.

In fact, more can be achieved:

Proposition 8: When output is observable, implementation of P_B on E^+ is possible.

Proof: In the construction developed in the proof of Proposition 4, choose $k_i(u) = u/N$ for all u in R^+, and for all i. Then, balance of the budget is achieved at equilibrium. Indeed, if $f_i = m_i$ for all i, then all the functions k_i have the same argument:

$$\Sigma h_i(f) = -\Sigma f_i(x_i(f)) + \Sigma k_i(f_i(x_i(f)) + \sum_{j\neq i} f_j(x_j(f)))x_j(f)$$

$$= -\Sigma f_i(x_i(f))(1 - \Sigma x_i(f)/N) = 0\ .$$

Note that for this choice of k_i, the incomes of the workers are not just equalized in each firm, but they are in fact equalized throughout the economy.

5.2 Nash Strategies

Nash implementation of P_A on E^+ is clearly possible since dominance implementation is possible. However, dominance implementation as described in the preceding section involves strategy spaces that are of large dimentionality, and it is worthwhile searching for mechanisms with simpler strategy spaces, Euclidean spaces for instance. In that respect, note that the IL scheme uses one-dimensional spaces. Since their scheme does not achieve budget balance, the question arises as to the existence of simple schemes having this additional property.

In what follows, in order to remain as close as possible to the IL formulation, we will consider auctioneer mechanisms where each agent's message is restricted to belong to R_+. The message of F_i is interpreted as its demand for labor, so that its payoff is simply

$$P_i(x_i,w) = \frac{f_i(x_i) + h_i(x_i,w)}{x_i} ,$$

which it attempts to maximize in x_i, taking w as a fixed parameter (Nash-Walras behavior). We will refer to such a mechanism as an elementary auctioneer mechanism. It is specified by the list $(R_+;(R_+,g_i), i = 1,\ldots,n; G)$ where $g_i(x_i,w) = (x_i,h_i(x_i,w))$ and $G = \{(x,w) \in R_+^{n+1} | \Sigma x_i = N\}$.

<u>Lemma 6</u>: An elementary auctioneer mechanism implement P_A on E^+ in Nash strategies iff, for all i, h_i has the form

$$h_i(x_i,w) = -f_i(x_i) + k_i(f_i(x_i) - v(w)x_i)x_i , \quad \forall x_i, \forall w ,$$

where v: $R_+ \to R_+$ is onto, and k_i: $R \to R$ is an increasing function.

<u>Proof</u>: Let us consider the class $E_D \subset E$ of differentiable

economies. Without loss of generality, since output is observable, $h_i(x_i,w)$ can be written as $h_i(x_i,w) = -f_i(x_i) + g_i(x_i,w)$, for some function g_i: $M_i \times W \to X_i$; then $P_i(x_i,w)$ takes the form $P_i(x_i,w) = g_i(x_i,w)/x_i$. We now claim that there exists a function v: $R_+ \to R_+$, which is onto, and such that, for all i, for all f_i in $\mathfrak{F}_{D_i}$, if $x_i(w)$ maximizes $P_i(x_i,w)$ in x_i, and $x_i(w) < N$, then $f_i'(x_i(w)) = v(w)$. This follows from the fact that at an interior optimal allocation of labor, the marginal productivities of the various firms are equal, and that any "shadow wage" is a possible equilibrium wage.

Then, for all $w \in W$, for all $f_i \in \mathfrak{F}_{D_i}$, the functions $P_i(x_i,w)$ and $f_i(x_i) - v(w)x_i$ are maximized in x_i at the same point. This is possible only if $P_i(x_i,w)$ is an increasing function k_i of $f_i(x_i) - v(w)x_i$:

$$P_i(x_i,w) = k_i(f_i(x_i) - v(w)x_i) .$$

Proposition 9: There exists no elementary auctioneer mechanism implementing P_B on E^+ in Nash strategies.

Proof: With an auctioneer mechanism, balance out of equilibrium is clearly impossible, since by changing his message, an agent only affects the value of the transfer payment he receives.

There remains the question of whether the budget can be balanced at equilibrium. On the basis of the preceding Lemma, this will be possible only if a list k_i: $R_+ \to R_+$, $i = 1,\ldots,n$, can be found such that

$$\Sigma\{-f_i(x_i(f)) + k_i(f_i(x_i(f)) - v(w^*)x_i(f))x_i(f)\} = 0$$

where $x(f)$ is an optimal allocation of labor for the economy f, and $v(w^*)$ is the corresponding marginal productivity of labor.

Suppose that balance were achieved for some economy f, and consider an economy $\tilde{f}$ such that for all i, $\tilde{f}_i = f_i + a_i$, where the a_i are elements of R adding up to 0. Then, for $\tilde{f}$, the same allocation of labor is optimal, and $v(w^*)$ remains the marginal productivity of labor of all the firms at optimality. However, balance now requires that $\Sigma k_i(b_i(f) + a_i)x_i(f) = 0$, where $b_i(f) = f_i(x_i(f)) - v(w^*)x_i(f)$. The domain E^+ is sufficiently rich to permit us to choose the b_i and the x_i as we want. Then, it is easy to see that changes of the a_i subject to $\Sigma a_i = 0$ are incompatible with the equality.

Q.E.D.

The specification used here where each agent's stragegy space is the set of non-negative real numbers is in that respect identical to the one used by IL. As just proved, this formulation is too restrictive to permit the construction of balanced schemes. On the other hand, from Proposition 9, we know that balanced schemes can be constructed if no restrictions are imposed on the strategy spaces. It would be interesting to inquire about the minimum dimensionality of the strategy spaces that would permit balance. Techniques to answer this question have been developed by Hurwicz, Mount and Reiter, Walker and Osana (see Osana [25] for references), but their application to our problem goes beyond the limits of the present paper.

5.3 Maximin Strategies

Since implementation in dominant strategies is possible, so is implementation in maximin strategies. In addition to being behaviorally less satisfactory, maximin implementation offers no hope (by opposition to Nash implementation) to save on the dimensionality of the message spaces. We will therefore not elaborate on this issue.

6. Historical Note

The classic studies of labor-managed economies are due to Ward [34], Domar [35], Vanek [31], Meade [21], [22]. Recently, there has been a considerable renewed interest in the topic. The labor-managed firm under uncertainty has been examined by Muzondo [24], Ramachandran, Russell and Seo [26] and Kihlstrom and Laffont [36]. Alternative formulations of the firms' and workers' objective functions have been proposed by Furubotn [9], Steinherr [28], Drèze [7], Ichiishi [18], Greenberg [11], Miyazaki and Neary [23], and Bennett and Wooders [2], where the emphasis is placed on firm formation. The question of incentives is touched upon by several of these authors, but mainly with respect to the analysis of effort, as opposed to the informational incentive, considered in the present paper. Effort incentives are the object of d'Andrea Tyson's contribution [1].

REFERENCES

[1] d'Andrea Tyson, L., "Incentives, Income Sharing and Institutional Innovation in the Yugoslav Self-Managed Firm," Journal of Comparative Economics, 3(1979), 285-301.

[2] Bennett, E. and M. Wooders, "Income Distribution and Firm Formation," Journal of Comparative Economics, 3(1979), 304-317.

[3] Clarke, E., "Multipart Pricing of Public Goods," Public Choice, 11(1971), 17-33.

[4] Crawford, V. P., "Maximin Behavior and Efficient Allocation," University of California, San Diego Discussion Paper 79-12, 1979.

[5] Dasgupta, P., P. Hammond, and E. Maskin, "The Implementation of Social Choice Rules: Some General Results on Incentive-Compatibility," Review of Economic Studies, 46(1979), 185-216.

[6] Drèze, J. and D. de la Vallée Poussin, "A Tâtonnement Process for Public Goods," Review of Economic Studies, 38(1970), 133-150.

[7] Drèze, J., "The Price Theory of Labour-Managed and Participatory Economies, Part I: Certainty," CORE Discussion Paper No. 7422, 1974.

[8] Dubins, L., "Group Decision Devices," mimeo, Berkeley, 1974.

[9] Furubotn, E., "The Long-Run Analysis of the Labour-Managed Firm: An Alternative Interpretation," American Economic Review, 66(1976), 275-302.

[10] Green, J. and J.-J. Laffont, Incentives in Public Decision-Making, North-Holland, 1979.

[11] Greenberg, J., "Existence and Optimality of Equilibrium in Labor-Managed Economies," Review of Economic Studies, 46(1979), 419-433.

[12] Groves, T., "Incentives in Teams," Econometrica, 41(1973), 617-631.

[13] Groves, T. and J. Ledyard, "Optimal Allocation of Public Goods: A Solution to the 'Free Rider' Problem," Econometrica, 4(1977), 783-809.

[14] Groves, T. and M. Loeb, "Incentives and Public Inputs," Journal of Public Economics, 4(1975), 311-326.

[15] Hurwicz, L., "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points," Review of Economic Studies, 46(1979), 217-225.

[16] Ichiishi, T., "Coalition Structure in a Labor-Managed Market Economy," Econometrica, 45(1977), 341-360.

[17] Ireland, N. J. and P. J. Law, "An Enterprise Fund for Labour Mobility in the Cooperative Economy," Economica, 45(1978), 143-151.

[18] Laffont, J.-J. and E. Maskin, "A Differential Approach to Dominant Strategy Mechanisms," Econometrica, forthcoming.

[19] Ledyard, J., "Dominant Strategy Mechanisms and Incomplete Information," in Aggregation and Revelation of Preferences, J.-J. Laffont (ed.), (1979), 309-319.

[20] Maskin, E., "Nash Equilibrium and Welfare Optimality," Mathematics of Operations Research, forthcoming.

[21] Meade, J. E., "The Theory of Labour-Managed Firms and of Profit-Sharing," Economic Journal, 82(1972), 402-428.

[22] Meade, J. E., "Labour-Managed Firms in Conditions of Imperfect Competition," Economic Journal, 84(1974), 817-824.

[23] Miyazaki, H. and H. M. Neary, "The Illyrian Firm Revisited," mimeo, July 1979.

[24] Muzondo, T., "On the Theory of the Competitive Labor-Managed Firm Under Price Uncertainty," Journal of Comparative Economics, 3(1979), 127-144.

[25] Osana, H., "On the Informational Size of Message Spaces for Resource Allocation Processes," Journal of Economic Theory, 17(1978), 66-78.

[26] Ramachandran, R., W. R. Russell, and T. K. Seo, "Risk-Bearing in a Yugoslavian Labor-Managed Firm," Oxford Economic Papers, 31(1979), 270-282.

[27] Schmeidler, D., "A Remark on Microeconomic Models of an Economy and on a Game Theoretic Interpretation of Walras Equilibria," mimeograph, Minneapolis, March 1976.

[28] Steinherr, A., "On the Efficiency of Profit Sharing and Labor Participation in Management," Bell Journal of Economics, 8 (Autumn 1977), 545-555.

[29] Thomson, W., "Maximin Strategies and Elicitation of Preferences," in Aggregation and Revelation of Preferences, J.-J. Laffont (ed.), North-Holland, 1979, 245-268.

[30] Vanek, J., The General Theory of Labor-Managed Market Economies, Cornell University Press, Ithaca, New York, 1970.

[31] Vanek, J., The Labor-Managed Economy, Cornell University Press, Ithaca, New York, 1977.

[32] Vickrey, W., "Counterspeculation, Auctions, and Competitive Sealed Tenders," The Journal of Finance, 16(1961), 1-17.

[33] Walker, M., "An Informationally Efficient Auctioneerless Mechanism for Attaining Lindahl Allocations," SUNY at Stony Brook, mimeograph, 1977.

[34] Ward, B., "The Firm in Illyria: Market Syndicalism," American Economic Review, 48(1958), 566-589.

[35] Domar, E. "The Soviet Collective Farm as a Producer Cooperative," American Economic Review, 56(1966), 734-757.

[36] Kihlstrom, R. E. and J.-J. Laffont, forthcoming.